24/06/2022

Morning

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Memory Based
Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

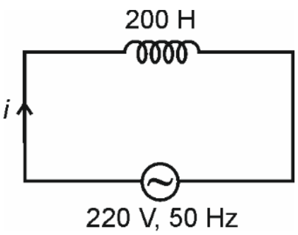
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Find I_{rms} in the following circuit.



(1) 3.5 mA

(2) 35 mA

(3) 350 mA

(4) 3500 mA

Answer (1)

Sol. $I_{rms} = \dfrac{V_{rms}}{X_L} = \dfrac{220}{2\pi fL}$

$= \dfrac{220}{2 \times \left(\dfrac{22}{7}\right) \times 50 \times 200}$

$= 3.5 \times 10^{-3}$ A

= 3.5 mA

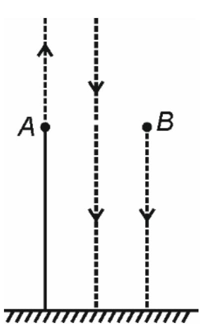
2. A ball is thrown vertically upwards from a tower and reaches ground in 6 seconds. Another ball is thrown downward with same position and with same speed reaches ground in 1.5 s. Time taken by the ball to reach ground if dropped from same height, is

(1) 3 s (2) 4 s

(3) 5 s (4) 2 s

Answer (1)

Sol.



$$A: -H = U(6) - \frac{1}{2}g(6)^2 \quad ...(1)$$

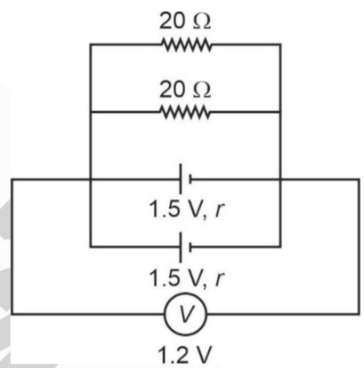
$$B: -H = -U(1.5) - \frac{1}{2}g(1.5)^2 \quad ...(2)$$

$\Rightarrow$ From (1) and (2), $H = 45$ m

On dropping from top of tower, time taken $= \sqrt{\dfrac{2H}{g}}$

= 3 s

3. For the circuit shown in figure, if ideal voltmeter reads 1.2 V, then find value of r.

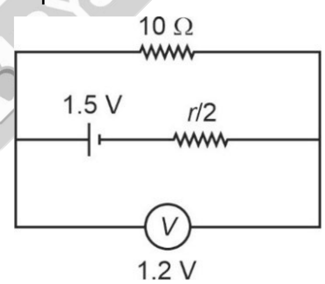


(1) 4 Ω (2) 5 Ω

(3) 6 Ω (4) 8 Ω

Answer (2)

Sol. Equivalent circuit is as shown below



$$\Rightarrow \frac{1.5 \times 10}{10 + \frac{r}{2}} = 1.2$$

On solving $r = 5\ \Omega$

4. At height h above the earth surface, weight of the person becomes $\dfrac{1}{3}$, find height. (Re = 6400 km)

(1) 4.68×10^6 m

(2) 2.68×10^6 m

(3) 3.50×10^6 m

(4) 4.20×10^6 m

Answer (1)

Sol. According to the statement

$$mg' = \frac{1}{3}mg$$

$$\Rightarrow \quad \frac{R^2}{(R+h)^2} = \frac{1}{3}$$

$$\Rightarrow \quad h = (\sqrt{3}-1)R$$

$$= 0.73 \times 6400 \times 10^3$$

$$= 4.685 \times 10^6 \text{ m}$$

5. A projectile is projected with horizontal velocity 25 m/s. If the range of projectile is 75 m, then find angle of projection of projectile

(1) $\tan^{-1}\left(\frac{4}{5}\right)$ (2) $\tan^{-1}\left(\frac{1}{2}\right)$

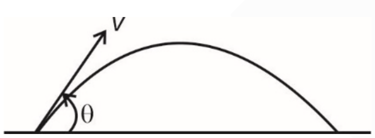
(3) $\tan^{-1}\left(\frac{3}{5}\right)$ (4) $\tan^{-1}(2)$

Answer (3)

Sol. According to given information:

$v\cos\theta = 25$ m/s ...(i)

and $\frac{v^2 \sin 2\theta}{g} = 75$ m ...(ii)



From (i) and (ii),

$v\sin\theta = 15$

$$\Rightarrow \quad \tan\theta = \frac{15}{25} = \frac{3}{5}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{3}{5}\right)$$

6. Find the effective focal length in the given combination. Two biconvex lens of focal length 10 cm and refractive index 1.5 are kept in contact with space between the lenses filled with a medium of refractive index 1.25.

(1) $+\frac{10}{3}$ cm (2) $+\frac{20}{3}$ cm

(3) + 10 cm (4) + 20 cm

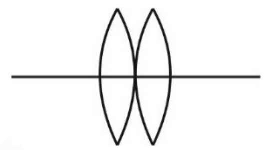
Answer (2)

Sol. 

For biconvex lens let the radius of curvature is R so using lens maker formula we can say.

$$\frac{1}{10} = \left(\frac{1.5}{1} - 1\right)\left(\frac{2}{R}\right)$$

$\Rightarrow$ R = 10 cm



For the given combination the focal length of lens made by medium of refractive index 1.25 is

$$\frac{1}{f'} = (1.25-1)\left(-\frac{2}{10}\right)$$

$\Rightarrow$ f' = – 20 cm

So focal length of combination is

$$\frac{1}{f_{net}} = \frac{1}{10} + \frac{1}{10} - \frac{1}{20} \Rightarrow f_{net} = \frac{20}{3} \text{ cm}$$

7. Find the depth below the surface of earth where weight of object is $\frac{1}{3}$rd of its original weight. [R: Radius of earth]

(1) $\frac{R}{3}$

(2) $\frac{2R}{3}$

(3) $\frac{R}{2}$

(4) $\frac{R}{6}$

Answer (2)

Sol. We know

$$g = g_0\left(1 - \frac{d}{R_e}\right)$$

Given $g = \frac{g_0}{3}$

On solving

$$d = \frac{2R_e}{3}$$

8. Two soap bubbles of radii 4 cm and 5 cm are placed in contact with each other. Radius of curvature of interface is

(1) 10 cm (2) 16 cm

(3) 15 cm (4) 20 cm

Answer (4)

Sol. Let the radius of curvature of interface be R.

Pressure on one side $= P_0 + \frac{4S}{R_1}$

Where P_0 : atmospheric pressure

S : surface tension

Pressure on other side $= P_0 + \frac{4S}{R_2}$

$$\Rightarrow \left[P_0 + \frac{4S}{R_1}\right] - \left[P_0 + \frac{4S}{R_2}\right] = \frac{4S}{R}$$

$$\Rightarrow R = \frac{R_1R_2}{R_2 - R_1} = 20 \text{ cm}$$

9. A ball dropped from height 'h' falls on spring of spring constant, k. Ball sticks to the spring and comes to rest when spring compressed by $\frac{h}{2}$. Value of spring constant, is

(1) $\frac{8mg}{h}$ (2) $\frac{6mg}{h}$

(3) $\frac{4mg}{h}$ (4) $\frac{12mg}{h}$

Answer (4)

Sol. Potential energy lost by the ball $= \frac{3}{2}mgh$

Potential energy stored in spring $= \frac{1}{2}k\left(\frac{h}{2}\right)^2$

$$\Rightarrow \frac{3}{2}mgh = \frac{1}{2}k\left(\frac{h}{2}\right)^2$$

$$\Rightarrow k = \frac{12mg}{h}$$

10. If a charged ball of mass m = 0.1 g is held stationary by an electric field $E = 2 \times 10^9$ V/m. The charge on the ball is

(1) 5×10^{-9} C (2) 5×10^{-11} C

(3) 5×10^{-12} C (4) 5×10^{-13} C

Answer (4)

Sol. $qE = mg$

$$q = \frac{mg}{E} = \frac{1\times10^{-4}\times10}{2\times10^9}$$

$= 5 \times 10^{-13}$ C

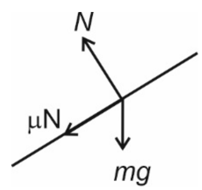
11. The normal reaction N for a vehicle of 800 kg negotiating a turn on a 30° banked road at maximum possible speed is ________ × 10^3 kg m/s²

given cos 30° = $\frac{\sqrt{3}}{2}$, $\mu_s = 0.2$

(1) 9.0 (2) 10.44

(3) 9.6 (4) 9.8

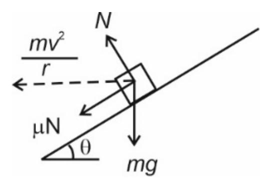
Answer (2)



Sol. $N\cos30° = mg + \mu N\sin30°$

$N[\cos30° - \mu\sin30°] = mg$

For maximum speed



$$v = \sqrt{rg\left(\frac{\mu+\tan\theta}{1-\mu\tan\theta}\right)}$$

$$\frac{v^2}{r} = g\left(\frac{0.2+\frac{1}{\sqrt{3}}}{1-0.2\times\frac{1}{\sqrt{3}}}\right) \quad \text{..... (i)}$$

$N = mg\cos\theta + \frac{mv^2}{r}\sin\theta$

$$= mg\left[\frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{0.2\sqrt{3}+1}{\sqrt{3}-0.2}\right)\right]$$

$= 8 \times 10^3(1.305)$

$= 10.443 \times 10^3$ N

12. Stopping potential for wavelength (λ) = 491 nm is 0.41 V. If wavelength is changed so that stopping potential becomes 1.02 V, then wavelength of new wave is

(1) 4500 Å (2) 3955 Å

(3) 6000 Å (4) 4276 Å

Answer (2)

Sol. As per the statement

$$\frac{1242}{491}\text{ eV} = \phi + 0.41\text{ eV}$$

ϕ = 2.12 eV

Now

$$\frac{1242}{\lambda} = (2.12 + 1.02)$$

$$\Rightarrow \lambda = \frac{1242}{3.14}\text{ nm}$$

= 3955 Å

13. Efficiency of Carnot engine was 25% at 27°C. What will be the increase in temperature required to increase its efficiency by 100%?

(1) 150°C

(2) 300°C

(3) 200°C

(4) 400°C

Answer (1)

Sol. $\eta = \left(1 - \frac{T}{300}\right) = \frac{1}{4} \Rightarrow T = 225\text{ K}$

$$\eta' = \left(1 - \frac{T}{T'}\right) = \frac{1}{2}$$

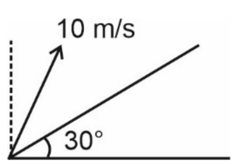
$$\Rightarrow \frac{T}{T'} = \frac{1}{2}$$

or T' = 2T = 450 K

$\Rightarrow$ ΔT_H = 450 K – 300 K

= 150 K or 150°C

14. A projectile is projected with speed 10 m/s for maximum range on inclined plane of angle of inclination, 30°. The maximum range of projectile is



(1) $\frac{40}{3}$ m

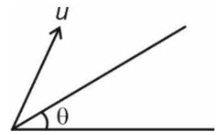
(2) $\frac{10}{3}$ m

(3) $\frac{20}{3}$ m

(4) 5 m

Answer (3)

Sol. Maximum range over an incline in the case shown is equal to



$$R_{\max} = \frac{u^2}{g(1 + \sin\theta)}$$

$$\Rightarrow R_{\max} = \frac{10}{1 + 1/2} = \frac{20}{3}\text{m}$$

15. If a block is displaced from (1, 2) to (2, 3) on applying a force $\vec{F} = 4x^2\hat{i} + 3y^2\hat{j}$, find the change in kinetic energy of block.

(1) 50.55 (2) 60.55

(3) 28.33 (4) 68.67

Answer (3)

Sol. By work energy theorem,

$$\Delta KE = \int \vec{F}.d\vec{r}$$

$$= \int \left(4x^2\hat{i} + 3y^2\hat{j}\right).\left(dx\,\hat{i} + dy\,\hat{j}\right)$$

$$= \int_1^2 4x^2 dx + \int_2^3 3y^2 dy$$

$$= 4\left[\frac{x^3}{3}\right]_1^2 + 3\left[\frac{y^3}{3}\right]_2^3$$

$$= \frac{4}{3} \times 7 + (27 - 8)$$

$$= \frac{28}{3} + 19$$

= 28.33 J

16. If B = 10^9 Nm^{-2} and fractional change in volume is 2%, find volumetric stress required

(1) 1 × 10^7 Pa (2) 2 × 10^7 Pa

(3) 3 × 10^7 Pa (4) 4 × 10^7 Pa

Answer (2)

Sol. $B = \frac{\text{Stress}}{\text{Strain}} = \frac{\Delta P}{\left(-\frac{\Delta V}{V}\right)}$

$$\Rightarrow \text{Stress} = B \times \left(\frac{-\Delta V}{V}\right)$$

$$= 10^9 \times \left(\frac{2}{100}\right)$$

= 2 × 10^7 Pa

17. If at the centre of circular current carrying coil, magnetic field is B_0, then magnetic field at distance $\frac{R}{2}$ from the axis of coil from centre is

(1) $\frac{2B_0}{\sqrt{5}}$ (2) $\frac{4B_0}{5\sqrt{2}}$

(3) $\frac{8B_0}{5\sqrt{5}}$ (4) $\frac{4B_0}{5\sqrt{5}}$

Answer (3)

Sol. $B_c = \frac{\mu_0 I}{2R} = B_0$ (given)

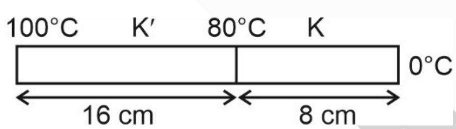
and, $B_{axis} = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}$

$\therefore$ at $x = \frac{R}{2}$,

$$B_{axis} = \frac{\mu_0 I R^2}{2\left(R^2 + \frac{R^2}{4}\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2}{2R^3\left(\frac{5}{4}\right)^{\frac{3}{2}}}$$

$$= \frac{B_0 \times 8}{5\sqrt{5}}$$

18. In the given figure if the temperature of interface is 80°C, then value of K′ is

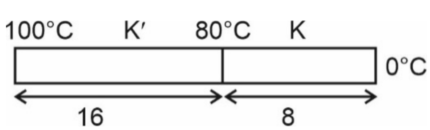


(1) 16 K (2) 4 K

(3) 8 K (4) 12 K

Answer (3)

Sol.



From heat current relation

$$\frac{K'A(100-80)}{16} = \frac{KA(80-0)}{8}$$

$$\Rightarrow \frac{K' \times 20}{16} = \frac{K \times 80}{8}$$

$\Rightarrow K' = 8$ K

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If frequency of light is double of the threshold frequency, then photoelectrons have max. velocity v_1 and if frequency is five times the threshold frequency then photoelectrons have max velocity v_2, $v_2 = xv_1$. Find the value of x $\left(\nu_{th} = \frac{\phi}{h}\right)$

(1) 1

(2) 2

(3) 3

(4) 4

Answer (2)

$\frac{1}{2}mv_1^2 = h\nu - \phi$ where $h\nu = 2\phi$

$\frac{1}{2}mv_2^2 = h\nu' - \phi$ where $h\nu' = 5\phi$

$\Rightarrow \frac{1}{2}mv_2^2 = 4\left(\frac{1}{2}mv_1^2\right)$

$\Rightarrow$ $v_2 = 2v_1$

22.

23.

24.

25.

26.

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which of the following is a stable nitrogen halide?

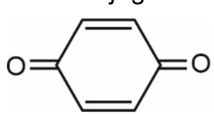
(1) NF_3 (2) NCl_3

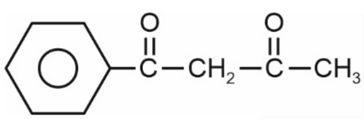
(3) NBr_3 (4) NF_3

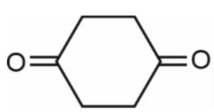
Answer (1)

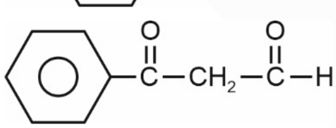
Sol. The stable nitrogen halide is NF_3 due to large difference in the electronegativities of N and F. As the size of halogen increases, stability decreases.

2. Which is conjugated dione?

(1) 

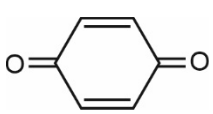
(2) 

(3) 

(4) 

Answer (1)

Sol. Conjugated dione has two carbonyl groups which are in resonance with carbon-carbon double bond.



3. Match the ore correctly with their formula?

(A) Calamine (P) PbS

(B) Galena (Q) $ZnCO_3$

(C) Sphalerite (R) $FeCO_3$

(D) Siderite (S) ZnS

(1) A(P); B(Q); C(R); D(S)

(2) A(Q); B(P); C(S); D(R)

(3) A(Q); B(P); C(R); D(S)

(4) A(P); B(Q); C(S); D(R)

Answer (2)

Sol. Calamine is an ore of zinc → $ZnCO_3$

Galena is an ore of lead → PbS

Sphalerite is an ore of zinc → ZnS

Siderite is an ore of iron → $FeCO_3$

4. Which of the following is correct statement?

(1) B_2H_6 is Lewis acid

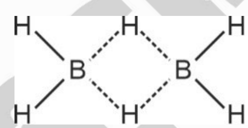
(2) All the B-H bonds in B_2H_6 are equal.

(3) B_2H_6 has the planar structure

(4) Maximum number of hydrogen in one plane is 6

Answer (1)

Sol. Diborane (B_2H_6) is an electron deficient compound. It is a Lewis acid. It has a bridged structure with four 2c – 2e bonds and two 3c – 2e bonds or banana bonds.



B----H----B 3c – 2e bond

B—H 2c – 2e bond

5. $\text{2,7 dimethyl-2,6-octadiene} \xrightarrow[\Delta]{H^+} A$

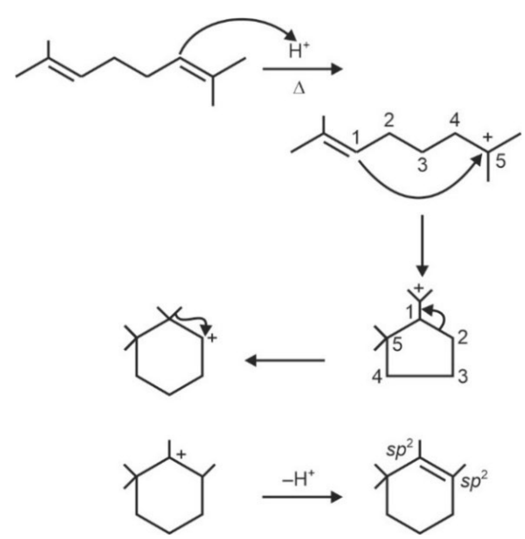
Find the number of sp^2 hybridised carbon in the product 'A'.

(1) 2 (2) 4

(3) 6 (4) 5

Answer (1)

Sol.



6. Which of the following is polyester?

(1) Dacron

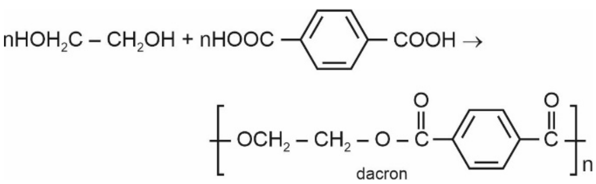
(2) Polyethene

(3) Teflon

(4) DNA

Answer (1)

Sol. Dacron is a polyester. It is a polymer of ethylene glycol and Terephthalic acid.



7. Which of the following has maximum melting point?

(1) Acetic acid

(2) Formic acid

(3) Propanoic acid

(4) Butanoic acid

Answer (1)

Sol. Approximate melting points are,

Formic acid = 8°C

Acetic acid = 17°C

Propanoic acid = – 22°C

Butanoic acid = – 5°C

Hence, acetic acid has maximum melting point.

8. Find the difference in oxidation number of chromium in chromate and in dichromate?

(1) 4

(2) 6

(3) 0

(4) 2

Answer (3)

Sol. Chromate ion is CrO_4^{2-}

Oxidation number of Cr is +6 in chromate.

Dichromate ion is $Cr_2O_7^{2-}$

Oxidation number of Cr is +6 in dichromate.

Difference in oxidation number of chromium is 0

9. In the production of which of the following compound, H_2 (Hydrogen gas) is used?

(1) CO_2 (2) NH_3

(3) P_4 (4) SO_2

Answer (2)

Sol. H_2 gas is used in the production of NH_3 by Haber's process using Fe as catalyst

$$N_2(g) + H_2(g) \rightleftharpoons 2NH_3(g)$$

10. Among LiF and $MgCl_2$, which is more soluble in ethanol.

(1) LiF more soluble in ethanol

(2) $MgCl_2$ is more soluble in ethanol

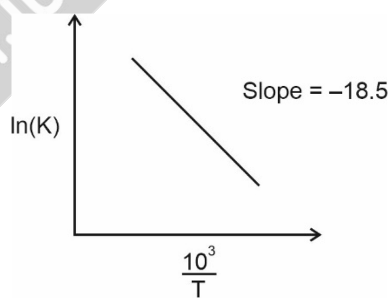
(3) Both are equally soluble in ethanol

(4) Both are not soluble in ethanol

Answer (2)

Sol. $MgCl_2$ is more soluble than LiF in ethanol due to higher covalent character. LiF is almost insoluble in ethanol.

11. What is the value of E_a for the given below graph (in kJ)?



(1) 190.6 (2) 253.55

(3) 153.55 (4) 89.5

Answer (3)

Sol. As per Arrhenius equation,

$$\ln K = \ln A \frac{-E_a \times 10^3}{RT \times 10^3}$$

On comparing,

$$\text{Slope} = -18.5 = \frac{-E_a}{R \times 10^3}$$

$E_a = -18.5 \times 10^3 \times 8.314 = -153.55$ kJ

12. Which has minimum role in formation of photochemical smog?

(1) HCHO (2) N_2

(3) NO (4) O_3

Answer (2)

Sol. N_2 has minimum role in formation of photochemical smog.

13. Which of the following vitamin can not be given to the living organism through food?

(1) C (2) K

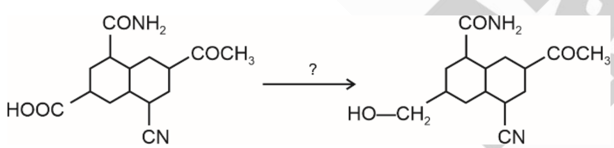
(3) D (4) B_5

Answer (3)

Sol. Vitamin D, also known as calciferol. It is a fat soluble vitamin produced by human body when skin is exposed to sunlight.

Since, vitamin D is hard to eat through food vitamin D supplements are available in two forms. One is vitamin D_2 also known as ergocalciferol and other is vitamin D_3 also known as cholecalciferol. Both are naturally occurring forms that are produced in presence of sunlight, this is why vitamin D is also known as sunshine vitamin.

14. The most suitable reagent for the given conversion is

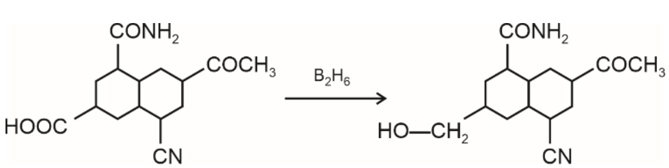


(1) $LiAlH_4$ (2) $NaBH_4$

(3) H_2/Pd (4) B_2H_6

Answer (4)

Sol. In the given conversion, carboxylic acid is selectively reduced to alcohol without affecting other functional groups like ketone, cyanide and amide. Out of the given reducing agents, the best choice is B_2H_6. Even though B_2H_6 is known to reduce amide but the rate is rather slow.



15. Two isomer can be metamers if they have

(1) Different functional group

(2) Carbon skeleton is different

(3) Number of carbon atom on either side of groups are different

(4) Different molecular formula

Answer (3)

Sol. Metamerism arises due to different alkyl chains on either side of the functional group in a molecule.

16. Find the number of lone pair in melamine structure?

(1) 2

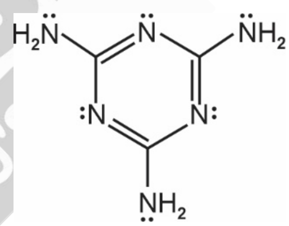
(2) 3

(3) 9

(4) 6

Answer (4)

Sol. Melamine –



Number of lone pairs = 6

17. A gaseous phase reaction

$$A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$$

Find the relation between k (equilibrium constant), α(degree of dissociation) and equilibrium pressure (P)

(1) $K = \dfrac{\alpha^{3/2} P^{1/2}}{(1-\alpha)(2+\alpha)^{1/2}}$

(2) $K = \dfrac{\alpha^{3/2} P^{1/2}}{1-\alpha^2}$

(3) $K = \dfrac{\alpha^2 P}{1-\alpha^2}$

(4) $K = \dfrac{\alpha^3 P^{1/2}}{1-\alpha^2}$

Answer (1)

$$A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$$

Sol.

	$A(g)$	$B(g)$	$\frac{1}{2}C(g)$
$t = 0$	P	–	–
$t = t$	$P(1-\alpha)$	$P\alpha$	$\frac{P\alpha}{2}$

$$P_{eq} = P\left(1+\frac{\alpha}{2}\right)$$

$$P = \frac{P_{eq}}{\left(1+\frac{\alpha}{2}\right)}$$

$$K_P = \frac{P_{eq}\alpha \cdot P_{eq}^{1/2}\,\alpha^{1/2}\left(1+\frac{\alpha}{2}\right)}{\left(1+\frac{\alpha}{2}\right)\cdot 2^{1/2}\left(1+\frac{\alpha}{2}\right)^{1/2}\cdot P_{eq}(1-\alpha)}$$

$$= \frac{P_{eq}^{1/2}\,\alpha^{3/2}}{(2+\alpha)^{1/2}(1-\alpha)}$$

18. Find the number of amphoteric oxides in the given compounds? Na_2O, Cl_2O_7, As_2O_3, N_2O, NO

(1) 1
(2) 3
(3) 2
(4) 5

Answer (1)

Sol.

Na_2O	–	Basic oxide
Cl_2O_7	–	Acidic oxide
N_2O, NO	–	Neutral oxide
As_2O_3	–	Amphoteric oxide

Hence correct option (1)

19. The process of removing sulphur from the ore is?

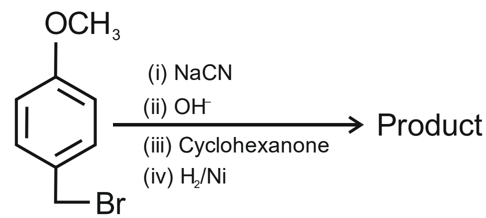
(1) Roasting
(2) Calcination
(3) Leaching
(4) Zone Refining

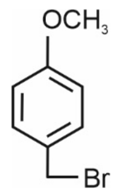
Answer (1)

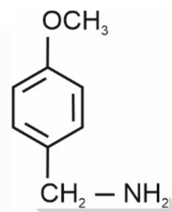
Sol. Roasting is mainly applicable for sulphides ores to get the corresponding metal oxides.

Metal sulphide + $O_2 \rightarrow$ Metal oxide + $SO_2\uparrow$

20. The product in the following sequence of reaction is?



(1)


(2)


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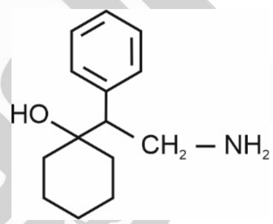
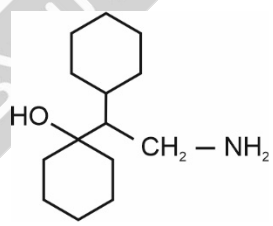
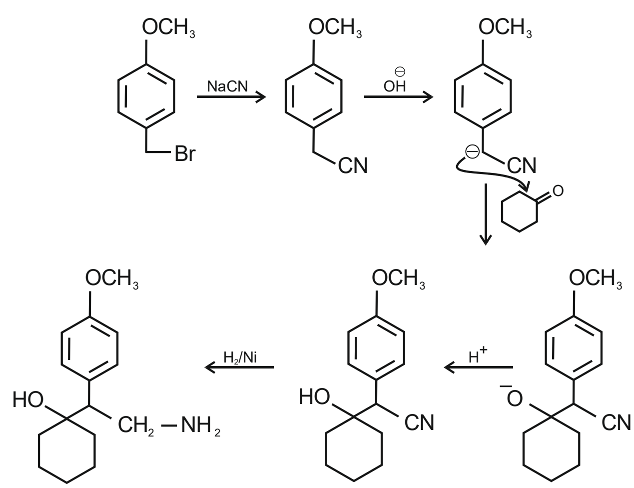


(4)


Answer (1)

Sol.



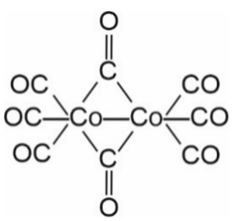
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. In the structure of $(Co)_2(CO)_8$, x is Co–Co bonds and y is number of Co–CO terminal bonds. Then find the value of $x + y$?

Answer (07.00)

Sol. The structure of $[Co_2(CO)_8]$ is



$\therefore$ x = No. of Co–Co bonds = 1

y = No. of terminal Co–CO bonds = 6

x + y = 1+ 6 = 7

22. In a lattice, atom "X" occupied all the lattice point of HCP and "Y" is present in all the tetrahedral voids. Then formula of lattice will be

(1) X_2Y (2) XY_2

(3) XY_3 (4) X_2Y_3

Answer (2)

Sol. Since, atom 'X' occupies all the lattice points of HCP. Hence, total X, atoms in a unit cell are 6. Atom 'Y' occupies all the tetrahedral voids. Hence, total Y atoms in a unit cell are 12.

X : Y

6 : 12

Hence simplest formula of the lattice is XY_2.

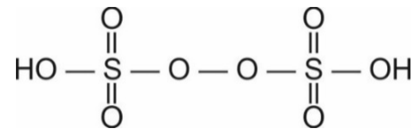
23. Number of π- bonds in Marshall acid is

(1) 2 (2) 6

(3) 4 (4) 8

Answer (3)

Sol. Marshall acid – $H_2S_2O_8$



π bonds = 4

24. Which of the following is not a broad spectrum antibiotics?

(1) Amoxycillin (2) Ofloxacin

(3) Penicillin (4) Chloramphenicol

Answer (3)

Sol. Penicillin G has a narrow spectrum. All the other antibiotics mentioned are broad spectrum antibiotic.

25. Enamel does not contain which of the following ion?

(1) P^{+5} (2) P^{+3}

(3) F^- (4) Ca^{2+}

Answer (2)

Sol. The F^- ions make the enamel on teeth much harder by converting hydroxyapatite, $[3(Ca_3(PO_4)_2 \cdot Ca(OH)_2]$, the enamel on the surface into much harder Fluorapatite, $[3(Ca_3(PO_4)_2 \cdot CaF_2]$. So, P^{+3} is not present in enamel.

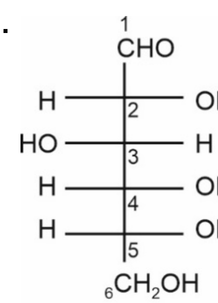
26. Galactose is which of the following epimer of glucose?

(1) C_1 – epimer (2) C_2 - epimer

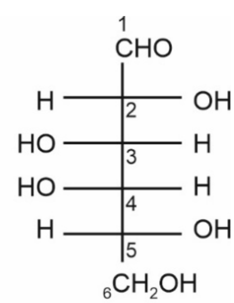
(3) C_3 – epimer (4) C_4 - epimer

Answer (4)

Sol.



D-Glucose



D-Galactose

Galactose is a C_4 - epimer of Glucose.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Find the remainder when 3^{2022} is divided by 5.

(1) 1
(2) 2
(3) 4
(4) 0

Answer (3)

Sol. $3^{2022} = 9^{1011} = (10 - 1)^{1011}$

$= {}^{1011}C_0 \cdot 10^{1011} - {}^{1011}C_1 \cdot 10^{1010} + \ldots..$

$+ {}^{1011}C_{1010} \cdot 10^1 - {}^{1011}C_{1011}$

= 5 (Integer) – 1

Hence 3^{2022} leaves the remainder –1 (i.e. 4) when divided by 5.

2. If sum of square of reciprocal of roots 'α' and 'β' of equation $3x^2 - \lambda x + 1 = 0$ is 15, then find $6(\alpha^3 + \beta^3)^2$

(1) $\frac{202}{3}$
(2) $\frac{200}{9}$
(3) $\frac{224}{9}$
(4) $\frac{221}{3}$

Answer (3)

Sol. $\alpha+\beta = \frac{\lambda}{3}$ and $\alpha\beta = \frac{1}{3}$

Given, $\frac{1}{\alpha^2}+\frac{1}{\beta^2} = 15 \Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = 15$

$$\Rightarrow \frac{\frac{\lambda^2}{9}-\frac{2}{3}}{\frac{1}{9}} = 15$$

$\Rightarrow \lambda^2 - 6 = 15 \Rightarrow \lambda^2 = 21$

Now $6(\alpha^3 + \beta^3)^2 = 6[(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)]^2$

$$= 6\left[\frac{\lambda^3}{27} - 3\left(\frac{1}{3}\right)\frac{\lambda}{3}\right]^2$$

$$= 6\lambda^2\left[\frac{\lambda^2}{27}-\frac{1}{3}\right]^2 = 6(21)\left(\frac{7}{9}-\frac{1}{3}\right)^2$$

$$= \frac{224}{9}$$

3. If $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = k\pi^3$, then find the range of k.

(1) $\left(\frac{1}{32}, \frac{9}{8}\right)$
(2) $\left[\frac{1}{32}, \frac{7}{8}\right)$
(3) $\left[\frac{1}{32}, \frac{9}{8}\right]$
(4) $\left[\frac{1}{32}, 1\right]$

Answer (2)

Sol. $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = (\tan^{-1}x + \cot^{-1}x)^3 - 3\tan^{-1}x \cdot \cot^{-1}x(\tan^{-1}x + \cot^{-1}x)$

$$= \left(\frac{\pi}{2}\right)^3 - 3\tan^{-1}x \cdot \cot^{-1}x \cdot \left(\frac{\pi}{2}\right)$$

$$= \left(\frac{\pi}{2}\right)\left[\frac{\pi^2}{4} - 3\tan^{-1}x\left(\frac{\pi}{2} - \tan^{-1}x\right)\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi^2}{4} + 3(\tan^{-1}x)^2 - \frac{3\pi}{2}\tan^{-1}x\right]$$

$$= \frac{\pi}{2}\left[3\left(\tan^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$$

$\because \tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so $\left(\tan^{-1}x - \frac{\pi}{4}\right)^2 \in \left[0, \frac{9\pi^2}{16}\right)$

Hence $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8}\right)$

$\Rightarrow k \in \left[\frac{1}{32}, \frac{7}{8}\right)$

4. If $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)\, d\theta$, then

$\left|\int_0^{\pi/2} f(\theta)\, d\theta\right|$ is

(1) $1 + \pi t f(t)$
(2) $1 - \pi t f(t)$
(3) $1 + \pi^2 t f(t)$
(4) $-1 + \pi t f(t)$

Answer (1)

Sol. $\because$ $f(\theta) = \sin\theta + f(t)\int_{-\pi/2}^{\pi/2} \sin\theta d\theta + tf(t)\int_{-\pi/2}^{\pi/2} \cos\theta d\theta$

$$\left\{\because \int_{-\pi/2}^{\pi/2} \sin\theta d\theta = 0 \text{ and } \int_{-\pi/2}^{\pi/2} \cos\theta d\theta = 2\right\}$$

So, $f(\theta) = \sin\theta + 2tf(t)$

Now, $\int_0^{\pi/2} f(\theta)d\theta = \int_0^{\pi/2}[\sin\theta + 2tf(t)]d\theta$

$$= [-\cos\theta + 2tf(t)\cdot\theta]_0^{\pi/2}$$

$$= 1 + \pi tf(t)$$

5. $\langle a_i \rangle$ sequence is an A.P. with common difference 1 and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then find the value of n, where n is an even integer.

(1) 48 (2) 96

(3) 18 (4) 36

Answer (2)

Sol. $a_1 + a_2 + ... + a_n = 192 \Rightarrow \frac{n}{2}(a_1 + a_n) = 192$

$\Rightarrow a_1 + a_n = \frac{384}{n}$ ...(1)

$a_2 + a_4 + ... + a_n = 120 \Rightarrow \frac{n}{4}(a_1 + 1 + a_n) = 120$

$\Rightarrow a_1 + 1 + a_n = \frac{480}{n}$ ...(2)

From (2) – (1)

$1 = \frac{480}{n} - \frac{384}{n}$

$n = 96$

6. If $A = \begin{bmatrix} 1 & 0 & a \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, where '$a$' is odd value from 1 to 50 and $\sum_{a=1}^{50} |\text{adj } A| = 100K$, then value of K is

(1) $\frac{1723}{2}$ (2) $\frac{1717}{2}$

(3) 221 (4) $\frac{1821}{4}$

Answer (3)

Sol. $\because$ $\det(A) = a + 1$

So $|\text{adj } A| = (\det A)^2 = (a + 1)^2$

Now $\sum_{\substack{a=1 \\ (a \in \text{odd})}}^{50} |\text{adj } A| = 2^2 + 4^2 + 6^2 + ... + 50^2$

$= 4[1^2 + 2^2 + 3^2 + ... + 25^2]$

$= 4\cdot\left(\frac{25\cdot 26\cdot 51}{6}\right)$

$= 100(221)$

Clearly $K = 221$

Note :

- If $a \in N$, then $\sum_{a=1}^{50} |\text{adj } A| = 2^2 + 3^2 + ... + 51^2$

$= 45525$

So, $K = \frac{45525}{100} = \frac{1821}{4}$

7. A tangent $ax - \mu y = 2$ to hyperbola $\frac{a^4x^2}{\lambda^2} - \frac{b^2y^2}{1} = 4$, then the value of $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is

(1) 0 (2) 1

(3) 2 (4) 3

Answer (2)

Sol. Equation of tangent is : $ax - \mu y = 2$

$\Rightarrow y = \frac{a}{\mu}x - \frac{2}{\mu}$ ...(i)

Equation of hyperbola is $\frac{a^4x^2}{\lambda^2} - \frac{b^2y^2}{1} = 4$

$\Rightarrow \frac{x^2}{\left(\frac{4\lambda^2}{a^4}\right)} - \frac{y^2}{\frac{4}{b^2}} = 1$ ...(ii)

if line $y = mx + c$ is tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

then $c^2 = a^2m^2 - b^2$

$\therefore \frac{4}{\mu^2} = \frac{4\lambda^2}{a^4}\cdot\frac{a^2}{\mu^2} - \frac{4}{b^2}$

$\Rightarrow a^2 = \lambda^2 - \frac{a^2\mu^2}{b^2}$

$\therefore \left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2 = 1$

8. A tangent at (x_1, y_1) to the curve $y = x^3 + 2x^2 + 4$ and passes through origin then (x_1, y_1) is

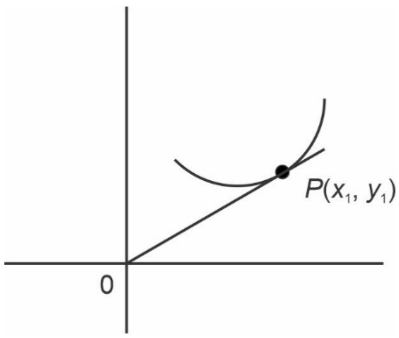
(1) (0, 4) (2) (–1, 5)

(3) (1, 7) (4) (2, 20)

Answer (3)

Sol. $\because$ $P(x_1, y_1)$ lies on the given curve

So, $y_1 = x_1^3 + 2x_1^2 + 4$ ...(i)



Also slope of $OP = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}$

$= \frac{y_1 - 0}{x_1 - 0}$

$\Rightarrow \frac{y_1}{x_1} = 3x_1^2 + 4x_1$

$\Rightarrow y_1 = 3x_1^3 + 4x_1^2$ ...(ii)

from (i) and (ii), we get

$x_1^3 + 2x_1^2 + 4 = 3x_1^3 + 4x_1^2$

$\Rightarrow x_1^3 + x_1^2 - 2 = 0$

$\Rightarrow (x_1 - 1)\left(x_1^2 + 2x_1 + 2\right) = 0$

$\Rightarrow$ $x_1 = 1$ hence $y_1 = 7$

Point P is (1, 7)

9. Find the domain of $\cos^{-1}\frac{\left(\frac{x^2 - 5x + 6}{x^2 - 9}\right)}{\ln(x^2)}$.

(1) $\left[-\frac{1}{2}, \infty\right) - \{0, 1, 3\}$

(2) $x \in \left(-\infty, \frac{5}{2}\right] - \{-3, 0\}$

(3) $x \in \left(-\infty, \frac{5}{2}\right] - \{0\}$

(4) $x \in \left(-\infty, \frac{5}{2}\right] - \{-3\}$

Answer (1)

Sol. $-1 \le \frac{x^2 - 5x + 6}{x^2 - 9} \le 1$

So $\frac{x^2 - 5x + 6}{x^2 - 9} + 1 \ge 0$ and $\frac{x^2 - 5x + 6}{x^2 - 9} - 1 \le 0$

$\Rightarrow \frac{2x^2 - 5x - 3}{x^2 - 9} \ge 0$ and $\frac{-5x + 15}{x^2 - 9} \le 0$

$\Rightarrow \frac{(2x + 1)(x - 3)}{(x - 3)(x + 3)} \ge 0$ and $\frac{x - 3}{x^2 - 9} \ge 0$

Solving these 2 inequalities and taking intersection

$x \in \left[-\frac{1}{2}, \infty\right) - \{3\}$

Again, $\ln(x^2) \ne 0$ and x should not be 0

So, $x \ne \pm 1, 0$

So, domain of $f(x)$ is $x \in \left[-\frac{1}{2}, \infty\right) - \{0, 1, 3\}$

10. Solution of differential equation $x\frac{dy}{dx} = 2y$ is

(1) $xy = c$ (2) $y = cx^2$

(3) $cx = y^2$ (4) $x^2 = cy^2$

Answer (2)

Sol. $x\frac{dy}{dx} = 2y$

$\Rightarrow \frac{dy}{y} = 2\frac{dx}{x}$

$\Rightarrow$ $\ln y = 2\ln x + \ln c$

$\Rightarrow$ $y = cx^2$

11. Consider a set $\Delta \in \{\vee, \wedge, \Rightarrow, \Leftrightarrow\}$ and $p \,\Delta\, q \Rightarrow (\sim p \,\Delta\, q) \,\Delta\, (\sim q \,\Delta\, p)$ is a tautology. Then number of arrangement is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (3)

Sol. If $p \,\Delta\, q \Rightarrow (\sim p \,\Delta\, q) \,\Delta\, (\sim q \,\Delta\, p)$ is tautology,

then $\sim(p \,\Delta\, q) \vee ((\sim p \,\Delta\, q) \,\Delta\, (\sim q \,\Delta\, p))$ is tautology.

Hence $(p \,\Delta\, q) \wedge (\sim((\sim p \,\Delta\, q) \,\Delta\, (\sim q \,\Delta\, p)))$ is fallacy.

So either $(p \,\Delta\, q)$ is always false or

$(\sim p \,\Delta\, q) \,\Delta\, (\sim q \,\Delta\, p)$ is a tautology.

Clearly $\Delta \equiv \wedge$ but $\Delta \not\equiv \vee$

$\rightarrow$ Now we check for $\Delta \equiv \Rightarrow$

$(\sim p \Rightarrow q) \Rightarrow (\sim q \Rightarrow p)$ is same as $(p \vee q)$

$\Rightarrow (p \vee q)$

Which is a tautology.

$\rightarrow$ Now we check for $\Delta \equiv \Leftrightarrow$

$(\sim p \Leftrightarrow q) \Leftrightarrow (\sim q \Leftrightarrow p)$ will be a tautology if $(\sim p \Leftrightarrow q)$ and $(\sim q \Leftrightarrow p)$ will have same truth values.

p	q	$\sim p$	$\sim q$	$\sim p \Leftrightarrow q$	$\sim q \Leftrightarrow p$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

So it's also a tautology.

Clearly Δ can be $\vee$, $\Rightarrow$ or $\Leftrightarrow$

12. The Boolean expression: $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to

(1) $\sim q$ (2) q

(3) $\sim p$ (4) p

Answer (3)

Sol. $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is $\sim p$

p	q	$\sim p$	$p \Rightarrow q$	$q \Rightarrow \sim p$	$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

13. Find number of solution in $\left[0, \frac{\pi}{2}\right]$ for

$$81^{\sin^2 x} + 81^{\cos^2 x} = 9$$

(1) Zero

(2) Two

(3) Three

(4) One

Answer (1)

Sol. $81^{\sin^2 x} + \dfrac{81}{81^{\sin^2 x}} = f(x)$

Applying AM $\geq$ GM,

$$\frac{81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}}}{2} \geq (81)^{\frac{1}{2}}$$

$$\Rightarrow 81^{\sin^2 x} + 81^{\cos^2 x} \geq 18$$

So, there is no solution for given equation.

14. The coefficient of x^{20} in $(1+x)(1+2x)(1+4x)(1+8x)............(1+2^{20}x)$ is

(1) $2^{211} - 2^{190}$ (2) $2^{191} - 2^{171}$

(3) $2^{231} - 2^{209}$ (4) $2^{161} - 2^{142}$

Answer (1)

Sol. Coefficient of x^{20} in

$$(1+2^0 x)(1+2^1 x)(1+2^2 x).....(1+2^{20} x)$$

$$= 2^0 \cdot 2^1 \cdot 2^2 .... \cdot 2^{20}\left[\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + ... + \frac{1}{2^{20}}\right]$$

$$= 2^{\frac{20 \times 21}{2}}\left[\frac{1 - \frac{1}{2^{21}}}{1 - \frac{1}{2}}\right]$$

$$= 2^{211}\left[1 - \frac{1}{2^{21}}\right]$$

$$= 2^{211} - 2^{190}$$

15. Given, $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$, find minimum value of $f(x)$

(1) 0 (2) 1

(3) –1 (4) 2

Answer (3)

Sol. $f(x) = \dfrac{x^2 - 1}{x^2 + 1} = 1 - \dfrac{2}{x^2 + 1}$

$\because$ $x^2 \in [0, \infty)$

So, $\dfrac{2}{1 + x^2} \in (0, 2]$

Hence, $1 - \dfrac{2}{x^2 + 1} \in [-1, 1)$

16. For a binomial probability distribution (33, P), $3P(X = 0) = P(X = 1)$ then $\dfrac{P(X = 15)}{P(X = 18)} - \dfrac{P(X = 16)}{P(X = 17)}$ is

(1) 1000

(2) 1320

(3) 1221

(4) 1121

Answer (2)

Sol. $P(X = r)$ for binomial distribution is $= {}^nC_r P^r (1-P)^{n-r}$

$n = 33$

Given, $3P(X = 0) = P(X = 1)$

$\Rightarrow \quad 3.{}^nC_0 P^0 (1-P)^n = {}^nC_1 P^1 (1-P)^{n-1}$

$\Rightarrow \quad \frac{3}{n} = \frac{P}{1-P}$

$\Rightarrow \quad P = \frac{1}{12}, 1-P = \frac{11}{12}$

$$\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$$

$$= \frac{{}^{33}C_{15}P^{15}(1-P)^{18}}{{}^{33}C_{18}P^{18}(1-P)^{15}} - \frac{{}^{33}C_{16}P^{16}(1-P)^{17}}{{}^{33}C_{17}(1-P)^{16}}$$

$$= \left(\frac{1-P}{P}\right)^3 - \left(\frac{1-P}{P}\right) = 11^3 - 11 = 1320$$

17. $S = \{\theta : \theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\}$ and $\sin\theta\tan\theta + \tan\theta = \sin 2\theta\}$

Let $T = \Sigma\cos 2\theta$ where $\theta \in S$, then $T + n(S) =$

(1) 6 (2) 7

(3) 9 (4) 8

Answer (3)

Sol. $\tan\theta(\sin\theta + 1) - 2\sin\theta\cos\theta = 0$

$\Rightarrow \quad \sin\theta(\sin\theta + 1) - 2\sin\theta\cos^2\theta = 0$

$\Rightarrow \quad \sin\theta(\sin\theta + 1 - 2(1 - \sin^2\theta)) = 0$

$\Rightarrow \quad \sin\theta = 0 \quad$ OR $\quad (\sin\theta + 1)(1 - 2 + 2\sin\theta) = 0$

$\Rightarrow \quad \sin\theta = 0 \quad$ OR $\quad \sin\theta = -1, \frac{1}{2}$

$\sin\theta \neq -1$, $\theta = 0, \pm\pi, \frac{5\pi}{6}, \frac{\pi}{6}$

$n(S) = 5$

$T = \Sigma\cos 2\theta = \cos 0 + \cos 2\pi + \cos(-2\pi)$

$$+\cos\left(\frac{5\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right)$$

$= 4$

$T + n(S) = 9$

18. A circle of equation $x^2 + y^2 + ax + by + c = 0$, passes through (0, 6) and touches $y = x^2$ at (2, 4). Find $a + c$.

(1) 17 (2) 15

(3) 19 (4) 16

Answer (4)

Sol. Equation of tangent to $y = x^2$ at (2, 4) is

$\frac{y+4}{2} = 2x$

$\Rightarrow \quad 4x - y - 4 = 0$

So, equation of required circle can be written in form of family of circles as

$(x-2)^2 + (y-4)^2 + \lambda(4x - y - 4) = 0$

$\because \quad$ This circle passes through (0, 6), then

$4 + 4 + \lambda[-10] = 0$

$\Rightarrow \quad \lambda = \frac{4}{5}$

So, equation of required circles is

$$(x^2 + y^2 - 4x - 8y + 20) + \frac{4}{5}(4x - y - 4) = 0$$

$$\Rightarrow \quad x^2 + y^2 - \frac{4}{5}x - \frac{44}{5}y + \frac{84}{5} = 0$$

Clearly, $a = -\frac{4}{5}$ and $c = \frac{84}{5}$, so $a + c = 16$

19. The equation of plane passing through the line of intersection of planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from (3, 1, –1) is

(1) $5x + 11y - z + 17 = 0$

(2) $5x + 11y - z - 17 = 0$

(3) $5x - 11y + z + 17 = 0$

(4) $5x - 11y + z - 17 = 0$

Answer (4)

Sol. Equation of plane passing through line of intersection of given planes $P_1 = 0$ and $P_2 = 0$ is given by $P_1 + \lambda P_2 = 0$

i.e. $(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0 \quad \ldots(1)$

i.e. $x(1 + \lambda) + y(2 - \lambda) + z(3 + \lambda) - (2 + 3\lambda) = 0$

Distance from (3, 1, –1) $= \frac{2}{\sqrt{3}}$

$$\left|\frac{3(1+\lambda) + 1(2-\lambda) - 1(3+\lambda) - (2+3\lambda)}{\sqrt{(1+\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \left|\frac{0-2\lambda}{\sqrt{3\lambda^2+4\lambda+14}}\right| = \frac{2}{\sqrt{3}}$$

$\Rightarrow$ $12\lambda^2 = 4(3\lambda^2 + 4\lambda + 14)$

$\lambda = -\frac{7}{2}$

Putting $\lambda = -\frac{7}{2}$ in ...(1)

$$x\left(\frac{-5}{2}\right) + y\left(\frac{11}{2}\right) + z\left(\frac{-1}{2}\right) - \left(\frac{-17}{2}\right) = 0$$

$\Rightarrow$ $5x - 11y + z - 17 = 0$

20. If the sides of a triangle are $x^2 + x + 1$, $x^2 - 1$, $2x + 1$, find the greatest angle of the triangle.

(1) 72°

(2) 104°

(3) 120°

(4) 108°

Answer (3)

Sol. Clearly, $x > 1$ because $x^2 - 1$ is a positive real number.

So, $(x^2 + x + 1)$ will be the greatest side.

Let the largest angle be θ, so using cosine rule

$$\cos\theta = \frac{(x^2-1)^2 + (2x+1)^2 - (x^2+x+1)^2}{2(x^2-1)(2x+1)}$$

$$= \frac{(x^4 - 2x^2 + 1) + (4x^2 + 4x + 1) - (x^4 + 2x^3 + 3x^2 + 2x + 1)}{2(2x^3 + x^2 - 2x - 1)}$$

$$= \frac{-(2x^3 + x^2 - 2x - 1)}{2(2x^3 + x^2 - 2x - 1)} = -\frac{1}{2}$$

So, $\theta = 120°$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A and B are two 3 × 3 matrices such that $AB = BA$ then

S-I: If A^3 is symmetric and B^2 is skew symmetric matrix, then $(AB)^6$ is a skew symmetric matrix.

S-II: If A^3 is a skew symmetric and B^2 is symmetric, then, $(AB)^6$ is symmetric

(1) S-I is true and S-II is false

(2) S-I and S-II both are true

(3) S-I and S-II both are false

(4) S-I is false and S-II is true

Answer (2)

Sol. $\because$ $AB = BA$ so $(A \cdot B)^6 = A^6 \cdot B^6$

S-I : If $\left(A^3\right)^T = A^3$ and $\left(B^2\right)^T = -B^2$

So $\left((A \cdot B)^6\right)^T = \left(A^6 \cdot B^6\right)^T = \left(\left(A^3\right)^T\right)^2 \cdot \left(\left(B^2\right)^T\right)^3$

$= \left(A^3\right)^2 \cdot \left(-B^2\right)^3$

$= -A^6 \cdot B^6 = -(AB)^6$

Hence, $(A \cdot B)^6$ is skew symmetric.

S-II : If $\left(A^3\right)^T = -A^3$ and $\left(B^2\right)^T = B^2$

So, $\left((A \cdot B)^6\right)^T = \left(A^6 \cdot B^6\right)^T = \left(\left(A^3\right)^T\right)^2 \cdot \left(\left(B^2\right)^T\right)^3$

$= \left(-A^3\right)^2 \cdot \left(B^2\right)^3$

$= A^6 \cdot B^6 = (AB)^6$

Hence, $(A \cdot B)^6$ is symmetric.

22. Image of $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$ in y-axis is B and image of B in x-axis is C. Point D $(3\cos\theta, a\sin\theta)$ lies in 4[th] quadrant and the maximum area of ΔACD = 12 sq. units. Then find a.

Answer (08)

Sol. $\because$ $A\left(\frac{3}{\sqrt{a}},\sqrt{a}\right)$ then $C\left(-\frac{3}{\sqrt{a}},-\sqrt{a}\right)$

$$\text{Area of } \Delta ACD = \left|\frac{1}{2}\begin{vmatrix} \frac{3}{\sqrt{a}} & \sqrt{a} & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}\right|$$

$$\Rightarrow \quad \Delta = \left|\frac{1}{2}\begin{vmatrix} 0 & 0 & 2 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}\right|$$

$$\Rightarrow \quad \Delta = \left|\, 3\sqrt{a}\cos\theta - 3\sqrt{a}\sin\theta \,\right|$$

$$\Rightarrow \quad \Delta = 3\sqrt{a}\,|\cos\theta - \sin\theta|$$

$$\Rightarrow \quad \Delta_{max} = 3\sqrt{a}\cdot\sqrt{2} = 12$$

$\Rightarrow$ $a = 8$

23.

24.

25.

26.

27.

28.

29.

30.

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