Answers & Solutions for JEE (MAIN)-2021 (Online) Phase-2
(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each part has two sections.

   (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.

   (ii) Section-II : This section contains 10 questions. In Section-II, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and there is no negative marking for wrong answer.
PART-A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. One main scale division of a vernier calipers is 'a' cm and n\textsuperscript{th} division of the vernier scale coincide with (n – 1)\textsuperscript{th} division of the main scale. The least count of the calipers in mm is:

   (1) \(\frac{10 n a}{(n – 1)}\)
   (2) \(\frac{10 n a}{n}\)
   (3) \(\frac{(n – 1) a}{10 n}\)
   (4) \(\frac{10 a}{n}\)

   Answer (4)

Sol. \(1 \text{ VSD} = \frac{(n – 1) \times (a \text{ cm})}{n}\)

   \(\therefore \text{ least count} = 1 \text{ MSD} - 1 \text{ VSD}
   = a \frac{(n – 1) a}{n}
   = \frac{a(n – n + 1)}{n}
   = \frac{a}{n} \text{ cm}
   = \frac{a}{n} \times 10 \text{ mm}

2. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is \(\frac{3d}{4}\), where ‘d’ is the separation between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C\textsubscript{0}) is given by the following relation:

   (1) \(C' = \frac{4 + K}{3} C_0\)
   (2) \(C' = \frac{4}{3 + K} C_0\)
   (3) \(C' = \frac{3 + K}{4K} C_0\)
   (4) \(C' = \frac{4K}{K + 3} C_0\)

   Answer (4)

Sol. \(C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}\)

   \(\frac{\varepsilon_0 A}{d - \frac{3d}{4} + \frac{3d}{4K}} = \frac{\varepsilon_0 A}{d - 3d + 3d} = \frac{\varepsilon_0 A}{\frac{d}{4} + \frac{3d}{4K}}\)

   \(= \frac{4K \times \varepsilon_0 A}{K + 3d}\)

   \(= \frac{4K}{K + 3}\)

3. The maximum and minimum distances of a comet from the Sun are \(1.6 \times 10^{12}\) m and \(8.0 \times 10^{10}\) m respectively. If the speed of the comet at the nearest point is \(6 \times 10^4\) ms\(^{-1}\), the speed at the farthest point is:

   (1) \(6.0 \times 10^3\) m/s
   (2) \(3.0 \times 10^3\) m/s
   (3) \(4.5 \times 10^3\) m/s
   (4) \(1.5 \times 10^3\) m/s

   Answer (2)

Sol. \(\therefore mv_1r_1 = mv_2r_2\)

   \(\Rightarrow 6 \times 10^4 \times 8 \times 10^{10} = v_2 \times 1.6 \times 10^{12}\)

   \(\Rightarrow v_2 = \frac{6 \times 8}{1.6} \times 10^2\)

   \(= 3 \times 10^3\) m/s

4. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is:

   (1) \(9.859 \times 10^{-2}\) N
   (2) \(9.859 \times 10^{-4}\) N
   (3) \(6.28 \times 10^{-3}\) N
   (4) \(0.0314\) N

   Answer (2)

Sol. \(N = \frac{mv^2}{r}\)

   \(\frac{(200 \times 10^{-3}) \times (\frac{2\pi \times 0.2}{40})^2}{0.2}\)

   \(= 9.859 \times 10^{-4}\) N
5. Four equal masses, \( m \) each are placed at the corners of a square of length \( l \) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be:

\[
I = m \left( \frac{l}{\sqrt{2}} \right)^2 \times 2 + m \times (\sqrt{2}l)^2
\]

\[
= \frac{m l^2}{2} \times 2 + 2ml^2
\]

\[
= 3ml^2
\]

(1) 3 \( ml^2 \)
(2) 2 \( ml^2 \)
(3) \( \sqrt{3} \) \( ml^2 \)
(4) \( ml^2 \)

**Answer (1)**

6. The angle of deviation through a prism is minimum when

\( \delta = i = e \)

and refracted ray is parallel to the base.

7. The volume \( V \) of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature \( T \). Consider \( R \) as universal gas constant. The pressure of the mixture of gases is:

\[
\left( \begin{array}{c}
\frac{88RT}{V} \\
\frac{4RT}{V} \\
\frac{3RT}{V} \\
\frac{5RT}{2V}
\end{array} \right)
\]

**Answer (4)**

8. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation:

(1) Intensity
(2) Amplitude
(3) Frequency
(4) Phase

**Answer (3)**

9. A conducting wire of length \( l' \), area of cross-section \( A \) and electric resistivity \( \rho \) is connected between the terminals of a battery. A potential difference \( V \) is developed between its ends, causing an electric current.

If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be:

\[
\left( \begin{array}{c}
\frac{4VA}{\rho l} \\
\frac{3VA}{4\rho l} \\
\frac{1\rho l}{4VA} \\
\frac{1VA}{4\rho l}
\end{array} \right)
\]

**Answer (4)**
10. A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be:
(1) 400 (2) 100 (3) 300 (4) 200
Answer (2)

Sol. \[ \lambda = 4 \times \text{antenna length} \]
\[ \Rightarrow \lambda = 4 \times 25 = 100 \text{ m}. \]

11. An RC circuit as shown in the figure is driven by an AC source generating a square wave. The output wave pattern monitored by CRO would look close to:
(1) \[\text{Graph 1}\]
(2) \[\text{Graph 2}\]
(3) \[\text{Graph 3}\]
(4) \[\text{Graph 4}\]
Answer (3)

Sol. For positive cycle capacitor will get charge and for zero input capacitor will discharge.
Sol. \( a = \frac{vdv}{dx} \)

\[ v = 10 + \frac{x}{5} \]

\[ a = \left(10 + \frac{x}{5}\right)\left(\frac{1}{5}\right) = 2 + \frac{x}{25} \]

\( a(x = 0) = 2 \text{ m/s}^2 \)
\( a(x = 200) = 10 \text{ m/s}^2 \)

In graph [18 should be marked as 10 on y-axis in given options]

13. In thermodynamics, heat and work are:

(1) Point functions
(2) Extensive thermodynamic state variables
(3) Path functions
(4) Intensive thermodynamic state variables

Answer (3)

Sol. Heat and work are path functions.

14. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration \( g/2 \), the time period of pendulum will be:

(1) \( \frac{2}{3} T \)
(2) \( \sqrt{3} T \)
(3) \( \frac{3}{2} T \)
(4) \( \frac{T}{\sqrt{3}} \)

Answer (1)

Sol. \( T = 2\pi \sqrt{\frac{l}{g_{eff}}} \)

\( T' = \frac{2}{\sqrt{3}} T \)

15. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, \( \vec{B} = 8.0 \times 10^{-5} \text{ T} \).

The value of electric field at this point is:

(speed of light = \( 3 \times 10^8 \text{ ms}^{-1} \))

\( \hat{x}, \hat{y}, \hat{z} \) are unit vectors along x, y and z directions.

(1) \( -24 \hat{x} \text{ V/m} \)
(2) \( 2.6 \hat{x} \text{ V/m} \)
(3) \( 24 \hat{x} \text{ V/m} \)
(4) \( -2.6 \hat{y} \text{ V/m} \)

Answer (1)

Sol. \( \vec{E} \cdot \vec{B} = 0 \)

\( \vec{E} \times \vec{B} \) is along positive y-direction

\[ \frac{|\vec{E}|}{|\vec{B}|} = 3 \times 10^8 \]

16. A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure.

![Diagram](image)

Two resistors \( R_1 \) and \( R_2 \) are connected across the ends of the rails. There is a uniform magnetic field \( \vec{B} \) pointing into the page. An external agent pulls the bar to the left at a constant speed \( v \).

The correct statement about the directions of induced currents \( I_1 \) and \( I_2 \) flowing through \( R_1 \) and \( R_2 \) respectively is:

(1) \( I_1 \) is in clockwise direction and \( I_2 \) is in anticlockwise direction
(2) \( I_1 \) is in anticlockwise direction and \( I_2 \) is in clockwise direction
(3) Both \( I_1 \) and \( I_2 \) are in clockwise direction
(4) Both \( I_1 \) and \( I_2 \) are in anticlockwise direction

Answer (1)

Sol. It is based on Lenz Law. \( I_1 \) is in clockwise direction and \( I_2 \) is in anticlockwise direction.

17. A block of mass \( m \) slides along a floor while a force of magnitude \( F \) is applied to it at an angle \( \theta \) as shown in figure. The coefficient of kinetic friction is \( \mu_K \). Then, the block’s acceleration ‘\( a \)’ is given by:

\( g \) is acceleration due to gravity.

![Diagram](image)

(1) \( \frac{F \cos \theta + \mu_K (g - \frac{F \sin \theta}{m})}{m} \)
(2) \( \frac{F \cos \theta - \mu_K (g - \frac{F \sin \theta}{m})}{m} \)
(3) \( \frac{F \cos \theta - \mu_K (g + \frac{F \sin \theta}{m})}{m} \)
(4) \( \frac{F \cos \theta - \mu_K (g + \frac{F \sin \theta}{m})}{m} \)

Answer (2)

Sol. \( F \cos \theta - \mu N = ma \)

\( N = (mg - F \sin \theta) \)
\( F \cos \theta - \mu mg + \mu F \sin \theta = ma \)
18. For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric ($U_e$) and magnetic ($U_m$) fields is:

(1) $U_e > U_m$
(2) $U_e = U_m$
(3) $U_e \neq U_m$
(4) $U_e < U_m$

Answer (2)

Sol. $U_e = U_m$

19. The pressure acting on a submarine is $3 \times 10^5$ Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be:

(Assume that atmospheric pressure is $1 \times 10^5$ Pa, density of water is $10^3$ kg m$^{-3}$, $g = 10$ ms$^{-2}$)

(1) $500\%$
(2) $200\%$
(3) $50\%$
(4) $200\%$

Answer (3)

Sol. $P_1 = P_0 + \rho gh_1$

$h_1 = \frac{2P_0}{\rho g}$

$P_2 = P_0 + 4P_0 = 5P_0$

% increase $= \frac{P_2 - P_1}{P_1} \times 100 = \frac{200}{3} \%$

20. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If $B_H = 0.4$ G, the magnetic moment of the magnet is ($1$ G = $10^{-4}$ T)

(1) $2.880 \times 10^2$ J T$^{-1}$
(2) $2.880$ J T$^{-1}$
(3) $2.880 \times 10^3$ J T$^{-1}$
(4) $28.80$ J T$^{-1}$

Answer (2)

Sol. Neutral point will lie on equatorial plane

$$B_1 = \frac{\mu_0 m}{4\pi} \frac{1}{(d^2 + x^2)}$$

$$B_{net} = 2B_1 \cos \theta$$

$$B_{net} = 0.4 \text{ G}$$

Putting the values we get $m \times 2x = M = 2.881 \text{ JT}^{-1}$

**SECTION - II**

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The value of power dissipated across the zener diode ($V_z = 15$ V) connected in the circuit as shown in the figure is $x \times 10^{-1}$ watt

   ![Circuit Diagram]

   The value of $x$, to the nearest integer, is _____

Answer (5)

Sol. $P = V_z i_z$

$$i = \frac{22 - 15}{35} = \frac{7}{35} = \frac{1}{5} \text{ A} \quad \text{and} \quad i' = \frac{15}{90} = \frac{1}{6} \text{ A}$$

$$i_z = \frac{1}{5} - \frac{1}{6} = \frac{1}{30} \text{ A}$$

$$P = 15 \times \frac{1}{30} = 5 \times 10^{-1} \text{ W}$$
2. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force $F = 20 \text{ N}$ through a massless string wrapped around its periphery as shown in the figure.

Suppose the disk makes $n$ number of revolutions to attain an angular speed of $50 \text{ rad s}^{-1}$.

The value of $n$, to the nearest integer, is ______

[Given : In one complete revolution, the disk rotates by 6.28 rad]

Answer (20)

Sol. $\alpha = \frac{FR}{l}$

$= \frac{20 \times 0.2}{\frac{1}{2} \times 20 \times 0.2^2}$

$= 10 \text{ rad} / \text{s}^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$n = \frac{\theta}{2\pi} + \frac{\omega^2}{4\pi\alpha} = \frac{2500}{4 \times 3.14 \times 10} = 19.90$

3. The first three spectral lines of H-atom in the Balmer series are given $\lambda_1$, $\lambda_2$, $\lambda_3$ considering the Bohr atomic model, the wave lengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately $x \times 10^{-1}$.

The value of $x$, to the nearest integer, is ______

Answer (15)

Sol. $\frac{1}{\lambda_1} = R \left(\frac{1}{4} - \frac{1}{9}\right)$

$\frac{1}{\lambda_2} = R \left(\frac{1}{4} - \frac{1}{16}\right)$

$\frac{1}{\lambda_3} = R \left(\frac{1}{4} - \frac{1}{25}\right)$

$\frac{\lambda_1}{\lambda_3} = \frac{21 \times 36}{100 \times 5} = 1.512$

4. In the figure given, the electric current flowing through the 5 k$\Omega$ resistor is $x$ mA.

The value of $x$ to the nearest integer is ______

Answer (3)

Sol. $i = \frac{E}{R_{eq}} = \frac{21}{5 + 1 + 1} \text{ mA} = 3 \text{ mA}$

5. A sinusoidal voltage of peak value 250 V is applied to a series LCR circuit, in which $R = 8 \Omega$, $L = 24 \text{ mH}$ and $C = 60 \mu \text{F}$. The value of power dissipated at resonant conditions is $x$ kW.

The value of $x$ to the nearest integer is ______

Answer (4)

Sol. Power dissipated at resonance

$= \frac{V_{rms}^2}{R} = \frac{(250)^2}{2 \times 8} = 3.906 \times 10^3 \text{ W}$

6. A ball of mass 10 kg moving with a velocity $10\sqrt{3} \text{ m s}^{-1}$ along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle $\theta$ (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure.
The value of $\theta$ to the nearest integer is ______.

After Collision

Y

X axis

\[ \theta = 30^\circ \]

Answer (30)

Sol. Using conservation of linear momentum

\[ -mv_1\sin\theta + mv_2 = 0 \]

\[ \sin\theta = \frac{v_2}{v_1} = \frac{10}{20} \]

\[ \theta = 30^\circ \]

7. The resistance $R = \frac{V}{I}$, where $V = (50 \pm 2)V$ and $I = (20 \pm 0.2)A$. The percentage error in $R$ is ‘x’ %. 
The value of ‘x’ to the nearest integer is ______.

Answer (5)

Sol. $R = \frac{V}{I}$

\[ \Rightarrow \frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 \]

\[ = \frac{2}{50} \times 100 + \frac{0.2}{20} \times 100 = 5\% \]

8. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $P$ of magnitude 100 N is applied at point A of the frame.

Suppose the force is $P$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is $x$ N.

The value of $x$, to the nearest integer, is ______.

[Given : $\sin(35^\circ) = 0.573$,
$\cos(35^\circ) = 0.819$,
$\sin(110^\circ) = 0.939$,
$\cos(110^\circ) = -0.342$]

Answer (164)

Sol. Let component be $x$, $y$

\[ x\cos35^\circ + y\cos70^\circ = P \]

\[ x\sin35^\circ + y\sin70^\circ = 0 \]

\[ y = -\frac{x}{2\cos35^\circ} \]

\[ x = 2P\cos35^\circ \]

9. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is ‘x’ nm.

The value of ‘x’ to the nearest integer is ______.

Answer (600)

Sol. $\beta = \frac{D\lambda}{d}$

\[ 6 \times 10^{-3} = \frac{10\lambda}{10^{-3}} \]

$\lambda = 600$ nm

10. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be ‘x’.

The value of x is ______.

Answer (0)

Sol.

\[ A = 0 \]

\[ B = 1 \]
SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. In the above chemical reaction, intermediate “X” and reagent/condition “A” are

   (1) \( \text{NO}_2^- \cdot \text{H}_2\text{O}/\text{NaOH} \)
   
   (2) \( \text{N}^\text{+}\text{Cl}^- \cdot \text{H}_2\text{O}/\text{NaOH} \)
   
   (3) \( \text{NO}_2^- \cdot \text{H}_2\text{O}/\Delta \)
   
   (4) \( \text{N}^\text{+}\text{Cl}^- \cdot \text{H}_2\text{O}/\Delta \)

Answer (3)

Sol. In chromatography technique, the purification of compound is independent of physical state of the pure compound.

3. Which among the following pairs of Vitamins is stored in our body relatively for longer duration?
   (1) Ascorbic acid and Vitamin D
   (2) Vitamin A and Vitamin D
   (3) Thiamine and Ascorbic acid
   (4) Thiamine and Vitamin A

Answer (2)

Sol. Vitamins which are soluble in fat and oils but insoluble in water are fat soluble vitamins, which are stored in our body relatively for longer time.

   e.g. Vitamin A and Vitamin D
   
   Thiamine (Vit B\(_1\)) and Ascorbic acid (Vit C) are water soluble.

4. Which of the following reaction DOES NOT involve Hoffmann bromamide degradation?

   (1)
   
   (2)
   
   (3)
   
   (4)

Answer (1)

Sol. In chromatography technique, the purification of compound is independent of

   (1) Solubility of the compound
   (2) Mobility or flow of solvent system
   (3) Length of the column or TLC plate
   (4) Physical state of the pure compound

Answer (4)
5. Match List-I with List-II:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial process</td>
<td>Application</td>
</tr>
<tr>
<td>(a) Haber’s process</td>
<td>(i) HNO₃ synthesis</td>
</tr>
<tr>
<td>(b) Ostwald’s process</td>
<td>(ii) Aluminium extraction</td>
</tr>
<tr>
<td>(c) Contact process</td>
<td>(iii) NH₃ synthesis</td>
</tr>
<tr>
<td>(d) Hall-Heroult process</td>
<td>(iv) H₂SO₄ synthesis</td>
</tr>
</tbody>
</table>

Choose the correct answer from the options given below.

(1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)
(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
(3) (a)-(iv), (b)-(i), (c)-(iv), (d)-(ii)
(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Answer (4)

Sol. Process               Application
Haber’s process            → NH₃ synthesis
Ostwald’s process          → HNO₃ synthesis
Contact process            → H₂SO₄ synthesis
Hall Heroult process       → Aluminium extraction
(a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

6. The process that involves the removal of sulphur from the ores is

(1) Smelting
(2) Refining
(3) Roasting
(4) Leaching

Answer (3)

Sol. Removal of sulphur from the ore is done by Roasting.

7. Given below are two statement: one is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: Size of Bk³⁺ ion is less than Np³⁺ ion.
Reason R: The above is a consequence of the lanthanoid contraction.

In the light of the above statements, choose the correct answer from the options given below.

(1) A is false but R is true
(2) A is true but R is false
(3) Both A and R are true but R is not the correct explanation of A
(4) Both A and R are true and R is the correct explanation of A

Answer (2)

Sol. Size of Bk³⁺ is 98 pm
Size of Np³⁺ is 101 pm
So size of Np³⁺ is more than Bk³⁺ ion.
there is a gradual decrease in the size of M³⁺ ions across the series. This may be referred to as the actinoid contraction.

8. The type of pollution that gets increased during the day time and in the presence of O₃ is

(1) Reducing smog
(2) Acid rain
(3) Global warming
(4) Oxidising smog

Answer (4)

Sol. Photochemical smog occurs in warm, dry and sunny climate. It is also called oxidising smog. The main components of photochemical smog are ozone, nitric oxide, acrolein, formaldehyde and PAN

9. Given below are two statements:

Statement I: H₂O₂ can act as both oxidising and reducing agent in basic medium.
Statement II: In the hydrogen economy, the energy is transmitted in the form of dihydrogen.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both statement I and statement II are false
(2) Statement I is true but statement II is false
(3) Both statement I and statement II are true
(4) Statement I is false but statement II is true

Answer (3)
Sol. Oxidising action in basic medium
\[ 2\text{Fe}^{2+} + \text{H}_2\text{O}_2 \rightarrow 2\text{Fe}^{3+} + 2\text{OH}^- \]

Reducing action in basic medium
\[ \text{I}_2 + \text{H}_2\text{O}_2 + 2\text{OH}^- \rightarrow 2\text{I}^- + 2\text{H}_2\text{O} + \text{O}_2 \]

Advantage of hydrogen economy is that energy is transmitted in the form of dihydrogen and not as electric power

10. Match List-I with List-II:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name of oxo acid</td>
<td>Oxidation state of ‘P’</td>
</tr>
<tr>
<td>(a) Hypophosphorous acid</td>
<td>(i) +5</td>
</tr>
<tr>
<td>(b) Orthophosphoric acid</td>
<td>(ii) +4</td>
</tr>
<tr>
<td>(c) Hypophosphoric acid</td>
<td>(iii) +3</td>
</tr>
<tr>
<td>(d) Orthophosphorous acid</td>
<td>(iv) +2</td>
</tr>
<tr>
<td>(v)</td>
<td>+1</td>
</tr>
</tbody>
</table>

Choose the correct answer from the options given below:

(1) (a)-(iv), (b)-(v), (c)-(ii), (d)-(iii)
(2) (a)-(v), (b)-(iv), (c)-(ii), (d)-(iii)
(3) (a)-(v), (b)-(i), (c)-(ii), (d)-(iii)
(4) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

Answer (3)

Sol. Hypophosphorous acid \( \text{H}_3\text{PO}_2 \) +1
Orthophosphorous acid \( \text{H}_3\text{PO}_3 \) +3
Hypophosphoric acid \( \text{H}_4\text{P}_2\text{O}_6 \) +4
Orthophosphoric acid \( \text{H}_3\text{PO}_4 \) +5
(a)-(v), (b)-(i), (c)-(ii), (d)-(iii)

11. Which of the following is Lindlar catalyst?

(1) Cold dilute solution of KMnO
(2) Partially deactivated palladised charcoal
(3) Zinc chloride and HCl
(4) Sodium and Liquid NH

Answer (2)

Sol. Partially deactivated palladised charcoal is called as Lindlar’s catalyst.

12. 

\[ \begin{align*}
\text{H}_3\text{C} & \quad \text{OH} \\
& \quad \text{H}_2\text{O}_2 \\
& \quad 358 \text{ K}
\end{align*} \]

\[ \text{H}_3\text{C} \quad \text{Cl} \]

\[ \text{CH}_2 \]

\[ \text{H}_2\text{O}_2 \]

\[ \text{H}_2\text{O} \]

\[ \text{O}_2 \]

\[ \text{A} \]

(Major Product)

\[ \text{B} \]

(Major Product)

The products “A” and “B” formed in above reactions are

13. **Assertion A**: Enol form of acetone \([\text{CH}_3\text{COCH}_3]\) exists in < 0.1% quantity. However, the enol form of acetyl acetone \([\text{CH}_3\text{COCH}_2\text{OCCH}_3]\) exists in approximately 15% quantity.

**Reason R**: Enol form of acetyl acetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone.

Choose the correct statement:

(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) A is true but R is false
(4) Both A and R are true but R is not the correct explanation of A

Answer (2)
14. The functions of antihistamine are
(1) Antiallergic and Analgesic
(2) Analgesic and antacid
(3) Antiallergic and antidepressant
(4) Antacid and antiallergic
Answer (4)
Sol. The functions of antihistamine are antacid and antiallergic.

15. A group 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is
(1) As  (2) P
(3) Bi  (4) Sb
Answer (3)
Sol. The stability of hydrides decreases from NH₃ to BiH₃ which can be observed from their bond dissociation enthalpy. Consequently, the reducing character of the hydrides increases. Ammonia is only a mild reducing agent while BiH₃ is the strongest reducing agent amongst all the hydrides.

16. Given below are two statements:
Statement I: The E° value for Ce⁴⁺/Ce³⁺ is +1.74 V.
Statement II: Ce is more stable in Ce⁴⁺ state than Ce³⁺ state.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Both statement I and statement II are correct
(2) Statement I is correct but statement II is incorrect
(3) Both statement I and statement II are incorrect
(4) Statement I is incorrect but statement II is correct
Answer (2)
Sol. Ce⁴⁺ → e⁻ → Ce³⁺  E° = +1.74 V
Positive SRP and higher SRP means greater oxidising power. So, Ce⁴⁺ wants to reduce to Ce³⁺. Indicates Ce⁴⁺ is less stable than Ce³⁺.

17. Given below are two statements:
Statement I: Both CaCl₂ · 6H₂O and MgCl₂ · 8H₂O undergo dehydration on heating.
Statement II: BeO is amphoteric whereas the oxides of other elements in the same group are acidic.
In the light of the above statements, choose the correct answer from the options given below.
(1) Statement I is false but statement II is true
(2) Both statement I and statement II are true
(3) Both statement I and statement II are false
(4) Statement I is true but statement II is false
Answer (3)
Sol. CaCl₂ · 6H₂O undergoes dehydration on heating, but MgCl₂ · 8H₂O undergoes hydrolysis on heating. BeO is amphoteric and other metal oxides of II A group are basic in nature.
Repulsion between lone pair - lone pair electrons is higher than bond pair - bond pair electrons. Because bond pair electrons are stuck between two nuclei.

19. Among the following, the aromatic compounds are:

(A) \[\text{CH}_2\]

(B) \[\text{ }\]

(C) \[\text{ }\]

(D) \[\text{ }\]

Choose the correct answer from the following options:

(1) (A), (B) and (C) only

(2) (B), (C) and (D) only

(3) (B) and (C) only

(4) (A) and (B) only

Answer (3)

Sol. Conditions for aromaticity are

(i) Planarity

(ii) Complete delocalisation of the \(\pi\) electrons in the ring

(iii) Presence of \((4n + 2)\pi\) electrons in the ring where \(n\) is an integer \((n = 0, 1, 2...).\)

Compounds B and C are aromatic

Compound A is non-aromatic

Compound D is anti-aromatic

20. The product “P” in the above reaction is:

(1) \[\text{CHO}\]

(2) \[\text{COOH}\]

(3) \[\text{CHO}\]

(4) \[\text{O–C–H}\]

Answer (3)

Sol. DIBAL-H – diisobutylaluminium hydride selectively reduces nitriles and esters to aldehydes.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. When light of wavelength 248 nm falls on a metal of threshold energy 3.0 eV, the de-Broglie wavelength of emitted electrons is ____ Å. (Round off to the Nearest Integer).

\[
\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{3.0 \times 1.6 \times 10^{-19} \text{ J}} = \frac{6.63 \times 3 \times 10^{-34} \times 3 \times 10^8}{3.0 \times 1.6 \times 10^{-19}} = \frac{6.63 \times 9 \times 10^{-26}}{4.8 \times 10^{-19}} = \frac{6.63 \times 9 \times 10^{-26}}{4.8 \times 10^{-19}} = 13.75 \text{ Å} 
\]

Answer (9)
Incident energy of photon = Work function of metal + KE of photoelectron

\[ h = h\nu_0 + KE \]

\[ 6.63 \times 10^{-34} \text{Js} \times 3 \times 10^8 \text{ms}^{-1} = 3.0 + \text{K.E.} \]

K. E. = 2.0 eV

\[ \lambda = \frac{h}{\sqrt{2m\text{K.E.}}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 10^{-19} \times 1.6}} \]

\[ = 8.68 \times 10^{-10} \text{m} = 9 \text{Å} \]

2. A 6.50 molal solution of KOH (aq.) has a density of 1.89 g cm\(^{-3}\). The molarity of the solution is ____ mol dm\(^{-3}\). (Round off to the Nearest Integer).

[Atomic masses : K : 39.0 u; O : 16.0 u; H : 1.0 u]

Answer (9)

Sol. Molality = \( \frac{\text{Moles of solute}}{\text{Mass of solvent (in kg)}} \)

6.50 molal solution of KOH means

6.50 moles of KOH in 1000 g of water (solvent)

364 g of KOH in 1364 g of solution

Volume of solution = \( \frac{\text{Mass of solution(g)}}{\text{density of solution (g mL}^{-1})} \)

\[ = \frac{1364}{1.89} \]

Molarity = \( \frac{\text{Moles of solute}}{\text{Volume of solution (inL)}} \)

\[ = \frac{6.50 \times 1.89}{1364 \times 10^{-3}} = 9.00 \text{M} \]

3. The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the trans-complex of CoCl\(_3\) \cdot 4NH\(_3\) is ____ (Round off to the Nearest Integer).

Answer (2)

Sol. CoCl\(_3\) \cdot 4NH\(_3\) \Rightarrow \text{trans-complex means octahedral with one} Cl^- \text{out of the coordination sphere i.e., [Co(NH}_3)_4Cl]^- \text{Cl. NH}_3 \text{are neutral ligands. Each ethylene diamine replaces two ammines ligands. So two ethylene diamine are required to replace all neutral monodentate (NH}_3\) ligands.}

4. For the reaction A(g) \overset{\text{495 K}}{\rightleftharpoons} B(g) at 495 K, \Delta G^o = -9.478 \text{ kJ mol}^{-1}.

If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B in the equilibrium mixture is ____ millimoles. (Round off to the Nearest Integer).

[ R = 8.314 J mol\(^{-1}\) K\(^{-1}\); \ln 10 = 2.303]

Answer (20)

Sol. \Delta G^o = -2.303 RT \ln K = -RT \ln K

\[ +9.478 \times 10^3 \]

\[ 8.314 \times 495 \]

\[ = \ln K = 2.303 \]

\[ \ln K = \ln 10 \]

K = 10

A(g) \rightleftharpoons B(g)

\[ 22 - x \]

\[ x \]

\[ 10 = \frac{x}{22 - x} \Rightarrow x = 20 \]

5. A certain element crystallises in a bcc lattice of unit cell edge length 27 Å. If the same element under the same conditions crystallises in the fcc lattice, the edge length of the unit cell in Å will be ____ (Round off to the Nearest Integer.)

[Assume each lattice point has a single atom]

[Assume \( \sqrt{3} = 1.73, \sqrt{2} = 1.41 \)]

Answer (33)

Sol. In BCC the relationship between edge length and radius of an atom is given by.

\[ 4r = \sqrt{3}a \]

\[ 4r = \sqrt{3} \times 27 \]

\[ 4r = 1.73 \times 27 = 46.71 \text{ Å} \]

In FCC the relationship between edge length and radius of atom is given by

\[ 4r = \sqrt{2} \times a \Rightarrow 46.71 = a \text{ (edge length)} \]

\[ \Rightarrow a = 33 \text{ Å} \]
6. The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at 300 K is $1.0 \times 10^{-3}$ s$^{-1}$ and the activation energy $E_a = 11.488$ kJ mol$^{-1}$, the rate constant at 200 K is ____ $\times 10^{-5}$ s$^{-1}$. (Round off to the Nearest Integer)

Answer (10)

Sol. $$\log \left( \frac{k_2}{k_1} \right) = \frac{E_a}{2.303R} \left[ \frac{1}{T_1} - \frac{1}{T_2} \right]$$

$k_1$ (at 200 K) = ?

$k_2$ (at 300 K) = $1 \times 10^{-3}$ s$^{-1}$

$$\log \left( \frac{1 \times 10^{-3}}{k_1} \right) = \frac{11.488 \times 10^3}{2.303 \times 8.314 \times 600} \left[ \frac{1}{200} - \frac{1}{300} \right] = 1$$

$$\frac{1 \times 10^{-3}}{k_1} = 10$$

$k_1 = 10 \times 10^{-5}$ s$^{-1}$

7. AB$_2$ is 10% dissociated in water to A$^{2+}$ and B$^-$. The boiling point of a 10.0 molal aqueous solution of AB$_2$ is _________ °C. (Round off to the Nearest Integer)

[Given : Molal elevation constant of water $K_b = 0.5$ K kg mol$^{-1}$boiling point of pure water = 100°C]

Answer (106)

Sol. $AB_2 \rightleftharpoons A^{2+} + 2B^-$

$1 - \alpha \quad \alpha \quad 2\alpha$

i = 1 + $\alpha$

i = 1.1

$\Delta T_b = T_s - T^o = i \times K_b \times$ molality

$T_s - 100 = 1.1 \times 0.5 \times 10$

$T_s = 105.5$

= 106°C

8. $2MnO_4^- + bC_2O_4^{2-} + cH^+ \rightarrow x Mn^{2+} + yCO_2 + zH_2O$

If the above equation is balanced with integer coefficients, the value of $c$ is ____.

_anser (16)

9. Two salts A$_2$X and MX have the same value of solubility product of $4.0 \times 10^{-12}$. The ratio of their molar solubilities i.e. $\frac{S(A_2X)}{S(MX)} = ____$. (Round off to the Nearest Integer)

Answer (50)

Sol. $A_2X \rightleftharpoons 2A^+ + X^{2-}$

$$2s_1 \quad s_1$$

$b = 5$

$c = 16$

$x = 2$

$y = 10$

$z = 8$

$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

$K_{sp} = 4s_1^3 \Rightarrow s_1 = \sqrt[3]{\frac{K_{sp}}{4}} = 10^{-4}$

$MX \rightleftharpoons M^{2+} + X^{2-}$

$s_2 \quad s_2$

$K_{sp} = s_2^2 \Rightarrow s_2 = \sqrt{K_{sp}} = 2 \times 10^{-6}$

$s_1 = s(A_2X)$

$s_2 = s(MX)$

$$\frac{s_1}{s_2} = \frac{4 \times 10^{-4}}{2 \times 10^{-6}} = 50$$

10. Complete combustion of 750 g of an organic compound provides 420 g of CO$_2$ and 210 g of H$_2$O. The percentage composition of carbon and hydrogen in organic compound is 15.3 and ____ respectively. (Round off to the Nearest Integer).

Answer (3)

Sol. Weight of H = $\frac{210}{18} \times 2 = 23.333$ g (in 750 g compound)

% of H = $\frac{23.333}{750} \times 100$ = 3.111

= 3
SECTION I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. If \( y = y(x) \) is the solution of the differential equation,
   \[
   \frac{dy}{dx} + 2y\tan x = \sin x, \quad y\left(\frac{\pi}{3}\right) = 0,
   \]
   then the maximum value of the function \( y(x) \) over \( \mathbb{R} \) is equal to:
   (1) \( \frac{15}{4} \)  
   (2) \( \frac{1}{8} \)  
   (3) \( \frac{1}{2} \)  
   (4) 8

   Answer (2)

   Sol. \( \frac{dy}{dx} + 2y\tan x = \sin x \)
   \[ I. F. = e^{\int 2\tan x \, dx} = \sec^2 x \]
   \[ \Rightarrow y\sec^2 x = \int \sin x \cdot \sec^2 x \, dx \]
   \[ \Rightarrow y\sec^2 x = \sec x + c \]
   \[ \therefore y\left(\frac{\pi}{3}\right) = 0 \]
   \[ \Rightarrow c = -2 \]
   \[ \Rightarrow y = \cos x - 2\cos^2 x = \frac{1}{8} - \left(\frac{2\cos x - \frac{1}{2\sqrt{2}}}{2\sqrt{2}}\right)^2 \]
   Maximum value of \( f(x) \) is \( \frac{1}{8} \).

2. Let \( S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right) \). Then \( \lim_{k \to \infty} S_k \) is equal to:
   (1) \( \tan^{-1}(3) \)  
   (2) \( \tan^{-1}\left(\frac{3}{2}\right) \)  
   (3) \( \cot^{-1}\left(\frac{3}{2}\right) \)  
   (4) \( \frac{\pi}{2} \)

   Answer (3)

   Sol. Let \( T_k = \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right) = \tan^{-1}\left(\frac{2^r}{3^{r+1}}\right) \)
   \[ = \tan^{-1}\left(\frac{2^r}{3^{r+1}}\right) - \tan^{-1}\left(\frac{2^r}{3}\right) \]
   \[ \therefore S_k = \sum_{r=1}^{k} T_k = \tan^{-1}\left(\frac{2^r}{3^{r+1}}\right) - \tan^{-1}\left(\frac{2^r}{3}\right)^{k+1} \]
   \[ \lim_{k \to \infty} S_k = \tan^{-1}\left(\frac{2}{3}\right) \]

3. If for \( a > 0 \), the feet of perpendiculars from the points \( A(a, -2a, 3) \) and \( B(0, 4, 5) \) on the plane \( lx + my + nz = 0 \) are points \( C(0, -a, -1) \) and \( D \) respectively, then the length of line segment \( CD \) is equal to:
   (1) \( \frac{41}{2} \)  
   (2) \( \frac{66}{2} \)  
   (3) \( \frac{55}{2} \)  
   (4) \( \frac{31}{2} \)

   Answer (2)

   Sol. DR’s of \( AC = a, -a, 4 \)
   So equation of the plane will be \( ax - ay + 4z = 0 \).
   \[ \therefore \] Point \( C \) lies on this plane, so \( a^2 = 4 \Rightarrow a = 2 \)
   Equation of plane : \( x - y + 2z = 0 \).
   Projection of \( B(0, 4, 5) \) on this plane is \( D(-1, 5, 3) \).
   \[ CD = \frac{\sqrt{66}}{2} \]

4. Consider three observations \( a, b \) and \( c \) such that \( b = a + c \). If the standard deviation of \( a + 2, b + 2, c + 2 \) is \( d \), then which of the following is true?
   (1) \( b^2 = a^2 + c^2 + 3d^2 \)  
   (2) \( b^2 = 3(a^2 + c^2) - 9d^2 \)  
   (3) \( b^2 = 3(a^2 + c^2) + 9d^2 \)  
   (4) \( b^2 = 3(a^2 + c^2 + d^2) \)

   Answer (2)

   Sol. \[ \therefore \] Standard deviation of \( a, b, c \) is \( d \).
   \[ d^2 = \frac{a^2 + b^2 + c^2}{3} - \left(\frac{a + b + c}{3}\right)^2 \]
   \[ \Rightarrow 9d^2 = 3(a^2 + b^2 + c^2) - 4b^2 \]
   \[ \Rightarrow b^2 = 3(a^2 + c^2) - 9d^2 \]
5. Let P be a plane \(lx + my + nz = 0\) containing the line, \(\frac{x-1}{1} = \frac{y+4}{2} = \frac{z+2}{3}\). If plane P divides the line segment AB joining points A(–3, –6, 1) and B(2, 4, –3) in ratio \(k : 1\) then the value of \(k\) is equal to:

(1) 2  (2) 4  (3) 1.5  (4) 3

Answer (1)

Sol. L : \(\frac{x-1}{1} = \frac{y+4}{2} = \frac{z+2}{3}\)

\[\therefore \text{ Plane P passes through origin and contains line L, then equation of plane P is } \frac{x}{1} = \frac{y}{2} = \frac{z}{3} = 0\]

\[\Rightarrow 8x + y + 2z = 0\]

\[\therefore \text{ Point } \left(2k – 3, 4k – 6, –3k + 1\right) \text{ lies on plane P, then } 8(2k – 3) + (4k – 6) + 2(–3k + 1) = 0\]

\[\Rightarrow k = 2\]

6. Let a complex number \(z\), \(|z| \neq 1\), satisfy \(|z|^{1/2} \leq 2\). Then, the largest value of \(|z|\) is equal to _____.

(1) 8  (2) 7  (3) 6  (4) 5

Answer (2)

Sol. \(\log_{\sqrt{2}} \left(\frac{|z| + 11}{|z|^2 - 2|z| + 1}\right) \leq 2\)

\[\Rightarrow \frac{|z| + 11}{|z|^2 - 2|z| + 1} \geq \frac{1}{2}\]

\[\Rightarrow |z|^2 - 2|z| + 1 \geq 2\]

\[\Rightarrow |z|^2 - 2|z| + 1 \leq 2\]

\[\Rightarrow (|z| - 1)^2 \leq 0 \Rightarrow |z| \leq 1\]

7. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is:

(1) \(\frac{52}{867}\)  (2) \(\frac{22}{425}\)  (3) \(\frac{3}{4}\)  (4) \(\frac{39}{50}\)

Answer (4)

Sol. Consider the events,

\(E_1 = \text{missing card is spade}\)

\(E_2 = \text{missing card is not a spade}\)

\(A = \text{Two spade cards are drawn}\)

\[\Rightarrow P(A) = \frac{\begin{pmatrix} 12 \end{pmatrix}}{\begin{pmatrix} 51 \end{pmatrix}}\]

\[\Rightarrow P(E_1) = \frac{\begin{pmatrix} 13 \end{pmatrix}}{\begin{pmatrix} 51 \end{pmatrix}}\]

\[\Rightarrow P(E_2) = \frac{\begin{pmatrix} 12 \end{pmatrix}}{\begin{pmatrix} 51 \end{pmatrix}}\]

\[\Rightarrow P(E_2|A) = \frac{\begin{pmatrix} 13 \end{pmatrix}}{\begin{pmatrix} 51 \end{pmatrix}}\]

8. The number of elements in the set \(\{x \in R : (|x| – 3)(|x| + 4) = 6\}\) is equal to:

(1) 4  (2) 2  (3) 1  (4) 3

Answer (2)

Sol. If \(x \in (–\infty, –4)\)

\((x + 3)(x + 4) = 6 \Rightarrow x^2 + 7x + 6 = 0 \Rightarrow x = –6 \text{ only}\)

If \(x \in (–4, 0)\)

\((x + 3)(x + 4) = –6 \Rightarrow x^2 + 7x + 18 = 0 \Rightarrow \text{No solution.}\)

If \(x \in (0, \infty)\)

\((x – 3)(x + 4) = 6 \Rightarrow x^2 + x – 18 = 0 \Rightarrow x = \frac{\sqrt{73} – 1}{2}\)

only.

So the given set contains only 2 elements.

9. Which of the following Boolean expression is a tautology?

(1) \((p \land q) \rightarrow (p \rightarrow q)\)  (2) \((p \land q) \land (p \rightarrow q)\)

(3) \((p \land q) \lor (p \land q)\)  (4) \((p \land q) \lor (p \rightarrow q)\)

Answer (1)

Sol. \(p \rightarrow q = \sim p \lor q\)

so \((p \land q) \rightarrow (p \rightarrow q) = \sim (p \land q) \lor (p \rightarrow q)\)

\[= \sim p \lor \sim q \lor \sim p \lor q\]

\[= \sim p \lor q \lor \sim q\]

is a tautology
10. If \( n \) is the number of irrational terms in the expansion
\[
\left( \frac{1}{3^4} + \frac{1}{5^8} \right)^{60}
\]
then \( (n - 1) \) is divisible by:

(1) 7  (2) 26
(3) 8   (4) 30

Answer (2)

Sol. 
\[
T_{r+1} = \binom{60}{r} \left( \frac{1}{3^4} \right)^{60-r} \left( \frac{1}{5^8} \right)^r
\]

Term will be rational if \( r \) is divisible by 8.

\( r = 0, 8, 16, 24, 32, 40, 48, 56 \)

Total number of irrational terms = \( n = 61 - 8 = 53 \)

hence \( n - 1 \) is divisible by 26.

11. Let the function \( f : \mathbb{R} \to \mathbb{R} \) and \( g : \mathbb{R} \to \mathbb{R} \) be defined as:

\[
f(x) = \begin{cases} 
x + 2, & x < 0 \\
x^2, & x \geq 0
\end{cases}, \quad g(x) = \begin{cases} 
x^3, & x < 1 \\
3x - 2, & x \geq 1
\end{cases}
\]

Then, the number of points in \( \mathbb{R} \) where \((f \circ g)(x)\) is NOT differentiable is equal to:

(1) 3  (2) 1
(3) 2   (4) 1

Answer (4)

Sol. 
\( g(x) \) is always differentiable and \( f(x) \) is non-differentiable at \( x = 0 \) (i.e. \( x = 0 \))

Clearly \( f(g(x)) \) is non-differentiable when \( g(x) = 0 \)

12. The range of \( a \in \mathbb{R} \) for which the function
\[
f(x) = (4a - 3) (x + \log_e 5) + 2(a - 7) \\
cot \left( \frac{x}{2} \right) \sin^2 \left( \frac{x}{2} \right), \quad x \neq 2\pi n, \quad n \in \mathbb{N}
\]

has critical points, is:

(1) \((-3, 1)\)  (2) \([1, \infty]\)
(3) \((-\infty, -1)\)  (4) \(\left[ -\frac{4}{3}, 2 \right] \)

Answer (4)

Sol. 
\( f(x) = (4a - 3) (x + \ln 5) + (a - 7) \sin x \)

\( f(x) \) is always continuous and differentiable in its domain,

then \( f(x) \) has critical points if

\[ f'(x) = 0 \] has solutions
\[ (4a -3) + (a -7)\cos x = 0 \]

\[ \Rightarrow \cos x = \frac{3 - 4a}{a - 7} \] has solutions

\[ \Rightarrow -1 \leq \frac{3 - 4a}{a - 7} \leq 1 \]

\[ \Rightarrow a \in \left[ -\frac{4}{3}, 2 \right] \]

13. Let \([x]\) denote greatest integer less than or equal to \( x \). If for \( n \in \mathbb{N} \),
\[
(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j,
\]

then \( \sum_{j=0}^{\frac{3n}{2}} a_{2j} + 4 \sum_{j=0}^{\frac{3n-1}{2}} a_{2j+1} \) is equal to:

(1) 1  (2) \( 2^{n-1} \)
(3) \( n \)  (4) 2

Answer (1)

Sol. 
\( f(x) = (1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j \)

\[ \sum_{j=0}^{\frac{3n}{2}} a_{2j} = \frac{1}{2} (f(1) + f(-1)) = \frac{1}{2} (1+1) = 1 \]

\[ \sum_{j=0}^{\frac{3n-1}{2}} a_{2j+1} = \frac{1}{2} (f(1) - f(-1)) = 0 \]

Clearly \( \sum_{j=0}^{\frac{3n}{2}} a_{2j} + 4 \sum_{j=0}^{\frac{3n-1}{2}} a_{2j+1} = 1 \)

14. Let a vector \( \alpha \hat{i} + \beta \hat{j} \) be obtained by rotating the vector \( \sqrt{3} \hat{i} + \hat{j} \) by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices \((\alpha, \beta)\), \((0, \beta)\) and \((0, 0)\) is equal to:

(1) \( \frac{1}{2} \)  (2) \( 2\sqrt{2} \)
(3) 1  (4) \( \frac{1}{\sqrt{2}} \)

Answer (1)
Sol. \( \therefore |\alpha \hat{i} + \beta \hat{j}| = \sqrt{3} \hat{i} + \hat{j} \Rightarrow \alpha^2 + \beta^2 = 4 \quad \ldots (i) \)

Also \( \frac{\sqrt{3} \alpha + \beta}{2} = \frac{1}{\sqrt{2}} \Rightarrow \sqrt{3} \alpha + \beta = 2\sqrt{2} \quad \ldots (ii) \)

\( \therefore \alpha, \beta > 0, \) then from (i) and (ii)

\( \alpha = \frac{\sqrt{3} - 1}{\sqrt{2}} \) and \( \beta = \frac{\sqrt{3} + 1}{\sqrt{2}} \)

Area of required triangle \( \frac{1}{2} \alpha \beta = \frac{1}{2} \left( \frac{\sqrt{3} - 1}{\sqrt{2}} \right) \left( \frac{\sqrt{3} + 1}{\sqrt{2}} \right) \)

\( = \frac{1}{2} \)

15. Let the position vectors of two points P and Q be \( 3 \hat{i} - \hat{j} + 2 \hat{k} \) and \( \hat{i} + 2 \hat{j} - 4 \hat{k}, \) respectively. Let R and S be two points such that the direction ratios of lines PR and QS are \( (4, -1, 2) \) and \( (-2, 1, -2), \) respectively. Let lines PR and QS intersect at T. If the vector \( \vec{T}A \) is perpendicular to both \( PR \) and \( QS \) and the length of vector \( \vec{T}A \) is \( \sqrt{5} \) units, then the modulus of a position vector of A is:

(1) \( \sqrt{482} \)

(2) \( \sqrt{227} \)

(3) \( \sqrt{5} \)

(4) \( \sqrt{171} \)

Answer (4)

Sol. Equation of \( PR: \frac{x - 3}{4} = \frac{y + 1}{-1} = \frac{z - 2}{2} \)

Equation of \( QS: \frac{x - 1}{-2} = \frac{y - 2}{1} = \frac{z + 4}{-2} \)

Their point of intersection of PR and QS is T(11, -3, 6)

\[ \overrightarrow{PQ} \times \overrightarrow{QS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -1 \end{vmatrix} = 2\hat{j} + \hat{k} \]

Clearly \( \overrightarrow{T}A = \pm (2\hat{j} + \hat{k}) \)

Position vector of \( A = 11\hat{i} - 3\hat{j} + 6\hat{k} \pm (2\hat{j} + \hat{k}) \)

\( 11\hat{i} - 3\hat{j} + 7\hat{k} \) or \( 11\hat{i} - 5\hat{j} + 5\hat{k} \)

Modulus of P. V. of \( A = \sqrt{171} \)

16. If the three normals drawn to the parabola \( y^2 = 4bx \) from \( (a, 0) \) if \( a > 2b. \)

So, \( a > 1 \)

17. If \( x \in \left( 0, \frac{\pi}{2} \right), \)

\( \log_{10} \sin x + \log_{10} \cos x = -1 \)

\( \log_{10} (\sin x + \cos x) = \frac{1}{2} (\log_{10} n - 1), n > 0, \) then the value of \( n \) is equal to

(1) \( 9 \)

(2) \( 16 \)

(3) \( 12 \)

(4) \( 20 \)

Answer (3)

Sol. Given, \( \log_{10} (\sin x \cos x) = -1 \)

\( \Rightarrow \sin 2x = \frac{2}{10} \Rightarrow 1 + \sin 2x = \frac{6}{5} \)

Also \( \log_{10} (\sin x + \cos x) = \frac{1}{2} (\log_{10} n - 1) \)

\( \Rightarrow \frac{1}{2} \log_{10} (1 + \sin 2x) = \frac{1}{2} (\log_{10} n - \log_{10} 10) \)

\( \Rightarrow \frac{6}{5} = \frac{n}{10} \Rightarrow n = 12 \)

18. The number of roots of the equation,

\( (81)^{\sin^2 x} + (81)^{\cos^2 x} = 30 \)

in the interval \( [0, \pi] \) is equal to

(1) \( 4 \)

(2) \( 2 \)

(3) \( 8 \)

(4) \( 3 \)

Answer (1)

Sol. \( 81^{\sin^2 x} + 81^{1 - \sin^2 x} = 30 \)

Let \( 81^{\sin^2 x} = t \)

\( \Rightarrow t + \frac{81}{t} = 30 \Rightarrow t^2 - 30t + 81 = 0 \)

\( \Rightarrow t = 3 \) or 27

i.e. \( 81^{\sin^2 x} = 3 \) or \( 3^3 \)

\( \Rightarrow 3^{4\sin^2 x} = 3^1 \) or \( 3^3 \)

\( \Rightarrow \sin^2 x = \frac{1}{4} \) or \( \frac{3}{4} \)

\( \Rightarrow \sin x = \pm \frac{1}{2} \) or \( \pm \frac{\sqrt{3}}{2} \)

If \( x \in (0, \pi) \) then \( \sin x = \frac{1}{2} \) or \( \frac{\sqrt{3}}{2} \) only

Hence 4 solutions.
19. Let \( \mathbf{A} = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} \), \( i = \sqrt{-1} \). Then, the system of
linear equations \( \mathbf{A}^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix} \) has

(1) Exactly two solutions
(2) No solution
(3) A unique solution
(4) Infinitely many solutions

Answer (2)

Sol.

\[
\mathbf{A}^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}
\]

\[
\mathbf{A}^4 = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}
\]

\[
\mathbf{A}^8 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}
\]

Given \( \mathbf{A}^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix} \)

\[
\begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}
\]

\[
128(x - y) = 8 \quad \text{and} \quad -128(x - y) = 64
\]

\[
x - y = \frac{1}{16} \quad \text{and} \quad x - y = -\frac{1}{2}
\]

Which cannot be equal on same time.

Hence no solution.

20. The locus of the mid-points of the chord of the circle, \( x^2 + y^2 = 25 \) which is tangent to the
hyperbola, \( \frac{x^2}{9} - \frac{y^2}{16} = 1 \) is

(1) \( (x^2 + y^2)^2 - 16x^2 + 9y^2 = 0 \)
(2) \( (x^2 + y^2)^2 - 9x^2 + 144y^2 = 0 \)
(3) \( (x^2 + y^2)^2 - 9x^2 - 16y^2 = 0 \)
(4) \( (x^2 + y^2)^2 - 9x^2 + 16y^2 = 0 \)

Answer (4)

Sol.

\[
\text{Let mid-point be } (h, k)
\]

\[
\therefore \text{Chord of circle is } hx + ky = h^2 + k^2
\]

\[
y = \frac{-h}{k} x + \left( \frac{h^2 + k^2}{k} \right) \quad \ldots (i)
\]

\[
\Rightarrow \text{Tangent to hyperbola (in slope form)}
\]

\[
y = mx \pm \sqrt{9m^2 - 16} \quad \ldots (ii)
\]

Comparing (i) and (ii) we get,

\[
\left( \frac{h^2 + k^2}{k} \right)^2 = 9 \left( \frac{h^2 + k^2}{k} \right) - 16
\]

\[
\Rightarrow (h^2 + k^2)^2 = 9h^2 - 16k^2
\]

\[
\Rightarrow (x^2 + y^2)^2 - 9x^2 + 16y^2 = 0
\]
1. \[ \lim_{n \to \infty} \frac{2}{n} \left( f \left( \frac{1}{n} \right) + f \left( \frac{2}{n} \right) + \ldots + f \left( \frac{n}{n} \right) \right) \]

\[ \Rightarrow \lim_{n \to \infty} \frac{1}{3} \sum_{n=1}^{n} \log_{2} \left( 1 + \tan \frac{\pi x}{4n} \right) \]

\[ \Rightarrow I = \int_{0}^{1} \log_{2} \left( 1 + \tan \frac{\pi x}{4} \right) dx \]

Using \( \int f(x) \, dx = \int f(a + b - x) \, dx \) we get,

\[ I = \int_{0}^{1} \log_{2} \left( 1 + \tan \frac{\pi (1-x)}{4} \right) dx \]

\[ \Rightarrow 2I = \int_{0}^{1} \log_{2} \left( 1 + \tan \frac{\pi x}{4} \right) \left( 1 + \tan \frac{\pi (1-x)}{4} \right) dx \]

\[ \Rightarrow 2I = \int_{0}^{1} \log_{2}^{2} \, dx \]

\[ \Rightarrow I = \int_{1}^{0} dx = 1 \]

3. Let ABCD be a square of side of unit length. Let a circle, centered at A with unit radius be drawn. Another circle, which touches \( C_1 \) and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle \( C_2 \) meet the side AB at E. If the length of EB is \( \alpha + \beta \), where \( \alpha, \beta \) are integers, then \( \alpha + \beta \) is equal to ______.

Answer (1)

Sol. Let the centre of \( C_2 \) be O.

\[ OC = \sqrt{2} - (1-r) \]

where \( r \) is the radius of \( C_2 \).

\[ 1-r = \sqrt{r^2 + r^2} \]

\[ r = \sqrt{2} - 1 \]

hence \( OC = \sqrt{2} - 1 \), \( r = 2r \)

\[ \sin \theta = \frac{r}{2r} = \frac{1}{2} \Rightarrow \theta = 30^\circ \]

then \( \triangle BCE = 15^\circ \)

So, \( EB = \tan 15^\circ = 2 - \sqrt{3} \)

4. If the normal to the curve \( y(x) = \int_{0}^{x} (2t^2 - 15t + 10) \, dt \)

at a point \( (a, b) \) is parallel to the line \( x + 3y = -5 \), \( a > 1 \), then the value of \( |a + 6b| \) is equal to ______.

Answer (406)

Sol. \( y(x) = \int_{0}^{x} (2t^2 - 15t + 10) \, dt \)

by Leibnitz's rule

\[ y'(x) = 2x^2 - 15x + 10 \, (\text{given } m_N = 3) \]

\[ \Rightarrow 2x^2 - 15x + 10 = 3 \Rightarrow x = \frac{1}{2} \text{ or } 7 \, (\text{but } a > 1) \]

\[ \therefore a = 7 \]

\[ b = y(7) = \int_{0}^{7} (2t^2 - 15t + 10) \, dt \]

\[ = \left[ \frac{2t^3}{3} - \frac{15t^2}{2} + 10t \right]_{0}^{7} = \frac{686}{3} - \frac{735}{2} + 70 \]

\[ = \frac{1372 - 2205 + 420}{6} = 406 \]

\[ |a + 6b| = |7 - 413| = 406 \]

5. Consider an arithmetic series and a geometric series having four initial terms from the set \{11, 8, 21, 16, 26, 32, 4\}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ______.

Answer (03)

Sol. G.P. from the set will be 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, ....

and A.P. from the set will be 11, 16, 21, 26, ....

Common terms will be the terms of G.P. having unit digit 1 or 6

i.e. common terms 16, 256, 409

6. Let the curve \( y = y(x) \) be the solution of the differential equation, \( \frac{dy}{dx} = 2(2x + 1) \). If the numerical value of area bounded by the curve \( y = y(x) \) and x-axis is \( \frac{4\sqrt{8}}{3} \), then the value of \( y(1) \) is equal to ______.

Answer (02)
Sol. \( \frac{dy}{dx} = 2x + 2 \Rightarrow y = x^2 + 2x + c \) \ldots (1)

Represents parabola with vertex at \( x = -1 \)

Given area \( \frac{4\sqrt{8}}{3} = A \)

\[ \int_{-1}^{-\sqrt{1-c}/3} (x^2 + (c-1))dx = \frac{4\sqrt{8}}{3} \]

\[ \sqrt{1-c}(1-c+3(-3)) = 2\sqrt{p} \]

\[ \sqrt{1-c}(c-1) = \sqrt{6} \Rightarrow c = -1 \]

\[ y = x^2 + 2x - 1 \Rightarrow y(1) = 2 \]

7. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be a continuous function such that

\[ f(x) + f(x + 1) = 2, \text{ for } x \in \mathbb{R} \]

If \( I_1 = \int f(x)dx \) and \( I_2 = \int f(x)dx \), then the value of \( I_1 + 2I_2 \) is equal to \(-1\).

Answer (16)

\[ f(x) + f(x + 1) = 2 \]

\[ f(x) + f(x + 2) = 2 \]

\[ f(x) - f(x + 2) = 0 \Rightarrow f(x) \text{ has fundamental period } 2 \]

\[ f(x) = 2 - f(x + 1) \]

\[ I_1 = \int f(x)dx = \int \left[ 2 - \frac{f(x + 1)}{2} \right]dx \]

\[ \Rightarrow I_1 = \int f(t)dt \]}

8. Let \( P = \begin{pmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{pmatrix} \) and \( A = \begin{pmatrix} 2 & \omega^2 \\ -\omega & 1 \end{pmatrix} \) where \( \omega = \frac{-1+i\sqrt{3}}{3} \), and \( I_3 \) be the identity matrix of order 3. If the determinant of the matrix \( (P^{-1}AP - I_3)^2 \) is \( \alpha\omega^2 \), then the value of \( \alpha \) is equal to \( 36 \).

Answer (36)

\[ \Rightarrow I_1 = \int f(x)dx = \int \left[ 2 - \frac{f(x + 1)}{2} \right]dx \]

\[ \Rightarrow \int f(t)dt \]

9. Let \( z \) and \( w \) be two complex numbers such that

\[ z \bar{z} - 2z + 2 \left| \frac{z + i}{z - 3i} \right| = 1 \] and \( \text{Re}(w) \) has minimum value. Then, the minimum value of \( n \in \mathbb{N} \) for which \( w^n \) is real, is equal to \( 4 \).

Answer (04)

\[ w = x^2 + y^2 - 2x - 2yi + 2 \]

\[ \Rightarrow \text{Re}(w) = (x - 1)^2 + y^2 + 1 \] \ldots (i)

Also \( |z + i| = |z - 3i| \)

\[ (y + 1)^2 = (y - 3)^2 \]

\[ \Rightarrow 2y + 1 = -6y + 9 \]

\[ \Rightarrow y = 1 \] \ldots (ii)

by (i) and (ii)

\[ \text{Re}(w)_{\text{min}} \Rightarrow x = 1 \text{ and } y = 1 \]

\[ \Rightarrow w = 1 + i \]

\[ (1 + i)^n = \text{real} \Rightarrow n_{\text{min}} = 4 \]
10. If \( \lim_{x \to 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2 \), then \( a + b + c \) is equal to ______.

**Answer (04)**

**Sol.** Put \( x = 0 \) we get \( N_r = a - b + c = 0 \) ....(i)

(for indeterminancy to be present)

\[
\lim_{x \to 0} \frac{ae^x - b \cos x + ce^{-x}}{x^2} = 2 \left( \lim_{x \to 0} \frac{\sin x}{x} = 1 \right)
\]

using L’ Hospital rule we get

\[
\lim_{x \to 0} \frac{ae^x + b \sin x - ce^{-x}}{2x} = 2
\]

Put \( x = 0 \Rightarrow a - c = 0 \) ... (ii)

Again by L’ hospital rule we get

\[
\lim_{x \to 0} \frac{ae^x + b \cos x + ce^{-x}}{2} = 2
\]

\[\Rightarrow a + b + c = 4 \] ... (iii)