Date: 28/08/2022

Aakash + BYJU'S

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Time : 3 hrs. Max. Marks: 180

Answers & Solutions

for

JEE (Advanced)-2022 (Paper-1)

PART-I : PHYSICS

SECTION – 1 (Maximum marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	**ONLY** if the correct numerical value is entered;
Zero Marks	:	0	In all other cases.

1. Two spherical stars *A* and *B* have densities ρ_A and ρ_B, respectively. *A* and *B* have the same radius, and their masses M_A and M_B are related by $M_B = 2M_A$. Due to an interaction process, star *A* loses some of its mass, so that its radius is halved, while its spherical shape is retained, and its density remains ρ_A. The entire mass lost by *A* is deposited as a thick spherical shell on *B* with the density of the shell being ρ_A. If v_A and v_B are the escape velocities from *A* and *B* after the interaction process, the ratio $\frac{v_B}{v_A} = \sqrt{\frac{10n}{15^{1/3}}}$. The value of *n* is ______.

Answer (2.30)

Sol. $v_A = \sqrt{\frac{2GM_A}{8 \times \left(\frac{R}{2}\right)}} = \frac{v_0}{2}$

For *B*, $\frac{4}{3}\pi\left(r^3 - R^3\right) = \frac{4}{3}\pi R^3 \times \frac{7}{8}$

$\Rightarrow \quad r = \left(\frac{15}{8}\right)^{\frac{1}{3}} R$

$$\therefore \quad v_B = \sqrt{\frac{2G\times\left(2M_A+\frac{7}{8}M_A\right)}{\frac{(15)^{\frac{1}{3}}R}{2}}}$$

$$= v_0 \times \sqrt{\frac{23\times 2}{8\times(15)^{\frac{1}{3}}}}$$

$$\therefore \quad \frac{v_B}{v_A} = \sqrt{\frac{23}{(15)^{\frac{1}{3}}}} = \sqrt{\frac{2.30\times 10}{(15)^{\frac{1}{3}}}}$$

$\therefore \quad n = 2.30$

2. The minimum kinetic energy needed by an alpha particle to cause the nuclear reaction ${}^{16}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{1}_{1}\text{H} + {}^{19}_{8}\text{O}$ in a laboratory frame is n (in MeV). Assume that ${}^{16}_{7}\text{N}$ is at rest in the laboratory frame. The masses of ${}^{16}_{7}\text{N}$, ${}^{4}_{2}\text{He}$, ${}^{1}_{1}\text{H}$ and ${}^{19}_{8}\text{O}$ can be taken to be 16.006 u, 4.003 u, 1.008 u and 19.003 u, respectively, where 1 u = 930 MeVc^{-2}. The value of n is _______.

Answer (2.33)

Sol. $Q = (m_N + m_{He} - m_H - m_O) \times c^2$

$= (16.006 + 4.003 - 1.008 - 19.003) \times 930$ MeV

$= -1.86$ MeV

$= 1.86$ MeV energy absorbed.

And, $\frac{1}{2}\times\frac{m\times 4m}{5m}\times v^2 =$ max loss in kinetic energy

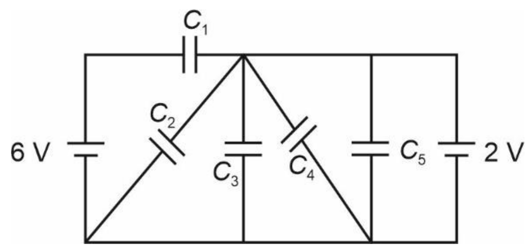
$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{5}{4}\times Q$$

$$= \frac{5}{4}\times(1.86)\text{ MeV}$$

$= 2.325$ MeV

$\therefore \quad n = 2.33$

3. In the following circuit $C_1 = 12$ μF, $C_2 = C_3 = 4$ μF and $C_4 = C_5 = 2$ μF. The charge stored in C_3 is ____ μC.



Answer (8.00)

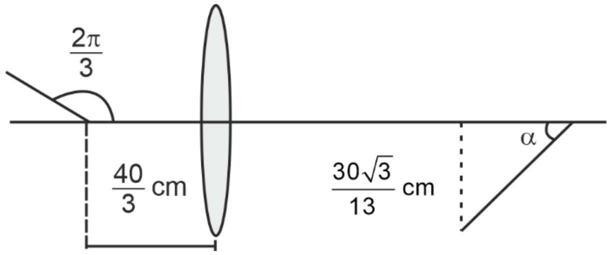
Sol. From circuit given,

Potential difference across C_3 is 2 V (constant)

$\therefore \quad Q_3 = 2 \times 4$ μC

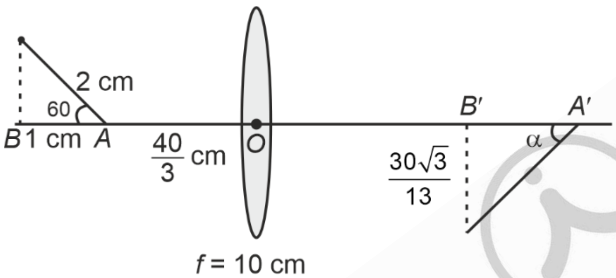
$= 8$ μC

4. A rod of length 2 cm makes an angle $\frac{2\pi}{3}$ rad with the principal axis of a thin convex lens. The lens has a focal length of 10 cm and is placed at a distance of $\frac{40}{3}$ cm from the object as shown in the figure. The height of the image is $\frac{30\sqrt{3}}{13}$ cm and the angle made by it with respect to the principal axis is α rad. The value of α is $\frac{\pi}{n}$ rad, where n is _______.



Answer (6.00)

Sol.



$$OA' = \frac{\frac{40}{3}\times 10}{\frac{43}{3}-10} = 40 \text{ cm}$$

$$OB' = \frac{\frac{43}{3}\times 10}{\frac{43}{3}-10} = \frac{430}{13} \text{ cm}$$

$$\therefore \quad A'B' = 40 - \frac{430}{13} = \frac{90}{13} \text{ cm}$$

$$\therefore \quad \tan\alpha = \frac{30\sqrt{3}}{13\times\left(\frac{90}{13}\right)} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \alpha = \frac{\pi}{6}$$

$\therefore \quad n = 6.00$

5. At time $t = 0$, a disk of radius 1 m starts to roll without slipping on a horizontal plane with an angular acceleration of $\alpha = \frac{2}{3}$ rad s^{-2}. A small stone is stuck to the disk. At $t = 0$, it is at the contact point of the disk and the plane. Later, at time $t = \sqrt{\pi}$ s, the stone detaches itself and flies off tangentially from the disk. The maximum height (in m) reached by the stone measured from the plane is $\frac{1}{2}+\frac{x}{10}$. The value of x is _____. [Take g = 10 ms^{-2}.]

Answer (00.52)

Sol. The angle rotated by disc in $t = \sqrt{\pi}$ s is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow \quad \theta = \frac{1}{2} \times \frac{2}{3}\left(\sqrt{\pi}\right)^2$$

$$= \frac{\pi}{3} \text{ rad}$$

and the angular velocity of disc is

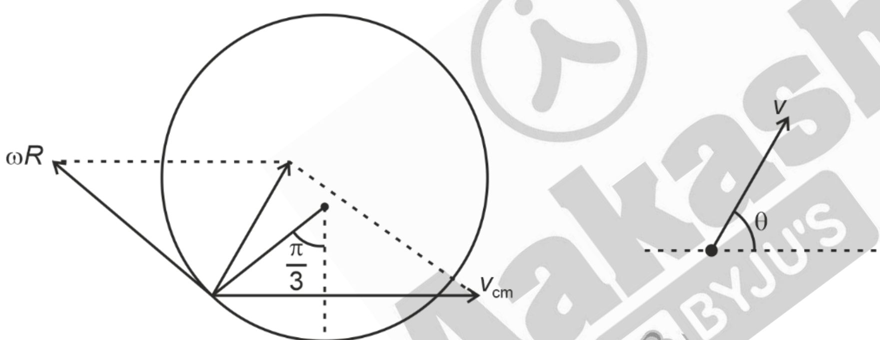
$$\omega = \omega_0 + \alpha t$$

$$= \frac{2\sqrt{\pi}}{3} \text{ rad/s}$$

and $v_{cm} = \omega R = \frac{2\sqrt{\pi}}{3} \times 1$

$$= \frac{2\sqrt{\pi}}{3} \text{ m/s}$$

So, at the moment it detaches the situation is



$$v = \sqrt{(\omega R)^2 + v_{cm}^2 + 2(\omega R)v_{cm}\cos 120^\circ}$$

$$= v_{cm} = \frac{2\sqrt{\pi}}{3} \text{ m/s}$$

and $\tan\theta = \dfrac{\omega R \sin 120^\circ}{v_{cm} + \omega R \cos 120^\circ}$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3} \text{ rad}$$

So, $H_{max} = \dfrac{u^2 \sin^2\theta}{2g}$

$$= \frac{\left(\frac{2\sqrt{\pi}}{3}\right)^2 \times \sin^2 60^\circ}{2 \times 10}$$

$$= \frac{4\pi \times 3}{9 \times 2 \times 10 \times 4}$$

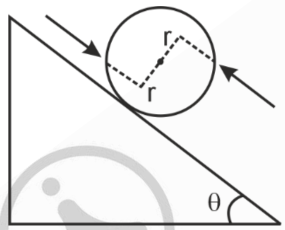
$$= \frac{\pi}{60} \text{ m}$$

So, height from ground will be

$$R(1 - \cos 60°) + \frac{\pi}{60} = \frac{1}{2} + \frac{x}{10}$$

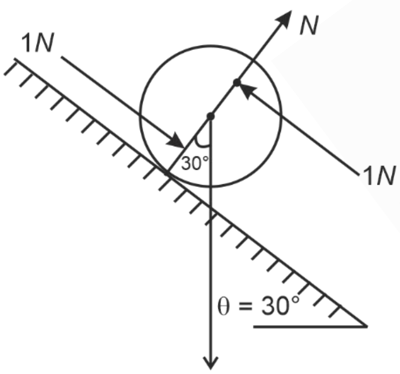
$$\Rightarrow \quad x = \frac{\pi}{6} = 0.52$$

6. A solid sphere of mass 1 kg and radius 1 m rolls without slipping on a fixed inclined plane with an angle of inclination θ = 30° from the horizontal. Two forces of magnitude 1 N each, parallel to the incline, act on the sphere, both at distance r = 0.5 m from the center of the sphere, as shown in the figure. The acceleration of the sphere down the plane is _______ ms^{-2}. (Take g = 10 ms^{-2}.)



Answer (02.86)

Sol.



Taking torque about contact point.

$$\vec{\tau} = mgR \sin 30^{\otimes} + 1 \times 1^{\odot}$$

taking $\otimes$ is positive

$$5 - 1 = \frac{7}{5} mR^2 \alpha$$

$$\Rightarrow \quad \alpha = \frac{20}{7} \text{ rad/s}^2$$

So, $a_{cm} = \alpha R = \frac{20}{7}$ m/s^2

$$\boxed{a_{cm} = 2.86 \text{ m/s}^2}$$

7. Consider an LC circuit, with inductance $L = 0.1$ H and capacitance $C = 10^{-3}$ F, kept on a plane. The area of the circuit is 1 m^2. It is placed in a constant magnetic field of strength B_0 which is perpendicular to the plane of the circuit. At time $t = 0$, the magnetic field strength starts increasing linearly as $B = B_0 + \beta t$ with $\beta = 0.04$ Ts^{-1}. The maximum magnitude of the current in the circuit is _____ mA.

Answer (04.00)

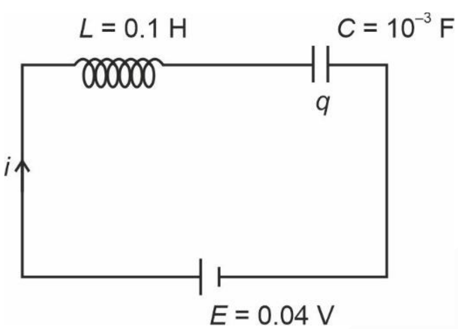
Sol. Emf induced in the circuit is

$$|E| = \left|\frac{d\phi}{dt}\right| = \frac{d}{dt}\left((B_0 + \beta t)A\right)$$

$$= \beta \times A$$

$$= 0.04 \text{ V}$$

So the circuit can be rearranged as



Using Kirchhoff's law we can write

$$E = L\frac{di}{dt} + \frac{q}{C}$$

$$L\frac{di}{dt} = E - \frac{q}{C}$$

Or $$\frac{d^2q}{dt^2} = -\frac{1}{LC}(q - CE)$$

Using SHM concept we can write

$q = CE + A\sin(\omega t + \phi)$ $\left(\text{where } \omega = \dfrac{1}{\sqrt{LC}}\right)$

at $t = 0$, $q = 0$ & $i = 0$

So $A = CE$ & $\phi = -\dfrac{\pi}{2}$

$q = CE - CE\cos\omega t$

so $i = \dfrac{dq}{dt} = CE\omega\sin\omega t$

so $$i_{max} = \frac{10^{-3} \times 0.04}{\sqrt{0.1 \times 10^{-3}}}$$

$$= 4 \text{ mA}$$

8. A projectile is fired from horizontal ground with speed v and projection angle θ. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81}$, then the new range is $d' = nd$. The value of n is _____.

Answer (00.95)

Sol. $d = \frac{u^2 \sin 2\theta}{g}$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

So, after entering in the new region, time taken by projectile to reach ground

$$t = \sqrt{\frac{2H}{g'}}$$

$$= \sqrt{\frac{2u^2 \sin^2 \theta \times 0.81}{2g \times g}}$$

$$= \frac{0.94 \sin \theta}{g}$$

So, horizontal displacement done by the projectile in new region is

$$x = \frac{0.9u \sin \theta}{g} \times u \cos \theta$$

$$= 0.9 \frac{u^2 \sin 2\theta}{2g}$$

So, $d' = \frac{d}{2} + x$

$= 0.95d$

So, $n = 0.95d$

SECTION – 2 (Maximum marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

9. A medium having dielectric constant $K > 1$ fills the space between the plates of a parallel plate capacitor. The plates have large area, and the distance between them is d. The capacitor is connected to a battery of voltage V, as shown in Figure (a). Now, both the plates are moved by a distance of $\frac{d}{2}$ from their original positions, as shown in Figure (b).

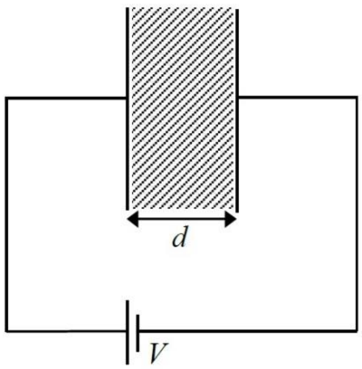


Figure (a)

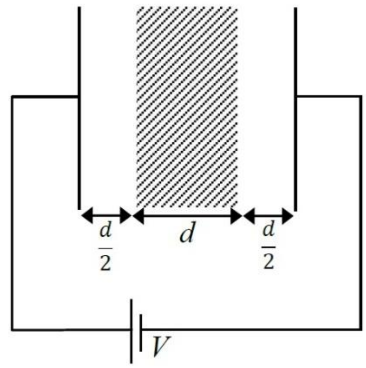


Figure (b)

In the process of going from the configuration depicted in Figure (a) to that in Figure (b), which of the following statement(s) is(are) correct?

(A) The electric field inside the dielectric material is reduced by a factor of $2K$.

(B) The capacitance is decreased by a factor of $\frac{1}{K+1}$.

(C) The voltage between the capacitor plates is increased by a factor of $(K + 1)$.

(D) The work done in the process **DOES NOT** depend on the presence of the dielectric material

Answer (B)

Sol.

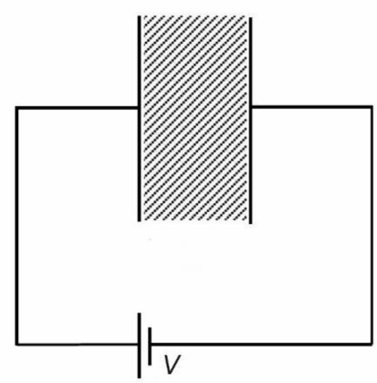


Figure (a)

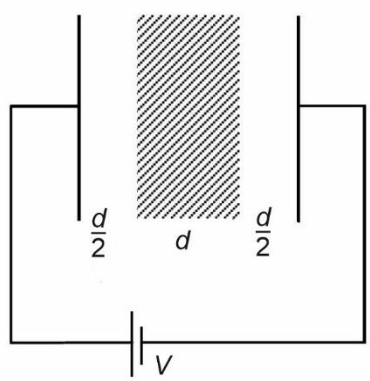


Figure (b)

$$C_a = \frac{K\varepsilon_0 A}{d}$$

$$C_b = \frac{\varepsilon_0 A}{d + \left(\frac{d}{K}\right)}$$

$$q_a = \frac{K\varepsilon_0 A}{d}V$$

$$= \frac{\varepsilon_0 AK}{d(K+1)}$$

$$E_a = \frac{q_a}{KA\varepsilon_0} = \frac{K\varepsilon_0 AV}{dK\varepsilon_0 A}$$

$$= \frac{V}{d}$$

$$q_b = \frac{\varepsilon_0 AKV}{d(K+1)}$$

$$(E_b)_{\text{dielectric}} = \frac{E_{\text{air}}}{K}$$

$$= \frac{q_b}{KA\varepsilon_0}$$

$$= \frac{\varepsilon_0 AKV}{d(K+1)(KA\varepsilon_0)}$$

$$= \frac{V}{d(K+1)}$$

Capacitance decrease by a factor of $\frac{1}{K+1}$

Work done in the process = $U_f - U_i$

$$= \frac{1}{2}(C_f - C_i)V^2$$

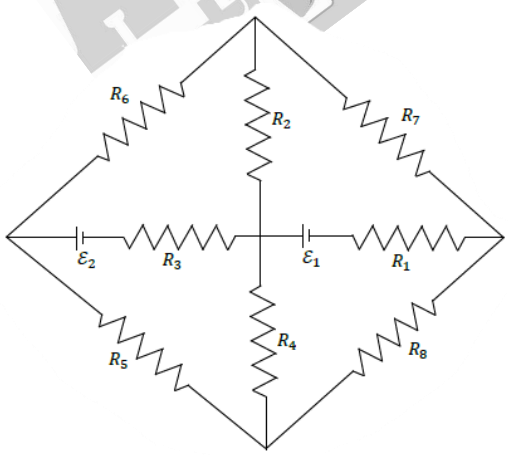
$$= \frac{1}{2}\left(\frac{\varepsilon_0 AK}{d(K+1)} - \frac{K\varepsilon_0 A}{d}\right)V^2$$

$$= \frac{1}{2}V^2 \frac{\varepsilon_0 AK}{d}\left(\frac{1}{K+1} - 1\right)$$

$$= \frac{1}{2}\frac{\varepsilon_0 AKV^2}{d}\frac{1-K-1}{K+1}$$

$$= \frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left(\frac{-K^2}{K+1}\right)$$

10. The figure shows a circuit having eight resistances of 1 Ω each, labelled R_1 to R_8, and two ideal batteries with voltages ε_1 = 12 V and ε_2 = 6 V.



Which of the following statement(s) is(are) correct?

(A) The magnitude of current flowing through R_1 is 7.2 A.

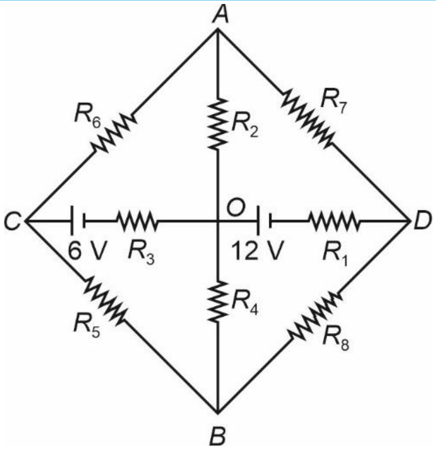
(B) The magnitude of current flowing through R_2 is 1.2 A.

(C) The magnitude of current flowing through R_3 is 4.8 A.

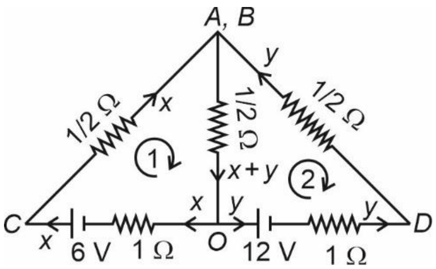
(D) The magnitude of current flowing through R_5 is 2.4 A.

Answer (A, B, C, D)

Sol.



Point *A* and *B* are at same potential so they can be merged/folded.



For loop 1

$$-\frac{1}{2}x-\frac{1}{2}(x+y)-x+6=0$$

$$-2x-\frac{y}{2}=-6$$

$-4x - y = -12$

$4x + y = 12$ …(1)

For loop 2

$$\frac{1}{2}y+1y+12+\frac{1}{2}(x+y)=0$$

$$\frac{3}{2}y+\frac{y}{2}+\frac{x}{2}=-12$$

$$2y+\frac{x}{2}=-12$$

$4y + x = -24$ …(2)

$4y + x - 16x - 4y$

$= -24 - 48$

$-15x = -72$

$$x=\frac{72}{15}$$

$$4\left(\frac{72}{15}\right)+y=12$$

$$y = 12 - \frac{288}{15}$$

$$= \frac{180 - 288}{15}$$

$$= \frac{-108}{15} = -7.2 \text{ A}$$

Current in R_1 = 7.2 A

$$\text{Current in } R_2 = \frac{x + y}{2} = \left(\frac{72}{15} - \frac{108}{15}\right)\frac{1}{2}$$

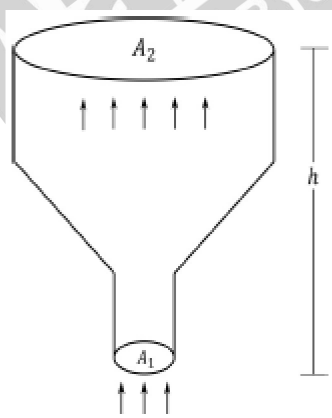
$$= \frac{1}{2}\left(\frac{36}{15}\right) = \frac{2.4}{2} \text{ A}$$

$$= 1.2 \text{ A}$$

Current in $R_3 = x$ = 4.8 A

$$\text{Current in } R_5 = \frac{1}{2}x = 2.4 \text{ A}$$

11. An ideal gas of density ρ = 0.2 kg m^{-3} enters a chimney of height h at the rate of α = 0.8 kg s^{-1} from its lower end, and escapes through the upper end as shown in the figure. The cross-sectional area of the lower end is A_1 = 0.1 m^2 and the upper end is A_2 = 0.4 m^2. The pressure and the temperature of the gas at the lower end are 600 Pa and 300 K, respectively, while its temperature at the upper end is 150 K. The chimney is heat insulated so that the gas undergoes adiabatic expansion. Take g = 10 ms^{-2} and the ratio of specific heats of the gas γ = 2. Ignore atmospheric pressure.



Which of the following statement(s) is(are) correct?

(A) The pressure of the gas at the upper end of the chimney is 300 Pa.

(B) The velocity of the gas at the lower end of the chimney is 40 ms^{-1} and at the upper end is 20 ms^{-1}.

(C) The height of the chimney is 590 m.

(D) The density of the gas at the upper end is 0.05 kg m^{-3} .

Answer (B, C)

Sol. $\frac{\rho_1}{\rho_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$

$\Rightarrow \quad \rho_2 = (0.2)\left(\frac{150}{300}\right)^{\frac{1}{1}} = 0.1\ \text{kg/m}^3$

Rate of mass flow = 0.8 kg/s

$\Rightarrow$ Volume flow rate at top = 8 m^3/s

$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \Rightarrow P_2 = 600\left(\frac{1}{2}\right)^{\gamma}$

$\Rightarrow \quad P_2 = 150\ \text{Pa}$

Velocity of gas at lower end $= \frac{V_1}{A_1} = \frac{0.8}{0.2\times0.1} = 40\ \text{m/s}$

Velocity of gas at upper end $= \frac{0.8\times2}{0.2\times0.4} = 20\ \text{m/s}$

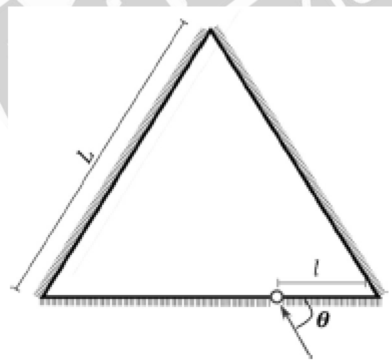
By applying energy conservation

$\frac{1}{2}\rho_1 v_1^2 + P_1 = \frac{1}{2}\rho_2 v_2^2 + P_2 + \rho_2\ g h_2$

$\Rightarrow \quad \frac{1}{2}(0.2)(1600) + 600 = \frac{1}{2}(0.1)(400) + 150 + (0.1)(10)h_2$

$\Rightarrow \quad h_2 = 590$ m

12. Three plane mirrors form an equilateral triangle with each side of length L. There is a small hole at a distance $l > 0$ from one of the corners as shown in the figure. A ray of light is passed through the hole at an angle θ and can only come out through the same hole. The cross section of the mirror configuration and the ray of light lie on the same plane.



Which of the following statement(s) is(are) correct?

(A) The ray of light will come out for θ = 30°, for $0 < l < L$.

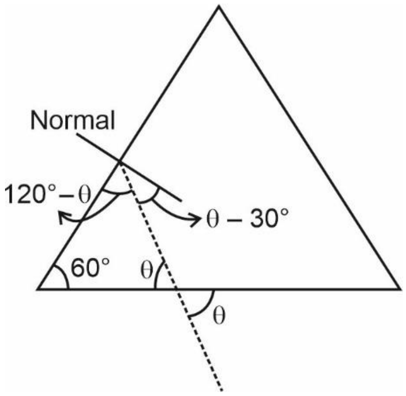
(B) There is an angle for $l = \frac{L}{2}$ at which the ray of light will come out after two reflections.

(C) The ray of light will **NEVER** come out for θ = 60°, and $l = \frac{L}{3}$.

(D) The ray of light will come out for θ = 60°, and $0 < l < \frac{L}{2}$ after six reflections.

Answer (A, B)

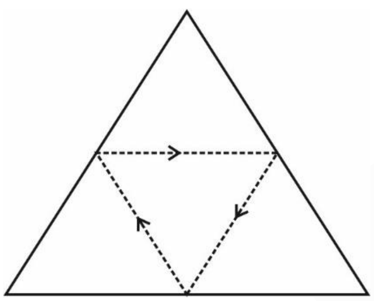
Sol.



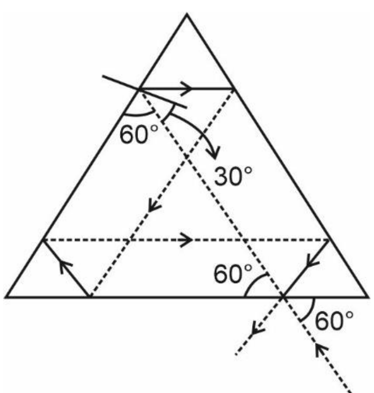
As we can see, for $\theta = 30°$, the ray will incident normally and hence will retrace its path.

$\Rightarrow$ (A) is correct

Considering the symmetry of the situation, we can have :



$\Rightarrow$ (B) is correct



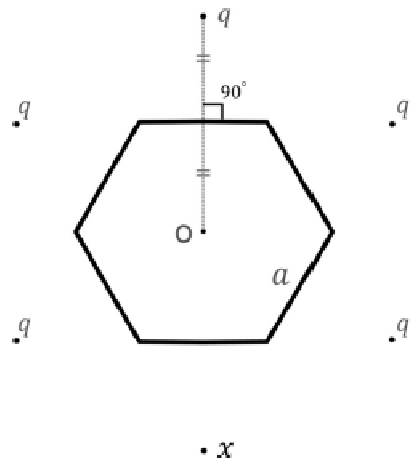
As is clear from the above diagram, ray comes out.

$\Rightarrow$ (C) is not correct

Also, as is clear from the above diagram, total number of reflections = 5.

$\Rightarrow$ (D) is not correct.

13. Six charges are placed around a regular hexagon of side length a as shown in the figure. Five of them have charge q, and the remaining one has charge x. The perpendicular from each charge to the nearest hexagon side passes through the center O of the hexagon and is bisected by the side.



Which of the following statement(s) is(are) correct in SI units?

(A) When $x = q$, the magnitude of the electric field at O is zero.

(B) When $x = -q$, the magnitude of the electric field at O is $\dfrac{q}{6\pi\epsilon_0 a^2}$.

(C) When $x = 2q$, the potential at O is $\dfrac{7q}{4\sqrt{3}\pi\epsilon_0 a}$

(D) When $x = -3q$, the potential at O is $-\dfrac{3q}{4\sqrt{3}\pi\epsilon_0 a}$

Answer (A, B, C)

Sol. When $x = q$, the situation is symmetric

$\Rightarrow$ Electric field at O would be zero.

$\Rightarrow$ (A) is correct.

When $x = -q$, we can think of x as $q + (-2q) \Rightarrow$ Magnitude of electric field

at O $= \dfrac{1}{4\pi\epsilon_0}\dfrac{(2q)}{\left(2\times\dfrac{\sqrt{3}a}{2}\right)^2}$

$$= \frac{1}{4\pi\epsilon_0}\frac{2q}{3a^2} = \frac{q}{6\pi\epsilon_0 a^2}$$

$\Rightarrow$ (B) is correct

For $x = 2q$, potential at O is

$$V_0 = 6\times\frac{1}{4\pi\epsilon_0}\times\frac{q}{\sqrt{3}a} + \frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{3}a}$$

$$= \frac{7q}{4\sqrt{3}\pi\epsilon_0 a}$$

$\Rightarrow$ (C) is correct

For $x = -3q$, $V_0 = 2\times\dfrac{1}{4\pi\epsilon_0}\times\dfrac{q}{\sqrt{3}a} = \dfrac{q}{2\sqrt{3}\pi\epsilon_0 a}$

$\Rightarrow$ (D) is not correct.

14. The binding energy of nucleons in a nucleus can be affected by the pairwise Coulomb repulsion. Assume that all nucleons are uniformly distributed inside the nucleus. Let the binding energy of a proton be E_b^p and the binding energy of a neutron be E_b^n in the nucleus.

Which of the following statement(s) is(are) correct?

(A) $E_b^p - E_b^n$ is proportional to $Z(Z-1)$ where Z is the atomic number of the nucleus.

(B) $E_b^p - E_b^n$ is proportional to $A^{-\frac{1}{3}}$ where A is the mass number of the nucleus.

(C) $E_b^p - E_b^n$ is positive.

(D) E_b^p increases if the nucleus undergoes a beta decay emitting a positron.

Answer (A, B, D)

Sol. Total binding energy (without considering repulsions),

$$E_b = [Zm_p + (A-Z)m_n - m_x]\,c^2$$

Where, ${}_Z^A X$ is the nuclei under consideration.

Now, considering repulsion :

Number of proton pairs $= {}^Z C_2$

$\Rightarrow$ This repulsion energy $\propto \dfrac{Z(Z-1)}{2} \times \dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{R}$

Where R is the radius of the nucleus

$\Rightarrow$ $E_b^p - E_b^n \propto Z(Z-1)$ $\therefore$ there will be no repulsion term for neutrons.

Also, since $R = R_0 A^{1/3}$

$\Rightarrow$ $E_b^p - E_b^n \propto A^{-1/3}$

Because of repulsion among protons,

$E_b^p < E_b^n$

Since in β^+ decay, number of protons decrease $\Rightarrow$ repulsion would decrease

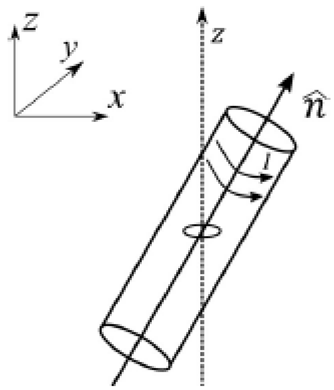
$\Rightarrow$ E_b^p increases

SECTION – 3 (Maximum marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- FOUR options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	**ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

15. A small circular loop of area A and resistance R is fixed on a horizontal xy-plane with the center of the loop always on the axis $\hat{n}$ of a long solenoid. The solenoid has m turns per unit length and carries current I counterclockwise as shown. In the figure. The magnetic filed due to the solenoid is in $\hat{n}$ direction. List-I gives time dependences of $\hat{n}$ in terms of a constant angular frequency ω. List-II gives the torques experienced by the circular loop at time $t = \frac{\pi}{6\omega}$, Let $\theta = \frac{A^2\mu_0^2 m^2 I^2 \omega}{2R}$.



List-I	List-II
(I) $\frac{1}{\sqrt{2}}(\sin\omega t\ \hat{j} + \cos\omega t\ \hat{k})$	(P) 0
(II) $\frac{1}{\sqrt{2}}(\sin\omega t\ \hat{i} + \cos\omega t\ \hat{j})$	(Q) $-\frac{\alpha}{4}\hat{i}$
(III) $\frac{1}{\sqrt{2}}(\sin\omega t\ \hat{i} + \cos\omega t\ \hat{k})$	(R) $\frac{3\alpha}{4}\hat{i}$
(IV) $\frac{1}{\sqrt{2}}(\cos\omega t\ \hat{j} + \sin\omega t\ \hat{k})$	(S) $\frac{\alpha}{4}\hat{j}$
	(T) $-\frac{3\alpha}{4}\hat{i}$

Which one of the following options is correct?

(A) I $\rightarrow$ Q, II $\rightarrow$ P, III $\rightarrow$ S, IV $\rightarrow$ T

(B) I $\rightarrow$ S, II $\rightarrow$ T, III $\rightarrow$ Q, IV $\rightarrow$ P

(C) I $\rightarrow$ Q, II $\rightarrow$ P, III $\rightarrow$ S, IV $\rightarrow$ R

(D) I $\rightarrow$ T, II $\rightarrow$ Q, III $\rightarrow$ P, IV $\rightarrow$ R

Answer (C)

Sol. I. $\boxed{\phi = B\,A\,\hat{k}\cdot\hat{n}}$

$$\phi = \frac{BA}{\sqrt{2}}\cos(\omega t)$$

$$\varepsilon = \frac{BA\omega}{\sqrt{2}}\sin(\omega t)$$

$$i = \frac{BA\omega}{\sqrt{2}R}\sin(\omega t)$$

$$\vec{m} = iA\,\hat{k} = \frac{BA^2\omega}{\sqrt{2}R}\sin(\omega t)\hat{k}$$

$$\vec{\tau} = \vec{m}\times\vec{B} = \frac{B^2A^2\omega}{\sqrt{2}R}\sin(\omega t)(\hat{k}\times\hat{n})$$

$$= -\frac{B^2A^2\omega}{2R}\left[\hat{i}\right]\sin^2(\omega t)$$

$$\tau = -\frac{B^2A^2\omega}{2R}\left[\sin^2\left(\frac{\pi}{6}\right)\right] = \frac{-\alpha}{4}\hat{i}$$

(I) $\rightarrow$ Q.

II. $\phi = O$

(II) $\rightarrow$ P

III. $\phi = \frac{BA}{\sqrt{2}}\cos(\omega t)$

$$i = \frac{BA\omega}{\sqrt{2}R}\sin(\omega t)$$

$$\vec{m} = \frac{BA^2\omega}{\sqrt{2}R}\sin(\omega t)\hat{k}$$

$$\vec{\tau} = \vec{m}\times\vec{B} = \frac{B^2A^2\omega}{\sqrt{2}\times\sqrt{2}R}\sin\omega t\left(\hat{k}\times\left(\sin\omega t\;\hat{i} + \cos\omega t\;\hat{k}\right)\right)$$

$$\tau = \frac{B^2A^2\omega\sin(\omega t)}{2R}\sin(\omega t)\hat{j}$$

$$= \frac{B^2A^2\omega}{2R}\sin^2(\omega t)\hat{j}$$

$$= \frac{\alpha}{4}\hat{j}.$$

III $\rightarrow$ S

IV. $\phi = \frac{BA}{\sqrt{2}}\sin(\omega t)$

$$i = -\frac{BA\omega}{\sqrt{2}R}\cos(\omega t)$$

$$\vec{m} = -\frac{BA^2\omega}{\sqrt{2}R}\cos\omega t\,(\hat{k})$$

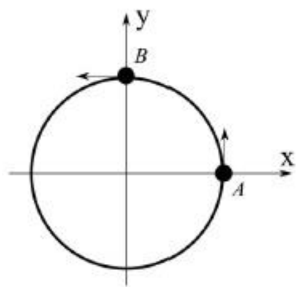
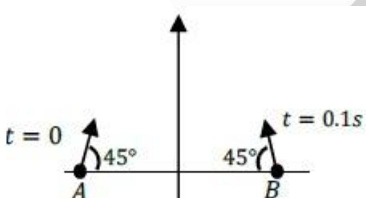
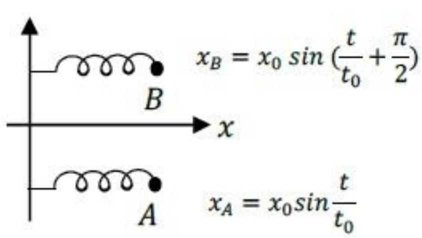
$$\vec{\tau} = \vec{m}\times\vec{B} = -\frac{B^2A^2\omega}{2R}(\hat{k}\times\hat{j})\cos^2(\omega t)$$

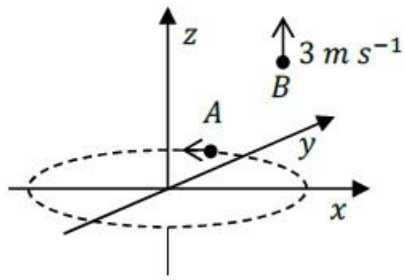
$$\tau = +\frac{B^2A^2\omega}{2R}(\hat{i})\cdot\cos^2\left(\frac{\pi}{6}\right)$$

$$= +\frac{3}{4}\alpha\hat{i}$$

(IV) $\rightarrow$ R.

16. List I describes four systems, each with two particles *A* and *B* in relative motion as shown in figures. List II gives possible magnitude of their relative velocities (in m s^{-1}) at time $t = \frac{\pi}{3}$ s.

	List-I		List-II
(I)	*A* and *B* are moving on a horizontal circle of radius 1 m with uniform angular speed ω = 1 rad s^{-1}. The initial angular positions of *A* *and* *B* at time $t = 0$ are $\theta = 0$ and $\theta = \frac{\pi}{2}$, respectively.  	(P)	$\frac{(\sqrt{3}+1)}{2}$
(II)	Projectiles *A* and *B* are fired (in the same vertical plane) at $t = 0$ and $t = 0.1$ s respectively, with the same speed $v = \frac{5\pi}{\sqrt{2}}$ ms^{-1} and at 45° from the horizontal plane. The initial separation between *A* and *B* is large enough so that they do not collide (g = 10 ms^{-2})  	(Q)	$\frac{(\sqrt{3}-1)}{\sqrt{2}}$
(III)	Two harmonic oscillators *A* and *B* moving in the *x* direction according to $x_A = x_0 \sin\frac{t}{t_0}$ and $x_B = x_0 \sin\left(\frac{t}{t_0} + \frac{\pi}{2}\right)$ respectively, starting from $t = 0$. Take x_0 = 1 m, t_0 = 1 s  	(R)	$\sqrt{10}$

(IV)	Particle *A* is rotating in a horizontal circular path of radius 1 m on the *xy* plane, with constant angular speed ω = 1 rad s^{-1}. Particle *B* is moving up at a constant speed 3 ms^{-1} in the vertical direction as shown in the figure. (Ignore gravity) 	(S)	$\sqrt{2}$
		(T)	$\sqrt{25\pi^2+1}$

Which one of the following options is correct?

(A) I $\rightarrow$ R, II $\rightarrow$ T, III $\rightarrow$ P, IV $\rightarrow$ S

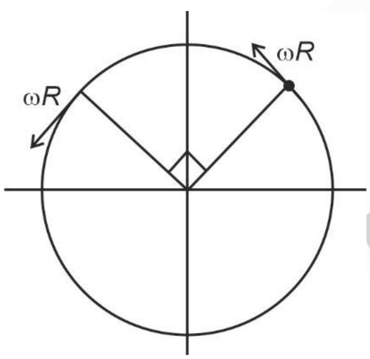
(B) I $\rightarrow$ S, II $\rightarrow$ P, III $\rightarrow$ Q, IV $\rightarrow$ R

(C) I $\rightarrow$ S, II $\rightarrow$ T, III $\rightarrow$ P, IV $\rightarrow$ R

(D) I $\rightarrow$ T, II $\rightarrow$ P, III $\rightarrow$ R, IV $\rightarrow$ S

Answer (C)

Sol. (I)



$$v_{\text{net}} = \sqrt{2}\omega R = \sqrt{2}$$

I $\rightarrow$ S

(II) $$v_A(t = 0.13) = \frac{5\pi}{\sqrt{2}}\cos 45\,\hat{i} + \left[\frac{5\pi}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - g \times 0.1\right]\hat{j}$$

$$= \frac{5\pi}{2}\hat{i} + \left(\frac{5\pi}{2} - 1\right)\hat{j}$$

$$v_B(t = 0.1\,\text{sec}) = \frac{-5\pi}{2}\hat{i} + \left(\frac{5\pi}{2}\right)\hat{j}$$

After t = 0.1, relative velocities should not change.

$$v_{\text{rel}}(t = 0.1\,\text{sec}) = \left|5\pi\hat{i} - \hat{j}\right|$$

$$= \sqrt{25\pi^2 + 1}$$

II $\rightarrow$ T

(III) $x = x_A - x_B$

$$= x_0 \sin t - x_0 \sin\left(t + \frac{\pi}{2}\right)$$

$$= \sqrt{2}x_0 \sin\left(t - \frac{\pi}{4}\right)$$

$$v_{rel} = \frac{dx}{dt} = \sqrt{2}x_0 \cos\left(t - \frac{\pi}{4}\right)$$

$$= \sqrt{2}\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \sqrt{2} \times \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2}$$

III $\rightarrow$ P

(IV) $v_{rel} = \sqrt{3^2 + 1^2} = \sqrt{10}$

IV $\rightarrow$ R

17. List I describes thermodynamic processes in four different systems. List II gives the magnitudes (either exactly or as a close approximation) of possible changes in the internal energy of the system due to the process.

	List-I		**List-II**
(I)	10^{-3} kg of water 100°C is converted to steam at the same temperature, at a pressure of 10^5 Pa. The volume of the system changes from 10^{-6} m^3 to 10^{-3} m^3 in the process. Latent heat of water = 2250 kJ/kg.	(P)	2 kJ
(II)	0.2 moles of a rigid diatomic ideal gas with volume V at temperature 500 K undergoes an isobaric expansion to volume 3 V. Assume R = 8.0 J mol^{-1} K^{-1}.	(Q)	7 kJ
(III)	One mole of a monoatomic ideal gas is compressed adiabatically from volume $V = \frac{1}{3}\text{m}^3$ and pressure 2 kPa to volume $\frac{V}{8}$.	(R)	4 kJ
(IV)	Three moles of a diatomic ideal gas whose molecules can vibrate, is given 9 kJ of heat and undergoes isobaric expansion.	(S)	5 kJ
		(T)	3 kJ

Which one of the following options is correct?

(A) I $\rightarrow$ T, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ Q

(B) I $\rightarrow$ S, II $\rightarrow$ P, III $\rightarrow$ T, IV $\rightarrow$ P

(C) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ T, IV $\rightarrow$ Q

(D) I $\rightarrow$ Q, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ T

Answer (C)

Sol. (I) $U = ML - P\Delta V$

$= 10^{-3} \times 2250 - 10^2\text{kP} \times (10^{-3} - 10^{-6})\ \text{m}^3$

$= 2.25\ \text{kJ} - 0.1\ \text{kJ}$

$= 2.15\ \text{kJ}$

$\text{I} \rightarrow \text{P}$

(II) $C_V = \dfrac{5R}{2}$ (rigid diatomic)

For isobasic expansion

$V \propto T$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \Rightarrow \frac{V}{3V} = \frac{500}{T_2} \Rightarrow T_2 = 1500\ \text{K}$$

$$\Delta U = nC_V\Delta T = 0.2 \times \frac{5\times 8}{2} \times (1500 - 500)\ \text{J}$$

$= 4\ \text{kJ}$

$\text{II} \rightarrow \text{R}$

(III) Adiabatic expansion $\left(\gamma = \dfrac{5}{3}\right)$

$$P_1 \cdot V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow (2\ \text{kPa}) \times V_0^{5/3} = P_2 \times \left(\frac{V_0}{8}\right)^{5/3}$$

$P_2 = 64\ \text{kPa}$

$\Delta U = nC_V\Delta T$

$$= \frac{3nR\Delta T}{2} = \frac{3}{2}(P_2V_2 - P_1V_1) = \frac{3}{2} \times \left(64 \times \frac{1}{3\times 8} - 2 \times \frac{1}{3}\right)$$

$$= \frac{3}{2} \times \left(\frac{8}{3} - \frac{2}{3}\right) = 3\ \text{kJ}$$

$\text{III} \rightarrow \text{T}$

(IV) For isobaric expansion,

$$\Delta U = nC_V\Delta T = \frac{7}{2}\ nR\Delta T$$

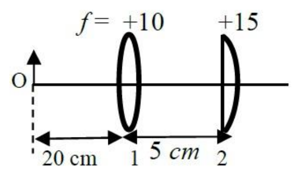
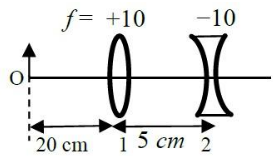
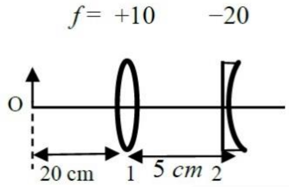
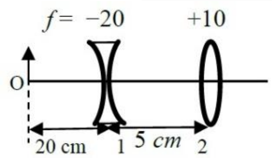
$$\Delta Q = nC_P\Delta T = \frac{9}{2}\ nR\Delta T$$

$$\frac{\Delta U}{\Delta Q} = \frac{7}{9}$$

$$\Delta U = \frac{7}{9}\Delta Q = 7\ \text{kJ}$$

$\text{IV} \rightarrow \text{Q}$

18. List I contains four combinations of two lenses (1 and 2) whose focal lengths (in cm) are indicated in the figures. In all cases, the object is placed 20 cm from the first lens on the left, and the distance between the two lenses is 5 cm. List II contains the positions of the final images.

	List I		**List II**
(I)		(P)	Final image is formed at 7.5 cm on the right side of lens 2.
(II)		(Q)	Final image is formed at 60.0 cm on the right side of lens 2.
(III)		(R)	Final image is formed at 30.0 cm on the left side of lens 2.
(IV)		(S)	Final image is formed at 6.0 cm on the right side of lens 2.
		(T)	Final image is formed at 30.0 cm on the right side of lens 2.

Which one of the following options is correct?

(A) (I) $\rightarrow$ P; (II) $\rightarrow$ R; (III) $\rightarrow$ Q; (IV) $\rightarrow$ T

(B) (I) $\rightarrow$ Q; (II) $\rightarrow$ P; (III) $\rightarrow$ T; (IV) $\rightarrow$ S

(C) (I) $\rightarrow$ P; (II) $\rightarrow$ T; (III) $\rightarrow$ R; (IV) $\rightarrow$ Q

(D) (I) $\rightarrow$ T; (II) $\rightarrow$ S; (III) $\rightarrow$ Q; (IV) $\rightarrow$ R

Answer (A)

Sol.

(I) $u = -20$ cm

$f = +10$ cm

$$\frac{1}{v}+\frac{1}{20}=\frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20}$$

$v = 20$ cm

$u = +15$ cm

$f = +15$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{15} = \frac{1}{15}$$

$$\frac{1}{v} = \frac{2}{15}$$

$v = 7.5$ cm (from lens 2)

$\mathrm{I} \rightarrow \mathrm{P}$

(II) $u = -20$ cm, $f = +10$ cm

$$\frac{1}{v} + \frac{1}{20} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{20} \Rightarrow v = 20 \text{ cm}$$

$u = +15$ cm

$f = -10$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{15} = \frac{-1}{10}$$

$$\frac{1}{v} = \frac{-1}{10} + \frac{1}{15} = \frac{-3+2}{30}$$

$$\frac{1}{v} = -\frac{1}{30}$$

$v = -30$ cm

$\mathrm{II} \rightarrow \mathrm{R}$

(III) $u = -20$

$f = +10$ cm

$$\frac{1}{v}+\frac{1}{20}=\frac{1}{10}$$

$$\frac{1}{v}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$$

$v = 20$ cm

$\Rightarrow \quad u = 15$ cm

$f = -20$ cm

$$\frac{1}{v}-\frac{1}{15}=\frac{-1}{20}$$

$$\frac{1}{v}=-\frac{1}{20}+\frac{1}{15}=\frac{-3+4}{60}=\frac{1}{60}$$

$v = 60$ cm

III $\rightarrow$ Q

(IV) $u = -20$ cm

$f = -20$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{20}=\frac{-1}{20}$$

$v = -10$ cm

$u = -15$ cm

$f = 10$ cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{15}=\frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{10}-\frac{1}{15}$$

$$=\frac{3-2}{30}=\frac{1}{30}$$

$v = 30$ cm

IV $\rightarrow$ T

PART-II : CHEMISTRY

SECTION – 1 (Maximum marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the correct numerical value is entered;

Zero Marks : 0 In all other cases.

1. 2 mol of Hg(g) is combusted in a fixed volume bomb calorimeter with excess of O_2 at 298 K and 1 atm into HgO(s). During the reaction, temperature increases from 298.0 K to 312.8 K. If heat capacity of the bomb calorimeter and enthalpy of formation of Hg(g) are 20.00 kJ K^{-1} and 61.32 kJ mol^{-1} at 298 K, respectively, the calculated standard molar enthalpy of formation of HgO(s) at 298 K is X kJ mol^{-1}. The value of |X| is _______.

[Given: Gas constant R = 8.3 J K^{-1} mol^{-1}]

Answer (90.39)

Sol. $2Hg(g) + O_2(g) \longrightarrow 2HgO(s)$

Heat capacity of calorimeter = 20 kJ K^{-1}

Rise in temperature = 14.8 K

Heat evolved = 20 × 14.8 = 296 kJ

$\Delta H° = \Delta U° + \Delta n_g RT$

$= -296 - 3 \times 8.3 \times 298 \times 10^{-3}$

$\simeq -303.42$ kJ

$\Delta H° = \Delta H_f^0(HgO(s)) - \Delta H_f^0(Hg(g))$

$-303.42 = \Delta H_f^0(HgO(s)) - 2 \times 61.32$

$\Delta H_f^0(HgO(s)) = -303.42 + 122.64$

$= -180.78$ kJ

$|\Delta H_f^0(HgO(s))| = 90.39 \text{ kJ mol}^{-1}$

2. The reduction potential (E^0, in V) of $MnO_4^-(aq)/Mn(s)$ is ______.

[Given: $E^0_{(MnO_4^-(aq)/MnO_2(s))} = 1.68$ V; $E^0_{(MnO_2(s)/Mn^{2+}(aq))} = 1.21$ V; $E^0_{(Mn^{2+}(aq)/Mn(s))} = -1.03$ V]

Answer (0.77)

Sol. (1) $MnO_4^-(aq) + 4H^+ + 3e \longrightarrow MnO_2(s) + 2H_2O;\quad E° = 1.68$ V

$\Delta G_1^0 = -3F\,(1.68) = -5.04\,F$

(2) $MnO_2(s) + 4H^+ + 2e \longrightarrow Mn^{2+} + 2H_2O;$ $E° = 1.21$ V

$\Delta G_2^o = -2F(1.21) = -2.42$ F

(3) $Mn^{2+}(aq) + 2e \longrightarrow Mn(s);$ $E° = -1.03$ V

$\Delta G_3^o = -2F(-1.03) = +2.06$ F

Adding (1), (2) and (3),

$$MnO_4^-(aq) + 8H^+ + 7e \longrightarrow Mn(s) + 4H_2O$$

$$\Delta G = \Delta G_1^o + \Delta G_2^o + \Delta G_3^o$$

$= (-5.04 - 2.42 + 2.06)$ F

$-7F\,E° = -5.4F$

$E° = 0.77$ V

3. A solution is prepared by mixing 0.01 mol each of H_2CO_3, $NaHCO_3$, Na_2CO_3, and NaOH in 100 mL of water. pH of the resulting solution is _______.

[Given: pK_{a_1} and pK_{a_2} of H_2CO_3 are 6.37 and 10.32, respectively; log2 = 0.30]

Answer (10.02)

Sol. First acid base reaction between H_2CO_3 and NaOH takes place.

$$\underset{\substack{0.01\text{ mole}\\ -}}{H_2CO_3} + \underset{\substack{0.01\text{ mole}\\ -}}{NaOH} \longrightarrow \underset{\substack{-\\ 0.01\text{ mole}}}{NaHCO_3} + \underset{-}{H_2O}$$

In the final solution, we have 0.01 mole Na_2CO_3 and 0.02 moles of $NaHCO_3$.

Here, we have a buffer of $NaHCO_3$ and Na_2CO_3.

$$\therefore \quad pH = pK_{a_2} + \log\frac{[\text{Salt}]}{[\text{Acid}]}$$

$$= 10.32 + \log\frac{\left(\frac{0.01}{0.1}\right)}{\left(\frac{0.02}{0.1}\right)}$$

$$= 10.32 + \log\frac{1}{2}$$

$= 10.32 - \log 2$

$= 10.32 - 0.3$

$= 10.02$

$\therefore$ pH = 10.02

4. The treatment of an aqueous solution of 3.74 g of $Cu(NO_3)_2$ with excess KI results in a brown solution along with the formation of a precipitate. Passing H_2S through this brown solution gives another precipitate **X**. The amount of **X** (in g) is ________.

[Given: Atomic mass of H = 1, N = 14, O = 16, S = 32, K = 39, Cu = 63, I = 127]

Answer (00.32)

Sol. Number of moles of $Cu(NO_3)_2 = \frac{3.74}{187} = 0.02$

$$\underset{0.02}{2Cu(NO_3)_2} + 4KI \rightarrow Cu_2I_2 \downarrow + \underset{0.01}{I_2} + 4KNO_3$$

$$\underset{0.01}{I_2} + KI \rightarrow \underset{\substack{0.01 \\ \text{(Brown solution)}}}{KI_3}$$

$$\underset{0.01}{KI_3} + H_2S \rightarrow KI + \underset{\substack{0.01 \\ (X)}}{S} \downarrow + 2HI$$

Number of moles of sulphur precipitated (X) = 0.01

Mass of sulphur precipitates (X) = 0.01 × 32

= 0. 32 gm

5. Dissolving 1.24 g of white phosphorous in boiling NaOH solution in an inert atmosphere gives a gas **Q**. The amount of $CuSO_4$ (in g) required to completely consume the gas **Q** is _______.

[Given: Atomic mass of H = 1, O = 16, Na = 23, P = 31, S = 32, Cu = 63]

Answer (2.38)

Sol. $$\underset{\substack{1.24\text{ g} \\ \text{or} \\ 0.01\,\text{mole}}}{P_4} + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$$

As NaOH is present in excess. So, amount of phosphine formed is 0.01 mole (as P_4 is limiting)

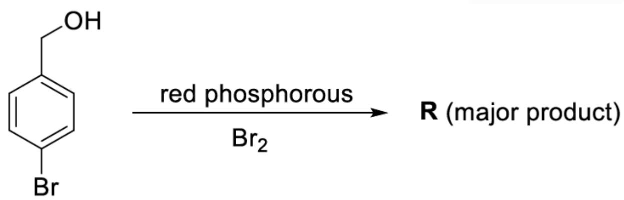
$$\underset{0.01\,\text{mole}}{2PH_3} + 3CuSO_4 \rightarrow Cu_3P_2 + 3H_2SO_4$$

Amount of $CuSO_4$ required = $\frac{3 \times 0.01}{2}$ mole

Mass of $CuSO_4$ (in g) required $= \frac{0.03}{2} \times (63 + 32 + 16 \times 4)$

$= \frac{0.03}{2} \times 159 = 2.38$ g

6. Consider the following reaction.

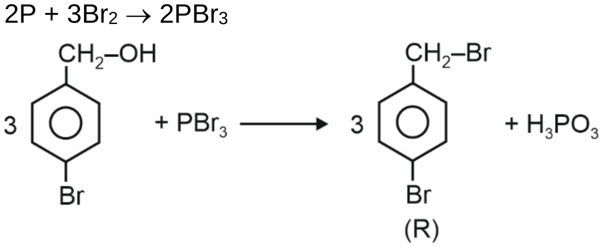


On estimation of bromine in 1.00 g of **R** using Carius method, the amount of AgBr formed (in g) is ________.

[Given: Atomic mass of H = 1, C = 12, O = 16, P = 31, Br = 80, Ag = 108]

Answer (01.50)

Sol. $2P + 3Br_2 \rightarrow 2PBr_3$

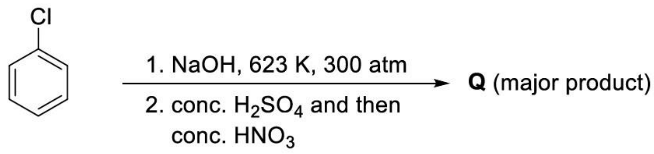


Number of moles in 1 gm of (R) = $\frac{1}{250}$

Number of moles of AgBr formed from (R) = $\frac{2}{250}$

Mass of AgBr formed = $\frac{2 \times 188}{250} = 1.50$ gm

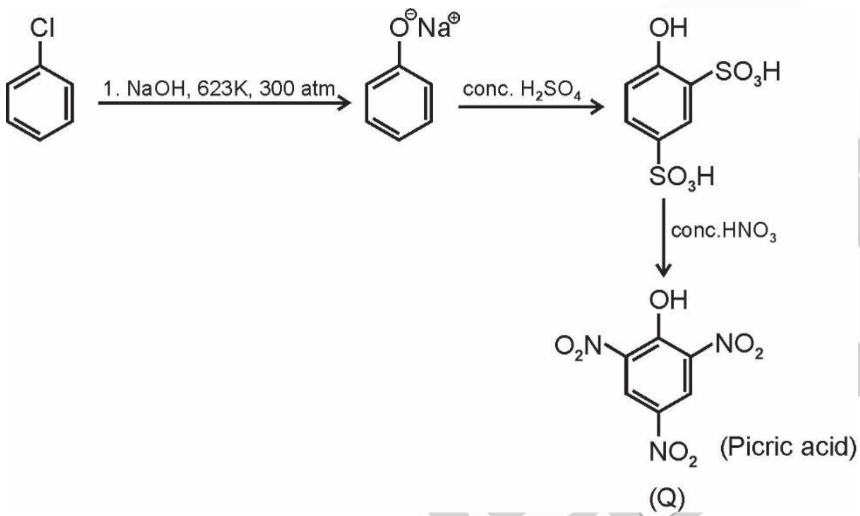
7. The weight percentage of hydrogen in **Q**, formed in the following reaction sequence, is ________.



[Given: Atomic mass of H = 1, C = 12, N = 14, O = 16, S = 32, Cl = 35]

Answer (1.31)

Sol.



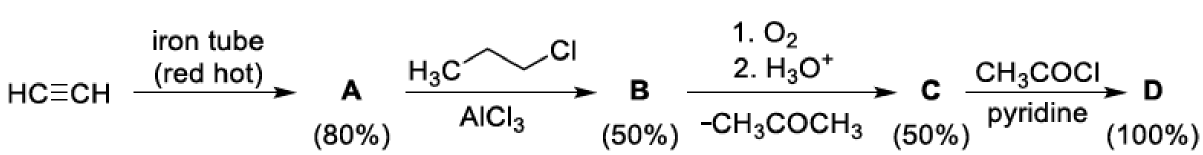
Formula of compound = $C_6H_3N_3O_7$

Molar Mass of compound = $(12 \times 6 + 3 + 14 \times 3 + 16 \times 7)$ g

= 229 g

Weight % of H = $\frac{3}{229} \times 100 = 1.31$

8. If the reaction sequence given below is carried out with 15 moles of acetylene, the amount of the product **D** formed (in g) is __________.

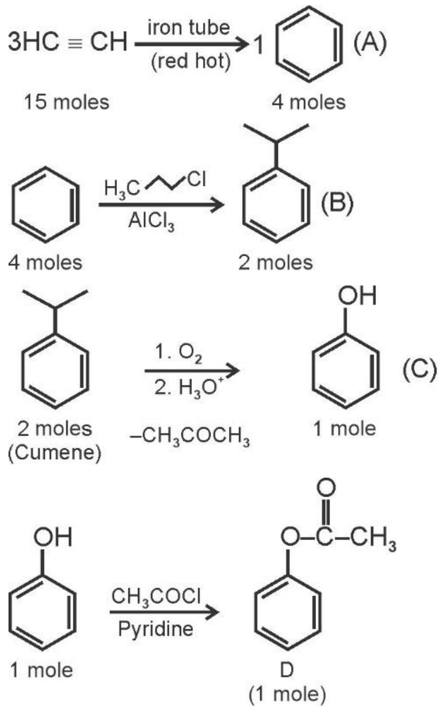


The yields of **A**, **B**, **C** and **D** are given in parentheses.

[Given: Atomic mass of H = 1, C = 12, O = 16, Cl = 35]

Answer (136.00)

Sol.



Molecular formula of D is $C_8H_8O_2$

Molar mass of D is $(12 \times 8 + 8 \times 1 + 16 \times 2) = 136$ g

$\therefore$ Mass of D is 136

SECTION – 2 (Maximum marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

---

9. For diatomic molecules, the correct statement(s) about the molecular orbitals formed by the overlap of two $2p_z$ orbitals is(are)

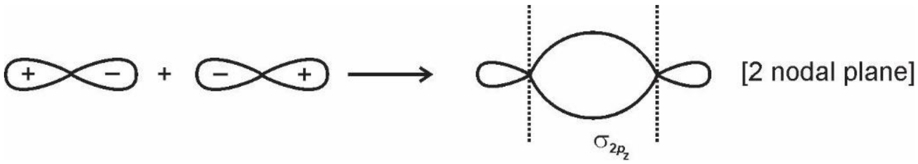
(A) σ orbital has a total of two nodal planes.

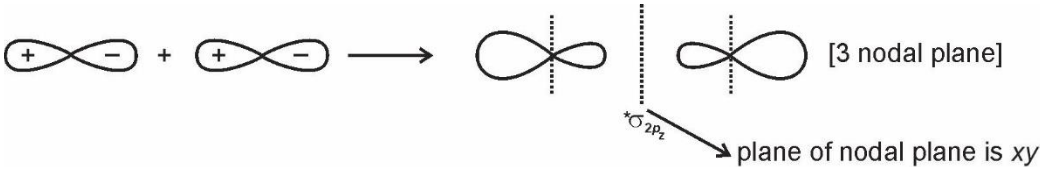
(B) σ^* orbital has one node in the xz-plane containing the molecular axis.

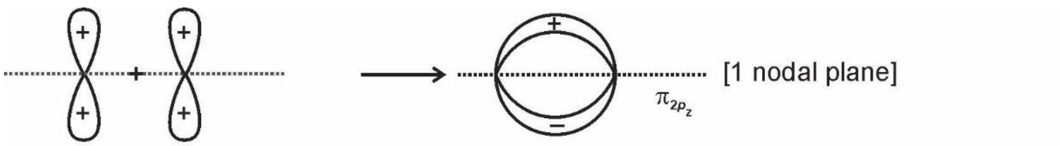
(C) π orbital has one node in the plane which is perpendicular to the molecular axis and goes through the center of the molecule.

(D) π^* orbital has one node in the xy-plane containing the molecular axis.

Answer (A, D)

Sol. (A) 

(B) 

(C) 

plane of nodal plane is *xy* plane which is parallel to molecular axis

(D) 

It has two nodal plane and its one nodal plane is in the xy-plane containing the molecular axis

10. The correct option(s) related to adsorption processes is(are)

(A) Chemisorption results in a unimolecular layer.

(B) The enthalpy change during physisorption is in the range of 100 to 140 kJ mol^{-1}.

(C) Chemisorption is an endothermic process.

(D) Lowering the temperature favours physisorption processes.

Answer (A, D)

Sol. (A) First statement is correct as chemisorption results in a unimolecular layer and physisorption result in a multimolecular layer.

(B) Second statement is incorrect as enthalpy change during physisorption is of the range of (20 – 40) kJ mol^{-1}.

(C) Chemisorption is an exothermic process with (80 – 240) kJ mol^{-1} as the enthalpy of adsorption.

(D) Lowering the temperature results in increase in the extent of physisorption.

Hence (A) and (D) are correct.

11. The electrochemical extraction of aluminium from bauxite ore involves

(A) the reaction of Al_2O_3 with coke (C) at a temperature > 2500°C.

(B) the neutralization of aluminate solution by passing CO_2 gas to precipitate hydrated alumina ($Al_2O_3.3H_2O$).

(C) the dissolution of Al_2O_3 in hot aqueous NaOH.

(D) the electrolysis of Al_2O_3 mixed with Na_3AlF_6 to give Al and CO_2.

Answer (B, C, D)

Sol. (A) The reduction of Al_2O_3 with coke (C) at a temperature > 2500°C is not carried out due to the formation of carbides.

(B) It is correct as neutralisation of aluminate solution is done by passing CO_2 gas to precipitate hydrated alumina.

(C) Reaction of powdered one is carried out with hot concentrated NaOH at 473 K – 523 K and 35 – 36 bar pressure.

(D) Electrolysis of Al_2O_3 is done mixed with Na_3AlF_6 to produce Al and CO_2. It is a correct statement.

12. The treatment of galena with HNO_3 produces a gas that is

(A) paramagnetic

(B) bent in geometry

(C) an acidic oxide

(D) colorless

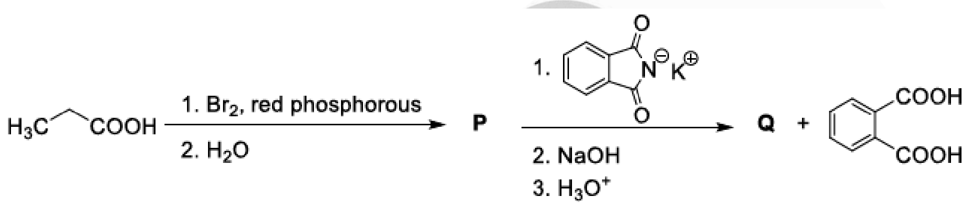
Answer (A, D)

Sol. $PbS + \text{dil. } HNO_3 \rightarrow Pb(NO_3)_2 + S + NO + H_2O$

NO is paramagnetic due to the presence of unpaired electron. It is a neutral oxide. It is colourless.

Hence, (A) and (D) are correct statements.

13. Considering the reaction sequence given below, the correct statement(s) is(are)



(A) **P** can be reduced to a primary alcohol using $NaBH_4$.

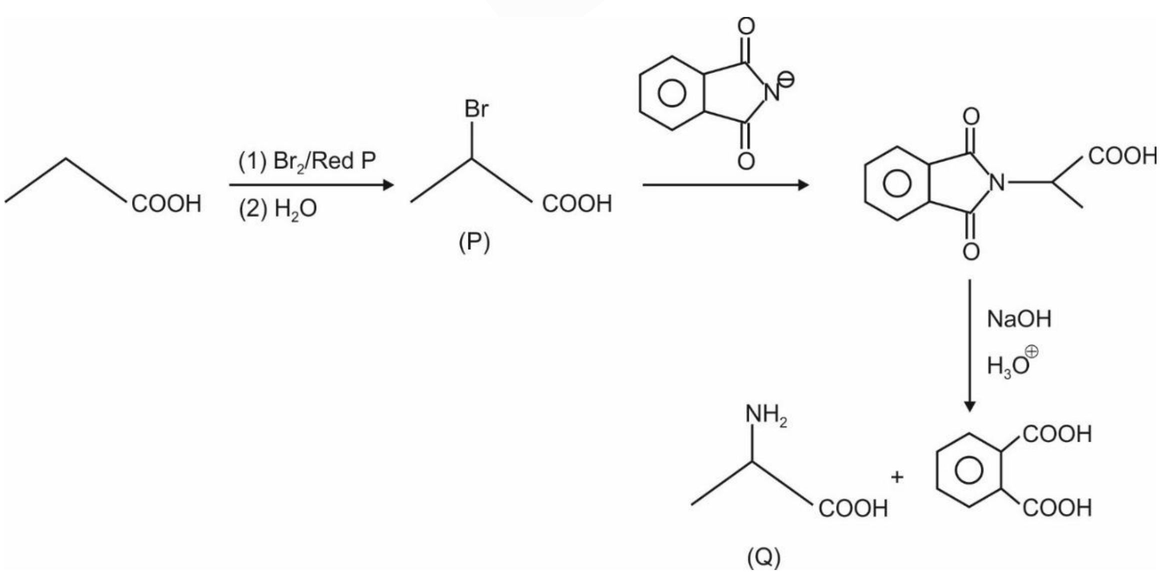
(B) Treating **P** with conc. NH_4OH solution followed by acidification gives Q.

(C) Treating **Q** with a solution of $NaNO_2$ in aq. HCl liberates N_2.

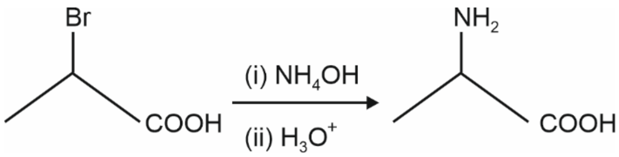
(D) **P** is more acidic than CH_3CH_2COOH.

Answer (B, C, D)

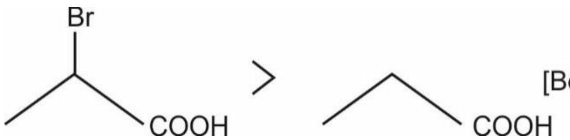
Sol.



→ $NaBH_4$ cannot reduce acids

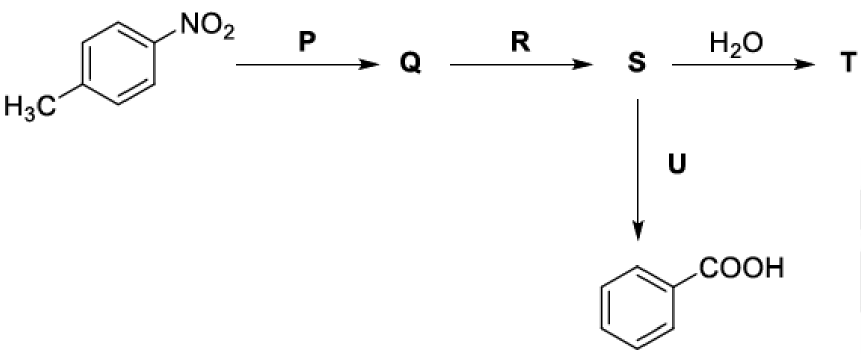
→ 

→ 1° amine on reaction with $NaNO_2$/HCl liberates N_2 gas.

→ 

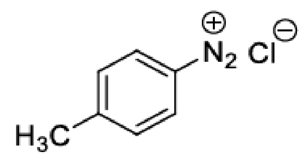
[Because of –I effect of Br]

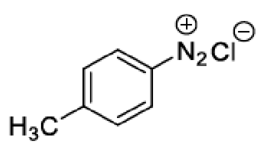
14. Considering the following reaction sequence,

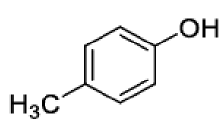


the correct option(s) is(are)

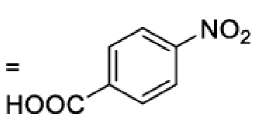
(A) **P** = H_2/Pd, ethanol **R** = $NaNO_2$/HCl **U** = 1. H_3PO_2
2. $KMnO_4$ – KOH, heat

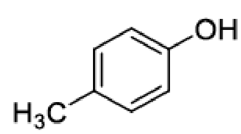
(B) **P** = Sn/HCl **R** = HNO_2 **S** = 

(C) **S** = 

T = 

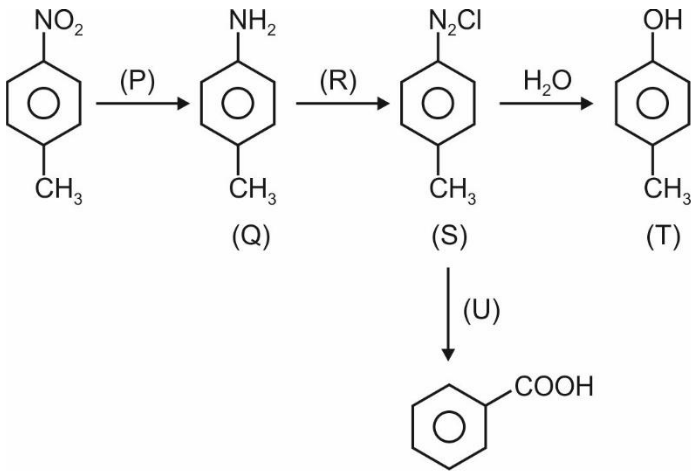
U = 1. CH_3CH_2OH
2. $KMnO_4$ – KOH, heat

(D) **Q** = 

R = H_2/Pd, ethanol **T** = 

Answer (A, B, C)

Sol.



→ P may be → H_2/Pd, ethanol; Sn/HCl

→ R may be → $NaNO_2$/HCl; HNO_2

→ U may be → (i) H_3PO_2, (ii) $KMnO_4$ – KOH, Δ or (i) $CH_3 – CH_2$ – OH, (ii) $KMnO_4$ – KOH, Δ

SECTION – 3 (Maximum marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- FOUR options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	**ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

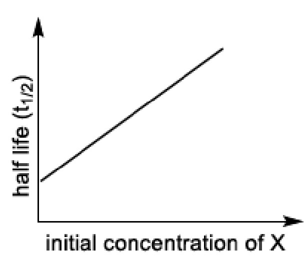
15. Match the rate expressions in LIST-I for the decomposition of X with the corresponding profiles provided in LIST-II. X_s and k are constants having appropriate units.

LIST-I | **LIST-II**

(I) $\text{rate} = \dfrac{k[X]}{X_s + [X]}$

under all possible initial concentrations of X

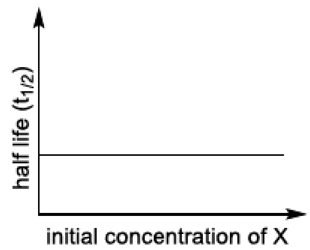
(P)



(II) $\text{rate} = \dfrac{k[X]}{X_S + [X]}$

where initial concentrations of X are much less than X_s

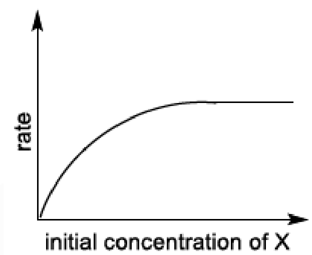
(Q)



(III) $\text{rate} = \dfrac{k[X]}{X_S + [X]}$

where initial concentrations of X are much higher than X_s

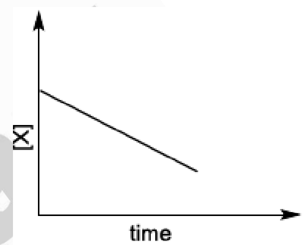
(R)



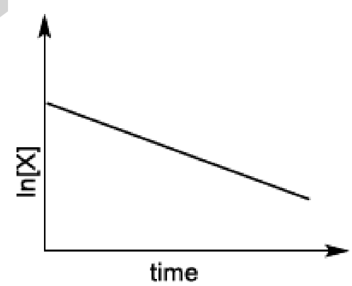
(IV) $\text{rate} = \dfrac{k[X]^2}{X_S + [X]}$

where initial concentration of X is much higher than X_s

(S)



(T)



(A) I→ P; II→ Q; III → S; IV → T

(B) I→ R; II→ S; III → S; IV → T

(C) I→ P; II→ Q; III → Q; IV → R

(D) I→ R; II→ S; III → Q; IV → R

Answer (A)

Sol. (I) $\text{rate} = \dfrac{k[X]}{X_S + [X]}$

Case-1: $[X] >> X_s$; $[X] + X_s \approx [X]$

$\text{rate} = \dfrac{k[X]}{[X]} = k$ (Zero order w.r.t. X)

I → P, S

Case-2: $[X] << X_s$; $[X] + X_s \approx X_s$

$\therefore$ $\text{rate} = \dfrac{k[X]}{X_s} = k'[X]$ (1st order w.r.t. X)

$\therefore$ I $\rightarrow$ Q, T

Case-3: $[X] \approx X_s$

$$\text{rate} = \frac{k[X]}{X_s + [X]}$$

In this case curve-R given in List-II will match.

$\therefore$ I $\rightarrow$ P, Q, R, S, T (The graph of half-life should start from origin)

(II) $\text{rate} = \dfrac{k[X]}{X_s + [X]}$

$\because$ $[X] << X_s$

$\therefore$ $X_s + [X] \approx X_s$

$\therefore$ $\text{rate} = \dfrac{k[X]}{X_s} = k'[X]$ (1st order w.r.t. X)

$\therefore$ II $\rightarrow$ Q, T

(III) $\text{rate} = \dfrac{k[X]}{X_s + [X]}$

$\because$ $[X] >> X_s$

$\therefore$ $X_s + [X] \approx [X]$

$\therefore$ $\text{rate} = \dfrac{k[X]}{[X]} = k$ (Zero order w.r.t. X)

$\therefore$ III $\rightarrow$ P, S

(IV) $\text{rate} = \dfrac{k[X]^2}{X_s + [X]}$

$\because$ $[X] >> X_s$

$\therefore$ $X_s + [X] \approx [X]$

$\therefore$ $\text{rate} = \dfrac{k[X]^2}{[X]} = k[X]$ (1st order w.r.t. X)

$\therefore$ IV $\rightarrow$ Q, T

16. LIST-I contains compounds and LIST-II contains reactions

LIST-I	**LIST-II**
(I) H_2O_2	(P) $Mg(HCO_3)_2 + Ca(OH)_2 \rightarrow$
(II) $Mg(OH)_2$	(Q) $BaO_2 + H_2SO_4 \rightarrow$
(III) $BaCl_2$	(R) $Ca(OH)_2 + MgCl_2 \rightarrow$
(IV) $CaCO_3$	(S) $BaO_2 + HCl \rightarrow$
	(T) $Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow$

Match each compound in LIST-I with its formation reaction(s) in LIST-II, and choose the correct option

(A) I → Q; II → P; III → S; IV → R

(B) I → T; II → P; III → Q; IV → R

(C) I → T; II → R; III → Q; IV → P

(D) I → Q; II → R; III → S; IV → P

Answer (D)

Sol. (P) $Mg(HCO_3)_2 + 2Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + Mg(OH)_2\downarrow + 2H_2O$

(Q) $BaO_2 + H_2SO_4 \longrightarrow BaSO_4 + H_2O_2$

(R) $Ca(OH)_2 + MgCl_2 \longrightarrow Mg(OH)_2 + CaCl_2$

(S) $BaO_2 + 2HCl \longrightarrow BaCl_2 + H_2O_2$

(T) $Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + 2H_2O$

I → Q

II → R

III → S

IV → P

Option (D) is correct.

17. LIST-I contains metal species and LIST-II contains their properties.

LIST-I	**LIST-II**
(I) $[Cr(CN)_6]^{4-}$	(P) t_{2g} orbitals contain 4 electrons
(II) $[RuCl_6]^{2-}$	(Q) μ(spin-only) = 4.9 BM
(III) $[Cr(H_2O)_6]^{2+}$	(R) low spin complex ion
(IV) $[Fe(H_2O)_6]^{2+}$	(S) metal ion in 4+ oxidation state
	(T) d^4 species

[Given: Atomic number of Cr = 24, Ru = 44, Fe = 26]

Match each metal species in LIST-I with their properties in LIST-II, and choose the correct option

(A) I → R, T; II → P, S; III → Q, T; IV → P, Q

(B) I → R, S; II → P, T; III → P, Q; IV → Q, T

(C) I → P, R; II → R, S; III → R, T; IV → P, T

(D) I → Q, T; II → S, T; III → P, T; IV → Q, R

Answer (A)

Sol. (I) $[Cr(CN)_6]^{4-}$

Cr^{+2} = [Ar] $3d^4\ 4s^0$

It is d^2sp^3 hybridised as CN^- is a strong field ligand.

(II) $[RuCl_6]^{2-}$

Ru^{+4} = [Kr] $4d^4\ 5s^0$

t_{2g} set contains 4 electron.

(III) $[Cr(H_2O)_6]^{2+}$

Cr^{+2} = [Ar] $3d^4\ 4s^0$

It has 4 unpaired e^- as H_2O is weak field ligand.

So, its μ = 4.9 B.M.

(IV) $[Fe(H_2O)_6]^{2+}$

Fe^{2+} = [Ar]$3d^6\ 4s^0$

$= t_{2g}{}^4\ e_g^2$

It has 4 unpaired e^-, its μ = 4.9 B.M.

18. Match the compounds in LIST-I with the observations in LIST-II, and choose the correct option.

LIST-I	LIST-II
(I) Aniline	(P) Sodium fusion extract of the compound on boiling with $FeSO_4$, followed by acidification with conc. H_2SO_4, gives Prussian blue color.
(II) o-Cresol	(Q) Sodium fusion extract of the compound on treatment with sodium nitroprusside gives blood red color.
(III) Cysteine	(R) Addition of the compound to a saturated solution of $NaHCO_3$ results in effervescence.
(IV) Caprolactam	(S) The compound reacts with bromine water to give a white precipitate.
	(T) Treating the compound with neutral $FeCl_3$ solution produces violet color.

(A) I → P, Q; II → S; III → Q, R; IV → P

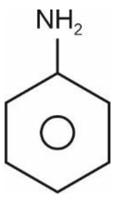
(B) I → P; II → R, S; III → R; IV → Q, S

(C) I → Q, S; II → P, T; III → P; IV → S

(D) I → P, S; II → T; III → Q, R; IV → P

Answer (D)

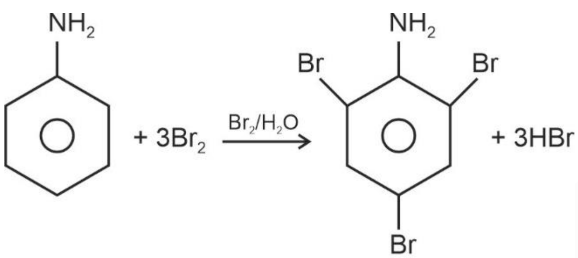
Sol. (I)



Since it contains both carbon and nitrogen so its sodium fusion extract with boiling $FeSO_4$, followed by acidification with conc. H_2SO_4 gives Prussian blue colour.

$Fe^{2+} + 6CN^- \rightarrow [Fe(CN)_6]^{4-}$

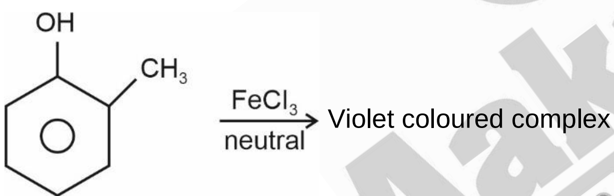
$$3[Fe(CN_6)]^{4-} + 4Fe^{3+} \xrightarrow{xH_2O} \underset{\text{Prussian blue}}{Fe_4[Fe(CN)_6]_3 \cdot xH_2O}$$



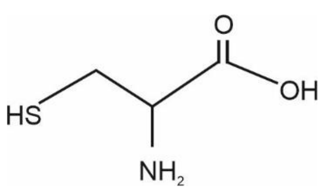
white ppt

I - (P, S)

(II) o-Cresol

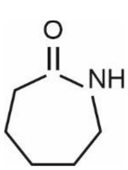


(III) Cysteine



Since it has both, sulphur and nitrogen, so its sodium fusion extract will give blood red colour with Fe^{3+} and it has carboxylic group so it will give effervescence with $NaHCO_3$.

(IV) Caprolactam



Its sodium fusion extract will give Prussian blue colour on boiling with $FeSO_4$ followed by acidification with conc. H_2SO_4.

PART-III : MATHEMATICS

SECTION – 1 (Maximum marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the correct numerical value is entered;

Zero Marks : 0 In all other cases.

1. Considering only the principal values of the inverse trigonometric functions, the value of

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$$

is ______.

Answer ($\simeq 2.36$)

Sol. $\frac{3}{2}\tan^{-1}\frac{\pi}{\sqrt{2}}+\frac{1}{4}\tan^{-1}\left(\frac{2\sqrt{2}\pi}{\pi^2-2}\right)+\tan^{-1}\frac{\sqrt{2}}{\pi}$

$$=\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\frac{\pi}{\sqrt{2}}-\frac{1}{4}\tan^{-1}\left(\frac{2\sqrt{2}\pi}{2-\pi^2}\right)$$

$$=\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)-\frac{1}{4}\tan^{-1}\left(\frac{2\cdot\left(\frac{\pi}{\sqrt{2}}\right)}{1-\left(\frac{\pi}{\sqrt{2}}\right)^2}\right)$$

$$=\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)-\frac{1}{4}\left(-\pi+2\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)\right)$$

$$=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$$

$$\simeq 2.36$$

2. Let α be a positive real number. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:(\alpha,\infty)\to\mathbb{R}$ be the functions defined by

$$f(x)=\sin\left(\frac{\pi x}{12}\right) \text{ and } g(x)=\frac{2\log_e(\sqrt{x}-\sqrt{\alpha})}{\log_e(e^{\sqrt{x}}-e^{\sqrt{\alpha}})}.$$

Then the value of $\lim\limits_{x\to\alpha^+} f(g(x))$ is ______.

Answer (00.50)

Sol. $$\lim_{x\to\alpha^+} g(x)=\lim_{x\to\alpha^+}\frac{\dfrac{2}{\sqrt{x}-\sqrt{\alpha}}\left(\dfrac{1}{2\sqrt{x}}\right)}{\dfrac{1}{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}\left(\dfrac{1}{2\sqrt{x}}\cdot e^{\sqrt{x}}\right)}$$

$$=\lim_{x\to\alpha^+}\frac{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}{\sqrt{x}-\sqrt{\alpha}}\cdot\frac{1}{e^{\sqrt{x}}}\cdot 2$$

$$=\lim_{x\to\alpha^+}\frac{e^{\sqrt{x}}\cdot\dfrac{1}{2\sqrt{x}}}{\dfrac{1}{2\sqrt{x}}}\cdot\frac{2}{e^{\sqrt{x}}}=2$$

$$\lim_{x\to\alpha^+} f(g(x))=f\left(\lim_{x\to\alpha^+} g(x)\right)=\sin\frac{\pi}{6}=\frac{1}{2}=00.50$$

3. In a study about a pandemic, data of 900 persons was collected. It was found that

190 persons had symptom of fever,

220 persons had symptom of cough,

220 persons had symptom of breathing problem,

330 persons had symptom of fever or cough or both,

350 persons had symptom of cough or breathing problem or both,

340 persons had symptom of fever or breathing problem or both,

30 persons had all three symptoms (fever, cough and breathing problem).

If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is ____________.

Answer (0.8)

Sol. We denote the set of people having symptoms of fever, cough and breathing problem by F, C and B respectively.

Given that n(F) = 190, n(B) = 220 and n(C) = 220

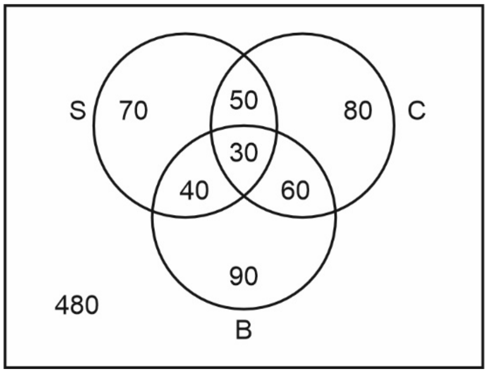
Also, $n(F\cup C)$ = 330, $n(C\cup B)$ = 350, $n(F\cup B)$ = 340 and $n(F\cap C\cap B)$ = 30

So $n(F\cap C)=n(F)+n(C)-n(F\cup C)$

= 80

Similarly, $n(F\cap B)=70$ and $n(C\cap B)=90$

So refer to Venn diagram



Number of people having at most one symptom

= 70 + 80 + 90 + 480 = 720

Required probability $= \dfrac{720}{900} = 0.8$.

4. Let z be a complex number with non-zero imaginary part. If

$$\frac{2+3z+4z^2}{2-3z+4z^2}$$

is a real number, then the value of $|z|^2$ is ________.

Answer (0.50)

Sol. Let $w = \dfrac{4z^2+3z+2}{4z^2-3z+2} = 1 + \dfrac{6z}{4z^2-3z+2}$

$$\Rightarrow \quad w = 1 + \frac{6}{2\left(2z+\frac{1}{z}\right)-3}$$

$$\because \quad w \in R \text{ then } 2z + \frac{1}{z} \in R$$

$$\Rightarrow \quad 2z + \frac{1}{z} = 2\bar{z} + \frac{1}{\bar{z}}$$

$$\Rightarrow \quad 2(z-\bar{z}) - \frac{z-\bar{z}}{|z|^2} = 0$$

$$\Rightarrow \quad (z-\bar{z})\left(2 - \frac{1}{|z|^2}\right) = 0$$

$$\because \quad z \neq \bar{z} \text{ (given)}$$

So $|z|^2 = \dfrac{1}{2}$

5. Let $\bar{z}$ denote the complex conjugate of a complex number z and let $i = \sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation

$$\bar{z} - z^2 = i(\bar{z} + z^2)$$

is ________.

Answer (4)

Sol. Given $\bar{z}(1-i) = z^2(1+i)$ ...(i)

So $|\bar{z}||1-i| = |z|^2|1+i|$

$\Rightarrow \quad |z| = |z|^2 \;\Rightarrow$ |z| = 0 or |z| = 1

Let arg $(z) = \theta$

So from (i) we get

$$2n\pi - \theta - \frac{\pi}{4} = 2\theta + \frac{\pi}{4}$$

$$\Rightarrow \quad \theta = \frac{1}{3}\left(\frac{4n-1}{2}\right)\pi = \frac{(4n-1)\pi}{6}$$

So we will get 3 distinct values of θ. Hence there will be total 4 possible values of complex number z.

6. Let $l_1, l_2, \ldots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1, w_2, \ldots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1d_2 = 10$. For each $i = 1, 2, \ldots, 100$, let R_i be a rectangle with length l_i, width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is ____________.

Answer (18900)

Sol. For A.P. $l_1, l_2, \ldots l_{100}$

Let $T_1 = a$ and common difference = d_1 and similarly for A.P. $w_1, w_2, \ldots w_{100}$

$T_1 = b$ and common difference = d_2

$A_{51} - A_{50} = l_{51}w_{51} - l_{50}w_{50}$

$= (a + 50d_1)(b + 50d_2) - (a + 49d_1)(b + 49d_2)$

$= 50bd_1 + 50ad_2 + 2500d_1d_2 - 49ad_2 - 49bd_1 - 2401d_1d_2$

$= bd_1 + ad_2 + 99d_1d_2 = 1000$

$\therefore \quad bd_1 + ad_2 = 10$ ...(i) (As $d_1d_2 = 10$)

$\therefore \quad A_{100} - A_{90} = l_{100}w_{100} - l_{90}w_{90}$

$= (a + 99d_1)(b + 99d_2) - (a + 89d_1)(b + 89d_2)$

$= 99bd_1 + 99ad_2 + 99^2d_1d_2 - 89bd_1 - 89ad_2 - 89^2d_1d_2$

$= 10(bd_1 + ad_2) + 1880d_1d_2$

$= 10(10) + 18800$

$= 18900$

7. The number of 4-digit integers in the closed interval [2022, 4482] formed by using the digits 0, 2, 3, 4, 6, 7 is ______.

Answer (569)

Sol. Counting integers starting from 2

Case-I: if zero on 2nd place

i.e., $2\;0\;\underline{2}\;\underset{5}{\uparrow} \to 5$ cases

or $2\;0\;\underset{4}{\uparrow}\;\underset{6}{\uparrow} \to 24$ cases

(Numbers except 0 or 2 in 3rd place)

Case-II: If non-zero number on 2nd place

i.e., $2\;\underset{5}{\uparrow}\;\underset{6}{\uparrow}\;\underset{6}{\uparrow} = 180$ cases

Counting integers starting from 3

$\underline{3}\,\underset{6}{\uparrow}\,\underset{6}{\uparrow}\,\underset{6}{\uparrow} = 216$ cases

Counting integers starting from 4

Case-I: If 0, 2 or 3 on 2[nd] place

i.e., $4\,\underset{3}{\uparrow}\,\underset{6}{\uparrow}\,\underset{6}{\uparrow} = 108$ cases

Case II: If 4 on 2[nd] place

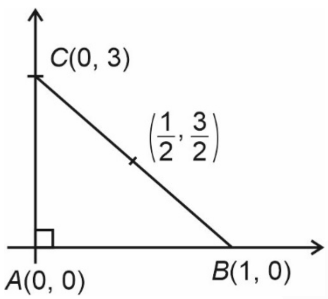
i.e., $4\;4\,\underset{6}{\uparrow}\,\underset{6}{\uparrow} = 36$ cases

$\therefore$ Total 5 + 24 + 180 + 216 + 108 + 36 = 569 numbers

8. Let ABC be the triangle with $AB = 1$, $AC = 3$ and $\angle BAC = \frac{\pi}{2}$. If a circle of radius $r > 0$ touches the sides AB, AC and also touches internally the circumcircle of the triangle ABC, then the value of r is _____________.

Answer (0.84)

Sol. Let A be the origin B on x-axis, C on y-axis as shown below



$\therefore$ Equation of circumcircle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\left(\frac{1}{2}\right)^2+\left(\frac{3}{2}\right)^2=\frac{5}{2} \quad \ldots(1)$$

Required circle touches AB and AC, have radius r

$\therefore$ Equation be $(x-r)^2+(y-r)^2=r^2$ $\quad\ldots(2)$

If circle in equation (2) touches circumcircle internally, we have

$$d_{c_1c_2}=|r_1-r_2|$$

$$\Rightarrow \left(\frac{1}{2}-r\right)^2+\left(\frac{3}{2}-r\right)^2=\left(\left|\sqrt{\frac{5}{2}}-r\right|\right)^2$$

$$\Rightarrow \frac{1}{4}+r^2-r+\frac{9}{4}+r^2-3r=\left(\sqrt{\frac{5}{2}}-r\right)^2 \text{ or } \left(r-\sqrt{\frac{5}{2}}\right)^2$$

$$\Rightarrow 2r^2-4r+\frac{5}{2}=\frac{5}{2}+r^2-\sqrt{10}\,r$$

$$\Rightarrow r=0 \text{ or } 4-\sqrt{10}$$

$$\Rightarrow r=0.837$$

= 0.84 (on rounding off)

SECTION – 2 (Maximum marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

9. Consider the equation

$$\int_1^e \frac{(\log_e x)^{1/2}}{x(a-(\log_e x)^{3/2})^2}\,dx = 1,\ a \in (-\infty, 0) \cup (1, \infty).$$

Which of the following statements is/are TRUE?

(A) No a satisfies the above equation

(B) An integer a satisfies the above equation

(C) An irrational number a satisfies the above equation

(D) More than one a satisfy the above equation

Answer (C, D)

Sol. Let $I = \int_1^e \frac{(\ln x)^{1/2}dx}{x(a-(\ln x)^{3/2})^2}$

Put $a - (\ln x)^{3/2} = t$

$$\Rightarrow\ -\frac{3}{2}(\ln x)^{1/2}\cdot\frac{1}{x}dx = dt$$

$$\therefore\ I = \int_a^{a-1}\frac{\left(-\frac{2}{3}\right)dt}{t^2}$$

$$= \left(-\frac{2}{3}\right)\frac{t^{-2+1}}{-2+1}\bigg|_a^{a-1}$$

$$= \frac{2}{3t}\bigg|_a^{a-1} = \frac{2}{3}\left(\frac{1}{a-1}-\frac{1}{a}\right)$$

$$\therefore\ I = \left(\frac{2}{3}\right)\frac{1}{a(a-1)} = 1$$

$\Rightarrow$ $2 = 3a^2 - 3a$

$\Rightarrow$ $3a^2 - 3a - 2 = 0$

$$\Rightarrow\ a = \frac{3 \pm \sqrt{9-4(3)(-2)}}{6}$$

$$a = \frac{3+\sqrt{33}}{6}, \frac{3-\sqrt{33}}{6}$$

10. Let a_1, a_2, a_3,… be an arithmetic progression with a_1 = 7 and common difference 8. Let T_1, T_2, T_3, … be such that T_1 = 3 and $T_{n+1} - T_n = a_n$ fo $n \geq 1$. Then, which of the following is/are TRUE ?

(A) $T_{20} = 1604$

(B) $\sum_{k=1}^{20} T_k = 10510$

(C) $T_{30} = 3454$

(D) $\sum_{k=1}^{30} T_k = 35610$

Answer (B, C)

Sol. Here $a_n = 7 + (n - 1)\,8$ and $T_1 = 3$

Also $T_{n+1} = T_n + a_n$

$T_n = T_{n-1} + a_{n-1}$

$\vdots$

$T_2 = T_1 + a_1$

$\therefore \quad T_{n+1} = (T_{n-1} + a_{n-1}) + a_n$

$= T_{n-2} + a_{n-2} + a_{n-1} + a_n$

$\vdots$

$\Rightarrow \quad T_{n+1} = T_1 + a_1 + a_2 + \ldots. + a_n$

$$\Rightarrow \quad T_{n+1} = T_1 + \frac{n}{2}\left[2(7) + (n-1)8\right]$$

$$\Rightarrow \quad T_{n+1} = T_1 + n(4n+3) \qquad \ldots (1)$$

$\therefore$ For $n = 19$ $\quad T_{20} = 3 + (19)(79) = 1504$

For $n = 29$ $\quad T_{30} = 3 + (29)(119) = 3454 \rightarrow$ (C)

$$\sum_{k=1}^{20} T_k = 3 + \sum_{k=2}^{20} T_k = 3 + \sum_{k=1}^{19}\left(3 + 4n^2 + 3n\right)$$

$$= 3 + 3(19) + \frac{3(19)(20)}{2} + \frac{4(19)(20)(39)}{6}$$

$= 3 + 10507 = 10510 \rightarrow$ (B)

And Similarly $\sum_{k=1}^{30} T_k = 3 + \sum_{k=1}^{29}\left(4n^2 + 3n + 3\right) = 35615$

11. Let P_1 and P_2 be two planes given by

P_1: $10x + 15y + 12z - 60 = 0$,

P_2: $-2x + 5y + 4z - 20 = 0$.

Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2?

(A) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$

(B) $\frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3}$

(C) $\frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4}$

(D) $\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3}$

Answer (A, B, D)

Sol. Equation of pair of planes is

S: $(10x + 15y + 12z - 60)(-2x + 5y + 4z - 20) = 0$

We will find a general point of each line and we will solve it with S. If we get more than one value of variable λ, then the line can be the edge of given tetrahedron.

(A) Point is $(1, 1, 5\lambda + 1)$

So, $(60\lambda - 23)(20\lambda - 17) = 0$

$\lambda = \frac{23}{60}$ and $\frac{17}{20}$

So, it can be the edge of tetrahedron.

(B) Point is $(-5\lambda + 6, 2\lambda, 3\lambda)$

So, $(16\lambda)(32\lambda - 32) = 0$

$\Rightarrow \lambda = 0$ and 1

So, it can be the edge of tetrahedron.

(C) Point is $(-2\lambda, 5\lambda + 4, 4\lambda)$

So, $(103\lambda)(45\lambda) = 0$

$\lambda = 0$ only

So, it cannot be the edge of tetrahedron.

(D) Point is $(\lambda, -2\lambda + 4, 3\lambda)$

$\Rightarrow (16\lambda)(0) = 0$

It is an identity in λ, so infinitely many solutions.

Hence, it can be the edge of tetrahedron.

12. Let S be the reflection of a point Q with respect to the plane given by

$\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$

where t, p are real parameters and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i} + 15\hat{j} + 20\hat{k}$ and $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ respectively, then which of the following is/are TRUE?

(A) $3(\alpha + \beta) = -101$

(B) $3(\beta + \gamma) = -71$

(C) $3(\gamma + \alpha) = -86$

(D) $3(\alpha + \beta + \gamma) = -121$

Answer (A, B, C)

Sol. Equation of plane is

$\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$

$\vec{r} = \hat{k} + t(-\hat{i} + \hat{j}) + p(-\hat{i} + \hat{k})$

Equation of plane in standard form is

$\left[\vec{r} - \hat{k} \quad -\hat{i} + \hat{j} \quad -\hat{i} + \hat{k}\right] = 0$

$\therefore$ $x + y + z = 1$ ...(1)

Coordinate of $Q = (10, 15, 20)$

Coordinate of $S = (\alpha, \beta, \gamma)$

$$\therefore \quad \frac{\alpha - 10}{1} = \frac{\beta - 15}{1} = \frac{\gamma - 20}{1} = \frac{-2(10 + 15 + 20 - 1)}{3}$$

$$\therefore \quad \alpha - 10 = \beta - 15 = \gamma - 20 = -\frac{88}{3}$$

$$\therefore \quad \alpha = -\frac{58}{3},\ \beta = -\frac{43}{3},\ \gamma = -\frac{28}{3}$$

$\therefore$ $3(\alpha + \beta) = -101$, $3(\beta + \gamma) = -71$

$3(\gamma + \alpha) = -86$ and $3(\alpha + \beta + \gamma) = -129$

$\therefore$ Ans. A, B, C

13. Consider the parabola $y^2 = 4x$. Let S be the focus of the parabola. A pair of tangents drawn to the parabola from the point $P = (-2, 1)$ meet the parabola at P_1 and P_2. Let Q_1 and Q_2 be points on the lines SP_1 and SP_2 respectively such that PQ_1 is perpendicular to SP_1 and PQ_2 is perpendicular to SP_2. Then, which of the following is/are TRUE ?

(A) $SQ_1 = 2$

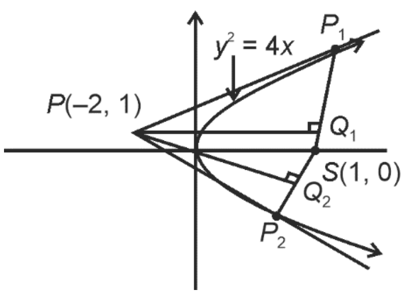
(B) $Q_1Q_2 = \dfrac{3\sqrt{10}}{5}$

(C) $PQ_1 = 3$

(D) $SQ_2 = 1$

Answer (B, C, D)

Sol.



Let $P_1(t^2, 2t)$ then tangent at P_1

$ty = x + t^2$

Since it passes through $(-2, 1)$

$\therefore$ $t^2 - t - 2 = 0$

$\therefore$ $t = 2, -1$

$\therefore$ $P_1(4, 4)$ and $P_2(1, -2)$

$\therefore$ $SP_1 : 4x - 3y - 4 = 0$

and $SP_2 : x - 1 = 0$

and for Q_1 : $\dfrac{x_1+2}{4} = \dfrac{y_1-1}{-3} = \dfrac{-(-8-3-4)}{25} = \dfrac{3}{5}$

$\therefore$ $x_1 = \dfrac{2}{5}, y_1 = \dfrac{-4}{5}$

and $Q_2 = (1, 1)$

So, $SQ_1 = \sqrt{\left(1-\dfrac{2}{5}\right)^2 + \left(\dfrac{4}{5}\right)^2} = 1$

$Q_1Q_2 = \sqrt{\dfrac{9}{25} + \dfrac{81}{25}} = \sqrt{\dfrac{90}{25}} = \dfrac{3\sqrt{10}}{5}$

$PQ_1 = \sqrt{\dfrac{144}{25} + \dfrac{81}{25}} = 3$

$SQ_2 = 1$

14. Let $|M|$ denote the determinant of a square matrix M. Let $g:\left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be the function defined by

$g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f\left(\frac{\pi}{2} - \theta\right) - 1}$ where

$$f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi \end{vmatrix}.$$

Let p(x) be a quadratic polynomial whose roots are the maximum and minimum values of the function $g(\theta)$, and $p(2) = 2 - \sqrt{2}$. Then, which of the following is/are TRUE ?

(A) $p\left(\frac{3+\sqrt{2}}{4}\right) < 0$

(B) $p\left(\frac{1+3\sqrt{2}}{4}\right) > 0$

(C) $p\left(\frac{5\sqrt{2}-1}{4}\right) > 0$

(D) $p\left(\frac{5-\sqrt{2}}{4}\right) < 0$

Answer (A and C)

Sol. $\because$ $$f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi \end{vmatrix}.$$

Here $\cos\left(\theta + \frac{\pi}{4}\right) = -\sin\left(\theta - \frac{\pi}{4}\right)$

and $\tan\left(\theta - \frac{\pi}{4}\right) = -\cot\left(\theta + \frac{\pi}{4}\right)$

and $\log_e\left(\frac{4}{\pi}\right) = -\log_e\left(\frac{\pi}{4}\right)$

Also $\sin\pi = -\cos\frac{\pi}{2} = \tan\pi = 0$

$\therefore$ $f(\theta) = 1 + \sin^2\theta$

$\therefore$ $g(\theta) = |\sin\theta| + |\cos\theta|$

$\therefore$ maximum and minimum values are $\sqrt{2}$ and 1 respectively.

$\therefore$ $P(x) = a\left(x - \sqrt{2}\right)(x - 1)$, where $a \in R - \{0\}$,

But $P(2) = 2 - \sqrt{2}$ then $a = 1$.

$\therefore$ $P(x) = \left(x - \sqrt{2}\right)(x - 1)$

$\therefore$ $P\left(\frac{3+\sqrt{2}}{4}\right) = \left(\frac{3-3\sqrt{2}}{4}\right)\cdot\left(\frac{\sqrt{2}-1}{4}\right) < 0$

$P\left(\frac{1+3\sqrt{2}}{4}\right) = \left(\frac{1-\sqrt{2}}{4}\right)\cdot\left(\frac{3\sqrt{2}-3}{4}\right) < 0$

$$P\left(\frac{5\sqrt{2}-1}{4}\right)=\left(\frac{\sqrt{2}-1}{4}\right)\cdot\left(\frac{5\sqrt{2}-5}{4}\right)>0$$

$$P\left(\frac{5-\sqrt{2}}{4}\right)=\left(\frac{5-5\sqrt{2}}{4}\right)\left(\frac{1-\sqrt{2}}{4}\right)>0$$

$\therefore$ (A) and (C) are correct.

SECTION – 3 (Maximum marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- FOUR options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	**ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

15. Consider the following lists:

List-I	**List-II**
(I) $\left\{x\in\left[-\frac{2\pi}{3},\frac{2\pi}{3}\right]:\cos x+\sin x=1\right\}$	(P) has two elements
(II) $\left\{x\in\left[-\frac{5\pi}{18},\frac{5\pi}{18}\right]:\sqrt{3}\tan 3x=1\right\}$	(Q) has three elements
(III) $\left\{x\in\left[-\frac{6\pi}{5},\frac{6\pi}{5}\right]:2\cos(2x)=\sqrt{3}\right\}$	(R) has four elements
(IV) $\left\{x\in\left[-\frac{7\pi}{4},\frac{7\pi}{4}\right]:\sin x-\cos x=1\right\}$	(S) has five elements
	(T) has six elements

The correct option is:

(A) (I) $\rightarrow$ (P); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (S)

(B) (I) $\rightarrow$ (P); (II) $\rightarrow$ (P); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (R)

(C) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (P); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (S)

(D) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (R)

Answer (B)

Sol. (i) $\left\{x\in\left[\frac{-2\pi}{3},\frac{2\pi}{3}\right],\cos x+\sin x=1\right\}$

$\cos x + \sin x = 1$

$$\sin\left(\frac{\pi}{4}+x\right)=\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{4}+x=n\pi+(-1)^n\frac{\pi}{4}$$

$$x=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$$

$\therefore$ x has 2 elements. $\rightarrow P$

(ii) $\left\{x \in \left[\frac{-5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$

$\sqrt{3}\tan 3x = 1$

$\tan 3x = \frac{1}{\sqrt{3}}$

$3x = n\pi + \frac{\pi}{6}$

$x = \frac{n\pi}{3} + \frac{\pi}{18}$

$\therefore$ x has 2 elements. $\rightarrow P$

(iii) $\left\{x \in \left[\frac{-6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos 2x = \sqrt{3}\right\}$

$2\cos 2x = \sqrt{3}$

$\cos 2x = \frac{\sqrt{3}}{2}$

$2x = 2n\pi \pm \frac{\pi}{6}$

$x = n\pi \pm \frac{\pi}{12}$

$\therefore$ x has 6 elements. $\rightarrow T$

(iv) $\left\{x \in \left[\frac{-7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$

$\sin x - \cos x = 1$

$\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$x - \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

$x = n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{4}$

$\therefore$ x has 4 elements. $\rightarrow R$

$\therefore$ option B is correct.

16. Two players, P_1 and P_2, play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by P_1 and P_2, respectively. If $x > y$, then P_1 scores 5 points and P_2 scores 0 point. If $x = y$, then each player scores 2 points. If $x < y$, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of P_1 and P_2, respectively, after playing the i^{th} round.

	List-I		List-II
(I)	Probability of $(X_2 \geq Y_2)$ is	(P)	$\frac{3}{8}$
(II)	Probability of $(X_2 > Y_2)$ is	(Q)	$\frac{11}{16}$
(III)	Probability of $(X_3 = Y_3)$ is	(R)	$\frac{5}{16}$
(IV)	Probability of $(X_3 > Y_3)$ is	(S)	$\frac{355}{864}$
		(T)	$\frac{77}{432}$

The correct option is:

(A) (I) → (Q); (II) → (R); (III) → (T); (IV) → (S)

(B) (I) → (Q); (II) → (R); (III) → (T); (IV) → (T)

(C) (I) → (P); (II) → (R); (III) → (Q); (IV) → (S)

(D) (I) → (P); (II) → (R); (III) → (Q); (IV) → (T)

Answer (A)

Sol. $P(X_i > Y_i) + P(X_i < Y_i) + P(X_i = Y_i) = 1$

and $P(X_i > Y_i) = P(X_i < Y_i) = p$

for $i = 2$

$P(X_2 = Y_2) = 2p(x > y).\ p(x < y) + (p(x = y))^2$

$$= 2.\frac{{}^6C_2}{36}.\frac{{}^6C_2}{36} + \left(\frac{{}^6C_1}{36}\right)^2$$

$$= \frac{25}{72} + \frac{1}{36} = \frac{27}{72} = \frac{3}{8}$$

$$P(X_2 > Y_2) = \frac{1}{2}\left(1 - \frac{3}{8}\right) = \frac{5}{16}$$

$$P(X_2 \geq Y_2) = \frac{5}{16} + \frac{3}{8} = \frac{11}{16}$$

I → Q, II → R

for $i = 3$

$P(X_3 = Y_3) = 6.\ p(x > y).\ p(x < y)\ p(x = y) + (p(x = y))^3$

$$= 6.\frac{{}^6C_2}{36}.\frac{{}^6C_2}{36}.\frac{{}^6C_1}{36} + \left(\frac{{}^6C_1}{36}\right)^3$$

$$= \frac{77}{432}$$

$$P(X_3 > Y_3) = \frac{1}{2}\left(1 - \frac{77}{432}\right)$$

$$= \frac{355}{864}$$

III → T, IV → S

17. Let p, q, r be non-zero real numbers that are, respectively, the 10^{th}, 100^{th} and 1000^{th} terms of a harmonic progression. Consider the system of linear equations

$x + y + z = 1$

$10x + 100y + 1000z = 0$

$qr\,x + pr\,y + pq\,z = 0$

	List-I		**List-II**
(I)	If $\frac{q}{r} = 10$, then the system of linear equations has	(P)	$x = 0,\ y = \frac{10}{9},\ z = -\frac{1}{9}$ as a solution
(II)	If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q)	$x = \frac{10}{9},\ y = -\frac{1}{9},\ z = 0$ as a solution
(III)	If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R)	infinitely many solutions
(IV)	If $\frac{p}{q} = 10$, then the system of linear equations has	(S)	no solution
		(T)	at least one solution

The **correct** option is:

(A) (I) $\rightarrow$ (T); (II) $\rightarrow$ (R); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (T)

(B) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (R)

(C) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (R)

(D) (I) $\rightarrow$ (T); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (T)

Answer (B)

Sol. $x + y + z = 1$ ____(1)

$10x + 100y + 1000z = 0$ ____(2)

$qrx + pry + pqz = 0$ ____(3)

Equation (3) can be re-written as

$$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 0 \qquad (\because p, q, r \neq 0)$$

Let $p = \frac{1}{a+9d},\ q = \frac{1}{a+99d},\ r = \frac{1}{a+999d}$

Now, equation (3) is

$(a + 9d)x + (a + 99d)y + (a + 999d)z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 100 & 1000 \\ a+9d & a+99d & a+999d \end{vmatrix} = 0$$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 100 & 1000 \\ 0 & a+99d & a+999d \end{vmatrix} = 900(d-a)$$

$$\Delta_y = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 0 & 1000 \\ a+9d & 0 & a+999d \end{vmatrix} = 990(a-d)$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 100 & 0 \\ a+9d & a+99d & 0 \end{vmatrix} = 90(d-a)$$

Option I: If $\frac{q}{r} = 10 \Rightarrow a = d$

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

And eq. (1) and eq. (2) represents non-parallel planes and eq. (2) and eq. (3) represents same plane

$\Rightarrow$ Infinitely many solutions

I $\rightarrow$ *P*, *Q*, *R*, *T*

Option II : $\frac{p}{r} \neq 100 \Rightarrow a \neq d$

$\Delta = 0$, Δ_x, Δ_y, $\Delta_z \neq 0$

No solution

II $\rightarrow$ S

Option III: $\frac{p}{q} \neq 10 \Rightarrow a \neq d$

No solution

III $\rightarrow$ S

Option IV: If $\frac{p}{q} = 10 \Rightarrow a = d$

Infinitely many solution

IV $\rightarrow$ *P*, *Q*, *R*, *T*

18. Consider the ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Let H(α, 0), 0< α < 2, be a point. A straight line drawn through *H* parallel to the *y*-axis crosses the ellipse and its auxiliary circle at points *E* and *F* respectively, in the first quadrant. The tangent to the ellipse at the point *E* intersects the positive *x*-axis at a point *G*. Suppose the straight line joining *F* and the origin makes an angle ϕ with the positive *x*-axis.

	List-I		**List-II**
(I)	If $\phi = \frac{\pi}{4}$, then the area of the triangle *FGH* is	(P)	$\frac{\left(\sqrt{3}-1\right)^4}{8}$
(II)	If $\phi = \frac{\pi}{3}$, then the area of the triangle *FGH* is	(Q)	1
(III)	If $\phi = \frac{\pi}{6}$, then the area of the triangle *FGH* is	(R)	$\frac{3}{4}$
(IV)	If $\phi = \frac{\pi}{12}$, then the area of the triangle *FGH* is	(S)	$\frac{1}{2\sqrt{3}}$
		(T)	$\frac{3\sqrt{3}}{2}$

The correct option is:

(A) (I) $\rightarrow$ (R); (II) $\rightarrow$ (S); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (P)

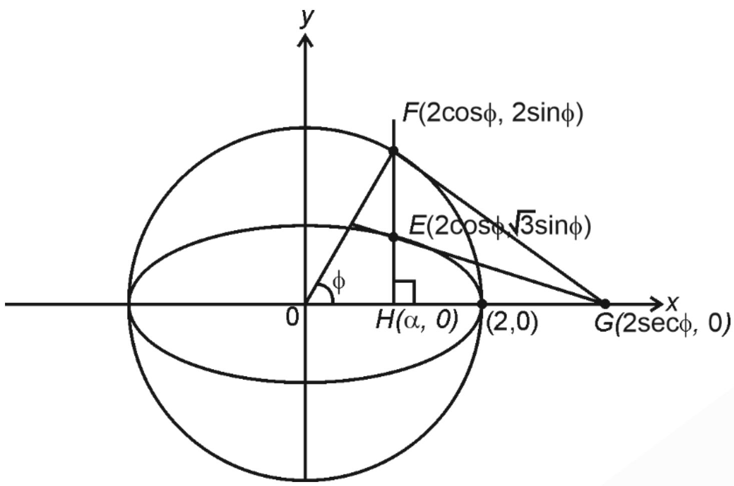
(B) (I) $\rightarrow$ (R); (II) $\rightarrow$ (T); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (P)

(C) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (T); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (P)

(D) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (P)

Answer (C)

Sol.



$\alpha \equiv 2\cos\phi$

Tangent at $E\left(2\cos\phi, \sqrt{3}\sin\phi\right)$ to ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{3}=1$

i.e. $\dfrac{x\cos\phi}{2}+\dfrac{y\sin\phi}{\sqrt{3}}=1$ intersect x-axis at $G(2\sec\phi, 0)$

Area of triangle FGH $=\dfrac{1}{2}(2\sec\phi-2\cos\phi)2\sin\phi$

$\Delta = 2\sin^2\phi.\ \tan\phi$

$\Delta = (1-\cos 2\phi).\tan\phi$

I. If $\phi=\dfrac{\pi}{4}, \Delta=1\rightarrow(Q)$

II. If $\phi=\dfrac{\pi}{3}, \Delta=2.\left(\dfrac{\sqrt{3}}{2}\right)^2.\sqrt{3}=\dfrac{3\sqrt{3}}{2}\rightarrow(T)$

III. If $\phi=\dfrac{\pi}{6}, \Delta=2.\left(\dfrac{1}{2}\right)^2.\dfrac{1}{\sqrt{3}}=\dfrac{1}{2\sqrt{3}}\rightarrow(S)$

IV. If $\phi=\dfrac{\pi}{12}, \Delta=\left(1-\dfrac{\sqrt{3}}{2}\right).\left(2-\sqrt{3}\right)=\dfrac{\left(2-\sqrt{3}\right)^2}{2}\rightarrow(P)$

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