Date: 28/08/2022

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Time : 3 hrs. Max. Marks: 180

Answers & Solutions

for

JEE (Advanced)-2022 (Paper-2)

PART-I : PHYSICS

SECTION – 1 (Maximum marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct integer is entered;
Zero Marks	:	0	If the question is unanswered;
Negative Marks	:	−1	In all other cases.

1. A particle of mass 1 kg is subjected to a force which depends on the position as $\vec{F} = -k(x\hat{i} + y\hat{j})$ kg ms^{-2} with k = 1 kg s^{-2}. At time t = 0, the particle's position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right)$ m and its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k}\right)$ ms^{-1}. Let v_x and v_y denote the x and the y components of the particle's velocity, respectively. **Ignore gravity.** When z = 0.5 m, the value of $(xv_y - yv_x)$ is _______ m^{2}s^{-1}.

Answer (3)

Sol. $F_x = -x = ma_x$.

So $a_x = \frac{d^2x}{dt^2} = -x$

$\Rightarrow x = A_x \sin(\omega t + \phi_x)$ $(\omega = 1 \text{ rad/s})$

and $v_x = A_x \omega \cos(\omega t + \phi_x)$

at $t = 0$, $x = \frac{1}{\sqrt{2}}$ m and $v_x = -\sqrt{2}$ m/s

So $\frac{1}{\sqrt{2}} = A_x \sin\phi_x$

and $-\sqrt{2} = A_x \cos\phi_x$

$\Rightarrow \tan\phi_x = -\frac{1}{2}$ … (1)

and $A_x = \sqrt{\frac{5}{2}}$ m … (2)

Similarly

$F_y = -y = ma_y$.

$\Rightarrow \frac{d^2y}{dt^2} = -y$

So, $y = A_y \sin(\omega t + \phi_y)$ (ω = 1 rad/s)

and $v_y = A_y \omega \cos(\omega t + \phi_y)$

at $t = 0$ $y = \sqrt{2}$ m and $v_y = \sqrt{2}$ m/s

So $\sqrt{2} = A_y \sin\phi$

and $\sqrt{2} = A_y \cos\phi$

$\Rightarrow \phi = \frac{\pi}{4}$ and $A_y = 2$ (3 and 4).

So, $(xv_y - yv_x) = \sqrt{\frac{5}{2}} \sin(\omega t + \phi_x) \times 2\cos(\omega t + \phi_y) - 2\sin(\omega t + \phi_y) \times \sqrt{\frac{5}{2}} \cos(\omega t + \phi_x)$

$= \sqrt{\frac{5}{2}} \times 2\,(\sin(\omega t + \phi_x)\cos(\omega t + \phi_y) - \sin(\omega t + \phi_y) \times \cos(\omega t + \phi_x))$

$= \sqrt{10} \sin(\phi_x - \phi_y)$

$= \sqrt{10} (\sin\phi_x \cos\phi_y - \cos\phi_x \sin\phi_y)$

$= \sqrt{10} \left(\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{2}} - \left(-\frac{2}{\sqrt{5}}\right) \times \frac{1}{\sqrt{2}}\right)$

$= 3$

2. In a radioactive decay chain reaction, ${}^{230}_{90}\text{Th}$ nucleus decays into ${}^{214}_{84}\text{Po}$ nucleus. The ratio of the number of α to number of β^- particles emitted in this process is _______.

Answer (2)

Sol. Let number of α particles are n_α and β particles are n_β so

$4n_\alpha = 230 - 214$

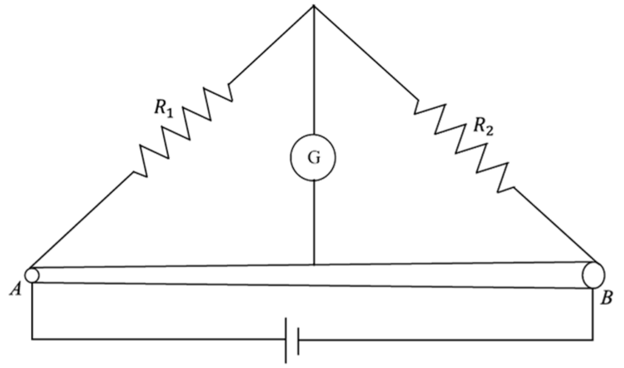
$\Rightarrow n_\alpha = 4$

$n_\beta = 84 - (90 - 2n_\alpha)$

$n_\beta = 2$

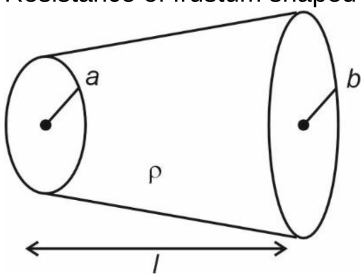
So $\frac{n_\alpha}{n_\beta} = 2$

3. Two resistances $R_1 = X\ \Omega$ and $R_2 = 1\ \Omega$ are connected to a wire *AB* of uniform resistivity, as shown in the figure. The radius of the wire varies linearly along its axis from 0.2 mm at *A* to 1 mm at *B*. A galvanometer (*G*) connected to the center of the wire, 50 cm from each end along its axis, shows zero deflection when *A* and *B* are connected to a battery. The value of *X* is _______.



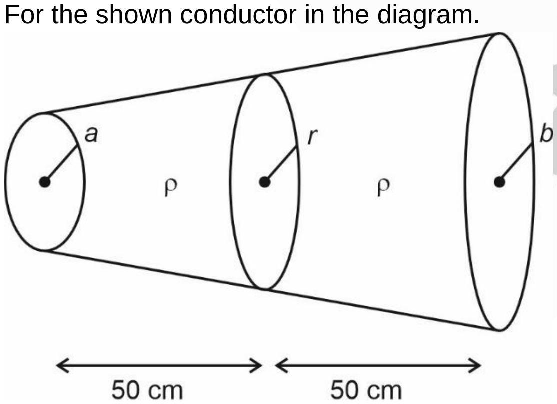
Answer (5)

Sol. Resistance of frustum shaped conductor shown is



$$R = \rho \frac{l}{\pi ab}$$

For the shown conductor in the diagram.



$$r = \frac{a+b}{2} = \frac{0.2+1}{2} = 0.6$$

thus, the resistance of left half is $P = \dfrac{\rho \times 0.5 \times 10^6}{\pi \times 0.2 \times 0.6}$

and the resistance of right half is $Q = \dfrac{\rho \times 0.5 \times 10^6}{\pi \times 0.6 \times 1}$

for Wheatstone to be balanced

$$\frac{R_1}{P} = \frac{R_2}{Q}$$

$$\frac{X\ \pi \times 0.2 \times 0.6}{\rho \times 0.5 \times 10^6} = \frac{1\,\pi \times 0.6 \times 1}{\rho \times 0.5 \times 10^6}$$

$\Rightarrow\ X = 5$

4. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and Coulomb's constant $k = \dfrac{1}{4\pi\varepsilon_0}$, where ε_0 is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^{\alpha}\,[m_e]^{\beta}\,[h]^{\gamma}\,[k]^{\delta}$. The value of $\alpha + \beta + \gamma + \delta$ is ______.

Answer (4)

Sol. $[B] = [e]^{\alpha}\,[m_e]^{\beta}\,[h^{\gamma}]\,[k]^{\delta}$

$[M^1T^{-2}I^{-1}] = [IT]^{\alpha}\,[M]^{\beta}\,[ML^2T^{-1}]^{\gamma}\,[ML^3T^{-4}]^{\delta}$

So, $\beta + \gamma + \delta = 1$ …(i)

$2\gamma + 3\delta = 0$ …(ii)

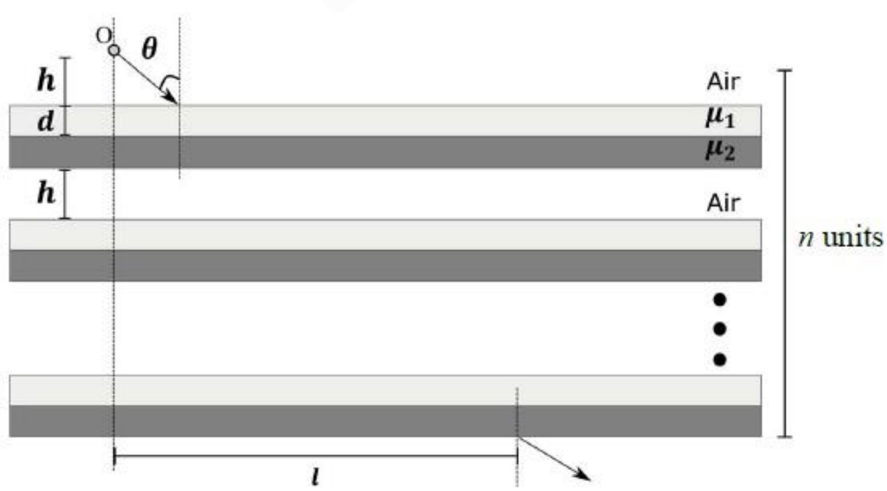
$\alpha - \gamma - 4\delta = -2$ …(iii)

$\alpha - 2\delta = -1$ …(iv)

On solving

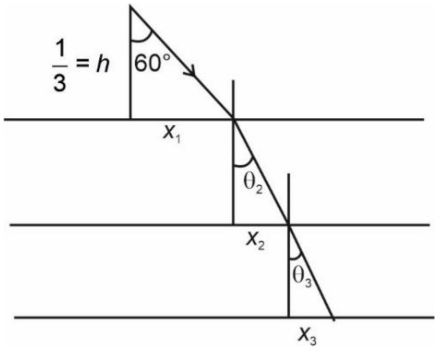
so, $\alpha + \beta + \gamma + \delta = 4$

5. Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height $h = \dfrac{1}{3}$ cm, and the second and third layers are of equal thickness $d = \dfrac{\sqrt{3}-1}{2}$ cm, and refractive indices $\mu_1 = \sqrt{\dfrac{3}{2}}$ and $\mu_2 = \sqrt{3}$, respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of $\theta = 60°$ to the normal. For a specific value of n, the ray of light emerges from the bottom of the configuration at a distance $l = \dfrac{8}{\sqrt{3}}$ cm, as shown in the figure. The value of n is _______.



Answer (4)

Sol.



$$x_1 = \frac{1}{3} \times \tan 60° = \frac{1}{\sqrt{3}} \text{ cm}$$

$$\text{and, } 1 \times \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{2}} \times \sin\theta_2$$

$$\Rightarrow \theta_2 = 45°$$

$$\Rightarrow x_2 = d$$

$$\text{and, } 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \times \sin\theta_2$$

$$\Rightarrow \theta_3 = 30°$$

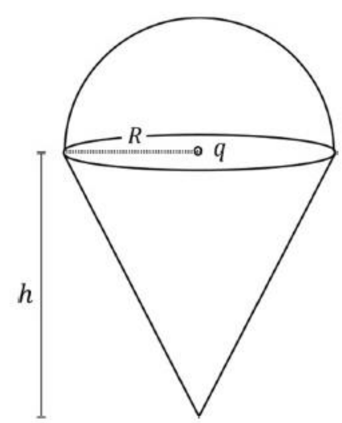
$$\Rightarrow x_3 = \frac{d}{\sqrt{3}}$$

$$\therefore \quad x_1 + x_2 + x_3 = \frac{1}{\sqrt{3}} + \frac{\left(\sqrt{3}-1\right)}{2}\left(1+\frac{1}{\sqrt{3}}\right)$$

$$= \frac{2}{\sqrt{3}} \text{ cm}$$

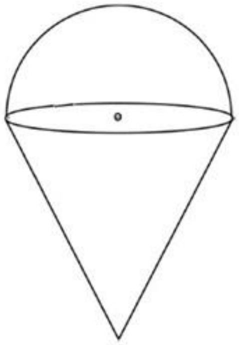
$$\therefore \quad n = \frac{l}{x_1 + x_2 + x_3} = \frac{8/\sqrt{3}}{2/\sqrt{3}} = 4$$

6. A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R, and a hemisphere of radius R as shown in the figure. The electric flux through the conical surface is $\frac{nq}{6\varepsilon_0}$ (in SI units). The value of n is _________.



Answer (3)

Sol.



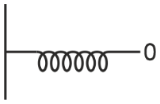
ϕ through cone $= \dfrac{q}{2\varepsilon_0}$

$\therefore \quad n = 3$

7. On a frictionless horizontal plane, a bob of mass $m = 0.1$ kg is attached to a spring with natural length $l_0 = 0.1$ m. The spring constant is $k_1 = 0.009$ Nm^{-1} when the length of the spring $l > l_0$ and is $k_2 = 0.016$ Nm^{-1} when $l < l_0$. Initially the bob is released from $l = 0.15$ m. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is $T = (n\,\pi)$ s, then the integer closest to n is _______.

Answer (6)

Sol.



$\omega_1 = \sqrt{\dfrac{k_1}{m}}$ and $\omega_2 = \sqrt{\dfrac{k_2}{m}}$

$\therefore \quad$ Time period $= \pi\sqrt{\dfrac{m}{k_1}} + \pi\sqrt{\dfrac{m}{k_2}}$

$= \pi\sqrt{\dfrac{0.1}{0.009}} + \pi\sqrt{\dfrac{0.1}{0.016}}$

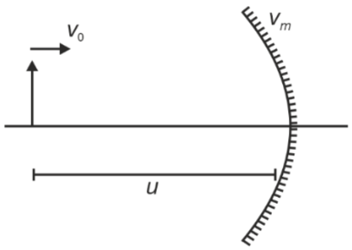
$= \dfrac{\pi}{0.3} + \dfrac{\pi}{0.4}$

$= \pi \times \left(\dfrac{4+3}{12}\right) \times 10$

$= \dfrac{70}{12}\pi$

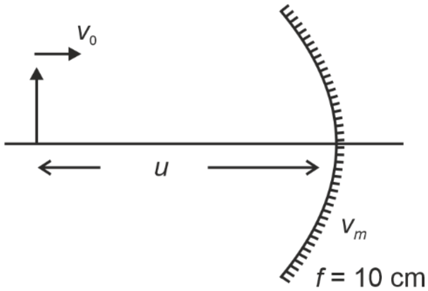
$= 5.83\pi$

8. An object and a concave mirror of focal length $f = 10$ cm both move along the principal axis of the mirror with constant speeds. The object moves with speed $V_0 = 15$ cm s^{-1} towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by u. When $u = 30$ cm, the speed of the mirror V_m is such that the image is instantaneously at rest with respect to the laboratory frame, and the object forms a real image. The magnitude of V_m is _____ cm s^{-1}.



Answer (3)

Sol.



$$m = \frac{f}{u - f}$$

$$= \frac{10}{30 - 10} = \frac{1}{2}$$

$$\therefore \quad (V_0 - V_m) \times m^2 - V_m = 0$$

$$\Rightarrow \quad (V_0 - V_m) \times \frac{1}{4} = V_m$$

$$\Rightarrow \quad V_0 = 5V_m$$

$$\Rightarrow \quad V_m = \frac{V_0}{5}$$

$$= \frac{15}{5}$$

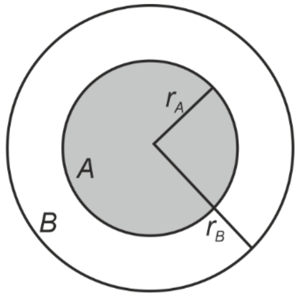
= 3 cm/s

SECTION – 2 (Maximum marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

9. In the figure, the inner (shaded) region A represents a sphere of radius $r_A = 1$, within which the electrostatic charge density varies with the radial distance r from the center as $\rho_A = kr$, where k is positive. In the spherical shell B of outer radius r_B, the electrostatic charge density varies as $\rho_B = \frac{2k}{r}$. Assume that dimensions are taken care of. All physical quantities are in their SI units.



Which of the following statement(s) is/(are) correct?

(A) If $r_B = \sqrt{\frac{3}{2}}$, then the electric field is zero everywhere outside B.

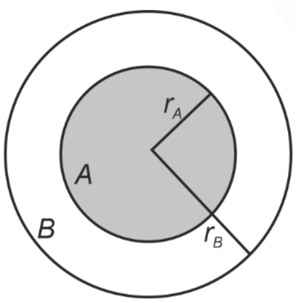
(B) If $r_B = \frac{3}{2}$, then the electric potential just outside B is $\frac{k}{\varepsilon_0}$.

(C) If $r_B = 2$, then the total charge of the configuration is $15\pi k$.

(D) If $r_B = \frac{5}{2}$, then the magnitude of the electric field just outside B is $\frac{13\pi k}{\varepsilon_0}$.

Answer (B)

Sol.



$$Q_{\text{Total}} = \int_0^{r_A} kr\left(4\pi r^2\right)dr + \int_{r_A}^{r_B} \frac{2k}{r}\left(4\pi r^2\right)dr$$

$$= \frac{4\pi k}{4} r_A^4 + \frac{8\pi k}{2}\left(r_B^2 - r_A^2\right)$$

$$= \pi k + 4\pi k\left(r_B^2 - r_A^2\right)$$

If $r_B = \sqrt{\frac{3}{2}}$

$$Q_{\text{Total}} = \pi k r_A^4 + 4\pi k\left(\frac{3}{2} - r_A^2\right)$$

$$= \pi k + 4\pi k\left(\frac{3}{2} - 1\right)$$

$$= \pi k + 2\pi k = 3\pi k$$

If $r_B = \dfrac{3}{2}$

$$Q_{\text{Total}} = \pi k + 4\pi k\left(\frac{9}{4} - 1\right)$$

$$= \pi k + 4\pi k\left(\frac{5}{4}\right) = 6\pi k$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{6\pi k}{r_B} = \frac{3k}{2}\frac{2}{3\varepsilon_0} = \frac{k}{\varepsilon_0}$$

(B) is correct

If $r_B = 2$

$$Q_{\text{Total}} = \pi k + 4\pi k(4-1)$$

$$= 13\pi k$$

Option (C) is incorrect

If $r_B = \dfrac{5}{2}$

$$Q_{\text{Total}} = \pi k + 4\pi k\left(\frac{25}{4} - 1\right)$$

$$= \pi k + \pi k(21)$$

$$= 22\pi k$$

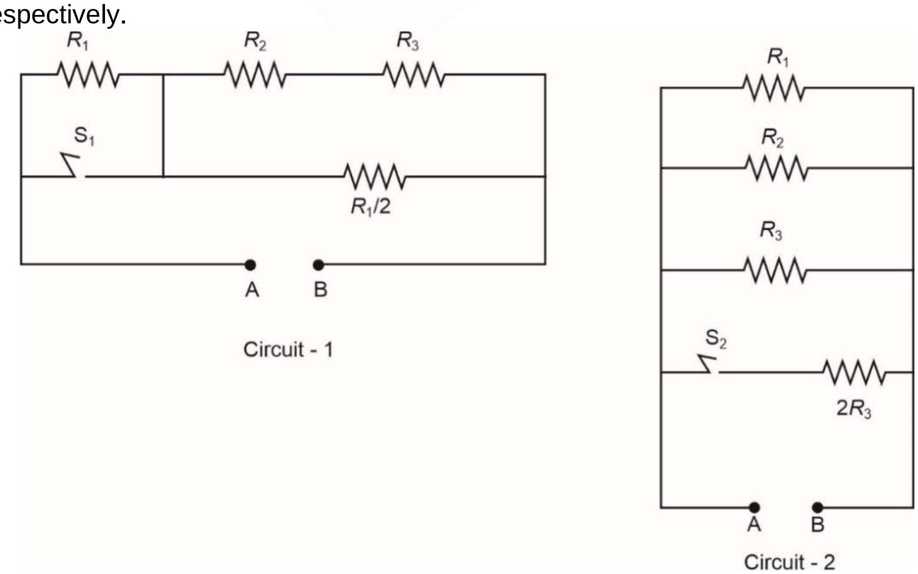
$$E = \frac{1}{4\pi\varepsilon_0}\frac{22\pi k}{25} \times 4$$

$$= \frac{22k}{25\varepsilon_0}$$

10. In Circuit-1 and Circuit-2 shown in the figures, $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 3\ \Omega$.

P_1 and P_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in open conditions, respectively.

Q_1 and Q_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in closed conditions, respectively.



Which of the following statement(s) is(are) correct?

(A) When a voltage source of 6 V is connected across A and B in both circuits, $P_1 < P_2$

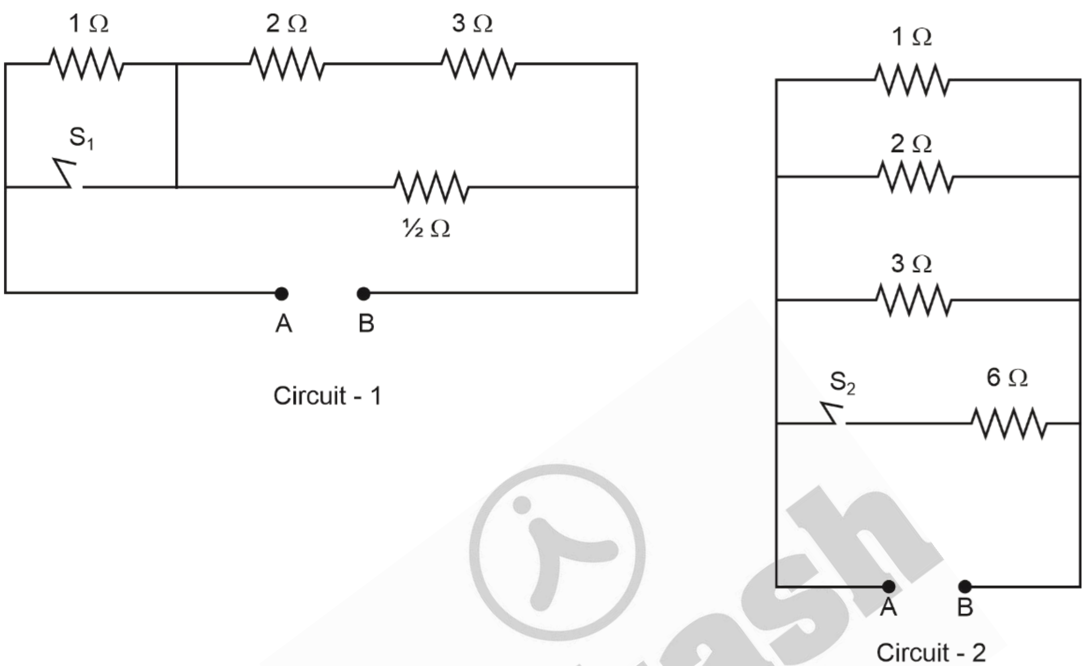
(B) When a constant current source of 2 Amp is connected across A and B in both circuits, $P_1 > P_2$

(C) When a voltage source 6 V is connected across A and B in Circuit-1, $Q_1 > P_1$

(D) When a constant current source of 2 Amp is connected across A and B in both circuits, $Q_2 < Q_1$

Answer (A, B, C)

Sol.



When S_1 and S_2 are open

$$(R_{eq})_1 = 1 + \frac{5\times\frac{1}{2}}{5+\frac{1}{2}} = 1 + \frac{5}{11} = \frac{16}{11}$$

$$P_1 = \frac{V^2}{R_{eq}} = \frac{(6)^2}{16}\times 11 = \frac{36\times 11}{16} = 24.75\text{ W}$$

$$(R_{eq})_2 = \frac{6}{11}\ \Omega$$

$$P_2 = \frac{V^2}{R_{eq}} = \frac{(6)^2}{6}\times 11 = \frac{36\times 11}{6} = 66\text{ W}$$

$P_2 > P_1$

Option (A) is correct.

$\Rightarrow$ If 2 A source is used in both the cases.

$$P_1 = i^2 (R_{eq})_1 = (2)^2 \times \frac{16}{11} = \frac{64}{11} = 5.818\text{ W}$$

$$P_2 = i^2 (R_{eq})_2 = (2)^2 \times \frac{6}{11} = \frac{24}{11} = 2.1818\text{ W}$$

$P_1 > P_2$

Option (B) is correct

For Q_1

$$R_{eq} = \frac{5}{11}\,\Omega$$

$$Q_1 = \frac{V^2}{R_{eq}} = \frac{(6)^2}{\frac{5}{11}} = \frac{36\times 11}{5} = 79.2\text{ W}$$

$P_1 = 24.75$ W

$Q_1 > P_1$

Option (C) is correct.

For option (D)

$$Q_1 = i^2R_{eq} = (2)^2\times\frac{5}{11} = \frac{20}{11} = 1.81\text{ W}$$

$$Q_2 = i^2R_{eq} = (2)^2\times\frac{1}{2} = \frac{4}{2} = 2\text{ W}$$

$Q_2 > Q_1$

Option (D) is incorrect.

11. A bubble has surface tension S. The ideal gas inside the bubble has ratio of specific heats $\gamma = \frac{5}{3}$. The bubble is exposed to the atmosphere and it always retains its spherical shape. When the atmospheric pressure is P_{a1}, the radius of the bubble is found to be r_1 and the temperature of the enclosed gas is T_1. When the atmospheric pressure is P_{a2}, the radius of the bubble and the temperature of the enclosed gas are r_2 and T_2, respectively. Which of the following statement(s) is(are) correct?

(A) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{r_1}{r_2}\right)^5 = \dfrac{P_{a2}+\frac{2S}{r_2}}{P_{a1}+\frac{2S}{r_1}}$

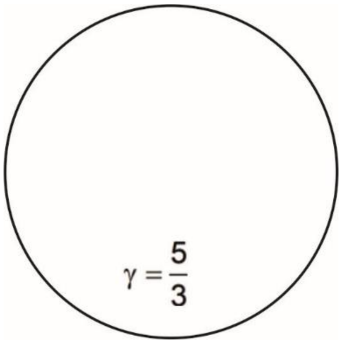
(B) If the surface of the bubble is a perfect heat insulator, then the total internal energy of the bubble including its surface energy does not change with the external atmospheric pressure.

(C) If the surface of the bubble is a perfect heat conductor and the change in atmospheric temperature is negligible, then $\left(\frac{r_1}{r_2}\right)^3 = \dfrac{P_{a2}+\frac{4S}{r_2}}{P_{a1}+\frac{4S}{r_1}}$

(D) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \dfrac{P_{a2}+\frac{4S}{r_2}}{P_{a1}+\frac{4S}{r_1}}$

Answer (C, D)

Sol. S : Surface tension



	Pressure		Radius		Temperature
When,	P_{a1}	$\longrightarrow$	r_1	$\longrightarrow$	T_1
	P_{a2}	$\longrightarrow$	r_2	$\longrightarrow$	T_2

For adiabatic process

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\left(P_{a1}+\frac{4T}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right)^{\frac{5}{3}} = \left(P_{a2}+\frac{4T}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)^{\frac{5}{3}}$$

$$\left(\frac{r_1}{r_2}\right)^5 = \frac{\left(P_{a2}+\frac{4T}{r_2}\right)}{\left(P_{a1}+\frac{4T}{r_1}\right)}$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{r_1}{r_2}\right)^{3\left(\frac{2}{3}\right)}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_{a2}+\frac{4T}{r_2}}{P_{a1}+\frac{4T}{r_1}}\right)^{\frac{2}{5}}$$

For option (B) Total internal energy + surface energy will not be same as word done by gas will be there.

Option (B) is incorrect.

For option (C)

$$P_1V_1 = P_2V_2$$

$$\left(P_{a1}+\frac{4T}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) = \left(P_{a2}+\frac{4T}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)$$

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{\left(P_{a2}+\frac{4T}{r_2}\right)}{\left(P_{a1}+\frac{4T}{r_1}\right)}$$

Option (C) is correct

12. A disk of radius R with uniform positive charge density σ is placed on the xy plane with its center at the origin. The Coulomb potential along the z-axis is

$$V(z)=\frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2}-z\right)$$

A particle of positive charge q is placed initially at rest at a point on the z-axis with $z = z_0$ and $z_0 > 0$. In addition to the Coulomb force, the particle experiences a vertical force $\vec{F}=-c\hat{k}$ with $c > 0$. Let $\beta=\frac{2c\epsilon_0}{q\sigma}$. Which of the following statement(s) is(are) correct?

(A) For $\beta=\frac{1}{4}$ and $z_0=\frac{25}{7}R$, the particle reaches the origin.

(B) For $\beta=\frac{1}{4}$ and $z_0=\frac{3}{7}R$, the particle reaches the origin.

(C) For $\beta=\frac{1}{4}$ and $z_0=\frac{R}{\sqrt{3}}$, the particle returns back to $z = z_0$

(D) For $\beta>1$ and $z_0>0$, the particle always reaches the origin.

Answer (A, C, D)

Sol. $V(z)=\frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2}-z\right)$

$U_z = cz$

$\Rightarrow \quad U(z)_{net}=\frac{\sigma q}{2\epsilon_0}\left(\sqrt{R^2+z^2}-z\right)+cz$

$=c\left[\frac{\sigma q}{2c\epsilon_0}\left(\sqrt{R^2+z^2}-z\right)+z\right]$

$= c\left[4\sqrt{R^2+z^2}-3z\right]$ at $\beta=\frac{1}{4}$

At $z = 0$, $\beta=\frac{1}{4}$

$U(z)_{net}=c[4R]=4Rc$ …(i)

At $z=z_0=\frac{25}{7}R, \beta=\frac{1}{4}$

$U(z)_{net}=c\left[4\times\frac{26R}{7}-3\times\frac{25R}{7}\right]=\frac{29}{7}Rc$ …(ii)

at $z=z_0=\frac{3}{7}R, \beta=\frac{1}{4}$

$U(z)_{net}=c\left[4\times\frac{\sqrt{58}}{7}R-\frac{9R}{7}\right]\approx 3Rc$ …(iii)

At $z=\frac{R}{\sqrt{3}}, \beta=\frac{1}{4}$

$U(z)_{net}=c\left[\frac{8R}{\sqrt{3}}-\frac{3R}{\sqrt{3}}\right]\approx 2.887Rc$

$\Rightarrow$ In option (A) particle reaches at origin with positive kinetic energy

$\frac{dU(z)}{dz} = 0$ at $z = \frac{3R}{\sqrt{7}}$

at $\beta = \frac{1}{4}$ and $z = \frac{3R}{\sqrt{7}}$

$U(z)_{\text{net}} = \sqrt{7}Rc = 2.645$

$\Rightarrow$ In option B at $U(z)_{\text{net}} \approx 3Rc$

$\Rightarrow$ The kinetic energy at origin will become negative

at $z = \frac{R}{\sqrt{3}}$

$\Rightarrow$ In option (C), $U(z)_{net}$ at $z = \frac{R}{\sqrt{3}} < U(z)_{net}$ at $z = 0$,

And $U(z)_{net}$ at $z = \frac{R}{\sqrt{3}} > U(z)_{net}$ at $z = \frac{3R}{\sqrt{7}}$

$\Rightarrow$ Particle will return back to z_0.

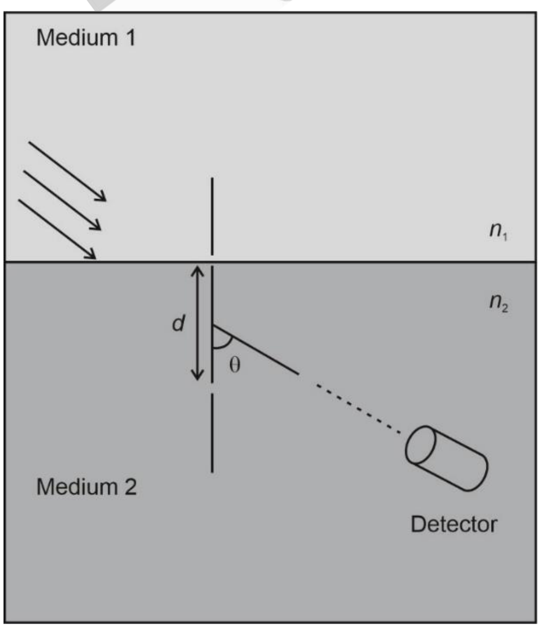
In option (D) $(\beta > 1, z_0 > 0)$

$U(z)_{net}$ will keep on increasing with z

$\Rightarrow$ Particle always reaches the origin.

$\Rightarrow$ Answer (A, C, D)

13. A double slit setup is shown in the figure. One of the slits is in medium 2 of refractive index n_2. The other slit is at the interface of this medium with another medium 1 of refractive index n_1 ($\neq n_2$). The line joining the slits is perpendicular to the interface and the distance between the slits is d. The slit widths are much smaller than d. A monochromatic parallel beam of light is incident on the slits from medium 1. A detector is placed in medium 2 at a large distance from the slits, and at an angle θ from the line joining them, so that θ equals the angle of refraction of the beam. Consider two approximately parallel rays from the slits received by the detector.



Which of the following statement(s) is(are) correct?

(A) The phase difference between the two rays is independent of d.

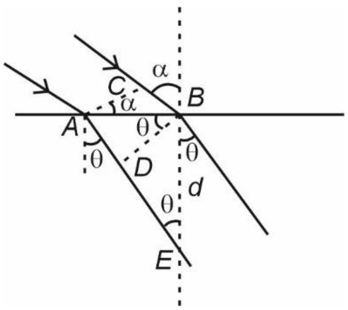
(B) The two rays interfere constructively at the detector.

(C) The phase difference between the two rays depends on n_1 but is independent of n_2.

(D) The phase difference between the two rays vanishes only for certain values of d and the angle of incidence of the beam, with q being the corresponding angle of refraction.

Answer (A, B)

Sol.



$AB = (d)(\tan\theta)$

and $BC = AB \sin\alpha = (d)(\tan\theta)(\sin\alpha)$

Also, $AD = AB \sin\theta$

$\Rightarrow$ Path difference (in vaccum)

$= n_1 BC - n_2 AD$

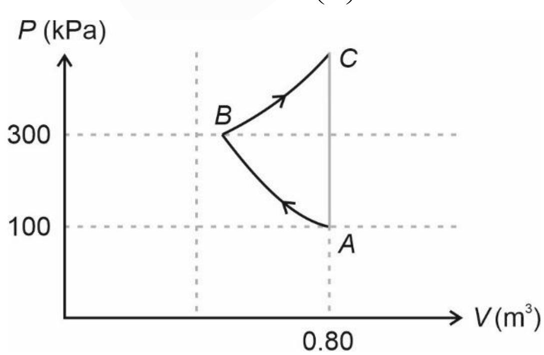
$= n_1(AB)\sin\alpha - n_2(AB\sin\theta)$

$= AB(n_1 \sin\alpha - n_2 \sin\underline{\theta}) = 0$

$\Rightarrow$ (A), (B) are correct

(C), (D) are incorrect.

14. In the given *P*-*V* diagram, a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is first compressed adiabatically from state *A* state *B*. Then it expands isothermally from state *B* to state *C*. [Given : $\left(\frac{1}{3}\right)^{0.6} \simeq 0.5, \ln 2 \simeq 0.7$].



Which of the following statement(s) is(are) correct?

(A) The magnitude of the total work done in the process $A \rightarrow B \rightarrow C$ is 144 kJ.

(B) The magnitude of the work done in the process $B \rightarrow C$ is 84 kJ.

(C) The magnitude of the work done in the process $A \rightarrow B$ is 60 kJ.

(D) The magnitude of the work done in the process $C \rightarrow A$ is zero.

Answer (B, C, D)

Sol. $Pv^{\gamma} = c$

$$\Rightarrow \quad 100(0.8)^{5/3} = 300(v)^{5/3}$$

$$\Rightarrow \quad V_B = \frac{0.8}{3^{3/5}}$$

$$\Rightarrow \quad W_{AB} = \frac{P_A V_A - P_B V_B}{\frac{5}{3} - 1} = \frac{80 - 300 \times \frac{0.8}{3^{3/5}}}{2/3} \text{ kJ}$$

$$= \frac{80 - 240(0.5)}{2/3} \text{ kJ}$$

$$= -60 \text{ kJ}$$

$\Rightarrow$ (C) is correct

$C \to A$ is isochoric $\Rightarrow$ (D) is correct

$$BC : W_{BC} = nRT \ln \frac{V_2}{V_1} = PV \ln \frac{V_2}{V_1}$$

$$= 300 \times 0.8 \times 0.5 \ln \left(\frac{0.8}{\frac{0.8}{3^{3/5}}} \right)$$

$$= 120 \ln 2 = 84 \text{ kJ}$$

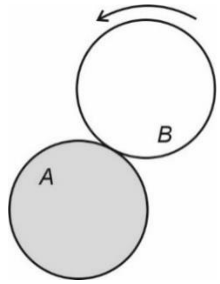
$\therefore$ Options (B, C, D) are correct.

SECTION – 3 (Maximum marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

15. A flat surface of a thin uniform disk A of radius R is glued to a horizontal table. Another thin uniform disk B of mass M and with the same radius R rolls without slipping on the circumference of A, as shown in the figure. A flat surface of B also lies on the plane of the table. The center of mass of B has fixed angular speed ω about the vertical axis passing through the center of A. The angular momentum of B is $nM\omega R^2$ with respect to the center of A. Which of the following is the value of n?



(A) 2
(B) 5
(C) $\frac{7}{2}$
(D) $\frac{9}{2}$

Answer (B)

Sol. Angular momentum of B with respect to center of A

$$\vec{L} = \vec{L}_{\text{CM}} + \vec{L}_{\text{Body about CM}}$$

$$= M(2R)^2\omega\hat{k} + \frac{MR^2}{2}(\omega_{\text{body}})\hat{k}$$

$$= M(2R)^2\omega\hat{k} + \frac{MR^2}{2}(2\omega)\hat{k}$$

$$= 5MR^2\omega\hat{k}$$

Comparing the magnitude with $nM\omega R^2$

$n = 5$

16. When light of a given wavelength is incident on a metallic surface, the minimum potential needed to stop the emitted photoelectrons is 6.0 V. This potential drops to 0.6 V if another source with wavelength four times that of the first one and intensity half of the first one is used. What are the wavelength of the first source and the work function of the metal, respectively? [Take $\frac{hc}{e} = 1.24 \times 10^{-6}$ J m C^{-1}.]

(A) 1.72×10^{-7} m, 1.20 eV
(B) 1.72×10^{-7} m, 5.60 eV
(C) 3.78×10^{-7} m, 5.60 eV
(D) 3.78×10^{-7} m, 1.20 eV

Answer (A)

Sol. $h\nu - \phi = 6$ eV

$$\frac{hc}{\lambda} - \phi = 6 \text{ eV} \quad \ldots\text{(i)}$$

$$\frac{hc}{4\lambda} - \phi = 0.6 \text{ eV}$$

$$\frac{3hc}{4\lambda} = 5.4 \text{ eV}$$

$$\therefore \quad \lambda = \frac{3hc}{4\times 5.4\text{ eV}} = \frac{3\times 1.24\times 10^{-6}}{4\times 5.4}$$

$= 1.72 \times 10^{-7}$ m

$\Rightarrow$ from equation (i)

$$\frac{hc}{1.72\times 10^{-7}} \times \frac{1}{1.6\times 10^{-19}} - \phi = 6\text{ eV}$$

$$\frac{2\times 10^{-25}}{2.75\times 10^{-26}} - \phi = 6$$

$$\Rightarrow \quad \phi = (7.27 - 6) \cong 1.2\text{ eV}$$

17. Area of the cross-section of a wire is measured using a screw gauge. The pitch of the main scale is 0.5 mm. The circular scale has 100 divisions and for one full rotation of the circular scale, the main scale shifts by two divisions. The measured readings are listed below.

Measurement condition	Main scale reading	Circular scale reading
Two arms of gauge touching each other without wire	0 division	4 divisions
Attempt-1: With wire	4 divisions	20 divisions
Attempt-2: With wire	4 divisions	16 divisions

What are the diameter and cross-sectional area of the wire measured using the screw gauge?

(A) 2.22 ± 0.02 mm, π (1.23 ± 0.02) mm^2

(B) 2.22 ± 0.01 mm, π (1.23 ± 0.01) mm^2

(C) 2.14 ± 0.02 mm, π (1.14 ± 0.02) mm^2

(D) 2.14 ± 0.01 mm, π (1.14 ± 0.01) mm^2

Answer (C)

Sol. Reading-1

MSR = 4 × 0.5 = 2 mm

$$\text{CSR} = \frac{20}{100}\text{ mm}$$

$$\text{Zero error} = \frac{4}{100} = 0.04\text{ mm}$$

R_1 = MSR + CSR – (Zero error)

= (2 + 0.20 – 0.04) mm

= 2.16 mm

Reading-2

MSR = 2 mm

CSR = 0.16 mm

Zero error = 0.04 mm

$R_2 = (2 + 0.16 - 0.04)$ mm = 2.12 mm

Reading/average $= \frac{R_1 + R_2}{2}$ = 2.14 mm = R_m (say)

Average mean error $= \frac{|R_m - R_1| + |R_m - R_2|}{2} = 0.02$ mm

$\Rightarrow$ Diameter = (2.14 ± 0.02) mm = d (say)

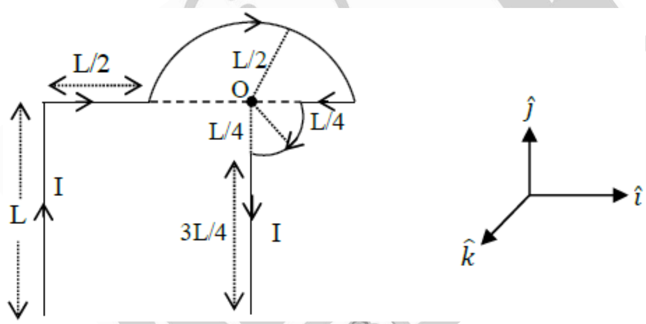
Area $= \frac{\pi d^2}{4}$

$\Rightarrow \quad \Delta \text{Area} = \frac{2\pi d}{4}(\Delta d)$

$\Rightarrow \quad \Delta \text{Area} \cong \pi(0.02)$

$\Rightarrow \quad \text{Area} = \frac{\pi d^2}{4} \pm \Delta\text{Area} = \pi\left(1.14 \pm 0.02\right)$ mm

18. Which one of the following options represents the magnetic field $\vec{B}$ at O due to the current flowing in the given wire segments lying on the xy plane?



(A) $\vec{B} = \frac{-\mu_0 I}{L}\left(\frac{3}{2} + \frac{1}{4\sqrt{2}\,\pi}\right)\hat{k}$

(B) $\vec{B} = \frac{-\mu_0 I}{L}\left(\frac{3}{2} + \frac{1}{2\sqrt{2}\,\pi}\right)\hat{k}$

(C) $\vec{B} = \frac{-\mu_0 I}{L}\left(1 + \frac{1}{4\sqrt{2}\,\pi}\right)\hat{k}$

(D) $\vec{B} = \frac{-\mu_0 I}{L}\left(1 + \frac{1}{4\pi}\right)\hat{k}$

Answer (C)

Sol. $B_{net} = B_{semicircle} + B_{quarter} + B_{straight}$

$$= \frac{\mu_0 I}{4\left(\frac{L}{2}\right)} + \frac{\mu_0 I}{8 \times \left(\frac{L}{4}\right)} + \frac{\mu_0 I}{4\pi \times L}\left(\frac{1}{\sqrt{2}}\right)\left(-\hat{k}\right)$$

$$= \left(\frac{\mu_0 I}{2L} + \frac{\mu_0 I}{2L} + \frac{\mu_0 I}{4\sqrt{2}\pi L}\right)\left(-\hat{k}\right)$$

$$= \frac{\mu_0 I}{L}\left(1 + \frac{1}{4\sqrt{2}\,\pi}\right)\left(-\hat{k}\right)$$

PART-II : CHEMISTRY

SECTION – 1 (Maximum marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct integer is entered;

Zero Marks : 0 If the question is unanswered;

Negative Marks : −1 In all other cases.

1. Concentration of H_2SO_4 and Na_2SO_4 in a solution is 1 M and 1.8×10^{-2} M, respectively. Molar solubility of $PbSO_4$ in the same solution is $X \times 10^{-Y}$ M (expressed in scientific notation). The value of Y is ________.

[Given: Solubility product of $PbSO_4$ (K_{sp}) = 1.6×10^{-8}. For H_2SO_4, K_{a1} is very large and $K_{a2} = 1.2 \times 10^{-2}$]

Answer (6)

Sol. $H_2SO_4 \longrightarrow \underset{1\text{ M}}{H^+} + \underset{1\text{ M}}{HSO_4^{2-}}$ (K_{a_1} is very large)

$HSO_4^- \rightleftharpoons H^+ + SO_4^{2-} \qquad K_{a_2} = 1.2 \times 10^{-2}$

$[SO_4^{2-}]$ coming from $Na_2SO_4 = 1.8 \times 10^{-2}$

$$\frac{[SO_4^{2-}][H^+]}{[HSO_4^-]} = \frac{1.8 \times 10^{-2} \times 1}{1} > K_{a_2}$$

$\therefore$ Rather than dissociation of HSO_4^- into H^+ and SO_4^{2-} ions, association between already present H^+ and SO_4^{2-} will take place.

Assuming 'x' mol/L of SO_4^{2-} and H^+ combines to form HSO_4^-

$\therefore \quad [SO_4^{2-}] = 1.8 \times 10^{-2} - x$

$$\left.\begin{aligned}&[H^+] = 1 - x \approx 1\\&[HSO_4^-] = 1 + x \approx 1\end{aligned}\right\} \text{(assuming } x << 1)$$

$$\frac{(1.8 \times 10^{-2} - x)1}{1} = 1.2 \times 10^{-2}$$

$\Rightarrow \quad x = 0.6 \times 10^{-2}$

$[SO_4^{2-}] = 1.2 \times 10^{-2}$ M

$PbSO_4(s) \rightleftharpoons Pb^{2+}(aq) + SO_4^{2-}(aq)$

If solubility of $PbSO_4$ = s M

$\therefore$ $[Pb^{2+}] = s$

$[SO_4^{2-}] = s + 1.2 \times 10^{-2} \approx 1.2 \times 10^{-2}$ (assuming s << 1.2×10^{-2})

$\therefore$ $s \times 1.2 \times 10^{-2} = 1.6 \times 10^{-8}$

$s = \frac{1.6}{1.2} \times 10^{-6} = 1.33 \times 10^{-6}$

On comparing with $X \times 10^{-Y}$

Y = 6

2. An aqueous solution is prepared by dissolving 0.1 mol of an ionic salt in 1.8 kg of water at 35ºC. The salt remains 90% dissociated in the solution. The vapour pressure of the solution is 59.724 mm of Hg. Vapor pressure of water at 35ºC is 60.000 mm of Hg. The number of ions present per formula unit of the ionic salt is _______.

Answer (5)

Sol. Number of ions present per formula unit of ionic salt = x

Van 't Hoff factor (i) = 0.9 × x + 0.1 × 1

(Assuming 90% dissociation)

$\therefore$ Relative lowering in vapour pressure = Mole fraction of solute

$$\Rightarrow \frac{60 - 59.724}{60} = \frac{i \times 0.1}{\frac{1800}{18} + 0.1}$$

$$\Rightarrow 0.0046 = \frac{i \times 0.1}{100 + 0.1}$$

$$0.0046 \approx \frac{(0.9x + 0.1) \times 0.1}{100}$$

$\Rightarrow$ 0.9x + 0.1 = 4.6

$$\Rightarrow x = \frac{4.5}{0.9} = 5$$

x = 5

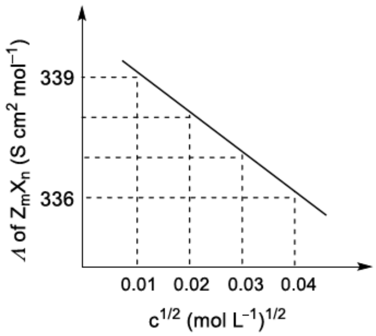
3. Consider the strong electrolytes Z_mX_n, U_mY_p and V_mX_n. Limiting molar conductivity (Λ^0) of U_mY_p and V_mX_n are 250 and 440 S cm² mol⁻¹, respectively. The value of (m + n + p) is _______.

Given:

Ion	Z^{n+}	U^{p+}	V^{n+}	X^{m-}	Y^{m-}
λ^0 (S cm^2 mol^{-1})	50.0	25.0	100.0	80.0	100.0

λ^0 is the limiting molar conductivity of ions

The plot of molar conductivity (Λ) of Z_mX_n vs $c^{1/2}$ is given below.



Answer (7)

Sol. $\lambda_m = \lambda_m^o - A\sqrt{C}$

For electrolyte Z_mX_n and from given curve

$\lambda_m(Z_mX_n) = \lambda_m^o(Z_mX_n) - A\sqrt{C}$

$$-A = \frac{336-339}{0.04-0.01} = -\frac{3}{0.03}$$

$\Rightarrow$ A = 100

$\therefore$ For $\lambda_m = 336\ \text{S cm}^2\ \text{mol}^{-1}$

$\Rightarrow$ $336 = \lambda_m^o(Z_mX_n) - 100 \times 0.04$

$\lambda_m^o = 336 + 4 = 340\ \text{S cm}^2\ \text{mol}^{-1}$

$Z_mX_n \longrightarrow mZ^{n+} + nX^{m-}$

$\therefore$ 50m + 80n = 340

$\Rightarrow$ 5m + 8n = 34 …(i)

$U_mY_p \longrightarrow mU^{p+} + pY^{m-}$

$\therefore$ 25m + 100p = $\lambda_m^o(U_mY_p) = 250$

$\Rightarrow$ m + 4p = 10 …(ii)

$V_mX_n \longrightarrow mV^{n+} + nX^{m-}$

$\therefore$ 100m + 80n = 440

$\Rightarrow$ 5m + 4n = 22 …(iii)

From equation (i) and (iii)

n = 3

m = 2

From equation (ii)

p = 2

$\therefore$ m + n + p = 2 + 3 + 2 = 7

4. The reaction of Xe and O_2F_2 gives a Xe compound **P**. The number of moles of HF produced by the complete hydrolysis of 1 mol of **P** is _______.

Answer (4)

Sol. $Xe + 2O_2F_2 \rightarrow \underset{(P)}{XeF_4} + 2O_2$

$$\underset{(P)}{6XeF_4} + 12H_2O \longrightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$$

Number of moles of HF formed by complete hydrolysis of 1 mole of (P) = 4

5. Thermal decomposition of $AgNO_3$ produces two paramagnetic gases. The total number of electrons present in the antibonding molecular orbitals of the gas that has the higher number of unpaired electrons is _______.

Answer (6)

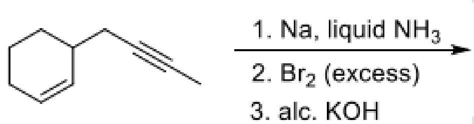
Sol. $2AgNO_3(s) \xrightarrow{\Delta} 2Ag(s) + 2NO_2(g) + O_2(g)$

Both the NO_2 and O_2 gases are paramagnetic. $NO_2(g)$ has 1 unpaired electron and $O_2(g)$ has 2 unpaired electrons. According to MOT, electronic configuration of O_2 is

$$\sigma_{1s}^2\ \sigma^*{}_{1s}^2\ \sigma_{2s}^2\ \sigma^*{}_{2s}^2\ \sigma_{2p_z}^2\ \pi_{2p_x}^2 = \pi_{2p_y}^2\ \pi^*{}_{2p_x}^1 = \pi^*{}_{2p_y}^1$$

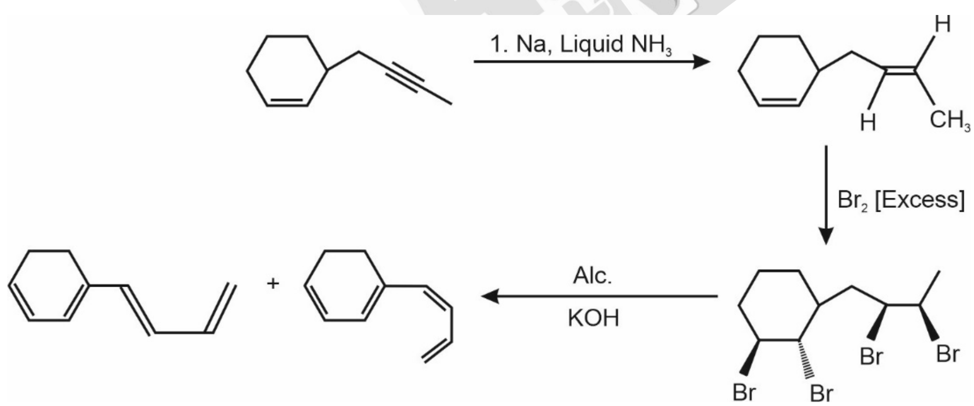
Total number of electrons present in antibonding molecular orbitals = 6

6. The number of isomeric tetraenes (NOT containing *sp*-hybridized carbon atoms) that can be formed from the following reaction sequence is ________.

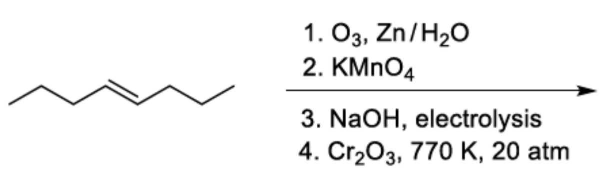


Answer (2)

Sol. :

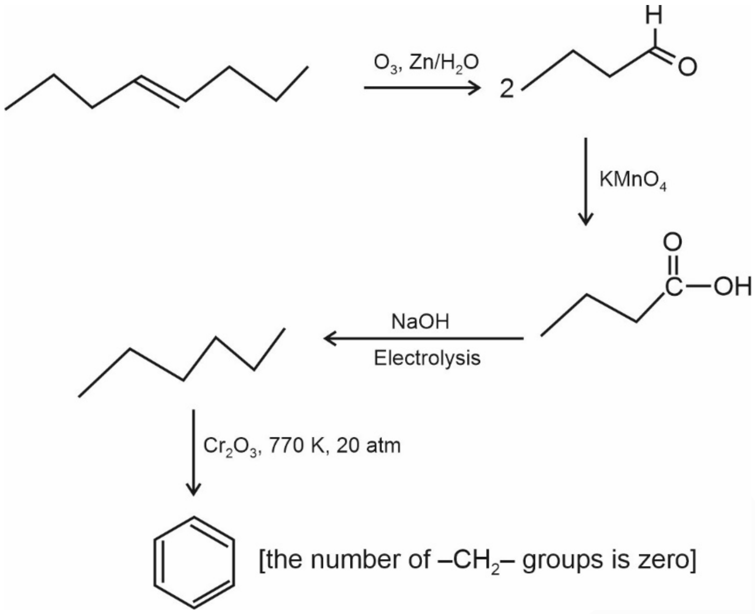


7. The number of -CH_2- (methylene) groups in the product formed from the following reaction sequence is ________.

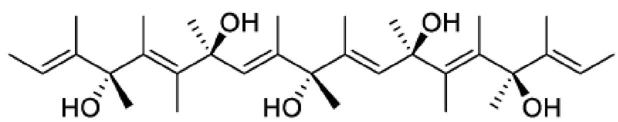


Answer (0)

Sol.



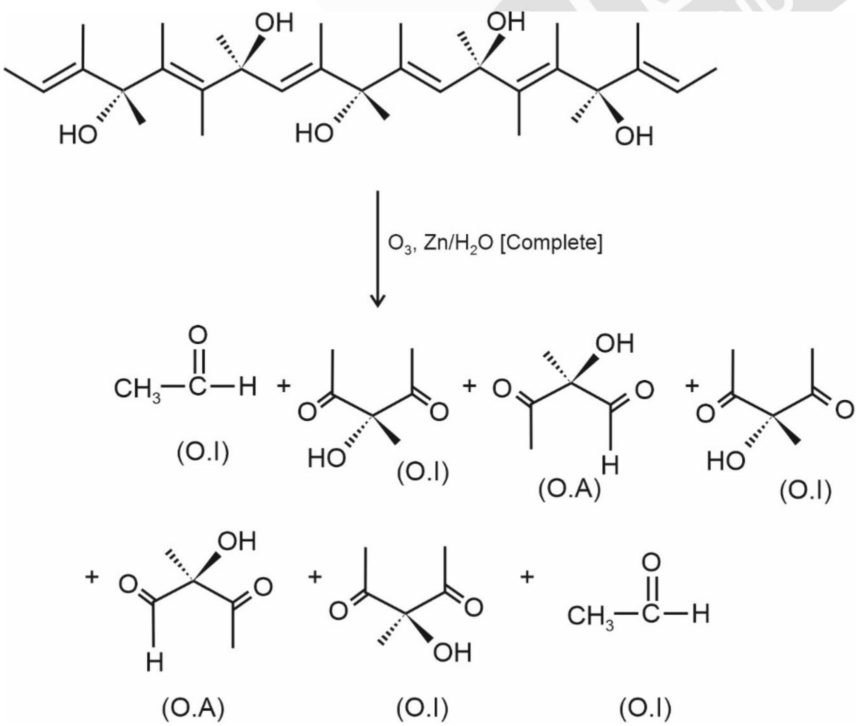
8. The total number of chiral molecules formed from one molecule of **P** on complete ozonolysis (O_3, Zn/H_2O) is ________.



P

Answer (2)

Sol.



$\therefore$ Two chiral molecules are obtained.

SECTION – 2 (Maximum marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

Partial Marks : + 2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –2 In all other cases.

9. To check the principle of multiple proportions, a series of pure binary compounds (P_mQ_n) were analyzed and their composition is tabulated below. The correct option(s) is(are)

Compound	Weight % of P	Weight % of Q
1	50	50
2	44.4	55.6
3	40	60

(A) If empirical formula of compound 3 is P_3Q_4, then the empirical formula of compound 2 is P_3Q_5.

(B) If empirical formula of compound 3 is P_3Q_2 and atomic weight of element P is 20, then the atomic weight of Q is 45.

(C) If empirical formula of compound 2 is PQ, then the empirical formula of the compound 1 is P_5Q_4.

(D) If atomic weight of P and Q are 70 and 35, respectively, then the empirical formula of compound 1 is P_2Q.

Answer (B, C)

Sol. (A) If empirical formula of 3 is P_3Q_4, then molecular formula is $(P_3Q_4)_n$

$$\Rightarrow \quad \frac{3M_P}{3M_P + 4M_Q} = \frac{40}{100} = \frac{2}{5}$$

$15M_P = 6M_P + 8M_Q$

$9M_P = 8M_Q$

$$\therefore \quad \text{For } P_3Q_5\text{, \% of P} = \left(\frac{3M_P}{3M_P + 5M_Q}\right)\times 100 = \frac{8M_Q}{23M_Q}\times 100 \simeq 34.78\%$$

(B) If empirical formula of compound 3 is P_3Q_2, we have

$$\frac{3M_P}{3M_P + 2M_Q} = \frac{2}{5}$$

$15M_P = 6M_P + 4M_Q$

$9M_P = 4M_Q$

If $M_P = 20$

$$M_Q = \frac{180}{4} = 45$$

(C) If empirical formula of 2 is PQ

$$\therefore \quad \frac{M_P}{M_P + M_Q} = \frac{4}{9} \simeq \frac{44.44}{100}$$

$9M_P = 5M_P + 4M_Q$

$5M_P = 4M_Q$

If empirical formula is assumed as P_5Q_4,

$$\% \text{ of P} = \left(\frac{5M_P}{5M_P + 4M_Q}\right) \times 100 = 50$$

Hence P_5Q_4 is the empirical formula of compound

(D) If empirical formula of I is P_2Q, we have

$$\% \text{ of P} = \left(\frac{2M_P}{2M_P + M_Q}\right) \times 100 = 50$$

$4M_P = 2M_P + M_Q$

$2M_P = M_Q$

Hence, atomic weight of P and Q cannot be 70 and 35 respectively.

Hence (B, C) are correct

10. The correct option(s) about entropy (S) is(are)

[R = gas constant, F = Faraday constant, T = Temperature]

(A) For the reaction, $M(s) + 2H^+(aq) \rightarrow H_2(g) + M^{2+}(aq)$, if $\frac{dE_{cell}}{dT} = \frac{R}{F}$ then the entropy change of the reaction is R (assume that entropy and internal energy changes are temperature independent).

(B) The cell reaction, $Pt(s) \mid H_2(g, 1bar) \mid H^+(aq, 0.01M) \parallel H^+(aq, 0.1M) \mid H_2(g, 1bar) \mid Pt(s)$, is an entropy driven process.

(C) For racemization of an optically active compound, $\Delta S > 0$

(D) $\Delta S > 0$, for $[Ni(H_2O)_6]^{2+} + 3\text{ en} \rightarrow [Ni(en)_3]^{2+} + 6H_2O$ (where en = ethylenediamine).

Answer (B, C, D)

Sol. (A) $M(s) + 2H^+(aq) \rightarrow H_2(g) + M^{2+}(aq)$

if $\frac{dE_{cell}}{dT} = \frac{R}{F}$

$$\Delta S = nF\frac{dE}{dT} = 2F\left(\frac{R}{F}\right) = 2R$$

(B) $E_{cell} = \frac{-2.303RT}{F}\log\frac{0.01}{0.1} = \frac{2.303RT}{F}$

$\frac{dE_{cell}}{dT} = \frac{2.303R}{F}$

$\therefore\ \Delta S = nF\frac{dE}{dT} > 0$

It is an entropy driven process.

(C) It is correct

During racemisation of optically active compound, disorder increases and hence, entropy increases.

(D) For $[Ni(H_2O)_6]^{2+}$ + 3en → $[Ni(en)_3]^{+3}$ + $6H_2O$,

Entropy increases when bidentate ligands replace monodentate ligands due to increase in the number of molecules on the product side.

Hence, (B, C, D) are correct.

11. The compound(s) which react(s) with NH_3 to give boron nitride (BN) is(are)

(A) B

(B) B_2H_6

(C) B_2O_3

(D) HBF_4

Answer (A, B, C)

Sol. The compound(s) which react with NH_3 to form Boron nitride (BN) is/are:

(A) Amorphous B + NH_3 $\xrightarrow[\text{Tempreature}]{\text{Very high}}$ BN + H_2

(B) $B_2H_6 + \underset{\text{(excess)}}{NH_3} \xrightarrow{\text{High Temp.}} (BN)_x$

(C) $B_2O_3 + 2NH_3 \xrightarrow{900°C} 2BN + 3H_2O$

(D) $HBF_4 + NH_3 \longrightarrow [NH_4^{\oplus}]\,[BF_4^{\ominus}]$

Hence, (A, B, C) are correct

12. The correct option(s) related to the extraction of iron from its ore in the blast furnace operating in the temperature range 900 – 1500 K is(are)

(A) Limestone is used to remove silicate impurity.

(B) Pig iron obtained from blast furnace contains about 4% carbon.

(C) Coke (C) converts CO_2 to CO.

(D) Exhaust gases consist of NO_2 and CO.

Answer (A, B, C)

Sol. (A) Limestone is added to remove silica as impurity.

(B) Pig iron obtained from blast furnace contains 4% carbon and many other impurities (eg. S, P, Si, Mn) in small amount.

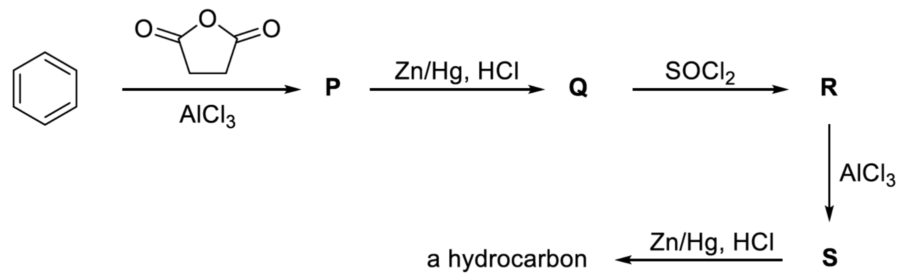
(C) Coke (C) converts CO_2 in CO.

$C + CO_2 \rightarrow 2CO$

(D) Exhaust gases consist of CO and CO_2.

Hence, (A, B, C) are correct.

13. Considering the following reaction sequence, the correct statement(s) is(are)



(A) Compounds P and Q are carboxylic acids.

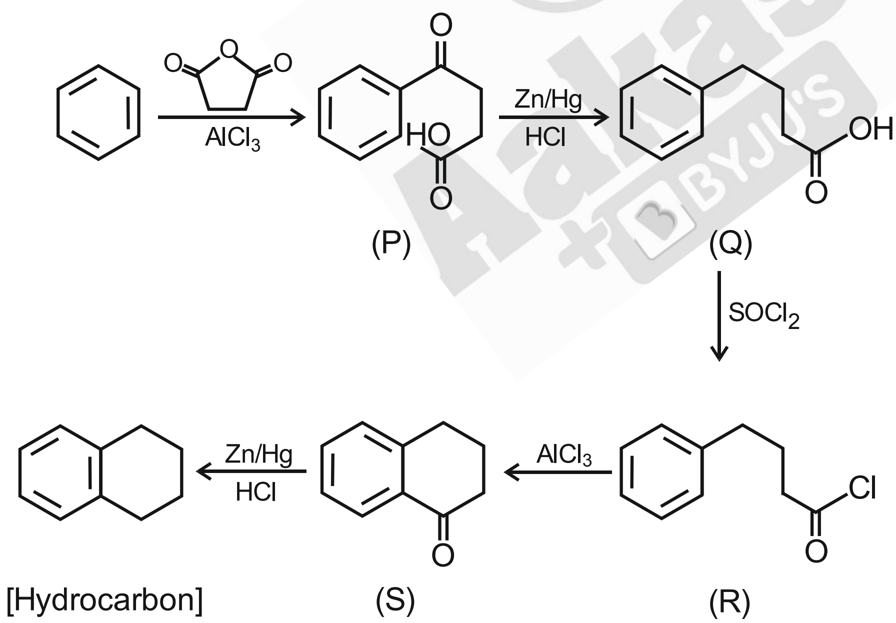
(B) Compound S decolorizes bromine water.

(C) Compounds P and S react with hydroxylamine to give the corresponding oximes.

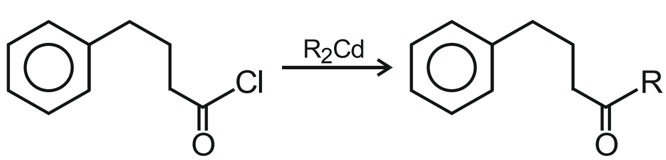
(D) Compound R reacts with dialkylcadmium to give the corresponding tertiary alcohol.

Answer (A, C)

Sol.



Compound (S) can not decolorizes bromine water.



R_2Cd is a less reactive nucleophile so reaction stops at carbonyl group.

14. Among the following, the correct statement(s) about polymers is(are)

(A) The polymerization of chloroprene gives natural rubber.

(B) Teflon is prepared from tetrafluoroethene by heating it with persulphate catalyst at high pressures.

(C) PVC are thermoplastic polymers.

(D) Ethene at 350-570 K temperature and 1000-2000 atm pressure in the presence of a peroxide initiator yields high density polythene.

Answer (B, C)

Sol. (A) Polymerization of chloroprene gives synthetic rubber

(B) Teflon is manufactured by heating tetrafluoroethene with a free radical or persulphate catalyst at hight pressure.

(C) Polyvinyl chloride (PVC) is a thermoplastic polymer and used in making hand bags, vinyl flooring and water pipes.

(D) Ethene at 350-570 K temperature and 1000-2000 atm pressure in the presence of a peroxide initiator yields low density polythene.

SECTION – 3 (Maximum marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

15. Atom X occupies the fcc lattice sites as well as alternate tetrahedral voids of the same lattice. The packing efficiency (in %) of the resultant solid is closest to

(A) 25 (B) 35

(C) 55 (D) 75

Answer (B)

Sol. Atom X occupies FCC lattice sites as well as alternate tetrahedral voids of FCC.

In FCC, tetrahedral voids are 8 (in a unit cell)

Hence

Atom X in a unit cell (FCC lattice sites) = $8\times\frac{1}{8}+6\times\frac{1}{2}=4$

Atom X in a unit cell (in T.V.) = $\frac{1}{2}\times 8=4$

Total atom X in one unit cell = 8

For relation between a and r, since T.V. forms at $\frac{1}{4}$ th of body diagonal,

$$\frac{a\sqrt{3}}{4} = 2r$$

$$a = \frac{8r}{\sqrt{3}}$$

$$\text{Packing efficiency} = \frac{8 \times \frac{4}{3}\pi r^3}{a^3} \times 100 = \frac{8 \times \frac{4}{3}\pi r^3}{\left(\frac{8r}{\sqrt{3}}\right)^3} \times 100 \simeq 35\%$$

16. The reaction of $HClO_3$ with HCl gives a paramagnetic gas, which upon reaction with O_3 produces

(A) Cl_2O

(B) ClO_2

(C) Cl_2O_6

(D) Cl_2O_7

Answer (C)

Sol. $HClO_3$ reacts with HCl according to the following equation,

$$2HClO_3 + 2HCl \longrightarrow 2ClO_2 + Cl_2 + 2H_2O$$

ClO_2 molecule is paramagnetic, as it contains odd number of electrons.

$$2ClO_2 + 2O_3 \longrightarrow Cl_2O_6 + 2O_2$$

17. The reaction of $Pb(NO_3)_2$ and NaCl in water produces a precipitate that dissolves upon the addition of HCl of appropriate concentration. The dissolution of the precipitate is due to the formation of

(A) $PbCl_2$

(B) $PbCl_4$

(C) $[PbCl_4]^{2-}$

(D) $[PbCl_6]^{2-}$

Answer (C)

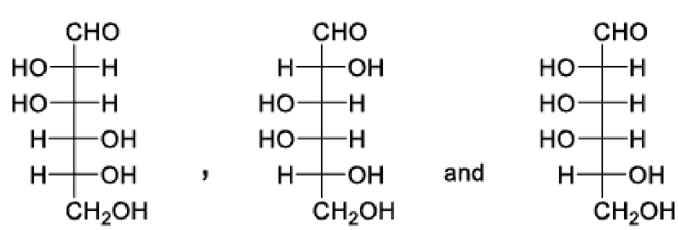
Sol. Pb^{2+} on reaction with Cl^-, produces white precipitate of $PbCl_2$

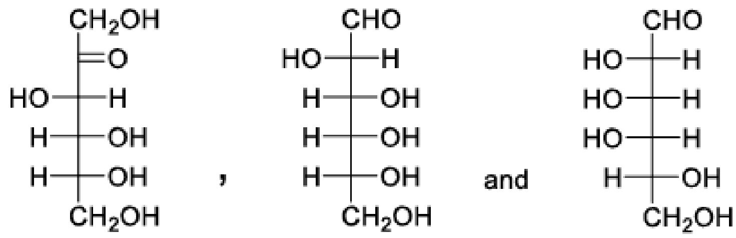
$$Pb^{2+} + 2Cl^- \longrightarrow \underset{\text{White}}{PbCl_2} \downarrow$$

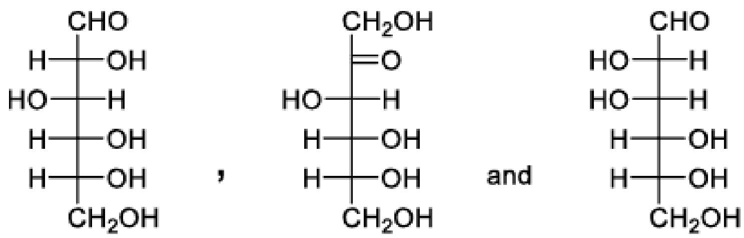
This precipitate is soluble in concentrated hydrochloric acid due to formation of tetrachloroplumbate (II) ion

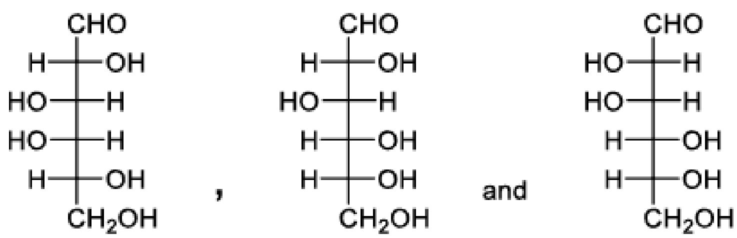
$$PbCl_2 \downarrow + 2Cl^- \longrightarrow [PbCl_4]^{2-}$$

18. Treatment of D-glucose with aqueous NaOH results in a mixture of monosaccharides, which are

(A) 

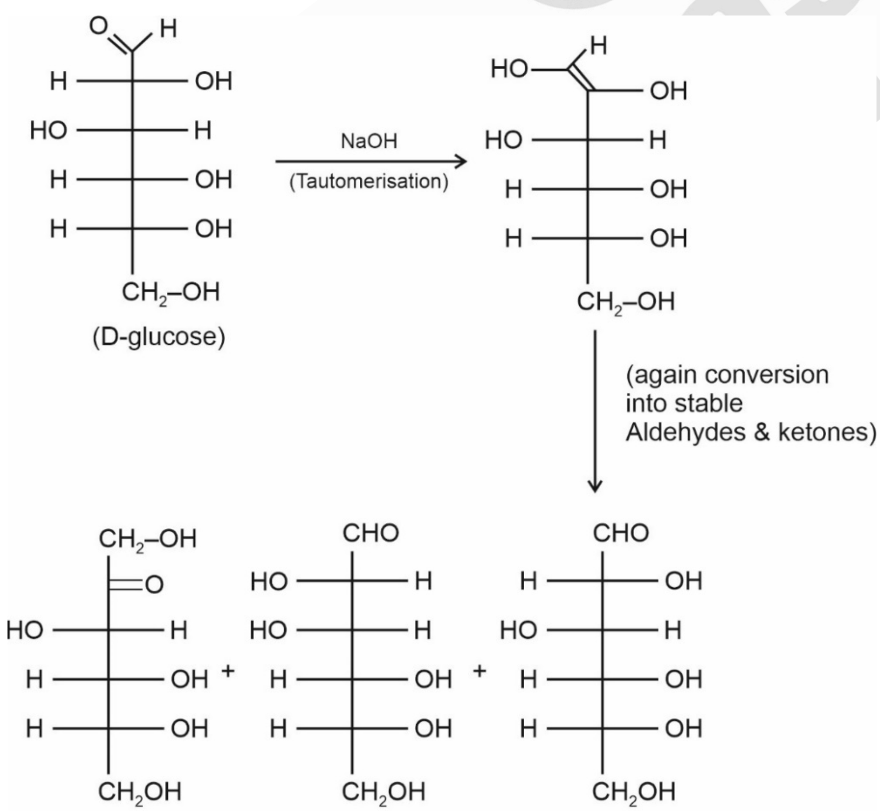
(B) 

(C) 

(D) 

Answer (C)

Sol.



[Mixture of 3 compounds will obtained]

PART-III : MATHEMATICS

SECTION – 1 (Maximum marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct integer is entered;
Zero Marks	:	0	If the question is unanswered;
Negative Marks	:	−1	In all other cases.

1. Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha+\beta) = \frac{1}{3}$ and $\cos(\alpha-\beta) = \frac{2}{3}$, then the greatest integer less than or equal to $\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2$ is _______.

Answer (1)

Sol. $\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\alpha}{\sin\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\sin\beta}{\cos\alpha}\right)^2$

$$= \left(\frac{\cos(\alpha-\beta)}{\sin\beta\cos\beta} + \frac{\cos(\alpha-\beta)}{\sin\alpha\cdot\cos\alpha}\right)^2$$

$$= \left(\frac{4}{3}\left\{\frac{1}{\sin 2\beta} + \frac{1}{\sin 2\alpha}\right\}\right)^2$$

$$= \frac{16}{9}\left(\frac{2\sin(\alpha+\beta)\cdot\cos(\alpha-\beta)}{\sin 2\alpha\cdot\sin 2\beta}\right)^2$$

$$= \frac{16}{9}\left(\frac{4\cdot\frac{1}{3}\cdot\frac{2}{3}}{\cos(2\alpha-2\beta)-\cos(2\alpha+2\beta)}\right)^2$$

$$= \frac{16}{9}\left(\frac{\frac{8}{9}}{2\cos^2(\alpha-\beta)-1-1+2\sin^2(\alpha+\beta)}\right)^2$$

$$= \frac{16}{9}\left(\frac{\frac{8}{9}}{\frac{8}{9}-2+\frac{2}{9}}\right)^2$$

$$= \frac{16}{9}$$

= 1

2. If $y(x)$ is the solution of the differential equation $xdy - (y^2 - 4y)dx = 0$ for $x > 0$, $y(1) = 2$, and the slope of the curve $y = y(x)$ is never zero, then the value of $10y(\sqrt{2})$ is ______.

Answer (8)

Sol. $\dfrac{dy}{y^2-4y} = \dfrac{dx}{x}$

$\Rightarrow \dfrac{1}{4}\left(\dfrac{1}{y-4} - \dfrac{1}{y}\right)dy = \dfrac{dx}{x}$

Integrating both sides

$\dfrac{1}{4}\ln\left|\dfrac{y-4}{y}\right| = \ln|x| + c$

As, $y(1) = 2$, $c = 0$

so, $\dfrac{y-4}{y} = \pm x^4$

or, $y = \dfrac{4}{1 \pm x^4}$

Considering $y = \dfrac{4}{1+x^4}$

$y\left(\sqrt{2}\right) = \dfrac{4}{5}$

So, $10y\left(\sqrt{2}\right) = 8$

3. The greatest integer less than or equal to $\int_1^2 \log_2(x^3+1)dx + \int_1^{\log_2 9}(2^x-1)^{\frac{1}{3}}dx$ is ______.

Answer (5)

Sol. $I = \int_1^2 \log_2(x^3+1)dx + \int_1^{\log_2 9}(2^x-1)^{\frac{1}{3}}dx$

Let $\int_1^{\log_2 9}(2^x-1)^{\frac{1}{3}}dx = I_2$

Let $2^x - 1 = t^3$

$\Rightarrow$ $2^x \ln 2 dx = 3t^2 dt$

or $dx = \dfrac{3t^2}{\ln 2(t^3+1)}dt$

So, $I = \int_1^2 \log_2(x^3+1)dx + \int_1^2 \frac{3t^3}{\ln 2(t^3+1)}dt$

or $I = \int_1^2 \left(\log_2(t^3+1) + t\cdot\frac{3t^2}{(t^3+1)\ln 2}\right)dt$

$= t\cdot\log_2(t^3+1)\Big|_1^2$

$= 2\log_2 9 - 1\log_2 2$

$= 2\log_2 9 - 1$

So, $[I] = 5$

4. The product of all positive real values of x satisfying the equation $x^{(16(\log_5 x)^3 - 68\log_5 x)} = 5^{-16}$ is ______.

Answer (01)

Sol. Taking log to the base 5 on both sides

$(16(\log_5 x)^3 - 68(\log_5 x))(\log_5 x) = -16$

Let $(\log_5 x) = t$

$16t^4 - 68t^2 + 16 = 0$

OR $4t^4 - 16t^2 - t^2 + 4 = 0$

OR $(4t^2 - 1)(t^2 - 4) = 0$

OR $t = \pm\frac{1}{2}, \pm 2$

So $\log_5 x = \pm\frac{1}{2}$ OR ± 2

$\Rightarrow x = 5^{\frac{1}{2}}, 5^{\frac{-1}{2}}, 5^2, 5^{-2}$

5. If $\beta = \lim_{x\to 0}\frac{e^{x^3} - (1-x^3)^{\frac{1}{3}} + ((1-x^2)^{\frac{1}{2}} - 1)\sin x}{x\sin^2 x}$, then the value of 6β is ______.

Answer (5)

Sol. $\beta = \lim_{x\to 0}\frac{e^{x^3} - (1-x^3)^{\frac{1}{3}} + ((1-x^2)^{\frac{1}{2}} - 1)\sin x}{x\cdot\frac{\sin^2 x}{x^2}\cdot x^2}$

$$\beta = \lim_{x\to 0}\left(\frac{\left(1+x^3+\frac{x^6}{2!}+\cdots\right) - \left(1-\frac{1}{3}x^3+\left(-\frac{1}{9}\right)x^6+\cdots\right) + \left(-\frac{1}{2}x^2 - \frac{1}{8}x^4+\cdots\right)\left(x-\frac{x^3}{3!}+\cdots\right)}{x^3}\right)$$

$$\beta = \lim_{x\to 0} \frac{x^3\left(1+\frac{1}{3}-\frac{1}{2}\right)}{x^3}$$ (Neglecting higher powers of x)

So, $\beta = \frac{5}{6}$

or $6\beta = 5$

6. Let β be a real number. Consider the matrix $A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix}$. If $A^7 - (\beta - 1)A^6 - \beta A^5$ is a singular matrix, then the value of 9β is ______.

Answer (3)

Sol. $A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix}$

det. $(A) = -1$ ...(1)

For $A^7 - (\beta - 1)A^6 - \beta A^5$ to be singular

$|A^5|\,|A^2 - (\beta - 1)A - \beta| = 0$

$\Rightarrow$ $|A^5|\,|(A + I)(A - \beta I)| = 0$ ...(2)

$\therefore$ $|A^5|\,|A + I|\,|A - \beta I| = 0$

As $|A| \neq 0$

$|A + I|$ or $|A - \beta I| = 0$

$\Rightarrow$ $\begin{pmatrix} \beta+1 & 0 & 1 \\ 2 & 2 & -2 \\ 3 & 1 & -1 \end{pmatrix} = 0$ $\quad \{|A+I| \neq 0\}$

Given, $-1 = 0$ (Rejected)

$\therefore$ $|A - \beta I| = \begin{vmatrix} 0 & 0 & 1 \\ 2 & 1-\beta & -2 \\ 3 & 1 & -2-\beta \end{vmatrix} = 0$

$\Rightarrow$ $2 - 3(1 - \beta) = 0$

$\Rightarrow$ $2 - 3 + 3\beta = 0$

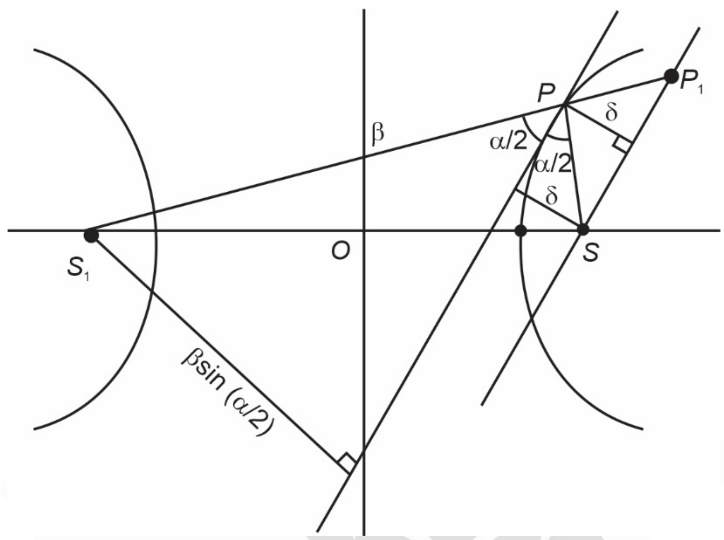
$\Rightarrow$ $\beta = \frac{1}{3}$

$\therefore$ $9\beta = 3$

7. Consider the hyperbola $\frac{x^2}{100}-\frac{y^2}{64}=1$ with foci at S and S_1, where S lies on the positive x-axis. Let P be a point on the hyperbola, in the first quadrant. Let $\angle SPS_1 = \alpha$, with $\alpha<\frac{\pi}{2}$. The straight line passing through the point S and having the same slope as that of the tangent at P to the hyperbola, intersects the straight line S_1P at P_1. Let δ be the distance of P from the straight line SP_1, and $\beta = S_1P$. Then the greatest integer less than or equal to $\frac{\beta\delta}{9}\sin\frac{\alpha}{2}$ is ______.

Answer (7)

Sol.



Product of distances of any tangent from two foci = b^2

$$\delta\times\beta\sin\frac{\alpha}{2}=b^2$$

So, $\frac{\beta\delta\sin\frac{\alpha}{2}}{9}=\frac{b^2}{9}=\frac{64}{9}$

Greatest integer less than or equal to

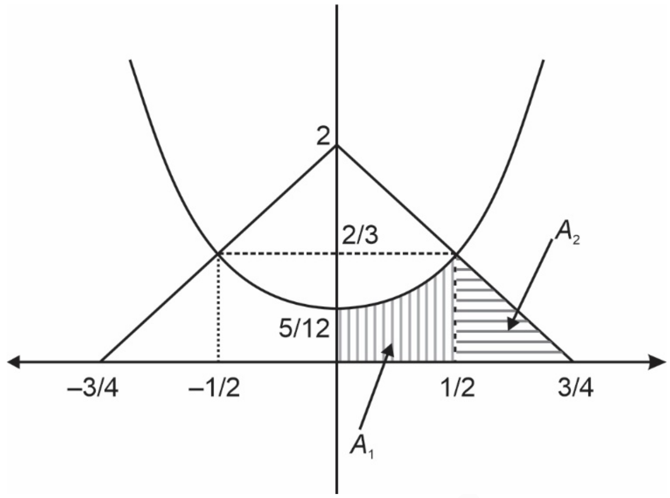
$\frac{\beta\delta\sin\frac{\alpha}{2}}{9}$ is 7.

8. Consider the functions $f,\ g:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^2+\frac{5}{12}$ and $g(x)=\begin{cases}2\left(1-\frac{4|x|}{3}\right), & |x|\le\frac{3}{4},\\ 0, & |x|>\frac{3}{4}.\end{cases}$

If α is the area of the region $\left\{(x,y)\in\mathbb{R}\times\mathbb{R}:|x|\le\frac{3}{4},\ 0\le y\le\min\{f(x),g(x)\}\right\}$, then the value of 9α is ______.

Answer (6)

Sol. Figure can be drawn as shown,



Required area = $2\cdot(A_1 + A_2)$

$$\Rightarrow \quad \alpha = \left(\int_0^{1/2}\left(x^2+\frac{5}{12}\right)dx+\frac{1}{2}\left(\frac{3}{4}-\frac{1}{2}\right)\cdot\frac{2}{3}\right)\times 2$$

$$\Rightarrow \quad \alpha = \left(\left(\frac{x^3}{3}+\frac{5x}{12}\right)\Bigg|_0^{1/2}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{2}{3}\right)\times 2$$

$$\Rightarrow \quad \alpha = \left(\frac{1}{4}+\frac{1}{12}\right)\times 2$$

$$\Rightarrow \quad \alpha = \frac{2}{3}$$

$$\Rightarrow \quad 9\alpha = 6$$

SECTION – 2 (Maximum marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

9. Let $PQRS$ be a quadrilateral in a plane, where $QR = 1$, $\angle PQR = \angle QRS = 70°$, $\angle PQS = 15°$ and $\angle PRS = 40°$. If $\angle RPS = \theta°$, $PQ = \alpha$ and $PS = \beta$, then the interval(s) that contain(s) the value of $4\alpha\beta\sin\theta°$ is/are

(A) $(0, \sqrt{2})$

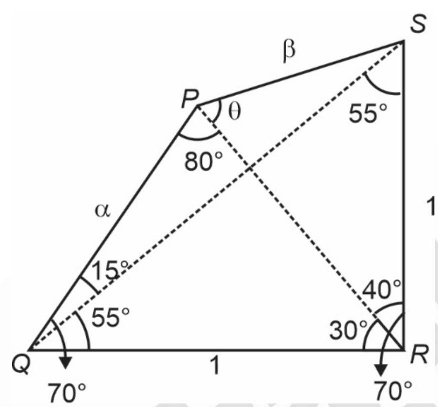
(B) $(1, 2)$

(C) $(\sqrt{2}, 3)$

(D) $(2\sqrt{2}, 3\sqrt{2})$

Answer (A, B)

Sol. Figure as shown, can be drawn on the basis of given data.



Applying sine rule in ΔPQR,

$$\frac{\alpha}{\sin 30°} = \frac{1}{\sin 80°}$$

$$\Rightarrow \quad \alpha = \frac{1}{2\sin 80°} \qquad \ldots \text{(i)}$$

Applying sine rule in ΔPRS,

$$\frac{\beta}{\sin 40°} = \frac{1}{\sin\theta}$$

$$\Rightarrow \quad \beta\sin\theta = \sin 40° \qquad \ldots \text{(ii)}$$

From (i) & (ii),

$$4\alpha\beta\sin\theta = 4\cdot\frac{1}{2\sin 80°}\cdot\sin 40° = \frac{1}{\cos 40°}$$

Using $\cos 0° = 1$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\cos 45° = \frac{1}{\sqrt{2}}$ and $\cos 60° = \frac{1}{2}$

Only Options (A) and (B) are correct.

10. Let

$$\alpha = \sum_{k=1}^{\infty} \sin^{2k}\left(\frac{\pi}{6}\right)$$

Let $g : [0, 1] \to \mathbb{R}$ be the function defined by

$g(x) = 2^{\alpha x} + 2^{\alpha(1-x)}$

Then, which of the following statements is/are TRUE?

(A) The minimum value of $g(x)$ is $2^{7/6}$

(B) The maximum value of $g(x)$ is $1 + 2^{1/3}$

(C) The function $g(x)$ attains its maximum at more than one point

(D) The function $g(x)$ attains its minimum at more than one point

Answer (A, B, C)

Sol. $\alpha = \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{2k} = \sum_{k=1}^{\infty}\left(\frac{1}{4}\right)^{k} = \frac{1/4}{1-\frac{1}{4}} = \frac{1}{3}$

Hence, $g(x) = 2^{x/3} + 2^{(1-x)/3}$

Now, $g'(x) = \frac{\ln 2}{3}\frac{(2^{2x/3} - 2^{1/3})}{2^{x/3}}$

$g'(x) = 0$ at $x = \frac{1}{2}$

And, derivative changes sign from negative to positive at $x = \frac{1}{2}$, hence $x = \frac{1}{2}$ is point of local minimum as well as absolute minimum of $g(x)$ for $x \in [0, 1]$

Hence, minimum value of $g(x) = g\left(\frac{1}{2}\right)$

$= 2^{1/6} + 2^{1/6} = 2^{7/6}$

$\Rightarrow$ Option (A) is correct

Maximum value of $g(x)$ is either equal to $g(0)$ or $g(1)$.

$g(0) = 1 + 2^{1/3}$

$g(1) = 2^{1/3} + 1$

Hence (B) and (C) are also correct

11. Let $\bar{z}$ denote the complex conjugate of a complex number z. If z is a non-zero complex number for which both real and imaginary parts of $(\bar{z})^2 + \frac{1}{z^2}$ are integers, then which of the following is/are possible value(s) of $|z|$?

(A) $\left(\frac{43+3\sqrt{205}}{2}\right)^{1/4}$

(B) $\left(\frac{7+\sqrt{33}}{4}\right)^{1/4}$

(C) $\left(\frac{9+\sqrt{65}}{4}\right)^{1/4}$

(D) $\left(\frac{7+\sqrt{13}}{6}\right)^{1/4}$

Answer (A)

Sol. Let $z = r.e^{i\theta}$

So, $(\bar{z})^2 + \dfrac{1}{z^2} = \left(r^2 + \dfrac{1}{r^2}\right)e^{-2i\theta} = a + ib$ (say), Where $a, b \in Z$

So, $\left(r^2 + \dfrac{1}{r^2}\right)^2 = a^2 + b^2$

$\Rightarrow$ $r^8 - (a^2 + b^2 - 2)r^4 + 1 = 0$

$\Rightarrow$ $r^4 = \dfrac{(a^2 + b^2 - 2) \pm \sqrt{(a^2 + b^2 - 2)^2 - 4}}{2}$

For $a^2 + b^2 = 45$ (*i.e* $(a, b) = (\pm 6, \pm 3)$ or $(\pm 3, \pm 6)$)

We get $r = \left(\dfrac{43 + 3\sqrt{205}}{2}\right)^{1/4}$

12. Let G be a circle of radius $R > 0$. Let $G_1, G_2, \ldots, G_n$ be n circles of equal radius $r > 0$. Suppose each of the n circles $G_1, G_2, \ldots, G_n$ touches the circle G externally. Also, for $i = 1, 2, \ldots, n - 1$, the circle G_i touches G_{i+1} externally, and G_n touches G_1 externally. Then, which of the following statements is/are TRUE ?

(A) If $n = 4$, then $(\sqrt{2} - 1)r < R$

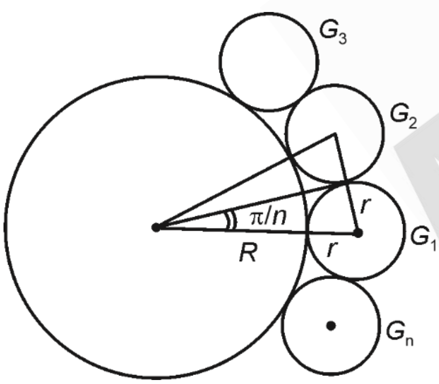
(B) If $n = 5$, then $r < R$

(C) If $n = 8$, then $(\sqrt{2} - 1)r < R$

(D) If $n = 12$, then $\sqrt{2}(\sqrt{3} + 1)r > R$

Answer (C, D)

Sol. Refer to diagram,



$\sin\left(\dfrac{\pi}{n}\right) = \dfrac{r}{R + r}$

$\Rightarrow$ $\dfrac{R}{r} + 1 = \operatorname{cosec}\left(\dfrac{\pi}{n}\right)$

$\Rightarrow$ $R = r\left[\operatorname{cosec}\left(\dfrac{\pi}{n}\right) - 1\right]$

(A) $n = 4,\ R = r(\sqrt{2} - 1)$

(B) $n = 5, R = r\left(\operatorname{cosec}\dfrac{\pi}{5} - 1\right)$

$\Rightarrow \quad R < r\left(\text{cosec}\frac{\pi}{6} - 1\right) \Rightarrow R < r$

(C) $n = 8, R = r\left[\text{cosec}\left(\frac{\pi}{8}\right) - 1\right]$

$\Rightarrow \quad R > r\left(\text{cosec}\left(\frac{\pi}{4}\right) - 1\right)$

$\Rightarrow \quad R > r\left(\sqrt{2} - 1\right)$

(D) $n = 12, R = r\left[\text{cosec}\left(\frac{\pi}{12}\right) - 1\right]$

$\Rightarrow \quad R = \left[\sqrt{2}\left(\sqrt{3} + 1\right) - 1\right]r$

$\Rightarrow \quad R < \sqrt{2}\left(\sqrt{3} + 1\right)r$

13. Let $\hat{i}, \hat{j}$ and $\hat{k}$ be the unit vectors along the three positive coordinate axes. Let

$$\vec{a} = 3\hat{i} + \hat{j} - \hat{k},$$
$$\vec{b} = \hat{i} + b_2\hat{j} + b_3\hat{k}, \quad b_2, b_3 \in \mathbb{R},$$
$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}, \; c_1, c_2, c_3 \in \mathbb{R}$$

be three vectors such that $b_2b_3 > 0$, $\vec{a} \cdot \vec{b} = 0$ and

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3 - c_1 \\ 1 - c_2 \\ -1 - c_3 \end{pmatrix}.$$

Then, which of the following is/are TRUE?

(A) $\vec{a} \cdot \vec{c} = 0$

(B) $\vec{b} \cdot \vec{c} - = 0$

(C) $\left|\vec{b}\right| > \sqrt{10}$

(D) $\left|\vec{c}\right| \le \sqrt{11}$

Answer (B, C, D)

Sol. $\left[\hat{i}\;\hat{j}\;\hat{k}\right]\begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ b_2 \\ b_3 \end{bmatrix} = \left[\hat{i}\;\hat{j}\;\hat{k}\right]\begin{bmatrix} 3 - c_1 \\ 1 - c_2 \\ -1 - c_3 \end{bmatrix}$

$\Rightarrow \quad \vec{c} \times \vec{b} = \vec{a} - \vec{c} \qquad \text{...(i)}$

$\Rightarrow \quad \left(\vec{c} \times \vec{b}\right) \cdot \vec{b} = \vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b} = 0$

$\Rightarrow \quad \vec{b} \cdot \vec{c} = 0$

Again from (i)

$$\left(\vec{c}\times\vec{b}\right)\cdot\vec{c}=\vec{a}\cdot\vec{c}-\left|\vec{c}\right|^2=0$$

$\Rightarrow\quad \left|\vec{c}\right|^2=\left|\vec{a}\right|\left|\vec{c}\right|\cos\theta$, where $\theta=\vec{a}\wedge\vec{c}$

$\Rightarrow\quad \left|\vec{c}\right|\geq\left|\vec{a}\right|\Rightarrow\left|\vec{c}\right|\leq\sqrt{11}$

Given that $\vec{a}\cdot\vec{b}=0\ \Rightarrow b_2-b_3+3=0$

$\Rightarrow b_3 - b_2 = 3$

Also $b_2 \cdot b_3 > 0$

Now $\left|\vec{b}\right|^2=1+b_2^2+b_3^2$

$= 1 + (b_3 - b_2)^2 + 2b_2b_3$

$= 10 + 2b_2b_3$

$\Rightarrow\quad \left|\vec{b}\right|^2>10\Rightarrow\left|\vec{b}\right|>\sqrt{10}$

14. For $x\in\mathbb{R}$, let the function $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx}+12y=\cos\left(\frac{\pi}{12}x\right),\ y(0)=0.$$

Then, which of the following statements is/are TRUE?

(A) $y(x)$ is an increasing function

(B) $y(x)$ is a decreasing function

(C) There exists a real number β such that the line $y = \beta$ intersects the curve $y = y(x)$ at infinitely many points

(D) $y(x)$ is a periodic function

Answer (C)

Sol. $\dfrac{dy}{dx}+12y=\cos\left(\dfrac{\pi x}{12}\right)$

$$I.F.=e^{12x}\Rightarrow y.e^{12x}=\int e^{12x}\cdot\cos\left(\frac{\pi x}{12}\right)dx+C$$

$$\Rightarrow\quad y\cdot e^{12x}=\frac{e^{12x}}{12^2+\left(\frac{\pi}{12}\right)^2}\left[12\cos\frac{\pi x}{12}+\frac{\pi}{12}\sin\left(\frac{\pi x}{12}\right)\right]+C$$

$$\because\quad y(0)=0\Rightarrow C=-\frac{12}{12^2+\left(\frac{\pi}{12}\right)^2}$$

So $y = \frac{1}{\lambda}\left[\underbrace{12\cos\left(\frac{\pi x}{12}\right) + \frac{\pi}{12}\sin\left(\frac{\pi x}{12}\right)}_{f_1(x)} - 12e^{-12x}\right]$

$$\frac{dy}{dx} = \frac{1}{\lambda}\left[\underbrace{-\pi\sin\left(\frac{\pi x}{12}\right) + \frac{\pi^2}{12^2}\cos\left(\frac{\pi x}{12}\right)}_{f_2(x)} + 12e^{-12x}\right]$$

When x is large then $12e^{-12x}$ tends to zero.

But $f_2(x)$ varies in $\left[-\sqrt{\pi^2 + \left(\frac{\pi}{12}\right)^4}, \sqrt{\pi^2 + \left(\frac{\pi}{12}\right)^4}\right]$

Hence $\frac{dy}{dx}$ is changing its sign.

So $y(x)$ is non monotonic for all real number.

Also when x is very large then again $-12e^{-12x}$ is almost zero but $f_1(x)$ is periodic, so there exist some β for which $y = \beta$ intersect $y = y(x)$ at infinitely many points.

SECTION – 3 (Maximum marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

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15. Consider 4 boxes, where each box contains 3 red balls and 2 blue balls. Assume that all 20 balls are distinct. In how many different ways can 10 balls be chosen from these 4 boxes so that from each box at least one red ball and one blue ball are chosen?

(A) 21816

(B) 85536

(C) 12096

(D) 156816

Answer (A)

Sol.

	Bag-1	Bag-2	Bag-3	Bag-4
	3R 2B	3R 2B	3R 2B	3R 2B
Case I	4 Balls	2 Balls	2 Balls	2 Balls
Case II	3 Balls (2R 1B, 2B 1R)	3 Balls (2R 1B, 2B 1R)	2 Balls	2 Balls

${}^4C_1$ [3 Red 1 Blue   2 Red 2 Blue] + Case II.

$$= {}^4C_1\left[{}^2C_1+{}^3C_1\right]\left({}^3C_1\,{}^2C_1\right)^3+{}^4C_2\left[{}^3C_2\,{}^2C_1+{}^3C_1\,{}^2C_2\right]^2\left[{}^3C_1\,{}^2C_1\right]^2$$

$$= 4(5)(6)^3+6(3\times 2+3)^2(6)^2$$

= 4320 + 17496

= 21816 (Option A)

16. If $M=\begin{pmatrix}\frac{5}{2} & \frac{3}{2}\\ -\frac{3}{2} & -\frac{1}{2}\end{pmatrix}$, then which of the following matrices is equal to M^{2022}?

(A) $\begin{pmatrix}3034 & 3033\\ -3033 & -3032\end{pmatrix}$

(B) $\begin{pmatrix}3034 & -3033\\ 3033 & -3032\end{pmatrix}$

(C) $\begin{pmatrix}3033 & 3032\\ -3032 & -3031\end{pmatrix}$

(D) $\begin{pmatrix}3032 & 3031\\ -3031 & -3030\end{pmatrix}$

Answer (A)

Sol. $A=\begin{pmatrix}\frac{5}{2} & \frac{3}{2}\\ -\frac{3}{2} & -\frac{1}{2}\end{pmatrix}$

$$A^2=\begin{pmatrix}4 & 3\\ -3 & -2\end{pmatrix}$$

$$A^3=\begin{pmatrix}\frac{11}{2} & \frac{9}{2}\\ -\frac{9}{2} & -\frac{7}{2}\end{pmatrix}$$

$$A^4=\begin{pmatrix}7 & 6\\ -6 & -5\end{pmatrix}$$

and so on

$$A^n = \begin{pmatrix} \frac{3n}{2}+1 & \frac{3n}{2} \\ -\frac{3n}{2} & -\frac{3n}{2}+1 \end{pmatrix}$$

Now,

$$A^{2022} = \begin{pmatrix} \frac{3\times 2022}{2}+1 & \frac{3\times 2022}{2} \\ \frac{-3\times 2022}{2} & \frac{-3\times 2022}{2}+1 \end{pmatrix}$$

$$= \begin{pmatrix} 3034 & 3033 \\ -3033 & -3032 \end{pmatrix}$$

$\therefore$ Option (A) is correct

17. Suppose that

Box-I contains 8 red, 3 blue and 5 green balls,

Box-II contains 24 red, 9 blue and 15 green balls,

Box-III contains 1 blue, 12 green and 3 yellow balls,

Box-IV contains 10 green, 16 orange and 6 white balls.

A ball is chosen randomly from Box-I; call this ball b. If b is red then a ball is chosen randomly from Box-II, if b is blue then a ball is chosen randomly from Box-III, and if b is green then a ball is chosen randomly from Box-IV. The conditional probability of the event 'one of the chosen balls is white' given that the event 'at least one of the chosen balls is green' has happened, is equal to

(A) $\frac{15}{256}$

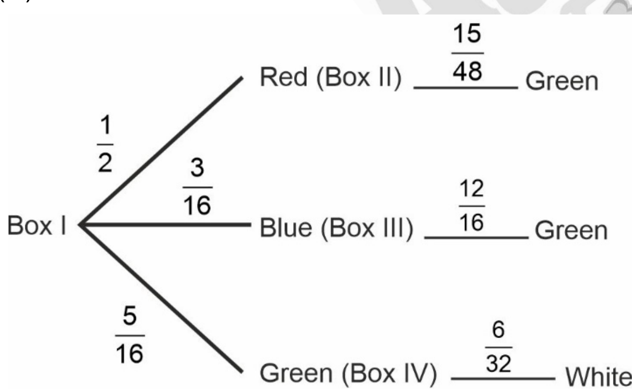
(B) $\frac{3}{16}$

(C) $\frac{5}{52}$

(D) $\frac{1}{8}$

Answer (C)

Sol.



Event A : One of the chosen ball is white

B : At least one of the chosen ball is green

$$P\left(\frac{A}{B}\right) = \frac{\frac{5}{16}\times\frac{6}{32}}{\frac{1}{2}\times\frac{15}{48}+\frac{3}{16}\times\frac{12}{16}+\frac{5}{16}\times 1}$$

$$= \frac{5}{52}$$

18. For positive integer n, define $f(n) = n + \frac{16+5n-3n^2}{4n+3n^2} + \frac{32+n-3n^2}{8n+3n^2} + \frac{48-3n-3n^2}{12n+3n^2} + \dots + \frac{25n-7n^2}{7n^2}$.

Then, the value of $\lim_{n\to\infty} f(n)$ is equal to

(A) $3 + \frac{4}{3}\log_e 7$

(B) $4 - \frac{3}{4}\log_e\left(\frac{7}{3}\right)$

(C) $4 - \frac{4}{3}\log_e\left(\frac{7}{3}\right)$

(D) $3 + \frac{3}{4}\log_e 7$

Answer (B)

Sol.

$$f(n) = n + \frac{16+5n-3n^2}{4n+3n^2} + \frac{32+n-3n^2}{8n+3n^2} + \dots\dots + \frac{25n-7n^2}{7n^2}$$

$$= \left(\frac{16+5n-3n^2}{4n+3n^2}+1\right) + \left(\frac{32+n-3n^2}{8n+3n^2}+1\right) + \dots\dots\dots + \left(\frac{25n-7n^2}{7n^2}+1\right)$$

$$f(n) = \frac{9n+16}{4n+3n^2} + \frac{9n+32}{8n+3n^2} + \dots\dots + \frac{25n}{7n^2}$$

$$= \sum_{r=1}^{n} \frac{9n+16r}{4rn+3n^2} = \frac{1}{n}\sum_{r=1}^{n} \frac{9+16\left(\frac{r}{n}\right)}{4\left(\frac{r}{n}\right)+3}$$

$$\lim_{n\to\infty} f(n) = \int_0^1 \frac{9+16x}{4x+3}dx$$

$$= \int_0^1 \frac{(16x+12)-3}{4x+3}dx$$

$$= \left(4x - \frac{3}{4}\ln|4x+3|\right)\Big|_0^1$$

$$= 4 - \frac{3}{4}\ln\frac{7}{3}$$

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