Answers & Solutions
for
JEE (Main)-2022 (Online) Phase-1
(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each part (subject) has two sections.

   (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

   (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Two balls \(A\) and \(B\) are placed at the top of 180 m tall tower. Ball \(A\) is released from the top at \(t = 0\) s. Ball \(B\) is thrown vertically down with an initial velocity \(u\) at \(t = 2\) s. After a certain time, both balls meet 100 m above the ground. Find the value of \(u\) in ms\(^{-1}\) [use \(g = 10\) ms\(^{-2}\)]

(A) 10
(B) 15
(C) 20
(D) 30

Answer (D)

Sol. Let us assume that they meet at \(t = t_0\)

\[A: \quad 80 = \frac{1}{2}gt_0^2 \quad \ldots (i)\]

\[B: \quad 80 = u(t_0 - 2) + \frac{1}{2}g(t_0 - 2)^2 \quad \ldots (ii)\]

From (i), \(t_0 = 4\)

\[\Rightarrow 80 = 2u + 5(2)^2\]

\[\Rightarrow u = 30\) m/s

2. A body of mass \(M\) at rest explodes into three pieces, in the ratio of masses 1 : 1 : 2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms\(^{-1}\) and 40 ms\(^{-1}\) respectively. The velocity of the third piece will be

(A) 15 ms\(^{-1}\)
(B) 25 ms\(^{-1}\)
(C) 35 ms\(^{-1}\)
(D) 50 ms\(^{-1}\)

Answer (B)

Sol. Conserving momentum:

\[m(30\hat{i}) + m(40\hat{j}) + 2m(\vec{v}) = \vec{0}\]

\[\Rightarrow \vec{v} = -15\hat{i} - 20\hat{j}\]

\[\Rightarrow |\vec{v}| = 25\) m/s

3. The activity of a radioactive material is \(2.56 \times 10^{-3}\) Ci. If the half life of the material is 5 days, after how many days the activity will become \(2 \times 10^{-5}\) Ci?

(A) 30 days
(B) 35 days
(C) 40 days
(D) 25 days

Answer (B)

Sol. \(A = A_0e^{-\lambda t}\)

\[\Rightarrow 2 \times 10^{-5} = 2.56 \times 10^{-3} e^{-\lambda t}\]

\[\Rightarrow e^{-\lambda t} = \frac{1}{128} = \left[\frac{1}{2}\right]^7\]

\[\Rightarrow t = 7t_1 = 35\] days

4. A spherical shell of 1 kg mass and radius \(R\) is rolling with angular speed \(\omega\) on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin \(O\) is \(\frac{2}{3}R^2\omega\). The value of \(a\) will be

(A) 2
(B) 3
(C) 5
(D) 4

Answer (C)

Sol. \(\vec{L}_0 = \vec{L}\) of cm + \(\vec{L}\) about cm

\[\Rightarrow \frac{2}{3}R^2\omega = mvR + \frac{2}{3}mR^2\omega = \frac{5}{3}mR^2\omega\]

\[\Rightarrow a = 5\]
5. A cylinder of fixed capacity of 44.8 litres contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperature of gas in the cylinder by 20.0°C will be (Given gas constant \( R = 8.3 \text{ JK}^{-1}\text{-mol}^{-1} \))

(A) 249 J  
(B) 415 J  
(C) 498 J  
(D) 830 J  

Answer (C)  

Sol. \( \Delta Q = nC_v\Delta T \) (Isochoric process)  
\[ = 2 \times \frac{3R}{2} \times 20 \]  
\[ = 498 \text{ J} \]

6. A wire of length \( L \) is hanging from a fixed support. The length changes to \( L_1 \) and \( L_2 \) when masses 1 kg and 2 kg are suspended respectively from its free end. Then the value of \( L \) is equal to

(A) \( \sqrt{L_1L_2} \)  
(B) \( \frac{L_1 + L_2}{2} \)  
(C) \( 2L_1 - L_2 \)  
(D) \( 3L_1 - 2L_2 \)  

Answer (C)  

Sol. \( y = \frac{FL}{A\Delta L} \)  
\[ \Rightarrow \Delta L = \frac{FL}{Ay} \]  
\[ \Rightarrow L_1 = L + \frac{(1g)L}{Ay} \ldots(i) \]  
and \( L_2 = L + \frac{(2g)L}{Ay} \ldots(ii) \)  
\[ \Rightarrow L = 2L_1 - L_2 \]

7. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A**: The photoelectric effect does not take place, if the energy of the incident radiation is less than the work function of a metal.

**Reason R**: Kinetic energy of the photoelectrons is zero, if the energy of the incident radiation is equal to the work function of a metal.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A  
(B) Both A and R are correct but R is not the correct explanation of A  
(C) A is correct but R is not correct  
(D) A is not correct but R is correct  

Answer (B)  

Sol. When energy of incident radiation is equal to the work function of the metal, then the KE of photoelectrons would be zero. But this statement does not comment on the situation when energy is less than the work function.

8. A particle of mass 500 gm is moving in a straight line with velocity \( v = bx^{5/2} \). The work done by the net force during its displacement from \( x = 0 \) to \( x = 4 \) m is: (Take \( b = 0.25 \text{ m}^{-3/2}\text{s}^{-1} \)).

(A) 2 J  
(B) 4 J  
(C) 8 J  
(D) 16 J  

Answer (D)  

Sol. \( W_{\text{total}} = \Delta K \)  
\[ = \frac{1}{2} \left[ \frac{1}{2} b (4^{5/2})^2 - 0 \right] \]  
\[ = \frac{b^2}{2} \times 4^5 \]  
\[ \Rightarrow W_{\text{total}} = 16 \text{ J} \]

9. A charge particle moves along circular path in a uniform magnetic field in a cyclotron. The kinetic energy of the charge particle increases to 4 times its initial value. What will be the ratio of new radius to the original radius of circular path of the charge particle

(A) 1 : 1  
(B) 1 : 2  
(C) 2 : 1  
(D) 1 : 4  

Answer (C)  

Sol. \( R = \frac{mv}{Bq} = \sqrt{\frac{2mK}{Bq}} \)  
\[ \Rightarrow R \propto \sqrt{K} \]  
\[ \Rightarrow \text{ratio} = 2 : 1 \]
10. For a series LCR circuit, \( I \) vs \( \omega \) curve is shown:
(a) To the left of \( \omega_r \), the circuit is mainly capacitive.
(b) To the left of \( \omega_r \), the circuit is mainly inductive.
(c) At \( \omega_r \), impedance of the circuit is equal to the resistance of the circuit.
(d) At \( \omega_r \), impedance of the circuit is 0.

Choose the most appropriate answer from the options given below

(A) (a) and (d) only
(B) (b) and (d) only
(C) (a) and (c) only
(D) (b) and (c) only

Answer (C)

Sol. We know that \( X_C = \frac{1}{\omega C} \) and \( X_L = \omega L \)

Also, at \( \omega = \omega_r \) : \( X_L = X_C \)

\( \Rightarrow \) For \( \omega < \omega_r \) : capacitive

and At \( \omega = \omega_r \) : 

\[ z = \sqrt{R^2 + (X_L - X_C)^2} = R \]

11. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1 kgs\(^{-1}\) and at a speed of 10 ms\(^{-1}\). Then, the initial acceleration of the block, in ms\(^{-2}\), will be:

(A) 3
(B) 6
(C) 5
(D) 4

Answer (C)

Sol. \( F = \rho v^2 a \)

\( \Rightarrow 10 \times 1 = 2 \times \text{acceleration} \)

\( \Rightarrow \text{Acc.} = 5 \text{ m/s}^2 \)

12. In van der Wall equation \[ P + \frac{a}{V^2} [V - b] = RT; P \]  

is pressure, \( V \) is volume, \( R \) is universal gas constant and \( T \) is temperature. The ratio of constants \( \frac{a}{b} \) is dimensionally equal to:

(A) \( \frac{P}{V} \)
(B) \( \frac{V}{P} \)
(C) \( PV \)
(D) \( PV^3 \)

Answer (C)

Sol. From the equation

\([a] = [PV^2]\)
\([b] = [V]\)

\( \Rightarrow \left[ \frac{a}{b} \right] = [PV] \)

13. Two vectors \( \vec{A} \) and \( \vec{B} \) have equal magnitudes. If magnitude of \( \vec{A} + \vec{B} \) is equal to two times the magnitude of \( \vec{A} - \vec{B} \), then the angle between \( \vec{A} \) and \( \vec{B} \) will be:

(A) \( \sin^{-1} \left( \frac{3}{5} \right) \)
(B) \( \sin^{-1} \left( \frac{1}{3} \right) \)
(C) \( \cos^{-1} \left( \frac{3}{5} \right) \)
(D) \( \cos^{-1} \left( \frac{1}{3} \right) \)

Answer (C)

Sol. \( \sqrt{A^2 + A^2 + 2A^2 \cos \theta} = 2\sqrt{A^2 + A^2 + 2A^2 (-\cos \theta)} \)

\( \Rightarrow 2A^2 + 2A^2 \cos \theta = 8A^2 \)

\( \Rightarrow 5 \cos \theta = 3 \)

\( \Rightarrow \theta = \cos^{-1} \left( \frac{3}{5} \right) \)
14. The escape velocity of a body on a planet ‘A’ is 12 kms\(^{-1}\). The escape velocity of the body on another planet ‘B’, whose density is four times and radius is half of the planet ‘A’, is:

(A) 12 kms\(^{-1}\)
(B) 24 kms\(^{-1}\)
(C) 36 kms\(^{-1}\)
(D) 6 kms\(^{-1}\)

Answer (A)

Sol. \(v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \times \rho \times \frac{4}{3} \pi R^3}\)

\[\Rightarrow v_{esc} \propto R \sqrt{\rho}\]

\[\Rightarrow \frac{(v_{esc})_B}{(v_{esc})_A} = 1\]

\[\Rightarrow (v_{esc})_B = 12 \text{ km/s}\]

15. At a certain place the angle of dip is 30° and the horizontal component of earth’s magnetic field is 0.5 G. The earth’s total magnetic field (in G), at that certain place, is :

(A) \(\frac{1}{\sqrt{3}}\)
(B) \(\frac{1}{2}\)
(C) \(\sqrt{3}\)
(D) 1

Answer (A)

Sol. \(B_h = B \cos 30^\circ\)

\[\Rightarrow B = \frac{1}{\sqrt{3}} G\]

16. A longitudinal wave is represented by \(x = 10 \sin 2\pi \left( nt - \frac{x}{\lambda} \right) \text{ cm}\). The maximum particle velocity will be four times the wave velocity if the determined value of wavelength is equal to :

(A) \(2\pi\)
(B) \(5\pi\)
(C) \(\pi\)
(D) \(\frac{5\pi}{2}\)

Answer (B)

Sol. \(v_{particle} = \frac{dx}{dt}\)

\[\Rightarrow \text{Maximum particle velocity} = (2\pi n)\ (10)\]

\[\Rightarrow (2\pi n)\ (10) = (n\lambda)\ (4)\]

\[\Rightarrow \lambda = 5\pi\]

17. A parallel plate capacitor filled with a medium of dielectric constant 10, is connected across a battery and is charged. The dielectric slab is replaced by another slab of dielectric constant 15. Then the energy of capacitor will :

(A) increased by 50%
(B) decrease by 15%
(C) increase by 25%
(D) increase by 33%

Answer (A)

Sol. \(U = \frac{1}{2} (kC) V^2\)

\[\Rightarrow \frac{U'}{U} = 1.5\]

\[\Rightarrow \text{Energy increases by 50%}\]

18. A positive charge particle of 100 mg is thrown in opposite direction to a uniform electric field of strength \(1 \times 10^5 \text{ NC}^{-1}\). If the charge on the particle is 40 \(\mu\text{C}\) and the initial velocity is 200 ms\(^{-1}\), how much distance it will travel before coming to the rest momentarily?

(A) 1 m
(B) 5 m
(C) 10 m
(D) 0.5 m

Answer (D)

Sol. \(v^2 - u^2 = 2as\)

\[\Rightarrow a^2 - 200^2 = 2 \left( \frac{-qE}{m} \right) (S)\]

\[\Rightarrow -200^2 = 2 \left[ \frac{-40 \times 10^{-6} \times 10^5}{100 \times 10^{-6}} \right] [S]\]

\[\Rightarrow S = \frac{4}{2 \times 4} \ m = 0.5 \ m\]
19. Using Young’s double slit experiment, a monochromatic light of wavelength 5000 Å produces fringes of fringe width 0.5 mm. If another monochromatic light of wavelength 6000 Å is used and the separation between the slits is doubled, then the new fringe width will be:
(A) 0.5 mm (B) 1.0 mm (C) 0.6 mm (D) 0.3 mm
Answer (D)
Sol. Fringe width \( \frac{\lambda D}{d} \)
\( \Rightarrow \) Fringe width \( \propto \frac{\lambda}{d} \)
\( \Rightarrow \) New fringe width = \( 0.5 \text{ mm} \times \frac{1.2}{2} = 0.3 \text{ mm} \)

20. Only 2% of the optical source frequency is the available channel bandwidth for an optical communicating system operating at 1000 nm. If an audio signal requires a bandwidth of 8 kHz, how many channels can be accommodated for transmission?
(A) \( 375 \times 10^7 \) (B) \( 75 \times 10^7 \) (C) \( 375 \times 10^8 \) (D) \( 75 \times 10^9 \)
Answer (B)
Sol. \( v = f \lambda \)
\( \Rightarrow f = \frac{v}{\lambda} = \frac{3 \times 10^8}{1000} \text{ Hz} = 3 \times 10^{14} \text{ Hz} \)
\( \Rightarrow \) Channels = \( \frac{2}{100} \times \frac{3 \times 10^{14}}{8 \times 10^3} = 75 \times 10^7 \)

SECTION - B
Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
3. As per the given figure, two plates $A$ and $B$ of thermal conductivity $K$ and $2K$ are joined together to form a compound plate. The thickness of plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 120 cm$^2$ for each plate. The equivalent thermal conductivity of the compound plate is $\left(1 + \frac{5}{\alpha}\right)K$, then the value of $\alpha$ will be ____.

![Diagram showing two plates joined together](image)

**Answer (21)**

**Sol.**  
\[
\frac{L_1}{K_1A_1} + \frac{L_2}{K_2A_2} = \frac{L_1 + L_2}{K_{\text{eff}}A_{\text{eff}}}
\]
\[
\Rightarrow \frac{4}{K} + \frac{2.5}{2K} = \frac{6.5}{K_{\text{eff}}}
\]
\[
\Rightarrow 10.5 = \frac{6.5}{2K}
\]
\[
\Rightarrow K_{\text{eff}} = \frac{13K}{10.5} = \left(1 + \frac{5}{21}\right)K
\]
\[
\Rightarrow \alpha = 21
\]

4. A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripped by air Jet when it is at 5 cm from its mean position. The new amplitude of vibration is $\sqrt{x}$ cm. The value of $x$ is ____.

**Answer (700)**

**Sol.**  
\[
\omega = \sqrt{\frac{\kappa}{\mu}}
\]
\[
\Rightarrow 3\omega\sqrt{10^2 - 5^2} = \omega\sqrt{(A')^2 - 5^2}
\]
\[
\Rightarrow 9 \times 75 = (A')^2 - 25
\]
\[
\Rightarrow A' = \sqrt{28 \times 25} \text{ cm}
\]
\[
\Rightarrow x = 700
\]

5. The variation of applied potential and current flowing through a given wire is shown in figure. The length of wire is 31.4 cm. The diameter of wire is measured as 2.4 cm. The resistivity of the given wire is measured as $x \times 10^{-3} \Omega$ cm. The value of $x$ is ____.

![Diagram showing potential and current](image)

**[Take $\pi = 3.14$]**

**Answer (144)**

**Sol.**  
\[
\text{Resistance} = \tan 45^\circ = 1 \Omega
\]
\[
\Rightarrow 1 = \frac{\rho l}{A}
\]
\[
\Rightarrow \rho = \frac{\pi (1.2 \text{ cm})^2}{31.4 \text{ cm}} = 1.44 \times 10^{-1} \Omega \text{ cm}
\]
\[
\Rightarrow x = 144
\]

6. 300 cal. of heat is given to a heat engine and it rejects 225 cal. of heat. If source temperature is 227°C, then the temperature of sink will be ____°C.

**Answer (102)**

**Sol.**  
\[
\eta = \frac{W}{Q} = \frac{300 - 225}{300}
\]
\[
\Rightarrow \frac{75}{300} = 1 - \frac{T_L}{T_H}
\]
\[
\Rightarrow T_L = \frac{3}{4} T_H = \frac{3}{4} (500) = 375 \text{ K}
\]
\[
\Rightarrow T_L = 102^\circ \text{C}
\]
7. $\sqrt{d_1}$ and $\sqrt{d_2}$ are the impact parameters corresponding to scattering angles $60^\circ$ and $90^\circ$ respectively, when an \( \alpha \) particle is approaching a gold nucleus. For $d_1 = x \cdot d_2$, the value of $x$ will be _______.

Answer (3)

Sol. Impact parameter $\propto \cot \frac{\theta}{2}$

\[ \Rightarrow \frac{\sqrt{d_1}}{\sqrt{d_2}} = \frac{\sqrt{3}}{1} \]

\[ \Rightarrow d_1 = 3d_2 \]

\[ \Rightarrow x = 3 \]

8. A transistor is used in an amplifier circuit in common emitter mode. If the base current changes by $100 \ \mu A$, it brings a change of $10 \ mA$ in collector current. If the load resistance is $2 \ k\Omega$ and input resistance is $1 \ k\Omega$, the value of power gain is $x \times 10^4$. The value of $x$ is _______.

Answer (2)

Sol. Power gain $= \frac{\Delta I_c}{\Delta I_B} \times \frac{R_o}{R_i}$

\[ = \frac{10^{-2}}{10^{-4}} \times \frac{2}{1} \]

\[ = 2 \times 10^4 \]

\[ \Rightarrow x = 2 \]

9. A parallel beam of light is allowed to fall on a transparent spherical globe of diameter $30 \ cm$ and refractive index $1.5$. The distance from the centre of the globe at which the beam of light can converge is_______ mm.

Answer (225)

Sol.

1st refraction: \( \frac{1.5}{v_1} - 0 = \frac{0.5}{15} \)

\[ \Rightarrow v_1 = 45 \ cm \]

2nd refraction: \( \frac{1}{v_2} - \frac{1.5}{15} = -\frac{0.5}{-15} \)

\[ \Rightarrow \frac{1}{v_2} = \frac{1}{30} + \frac{1}{10} \]

\[ = \frac{4}{30} \]

\[ \Rightarrow v_2 = +7.5 \ cm \]

Distance from centre $= 22.5 \ cm$

10. For the network shown below, the value of $V_B - V_A$ is _______ V.

Answer (10)

Sol. $V_B - V_A = i \times 2$

\[ = \frac{15}{1+2} \times 2 \]

\[ \Rightarrow V_B - V_A = 10 \ volts \]
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. Production of iron in blast furnace follows the following equation
   \[ \text{Fe}_3\text{O}_4(s) + 4\text{CO}(g) \rightarrow 3\text{Fe}(l) + 4\text{CO}_2(g) \]
   when 4.640 kg of \( \text{Fe}_3\text{O}_4 \) and 2.520 kg of CO are allowed to react then the amount of iron (in g) produced is:
   \[ \text{[Given : Molar Atomic mass (g mol}^{-1}\text{) : Fe = 56} \]
   \[ \text{ Molar Atomic mass (g mol}^{-1}\text{) : O = 16} \]
   \[ \text{ Molar Atomic mass (g mol}^{-1}\text{) : C = 12} \]
   (A) 1400
   (B) 2200
   (C) 3360
   (D) 4200

   Answer (C)

   Sol.
   \[ \text{Fe}_3\text{O}_4(s) + 4\text{CO}(g) \rightarrow 3\text{Fe}(l) + 4\text{CO}_2(g) \]
   \[ \frac{4.640\text{ kg}}{720\text{ g}} = 2520 \text{ moles} \]
   \[ \frac{2.520\text{ kg}}{28\text{ g}} = 82.857 \text{ moles} \]
   \[ \frac{60\text{ mol} \times 56}{3360\text{ g}} = 10 \text{ moles} \]
   Total number of moles = 90 moles
   Total number of moles = 30 moles

2. Which of the following statements are correct?
   (A) The electronic configuration of Cr is \([Ar]3d^64s^1\).
   (B) The magnetic quantum number may have a negative value.
   (C) In the ground state of an atom, the orbitals are filled in order of their increasing energies.
   (D) The total number of nodes are given by \( n - 2 \).

   Choose the most appropriate answer from the options given below:
   (A) (A) > (C) > (B) > (D)
   (B) (B) > (A) > (C) > (D)
   (C) (A) > (B) > (C) > (D)
   (D) (A) > (B) > (D) > (C)

   Answer (C)

   Sol. \( \text{Cr} = (Ar)3d^64s^1 \)
   M = +1 to –1
   As per Aufbau principle, orbitals are filled in increasing order of energy.
   Total number of nodes = \( (n - 1) \)

3. Arrange the following in the decreasing order of their covalent character:
   (A) LiCl
   (B) NaCl
   (C) KCl
   (D) CsCl

   Choose the most appropriate answer from the options given below:
   (A) (A) > (C) > (B) > (D)
   (B) (B) > (A) > (C) > (D)
   (C) (A) > (B) > (C) > (D)
   (D) (A) > (B) > (D) > (C)

   Answer (C)

   Sol. Covalent character \( \propto \) polarising power of cation
   Correct decreasing order of covalent character
   LiCl > NaCl > KCl > CsCl

4. The solubility of AgCl will be maximum in which of the following?
   (A) 0.01 M KCl
   (B) 0.01 M HCl
   (C) 0.01 M AgNO_3
   (D) Deionised water

   Answer (D)

   Sol. Solubility decreases with increasing the concentration of common ion. Therefore, the maximum solubility of AgCl will be in deionized water.

5. Which of the following is a correct statement?
   (A) Brownian motion destabilises sols.
   (B) Any amount of dispersed phase can be added to emulsion without destabilising it.
(C) Mixing two oppositely charged sols in equal amount neutralises charges and stabilises colloids.
(D) Presence of equal and similar charges on colloidal particles provides stability to the colloidal solution.

Answer (D)

Sol. Presence of equal and similar charges on colloidal particle provides stability to the colloidal solution.

6. The electronic configuration of Pt (atomic number 78) is:
(A) [Xe] 4f¹⁴ 5d⁹ 6s¹
(B) [Kr] 4f¹⁴ 5d¹⁰
(C) [Xe] 4f¹⁴ 5d¹⁰
(D) [Xe] 4f¹⁴ 5d⁶ 6s²

Answer (A)

Sol. Pt = [Xe] 4f¹⁴ 5d⁶ 6s¹

7. In isolation of which one of the following metals from their ores, the use of cyanide salt is not commonly involved?
(A) Zinc
(B) Gold
(C) Silver
(D) Copper

Answer (D)

Sol. In the extraction of Silver and Gold, NaCN is used to leach the metal.

\[ 4\text{Au}(s) + 8\text{CN}^-(aq) + 2\text{H}_2\text{O} + \text{O}_2(g) \rightarrow 4\text{[Au(CN)]}_2^-(aq) + 4\text{OH}^-(aq) \]

\[ 2[\text{Au(CN)}_2^-](aq) + \text{Zn} \rightarrow 2\text{Au}(s) + [[\text{Zn(CN)}_4]^{2-}](aq) \]

In case of ore containing ZnS and PbS, the depressant used is NaCN.

8. Which one of following reactions indicates the reducing ability of hydrogen peroxide in basic medium?
(A) HOCI + H₂O₂ → H₂O⁺ + Cl⁻ + O₂
(B) PbS + 4H₂O₂ → PbSO₄ + 4H₂O
(C) 2MnO₄⁻ + 3H₂O₂ → 2MnO₂ + 3O₂ + 2H₂O + 2OH⁻
(D) Mn²⁺ + H₂O₂ → Mn⁴⁺ + 2OH⁻

Answer (C)

Sol. 2MnO₄⁻ + 3H₂O₂ →

\[ 2\text{MnO}_2 + 3\text{O}_2 + 2\text{H}_2\text{O} + 2\text{OH}^- \]

In basic medium MnO₄⁻ is reduced to MnO₂, whereas in acidic medium it is reduced to Mn²⁺.

9. Match List-I with List-II.

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Metal)</td>
<td>(Emitted light wavelength (nm))</td>
</tr>
<tr>
<td>(A) Li</td>
<td>(I) 670.8</td>
</tr>
<tr>
<td>(B) Na</td>
<td>(II) 589.2</td>
</tr>
<tr>
<td>(C) Rb</td>
<td>(III) 780.0</td>
</tr>
<tr>
<td>(D) Cs</td>
<td>(IV) 455.5</td>
</tr>
</tbody>
</table>

Choose the most appropriate answer from the options given below:
(A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(B) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)
(C) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(D) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Answer (A)

Sol.

10. Match List-I with List-II.

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Metal)</td>
<td>(Application)</td>
</tr>
<tr>
<td>(A) Cs</td>
<td>(I) High temperature thermometer</td>
</tr>
<tr>
<td>(B) Ga</td>
<td>(II) Water repellent sprays</td>
</tr>
<tr>
<td>(C) B</td>
<td>(III) Photoelectric cells</td>
</tr>
<tr>
<td>(D) Si</td>
<td>(IV) Bullet proof vest</td>
</tr>
</tbody>
</table>

- 10 -
Choose the most appropriate answer from the options given below:

(A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

Answer (A)

Sol. Metal Application
 Cs Photoelectric cells
 Ga High temperature thermometer
 B Bullet proof vest
 Si Water repellent sprays

11. The oxoacid of phosphorus that is easily obtained from a reaction of alkali and white phosphorus and has two P-H bonds is:

(A) Phosphonic acid (B) Phosphinic acid

(C) Pyrophosphorus acid (D) Hypophosphoric acid

Answer (B)

Sol. White phosphorus + alkali → H₃PO₂

H₃PO₂ = phosphinic acid

12. The acid that is believed to be mainly responsible for the damage of Taj Mahal is:

(A) sulfuric acid (B) hydrofluoric acid

(C) phosphoric acid (D) hydrochloric acid

Answer (A)

Sol. CaCO₃ + H₂SO₄ → CaSO₄ + H₂O + CO₂

13. Two isomers ‘A’ and ‘B’ with molecular formula C₄H₈ give different products on oxidation with KMnO₄/H⁺ results in effervescence of a gas and gives ketone. The compound ‘A’ is:

(A) But-1-ene (B) cis-But-2-ene

(C) trans-But-2-ene (D) 2-methyl propene

Answer (D)

Sol. Esterification of carboxylic acid with an alcohol is nucleophilic acyl substitution and presence of electron withdrawing group in the carboxylic acid increases the rate of esterification reaction.
16. Consider the above reactions, the product A and product B respectively are

(A) [Image]

(B) [Image]

(C) [Image]

(D) [Image]

17. The polymer, which can be stretched and retains its original status on releasing the force is

(A) Bakelite

(B) Nylon 6, 6

(C) Buna-N

(D) Terylene

Answer (C)

Sol. Buna – N is a synthetic rubber which can be stretched and retains its original status on releasing the force.

18. Sugar moiety in DNA and RNA molecules respectively are

(A) β-D-2-deoxyribose, β-D-deoxyribose

(B) β-D-2-deoxyribose, β-D-ribose

(C) β-D-ribose, β-D-2-deoxyribose

(D) β-D-deoxyribose, β-D-2-deoxyribose

Answer (B)

Sol. DNA consists of β-D-2-deoxyribose sugar whereas RNA consists of β-D-ribose.

19. Which of the following compound does not contain sulphur atom?

(A) Cimetidine

(B) Ranitidine

(C) Histamine

(D) Saccharin

Answer (C)

Sol. Histamine – does not contain sulphur.

20. Given below are two statements.

Statement I : Phenols are weakly acidic.

Statement II : Therefore they are freely soluble in NaOH solution and are weaker acids than alcohols and water.

Choose the most appropriate option.
(A) Both Statement I and Statement II are correct.
(B) Both Statement I and Statement II are correct.
(C) Statement I is correct but Statement II is incorrect.
(D) Statement I is incorrect but Statement II is correct.

Answer (C)

Sol.

![Chemical Structure](image)

\[ \text{pK}_a = 10 \]

Alcohol \( \text{pK}_a \approx 16 \)

\[ \text{H}_2\text{O} \] \( \text{pK}_a \approx 15 \)

**SECTION B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Geraniol, a volatile organic compound, is a component of rose oil. The density of the vapour is 0.46 g L\(^{-1}\) at 257°C and 100 mm Hg. The molar mass of geraniol is ____ g mol\(^{-1}\). (Nearest Integer)

   [Given: \( R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1} \)]

   **Answer (152)**

   **Sol.**

   \[ \frac{\text{PM}}{\text{RT}} = d \]

   \[ M = \frac{0.46 \times 0.082 \times 530 \times 760}{100} \]

   \[ = 152 \text{ g mol}^{-1} \]

2. 1.2 mL of acetic acid is dissolved in water to make 2.0 L of solution. The depression in freezing point observed for this strength of acid is 0.0198°C. The percentage of dissociation of the acid is ____.

   [Given: Density of acetic acid is 1.02 g mL\(^{-1}\), Molar mass of acetic acid is 60 g mol\(^{-1}\), \( K_f(\text{H}_2\text{O}) = 1.85 \text{ K kg mol}^{-1} \)]

   **Answer (5)**

   **Sol.**

   \[ \Delta \text{T}_b = i \times K_b \times m \]

   Moles of solute (acetic acid) = \( \frac{1.2 \times 1.02}{60} \)

   As moles of solute are very less.

   So, take molarity and molality same.

   \[ i = 1.05 \]

   \[ \alpha = \frac{i-1}{n-1} = \frac{0.05}{1} = 0.05 \]

3. 1.2 mL of acetic acid is dissolved in water to make 2.0 L of solution. The depression in freezing point observed for this strength of acid is 0.0198°C. The percentage of dissociation of the acid is ____.

   [Given: Density of acetic acid is 1.02 g mL\(^{-1}\), Molar mass of acetic acid is 60 g mol\(^{-1}\), \( K_f(\text{H}_2\text{O}) = 1.85 \text{ K kg mol}^{-1} \)]

   **Answer (5)**

   **Sol.**

   \[ \Delta \text{T}_b = i \times K_b \times m \]

   Moles of solute (acetic acid) = \( \frac{1.2 \times 1.02}{60} \)

   As moles of solute are very less.

   So, take molarity and molality same.

   \[ i = 1.05 \]

   \[ \alpha = \frac{i-1}{n-1} = \frac{0.05}{1} = 0.05 \]

4. A dilute solution of sulphuric acid is electrolysed using a current of 0.10 A for 2 hours to produce hydrogen and oxygen gas. The total volume of gases produced at STP is ____ cm\(^3\). (Nearest integer)

   [Given : Faraday constant \( F = 96500 \text{ C mol}^{-1} \) at STP, molar volume of an ideal gas is 22.7 L mol\(^{-1}\)]

   **Answer (127 cm\(^3\))**

   **Sol.**

   \[ 2 \text{ F produces} = \frac{3}{2} \text{ mole of gas} \]

   \[ 0.10 \times 2 \times 3600 \text{ coulomb produces} \]

   \[ \frac{3}{2} \times 0.1 \times 2 \times 3600 \]

   \[ = \frac{2 

\[ = 0.0056 \text{ moles of gas} \]
Volume of gas produced = 0.0056 × 22.7 L
= 0.127 L
= 127 mL

5. The activation energy of one of the reactions in a biochemical process is 532611 J mol⁻¹. When the temperature falls from 310 K to 300 K, the change in rate constant observed is $k_{300} = x \times 10^{-3} k_{310}$. The value of $x$ is ______.

[Given: $\ln 10 = 2.3$, $R = 8.3$ JK⁻¹ mol⁻¹]

Answer (1)

Sol. $\log \left(\frac{K_{310}}{K_{300}}\right) = \frac{532611}{8.3} \left(\frac{1}{300} - \frac{1}{310}\right)$

$K_{310} = 10^3 \Rightarrow K_{300} = 1 \times 10^{-3} \times K_{310}$

6. The number of terminal oxygen atoms present in the product B obtained from the following reaction is ______.

$\text{FeCr}_2\text{O}_4 + \text{Na}_2\text{CO}_3 + \text{O}_2 \rightarrow \text{A} + \text{Fe}_2\text{O}_3 + \text{CO}_2$

$\text{A} + \text{H}^+ \rightarrow \text{B} + \text{H}_2\text{O} + \text{Na}^+$

Answer (6)

Sol. $\text{FeCr}_2\text{O}_4 + \text{Na}_2\text{CO}_3 + \text{O}_2 \rightarrow \text{Fe}_2\text{O}_3 + \text{CO}_2$

$+ \text{Na}_2\text{CrO}_4$

(A)

$\text{Na}_2\text{CrO}_4 + \text{H}^+ \rightarrow \text{Cr}_2\text{O}_7^{2-} + \text{H}_2\text{O} + \text{Na}^+$

7. An acidified manganate solution undergoes disproportionation reaction. The spin-only magnetic moment value of the product having manganese in higher oxidation state is _____ B.M. (Nearest integer)

Answer (00.00)

Sol. $\text{MnO}_4^2 - \text{H}^+ \rightarrow \text{MnO}_2 + \text{MnO}_4 + \text{H}_2\text{O}$

$\text{Mn}^{7+} = d^0$

Hence, magnetic moment = zero

8. Kjeldahl’s method was used for the estimation of nitrogen in an organic compound. The ammonia evolved from 0.55 g of the compound neutralised 12.5 mL of 1 M H₂SO₄ solution. The percentage of nitrogen in the compound is ______. (Nearest integer)

Answer (00.00)

Sol. $\% \text{N} = \frac{1.4 \times \text{N} \times \text{V}}{\text{Mass of organic compound}}$

$\text{Mass of organic compound} \times \frac{1.4 \times 2 \times 12.5}{0.55} = 63.63\% = 64$

9. Observe structures of the following compounds

The total number of structures/compounds which possess asymmetric carbon atoms is _____.

Answer (3)

Sol. $\text{C}_6\text{H}_4\text{O}_6 \rightarrow \text{C}_2\text{H}_5\text{OH}$

$\text{C}_6\text{H}_2\text{O}_6$ $\xrightarrow{\text{Zymase}}$ $\text{A}$ $\xrightarrow{\text{NaO}}$ $\text{B} + \text{CH}_3$

The number of carbon atoms present in the product B is ______.

Answer (1)

Sol. $\text{C}_6\text{H}_2\text{O}_6 \rightarrow \text{C}_2\text{H}_5\text{OH}$

$\text{C}_6\text{H}_2\text{O}_6 \xrightarrow{\text{Zymose}} \text{C}_2\text{H}_5\text{OH}$

$\text{C}_6\text{H}_2\text{O}_6 \xrightarrow{\text{NaO}} \text{HCOONa} + \text{CH}_3$
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. The probability that a randomly chosen 2 × 2 matrix with all the entries from the set of first 10 primes, is singular, is equal to:

   (A) \( \frac{133}{10^4} \)  
   (B) \( \frac{18}{10^3} \)  
   (C) \( \frac{19}{10^3} \)  
   (D) \( \frac{271}{10^4} \)

   Answer: (C)

   Sol. Let \( A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\} \)

   Let \( E \) be the event that matrix of order 2 × 2 is singular

   **Case-I**

   All entries are same example \[
   \begin{pmatrix}
   2 & 2 \\
   2 & 2
   \end{pmatrix}
   \]

   \( = 10C_1 \)

   **Case-II**

   Matrix with two prime numbers only example \[
   \begin{pmatrix}
   3 & 5 \\
   3 & 5
   \end{pmatrix}
   \]

   \( = 10C_2 \times 2! \times 2! \)

   \[ P(E) = \frac{10C_1 + 10C_2 \times 2! \times 2!}{10^4} = \frac{190}{10^4} = \frac{19}{10^3} \]

2. Let the solution curve of the differential equation

   \[ x \frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}, \ y(1) = 3 \] be \( y = y(x) \). Then \( y(2) \) is equal to:

   (A) 15  
   (B) 11  
   (C) 13  
   (D) 17

   Answer: (A)

   Sol. \[ x \frac{dy}{dx} - y = \sqrt{y^2 + 16x^2} \]

   \[ y = 4x \tan \theta \]

   \[ \frac{dy}{dx} = 4\tan \theta + 4x \sec^2 \theta \frac{d\theta}{dx} \]

   \[ 4x \tan \theta + 4x^2 \sec^2 \theta \frac{d\theta}{dx} - 4x \tan \theta = 4x \sec \theta \]

   \[ \int \sec \theta \frac{d\theta}{dx} = \int \frac{dx}{x} \]

   \[ \log |\sec \theta + \tan \theta| = \log |x| + C \]

   \[ y(1) = 3 \Rightarrow 3 = 4 \tan \theta \]

   \[ = \tan \theta = \frac{3}{4} \Rightarrow \sec \theta = \frac{5}{4} \]

   \[ \ln \left( \frac{8}{4} \right) = \ln |1| + C \]

   \[ \Rightarrow C = \ln 2 \]

   \[ \Rightarrow \sec \theta + \tan \theta = 2|\theta| \]

   \[ \Rightarrow y = 15 \]

3. If the mirror image of the point \((2, 4, 7)\) in the plane \(3x - y + 4z = 2\) is \((a, b, c)\), then \(2a + b + 2c\) is equal to:

   (A) 54  
   (B) 50  
   (C) -6  
   (D) -42

   Answer: (C)

   Sol. Mirror image of \((2, 4, 7)\) in \(3x - y + 4z = 2\) is \((a, b, c)\) then

   \[ a - \frac{2}{3} = \frac{b - 4}{-1} = \frac{c - 7}{4} \]

   \[ a - \frac{2}{3} = \frac{b - 4}{-1} = \frac{c - 7}{4} = \frac{-2(6 - 4 + 28 - 2)}{3^2 + (-1)^2 + 4^2} \]

   \[ a - \frac{2}{3} = \frac{b - 4}{-1} = \frac{c - 7}{4} = \frac{-28}{13} \]

   \[ a = \frac{-58}{13} \quad b = \frac{80}{13} \quad c = \frac{21}{13} \]

   \[ 2a + b + 2c = \frac{-116 + 80 - 42}{13} \]

   \[ = -6 \]
4. Let \( f : \mathbb{R} \to \mathbb{R} \) be a function defined by:
\[
f(x) = \begin{cases} 
  \max\{t^3 - 3t\} & ; x \leq 2 \\
  x^2 + 2x - 6 & ; 2 < x < 3 \\
  [x - 3] + 9 & ; 3 \leq x \leq 5 \\
  2x + 1 & ; x > 5 
\end{cases}
\]
where \([t]\) is the greatest integer less than or equal to \(t\). Let \(m\) be the number of points where \(f\) is not differentiable and \(I = \int_{-2}^{2} f(x) \, dx\). Then the ordered pair \((m, I)\) is equal to:

(A) \((3, \frac{27}{4})\)  
(B) \((3, \frac{23}{4})\)  
(C) \((4, \frac{27}{4})\)  
(D) \((4, \frac{23}{4})\)

Answer (C)

Sol. \( \max\{t^3 - 3t\} ; x \leq 2 \)
\[
g(t) = t^3 - 3t \Rightarrow g'(t) = 3t^2 - 3 = 3(t - 1)(t + 1)
\]

Points of non-differentiability = \(\{2, 3, 4, 5\}\)
\[
\Rightarrow m = 4
\]
\[
I = \frac{2}{-2} \int_{-2}^{2} f(x) \, dx = -\int_{-2}^{2} (x^3 - 3x) \, dx + \int_{-2}^{2} 2 \, dx
\]
\[
= \left[\frac{x^4}{4} - \frac{3x^2}{2}\right]_{-2}^{2} + 2(2 + 1) = \left(\frac{1}{4} - \frac{3}{2}\right) - (4 - 6) + 6
\]
\[
= \frac{27}{4}
\]

5. Let \( \vec{a} = \alpha \hat{i} + 3 \hat{j} - \hat{k} \), \( \vec{b} = 3 \hat{i} - \beta \hat{j} + 4 \hat{k} \) and \( \vec{c} = \hat{i} + 2 \hat{j} - 2 \hat{k} \) where \(\alpha, \beta \in \mathbb{R}\), be three vectors. If the projection of \(\vec{a}\) on \(\vec{c}\) is \(\frac{10}{3}\) and
\[
\vec{b} \times \vec{c} = -6 \hat{i} + 10 \hat{j} + 7 \hat{k},
\]
then the value of \(\alpha + \beta\) is equal to:

(A) 3  
(B) 4  
(C) 5  
(D) 6

Answer (A)

Sol. \( \vec{a} = \alpha \hat{i} + 3 \hat{j} - \hat{k} \)
\( \vec{b} = 3 \hat{i} - \beta \hat{j} + 4 \hat{k} \)
\( \vec{c} = \hat{i} + 2 \hat{j} - 2 \hat{k} \)

Projection of \(\vec{a}\) on \(\vec{c}\) is
\[
\frac{\vec{a} \cdot \vec{c}}{|\vec{b}|} = \frac{10}{3}
\]
\[
\frac{\alpha + 6 + 2}{\sqrt{1^2 + 2^2 + (-2)^2}} = \frac{\alpha + 8}{3} = \frac{10}{3}
\]
\[
\Rightarrow \alpha = 2
\]
\[
\vec{b} \times \vec{c} = -6 \hat{i} + 10 \hat{j} + 7 \hat{k}
\]
\[
\begin{vmatrix}
  \hat{i} & \hat{j} & \hat{k} \\
  3 & -\beta & 4 \\
  1 & 2 & -2
\end{vmatrix}
\]
\[
= (2\beta - 8) \hat{i} + (6 + \beta) \hat{k} - 6 \hat{i} - 10 \hat{j} + 7 \hat{k}
\]
\[
= 2\beta - 8 = -6 \iff 6 + \beta = 7
\]
\[
\Rightarrow \beta = 1
\]
\[
\alpha + \beta = 2 + 1 = 3
\]
6. The area enclosed by \( y^2 = 8x \) and \( y = \sqrt{2}x \) that lies outside the triangle formed by \( y = \sqrt{2}x, \ x = 1, \ y = 2\sqrt{2} \), is equal to:

(A) \( \frac{16\sqrt{2}}{6} \)  
(B) \( \frac{11\sqrt{2}}{6} \)  
(C) \( \frac{13\sqrt{2}}{6} \)  
(D) \( \frac{5\sqrt{2}}{6} \)

Answer (C)

Sol. The area enclosed by the given curves can be calculated by integrating the difference between the two functions from 0 to 1.

\[
\text{Area} = \int_0^1 \left( \frac{y^2}{\sqrt{2}} - \frac{y^2}{8} \right) dy - \text{area (\( \Delta BAC \))}
\]

\[
= \left[ \frac{y^2}{2\sqrt{2}} - \frac{y^3}{24} \right]_0^1 - \frac{1}{2} \times AB \times BC
\]

\[
= \frac{8\sqrt{2}}{2} - \frac{32\sqrt{2}}{24} - \frac{1}{2} \times 1 \times \sqrt{2}
\]

\[
= \frac{8\sqrt{2}}{3} - \frac{\sqrt{2}}{2}
\]

\[
= \frac{\sqrt{2}}{6} \times (48 - 32 - 3) = \frac{13\sqrt{2}}{6}
\]

7. If the system of linear equations

\( 2x + y - z = 7 \)
\( x - 3y + 2z = 1 \)
\( x + 4y + \delta z = k, \) where \( \delta, k \in \mathbb{R} \)

has infinitely many solutions, then \( \delta + k \) is equal to:

(A) -3  
(B) 3  
(C) 6  
(D) 9

Answer (B)

Sol. The system of linear equations can be solved to find \( \delta \) and \( k \).

\[
\begin{vmatrix}
2 & 1 & -1 \\
1 & 4 & \delta \\
1 & -3 & 2
\end{vmatrix}
= -7\delta - 21 = 0
\]

\[
\delta = -3
\]

8. Let \( \alpha \) and \( \beta \) be the roots of the equation \( x^2 + (2i - 1) = 0 \). Then, the value of \( |\alpha^8 + \beta^8| \) is equal to:

(A) 50  
(B) 250  
(C) 1250  
(D) 1500

Answer (A)

Sol. \( x^2 + (2i - 1) = 0 \)

\( \alpha^2 = \beta^2 = 1 - 2i \)

\( \alpha^4 = (1 - 2i)^2 = 1 + (2i)^2 - 4i = -3 - 4i \)

\( \alpha^8 = (-3 - 4i)^2 = 9 - 16 + 24i = -7 + 24i \)

\[
|\alpha^8 + \beta^8| = 2\sqrt{(-7)^2 + (24)^2} = 50
\]
10. Let \( A = [a_{ij}] \) be a square matrix of order 3 such that 
\[ a_{ij} = 2^{-i}, \text{for all } i, j = 1, 2, 3. \]
Then, the matrix \( A^2 + A^3 + \ldots + A^{10} \) is equal to:

\[
\begin{align*}
(A) & \left( \begin{array}{c}
3^{10} - 3 \\
2
\end{array} \right) A \\
(B) & \left( \begin{array}{c}
3^{10} - 1 \\
2
\end{array} \right) A \\
(C) & \left( \begin{array}{c}
3^{10} + 1 \\
2
\end{array} \right) A \\
(D) & \left( \begin{array}{c}
3^{10} + 3 \\
2
\end{array} \right) A
\end{align*}
\]

Answer (A)

Sol. \( A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 2^0 & 2^1 & 2^2 \\
2^{-1} & 2^0 & 2^1 \\
2^{-2} & 2^{-1} & 2^0 \end{bmatrix} \)

\[
\begin{align*}
A^2 &= \begin{bmatrix} 1 & 2 & 4 \\
1 & 2 & 4 \\
1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 12 \\
3 & 6 & 12 \\
3 & 6 & 12 \end{bmatrix} = 3A \\
A^3 &= \begin{bmatrix} 2^0 & 2^1 & 2^2 \\
2^{-1} & 2^0 & 2^1 \\
2^{-2} & 2^{-1} & 2^0 \end{bmatrix} = 3A
\end{align*}
\]

11. Let \( A = A_1 \cup A_2 \cup \ldots \cup A_k \), where \( A_i \cap A_j = \emptyset \) for \( i \neq j \). Define the relation \( R \) from \( A \) to \( A \) by \( R = \{(x, y) : y \in A_i \text{ if and only if } x \in A_i, 1 \leq i \leq k\} \). Then, \( R \) is:

(A) reflexive, symmetric but not transitive
(B) reflexive, transitive but not symmetric
(C) reflexive but not symmetric and transitive
(D) an equivalence relation

Answer (D)

Sol. \( R = \{(x, y) : y \in A_i \text{ if } x \in A_i, 1 \leq i \leq k\} \)

12. Let \( \{a_n\}_{n=0}^\infty \) be a sequence such that \( a_0 = a_1 = 0 \) and \( a_{n+2} = 2a_{n+1} - a_n + 1 \) for all \( n \geq 0 \).

Then \( \sum_{n=2}^{\infty} \frac{a_n}{7^n} \) is equal to:

(A) 6/343  \quad (B) 7/216
(C) 8/343  \quad (D) 49/216

Answer (B)

Sol. \( a_{n+2} = 2a_{n+1} - a_n + 1 \) & \( a_0 = a_1 = 0 \)

\[ a_2 = 2a_1 - a_0 + 1 = 1 \]
\[ a_3 = 2a_2 - a_1 + 1 = 3 \]
\[ a_4 = 2a_3 - a_2 + 1 = 6 \]
\[ a_5 = 2a_4 - a_3 + 1 = 10 \]
\[ \sum_{n=2}^{\infty} \frac{a_n}{7^n} = \frac{a_2}{7^2} + \frac{a_3}{7^3} + \frac{a_4}{7^4} + \ldots \]

\[
\begin{align*}
n1 &= 1 + 3 + 6 + 10 + \ldots \\
\frac{1}{7} &= 1 + 3 + 6 + 10 + \ldots \\
6s &= \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \ldots \\
6s &= \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \ldots \\
\frac{36s}{49} &= \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \ldots \\
\frac{36s}{49} &= \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \ldots \\
\end{align*}
\]
13. The distance between the two points $A$ and $A'$ which lie on $y = 2$ such that both the line segments $AB$ and $A'B$ (where $B$ is the point $(2, 3)$) subtend angle $\frac{\pi}{4}$ at the origin, is equal to

(A) 10  
(B) $\frac{48}{5}$  
(C) $\frac{52}{5}$  
(D) 3

Answer (C)

Sol. Let $A(\alpha, 2)$  
Given $B(2, 3)$

$m_{OA} = \frac{2}{\alpha}$  
$m_{OB} = \frac{3}{2}$

$\tan \frac{\pi}{4} = \left| \frac{\frac{2}{\alpha} - \frac{3}{2}}{1 + \frac{2}{\alpha} \cdot \frac{3}{2}} \right| \Rightarrow \frac{4 - 3\alpha}{2\alpha + 6} = \pm 1$

$4 - 3\alpha = 2\alpha + 6$  
$4 - 3\alpha = -2\alpha - 6$

$\alpha = -2 \Rightarrow \alpha = 10$

$A\left(\frac{2}{5}, 2\right)$  
$A'\left(10, 2\right)$  
and $B(2, 3)$

$AA' = 10 + \frac{2}{5} = \frac{52}{5}$}

14. A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is

(A) $\frac{22}{9 + 4\sqrt{3}}$  
(B) $\frac{66}{9 + 4\sqrt{3}}$  
(C) $\frac{22}{4 + 9\sqrt{3}}$  
(D) $\frac{66}{4 + 9\sqrt{3}}$

Answer (B)

Sol. $4a + 3b = 22$

Total area $A = a^2 + \frac{\sqrt{3}}{4}b^2$

$A = \left(\frac{22 - 3b}{4}\right)^2 + \frac{\sqrt{3}}{4}b^2$

$\frac{dA}{db} = 2\left(\frac{22 - 3b}{4}\right)(-3) + \frac{\sqrt{3}}{4} \cdot 2b = 0$

$\Rightarrow \frac{\sqrt{3}b}{2} = \frac{3}{8}(22 - 3b)$

$4\sqrt{3}b = 66 - 9b$

$b = \frac{66}{9 + 4\sqrt{3}}$

15. The domain of the function

$\cos^{-1}\left(2\sin^{-1}\left(\frac{1}{4x^2 - 1}\right)\right)$

(A) $\mathbb{R}$  
(B) $(\infty, -1] \cup [1, \infty) \cup \{0\}$  
(C) $\left(-\frac{1}{2}, \frac{1}{2}\right)$  
(D) $\left(-\frac{1}{2}, \infty\right) \cup \{0\}$

Answer (D)

Sol. $-1 \leq \frac{2}{\pi} \sin^{-1}\left(\frac{1}{4x^2 - 1}\right) \leq 1$

$-\frac{\pi}{2} \leq \sin^{-1}\left(\frac{1}{4x^2 - 1}\right) \leq \frac{\pi}{2}$

$-1 \leq \frac{1}{4x^2 - 1} \leq 1$

$\frac{1}{4x^2 - 1} \geq -1 \Rightarrow \frac{4x^2}{(2n+1)(2x-1)} \geq 0$
11. \( (0) \), \( 2x \) are in \([-\infty, \infty) \) \( \cup (-\infty, -\frac{1}{\sqrt{2}}) \) \( \cup \left( -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \) \( \cup \left( \frac{1}{\sqrt{2}}, \infty \right) \) \( \ldots (2) \)

From (1) & (2)

\( x \in \left( -\infty, -\frac{1}{\sqrt{2}} \right) \cup \left( -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \cup \left( \frac{1}{\sqrt{2}}, \infty \right) \cup \{0\} \)

16. If the constant term in the expansion of 

\( (3x^3 - 2x^2 + \frac{5}{x^5})^{10} \) is \( 2^k \cdot l \), where \( l \) is an odd integer, then the value of \( k \) is equal to

(A) 6  
(B) 7  
(C) 8  
(D) 9

Answer (D)

Sol. Constant term in

\( (3x^3 - 2x^2 + \frac{5}{x^5})^{10} \rightarrow x^0 \)

\( \Rightarrow (3x^8 - 2x^7 + 5)^{10} \rightarrow x^{50} \)

General term of \( (3x^8 - 2x^7 + 5)^{10} \) is

\[ \frac{10!}{p!q!r!} (3x^8)^p (\frac{-2x^7}{x^5})^q (5)^r \]

Here \( 8p + 7q = 50 \) and \( p + q + r = 10 \)

\( \Rightarrow p = 1, q = 6, r = 3 \) is only valid solution

\[ \therefore \frac{10!}{1!6!3!} 3^1 2^6 5^3 = 2^k \cdot l \]

\[ \Rightarrow 5 \cdot 3 \cdot 7 \cdot 5^3 \cdot 3 \cdot 2^9 = 2^k \cdot l \]

\[ \therefore k = 9 \]

17. \( \int_0^5 \cos \left( \pi \left( x - \frac{x}{2} \right) \right) \, dx \), where \([ \cdot ]\) denotes greatest integer less than or equal to \( t \), is equal to

(A) \(-3\)  
(B) \(-2\)  
(C) 2  
(D) 0

Answer (D)

Sol. \( \int_0^5 \cos \left( \pi \left( x - \frac{x}{2} \right) \right) \, dx \)

\[ = \int_0^2 \cos(\pi x) \, dx + \int_2^4 \cos(\pi(x-1)) \, dx + \int_4^5 \cos(\pi(x-2)) \, dx \]

\[ = \frac{2}{\pi} \sin\pi x \bigg|_0^2 + \frac{4}{\pi} \sin\pi(x-1) \bigg|_2^4 + \frac{5}{\pi} \sin\pi(x-2) \bigg|_4^5 \]

\[ = 0 + 0 + 0 = 0 \]

18. Let \( PQ \) be a focal chord of the parabola \( y^2 = 4x \) such that it subtends an angle of \( \frac{\pi}{2} \) at the point \((3, 0)\). Let the line segment \( PQ \) be also a focal chord of the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), \( a^2 > b^2 \). If \( e \) is the eccentricity of the ellipse \( E \), then the value of \( \frac{1}{e^2} \) is equal to

(A) \( 1 + \sqrt{2} \)  
(B) \( 3 + 2\sqrt{2} \)  
(C) \( 1 + 2\sqrt{3} \)  
(D) \( 4 + 5\sqrt{3} \)

Answer (B)
As \( \angle PRQ = \frac{\pi}{2} \)

\[
\begin{pmatrix}
\frac{2}{t} \\
\frac{3}{1 - t^2}
\end{pmatrix} \begin{pmatrix}
-2t \\
3 - t^2
\end{pmatrix} = -1
\]

\( \Rightarrow t = \pm 1 \)

\( \therefore P = (1, 2) \) & \( Q(1, -2) \)

\( \therefore \) for ellipse \( \frac{1}{a^2} + \frac{4}{b^2} = 1 \) and \( ae = 1 \)

\( \Rightarrow \frac{1}{a^2} + \frac{4}{a^2(1 - e^2)} = 1 \)

\( \Rightarrow 1 + \frac{4}{1 - e^2} = \frac{1}{e^2} \)

\( \Rightarrow (5 - e^2)e^2 = 1 - e^2 \)

\( \Rightarrow e^4 - 6e^2 + 1 = 0 \)

\( \Rightarrow e^2 = \frac{1}{3 - 2\sqrt{2}} \Rightarrow \frac{1}{e^2} = 3 + 2\sqrt{2} \)

19. Let the tangent to the circle \( C_1: x^2 + y^2 = 2 \) at the point \( M(–1, 1) \) intersect the circle \( C_2: (x – 3)^2 + (y – 2)^2 = 5 \), at two distinct points \( A \) and \( B \). If the tangents to \( C_2 \) at the points \( A \) and \( B \) intersect at \( N \), then the area of the triangle \( ANB \) is equal to

(A) \( \frac{1}{2} \)

(B) \( \frac{2}{3} \)

(C) \( \frac{1}{6} \)

(D) \( \frac{5}{3} \)

Answer (C)

Sol. Tangent to \( C_1 \) at \( M: –x + y = 2 = T \)

Intersection of \( T \) with \( C_2 \Rightarrow (x – 3)^2 + x^2 = 5 \)

\( \Rightarrow x = 1, 2 \)

\( A(1, 3) \) and \( B(2, 4) \)

Let \( N = (\alpha, \beta) \)

Then \( –x + y = 2 \) shall be chord of contact for \( x^2 + y^2 - 6x - 4y + 8 = 0 \)

\( \therefore \ \alpha x + \beta y - 3x - 3\alpha - 2y - 2\beta + 8 = 0 \)

is same as \( –x + y = 2 \)

\[
\begin{align*}
\alpha - 3 & = \beta - 2 \\
\beta - 1 & = 3\alpha - 8 + 2\beta
\end{align*}
\]

\( \Rightarrow (\alpha, \beta) = \left( \frac{4}{3}, \frac{11}{3} \right) \)

\( \Delta = \begin{vmatrix}
1 & 1 & 1 \\
4/3 & 11/3 & 1
\end{vmatrix} = \frac{1}{6} \) units

20. Let the mean and the variance of 5 observations \( x_1, x_2, x_3, x_4, x_5 \) be \( 24 \) and \( 5 \) respectively. If the mean and variance of the first 4 observation are \( \frac{7}{2} \) and \( a \) respectively, then \( (4a + x_5) \) is equal to

(A) \( 13 \)

(B) \( 15 \)

(C) \( 17 \)

(D) \( 18 \)

Answer (B)

Sol. \( \sum_{i=1}^{5} x_i = 24 \) \quad ...(i)

\[
\sum_{i=1}^{5} x_i^2 = \frac{194}{25}
\]

\( \Rightarrow \sum_{i=1}^{4} x_i^2 = 1154 \) \quad ...(ii)

\( \Rightarrow x_5 = 10 \)

\( \therefore \sum_{i=1}^{4} x_i = 14 \)

\( \Rightarrow \sum_{i=1}^{4} x_i^2 = \frac{54 - 49}{4} = \frac{5}{4} \)

\( \Rightarrow x_5 + 4a = 10 + 5 = 15 \)
1. Let \( S = \{z \in \mathbb{C} : |z - 2| \leq 1, z(1 + i) + \bar{z} (1 - i) \leq 2\} \)
   Let \( |z - 4i| \) attains minimum and maximum values, respectively, at \( z_1 \in S \) and \( z_2 \in S \). If
   \[ 5(|z_1|^2 + |z_2|^2) = \alpha + \beta \sqrt{5}, \]
   where \( \alpha \) and \( \beta \) are integers, then the value of \( \alpha + \beta \) is equal to ______.
   \[ \text{Answer (26)} \]
   \[ \text{Sol.} \]

2. Let \( y = y(x) \) be the solution of the differential equation \( \frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^2 x - \cos 2x} = x e^{\tan^{-1}(\sqrt{2}\cot 2x)} \),
   \[ 0 < x < \frac{\pi}{2} \text{ with } y \left( \frac{\pi}{4} \right) = \frac{\pi^2}{32}. \]
   If \( y \left( \frac{\pi}{3} \right) = \frac{\pi^2}{18} e^{\tan^{-1}(\alpha)} \),
   then the value of \( 3\alpha^2 \) is equal to ______.
   \[ \text{Answer (2)} \]
   \[ \text{Sol.} \]

3. Let \( d \) be the distance between the foot of perpendiculars of the point \( P(1, 2, -1) \) and \( Q(2, -1, 3) \)
   on the plane \( -x + y + z = 1 \). Then \( d^2 \) is equal to ______.
   \[ \text{Answer (26)} \]
Sol. Foot of perpendicular from P
\[
\begin{align*}
x - 1 &= y - 2 = z + 1 = \frac{-(1+2-1-1)}{3} \\
\Rightarrow p' &= \left(\frac{2}{3}, \frac{7}{3}, -\frac{2}{3}\right)
\end{align*}
\]
and foot of perpendicular from Q
\[
\begin{align*}
x - 2 &= y + 1 = z - 3 = \frac{-(2+1+3-1)}{3} \\
\Rightarrow Q' &= \left(\frac{5}{3}, -\frac{2}{3}, \frac{10}{3}\right)
\end{align*}
\]
PQ = \sqrt{(1)^2 + (3)^2 + (4)^2} = d = \sqrt{26}
\[
\Rightarrow d^2 = 26
\]
4. The number of elements in the set
\[
S = \{(0 \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2 \theta + 5 = 0)\}
\]
is __________.
Answer (32)
Sol. 3\cos^2 2\theta + 6\cos 2\theta - \frac{10(1+\cos 2\theta)}{2} + 5 = 0
\[
\Rightarrow 3\cos^2 2\theta + \cos 2\theta = 0
\]
\[
\Rightarrow \cos 2\theta = 0 \text{ or } \cos 2\theta = -\frac{1}{3}
\]
As \(\theta \in [0, \pi]\), \(\cos 2\theta = -\frac{1}{3}\) \(\Rightarrow\) 2 times
\[
\Rightarrow \theta \in [-4\pi, 4\pi], \cos 2\theta = -\frac{1}{3} \Rightarrow 16 \text{ times}
\]
Similarly, \(\cos 2\theta = 0 \Rightarrow 16 \text{ times}
\]
\[
\therefore \text{Total 32 solutions}
\]
5. The number of solutions of the equation
\[
2\theta - \cos^2 \theta + \sqrt{2} = 0 \text{ in } R \text{ is equal to __________.}
\]
Answer (1)
Sol. \(\cos^2 \theta = 2\theta + \sqrt{2}\)
\[
\begin{align*}
y &= 2x + \sqrt{2} \\
y &= \cos x
\end{align*}
\]
1 point of intersection = 1 solution.

6. \(50\tan \left(\frac{3\tan^{-1} \frac{1}{2} + 2\cos^{-1} \frac{1}{\sqrt{5}}}{2}\right) + 4\sqrt{2}\tan \left(\frac{\tan^{-1} \frac{1}{2}}{2}\right)\) is equal to __________.
Answer (29)
Sol. \(50\tan \left(\frac{1}{2} \right) + 2\tan^{-1} \frac{1}{2} + 2\tan^{-1} (2)\)
\[
\Rightarrow 50\tan \left(\pi + \tan^{-1} \frac{1}{2}\right) + 4\sqrt{2}\tan \left(\frac{\tan^{-1} \frac{1}{2}}{2}\right)
\]
\[
\Rightarrow 50 \left(\frac{1}{2}\right) + 4\sqrt{2}\tan \alpha
\]
\[
\text{Where } 2\alpha = \tan^{-1} 2\sqrt{2}
\]
\[
\Rightarrow \frac{2\tan \alpha}{1 - \tan^2 \alpha} = 2\sqrt{2} \quad \ldots (i)
\]
\[
2\sqrt{2}\tan^2 \alpha + 2\tan \alpha - 2\sqrt{2} = 0
\]
\[
\Rightarrow 2\sqrt{2}\tan^2 \alpha + 4\tan \alpha - 2\tan \alpha - 2\sqrt{2} = 0
\]
\[
\Rightarrow (2\sqrt{2}\tan \alpha - 2)(\tan \alpha - \sqrt{2}) = 0
\]
\[
\Rightarrow \tan \alpha = \sqrt{2} \text{ or } \frac{1}{\sqrt{2}}
\]
\[
\Rightarrow 25 + 4\sqrt{2} \frac{1}{2} = 29
\]
7. Let \(c, k \in R\). If \(f(x) = (c + 1)x^2 + (1 - c^2)x + 2k\) and \(f(x + y) = f(x) + f(y) - xy\), for all \(x, y \in R\), then the
\[
\text{value of } |2(f(1) + f(2) + f(3) + \ldots + f(20))| \text{ is equal to __________.}
\]
Answer (3395)
Sol. \(f(x)\) is polynomial
Put \(y = \frac{1}{x}\) in given functional equation we get
\[
f \left(x + \frac{1}{x}\right) = f(x) + f \left(\frac{1}{x}\right) - 1
\]
\[
\Rightarrow (c+1) \left(x + \frac{1}{x}\right)^2 + (1 - c^2) \left(x + \frac{1}{x}\right) + 2K
\]
\[
= (c+1)x^2 + (1 - c^2)x + 2K
\]
\[
+ (c+1) \frac{1}{x^2} + (1 - c^2) \frac{1}{x} + 2K - 1
\]
2(c + 1) = 2K − 1 \quad \ldots (1)

and put \( x = y = 0 \) we get

\[ f(0) = 2 + f(0) - 0 \Rightarrow f(0) = 0 \Rightarrow k = 0 \]

\[ k = 0 \) and \( 2c = -3 \Rightarrow c = -3/2 \]

\[ f(x) = -x^2 - 5x - \frac{1}{4}(5x + 2x^2) \]

\[ \left| 2 \sum_{i=1}^{20} f(i) \right| = \left| \frac{-2 (5.20.21 + 2.20.21.41)}{4} \right| \]

\[ = \left| - \frac{1}{2} (2730 + 5740) \right| \]

\[ = \left| - \frac{6790}{2} \right| = 3395. \]

8. Let \( H: x^2/a^2 - y^2/b^2 = 1, a > 0, b > 0 \), be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is \( 4(2\sqrt{2} + \sqrt{14}) \). If the eccentricity \( H \) is \( \frac{\sqrt{11}}{2} \), then the value of \( a^2 + b^2 \) is equal to \___________.

Answer (88)

Sol. \( 2a + 2b = 4(2\sqrt{2} + \sqrt{14}) \) \ldots (1)

\[ 1 + \frac{b^2}{a^2} = \frac{11}{4} \quad \ldots (2) \]

\[ \Rightarrow \frac{b^2}{a^2} = \frac{7}{4} \quad \ldots (3) \]

and \( a + b = 4\sqrt{2} + 2\sqrt{14} \quad \ldots (4) \)

By (3) and (4)

\[ a = \frac{\sqrt{17}}{2} \Rightarrow a = 4\sqrt{2} + 2\sqrt{14} \]

\[ \frac{a(2 + \sqrt{17})}{2} = 2\sqrt{2} (2 + \sqrt{17}) \]

\[ a = 4\sqrt{2} \Rightarrow a^2 = 32 \] and \( b^2 = 56 \)

\[ a^2 + b^2 = 32 + 56 = 88 \]

9. Let \( P_1: \hat{r} \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 4 \) be a plane. Let \( P_2 \) be another plane which passes through the points \( (2, -3, 2), \) \( (2, -2, -3) \) and \( (1, -4, 2) \). If the direction ratios of the line of intersection of \( P_1 \) and \( P_2 \) be \( \alpha, \beta, \) then the value of \( \alpha + \beta \) is equal to \___________.

Answer (28)

Sol. Direction ratio of normal to \( P_1 = < 2, 1, -3 > \)

and that of \( P_2 = \begin{vmatrix} 0 & 1 & -5 \\ -1 & -2 & 5 \end{vmatrix} = -5\hat{i} - \hat{j}(-5) + \hat{k}(1) \)

\[ i.e. < -5, 5, 1 > \]

d.r’s of line of intersection are along vector

\[ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ -5 & 5 & 1 \end{vmatrix} = \hat{i}(16) - \hat{j}(-13) + \hat{k}(15) \]

\[ i.e. < 16, 13, 15 > \]

\[ \therefore \alpha + \beta = 16 + 13 + 15 = 28 \]

10. Let \( b_1, b_2, \ldots, b_4 \) be a 4-element permutation with \( b_i \in \{1, 2, 3, \ldots, 100\} \) for \( 1 \leq i \leq 4 \) and \( b_i \neq b_j \) for \( i \neq j \), such that either \( b_1, b_2, b_3 \) are consecutive integers or \( b_2, b_3, b_4 \) are consecutive integers. Then the number of such permutations \( b_1b_2b_3b_4 \) is equal to \___________.

Answer (18915)

Sol. There are 98 sets of three consecutive integer and 97 sets of four consecutive integers.

Using principle of inclusion and exclusion,

Number of permutations of \( b_1b_2b_3b_4 = \) Number of permutations when \( b_1b_2b_3 \) are consecutive + Number of permutations when \( b_2b_3b_4 \) are consecutive – Number of permutations when \( b_1b_2b_3b_4 \) are consecutive

\[ = 97 \times 98 + 97 \times 98 - 97 = 97 \times 195 = 18915. \]