CHAPTER-WISE PREVIOUS YEARS' QUESTIONS

MATHEMATICS

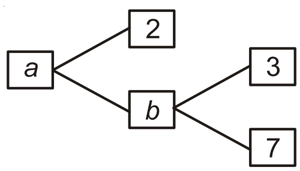
HINTS & SOLUTIONS

Class X (CBSE)

MATHEMATICS

1 : Real Numbers

1.



Let assume the missing entries be a, b.

$b = 3 \times 7 = 21$ **[½]**

$a = 2 \times b = 2 \times 21 = 42$ **[½]**

2. Given two numbers 100 and 190.

$\therefore$ HCF × LCM $= 100 \times 190$ **[½]**

$= 19000$ **[½]**

3. Given a rational number $\dfrac{441}{2^5 5^7 7^2}$.

$\therefore \quad \dfrac{441}{2^5 5^7 7^2} = \dfrac{9}{2^5 5^7}$ **[½]**

Since, the denominator is in the form of $2^m 5^n$. So, the rational number has terminating decimal expansion. **[½]**

4. Smallest prime number is 2.

Smallest composite number is 4.

Therefore, HCF is 2. **[1]**

5. Rational number lying between $\sqrt{2}$ and $\sqrt{3}$ is

$1.5 = \dfrac{15}{10} = \dfrac{3}{2}$ **[½]**

[$\because \sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.732$] **[½]**

6. Answer (b)

$144 = 2^4 \times 3^2$

$198 = 2 \times 3^2 \times 11$

HCF $= 2 \times 3^2$

$= 18$

Hence, option (b) is correct. **[1]**

7. Answer (c)

Prime factorisation of 225 is given below,

3	225
3	75
5	25
5	5
	1

$\therefore \quad 225 = 3^2 \times 5^2$

Option (c) is correct. **[1]**

8. Answer (b)

$2.\overline{35}$ is a non-terminating recurring decimal. **[1]**

9. Answer (c) **[1]**

Total number of factors of a prime number is 2

Hence, option (c) is correct.

10. Answer (c) **[1]**

$12 = 2 \times 2 \times 3$

$21 = 3 \times 7$

$15 = 5 \times 3$

$\therefore$ HCF = 3

L.C.M $= 2 \times 2 \times 3 \times 5 \times 7$

$= 420$

Hence, option (c) is correct.

11. Answer (a) **[1]**

$92 = 2 \times 2 \times 23$

$152 = 2 \times 2 \times 2 \times 19$

H. C. F $(92, 152) = 2 \times 2 = 4$

12. Answer (b) **[1]**

$$\frac{57}{300} = \frac{19}{100} = \frac{19}{2^2 5^2}$$

Since, denominator is of the form of $2^m 5^n$ and $m = n = 2$. So, fraction will terminate after 2 places of decimals.

13. Answer (a) **[1]**

$5.\overline{213} = 5.213213213...$

14. Answer (c) **[1]**

Let numbers be $2x$ and $2x + 2$.

$2x = 2 \times x$

$2x + 2 = 2(x + 1)$

H. C. F = 2

15. Answer (d) **[1]**

Since, the denominator of $\frac{13}{2 \times 5^2 \times 7}$ has 7.

So, it is not terminating but it is rational number.

So, given number is non-terminating but repeating.

16. Answer (a) **[1]**

HCF × LCM = Product of numbers

= 50 × 20

= 1000

17. Answer (d) **[1]**

For 6^n, where n belongs to natural number, the given number never ends with zero.

18. Answer (b)

3750 = 2 × 3 × 5 × 5 × 5 × 5

= $2^1 \times 3^1 \times 5^4$

19. Answer (b) **[1]**

95 = 5 × 19 and 171 = 9 × 19

$\Rightarrow$ HCF (95, 171) = 19

20. Answer (c) **[1]**

LCM (20, 25, 30) = 300 minutes

= 5 hours

21. Answer (d) **[1]**

Greatest number =

$\text{H.C.F.}\left[(1251-1), (9377-2) \text{ and } (15628-3)\right]$

$= \text{H.C.F.}[1250, 9375, 15625]$

= 625

22. Answer (a) **[1]**

a^3 and b^3 will be co-prime, if a, b are co-prime.

23. Answer (d) **[1]**

Unit digit of 5^n and 6^n are 5 and 6 respectively. [$\because$ n is a natural number]

$\therefore$ Unit's digit of $2(5^n + 6^n) = 2 \times (5 + 6)$

= 2 × 11

= 22 (*i.e.* 2)

24. Answer (c) **[1]**

2400 is not divisible by 500.

25. Let us assume that $\left(5 + 3\sqrt{2}\right)$ is rational. Then there exist co-prime positive integers a and b such that

$5 + 3\sqrt{2} = \frac{a}{b}$ **[½]**

$3\sqrt{2} = \frac{a}{b} - 5$

$\sqrt{2} = \frac{a-5b}{3b}$ **[½]**

$\Rightarrow$ $\sqrt{2}$ is irrational.

[$\because$ a, b are integers, $\therefore$ $\frac{a-5b}{3b}$ is rational].

[½]

This contradicts the fact that $\sqrt{2}$ is irrational.

So, our assumption is incorrect. **[½]**

Hence, $\left(5 + 3\sqrt{2}\right)$ is an irrational number.

26. Since 7344 > 1260

7344 = 1260 × 5 + 1044 **[½]**

Since remainder $\neq$ 0

1260 = 1044 × 1 + 216

1044 = 216 × 4 + 180 **[½]**

216 = 180 × 1 + 36

180 = 36 × 5 + 0 **[½]**

The remainder has now become zero.

$\therefore$ HCF of 1260 and 7344 is 36. **[½]**

27. Let a be positive odd integer.

Using division algorithm on a and $b = 4$ **[½]**

$a = 4q + r$

Since $0 \le r < 4$, the possible remainders are 0, 1, 2 and 3. **[½]**

$\therefore$ a can be $4q$ or $4q + 1$ or $4q + 2$ or $4q + 3$, where q is the quotient.

Since a is odd, a cannot be $4q$ and $4q + 2$.

[½]

$\therefore$ Any odd integer is of the form $4q + 1$ or $4q + 3$, where q is some integer. **[½]**

28. Let 'a' be any positive integer and $b = 3$.

We know $a = bq + r$, $0 \le r < b$.

Now, $a = 3q + r$, $0 \le r < 3$.

The possible remainder = 0, 1 or 2

Case (i) $a = 3q$

$a^2 = 9q^2$

$= 3 \times (3q^2)$

$= 3m$ (where $m = 3q^2$) **[1]**

Case (ii) $a = 3q + 1$

$a^2 = (3q + 1)^2$

$= 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

$= 3m + 1$ (where $m = 3q^2 + 2q$) **[1]**

Case (iii) $a = 3q + 2$

$a^2 = (3q + 2)^2$

$= 9q^2 + 12q + 4$

$= 3(3q^2 + 4q + 1) + 1$

$= 3m + 1$ (where $m = 3q^2 + 4q + 1$)

From all the above cases it is clear that square of any positive integer (as in this a^2) is either of the form $3m$ or $3m + 1$. **[1]**

29. Let assume $3+\sqrt{2}$ is a rational number.

$\therefore \quad 3+\sqrt{2}=\dfrac{p}{q}$

{p, q are co-prime integers and $q \neq 0$} **[1]**

$\Rightarrow \quad \sqrt{2}=\dfrac{p}{q}-3$

$\Rightarrow \quad \sqrt{2}=\dfrac{p-3q}{q}$ **[1]**

Since, $\dfrac{p-3q}{q}$ is a rational number but we know

$\sqrt{2}$ is an irrational.

$\therefore$ Irrational $\neq$ rational

$\therefore \quad 3+\sqrt{2}$ is not a rational number. **[1]**

30. Let assume $2 - 3\sqrt{5}$ is a rational number.

$\Rightarrow \quad 2 - 3\sqrt{5} = \dfrac{p}{q}$,

(where p, q are co-prime integers and $q \neq 0$)

$\Rightarrow \quad 2-\dfrac{p}{q}=3\sqrt{5}$ **[1]**

$\Rightarrow \quad \dfrac{2q-p}{3q}=\sqrt{5}$

Since, $\dfrac{2q-p}{3q}$ is a rational number but we also know $\sqrt{5}$ is an irrational **[1]**

$\therefore$ Rational $\neq$ irrational.

$\Rightarrow$ Our assumption is wrong.

$\therefore \quad 2 - 3\sqrt{5}$ is an irrational number. **[1]**

31. Using the factor tree for the prime factorization of 404 and 96, we have

$404 = 2^2 \times 101$ and $96 = 2^5 \times 3$

To find the HCF, we list common prime factors and their smallest exponent in 404 and 96 as under :

Common prime factor = 2, Least exponent = 2

$\therefore$ HCF $= 2^2 = 4$ **[1]**

To find the LCM, we list all prime factors of 404 and 96 and their greatest exponent as follows :

Prime factors of 404 and 96	**Greatest Exponent**
2	5
3	1
101	1

$\therefore$ LCM $= 2^5 \times 3^1 \times 101^1$

$= 2^5 \times 3^1 \times 101^1$

$= 9696$ **[1]**

Now,

HCF × LCM = 9696 × 4 = 38784

Product of two numbers = 404 × 96 = 38784

Therefore, HCF × LCM = Product of two numbers. **[1]**

32. Let $\sqrt{2}$ be rational. Then, there exist positive integers a and b such that $\sqrt{2}=\dfrac{a}{b}$. [where a and b are co-prime, $b \neq 0$]. **[½]**

$\Rightarrow \quad (\sqrt{2})^2=\left(\dfrac{a}{b}\right)^2$ **[½]**

$\Rightarrow \quad 2=\dfrac{a^2}{b^2}$

$\Rightarrow$ $2b^2 = a^2$

$\therefore$ 2 divides a^2

$\Rightarrow$ 2 divides a ...(i)

Let $a = 2c$ for some integer c. **[½]**

$a^2 = 4c^2$

$\Rightarrow$ $2b^2 = 4c^2$

$\Rightarrow$ $b^2 = 2c^2$

$\therefore$ 2 divides b^2

$\Rightarrow$ 2 divides b ...(ii) **[½]**

From (i) and (ii), we get

2 is common factor of both a and b.

But this contradicts the fact that a and b have no common factor other than 1. **[½]**

$\therefore$ Our supposition is wrong.

Hence, $\sqrt{2}$ is an irrational number. **[½]**

33. Let $5+2\sqrt{3}$ be a rational number.

$5+2\sqrt{3} = \frac{p}{q}$, where p and q are co-prime integers. **[½]**

$\Rightarrow$ $2\sqrt{3} = \frac{p}{q} - 5$

$= \frac{p-5q}{q}$ **[½]**

$\Rightarrow$ $\sqrt{3} = \frac{p-5q}{2q}$ **[½]**

Here, $\frac{p-5q}{2q}$ is rational as p and q are integers. **[½]**

But it is given that $\sqrt{3}$ is irrational.

$\Rightarrow$ LHS is irrational and RHS is rational. **[½]**

which contradicts our assumption that $5+2\sqrt{3}$ is a rational number.

$\therefore$ $5+2\sqrt{3}$ is an irrational number. **[½]**

OR

For maximum number of columns, we need to find highest common factor i.e., HCF of 612 and 48. **[½]**

Now,

$612 = 48 \times 12 + 36$ **[½]**

$48 = 36 \times 1 + 12$ **[½]**

$36 = 12 \times 3 + 0$ **[½]**

$\therefore$ HCF of 612 and 48 is 12. **[½]**

$\therefore$ Maximum number of columns in which they can march is 12. **[½]**

34. Let a be any positive integer and $b = 5$

Then, by Euclid's division Lemma

$a = 5m + r$ for some integer $m \geq 0$ and $r = 0$, 1, 2, 3, 4 **[½]**

So, $a = 5m$ or $5m + 1$ or $5m + 2$ or $5m + 3$ or $5m + 4$ **[½]**

$(5m)^2 = 25m^2 = 5(5m^2)$ **[½]**

$= 5q$, where q is any integer

$(5m + 1)^2 = 25m^2 + 10m + 1$

$= 5(5m^2 + 2m) + 1$ **[½]**

$= 5q + 1$, where, q is any integer

$(5m + 2)^2 = 25m^2 + 20m + 4$

$= 5(5m^2 + 4m) + 4$ **[½]**

$= 5q + 4$, where, q is any integer

$(5m + 3)^2 = 25m^2 + 30m + 9$

$= 5(5m^2 + 6m + 1) + 4$

$= 5q + 4$, where, q is any integer **[½]**

$(5m + 4)^2 = 25m^2 + 40m + 16$

$= 5(5m^2 + 8m + 3) + 1$ **[½]**

$= 5q + 1$, where q is any integer

Hence, square of any positive integer cannot be of the form **[½]**

$(5q + 2)$ or $(5q + 3)$ for any integer q.

OR

Let n, $(n + 1)$, $(n + 2)$ be three consecutive positive integers. **[1]**

Then by Euclid's division Lemma

$n = 3q + r$ for some integer $q \geq 0$ and $r = 0, 1, 2$ **[1]**

Case (i) when $n = 3q$:

In this case,

n is divisible by 3 but $(n + 1)$ and $(n + 2)$ are not divisible by 3 **[½]**

Case (ii) when $n = 3q + 1$,

In this case,

$n + 2 = 3q + 1 + 2 = 3(q + 1)$ is divisible by 3 but n and (n + 1) are not divisible by 3. **[½]**

Case (iii) when $n = 3q + 2$,

In this case,

$n + 1 = 3q + 2 + 1 = 3(q + 1)$ is divisible by 3 but n and $(n + 2)$ are not divisible by 3. **[½]**

Hence, one of n, $(n + 1)$ and $(n + 2)$ is divisible by 3. **[½]**

2 : Polynomials

1. $(x + a)$ is factor of the polynomial $p(x) = 2x^2 + 2ax + 5x + 10$.

   $\therefore$ $p(-a) = 0$ {By factor theorem}

   $2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$ **[½]**

   $2a^2 - 2a^2 - 5a + 10 = 0$ **[½]**

   $a = 2$

2. If $x = 1$ is the zero of the polynomial

   $\therefore$ $p(x) = ax^2 - 3(a - 1)x - 1$

   Then $p(1) = 0$ **[½]**

   $\therefore$ $a(1)^2 - 3(a - 1) - 1 = 0$

   $-2a + 2 = 0$

   $a = 1$ **[½]**

3. Given α and β are the zeroes of quadratic polynomial with $\alpha + \beta = 6$ and $\alpha\beta = 4$.

   Quadratic polynomial = $k[x^2 - 6x + 4]$, where k is real. **[1]**

4. Answer (d)

   2 is a zero of polynomial $p(x) = kx^2 + 3x + k$.

   $\Rightarrow$ $p(2) = 0$

   $\Rightarrow$ $k(2^2) + 3(2) + k = 0$

   $\Rightarrow$ $4k + 6 + k = 0$

   $\Rightarrow$ $5k = -6$

   $\therefore$ $k = \dfrac{-6}{5}$

   Option (d) is correct. **[1]**

5. Answer (a)

   Graph of given polynomial cuts the x-axis at 3 distinct points. **[1]**

   $\therefore$ Number of zeroes is 3.

6. Answer (b) **[1]**

   Let $f(x) = x^2 + 3x + k$

   $f(2) = (2)^2 + 3(2) + k = 0$

   $\Rightarrow$ $4 + 6 + k = 0$

   $\Rightarrow$ $k = -10$

   Hence, option (b) is correct.

7. Answer (a) **[1]**

   Quadratic polynomial

   $= x^2 -$ (sum of zeroes)x + product of zeroes

   $= x^2 - (-5)x + 6$

   $= x^2 + 5x + 6$

   Hence, option (a) is correct.

8. Answer (b) **[1]**

   $K[x^2 -$ (sum of zeroes) x + (product of zeroes], where K is a non-zero constant.

   $\therefore$ $p(x) = K[x^2 - 5x]$

9. Answer (c) **[1]**

   $p(x) = x^2 - 5x + 6 = 0$

   $(x - 2)(x - 3) = 0$

   $x = 2$, $x = 3$

10. Answer (a) **[1]**

    Polynomial, $p(x) = x^2 + 99x + 127$

    Sum of zeroes $= -\dfrac{b}{a} = -99 =$ negative

    Product of zeroes $= \dfrac{c}{a} = 127 =$ positive

    So, both zeroes must be negative.

11. Answer (c) **[1]**

    As, we can see from the graph maximum height is achieved at $t = 1$ s.

    Height attained at $t = 1$ s

    $h = -(1)^2 + 2(1) + 8 = 9$ m

12. Answer (b) **[1]**

    Quadratic polynomial

13. Answer (c) **[1]**

    As, we can see from the graph ball reach maximum height at $t = 1$ s.

14. Answer (b) **[1]**

    Since, it is a quadratic polynomial so, it will have 2 zeroes.

15. Answer (b) **[1]**

    Zeroes of the polynomial, $h = -t^2 + 2t + 8 = 0$

    $-(t^2 - 2t - 8) = 0$

    $\Rightarrow$ $(t - 4)(t + 2) = 0$

    $t = 4$ and $t = -2$

16. Answer (d) **[1]**

Zeroes of a polynomial $f(x)$ would be those points where the graph $f(x)$ will touch or cut the x-axis.

$\therefore$ Number of zeroes = 5

17. Answer (c) **[1]**

Graph intersects x-axis at 3 points.

18. Answer (a) **[1]**

Required polynomial = $k[x^2 - 8x + 5]$

19. Answer (b) **[1]**

$p(1) = 1 + a + 2b = 0$

$\Rightarrow a + 2b = -1$

and $a + b = 4$

$\Rightarrow b = -5$ and $a = 9$

20. Answer (b) **[1]**

$\alpha + \beta = k + 6$ and $\alpha\beta = 4k - 2$

$$\alpha+\beta = \frac{\alpha\beta}{2}$$

$\Rightarrow k + 6 = 2k - 1$

$\therefore k = 7$

21. $p(x) = x^4 + x^3 - 34x^2 - 4x + 120$

Let assume other two zeroes be α, β.

Sum of all zeroes = $\alpha + \beta + 2 - 2$

$= \alpha + \beta$

$\alpha + \beta = -1$

$\Rightarrow \boxed{\alpha = -1 - \beta}$ ...(i)

Product of zeroes = 120

$\alpha.\beta.2.(-2) = 120$

$\boxed{\alpha\beta = -30}$ ...(ii) **[1]**

Substituting (i) in (ii), we get

$\beta(-1 - \beta) = -30$

$\beta + \beta^2 = 30$

$\beta^2 + \beta - 30 = 0$

$\therefore \beta = -6, 5$

$\alpha = 5, -6$

Zeroes of the polynomial are –6, –2, 2, 5. **[1]**

22. $x^3 + 3x^2 - 2x - 6 = 0$

Given two zeros are $-\sqrt{2}, \sqrt{2}$

Sum of all zeros = –3 **[1]**

Let the third zero be x

$\therefore \; x + \sqrt{2} + \left(-\sqrt{2}\right) = -3$

$x = -3$

$\therefore$ All zeroes will be $-3, -\sqrt{2}, \sqrt{2}$ **[1]**

23. Given a polynomial

$x^3 - 4x^2 - 3x + 12$

Sum of all the zeroes of polynomial = –(–4) = 4

Given two zeroes are $\sqrt{3}, -\sqrt{3}$. **[1]**

Say the third zero = α

$\Rightarrow \alpha + \sqrt{3} - \sqrt{3} = 4$

$\therefore \boxed{\alpha = 4}$ **[1]**

$\Rightarrow$ Third zero is 4.

24. **[1½]**

$$\begin{array}{r|l} & 2x+1 \\ \hline 2x+1 & 4x^2+4x+5 \\ & \underline{-4x^2 \pm 2x} \\ & \quad\quad 2x+5 \\ & \quad\quad \underline{-2x \pm 1} \\ & \quad\quad\quad 4 \end{array}$$

$\therefore$ Quotient on dividing $(4x^2 + 4x + 5)$ by $(2x + 1)$ is $2x + 1$ and remiander = 4 **[½]**

25. It is given that $\left(2+\sqrt{3}\right)$ and $\left(2-\sqrt{3}\right)$ are two zeros of $f(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

$$\left\{x-\left(2+\sqrt{3}\right)\right\}\left\{x-\left(2-\sqrt{3}\right)\right\}$$

$$= \left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right)$$

$$= (x-2)^2 - \left(\sqrt{3}\right)^2$$

$= x^2 - 4x + 1$ **[1]**

$\therefore$ $(x^2 - 4x + 1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & 2x^2 - x - 1 \\ \hline x^2-4x+1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \\ & 2x^4 - 8x^3 + 2x^2 \\ & (-)\quad (+)\quad (-) \\ \hline & -x^3 + 3x^2 + 3x - 1 \\ & -x^3 + 4x^2 - x \\ & (+)\quad (-)\quad (+) \\ \hline & -x^2 + 4x - 1 \\ & -x^2 + 4x - 1 \\ & (+)\quad (-)\quad (+) \\ \hline & 0 \\ \hline \end{array}$$

We have,

$\therefore$ $f(x) = (x^2 - 4x + 1)(2x^2 - x - 1)$ **[1]**

Hence, other two zeros of $f(x)$ are the zeros of the polynomial $2x^2 - x - 1$.

We have,

$$2x^2 - x - 1 = 2x^2 - 2x + x - 1$$
$$= 2x(x - 1) + 1(x - 1)$$
$$= (2x + 1)(x - 1)$$

$$f(x) = \left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right)(2x + 1)(x - 1)$$

Hence, the other two zeros are $-\frac{1}{2}$ and 1. **[1]**

26. For given polynomial

$x^2 - (k + 6)x + 2(2k - 1),$ **[½]**

Let the zeroes be α and β.

So, $\alpha + \beta = -\frac{b}{a} = k + 6,\ \ \alpha\beta = \frac{c}{a} = \frac{4k-2}{1}$ **[1]**

$\because$ Sum of zeroes $= \frac{1}{2}$ (product of zeroes)

$\Rightarrow\ \ \alpha + \beta = \frac{1}{2}\alpha\beta$ **[½]**

$\Rightarrow\ \ k + 6 = \frac{1}{2}(4k - 2)$

$\Rightarrow\ \ k + 6 = 2k - 1$

$\therefore\ \ k = 7$

So, the value of k is 7. **[1]**

27. α and β are zeroes of the polynomial $f(x) = x^2 - 4x - 5$

$\Rightarrow\ \ \alpha + \beta = -\frac{b}{a} = 4$ and $\alpha\beta = \frac{c}{a} = -5,$ where a = 1, b = –4, c = –5 **[1]**

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ **[½]**

$= (4)^2 - 2(-5)$ **[½]**

$= 16 + 10$ **[½]**

$= 26$ **[½]**

28. Let α and β are the zeroes of the polynomial $f(x) = ax^2 + bx + c$.

$\therefore\ \ (\alpha + \beta) = \frac{-b}{a}$ ...(i) **[½]**

and $\alpha\beta = \frac{c}{a}$ ...(ii) **[½]**

According to the question, $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the zeroes of the required quadratic polynomial

$\therefore$ Sum of zeroes of required polynomial

$$S' = \frac{1}{\alpha} + \frac{1}{\beta}$$

$$= \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{-b}{c}$$ ...(iii) **[½]**

[From equation (i) and (ii)]

and product of zeroes of required polynomial $= \frac{1}{\alpha} \times \frac{1}{\beta}$.

$$P' = \frac{1}{\alpha\beta}$$

$$= \frac{a}{c}$$ ...(iv) **[½]**

[From equation (ii)]

$\therefore$ Equation of the required quadratic polynomial

$= k\left(x^2 - S'x + P'\right)$, where k is any non-zero constant **[½]**

$$= k\left(x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c}\right)$$

[From equation (iii) and (iv)]

$$= k\left(x^2 + \frac{b}{c}x + \frac{a}{c}\right)$$ **[½]**

OR

Using long division method,

$$\begin{array}{r|l} & x - 2 \\ -x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\ & -x^3 + x^2 - x \\ & \underline{+ \quad - \quad +} \\ & 2x^2 - 2x + 5 \\ & 2x^2 - 2x + 2 \\ & \underline{- \quad + \quad -} \\ & 3 \end{array}$$ **[1]**

Clearly, quotient $q(x) = (x - 2)$ and remainder $r(x) = 3$ **[1]**

Now,

(Quotient × Divisor) + Remainder **[½]**

$= (x - 2)(-x^2 + x - 1) + 3$

$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5 =$ Dividend **[½]**

Hence, the division algorithm is verified.

29. Let $f(x) = x^3 - 3x^2 - 10x + 24$

$f(x)$ is divisible by $(x - 4)$ **[½]**

$$
\begin{array}{r|l}
 & x^2 + x - 6 \\
x - 4 & x^3 - 3x^2 - 10x + 24 \\
 & \underline{\overset{-}{x^3} \overset{+}{- 4x^2}} \\
 & x^2 - 10x + 24 \\
 & \underline{\overset{-}{x^2} \overset{+}{- 4x}} \\
 & -6x + 24 \\
 & \underline{\overset{+}{-6x} \overset{-}{+ 24}} \\
 & 0
\end{array}
$$

[1½]

$\therefore$ $x^2 + x - 6 = x^2 + 3x - 2x - 6$ **[½]**

$= x(x + 3) -2(x + 3)$

$= (x - 2)(x + 3)$ **[½]**

$\therefore$ Other two zeroes of the given polynomial are 2 and –3.

3 : Pair of Linear Equations in Two Variables

1. $x + 2y - 8 = 0$

$2x + 4y - 16 = 0$

For any pair of linear equations

$a_1x + b_2y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then **[½]**

There exists infinite solutions

Here $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4}, \frac{c_1}{c_2} = \frac{-8}{-16}$

$\therefore$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

$\therefore$ Lines are coincident and will have infinite solutions. **[½]**

2. For any real number except $k = -6$ **[1]**

$kx - 2y = 3$ and $3x + y = 5$ represent lines intersecting at a unique point.

$\Rightarrow$ $\frac{k}{3} \neq \frac{-2}{1}$

$\Rightarrow$ $k \neq -6$

For any real number except $k = -6$

The given equation represent two intersecting lines at unique point.

3. Answer (d) **[1]**

For no solution; $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ $\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3}$

$\Rightarrow$ $\boxed{k = 2}$

Hence, option (d) is correct.

4. Answer (b) **[1]**

For no solution,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow$ $\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3}$

$\Rightarrow$ $k = 2$

5. Answer (d) **[1]**

Perimeter, $2(l + b) = 14$ …(i)

$l = 2b + 4$ …(ii)

6. Answer (a) **[1]**

(–5, 6) is the solution of $x = -5$ and $y = 6$.

7. Answer (b) **[1]**

For, infinitely many solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow$ $\frac{3}{k} = \frac{5}{15} = \frac{8}{24}$

$k = 9$

8. Answer (a) **[1]**

$32x + 33y = 34$ …(i)

$33x + 32y = 31$ …(ii)

Adding equation (i) and (ii) and subtracting equation (ii) from (i), we get

$65x + 65y = 65$ or $x + y = 1$ …(iii)

and $-x + y = 3$ …(iv)

Adding equation (iii) and (iv), we get

$y = 2$

Substituting the value of y in equation (iii),

$x = -1$

9. Answer (c) **[1]**

If two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

It can only possible between $3x - 2y = 5$ and $-12x + 8y = 7$.

Solution for 10 to 14 :

For Amruta, $x + (6 - 2)y = 22$

i.e., $x + 4y = 22$ …(i)

For Radhika, $x + (4 - 2)y = 16$

i.e., $x + 2y = 16$ …(ii)

Solving equation (i) and (ii), we get

$x = 10$ and $y = 3$

i.e., Fixed charges (x) = ₹10 …(iii)

and additional charges per subsequent day (y) = ₹3 …(iv)

10. Answer (d) **[1]**

$x + 2y = 16$ [From equation (ii)]

11. Answer (c) **[1]**

$x + 4y = 22$ [From equation (i)]

12. Answer (b) **[1]**

x = ₹10 [From equation (iii)]

13. Answer (d) **[1]**

y = ₹3 [From equation (iv)]

14. Answer (c) **[1]**

Total amount paid for 2 more days by both

$= (x + 4y) + 2y + (x + 2y) + 2y$

$= 2x + 10y$

$= 2 \times 10 + 10 \times 3$

= ₹50

15. $2x + 3y = 7$

$(k - 1)x + (k + 2)y = 3k$

For this pair of linear equations to have infinitely many solutions, they need to be coincident **[½]**

$$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2} = \frac{7}{3k}$$ **[½]**

Upon solving we get

$\boxed{k = 7}$ **[1]**

16. Since it is a rectangle

$\ell(AB) = \ell(CD)$

$x + y = 30$ ...(i) **[½]**

$\ell(AD) = \ell(BC)$

$x - y = 14$ ...(ii) **[½]**

Adding (i) and (ii), we get

$2x = 44$

$x = 22$ **[½]**

Putting $x = 22$ in equation (ii)

$22 - y = 14 \Rightarrow 22 - 14 = y$

$\therefore\ y = 8$

$\therefore\ x = 22$ and $y = 8$ **[½]**

17. For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ **[½]**

$$\underset{\text{I}}{\frac{c}{12}} = \underset{\text{II}}{\frac{3}{c}} = \underset{\text{III}}{\frac{3-c}{-c}}$$

(i) $c^2 = 12 \times 3$ [From I and II]

$c = \pm 6$ **[½]**

(ii) $\frac{3}{c} = \frac{3-c}{-c}$ [From II and III]

$-3c = 3c - c^2$

$c^2 - 6c = 0$

$c = 0, 6$

(iii) $c^2 = 12(c - 3)$ [From I and III] **[½]**

$c^2 - 12c + 36 = 0$

$(c - 6)^2 = 0$

$c = 6$

Hence the value of c is 6. **[½]**

18. $x + 3y = 6$

$2x - 3y = 12$

Graph of $x + 3y = 6$:

When $x = 0$, we have $y = 2$ and when $y = 0$, we have $x = 6$. **[½]**

Therefore, two points on the line are (0, 2) and (6, 0). **[½]**

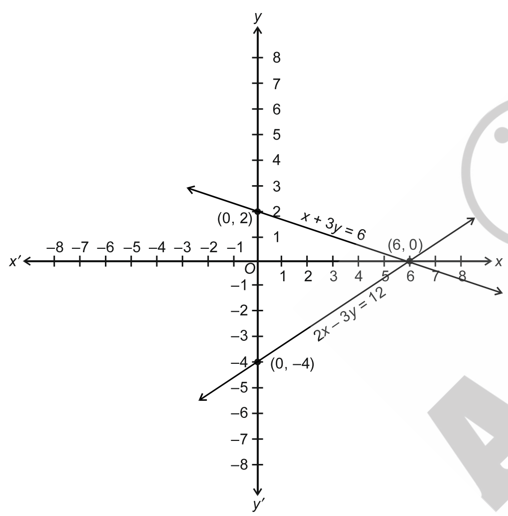
The line $x + 3y = 6$ is represented in the given graph.

Graph of $2x - 3y = 12$:

When $x = 0$, we have $y = -4$ and when $y = 0$, we have $x = 6$. **[½]**

Hence, the two points on the line are (0, –4) and (6, 0). **[½]**

The line $2x - 3y = 12$ is shown in the graph.



[½]

The line $x + 3y = 6$ intersects y-axis at (0, 2) and the line $2x - 3y = 12$ intersects y-axis at (0, –4). **[½]**

19. $\frac{ax}{b} - \frac{by}{a} = a + b$ ...(i)

$ax - by = 2ab$ ...(ii) **[½]**

Multiply (ii) with $\frac{1}{b}$ and subtract (i) from (ii)

$$\frac{a}{b}x - y = 2a$$

$$-\frac{ax}{b} \underset{+}{-} \frac{by}{a} = -a \underset{-}{+} b$$ **[1]**

$$y\left(\frac{b-a}{a}\right) = a - b$$ **[½]**

$y = -a$

Substituting $y = -a$ in (i)

$$\frac{a}{b}x - \frac{b}{a}(-a) = a + b$$ **[½]**

$$\frac{a}{b}x = a$$

$x = b$

$\therefore$ $x = b$ and $y = -a$ **[½]**

20. Lets say numerator = x

Denominator = y

Given $x + y = 2y - 3$

$\Rightarrow$ $\boxed{x - y + 3 = 0}$ ...(i) **[1]**

From the next condition

$$\frac{x-1}{y-1} = \frac{1}{2}$$

$\boxed{2x - y - 1 = 0}$ ...(ii) **[1]**

Solving (i) and (ii)

$x = 4$

$y = 7$

$\therefore$ Fraction $= \frac{4}{7}$ **[1]**

21. $\frac{4}{x} + 3y = 8$ ...(i) **[½]**

$\frac{6}{x} - 4y = -5$ ...(ii) **[½]**

Multiplying 4 to (i) and 3 to (ii)

$$\frac{16}{x} + 12y = 32$$

$$\frac{18}{x} - 12y = -15$$ **[½]**

$$\frac{34}{x} = 17$$

$\boxed{x = 2}$ **[½]**

Substitute

$x = 2$ in (i)

$2 + 3y = 8$

$3y = 6$

$y = 2$ **[½]**

$\therefore$ $x = 2$

$y = 2$ **[½]**

22. Let the present age of father be x years and the sum of present ages of his two children be y years. **[½]**

According to question

$x = 3y$ **[½]**

$\Rightarrow \quad x - 3y = 0 \quad$ ...(i)

After 5 years,

$x + 5 = 2(y + 10)$

$\Rightarrow \quad x - 2y = 15 \quad$ ...(ii) **[½]**

On subtracting equation (i) from (ii), we get :

$$\begin{array}{rcrcr} x & - & 2y & = & 15 \\ x & - & 3y & = & 0 \\ - & + & & & - \\ \hline & & y & = & 15 \end{array}$$ **[1]**

On substituting the value of $y = 15$ in (i), we get :

$x - 3 \times 15 = 0$

$\therefore \quad x = 45$ **[½]**

Hence, the present age of father is 45 years.

23. Let the numerator of required fraction be x and the denominator of required fraction be y $(y \neq 0)$

According to question; **[½]**

$$\frac{x-2}{y} = \frac{1}{3}$$

$\Rightarrow \quad 3x - 6 = y$ **[½]**

$\Rightarrow \quad 3x - y = 6 \quad$ ...(i)

and

$$\frac{x}{y-1} = \frac{1}{2}$$

$\Rightarrow \quad 2x = y - 1$ **[½]**

$\Rightarrow \quad 2x - y = -1 \quad$ ...(ii)

On subtracting (ii) from (i), we get :

$$\begin{array}{rcrcr} 3x & - & y & = & 6 \\ 2x & - & y & = & -1 \\ - & + & & & + \\ \hline & & x & = & 7 \end{array}$$ **[1]**

On substituting $x = 7$ in (i), we get :

$3(7) - y = 6$

$\Rightarrow \quad -y = 6 - 21$

$\therefore \quad y = 15$ **[½]**

Hence, the required fraction is $\frac{x}{y} = \frac{7}{15}$.

24. Given lines are $2x + 3y = 2$ and $x - 2y = 8$

$2x + 3y = 2$

$$\Rightarrow \quad y = \frac{2-2x}{3}$$

x	1	–2	4
y	0	2	–2

[½]

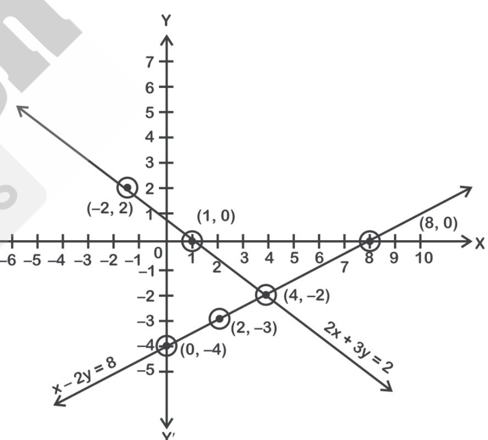
and $x - 2y = 8$

$$\Rightarrow \quad y = \frac{x-8}{2}$$

x	0	8	2
y	–4	0	–3

[½]

$\therefore$ We will plot the points (1, 0), (–2, 2) and (4, –2) and join them to get the graph of $2x + 3y = 2$ and we will plot the points (0, –4), (8, 0) and (2, –3) and join them to get the graph of $x - 2y = 8$



[1½]

The graph of two given equations intersect at (4, –2)

$\therefore$ Solution of $2x + 3y = 2$ and $x - 2y = 8$ is $x = 4$ and $y = -2$ **[½]**

25. $2y - x = 8$

x	0	–8
y	4	0

[½]

$5y - x = 14$

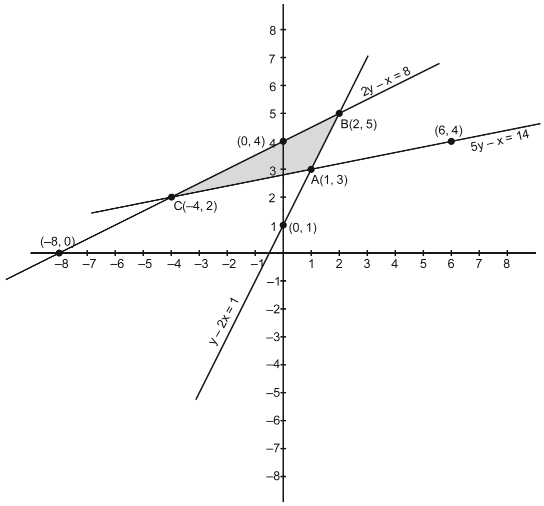
x	–4	6
y	2	4

[½]

$y - 2x = 1$

x	0	1
y	1	3

[½]



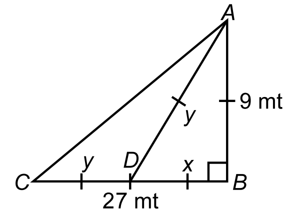
[1½]

26. Let AB be the pillar of height 9 meter. The peacock is sitting at point A on the pillar and B is the foot of the pillar. ($AB = 9$)

Let C be the position of the snake which is at 27 meters from B. ($BC = 27$ and $\angle ABC = 90°$)

As the speed of the snake and of the peacock is same they will travel the same distance in the same time

Now take a point D on BC that is equidistant from A and C (Please note that snake is moving towards the pillar) **[½]**



[½]

Hence by condition $AD = DC = y$(say)

Take $BD = x$

Now consider triangle ABD which is a right angled triangle

Using Pythagoras theorem ($AB^2 + BD^2 = AD^2$)

$9^2 + x^2 = y^2$ **[½]**

$81 = y^2 - x^2 = (y - x)(y + x)$ **[½]**

$81/(y + x) = (y - x)$ **[½]**

$y + x = BC = 27$

Hence, $81/27 = (y - x) = 3$ **[½]**

$y - x = 3$ ...(i)

$y + x = 27$ ...(ii) **[½]**

Adding (i) and (ii), gives $2y = 30$ or $y = 15$ **[1]**

$x = 12$, $y = 5$ **[1]**

Thus the snake is caught at a distance of x meters or 12 meters from the hole. **[½]**

4 : Quadratic Equations

1. $x^2 + 6x + 9 = 0$

$x^2 + 2.3x + (3)^2 = 0$ **[½]**

$(x + 3)^2 = 0$

$\Rightarrow$ $x = -3$ is the solution of $x^2 + 6x + 9 = 0$. **[½]**

2. $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0.$

Discriminant for $ax^2 + bx + c = 0$ will be $b^2 - 4ac$. **[½]**

$\therefore$ For the given quadratic equation

$= (10)^2 - 4\left(3\sqrt{3}\right)\left(\sqrt{3}\right)$

$= 100 - 36$

$= 64$ **[½]**

3. Answer (B)

Given a quadratic equation

$x^2 - 3x - m(m + 3) = 0$

$\Rightarrow$ $x^2 - (m + 3)x + mx - m(m + 3) = 0$ **[½]**

$x(x - (m + 3)) + m(x - (m + 3)) = 0$

$(x - (m + 3))(x + m) = 0$

$\therefore$ $x = -m, m + 3$ **[½]**

4. Answer (A)

It is given that 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$.

Therefore, $y = 1$ will satisfy both the equations.

$\therefore$ $a(1)^2 + a(1) + 3 = 0$

$\Rightarrow$ $a + a + 3 = 0$

$\Rightarrow$ $2a + 3 = 0$ **[½]**

$\Rightarrow$ $a = \frac{-3}{2}$

Also, $(1)^2 + (1) + b = 0$

$\Rightarrow$ $1 + 1 + b = 0$

$\Rightarrow$ $b = -2$

$\therefore$ $ab = \frac{-3}{2} \times -2 = 3$ **[½]**

5. Given quadratic equation is,

$px^2 - 2\sqrt{5}px + 15 = 0$

Here, $a = p, b = -2\sqrt{5}p, c = 15$

For real equal roots, discriminant = 0

$\therefore$ $b^2 - 4ac = 0$ **[½]**

$\therefore$ $\left(-2\sqrt{5}p\right)^2 - 4p(15) = 0$

$\therefore$ $20p^2 - 60p = 0$

$\therefore$ $20p(p - 3) = 0$

$\therefore$ $p = 3$ or $p = 0$

But, $p = 0$ is not possible.

$\therefore$ $p = 3$ **[½]**

6. $\because$ $x = 3$ is one of the root of $x^2 - 2kx - 6 = 0$

$(3)^2 - 2k(3) - 6 = 0$

$9 - 6k - 6 = 0$

$3 - 6k = 0$ **[½]**

$3 = 6k$

$k = \frac{3}{6} = \frac{1}{2}$ **[½]**

7. $x^2 + 4x + k = 0$

$\because$ Roots of given equation are real,

$D \geq 0$ **[½]**

$\Rightarrow$ $(4)^2 - 4 \times k \geq 0$

$\Rightarrow$ $-4k \geq -16$

$\Rightarrow$ $k \leq 4$

$\therefore$ k has all real values ≤ 4 **[½]**

8. $3x^2 - 10x + k = 0$

$\because$ Roots of given equation are reciprocal of each other.

Let the roots be α and $\frac{1}{\alpha}$. **[½]**

Product of roots $= \frac{c}{a}$

$\Rightarrow$ $\alpha . \frac{1}{\alpha} = \frac{k}{3}$

$\therefore$ $k = 3$ **[½]**

9 Quadratic equation $3x^2 - 4x + k = 0$ has equal roots

$\Rightarrow$ $D = b^2 - 4ac = 0$, where $a = 3$, $b = -4$ and $c = k$

$\Rightarrow$ $(-4)^2 - 4 \times 3 \times k = 0$

$\Rightarrow$ $16 - 12k = 0$

$\Rightarrow$ $k = \frac{16}{12} = \frac{4}{3}$ **[1]**

10. Given; $mx(x - 7) + 49 = 0$

$\Rightarrow$ $mx^2 - 7mx + 49 = 0$

$D = (7m)^2 - 4m \times 49$ **[1]**

$49m^2 - 4m \times 49 = 0$

$4\cancel{9}m^2 = 4m \times 4\cancel{9}$

$m = 4$ $[\because m \neq 0]$ **[1]**

11. Given quadratic equation is $3x^2 - 2kx + 12 = 0$

Here $a = 3$, $b = -2k$ and $c = 12$.

The quadratic equation will have equal roots if $\Delta = 0$

$\therefore$ $b^2 - 4ac = 0$

Putting the values of a, b and c we get

$(2k)^2 - 4(3)(12) = 0$ **[1]**

$\Rightarrow$ $4k^2 - 144 = 0$

$\Rightarrow$ $4k^2 = 144$

$\Rightarrow$ $k^2 = \frac{144}{4} = 36$

Considering square root on both sides,

$k = \sqrt{36} = \pm 6$

Therefore, the required values of k are 6 and –6. **[1]**

12. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow$ $4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow$ $4x\left(\sqrt{3}x + 2\right) - \sqrt{3}\left(\sqrt{3}x + 2\right) = 0$ **[1]**

$\Rightarrow$ $\left(4x - \sqrt{3}\right)\left(\sqrt{3}x + 2\right) = 0$

$\therefore$ $x = \frac{\sqrt{3}}{4}$ or $x = -\frac{2}{\sqrt{3}}$ **[1]**

13. Comparing the given equation with the standard quadratic equation ($ax^2 + bx + c = 0$), we get $a = 2$, $b = a$ and $c = -a^2$

Using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we get :

$x = \frac{-a \pm \sqrt{a^2 - 4 \times 2 \times (-a)^2}}{2 \times 2}$ **[1]**

$= \frac{-a \pm \sqrt{9a^2}}{4}$

$= \frac{-a \pm 3a}{4}$

$\Rightarrow$ $x = \frac{-a + 3a}{4} = \frac{a}{2}$ or $\frac{-a - 3a}{4} = -a$

So, the solutions of the given quadratic equation are $x = \frac{a}{2}$ or $x = -a$. **[1]**

14. $4x^2 + 4bx - (a^2 - b^2) = 0$

$\Rightarrow \quad x^2 + bx - \left(\frac{a^2 - b^2}{4}\right) = 0$

$\Rightarrow \quad x^2 + 2\left(\frac{b}{2}\right)x = \frac{a^2 - b^2}{4}$

$\Rightarrow \quad x^2 + 2\left(\frac{b}{2}\right)x + \left(\frac{b}{2}\right)^2 = \frac{a^2 - b^2}{4} + \left(\frac{b}{2}\right)^2$ **[1]**

$\Rightarrow \quad \left(x + \frac{b}{2}\right)^2 = \frac{a^2}{4}$

$\Rightarrow \quad x + \frac{b}{2} = \pm\frac{a}{2}$

$\Rightarrow \quad x = \frac{-b}{2} \pm \frac{a}{2}$

$\Rightarrow \quad x = \frac{-b - a}{2}, \frac{-b + a}{2}$

Hence, the roots are $-\left(\frac{a + b}{2}\right)$ and $\left(\frac{a - b}{2}\right)$. **[1]**

15. Given –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$.

$\therefore$ –5 satisfies the given equation.

$\therefore \quad 2(-5)^2 + p(-5) - 15 = 0$

$\therefore \quad 50 - 5p - 15 = 0$

$\therefore \quad 35 - 5p = 0$

$\therefore \quad 5p = 35$

$\Rightarrow \quad p = 7$ **[1]**

Substituting $p = 7$ in $p(x^2 + x) + k = 0$, we get

$7(x^2 + x) + k = 0$

$\therefore \quad 7x^2 + 7x + k = 0$

The roots of the equation are equal.

$\therefore$ Discriminant $= b^2 - 4ac = 0$

Here, $a = 7$, $b = 7$, $c = k$

$b^2 - 4ac = 0$

$\therefore \quad (7)^2 - 4(7)(k) = 0$

$\therefore \quad 49 - 28k = 0$

$\therefore \quad 28k = 49$

$\therefore \quad k = \frac{49}{28} = \frac{7}{4}$ **[1]**

16. Quadratic equation $px^2 - 14x + 8 = 0$

Also, one root is 6 times the other

Let say one root = x

Second root = $6x$

From the equation : Sum of the roots $= +\frac{14}{p}$

Product of roots $= \frac{8}{p}$

$\therefore \quad x + 6x = \frac{14}{p}.$

$x = \frac{2}{p}$ **[1]**

$\Rightarrow \quad 6x^2 = \frac{8}{p}$

$\Rightarrow \quad 6\left(\frac{2}{p}\right)^2 = \frac{8}{p}$

$\frac{6 \times 4}{p^2} = \frac{8}{p}$

$p = 3$ **[1]**

17. $4x^2 - 5x - 1 = 0$

$D = b^2 - 4ac$, where $a = 4$, $b = -5$ and $c = -1$ **[½]**

$\Rightarrow D = 25 + 16 = 41$ **[½]**

$\Rightarrow D > 0$ **[½]**

$\therefore$ The given equation has real and distinct roots **[½]**

18. $x^2 + 2\sqrt{2}x - 6 = 0$

$\therefore \quad x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$ **[1]**

$x\left(x + 3\sqrt{2}\right) - \sqrt{2}\left(x + 3\sqrt{2}\right) = 0$

$\left(x + 3\sqrt{2}\right)\left(x - \sqrt{2}\right) = 0$

$\Rightarrow \quad x = -3\sqrt{2}, \sqrt{2}$ **[1]**

19. Let assume two numbers be x, y.

Given, $x + y = 8 \Rightarrow x = 8 - y$ ...(i)

$\frac{1}{x} + \frac{1}{y} = \frac{8}{15}$ **[1]**

$\frac{x + y}{xy} = \frac{8}{15} \Rightarrow \frac{8}{xy} = \frac{8}{15}$

$\Rightarrow \quad xy = 15$ **[1]**

From (i) $xy = y(8 - y) = 15$

$\therefore \quad y^2 - 8y + 15 = 0$

$y = 3, 5 \Rightarrow x = 5, 3$

$\therefore$ The numbers are 3 and 5. **[1]**

20. $x^2 - 3\sqrt{5}x + 10 = 0$

For any quadratic equation

$ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **[1]**

$\therefore$ For the given equation

$x = \dfrac{3\sqrt{5} \pm \sqrt{45 - 40}}{2}$ **[1]**

$x = \dfrac{3\sqrt{5} \pm \sqrt{5}}{2}$

$\Rightarrow \boxed{x = \sqrt{5},\ 2\sqrt{5}}$ **[1]**

21. $4x^2 - 4ax + (a^2 - b^2) = 0$

$\Rightarrow (4x^2 - 4ax + a^2) - b^2 = 0$ **[1]**

$\Rightarrow [(2x^2) - 2.2x.a + a^2] - b^2 = 0$

$\Rightarrow [(2x - a)^2] - b^2 = 0$ **[1]**

$\Rightarrow [(2x - a) - b][(2x - a) + b] = 0$

$\Rightarrow [(2x - a) - b] = 0$ or $[(2x - a) + b] = 0$

$\Rightarrow x = \dfrac{a+b}{2};\ x = \dfrac{a-b}{2}$ **[1]**

22. $3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\Rightarrow \sqrt{3} \times \left[\sqrt{3}x - \sqrt{2}\right] - \sqrt{2}\left[\sqrt{3}x - \sqrt{2}\right] = 0$ **[1]**

$\Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right) = 0$

$\Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)^2 = 0$

$\therefore \sqrt{3}x - \sqrt{2} = 0$ **[1]**

$\Rightarrow \sqrt{3}x = \sqrt{2}$

$\Rightarrow x = \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{\sqrt{2} \times \sqrt{3}}{\left(\sqrt{3}\right)^2} = \dfrac{\sqrt{6}}{3}$ **[1]**

23. $(k + 4)x^2 + (k + 1)x + 1 = 0$

$a = k + 4,\ b + k + 1,\ c = 1$

For equal roots, discriminant, $D = 0$ **[1]**

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (k + 1)^2 - 4(k + 4) \times 1 = 0$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$ **[1]**

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow k(k - 5) + 3(k - 5) = 0$

$\Rightarrow (k - 5)(k + 3) = 0$

$\Rightarrow k = 5$ or $k = -3$

Thus, for $k = 5$ or $k = -3$, the given quadratic equation has equal roots. **[1]**

24. Given equation :

$\dfrac{4}{x} - 3 = \dfrac{5}{2x+3};\ x \neq 0, -\dfrac{3}{2}$

$\dfrac{4}{x} - 3 = \dfrac{5}{2x+3}$

$\Rightarrow \dfrac{4 - 3x}{x} = \dfrac{5}{2x+3}$ **[1]**

$\Rightarrow (4 - 3x)(2x + 3) = 5x$

$\Rightarrow -6x^2 + 8x - 9x + 12 = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$ **[1]**

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow (x + 2)(x - 1) = 0$

$\Rightarrow (x + 2) = 0$ or $(x - 1) = 0$

$\Rightarrow x = -2$ or $x = 1$

Thus, the solution of the given equation is –2 and 1. **[1]**

25. For the given equation, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{3},\ b = -2\sqrt{2},\ c = -2\sqrt{3}$

Now, $\sqrt{D} = \sqrt{b^2 - 4ac}$

$= \sqrt{\left(-2\sqrt{2}\right)^2 - 4\left(\sqrt{3}\right)\left(-2\sqrt{3}\right)}$

$= \sqrt{8 + 24 = \sqrt{32} = 4\sqrt{2}}$ **[1]**

Using quadratic formula, we obtain

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-\left(-2\sqrt{2}\right) \pm 4\sqrt{2}}{2\sqrt{3}}$

$\Rightarrow x = \dfrac{2\sqrt{2} + 4\sqrt{2}}{2\sqrt{3}}$ or $\dfrac{2\sqrt{2} - 4\sqrt{2}}{2\sqrt{3}}$ **[1]**

$\Rightarrow x = \dfrac{\sqrt{2} + 2\sqrt{2}}{\sqrt{3}}$ or $\dfrac{\sqrt{2} - 2\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \quad x = \dfrac{3\sqrt{2}}{\sqrt{3}}$ or $x = \dfrac{-\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \quad x = \sqrt{3}\sqrt{2}$ or $x = \dfrac{-\sqrt{2}}{\sqrt{3}}$

$\therefore \quad x = \sqrt{6}$ or $x = \dfrac{-\sqrt{2}}{\sqrt{3}}$ **[1]**

26. $\dfrac{1}{(x-1)(x-2)} + \dfrac{1}{(x-2)(x-3)} = \dfrac{2}{3}$

$\dfrac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)} = \dfrac{2}{3}$

$\dfrac{x-3+x-1}{(x^2-3x+2)(x-3)} = \dfrac{2}{3}$ **[1]**

$\dfrac{2x-4}{x^3-3x^2-3x^2+9x+2x-6} = \dfrac{2}{3}$

$\dfrac{2x-4}{x^3-6x^2+11x-6} = \dfrac{2}{3}$

$6x - 12 = 2x^3 - 12x^2 + 22x - 12$

$2x^3 - 12x^2 + 16x = 0$

$2x(x^2 - 6x + 8) = 0$

$x^2 - 6x + 8 = 0$ **[1]**

$x^2 - 4x - 2x + 8 = 0$

$x(x - 4) - 2(x - 4) = 0$

$(x - 4)(x - 2) = 0$

$x - 4 = 0$ or $x - 2 = 0$

$x = 4$ and $x = 2$ **[1]**

27. Given $ad \neq bc$ for the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$.

For this equation not to have real roots its discriminant < 0. **[1]**

$D = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

$D = 4a^2c^2 + 4b^2d^2 + 8acbd - 4a^2c^2 - 4b^2d^2 - 4b^2c^2 - 4a^2d^2$ **[1]**

$D = -4(a^2d^2 + b^2c^2 - 2acbd)$

$D = -4(ad - bc)^2$

Given $ad \neq bc$

$\therefore \quad D < 0$

Quadratic equation has no real roots. **[1]**

28. Let the usual speed of the plane be x km/hr.

Time taken to cover 1500 km with usual speed $= \dfrac{1500}{x}$ hrs

Time taken to cover 1500 km with speed of $(x + 100)$ km/hr $= \dfrac{1500}{x+100}$ hrs. **[1]**

$\therefore \quad \dfrac{1500}{x} = \dfrac{1500}{x+100} + \dfrac{1}{2}$

$\dfrac{1500}{x} - \dfrac{1500}{x+100} = \dfrac{1}{2}$

$1500\left(\dfrac{x+100-x}{x(x+100)}\right) = \dfrac{1}{2}$ **[1]**

$150000 \times 2 = x(x + 100)$

$x^2 + 100x - 300000 = 0$

$x^2 + 100x - 300000 = 0$

$x = -600$ or $x = 500$

But speed can't be negative.

Hence, usual speed 500 km/hr. **[1]**

29. Let the duration of the flight be x hours

$\text{Speed} = \dfrac{\text{Distance}}{\text{time}} = \dfrac{600}{x}$ km/h **[½]**

Duration of the flight due to slow down $= x + \dfrac{30}{60} = x + \dfrac{1}{2}$ According to question **[½]**

$\dfrac{600}{x} - \dfrac{600}{x+\frac{1}{2}} = 200$ **[½]**

$\Rightarrow \quad \dfrac{3}{x} - \dfrac{3}{x+\frac{1}{2}} = 1$

$\Rightarrow \quad \dfrac{3(2x+1)-6x}{x(2x+1)} = 1$ **[½]**

$\Rightarrow \quad \dfrac{6x+3-6x}{x(2x+1)} = 1$

$\Rightarrow \quad \dfrac{3}{x(2x+1)} = 1$

$\Rightarrow \quad 2x^2 + x - 3 = 0$ **[½]**

$\Rightarrow \quad 2x^2 + 3x - 2x - 3 = 0$

$\Rightarrow \quad x(2x + 3) - 1\ (2x + 3) = 0$

$\Rightarrow \quad (2x + 3)\ (x - 1) = 0$

$x = 1$ **[½]**

Original duration of the flight is 1 hour.

30. Let the sides of the two squares be x cm and y cm where $x > y$.

Then, their areas are x^2 and y^2 and their perimeters are $4x$ and $4y$.

By the given condition :

$x^2 + y^2 = 400$ ...(i)

and $4x - 4y = 16$

$\Rightarrow 4(x - y) = 16 \Rightarrow x - y = 4$

$\Rightarrow x = y + 4$ ...(ii) **[1]**

Substituting the value of x from (ii) in (i), we get :

$(y + 4)^2 + y^2 = 400$

$\Rightarrow y^2 + 16 + 8y + y^2 = 400$

$\Rightarrow 2y^2 + 16 + 8y = 400$

$\Rightarrow y^2 + 4y - 192 = 0$

$\Rightarrow y^2 + 16y - 12y - 192 = 0$

$\Rightarrow y(y + 16) - 12(y + 16) = 0$ **[1]**

$\Rightarrow (y + 16)(y - 12) = 0$

$\Rightarrow y = -16$ or $y = 12$ **[1]**

Since, y cannot be negative, $y = 12$.

So, $x = y + 4 = 12 + 4 = 16$

Thus, the sides of the two squares are 16 cm and 12 cm. **[1]**

31. $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$ **[1]**

$\Rightarrow \frac{2x-2a-b-2x}{2x(2a+b+2x)} = \frac{b+2a}{2ab}$

$\Rightarrow \frac{-2a-b}{2x(2a+b+2x)} = \frac{b+2a}{2ab}$ **[1]**

$\Rightarrow \frac{-1}{x(2a+b+2x)} = \frac{1}{ab}$

$\Rightarrow 2x^2 + 2ax + bx + ab = 0$

$\Rightarrow 2x(x + a) + b(x + a) = 0$

$\Rightarrow (x + a)(2x + b) = 0$ **[1]**

$\Rightarrow x + a = 0$ or $2x + b = 0$

$\Rightarrow x = -a,$ or $x = \frac{-b}{2}$ **[1]**

32. Let the two natural numbers be x and y such that $x > y$.

Given :

Difference between the natural numbers = 5

$\therefore x - y = 5$ ...(i)

Difference of their reciprocals $\frac{1}{10}$ (given)

$\frac{1}{y} - \frac{1}{x} = \frac{1}{10}$ **[1]**

$\Rightarrow \frac{x-y}{xy} = \frac{1}{10}$

$\Rightarrow \frac{5}{xy} = \frac{1}{10}$

$\Rightarrow xy = 50$ ...(ii) **[1]**

Putting the value of x from equation (i) in equation (ii), we get

$(y + 5)y = 50$

$\Rightarrow y^2 + 5y - 50 = 0$

$\Rightarrow y^2 + 10y - 5y - 50 = 0$

$\Rightarrow y(y + 10) - 5(y + 10) = 0$

$\Rightarrow (y - 5)(y + 10) = 0$

$\Rightarrow y = 5$ or -10 **[1]**

As y is a natural number, therefore $y = 5$

Other natural number = $y + 5 = 5 + 5 = 10$

Thus, the two natural numbers are 5 and 10. **[1]**

33. Given quadratic equation :

$(k + 4)x^2 + (k + 1)x + 1 = 0$

Since the given quadratic equation has equal roots, its discriminant should be zero.

$\therefore D = 0$ **[1]**

$\Rightarrow (k + 1)^2 - 4 \times (k + 4) \times 1 = 0$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow (k - 5)(k + 3) = 0$

$\Rightarrow k - 5 = 0$ or $k + 3 = 0$

$\Rightarrow k = 5$ or -3 **[1]**

Thus, the values of k are 5 and -3.

For $k = 5$, $(k + 4)x^2 + (k + 1)x + 1 = 0$

$\Rightarrow 9x^2 + 6x + 1 = 0$

$\Rightarrow (3x)^2 + 2(3x) + 1 = 0$

$\Rightarrow$ $(3x + 1)^2 = 0$

$\Rightarrow$ $x = -\frac{1}{3}, -\frac{1}{3}$

$\Rightarrow$ $x^2 - 2x + 1 = 0$ [For $k = -3$]

$\Rightarrow$ $(x - 1)^2 = 0$

$\Rightarrow$ $x = 1, 1$ **[1]**

Thus, the equal roots of the given quadratic equation is either 1 or $-\frac{1}{3}$. **[1]**

34. Let l be the length of the longer side and b be the length of the shorter side.

Given that the length of the diagonal of the rectangular field is 16 metres more than the shorter side.

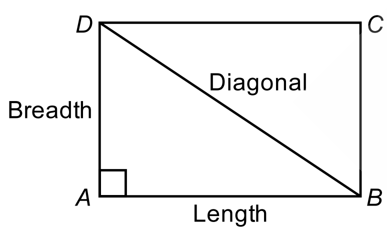
Thus, diagonal = $16 + b$

Since longer side is 14 metres more than shorter side, we have,

$l = 14 + b$

Diagonal is the hypotenuse of the triangle. **[1]**

Consider the following figure of the rectangular field.



By applying Pythagoras Theorem in ΔABD, we have,

Diagonal^2 = Length^2 + Breadth^2 **[1]**

$\Rightarrow$ $(16 + b)^2 = (14 + b)^2 + b^2$

$\Rightarrow$ $256 + b^2 + 32b = 196 + b^2 + 28b + b^2$

$\Rightarrow$ $256 + 32b = 196 + 28b + b^2$

$\Rightarrow$ $60 + 32b = 28b + b^2$

$\Rightarrow$ $b^2 - 4b - 60 = 0$ **[1]**

$\Rightarrow$ $b^2 - 10b + 6b - 60 = 0$

$\Rightarrow$ $b(b - 10) + 6(b - 10) = 0$

$\Rightarrow$ $(b + 6)(b - 10) = 0$

$\Rightarrow$ $(b + 6) = 0$ or $(b - 10) = 0$

$\Rightarrow$ $b = -6$ or $b = 10$

As breadth cannot be negative, breadth = 10 m

Thus, length of the rectangular field = 14 + 10 = 24 m. **[1]**

35. Let x be the first speed of the train.

We know that, $\frac{\text{Distance}}{\text{Speed}} = \text{time}$

Thus, we have,

$\frac{54}{x} + \frac{63}{x+6} = 3$ **[1]**

$\Rightarrow$ $\frac{54(x+6)+63x}{x(x+6)} = 3$

$\Rightarrow$ $54(x + 6) + 63x = 3x(x + 6)$

$\Rightarrow$ $54x + 324 + 63x = 3x^2 + 18x$

$\Rightarrow$ $117x + 324 = 3x^2 + 18x$ **[1]**

$\Rightarrow$ $3x^2 - 117x - 324 + 18x = 0$

$\Rightarrow$ $3x^2 - 99x - 324 = 0$

$\Rightarrow$ $x^2 - 33x - 108 = 0$

$\Rightarrow$ $x^2 - 36x + 3x - 108 = 0$

$\Rightarrow$ $x(x - 36) + 3(x - 36) = 0$

$\Rightarrow$ $(x + 3)(x - 36) = 0$ **[1]**

$\Rightarrow$ $(x + 3) = 0$ or $(x - 36) = 0$

$\Rightarrow$ $x = -3$ or $x = 36$

Speed cannot be negative. Hence, initial speed of the train is 36 km/hour. **[1]**

36. $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

L.C.M. of all the denominators is $(x + 1)(x + 2)(x + 4)$ **[1]**

Multiply throughout by the L.C.M., we get

$(x + 2)(x + 4) + 2(x + 1)(x + 4) = 4(x + 1)(x + 2)$

$(x + 4)(x + 2 + 2x + 2) = 4(x^2 + 3x + 2)$

$(x + 4)(3x + 4) = 4x^2 + 12x + 8$

$3x^2 + 16x + 16 = 4x^2 + 12x + 8$ **[1]**

$\therefore$ $x^2 - 4x - 8 = 0$

Now, $a = 1$, $b = -4$, $c = -8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16+32}}{2}$$

$$= \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$ **[1]**

$\therefore$ $x = 2 \pm 2\sqrt{3}$ **[1]**

37. Let the speed of the stream be s km/h.

Speed of the motor boat 24 km/h

Speed of the motor boat (upstream) = $24 - s$

Speed of the motor boat (downstream) = $24 + s$ **[1]**

According to the given condition,

$$\frac{32}{24-s} - \frac{32}{24+s} = 1$$

$\therefore \quad 32\left(\frac{1}{24-s} - \frac{1}{24+s}\right) = 1$ **[1]**

$\therefore \quad 32\left(\frac{24+s-24+s}{576-s^2}\right) = 1$

$\therefore \quad 32 \times 2s = 576 - s^2$

$\therefore \quad s^2 + 64s - 576 = 0$

$\therefore \quad (s + 72)(s - 8) = 0$ **[1]**

$\therefore \quad s = -72$ or $s = 8$

Since, speed of the stream cannot be negative, the speed of the stream is 8 km/h. **[1]**

38. $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, x \neq -1, \frac{-1}{5}, -4$

Take L.C.M. on the left hand side of equation

$\frac{5x+1+3(x+1)}{(x+1)(5x+1)} = \frac{5}{x+4}$ **[1]**

$8x^2 + 4x + 32x + 16 = 25x^2 + 5 + 5x + 25x$

$17x^2 - 6x - 11 = 0$ **[1]**

$17x^2 - 17x + 11x - 11 = 0$

$17x(x - 1) + 11(x - 1) = 0$

$(x - 1)(17x + 11) = 0$ **[1]**

$\therefore \quad x = \frac{-11}{17}, 1$ **[1]**

39. Two taps when run together fill the tank in $3\frac{1}{13}$ hrs

Say taps are A, B and

A fills the tank by itself in x hrs

B fills tank in $(x + 3)$ hrs **[1]**

Portion of tank filled by A (in 1 hr) $= \frac{1}{x}$

Portion of tank filled by B (in 1hr) $= \frac{1}{x+3}$

Portion of tank filled by A and B (both in 1hr) $= \frac{13}{40}$

$\therefore \quad \frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$ **[1]**

$(x + 3 + x)40 = 13(x)(x + 3)$

$80x + 120 = 13x^2 + 39x$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0$

$\Rightarrow \quad 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow \quad x = 5$ or $\frac{-24}{13}$

[But negative value not be taken] **[1]**

$\therefore \quad A$ fills tank in 5 hrs

B fills tank in 8 hrs **[1]**

40. Let the speed of stream be x km/ hr.

Now, for upstream: speed = $(18 - x)$ km/hr

$\therefore \quad$ Time taken $= \left(\frac{24}{18-x}\right)$ hr **[½]**

Now, for downstream: speed = $(18 + x)$ km/hr

$\therefore \quad$ Time taken $= \left(\frac{24}{18+x}\right)$ hr **[½]**

Given that,

$\frac{24}{18-x} = \frac{24}{18+x} + 1$ **[½]**

$-1 = \frac{24}{18+x} - \frac{24}{18-x}$

$-1 = \frac{24[(18-x)-(18+x)]}{(18)^2 - x^2}$ **[½]**

$-1 = \frac{24[-2x]}{324 - x^2}$ **[½]**

$-324 + x^2 = -48x$

$x^2 + 48x - 324 = 0$ **[½]**

$x^2 + 54x - 6x - 324 = 0$

$(x + 54)(x - 6) = 0$

$x = -54$ or $x = 6$ **[½]**

$x = -54$ km/hr (not possible) **[½]**

Therefore, speed of the stream = 6 km/hr.

41. Let x be the original average speed of the train for 63 km.

Then, $(x + 6)$ will be the new average speed for remaining 72 km. **[½]**

Total time taken to complete the journey is 3 hrs.

$\therefore \quad \frac{63}{x}+\frac{72}{(x+6)}=3$ **[½]**

$\left(\therefore \text{ Time} = \frac{\text{Distance}}{\text{Speed}}\right)$

$\therefore \quad \frac{63x+378+72x}{x(x+6)}=3$ **[½]**

$\Rightarrow$ $135x + 378 = 3x^2 + 18x$ **[½]**

$\Rightarrow$ $x^2 - 39x - 126 = 0$ **[½]**

$\Rightarrow$ $(x - 42)(x + 3) = 0$ **[½]**

$\Rightarrow$ $\boxed{x=42}$ OR $\boxed{x=-3}$ **[½]**

Since, speed cannot be negative.

Therefore x = 42 km/hr. **[½]**

42. Let the time in which tap with longer and smaller diameter can fill the tank separately be x hours and y hours respectively. **[½]**

According to the question

$\frac{1}{x}+\frac{1}{y}=\frac{8}{15}$ ...(i) **[½]**

and $x = y - 2$ ...(ii) **[½]**

On substituting $x = y - 2$ from (ii) in (i), we get

$\frac{1}{y-2}+\frac{1}{y}=\frac{8}{15}$ **[½]**

$\Rightarrow \frac{y+y-2}{y^2-2y}=\frac{8}{15}$

$\Rightarrow$ $15(2y - 2) = 8(y^2 - 2y)$

$\Rightarrow$ $30y - 30 = 8y^2 - 16y$

$\Rightarrow$ $8y^2 - 46y + 30 = 0$ **[½]**

$\Rightarrow$ $4y^2 - 20y - 3y + 15 = 0$

$\Rightarrow$ $(4y - 3)(y - 5) = 0$

$\Rightarrow \quad y=\frac{3}{4}, y=5$ **[½]**

Substituting values of y in (ii), we get

$x=\frac{3}{4}-2$	$x=5-2$
$x=\frac{-5}{4}$	$x=3$
$\therefore \quad x \neq \frac{-5}{4}$	

[½]

(time cannot be negative)

Hence, the time taken by tap with longer diameter is 3 hours and the time taken by tap with smaller diameter is 5 hours, in order to fill the tank separately. **[½]**

43. Let the units digit of the two digit number be x.

$\therefore$ Ten's digit will be $\frac{14}{x}$ **[½]**

According to question,

$10\times\frac{14}{x}+x+45=10x+\frac{14}{x}$ **[1]**

$\Rightarrow \frac{140}{x}+x+45=\frac{10x^2+14}{x}$

$\Rightarrow \frac{140+x^2+45x}{x}=\frac{10x^2+14}{x}$ **[½]**

$\Rightarrow$ $9x^2 - 45x - 126 = 0$

$\Rightarrow$ $9x^2 - 63x + 18x - 126 = 0$

$\Rightarrow$ $9x(x - 7) + 18(x - 7) = 0$ **[½]**

$\Rightarrow$ $(x - 7)(9x + 18) = 0$

$\Rightarrow$ Either $x = 7$ or $x = -2$ **[½]**

$\therefore$ $x = 7$ [$\because x \neq -2$]

$\therefore$ Ten's digit $=\frac{14}{7}=2$ **[½]**

So, the number is 27. **[½]**

44. Let age of boy be x years, then age of his sister will be $(25 - x)$ years **[½]**

Product of their ages, $(x)(25 - x) = 150$ **[½]**

$\Rightarrow 25x - x^2 = 150$ **[½]**

$\Rightarrow x^2 - 25x + 150 = 0$ **[½]**

$\Rightarrow (x - 15)(x - 10) = 0$ **[1]**

$\Rightarrow x = 10$ and 15 **[½]**

Hence, their present age's are 10 years and 15 years. **[½]**

45. (a) Let x be the digit at 10th place of given two digit number and y be the unit's place of given two digit number.

According to the question,

$xy = 24$

$\Rightarrow \quad y=\frac{24}{x}$ …(i) **[1]**

and

$10x + y - 18 = 10y + x$

$\Rightarrow$ $9x - 9y = 18$

$\Rightarrow$ $x - y = 2$ …(ii) **[1]**

From equation (i) and (ii), we get

$x-\frac{24}{x}=2$

or $x^2 - 2x - 24 = 0$

or $x^2 - 6x + 4x - 24 = 0$

or $(x - 6)(x + 4) = 0$

$x = 6$ or $x = -4$ **[1]**

$\therefore$ $x = 6$ [Because x can't be negative]

From (i),

$y = 4$

$\therefore$ Original number is 64. **[1]**

OR

(b) Let x and y be the two numbers such that $x > y$

According to question,

$x^2 - y^2 = 180$ …(i) **[½]**

and $y^2 = 8x$ …(ii) **[½]**

From (i) and (ii), we get

$x^2 - 8x - 180 = 0$ **[½]**

or $(x - 18)(x + 10) = 0$ **[½]**

$x = 18, -10$

$x = 18$ **[**Because x cannot be negative**]** **[½]**

From (ii)

Put $x = 18$ in equation (ii), we get

$y^2 = 144$ **[½]**

or $y = \pm 12$ **[½]**

$\therefore$ Required numbers are (18, 12) and (18, –12) **[½]**

46. Let assume the two numbers to be x, y ($y > x$)

Given that $y - x = 4 \Rightarrow y = 4 + x$ ...(i) **[1]**

$$\frac{1}{x} - \frac{1}{y} = \frac{4}{21}$$ **[1]**

$$\Rightarrow \frac{y - x}{xy} = \frac{4}{21}$$

$$\Rightarrow \frac{4}{xy} = \frac{4}{21}$$ **[1]**

$\Rightarrow xy = 21$

$x(4 + x) = 21$ **[1]**

$x^2 + 4x - 21 = 0$

$(x + 7)(x - 3) = 0$

$x = -7, 3$ **[1]**

$y = -3, 7$

$\therefore$ Numbers are –7, –3 or 3, 7 **[1]**

47. $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Discriminant

$D = 81(a + b)^2 - 36(2a^2 + 5ab + 2b^2)$ **[1]**

$D = 9[9a^2 + 9b^2 + 18ab - 8a^2 - 8b^2 - 20ab]$

$D = 9[a^2 + b^2 - 2ab]$ **[1]**

$\therefore$ $\boxed{D = 9(a-b)^2}$ **[1]**

$$\therefore \quad x = \frac{+9(a+b) \pm \sqrt{9(a-b)^2}}{2 \times 9}$$ **[1]**

$$x = \frac{9(a+b) \pm 3(a-b)}{18}$$

$$x = \frac{3a+3b+a-b}{6}, \frac{3a+3b-a+b}{6}$$ **[1]**

$$\therefore \quad x = \frac{2a+b}{3}; \frac{a+2b}{3}$$ **[1]**

48. –5 is root of $2x^2 + px - 15 = 0$

$\therefore$ $2(-5)^2 + p(-5) - 15 = 0$ **[1]**

$10 - p - 3 = 0$

$\therefore$ $p = 7$ **[1]**

$p(x^2 + x) + k = 0$ has equal roots. **[1]**

$\therefore$ $7x^2 + 7x + k = 0$ [As we know $p = 7$] **[1]**

$\therefore$ Discriminant = 0

$D = 49 - 28k$ **[1]**

$28k = 49$

$$k = \frac{7}{4}$$ **[1]**

49. Let the required three integers be $(x - 1)$, x and $(x + 1)$. **[1]**

Now, $(x - 1)^2 + [x.(x + 1)] = 46$

$(x^2 - 2x + 1) + [x^2 + x] = 46$ **[1]**

$2x^2 - x - 45 = 0$

$2x^2 - 10x + 9x - 45 = 0$ **[1]**

$2x(x - 5) + 9(x - 5) = 0$

$(x - 5)(2x + 9) = 0$ **[1]**

$x = 5$ or $x = -9/2$

So, $x = 5$ [Because it is given that x is a positive integer] **[1]**

Thus, the required integers are (5 –1), *i.e.* 4, 5 and 6. **[1]**

50. Let the smaller number be x and larger number be y.

$y^2 - x^2 = 88$ ...(i)

$y = 2x - 5$ ...(ii) **[1]**

In equation (i)

$(2x - 5)^2 - x^2 = 88$ **[1]**

$4x^2 - 20x + 25 - x^2 = 88$

$3x^2 - 20x - 63 = 0$ **[1]**

By splitting the middle term,

$3x^2 - 27x + 7x - 63 = 0$

$3x(x - 9) + 7(x - 9) = 0$ **[1]**

$(x - 9)(3x + 7) = 0$

$\Rightarrow$ $x = 9$ and $x = -7/3$ **[1]**

We cannot take negative value because x must be greater than 5.

So, smaller number = 9

And larger number = $2x - 5 = 18 - 5 = 13$ **[1]**

51. A ———————— B
180 km

Distance travelled by train = 180 km, let say speed = s km/hr

$\boxed{\text{Time taken } (t) = \frac{180}{s}}$ **[1]**

It is given if speed had been $(s + 9)$ km/hr

Train would have travelled AB in $(t - 1)$ hrs. **[1]**

$\therefore \quad t - 1 = \frac{180}{s+9}$

$\Rightarrow \quad \boxed{t = \frac{180}{s+9} + 1}$ **[1]**

$\therefore \quad \frac{180}{s+9} + 1 = \frac{180}{s}$

$(189 + s)s = 180s + 1620$ **[1]**

$189s + s^2 = 180s + 1620$

$s^2 + 9s - 1620 = 0$ **[1]**

$\Rightarrow$ $s^2 + 45s - 36s - 1620 = 0$

$\Rightarrow$ $s = -45, 36$ [$\because$ s cannot negative] **[1]**

$\therefore \quad \boxed{s = 36 \text{ km/hr}}$

52. $\frac{1}{2x-3} + \frac{1}{x-5} = 1, \; x \neq \frac{3}{2}, \; 5.$

Taking L.C.M on left side of equality

$\frac{x-5+2x-3}{(2x-3)(x-5)} = 1$ **[1]**

$3x - 8 = 2x^2 - 3x - 10x + 15$ **[1]**

$2x^2 - 16x + 23 = 0$

$x = \frac{16 \pm \sqrt{256 - 4 \times 2 \times 23}}{4}$ **[1]**

$x = \frac{16 \pm \sqrt{72}}{4}$ **[1]**

$x = \frac{16 \pm 6\sqrt{2}}{4}$ **[1]**

$x = \left(4 \pm \frac{3\sqrt{2}}{2}\right)$ **[1]**

53. Total cost of books = ₹80

Let the number of books be x.

So, the cost of each book = ₹$\frac{80}{x}$ **[1]**

Cost of each book if he buy 4 more book

= ₹$\frac{80}{x+4}$ **[1]**

As per given in question :

$\frac{80}{x} - \frac{80}{x+4} = 1$ **[1]**

$\Rightarrow \quad \frac{80x + 320 - 80x}{x(x+4)} = 1$

$\Rightarrow \quad \frac{320}{x^2 + 4x} = 1$

$\Rightarrow$ $x^2 + 4x - 320 = 0$ **[1]**

$\Rightarrow$ $(x + 20)(x - 16) = 0$

$\Rightarrow$ $x = -20, 16$ **[1]**

Since, number of books cannot be negative.

So, the number of books he bought is 16. **[1]**

54. Let the first number be x then the second number be $(9 - x)$ as the sum of both numbers is 9. **[1]**

Now, the sum of their reciprocals is $\frac{1}{2}$, therefore

$\frac{1}{x} + \frac{1}{9-x} = \frac{1}{2}$ **[1]**

$\Rightarrow \quad \frac{9 - x + x}{x(9-x)} = \frac{1}{2}$ **[1]**

$\Rightarrow \quad \frac{9}{9x - x^2} = \frac{1}{2}$

$\Rightarrow$ $18 = 9x - x^2$ **[1]**

$\Rightarrow$ $x^2 - 9x + 18 = 0$

$\Rightarrow$ $(x - 6)(x - 3) = 0$

$\Rightarrow$ $x = 6, 3$ **[1]**

If $x = 6$ then other number is 3.

and if $x = 3$ then other number is 6.

Hence, numbers are 3 and 6. **[1]**

5 : Arithmetic Progressions

1. First term of an AP = p

   Common difference = q

   $T_{10} = p + (10 - 1)q$ **[½]**

   $T_{10} = p + 9q$ **[½]**

2. Given $\frac{4}{5}$, a, 2 are in AP

   $\therefore \quad a - \frac{4}{5} = 2 - a$ **[½]**

   $\Rightarrow \quad 2a = \frac{4}{5} + 2$

   $2a = \frac{14}{5}$

   $\therefore \quad a = \frac{7}{5}$ **[½]**

3. Given an AP which has sum of first p terms $= ap^2 + bp$

   Lets say first term = k & common difference = d

   $\therefore \quad ap^2 + bp = \frac{p}{2}[2k + (p-1)d]$

   $2ap + 2b = 2k + (p - 1)d$

   $2b + 2ap = (2k - d) + pd$ **[½]**

   Comparing terms on both sides,

   $\Rightarrow \quad \boxed{2a = d}$

   $2k - d = 2b$

   $2k = 2b + 2a$

   $\boxed{k = a + b}$

   Common difference = $2a$

   First term = $a + b$ **[½]**

4. Answer (C)

   Given common difference of the

   $AP = d = 3$

   Lets say the first term = a

   $a_{20} = a + 19d = a + 19 \times 3$

   $= a + 57$

   $a_{15} = a + 14d = a + 14 \times 3$ **[½]**

   $= a + 42$

   $a_{20} - a_{15} = a + 57 - a - 42$

   $= 15$ **[½]**

5. Answer (C)

   The first 20 odd numbers are 1, 3, 5, ... 39

   This is an AP with first term 1 and the common difference 2. **[½]**

   Sum of 20 terms = S_{20}

   $S_{20} = \frac{20}{2}[2(1) + (20-1)(2)] = 10[2 + 38] = 400$ **[½]**

   Thus, the sum of first 20 odd natural numbers is 400.

6. Answer (C)

   Common difference =

   $\frac{1-6q}{3q} - \frac{1}{3q} = \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$ **[1]**

7. Answer (C)

   The first three terms of an AP are $3y - 1$, $3y + 5$ and $5y + 1$, respectively.

   We need to find the value of y.

   We know that if a, b and c are in AP, then :

   $b - a = c - b$

   $\Rightarrow \quad 2b = a + c$

   $\therefore \quad 2(3y + 5) = 3y - 1 + 5y + 1$ **[½]**

   $\Rightarrow \quad 6y + 10 = 8y$

   $\Rightarrow \quad 10 = 8y - 6y$

   $\Rightarrow \quad 2y = 10$

   $\Rightarrow \quad y = 5$

   Hence the correct option is C. **[½]**

8. If $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of AP, then the common difference will be the same.

   $\therefore \quad (2k - 1) - (k + 9) = (2k + 7) - (2k - 1)$ **[½]**

   $\therefore \quad k - 10 = 8$

   $\therefore \quad k = 18$ **[½]**

9. Given

   $a_{21} - a_7 = 84$ ...(i)

   In an AP a_1, a_2, a_3, a_4 .....

   $a_n = a_1 + (n - 1)d$ $\quad d$ = common difference

   $a_{21} = a_1 + 20d$ ...(ii)

   $a_7 = a_1 + 6d$ ...(iii) **[½]**

Substituting (ii) and (iii) in (i)

$a_1 + 20d - a_1 - 6d = 84$

$14d = 84$

$d = 6$

$\therefore$ Common difference = 6 **[½]**

10. $a_7 = 4$

$a + 6d = 4$ (as $a_n = a + (n-1)d$)

but $d = -4$

$a + 6(-4) = 4$ **[½]**

$a + (-24) = 4$

$a = 4 + 24 = 28$

Therefore first term $a = 28$ **[½]**

11. Two digit numbers divisible by 3 are

12, 15, 18, ..., 99.

$a = 12$, $d = 15 - 12 = 3$ **[½]**

$\Rightarrow T_n = 99$

$\Rightarrow a + (n-1)d = 99$

$\Rightarrow 12 + (n-1)3 = 99$

$\Rightarrow n = 30$

$\therefore$ Number of two digit numbers divisible by 3 are 30. **[½]**

12. $T_n = 7 - 4n$

$T_1 = 7 - 4(1) = 3$

$T_2 = 7 - 4(2) = 7 - 8 = -1$ **[½]**

$\therefore$ Common difference = $T_2 - T_1$

$= -1 - 3 = -4$ **[½]**

13. Answer (a) **[1]**

$2x$, $(x + 10)$, $(3x + 2)$ are in A.P.

$\therefore x + 10 - 2x = 3x + 2 - x - 10$

$\Rightarrow x = 6$

Hence, option (a) is correct.

14. Answer (c) **[1]**

$\therefore$ 10^{th} term = $p + (10 - 1)q$

$$\boxed{a_{10} = p + 9q}$$

Hence, option (c) is correct.

15. Given an AP 3, 15, 27, 39, ...

Lets say n^{th} term is 120 more than 21^{st} term

$\therefore T_n = 120 + T_{21}$

$a + (n-1)d = 120 + (a + 20d)$ **[1]**

$(n-1)12 = 120 + 20 \times 12$

$n - 1 = 30$

$\therefore$ 31^{st} term is 120 more than 12^{th} term. **[1]**

16. Given an AP with first term $(a) = 2$

Last term $(\ell) = 29$

Sum of the terms = 155

Common difference (d) = ?

Sum of the n terms $= \dfrac{n}{2}(a + \ell)$ **[½]**

$\Rightarrow 155 = \dfrac{n}{2}(2 + 29)$

$\Rightarrow \boxed{n = 10}$ **[½]**

Last term which is T_n

$= a + (n-1)d$ **[½]**

$= a + (9)d$

$\therefore 29 = 2 + 9d$

$\boxed{d = 3}$

Common difference = 3 **[½]**

17. Two digit numbers divisible by 6 are,

12, 18..... 96 **[1]**

$\Rightarrow 96 = 12 + (n-1) \times 6$

$[\because a_n = a + (n-1)d]$

$\Rightarrow n = \dfrac{96 - 12}{6} + 1 = 15$ **[½]**

$\therefore$ Two digit numbers divisible by 6 are 15. **[½]**

18. First three– digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore the series is 105, 112, 119,…

The maximum possible three digit number is 999.

When we divide by 7, the remainder will be 5.

Clearly, 999 – 5 = 994 is the maximum possible three – digit number divisible by 7.

The series is as follows :

105, 112, 119, ..., 994 **[½]**

Here $a = 105$, $d = 7$

Let 994 be the nth term of this AP.

$a_n = a + (n - 1)d$

$\Rightarrow$ $994 = 105 + (n - 1)7$

$\Rightarrow$ $(n - 1)7 = 889$

$\Rightarrow$ $(n - 1) = 127$

$\Rightarrow$ $n = 128$ **[½]**

So, there are 128 terms in the AP.

$\therefore$ $\text{Sum} = \frac{n}{2}\{\text{first term + last term}\}$

$= \frac{128}{2}\{a_1 + a_{128}\}$

$64\{105 + 994\} = (64)(1099) = 70336$ **[1]**

19. Let a be the first term and d be the common difference.

Given : $a = 5$

$T_n = 45$

$S_n = 400$

We know :

$T_n = a + (n - 1)d$

$\Rightarrow$ $45 = 5 + (n - 1)d$

$\Rightarrow$ $40 = (n - 1)d$ ...(i) **[1]**

And $S_n = \frac{n}{2}(a + T_n)$

$\Rightarrow$ $400 = \frac{n}{2}(5 + 45)$

$\Rightarrow$ $\frac{n}{2} = \frac{400}{50}$

$\Rightarrow$ $n = 2 \times 8 = 16$ **[½]**

On substituting $n = 16$ in (i), we get :

$40 = (16 - 1)d$

$\Rightarrow$ $40 = (15)d$

$\Rightarrow$ $d = \frac{40}{15} = \frac{8}{3}$

Thus, the common difference is $\frac{8}{3}$. **[½]**

20. $S_5 + S_7 = 167$ and $S_{10} = 235$

Now, $S_n = \frac{n}{2}\{2a + (n - 1)d\}$

$\therefore$ $S_5 + S_7 = 167$

$\Rightarrow$ $\frac{5}{2}\{2a + 4d\} + \frac{7}{2}\{2a + 6d\} = 167$

$\Rightarrow$ $5a + 10d + 7a + 21d = 167$

$\Rightarrow$ $12a + 31d = 167$ ...(i) **[½]**

Also, $S_{10} = 235$

$\therefore$ $\frac{10}{2}\{2a + 9d\} = 235$

$\Rightarrow$ $10a + 45d = 235$

$\Rightarrow$ $2a + 9d = 47$ ...(ii) **[½]**

Multiplying equation (ii) by 6, we get

$12a + 54d = 282$ ...(iii)

Subtracting (i) from (iii), we get

$$\begin{array}{rcrcr} 12a & + & 54d & = & 282 \\ (-)12a & \underset{-}{+} & 31d & = & -167 \\ \hline & & 23d & = & 115 \\ \hline \end{array}$$

$\therefore$ $d = 5$ **[½]**

Substituting value of d in (ii), we have

$2a + 9(5) = 47$

$\Rightarrow$ $2a + 45 = 47$

$\Rightarrow$ $2a = 2$

$\Rightarrow$ $a = 1$

Thus, the given AP is 1, 6, 11, 16,..... **[½]**

21. 4^{th} term of an AP $= a_4 = 0$

$\therefore$ $a + (4 - 1)d = 0$

$\therefore$ $a + 3d = 0$

$\therefore$ $a = -3d$ ...(i) **[½]**

25^{th} term of an AP $= a_{25}$

$= a + (25 - 1)d$

$= -3d + 24d$ ...[From (i)] **[½]**

$= 21d$

3 times 11^{th} term of an AP $= 3a_{11}$

$= 3[a + (11 - 1)d]$

$= 3[a + 10d]$

$= 3[-3d + 10d]$

$= 3 \times 7d$

$= 21d$ **[½]**

$\therefore$ $a_{25} = 3a_{11}$

i.e., the 25^{th} term of the AP is three times its 11^{th} term. **[½]**

22. Given progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$

This is an Arithmetic progression because

Common difference

$$(d) = 19\frac{1}{4} - 20 = 18\frac{1}{2} - 19\frac{1}{4} = ...$$

$d = \frac{-3}{4}$ **[1]**

Any n^{th} term $a_n = 20 + (n-1)\left(\frac{-3}{4}\right) = \frac{83-3n}{4}$

Any term $a_n < 0$ when $83 < 3n$

$\Rightarrow \quad n > \frac{83}{3}$

$\Rightarrow \quad n = 28$

$\therefore$ 28^{th} term will be the first negative term. **[1]**

23. First 8 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24

The above sequence is an AP **[1]**

$a = 3$, $d = 3$ and last term $l = 24$

$$S_n = \frac{n}{2}(a+l) = \frac{8}{2}[3+24] = 4(27)$$

$S_n = 108$ **[1]**

24. $S_n = 3n^2 - 4n$

Let S_{n-1} be sum of $(n - 1)$ terms

$t_n = S_n - S_{n-1}$ **[½]**

$= (3n^2 - 4n) - [3(n - 1)^2 - 4(n - 1)]$ **[½]**

$= (3n^2 - 4n) - [3n^2 - 6n + 3 - 4n + 4]$ **[½]**

$= 3n^2 - 4n - 3n^2 + 10n - 7$

$\therefore \quad t_n = 6n - 7$

So, required n^{th} term $= 6n - 7$ **[½]**

25. Common difference must be equal

$\therefore \quad (a^2 + b^2) - (a - b)^2 = (a + b)^2 - (a^2 + b^2)$ **[½]**

$\Rightarrow \quad (a^2 + b^2) - (a^2 + b^2 - 2ab) = (a^2 + b^2 + 2ab) - a^2 - b^2$ **[½]**

$\Rightarrow \quad a^2 + b^2 - a^2 - b^2 + 2ab = a^2 + b^2 + 2ab - a^2 - b^2$ **[½]**

$\Rightarrow \quad 2ab = 2ab$ **[½]**

Hence, $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P.

26. **(a)** Given A.P. is 3, 8, 13, 18, … .

Here, $a = 3$ and $d = 8 - 3 = 5$ **[½]**

$a_n = a + (n - 1)d$ $\quad$ [n^{th} term] **[½]**

$\Rightarrow \quad 78 = 3 + (n - 1)5$

$\Rightarrow \quad \frac{75}{5} = n - 1$ **[½]**

$\Rightarrow n = 16$

$\therefore$ 78 is 16^{th} term of the given A.P. **[½]**

OR

(b) n^{th} term of A.P. is

$a_n = 6n - 5$

if $n = 1$,

$\Rightarrow \quad a_1 = 6 - 5 = 1$ **[½]**

if $n_2 = 1$,

$a_2 = 6 \times 2 - 5 = 7$ **[½]**

$\therefore$ Common difference $(d) = 7 - 1$ **[½]**

$= 6$ **[½]**

27. First fifteen multiples of 8 are

8, 16, 24….. **[½]**

Here, $a = 8$ and $d = 8$

$S_{15} = \frac{15}{2}[2\times 8 + (15-1)8]$ **[½]**

$= \frac{15}{2}[16 + 112]$ **[½]**

$= \frac{15 \times 128}{2}$

$= 960$

$\therefore$ Sum of first fifteen multiples of 8 is 960. **[½]**

28. (a) Given A.P.

$-\frac{11}{2}, -3, -\frac{1}{2}, ...$

Here,

$a = -\frac{11}{2}$, $d = -3 + \frac{11}{2} = \frac{11-6}{2} = \frac{5}{2}$ **[½]**

$t_n = \frac{49}{2}$

$a + (n-1)d = \frac{49}{2}$ **[½]**

or $\quad -\frac{11}{2} + (n-1)\left(\frac{5}{2}\right) = \frac{49}{2}$

or $\quad -11 + 5n - 5 = 49$ **[½]**

$\Rightarrow \quad 5n = 49 + 16$

$\Rightarrow$ $5n = 65$

$\Rightarrow$ $n = \frac{65}{5} = 13$

$\Rightarrow$ $n = 13$ [½]

OR

(b) Given,

$a, 7, b, 23$ are in A.P.

$\therefore$ $7 - a = b - 7 = 23 - b$ [½]

$\Rightarrow$ $7 - a = b - 7$

$\Rightarrow$ $a + b = 14$ ...(i) [½]

and $b - 7 = 23 - b$

$\Rightarrow$ $2b = 30$

$\Rightarrow$ $b = 15$ [½]

From (i)

$a = 14 - 15$

$a = -1$ [½]

29. Sum of n terms of A.P. if nth term of A.P. is given by,

$$S_n = \frac{n}{2}[a + a_n]$$

If $n = 1$

$a_1 = 5 - 2 = 3$ [½]

and if $n = 20$

$a_{20} = 5 - 40 = -35$ [½]

$$\therefore \quad S_{20} = \frac{20}{2}[a_1 + a_{20}]$$

$$= \frac{20}{2}[3 + (-35)]$$

$= 10[-32]$ [½]

$S_{20} = -320$ [½]

30. n^{th} term of 63, 65, 67, ...

$= 63 + (n - 1)(2)$

$= 63 + 2n - 2$

$= 61 + 2n$ ...(i) [1]

n^{th} term of 3, 10, 17, ...

$= 3 + (n - 1)7$

$= 3 + 7n - 7$

$= 7n - 4$ ...(ii) [1]

Given that n^{th} terms of two AP's are equal.

$61 + 2n = 7n - 4$ [Using (i) and (ii)]

$65 = 5n$

$\boxed{n = 13}$ [1]

31. Lets assume first term = a

Common difference = d

$T_m = a + (m - 1)d$

$T_n = a + (n - 1)d$

Given $m.T_m = n.T_n$ [1]

$m(a + (m - 1)d) = n(a + (n - 1)d)$

$ma + m(m - 1)d = na + n(n - 1)d$

$(m - n)a + d(m^2 - m - n^2 + n) = 0$ [1]

$a(m - n) + d(m - n)(m + n - 1) = 0$

$(m - n)[a + (m + n - 1)d] = 0$

$m \neq n$

$\therefore$ $a + (m + n - 1)d = 0$

$\boxed{T_{m+n} = 0}$ [1]

32. First term (a) = 5

$T_n = 33$

Sum of first n terms = 123

$\therefore$ $\frac{n}{2}[a + T_n] = 123$ [1]

$\frac{n}{2}[8 + 33] = 123$

$\boxed{n = 6}$ [1]

$T_n = a + (n - 1)d$

$33 = 8 + (5)d$

$\boxed{d = 5}$ [1]

33. Lets say first term of given AP = a

Common difference = d

Sum of first six terms = 42

$\therefore$ $\frac{6}{2}(2a + 5d) = 42$

$2a + 5d = 14$ ...(i) [1]

Also given $T_{10} : T_{30} = 1 : 3$

$\Rightarrow$ $\frac{a + 9d}{a + 29d} = \frac{1}{3}$

$3a + 27d = a + 29d$

$\Rightarrow$ $2a = 2d$

$\Rightarrow$ $\boxed{a = d}$ ...(ii) [1]

Substituting (ii) in (i)

$\Rightarrow$ $2a + 5a = 14$

$a = 2$ and $d = 2$

$T_{13} = a + 12d$

$= 2 + 24$

$T_{13} = 26$ **[1]**

34. Sum of first ten terms = –150

Sum of next ten terms = 550

Lets say first term of AP = a

Common difference = d

Sum of first ten terms $= \frac{10}{2}[2a + 9d]$

$-150 = 5[2a + 9d]$

$\boxed{2a + 9d = -30}$ ...(i) **[1]**

For sum of next ten terms the first term would be $T_{11} = a + 10d$

$\Rightarrow$ $-550 = \frac{10}{2}[2(a + 10d) + 9d]$

$\Rightarrow$ $\boxed{-110 = 2a + 29d}$ ...(ii) **[1]**

Solving (i) and (ii)

$d = -4$

$a = 3$

$\therefore$ AP will be 3, –1, –5, –9, –13, ... **[1]**

35. Given an AP

Say first term = a

Common difference = d

Given $T_4 = 9$

$a + 3d = 9$ ...(i) **[1]**

Also $T_6 + T_{13} = 40$

$a + 5d + a + 12d = 40$

$2a + 17d = 40$ ...(ii) **[1]**

Solving (i) and (ii)

$a = 3$ $d = 2$

$\therefore$ AP will be 3, 5, 7, 9, ... **[1]**

36. Let a and d respectively be the first term and the common difference of the AP.

We know that the n^{th} term of an AP is given by $a_n = a + (n - 1)d$

According to the given information,

$A_{16} = 1 + 2a_8$

$\Rightarrow$ $a + (16 - 1)d = 1 + 2[a + (8 - 1)d]$

$\Rightarrow$ $a + 15d = 1 + 2a + 14d$

$\Rightarrow$ $-a + d = 1$ ...(i) **[1]**

Also, it is given that, $a_{12} = 47$

$\Rightarrow$ $a + (12 - 1)d = 47$

$\Rightarrow$ $a + 11d = 47$ ...(ii) **[1]**

Adding (i) and (ii), we have :

$12d = 48$

$\Rightarrow$ $d = 4$

From (i),

$-a + 4 = 1$

$\Rightarrow$ $a = 3$ **[½]**

Hence, $a_n = a + (n - 1)d = 3 + (n - 1)(4) = 3 + 4n - 4 = 4n - 1$

Hence, the n^{th} term of the AP is $4n - 1$. **[½]**

37. $S_n = 3n^2 + 4n$

First term $(a_1) = S_1 = 3(1)^2 + 4(1) = 7$

$S_2 = a_1 + a_2 = 3(2)^2 + 4(2) = 20$ **[1]**

$a_2 = 20 - a_1 = 20 - 7 = 13$

So, common difference $(d) = a_2 - a_1 = 13 - 7 = 6$ **[1]**

Now, $a_n = a + (n - 1)d$

$\therefore$ $a_{25} = 7 + (25 - 1) \times 6 = 7 + 24 \times 6 = 7 + 144 = 151$ **[1]**

38. Let a be the first term and d be the common difference of the given AP

Given :

$a_7 = \frac{1}{9}$

$a_9 = \frac{1}{7}$

$a_7 = a + (7 - 1)d = \frac{1}{9}$

$\Rightarrow$ $a + 6d = \frac{1}{9}$ ...(i) **[1]**

$a_9 = a + (9 - 1)d = \frac{1}{7}$

$\Rightarrow$ $a + 8d = \frac{1}{7}$ ...(ii) **[1]**

Subtracting equation (i) from (ii), we get :

$2d = \frac{2}{63}$

$\Rightarrow \quad d = \frac{1}{63}$ **[½]**

Putting $d = \frac{1}{63}$ in equation (i), we get :

$a + \left(6 \times \frac{1}{63}\right) = \frac{1}{9}$

$\Rightarrow \quad a = \frac{1}{63}$

$\therefore \quad a_{63} = a + (63-1)d = \frac{1}{63} + 62\left(\frac{1}{63}\right) = \frac{63}{63} = 1$

Thus, the 63rd term of the given AP is 1. **[½]**

39. Here it is given that,

$T_{14} = 2(T_8)$

$\Rightarrow$ $a + (14 - 1)d = 2[a + (8 - 1)d]$

$\Rightarrow$ $a + 13d = 2[a + 7d]$

$\Rightarrow$ $a + 13d = 2a + 14d$

$\Rightarrow$ $13d - 14d = 2a - a$

$\Rightarrow$ $-d = a$ ...(i) **[1]**

Now, it is given that its 6th term is –8.

$T_6 = -8$

$\Rightarrow$ $a + (6 - 1)d = -8$

$\Rightarrow$ $a + 5d = -8$

$\Rightarrow$ $-d + 5d = -8$ [$\because$ Using (i)]

$\Rightarrow$ $4d = -8$

$\Rightarrow$ $d = -2$

Substituting this in eq. (i), we get $a = 2$ **[1]**

Now, the sum of 20 terms,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2a + (20-1)d]$

$= 10[2(2) + 19(-2)]$

$= 10[4 - 38]$

$= -340$ **[1]**

40. Let a_1, a_2 be the first terms and d_1, d_2 the common differences of the two given AP's.

Thus, we have $S_n = \frac{n}{2}[2a_1 + (n-1)d_1]$ and

$S_n' = \frac{n}{2}[2a_2 + (n-1)d_2]$

$\therefore \quad \frac{S_n}{S_n'} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2}$ **[½]**

It is given that $\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$

$\therefore \quad \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27}$ ...(i) **[½]**

To find the ratio of the m^{th} term of the two given AP's, replace n by $(2m - 1)$ in equation (i).

$\therefore \quad \frac{2a_1 + (2m-1-1)d_1}{2a_2 + (2m-1-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27}$

$\therefore \quad \frac{2a_1 + (2m-2)d_1}{2a_2 + (2m-2)d_2} = \frac{14m-7+1}{8m-4+27}$ **[1]**

$\therefore \quad \frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2} = \frac{14m-6}{8m+23}$

Hence, the ratio of the m^{th} term of the two AP's is $14m - 6 : 8m + 23$. **[1]**

41. Given an A.P with first $(a) = 8$

Last term $(\ell) = 350$

Common difference $(d) = 9$

$T_n = a + (n - 1)d$

$= a + (n - 1)d = 350$

$\Rightarrow$ $8 + (n - 1)9 = 350$ **[1]**

$\boxed{n = 39}$

$\therefore$ Number of terms = 39 **[1]**

Sum of the terms

$= \frac{n}{2}[a + \ell]$

$= \frac{39}{2}[8 + 350]$ **[1]**

$= 6981$ **[1]**

42. Multiples of 4 between 10 and 250 are 12, 16, ... 248. **[1]**

We now have an A.P with first term = 12 and last term = 248 **[1]**

Common difference = 4

$\therefore$ $248 = 12 + (n - 1)4$

[$\because a_n = a + (n - 1)d$] **[1]**

$\Rightarrow$ $\boxed{n = 60}$

$\therefore$ Multiples of 4 between 10 and 250 are 60. **[1]**

43. Given : $S_{20} = -240$ and $a = 7$

Consider, $S_{20} = -240$

$\Rightarrow \quad \frac{20}{2}(2 \times 7 + 19d) = -240$ **[1]**

$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow$ $10(14 + 19d) = –240$

$\Rightarrow$ $14 + 19d = –24$ **[1]**

$\Rightarrow$ $19d = –38$

$\Rightarrow$ $d = –2$ **[1]**

Now, $a_{24} = a + 23d = 7 + 23 \times –2 = –39$

[$\because$ $a_n = a + (n – 1)d$]

Hence, $a_{24} = –39$ **[1]**

44. Given AP is –12, –9, –6, ..., 21

First term, $a = –12$

Common difference, $d = 3$ **[1]**

Let 12 be the n^{th} term of the AP.

$12 = a + (n – 1)d$

$\Rightarrow$ $12 = –12 + (n – 1) \times 3$ **[1]**

$\Rightarrow$ $24 = (n – 1) \times 3$

$\Rightarrow$ $n = 9$

Sum of the terms of the AP = S_9

$= \frac{n}{2}\left(2a+(n-1)d\right) = \frac{9}{2}\left(-24+8\times 3\right) = 0$ **[1]**

If 1 is added to each term of the AP, the sum of all the terms of the new AP will increase by n, *i.e.*, 9.

$\therefore$ Sum of all the terms of the new AP = 0 + 9 = 9 **[1]**

45. Let a and d be the first term and the common difference of an AP respectively.

n^{th} term of an AP, $a_n = a + (n – 1)d$

Sum of n terms of an AP, $S_n = \frac{n}{2}[2a+(n-1)d]$

We have :

Sum of the first 10 terms $= \frac{10}{2}[2a+9d]$

$\Rightarrow$ $210 = 5[2a + 9d]$

$\Rightarrow$ $42 = 2a + 9d$ ...(i) **[1]**

15^{th} term from the last = $(50 – 15 + 1)^{th}$ = 36^{th} term from the beginning

Now, $a_{36} = a + 35d$

$\therefore$ Sum of the last 15 terms

$= \frac{15}{2}(2a_{36}+(15-1)d)$ **[1]**

$= \frac{15}{2}[2(a+35d)+14d]$

$= 15[a + 35d + 7d]$

$\Rightarrow$ $2565 = 15[a + 42d]$

$\Rightarrow$ $171 = a + 42d$ ...(ii) **[1]**

From (i) and (ii), we get,

$d = 4$

$a = 3$

So, the AP formed is 3, 7, 11, 15... and 199. **[1]**

46. Consider the given AP 8, 10, 12, ...

Here the first term is 8 and the common difference is 10 – 8 = 2

General term of an AP is t_n is given by,

$t_n = a + (n – 1)d$

$\Rightarrow$ $t_{60} = 8 + (60 – 1) \times 2$

$\Rightarrow$ $t_{60} = 8 + 59 \times 2$

$\Rightarrow$ $t_{60} = 8 + 118$

$\Rightarrow$ $t_{60} = 126$ **[1]**

We need to find the sum of the last 10 terms.

Thus,

Sum of last 10 terms = Sum of first 60 terms – Sum of first 50 terms

[½]

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow$ $S_{60} = \frac{60}{2}[2\times 8+(60-1)\times 2]$

$\Rightarrow$ $S_{60} = 30[16 + 59 \times 2]$

$\Rightarrow$ $S_{60} = 30[134]$

$\Rightarrow$ $S_{60} = 4020$ **[1]**

Similarly,

$\Rightarrow$ $S_{50} = \frac{50}{2}[2\times 8+(50-1)\times 2]$

$\Rightarrow$ $S_{50} = 25[16 + 49 \times 2]$

$\Rightarrow$ $S_{50} = 25[114]$

$\Rightarrow$ $S_{50} = 2850$ **[1]**

Thus the sum of last 10 terms = $S_{60} – S_{50}$ = 4020 – 2850 = 1170 **[½]**

47. Let there be a value of X such that the sum of the numbers of the houses preceding the house numbered X is equal to the sum of the numbers of the houses following it.

That is, $1 + 2 + 3 + ... + (X – 1) = (X + 1) + (X + 2) ..... + 49$

$\therefore$ $[1 + 2 + 3 + ... + (X – 1)$

$= [1 + 2 + ... + X + (X - 1) + ... + 49]$
$- (1 + 2 + 3 + ... + X)$ **[1]**

$\therefore \quad \frac{X-1}{2}[1+X-1] = \frac{49}{2}[1+49] - \frac{X}{2}[1+X]$

$\therefore \quad X(X - 1) = 49 \times 50 - X(1 + X)$

$\therefore \quad X(X - 1) + X(1 + X) = 49 \times 50$ **[1]**

$\therefore \quad X^2 - X + X + X^2 = 49 \times 50$

$\therefore \quad 2X^2 = 49 \times 50$ **[1]**

$\therefore \quad X^2 = 49 \times 25$

$\therefore \quad X = 7 \times 5 = 35$

Since X is not a fraction, the value of x satisfying the given condition exists and is equal to 35. **[1]**

48. Let the numbers be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$

$\therefore \quad (a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow \quad 4a = 32$

$a = 8$ **[1]**

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow \quad 15a^2 - 135d^2 = 7a^2 - 7d^2$

$\Rightarrow \quad 8a^2 = 128d^2$ **[1]**

$d^2 = \frac{8a^2}{128} = \frac{8 \times 8 \times 8}{128}$

$d^2 = 4$

$d = \pm 2$ **[1]**

If $d = 2$ numbers are : 2, 6, 10, 14

If $d = -2$ numbers are 14, 10, 6, 2 **[1]**

49. Let the first four terms be a, $a + d$, $a + 2d$, $a + 3d$

$a + a + d + a + 2d + a + 3d = 40$ **[½]**

$\Rightarrow \quad 2a + 3d = 20 \quad ...(i)$ **[½]**

Sum of first 14 terms = 280

$\frac{n}{2}[2a+(n-1)d] = 280$ **[½]**

$\Rightarrow \quad \frac{14}{2}[2a+13d] = 280$

$\Rightarrow \quad 2a + 13d = 40 \quad ...(ii)$ **[1]**

On subtracting (i) from (ii), we get $d = 2$

Substituting the value of d in (i) **[½]**

$a = 7$

$\therefore \quad$ Sum of n terms $= \frac{n}{2}[2a+(n-1)d]$ **[½]**

$= \frac{n}{2}[14+(n-1)2]$

$= n^2 + 6n$ **[½]**

50. Let the first term and common difference be a and d.

According to the question,

$4(a + 3d) = 18 \times (a + 17d)$ **[1]**

$\Rightarrow \quad 4a + 12d = 18a + 306d$

$\Rightarrow \quad 14a + 294d = 0$ **[1]**

$\Rightarrow \quad a + 21d = 0$ **[1]**

$\therefore \quad a_{22} = a + 21d$

$= 0$ **[1]**

OR

Given A.P.

24, 21, 18,.....

$\therefore \quad$ First term = 24 = a

and common difference = $-3 = d$ ...(i) **[1]**

Let number of terms is n.

$\therefore \quad$ Sum of n terms $= \frac{n}{2}[2a+(n-1)d]$ **[1]**

According to question

$\Rightarrow \quad 78 = \frac{n}{2}[2 \times 24 - 3x(n-1)]$ [from (i) and given]

$\Rightarrow \quad 78 = \frac{n}{2}[51 - 3n]$

$\Rightarrow \quad n^2 - 17n + 52 = 0$ **[1]**

$\Rightarrow \quad n^2 - 13n - 4n + 52 = 0$

$\Rightarrow \quad n(n - 13) - 4(n - 13) = 0$

$(n - 13)(n - 4) = 0$

$n = 13, 4$

For first 4 terms and first 13 terms in both case we get sum 78. **[1]**

51. Let the four consecutive numbers in A.P. are $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$. **[½]**

$\therefore \quad$ According to the condition given,

$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow \quad 4a = 32$

$\Rightarrow \quad a = 8 \quad ...(i)$ **[1]**

and, according to the 2nd condition given,

$$\frac{(a-3d)\times(a+3d)}{(a-d)\times(a+d)}=\frac{7}{15}$$ **[½]**

$$\Rightarrow \frac{(8-3d)\times(8+3d)}{(8-d)\times(8+d)}=\frac{7}{15}$$

$$\Rightarrow \frac{64-9d^2}{64-d^2}=\frac{7}{15}$$ **[½]**

$\Rightarrow$ $15(64 - 9d^2) = 7(64 - d^2)$

$\Rightarrow$ $128d^2 = 512$

$\Rightarrow$ $d^2 = 4$

$\Rightarrow$ $d = \pm 2$ **[½]**

$\therefore$ Numbers are 2, 6, 10 and 14 or 14, 10, 6 and 2. **[1]**

OR

Here 1, 4, 7, 10, ... x is an A.P.

With first term a = 1 and common difference d = 3. **[½]**

Let there be n terms in the A.P. Then,

$x = n^{th}$ term

$\Rightarrow$ $x = 1 + (n - 1) \times 3$ **[½]**

$= 3n - 2$ ...(i)

Now, $1 + 4 + 7 + 10 + ... + x = 287$

$$\Rightarrow \frac{n}{2}[1+x]=287 \quad \left[S_n=\frac{n}{2}(a+l)\right]$$ **[½]**

$$\Rightarrow \frac{n}{2}[1+3n-2]=287$$

$\Rightarrow$ $3n^2 - n - 574$

$\Rightarrow$ $3n^2 - n - 574 = 0$ **[1]**

$\Rightarrow$ $3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow$ $3n(n - 14) + 41(n - 14) = 0$

$\Rightarrow$ $(n - 14)(3n + 41) = 0$

$\Rightarrow$ $n - 14 = 0$ $\quad [\because 3n+41\neq 0]$

$\Rightarrow$ $n = 14$ **[½]**

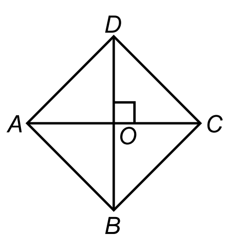
Putting $n = 14$ in eqn (i), we get

$x = 3 \times 14 - 2$

$x = 40$ **[1]**

6 : Triangles

1. Length of the diagonals of a rhombus are 30 cm and 40 cm.



i.e., $BD = 30$ cm

$AC = 40$ cm

$\therefore$ $OD = OB = 15$ cm

$OA = OC = 20$ cm **[½]**

In ΔAOD,

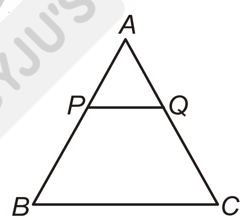
$OA^2 + OD^2 = AD^2$

$(20)^2 + (15)^2 = AD^2$

$AD = 25$ cm

Side of rhombus = 25 cm **[½]**

2.



$PQ \parallel BC$

$$\frac{AP}{PB}=\frac{1}{2}$$

$$\frac{PB}{AP}=\frac{2}{1}$$

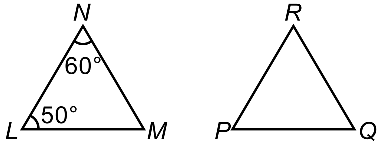
$$\frac{PB}{AP}+1=\frac{2}{1}+1$$ **[½]**

$$\frac{PB+AP}{AP}=\frac{3}{1}$$

$$\boxed{\frac{AP}{AB}=\frac{1}{3}}$$

$$\therefore \frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)}=\left(\frac{AP}{AB}\right)^2=\frac{1}{9}$$ **[½]**

3.



Given $\Delta LMN \sim \Delta PQR$

In similar triangles, corresponding angles are equal.

$\therefore \quad \angle L = \angle P$

$\angle M = \angle Q$

$\angle N = \angle R$

In ΔLMN,

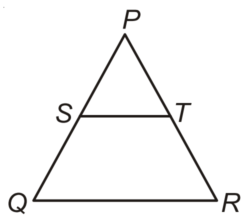
$\angle L + \angle M + \angle N = 180°$

$\angle M = 180° - 50° - 60°$

$\angle M = 70°$

$\therefore \quad \angle Q = 70°$

4.



Given : $PT = 2$ cm, $TR = 4$ cm. So, $PR = 6$ cm

$ST \parallel QR$

As it is given that $ST \parallel QR$

$\Delta PST \sim \Delta PQR$

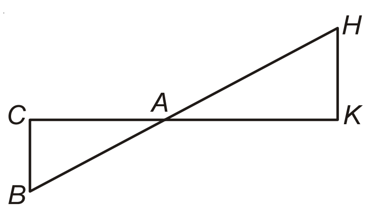
$$\therefore \quad \frac{PS}{PQ} = \frac{PT}{PR} = \frac{ST}{QR}$$ **[½]**

Also, $\dfrac{\text{ar}(\Delta PST)}{\text{ar}(\Delta PQR)} = \left(\dfrac{PS}{PQ}\right)^2 = \left(\dfrac{PT}{PR}\right)^2 = \left(\dfrac{ST}{QR}\right)^2$

$$\therefore \quad \frac{\text{ar}(\Delta PST)}{\text{ar}(\Delta PQR)} = \left(\frac{PT}{PR}\right)^2 = \left(\frac{2}{6}\right)^2$$

Ratio : 1 : 9 **[½]**

5.



Given $\Delta AHK \sim \Delta ABC$

$$\Rightarrow \quad \frac{AH}{AB} = \frac{HK}{BC} = \frac{AK}{AC}$$ **[½]**

Also, we know $AK = 10$ cm, $BC = 3.5$ cm and $HK = 7$ cm.

$$\Rightarrow \quad \frac{AK}{AC} = \frac{HK}{BC}$$

$$\Rightarrow \quad \frac{10}{AC} = \frac{7}{3.5}$$

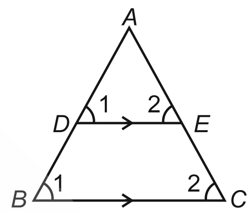
$\boxed{AC = 5 \text{ cm}}$ **[½]**

6. $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{AB^2}{PQ^2}$ **[½]**

(Ratio of area of similar triangle is equal to square of their proportional sides)

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$ **[½]**

7.



$DE \parallel BC$

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By AA similarity] **[½]**

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta ADE)} = \left(\frac{AB}{AD}\right)^2$$

[By area similarity theorem]

$$= \left(\frac{3}{1}\right)^2$$

$$= \frac{9}{1}$$ **[½]**

8. Let perimeters of two similar triangles be P_1 and P_2 and their corresponding sides be a_1 and a_2

$$\therefore \quad \frac{P_1}{P_2} = \frac{a_1}{a_2}$$

$$\Rightarrow \quad \frac{25}{15} = \frac{9}{a_2}$$

$\Rightarrow \quad a_2 = 5.4$ cm **[1]**

9. $\because \quad DE \parallel BC$

$$\therefore \quad \frac{AD}{AB} = \frac{AE}{AC}$$ **[½]**

$$\Rightarrow \quad \frac{2.4}{3.2} = \frac{AE}{8}$$

$$\therefore \quad AE = \frac{24}{32} \times 8 = 6 \text{cm}$$ **[½]**

10. $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{(AB)^2}{(PQ)^2} = \left(\dfrac{1}{3}\right)^2$

$= \dfrac{1}{9}$ **[1]**

11. Length of altitude of an equilateral triangle $= \dfrac{\sqrt{3}}{2} \times \text{side}$

$\therefore \quad \dfrac{\sqrt{3}}{2} \times 2a = \sqrt{3}a$ **[1]**

12. Answer (c)

Applying B.P.T. in ΔABC, $DE \parallel BC$

$\dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\Rightarrow \quad \dfrac{4}{6} = \dfrac{5}{EC}$

$\Rightarrow \quad EC = 7.5$ cm **[1]**

13. Answer (d)

$\dfrac{\text{ar}(\Delta DEF)}{\text{ar}(\Delta PQR)} = \left(\dfrac{EF}{QR}\right)^2 = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

$\therefore$ ar(ΔDEF) : ar(ΔPQR) = 9 : 4 **[1]**

14. Answer (a)

Two congruent figures are always similar.

15. Answer (b)

$DE \parallel BC$

$\therefore \quad \dfrac{DE}{BC} = \dfrac{AD}{AB}$

$\Rightarrow \quad DE : BC = 2 : 5$ **[1]**

16. Answer (d)

Because, according to criteria of similarity RHS similarity is not possible. **[1]**

17. Answer (a)

$BC = \sqrt{AC^2 + AB^2} = x\sqrt{2}$ units **[1]**

18. Answer (c)

$\dfrac{BC}{PR} = \dfrac{x\sqrt{2}}{2x\sqrt{2}} = \dfrac{1}{2}$

$\therefore \quad BC : PR = 1 : 2$ **[1]**

19. Answer (c)

$\dfrac{\text{ar}(\Delta PQR)}{\text{ar}(\Delta ABC)} = \left(\dfrac{PQ}{AC}\right)^2 = \left(\dfrac{2}{1}\right)^2 = \dfrac{4}{1}$

ar(ΔPQR) : ar(ΔABC) = 4 : 1 **[1]**

20. Answer (d)

$\because \quad \angle A = 90°$ and $\angle P = 45°$

$\therefore \quad \Delta PQR$ is not similar to ΔABC. **[1]**

21. Answer (d)

$\dfrac{ar(\Delta ABC)}{ar(\Delta ADE)} = \left(\dfrac{AB}{AD}\right)^2$ [By area theorem]

$= \left(\dfrac{3+2}{2}\right)^2$

$= \dfrac{25}{4}$ **[1]**

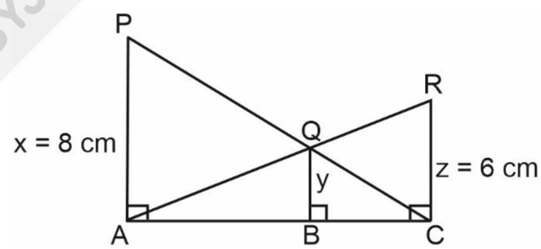
22. Answer (b)

Here, $\angle F = \angle C$, $\angle B = \angle E$ and $AB = \dfrac{1}{2}DE$

Since, AB and DE are not equal.

So, $\Delta ABC \sim \Delta DEF$. **[1]**

23. Answer (d)



$\dfrac{y}{x} = \dfrac{BC}{AC}$ [By BPT]

$\Rightarrow \quad \dfrac{x}{y} = \dfrac{AB+BC}{BC} = \dfrac{AB}{BC} + 1$ ...(i)

and $\dfrac{z}{y} = \dfrac{AC}{AB} = 1 + \dfrac{BC}{AB} = 1 + \dfrac{y}{x-y}$ [By BPT]

[From (i)]

$\Rightarrow \quad \dfrac{6}{y} = 1 + \dfrac{y}{8-y}$

$\Rightarrow \quad 8y = 48 - 6y$

$\Rightarrow \quad y = \dfrac{24}{7}$ cm **[1]**

24. Answer (a)

$y° - (3x - 2)° = 9°$

$\Rightarrow\ 3x° - y° = -7$ ...(i)

and $x° + (3x - 2)° + y° = 180°$

$\Rightarrow\ 4x° + y° = 182°$ ...(ii)

$\Rightarrow\ x° = \dfrac{182° - 7°}{7} = 25°$ and $y° = 82°$

$\Rightarrow\ \angle A = 25°, \angle B = 73°$ and $\angle C = 82°$

$\therefore$ Sum of greatest and smallest angles

$= 82° + 25° = 107°$ **[1]**

25. Answer (a)

$\Delta AOB \sim \Delta COD$ [By AA similarity]

26. Answer (b)

$$\frac{\text{ar}(\Delta\text{AOB})}{\text{ar}(\Delta\text{COD})} = \left(\frac{AB}{CD}\right)^2 = \left(\frac{5}{10}\right)^2 = \frac{1}{4}$$ **[1]**

27. Answer (d)

$$\frac{\text{Perimeter of } \Delta AOB}{\text{Perimeter of } \Delta COD} = \frac{AB}{CD}$$

or $\dfrac{AB}{CD} = \dfrac{1}{4}$ **[1]**

28. Answer (b)

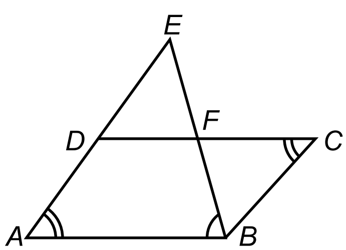
If $\dfrac{AO}{BC} = \dfrac{AD}{BO} = \dfrac{OD}{OC}$

or $\Delta AOD \sim \Delta BCO$ **[1]**

29. Answer (b)

$$\frac{\text{ar}(\Delta AOB)}{\text{ar}(\Delta COD)} = \left(\frac{\text{Corresponding altitude of } \Delta AOB}{\text{Corresponding altitude of } \Delta COD}\right)^2$$ **[1]**

30.



In ΔABE and ΔCFB,

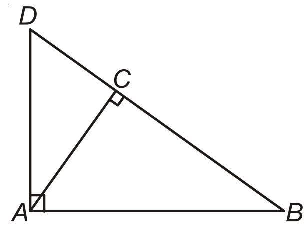
$\angle A = \angle C$ (Opposite angles of a parallelogram) **[1]**

$\angle AEB = \angle CBF$

(Alternate interior angles as $AE \parallel BC$)

$\therefore\ \Delta ABE \sim \Delta CFB$ (By AA similarly criterion) **[1]**

31.



In ΔABC

$AB^2 + AD^2 = BD^2$ ...(i)

In ΔABC

$AC^2 + BC^2 = AB^2$ ...(ii)

In ΔACD

$AC^2 + CD^2 = AD^2$ ...(iii)

Subtracting (iii) from (ii)

$AB^2 - AD^2 = BC^2 - CD^2$ ...(iv) **[1]**

Adding (i) and (iv)

$2AB^2 = BD^2 + BC^2 - CD^2$

$2AB^2 = (BC + CD)^2 + BC^2 - CD^2$

$2AB^2 = BC^2 + CD^2 + 2BC.CD + BC^2 - CD^2$

$AB^2 = BC(BC + CD)$

$AB^2 = BC.BD$ **[1]**

32. In ΔBAC; $DE \parallel AC$

$\dfrac{BE}{EC} = \dfrac{BD}{DA}$ ...(i) {By B.P.T} **[½]**

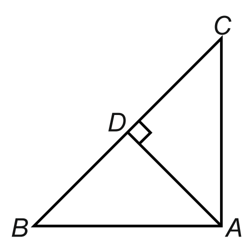
In ΔBAP; $DC \parallel AP$

$\dfrac{BC}{CP} = \dfrac{BD}{DA}$ ...(ii) {By B.P.T} **[½]**

From (i) and (ii), we have **[½]**

$\dfrac{BE}{EC} = \dfrac{BC}{CP}$ Hence proved. **[½]**

33.



In ΔABD,

By Pythagoras theorem,

$AB^2 = BD^2 + AD^2$ ...(i)

And in ΔADC, **[1]**

By Pythagoras theorem,

$AC^2 = CD^2 + AD^2$

$CD^2 = AC^2 - AD^2$ ...(ii) **[1]**

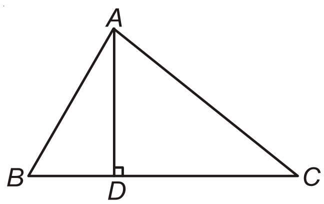
On adding (i) and (ii), we get,

$\Rightarrow AB^2 + CD^2 = BD^2 + AD^2 + AC^2 - AD^2$

$\Rightarrow AB^2 + CD^2 = BD^2 + AC^2$ **[1]**

Hence proved.

34.



$BD = \frac{1}{3}CD;$

$BD + CD = BC$

$CD = \frac{3}{4}BC$

$BD = \frac{1}{4}BC$

In right ΔACD,

$AC^2 = AD^2 + CD^2$ ...(i) **[1]**

(Pythagoras Theorem)

In right ΔABD,

$AB^2 = AD^2 + BD^2$ ...(ii)

(Pythagoras Theorem)

From (i) and (ii), we get

$AC^2 = AB^2 - BD^2 + CD^2$

$$\Rightarrow AC^2 = AB - \left(\frac{BC}{4}\right)^2 + \left(\frac{3BC}{4}\right)^2$$ **[1]**

$$\Rightarrow AC^2 = AB^2 - \frac{BC^2}{16} + \frac{9BC^2}{16}$$

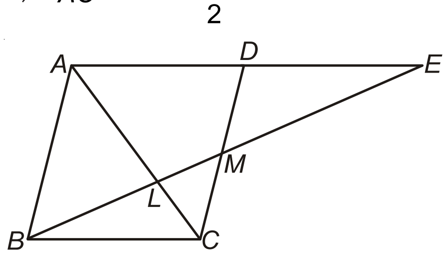
$$\Rightarrow AC^2 = AB^2 + \frac{9BC^2 - BC^2}{16}$$

$$\Rightarrow AC^2 = AB^2 + \frac{8BC^2}{16}$$

$$\Rightarrow AC^2 = AB^2 + \frac{BC^2}{2}$$

$$\Rightarrow AC^2 = \frac{2AB^2 + BC^2}{2}$$ **[1]**

35.



In ΔDME and ΔCMB

$\angle EDM = \angle MCB$ [Alternate angles]

$DM = CM$ [M is mid-point of CD]

$\angle DME = \angle BMC$ [Vertically opposite angles]

By ASA congruency $\Delta DME \cong \Delta CMB$ **[1]**

By CPCT

$BM = ME$

$DE = BC$

Now in

ΔALE and ΔBLC

$\angle ALE = \angle BLC$ [VOA]

$\angle LAE = \angle LCB$ [Alternate angles]

By AA similarly

$\Delta ALE \sim \Delta CLB$ **[1]**

$$\Rightarrow \frac{AE}{BC} = \frac{AL}{CL} = \frac{LE}{LB}$$

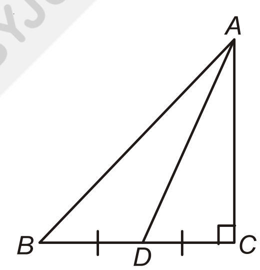
$$\Rightarrow \frac{EL}{BL} = \frac{AE}{BC}$$

$$\Rightarrow \frac{EL}{BL} = \frac{AD + DE}{BC}$$

$$\Rightarrow \frac{EL}{BL} = \frac{BC + BC}{BC}$$

$$\Rightarrow \boxed{EL = 2BL}$$ **[1]**

36.



Given that $BD = CD$

$AC \perp BC$

In ΔABC, $AB^2 = BC^2 + AC^2$

$AB^2 = (BD + CD)^2 + AC^2$

$AB^2 = (2CD)^2 + AC^2$

$AB^2 = 4CD^2 + AC^2$ ...(i) **[1]**

In ΔADC, $AD^2 = CD^2 + AC^2$

$CD^2 = AD^2 - AC^2$ **[1]**

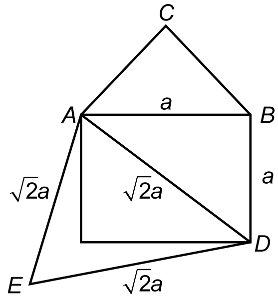
Substituting CD^2 in (i), we get

$\Rightarrow AB^2 = 4AD^2 - 4AC^2 + AC^2$

$$\Rightarrow \boxed{AB^2 = 4AD^2 - 3AC^2}$$ **[1]**

Hence proved.

37.



$$A(\Delta ABC)=\frac{\sqrt{3}}{4}\times \text{side}^2=\frac{\sqrt{3}}{4}\times a^2 \quad \text{...(i)}$$

Using Pythagoras theorem

$AD^2 = AB^2 + BD^2 = a^2 + a^2 = 2a^2$ **[1]**

$AD=\sqrt{2}a$

$$\therefore \quad A(\Delta ADE)=\frac{\sqrt{3}}{4}\times\left(\sqrt{2}a\right)^2=\frac{\sqrt{3}}{4}\times 2a^2 \quad \text{...(ii)}$$

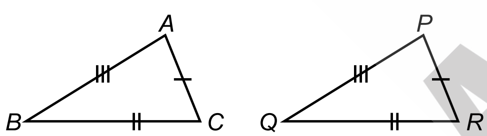
$$\frac{A(\Delta ABC)}{A(\Delta ADE)}=\frac{\sqrt{3}/4\times a^2}{\sqrt{3}/4\times 2a^2}$$ **[1]**

$$A(\Delta ABC)=\frac{1}{2}A(\Delta ADE)$$

Area of equilateral triangle described on one side

$=\frac{1}{2}$ (area of equilateral Δ described on one of its diagonal) **[1]**

38.



Let ΔABC be similar to ΔPQR.

$$\therefore \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)}=\frac{AB^2}{PQ^2}=\frac{BC^2}{QR^2}=\frac{AC^2}{PR^2}$$ **[1]**

Given that ar(ΔABC) = ar(ΔPQR)

$$\therefore \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)}=1$$

$$1=\frac{AB^2}{PQ^2}=\frac{BC^2}{QR^2}=\frac{AC^2}{PR^2}$$ **[1]**

$\therefore \quad AB = PQ$

$BC = QR$

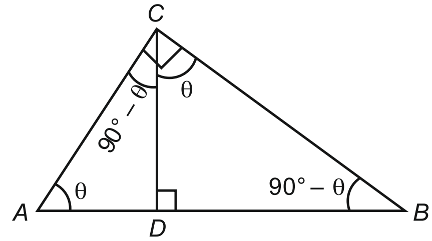
$AC = PR$

Hence, corresponding sides are equal.

$\therefore \quad \Delta ABC \cong \Delta PQR$ (SSS rule) **[1]**

Hence proved.

39.



Let $\angle A = \theta$

$\therefore \quad \angle ACD = 90° - \theta,\ \angle BCD = \theta,\ \angle CBD = 90° - \theta$

[½]

$\because \quad \angle CAD = \angle BCD$

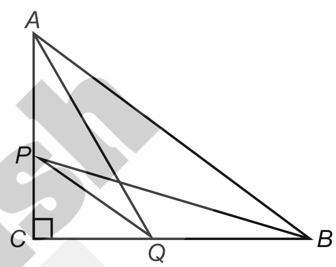
and $\angle ACD = \angle CBD$ **[½]**

$\Delta CAD \sim \Delta BCD$ [By AA similarity] **[1]**

$$\therefore \quad \frac{AD}{CD}=\frac{CD}{BD}$$ **[½]**

$\therefore \quad CD^2 = AD \times BD$ **[½]**

40.



In right ΔACQ,

$AQ^2 = AC^2 + CQ^2$ ...(i)

[By Pythagoras theorem] **[1]**

In right ΔPCB,

$BP^2 = PC^2 + CB^2$ ...(ii)

[By Pythagoras theorem] **[1]**

On adding equations (i) and (ii), we get

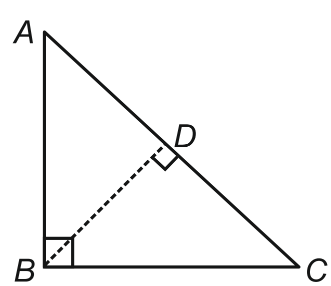
$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + CB^2$ **[½]**

$= (AC^2 + CB^2) + (CQ^2 + PC^2)$

$= AB^2 + PQ^2$

[By Pythagoras theorem] **[½]**

41. Given : ΔABC is a right triangle right angled at B.



To prove : $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$.

Proof : In ΔABC and ΔADB, **[½]**

$\angle ABC = \angle ADB$ [Each 90°]

and $\angle BAC = \angle DAB$ [common]

$\therefore \Delta ABC \sim \Delta ADB$ [By AA] **[½]**

$\Rightarrow \dfrac{AB}{AD} = \dfrac{AC}{AB}$

[Corresponding sides of similar triangles are proportional]

$\therefore AB^2 = AC \times AD$ ...(i) **[½]**

Now,

In ΔABC and ΔBDC,

$\angle ABC = \angle BDC$ [Each 90°]

and $\angle ACB = \angle BCD$ [common]

$\therefore \Delta ABC \sim \Delta BDC$ [By AA] **[½]**

$\Rightarrow \dfrac{AC}{BC} = \dfrac{BC}{CD}$

[Corresponding sides of similar triangles are proportional]

$\therefore BC^2 = AC \times CD$ ...(ii) **[½]**

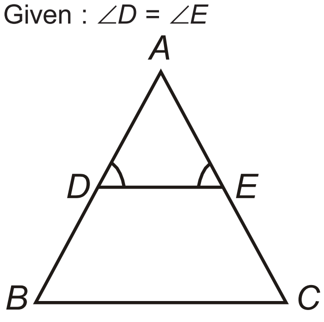
Adding equation (i) and (ii), we get

$$AB^2 + BC^2 = AC \times AD + AC \times CD$$
$$= AC(AD + CD)$$
$$= AC \times AC = AC^2$$

$\therefore AC^2 = AB^2 + BC^2$

Hence, proved. **[½]**

42. Given : $\angle D = \angle E$



$\dfrac{AD}{DB} = \dfrac{AE}{EC}$ **[½]**

To Prove : ΔBAC is an isosceles triangle.

Proof : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ (Given) **[½]**

$\therefore DE \parallel BC$ [By converse of B.P.T]

$\Rightarrow \angle D = \angle B$ ...(i) **[½]**

[Corresponding angles]

$\angle E = \angle C$ ...(ii)

[Corresponding angles]

But $\angle D = \angle E$ (Given) **[½]**

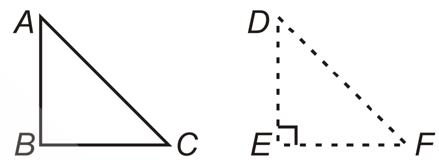
From (i) and (ii)

$\therefore \angle B = \angle C \Rightarrow AB = AC$ **[½]**

Hence, ΔBAC is an isosceles triangle. **[½]**

43. Given : A triangle ABC such that $AC^2 = AB^2 + BC^2$

To prove : $\angle ABC = 90°$



[½]

Construction : Construct a ΔDEF such that $DE = AB$, $EF = BC$ and $\angle E = 90°$ **[½]**

Proof : In right ΔDEF

$DE^2 + EF^2 = DF^2$ **[½]**

[By pythagoras theorem]

$\Rightarrow AB^2 + BC^2 = DF^2$ [$\because DE = AB$, EF = BC]

But $AB^2 + BC^2 = AC^2$ [Given] **[½]**

$\therefore AC^2 = DF^2$

$\Rightarrow AC = DF$

Thus in ΔABC and ΔDEF, we have

$AB = DE$, $BC = EF$ and $AC = DF$ **[½]**

$\therefore \Delta ABC \cong \Delta DEF$ [By SSS congruency]

$\Rightarrow \angle B = \angle E = 90°$ **[½]**

Therefore, ΔABC is right triangle, right angled at B.

Hence proved.

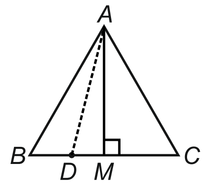
44. Let the each side of ΔABC be 'a' unit.

$\therefore BD = \dfrac{a}{3}$

To prove : $9(AD)^2 = 7(AB)^2$

Construction : Draw $AM \perp BC$:

$DM = \dfrac{a}{2} - \dfrac{a}{3} = \dfrac{a}{6}$



[1]

$\therefore$ In ΔABM

$AB^2 = BM^2 + AM^2$ ...(i)

and in ΔADM

$AD^2 = AM^2 + DM^2$ ...(ii)

In ΔABM, $\sin 60° = \dfrac{AM}{AB}$ **[1]**

$\Rightarrow$ $AM = AB\sin 60°$

$= a\dfrac{\sqrt{3}}{2}$

Now, taking $9(AD)^2$

$9(AM^2 + DM^2)$ **[1]**

$$9\left(\left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2\right)$$

$$9\left[\frac{3a^2}{4} + \frac{a^2}{36}\right] = 9 \times \frac{28a^2}{36}$$

$7(AB)^2 = 7a^2$

or

$\therefore$ $9(AD^2) = 7(AB^2)$

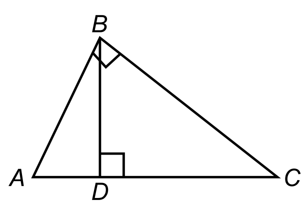
Hence proved. **[1]**

45. Given : A right-angled triangle ABC in which $\angle B = 90°$.

To Prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $AC^2 = AB^2 + BC^2$

Construction : From B draw $BD \perp AC$.



[1]

Proof : In triangle ADB and ABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90°]

and, $\angle A = \angle A$ [Common]

So, by AA-similarity criterion, we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow$ $\dfrac{AD}{AB} = \dfrac{AB}{AC}$ **[1]**

[$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow$ $AB^2 = AD \times AC$ ...(i)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ [Each equal to 90°]

and, $\angle C = \angle C$ [Common]

So, by AA-similarity criterion, we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow$ $\dfrac{DC}{BC} = \dfrac{BC}{AC}$ **[1]**

[$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow$ $BC^2 = AC \times DC$ ...(ii)

Adding equation (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

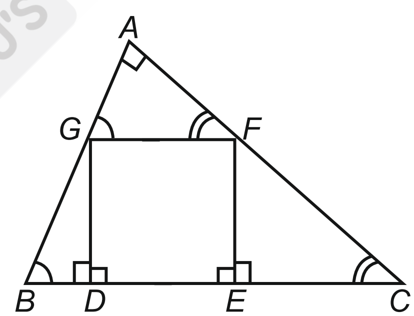
$\Rightarrow$ $AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow$ $AB^2 + BC^2 = AC \times AC$

$\Rightarrow$ $AB^2 + BC^2 = AC^2$

Hence, $AC^2 = AB^2 + BC^2$ **[1]**

46. **Given :** $DEFG$ is a square and ΔABC is a right triangle right angled at A.



To prove : (i) $\Delta AGF \sim \Delta DBG$

(ii) $\Delta AGF \sim \Delta EFC$

Proof :

(i) In ΔAGF and ΔDBG

$\angle A = \angle D = 90°$

and $\angle AGF = \angle GBD = 90°$

($\because$ $GF \parallel BC \Rightarrow$ Corresponding angles) **[1]**

By AA similarity

$\Delta AGF \sim \Delta DBG$ **[1]**

(ii) In ΔAGF and ΔEFC

$\angle A = \angle E = 90°$

$\angle AFG = \angle ECF = 90°$

($\because$ $GF \parallel BC \Rightarrow$ Corresponding angles) **[1]**

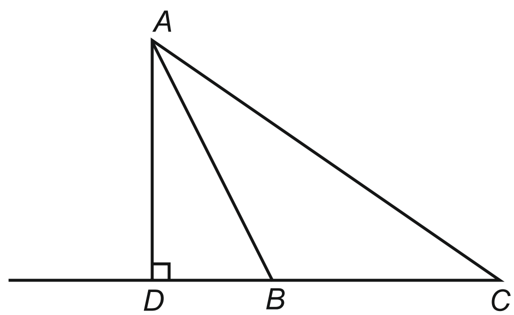
By AA similarity

$\Delta AGF \sim \Delta EFC$ **[1]**

Hence proved.

OR

Given : In ΔABC, $\angle B$ is obtuse angle.



$AD \perp CB$ produced.

To prove : $AC^2 = AB^2 + BC^2 + 2BC \times BD$ **[1]**

Proof : In ΔADC, $\angle D = 90°$

$AC^2 = AD^2 + DC^2$ ... (1) **[½]**

In ΔABD, $\angle D = 90°$

$AB^2 = AD^2 + BD^2$

$\Rightarrow$ $AD^2 = AB^2 - BD^2$ ... (2) **[½]**

From (1) and (2)

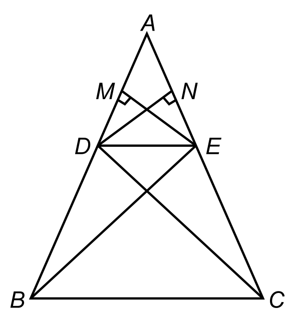
$AC^2 = AB^2 - BD^2 + DC^2$ **[½]**

$= AB^2 - BD^2 + (BD + BC)^2$ **[½]**

$= AB^2 - BD^2 + BD^2 + BC^2 + 2BC \times BD$

$AC^2 = AB^2 + BC^2 + 2BC \times BD$ **[1]**

47.



Construction: Join BE and CD and draw perpendicular DN and EM to AC and AB respectively.

$$\frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\frac{1}{2} \times EM \times AD}{\frac{1}{2} \times BD \times EM}$$

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{AD}{BD} \quad \text{...(i)}$$ **[1]**

Similarly,

$$\frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN}$$

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CDE)} = \frac{AE}{EC} \quad \text{...(ii)}$$ **[1]**

But ar(ΔBDE) = ar(ΔCDE) ($\because$ Triangles on same base DE and between the same parallels DE and BC)

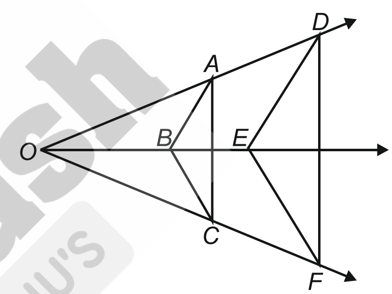
Thus, equation (ii) becomes,

$$\frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{AE}{EC} \quad \text{...(iii)}$$ **[1]**

From equations (i) and (iii), we have,

$$\frac{AD}{BD} = \frac{AE}{EC}$$ **[1]**

In the given figure, $AB \parallel DE$ and $BC \parallel EF$.



In ΔODE, $AB \parallel DE$ (Given)

$\therefore$ By basic proportionality theorem,

$$\frac{OA}{AD} = \frac{OB}{BE} \quad \text{...(i)}$$ **[1]**

Similarly, in ΔOEF, $BC \parallel EF$ (Given)

$$\therefore \frac{OB}{BE} = \frac{OC}{CF} \quad \text{....(ii)}$$

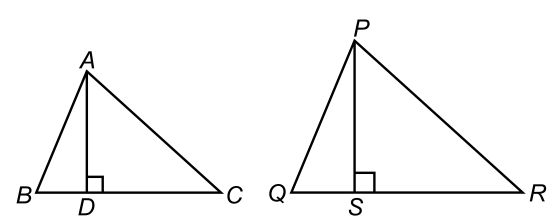
Comparing (i) and (ii), we get

$$\frac{OA}{AD} = \frac{OC}{CF}$$

Hence, $AC \parallel DF$ **[1]**

[By the converse of BPT]

48.



Proof : Given $\Delta ABC \sim \Delta PQR$

$\Rightarrow \quad \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \qquad \text{...(i)}$$

Ratio of areas of ΔABC and ΔPQR will be

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{\frac{1}{2}\times BC\times AD}{\frac{1}{2}\times QR\times PS} \qquad \text{...(ii)} \qquad \textbf{[1]}$$

In ΔABD and ΔPQS

$\angle B = \angle Q$

$\angle ADB = \angle PSQ = 90°$

By AA similarity $\Delta ABD \sim \Delta PQS$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PS} = \frac{BD}{QS} \qquad \text{...(iii)} \qquad \textbf{[1]}$$

From (i) and (iii) we get

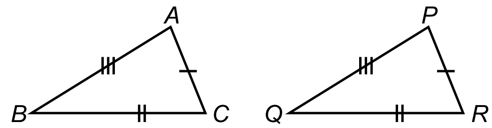
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR} = \frac{AD}{PS}$$

$$\therefore \frac{BC}{QR} = \frac{AD}{PS} \qquad \text{...(iv)}$$

From (ii) and (iv)

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC\times BC}{QR\cdot QR}$$

$$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{(BC)^2}{(QR)^2} = \frac{(AB)^2}{(PQ)^2} = \frac{(CA)^2}{(PR)^2} \qquad \textbf{[1]}$$



Let ΔABC be similar to ΔPQR.

$$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} \qquad \textbf{[1]}$$

Given that ar(ΔABC) = ar(ΔPQR)

$$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$$

$$1 = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} \qquad \textbf{[1]}$$

$\therefore \quad AB = PQ$

$BC = QR$

$AC = PR$

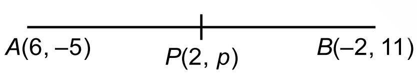
Hence, corresponding sides are equal.

$\therefore \quad \Delta ABC \cong \Delta PQR$ (SSS rule) **[1]**

Hence proved.

7 : Coordinate Geometry

1.



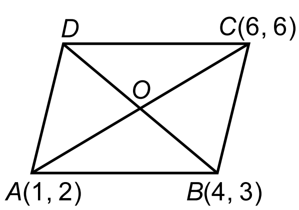
Given P is midpoint of AB

$$\therefore \quad (2, p) = \left(\frac{6-2}{2}, \frac{-5+11}{2}\right) \qquad \textbf{[½]}$$

$(2, p) = (2, 3)$

$\therefore \quad \boxed{p = 3}$ **[½]**

2.



Let O be the mid-point of diagonals AC and BD of the parallelogram $ABCD$ and coordinates of D is (x, y) then

$$\left(\frac{6+1}{2}, \frac{6+2}{2}\right) = \left(\frac{x+4}{2}, \frac{y+3}{2}\right) \qquad \textbf{[½]}$$

On comparing

$\frac{x+4}{2} = \frac{7}{2}, \qquad \frac{8}{2} = \frac{y+3}{2}$

$x = 7 - 4 \qquad 8 = y + 3$

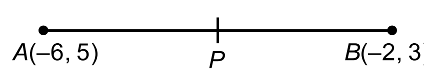
$x = 3 \qquad y = 8 - 3 = 5$

Hence coordinates of D = (3, 5) **[½]**

3. Answer (A)

Given a line segment joining

$A(-6, 5)$ and $B(-2, 3)$ **[½]**



Midpoint of A & B is $P\left(\frac{a}{2}, 4\right)$

$$\left(\frac{a}{2}, 4\right) = \left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$$

$\frac{a}{2} = -\frac{8}{2}$ [On comparing]

$\boxed{a = -8}$ **[½]**

4. Answer (B)

Given 2 points are $A(-6, 7)$ and $B(-1, -5)$

Distance between the points = AB

$$= \sqrt{(-6+1)^2 + (7+5)^2}$$ [½]

$$= \sqrt{25+144}$$

$\Rightarrow$ AB = 13

$\Rightarrow$ $2AB$ = 26 [½]

5. Answer (B)

It is given that the point P divides AB in the ratio 2 : 1.

Using section formula, the coordinates of the point P are

$$\left(\frac{1\times1+2\times4}{2+1}, \frac{1\times3+2\times6}{2+1}\right) = \left(\frac{1+8}{3}, \frac{3+12}{3}\right) = (3, 5)$$

[½]

Hence the coordinates of the point P are (3, 5). [½]

6. Answer (A)

Let the coordinates of the other end of the diameter be (x, y).

We know that the centre is the mid-point of the diameter. So, $O(-2, 5)$ is the mid-point of the diameter AB.

The coordinates of the point A and B are (2, 3) and (x, y) respectively.

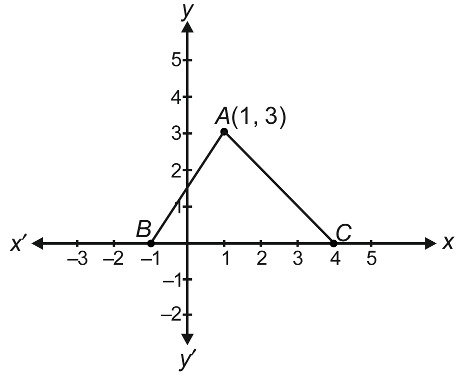
Using mid-point formula, we have,

$$-2 = \frac{2+x}{2} \Rightarrow -4 = 2 + x \Rightarrow x = -6$$

$$5 = \frac{3+y}{2} \Rightarrow 10 = 3 + y \Rightarrow y = 7$$ [½]

Hence, the coordinates of the other end of the diameter are (–6, 7). [½]

7. Answer (C)



From the figure, the coordinates of A, B, and C are (1, 3), (–1, 0) and (4, 0) respectively.

Area of ΔABC

$$= \frac{1}{2}\left|1(0-0) + (-1)(0-3) + 4(3-0)\right|$$ [½]

$$= \frac{1}{2}\left|0+3+12\right|$$

$$= \frac{1}{2}\left|15\right|$$

= 7.5 sq. units [½]

8. Answer (A)

It is given that the three points $A(x, 2)$, $B(-3, -4)$ and $C(7, -5)$ are collinear.

$\therefore$ Area of ΔABC = 0

$$\Rightarrow \frac{1}{2}\left|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\right| = 0$$

[½]

Here, $x_1 = x$, $y_1 = 2$, $x_2 = -3$, $y_2 = -4$, and $x_3 = 7$, $y_3 = -5$

$\Rightarrow$ $x[-4 - (-5)] - 3(-5 - 2) + 7[2 - (-4)] = 0$

$\Rightarrow$ $x(-4 + 5) - 3(-5 - 2) + 7(2 + 4) = 0$

$\Rightarrow$ $x - 3 \times (-7) + 7 \times 6 = 0$

$\Rightarrow$ $x + 21 + 42 = 0$ $\Rightarrow$ $x + 63 = 0$

$\Rightarrow$ $x = -63$

Thus, the value of x is –63. [½]

Hence, the correct option is A.

9. Using distance formula

$$\ell(OP) = \sqrt{(x-0)^2 + (y-0)^2}$$ [½]

$$\ell(OP) = \sqrt{x^2 + y^2}$$ [½]

10. Let the centre be O and coordinates of point A be (x, y)

$$\frac{x+1}{2} = 2$$ [By Mid-point formula]

$\Rightarrow$ $x = 3$ [½]

$$\frac{y+4}{2} = -3$$

$\Rightarrow$ $y = -10$ [½]

$\therefore$ Coordinates of A = (3, –10)

11. Answer (b) [1]

Distance of point (3, 4) from x-axis is its y-coordinate.

12. Answer (c) **[1]**

$A(4, p)$

$B(1, 0)$

$AB = 5$

$\therefore \quad \sqrt{(4-1)^2 + (p-0)^2} = 5$

$\Rightarrow \quad 9 + p^2 = 25$

$\Rightarrow \quad p^2 = 16$

$\Rightarrow \quad p = \pm 4$

13. $A(2, 6)$ —2— $C(k, 4)$ —3— $B(5, 1)$

$C(k, 4)$ divides AB in the ratio 2 : 3

$$\Rightarrow \quad C(k, 4) = \left(\frac{2\times 3 + 5\times 2}{2+3}, \frac{6\times 3 + 1\times 2}{2+3}\right)$$

$$\Rightarrow \quad (k, 4) = \left(\frac{16}{5}, \frac{20}{5}\right)$$

$$\Rightarrow \quad k = \frac{16}{5}$$ **[1]**

OR

Points $A(-3, 12)$, $B(7, 6)$ and $C(x, 9)$ are collinear.

$\Rightarrow \quad \text{ar}(\Delta ABC) = 0$

$$\Rightarrow \quad \frac{1}{2}\left|-3(6-9) + 7(9-12) + x(12-6)\right| = 0$$

$$\Rightarrow \quad |9 - 21 + 6x| = 0$$

$$\Rightarrow \quad |6x - 12| = 0$$

$$\Rightarrow \quad x = \frac{12}{6} = 2$$ **[1]**

14. Answer (c) **[1]**

Distance between $A(a\cos\theta + b\sin\theta, 0)$ and $B(0, a\sin\theta - b\cos\theta)$ is

$$AB = \sqrt{((a\cos\theta + b\sin\theta) - 0)^2 + (0 - (a\sin\theta - b\cos\theta))^2}$$

$$= \sqrt{(a\cos\theta + b\sin\theta)^2 + (b\cos\theta - a\sin\theta)^2}$$

$$= \sqrt{a^2 + b^2}$$

Option (c) is correct.

15. Answer (d) **[1]**

$A(2, -2)$ —1— $P(k, 0)$ —2— $B(-7, 4)$

$$\therefore \quad k = \frac{(1\times -7) + (2\times 2)}{1+2} \quad \text{[Using section formula]}$$

$$\boxed{k = -1}$$

Hence, option (d) is correct.

16. Answer (a)

Since, points are collinear, then area of triangle formed by these points is zero.

$$\frac{1}{2}\left|3(p+5) + 5(-5-1) + 7(1-p)\right| = 0$$

$\Rightarrow \quad p = -2$

Hence, option (a) is correct **[1]**

17. Answer (a)

$(x - 0)^2 + (1 - 0)^2 = (x - 2)^2 + (1 - 0)^2$

$x^2 + 1 = x^2 + 4 - 4x + 1$

$x = 1$ **[1]**

18. Answer (b) **[1]**

Let $P(4, 0)$ divides $A(4, 6)$ and $B(4, -8)$ in $k : 1$.

Applying section formula

$$\therefore \quad 4 = \frac{k(4) + 1(4)}{k+1}$$

A(4, 6) —k— P(4, 0) —1— B(4, –8)

$$0 = \frac{-8k + 1(6)}{k+1} \quad \Rightarrow \quad k = \frac{3}{4} \quad \text{or} \quad 3 : 4$$

19. Answer (b) **[1]**

$$OD = \frac{OB}{2} = 3 \text{ units}$$

$OA = 4$ units [Given]

$$\therefore \quad AD = \sqrt{OD^2 + OA^2} \qquad [\because \angle AOD = 90°]$$

$AD = 5$ units

20. Answer (b) **[1]**

Let (0, 0) divides the line segment AB in $k : 1$.

$$\therefore \quad \frac{1-3k}{k+1} = 0 \quad \text{and} \quad \frac{-3+9k}{k+1} = 0$$

$$\Rightarrow \quad k = \frac{1}{3}$$

Required ratio = 1 : 3

21. Answer (c) **[1]**

k belongs to any real number.

22. Answer (d) **[1]**

Let $O(0, 0)$ be centre of circle.

and $A(-1, -1)$, $B(0, 3)$, $C(1, 2)$, $D(3, 1)$

$$OA = \sqrt{(0+1)^2 + (0+1)^2} = \sqrt{2} \text{ units}$$

$$OB = \sqrt{(0-0)^2 + (3-0)^2} = 3 \text{ units}$$

$$OC = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{5} \text{ units}$$

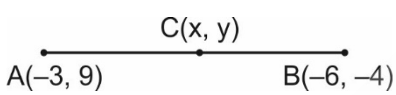
$$OD = \sqrt{(3-0)^2 + (1-0)^2} = \sqrt{10} \text{ units} > 3 \text{ units}$$

So, (3, 1) lies outside the circle.

23. Answer (c) **[1]**

Let $C(x, y)$ be the mid-point.

Applying mid-point formula



$$x = \frac{-3-6}{2} = \frac{-9}{2}$$

$$y = \frac{9-4}{2} = \frac{5}{2}$$

So, mid-point is $\left(\frac{-9}{2}, \frac{5}{2}\right)$.

24. Answer (c) **[1]**

A, B and C are the vertices of an equilateral triangle, then $AB = BC$

$$\sqrt{(3-0)^2 + (\sqrt{3}-0)^2} = \sqrt{(3-0)^2 + (k-0)^2}$$

$$\Rightarrow \quad \sqrt{9+3} = \sqrt{9+k^2}$$

Squaring both sides,

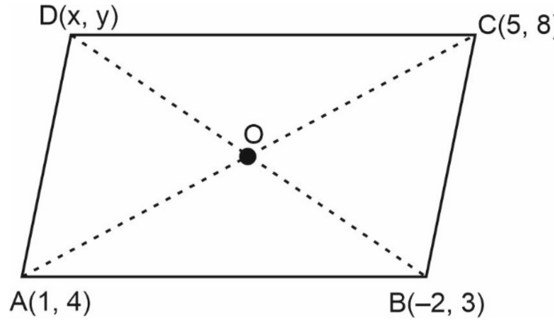
$$12 = 9 + k^2$$

$$k^2 = 3 \text{ or } k = \pm\sqrt{3}$$

25. Answer (b) **[1]**

$ABCD$ is a parallelogram.

Hence, O is the mid-point of both AC and BD.



$\therefore$ For ordinate of point D,

$$\frac{y+3}{2} = \frac{4+8}{2}$$

$$\Rightarrow \quad y = 9$$

26. Answer (b) **[1]**

$$\sqrt{(2+1)^2 + (y+3y)^2} = \sqrt{(5-2)^2 + (7+3y)^2}$$

$\Rightarrow$ $9 + 16y^2 = 9 + 49 + 9y^2 + 42y$

$\Rightarrow$ $7y^2 - 42y - 49 = 0$

$\Rightarrow$ $y^2 - 6y - 7 = 0$

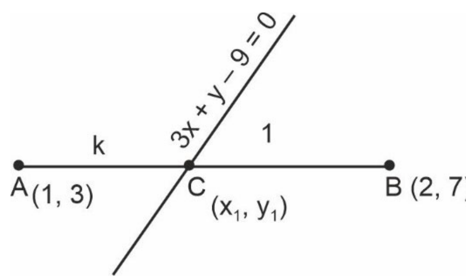
$\Rightarrow$ $(y - 7)(y + 1) = 0$

$\therefore$ $y = 7, -1$

27. Answer (c) **[1]**

Let $3x + y - 9 = 0$ divides the line segment formed by joining the point $A(1, 3)$ and $B(2, 7)$ in $k : 1$

(i.e., at point C).



Now, $x_1 = \frac{2k+1}{k+1}$ *and* $y_1 = \frac{7k+3}{k+1}$

Point C lies on $3x + y - 9 = 0$, then

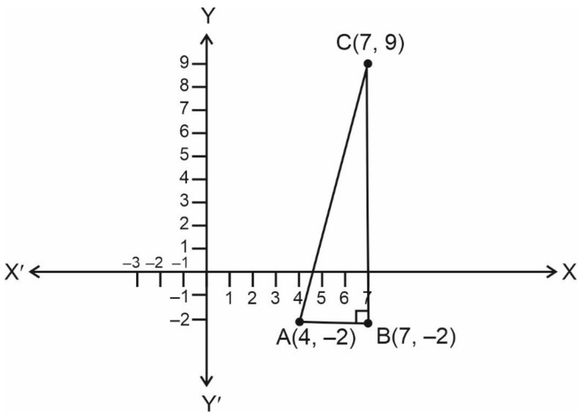
$$3\left(\frac{2k+1}{k+1}\right) + \left(\frac{7k+3}{k+1}\right) - 9 = 0$$

$\Rightarrow$ $6k + 3 + 7k + 3 - 9k - 9 = 0$

$\Rightarrow$ $4k - 3 = 0$

$$\Rightarrow \quad k = \frac{3}{4}$$

28. **Answer (c)** **[1]**



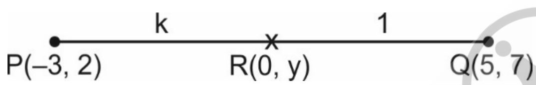
$\therefore$ ΔABC is a right angled triangle

29. Answer (d) **[1]**

Any point on y-axis is of the form $(0, y)$

Let R divides PQ in the ratio $k : 1$

$$\therefore \quad 0 = \frac{5k + 1(-3)}{k+1}$$



$\Rightarrow$ $5k - 3 = 0$

or $k = \dfrac{3}{5}$

30. Answer (c) **[1]**

Let coordinates of B be (x, y)

$$\Rightarrow \left(\frac{x+0}{2}, \frac{y-3}{2}\right) = (0, 0)$$

$\Rightarrow$ $x = 0$, $y = 3$

$$\Rightarrow \quad OA = \frac{\sqrt{3}}{2} BC = 3\sqrt{3} \text{ units}$$

$\therefore$ Coordinates of A are $\left(\pm 3\sqrt{3}, 0\right)$

31. Given points $(k, 3)$, $(6, -2)$, $(-3, 4)$ are collinear

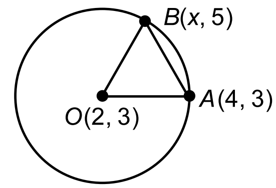
$\therefore$ Area of the triangle formed by these points = 0 **[½]**

$$\frac{1}{2}\left|k(-2-4) + 6(4-3) - 3(3+2)\right| \quad \textbf{[½]}$$

$-6k + 6 - 15 = 0$ **[½]**

$$k = \frac{-3}{2} \quad \textbf{[½]}$$

32.



$$OA = \sqrt{(2-4)^2 + (3-3)^2} = 2 \quad \textbf{[½]}$$

$$OB = \sqrt{(2-x)^2 + (3-5)^2} = \sqrt{(2-x)^2 + 4} \quad \textbf{[½]}$$

$\Rightarrow$ $2 = \sqrt{(2-x)^2 + 4}$ [$\because$ $OA = OB$ (radii)]

$4 = (2 - x)^2 + 4$ **[½]**

$\Rightarrow$ $\boxed{x = 2}$ **[½]**

33. Distance between the points $A(3, -1)$ and $B(11, y)$ is 10 units

$AB = 10$

$$\sqrt{(3-11)^2 + (-1-y)^2} = 10 \quad \textbf{[½]}$$

$64 + (y + 1)^2 = 100$ **[½]**

$(y + 1)^2 = 36$

$y + 1 = 6$ or $y + 1 = -6$ **[½]**

$\therefore$ $\boxed{y = -7, 5}$ **[½]**

34. It is given that the point $A(0, 2)$ is equidistant from the points $B(3, p)$ and $C(p, 5)$.

So, $AB = AC \Rightarrow AB^2 = AC^2$ **[½]**

Using distance formula, we have :

$\Rightarrow$ $(0 - 3)^2 + (2 - p)^2 = (0 - p)^2 + (2 - 5)^2$ **[½]**

$\Rightarrow$ $9 + 4 + p^2 - 4p = p^2 + 9$

$\Rightarrow$ $4 - 4p = 0$ **[½]**

$\Rightarrow$ $4p = 4$

$\Rightarrow$ $p = 1$ **[½]**

35. ΔABC is right angled at B.

$\therefore$ $AC^2 = AB^2 + BC^2$ ...(i) [Pythagoras]

Now, $AC^2 = (7 - 4)^2 + (3 - 7)^2 = (3)^2 + (-4)^2 = 9 + 16 = 25$

$AB^2 = (p - 4)^2 + (3 - 7)^2 = p^2 - 8p + 16 + (-4)^2$

$= p^2 - 8p + 16 + 16$

$= p^2 - 8p + 32$

$BC^2 = (7 - p)^2 + (3 - 3)^2 = 49 - 14p + p^2 + 0$

$= p^2 - 14p + 49$ **[1]**

From (i), we have

$25 = (p^2 - 8p + 32) + (p^2 - 14p + 49)$

$\Rightarrow$ $25 = 2p^2 - 22p + 81$

$\Rightarrow$ $2p^2 - 22p + 56 = 0$

$\Rightarrow$ $p^2 - 11p + 28 = 0$

$\Rightarrow$ $p^2 - 7p - 4p + 28 = 0$

$\Rightarrow$ $p(p - 7) - 4(p - 7) = 0$

$\Rightarrow$ $(p - 7)(p - 4) = 0$

$\Rightarrow$ $p = 7$ and $p = 4$ **[1]**

36. Given, the points $A(x, y)$, $B(-5, 7)$ and $C(-4, 5)$ are collinear.

So, the area formed by these vertices is 0.

$\therefore$ $\frac{1}{2}|x(7-5)+(-5)(5-y)+(-4)(y-7)| = 0$ **[½]**

$\Rightarrow$ $\frac{1}{2}|2x - 25 + 5y - 4y + 28| = 0$ **[½]**

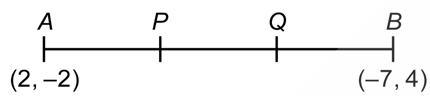
$\Rightarrow$ $\frac{1}{2}|2x + y + 3| = 0$

$\Rightarrow$ $2x + y + 3 = 0$ **[½]**

$\Rightarrow$ $y = -2x - 3$ **[½]**

37. Since P and Q are the points of trisection of AB, $AP = PQ = QB$

Thus, P divides AB internally in the ratio 1 : 2 and Q divides AB internally in the ratio 2 : 1.



$\therefore$ By section formula,

Coordinates of $P = \left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right)$

$= \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$

$= \left(\frac{-3}{3}, 0\right) = (-1, 0)$ **[1]**

Coordinates of $Q = \left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right)$

$= \left(\frac{-14+2}{3}, \frac{8-2}{3}\right)$

$= \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$ **[1]**

38. Let $A(3, 0)$, $B(6, 4)$ and $C(-1, 3)$ be the given points of the vertices of triangle.

Now,

$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9+16} = \sqrt{25}$ ...(i) **[½]**

$BC = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{(-7)^2 + (-1)^2}$

$= \sqrt{49+1} = \sqrt{50}$ ...(ii) **[½]**

$AC = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{(-4)^2 + (3)^2}$

$= \sqrt{16+9} = \sqrt{25}$ ...(iii) **[½]**

$\therefore$ $BC^2 = AB^2 + AC^2$ and $AB = AC$

Hence triangle is isosceles right triangle. **[½]**

Thus, ΔABC is a right-angled isosceles triangle.

39. Let the coordinates of points P and Q be $P(0, a)$ and $Q(b, 0)$ respectively.

[$\because$ P on y-axis Q on x-axis] **[½]**

Coordinates of mid-point of PQ

$= \left(\frac{0+b}{2}, \frac{0+a}{2}\right)$

$= \left(\frac{b}{2}, \frac{a}{2}\right)$ **[½]**

On comparing with (2, –5)

$\frac{b}{2} = 2$ and $\frac{a}{2} = -5$

$b = 4$, $a = -10$ **[½]**

Hence coordinates of $P = (0, -10)$

Hence coordinates of $Q = (4, 0)$ **[½]**

40. Given that

$PA = PB$

By using distance formula

$\sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y-5)^2}$ **[½]**

Squaring on both sides

$\Rightarrow$ $x^2 + 25 - 10x + y^2 - 2y + 1$

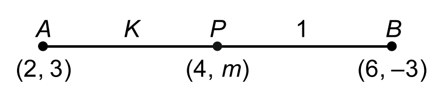
$= x^2 + 2x + 1 + y^2 - 10y + 25$ **[½]**

$\Rightarrow$ $-10x - 2y = 2x - 10y$ **[½]**

$\Rightarrow$ $8y = 12x$

$\therefore$ $3x = 2y$ **[½]**

41. Suppose the point $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$ in the ratio $K : 1$.



Co-ordinates of point $P = \left(\frac{6K+2}{K+1}, \frac{-3K+3}{K+1}\right)$ **[½]**

But the co-ordinates of point P are given as $(4, m)$

$\frac{6K+2}{K+1} = 4$ ...(i)

$$\frac{-3K+3}{K+1} = m \qquad \text{...(ii)} \qquad [½]$$

$\Rightarrow$ $6K + 2 = 4K + 4$ [From (i)]

$\Rightarrow$ $2K = 2$

$\Rightarrow$ $K = 1$

Putting $K = 1$ in equation (ii)

$$\frac{-3(1)+3}{1+1} = m$$

$\therefore$ $m = 0$ **[½]**

Ratio is 1 : 1 and $m = 0$

i.e. P is the mid-point of AB **[½]**

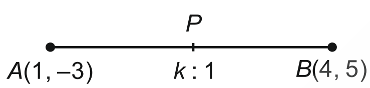
42. Let $P(x, y)$ divides the line segment joining the points $A(1, -3)$ and $B(4, 5)$ internally in the ratio $k : 1$.

Using section formula, we get

$$x = \frac{4k+1}{k+1} \qquad \text{...(i)}$$

$$y = \frac{5k-3}{k+1} \qquad \text{...(ii)} \qquad [½]$$

Since, P lies on x-axis. So its ordinate will be zero.



$$\Rightarrow \frac{5k-3}{k+1} = 0$$

$$\Rightarrow k = \frac{3}{5}$$

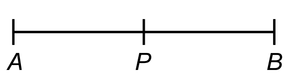
Hence, the required ratio is 3 : 5. **[½]**

Now putting the value of k in (i) and (ii), we get

$$x = \frac{17}{8} \text{ and } y = 0$$

So, coordinates of point P are $\left(\frac{17}{8}, 0\right)$ **[1]**

43.



$$\frac{AP}{AB} = \frac{3}{7}$$

As, $AB = 7a$, $AP = 3a$

$\Rightarrow$ $AB = AP + PB$

$\Rightarrow$ $7a = 3a + PB$

$\Rightarrow$ $PB = 7a - 3a = 4a$ **[1]**

Let the point $P(x, y)$ divide the line segment joining the points $A(-2, -2)$ and $B(2, -4)$ in the ratio $AP : PB = 3 : 4$ **[½]**

$$\Rightarrow x = \frac{2(3)+(-2)(4)}{3+4} \text{ and } y = \frac{(-4)(3)+(4)(-2)}{3+4}$$

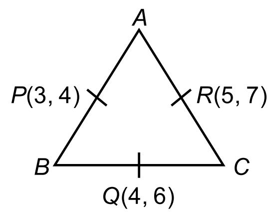
[1]

$$\Rightarrow x = \frac{6-8}{7} \text{ and } y = \frac{-12-8}{7}$$

$$\Rightarrow x = \frac{-2}{7} \text{ and } y = \frac{-20}{7}$$

$\Rightarrow$ The coordinate of $P(x, y) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$ **[½]**

44.



Consider a ΔABC with $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, $P(3, 4)$, $Q(4, 6)$ and $R(5, 7)$ are the mid-points of AB, BC and CA. Then,

$$3 = \frac{x_1+x_2}{2} \Rightarrow x_1 + x_2 = 6 \quad \text{...(i)}$$

$$4 = \frac{y_1+y_2}{2} \Rightarrow y_1 + y_2 = 8 \quad \text{...(ii)}$$

$$4 = \frac{x_2+x_3}{2} \Rightarrow x_2 + x_3 = 8 \quad \text{...(iii)}$$

$$5 = \frac{y_2+y_3}{2} \Rightarrow y_2 + y_3 = 12 \quad \text{...(iv)}$$

$$6 = \frac{x_3+x_1}{2} \Rightarrow x_3 + x_1 = 10 \quad \text{...(v)}$$

$$7 = \frac{y_3+y_1}{2} \Rightarrow y_2 + y_1 = 14 \quad \text{...(vi)} \qquad [½]$$

On adding (i), (iii) and (v) we get

$2(x_1 + x_2 + x_3) = 6 + 8 + 10 = 24$

$\Rightarrow$ $x_1 + x_2 + x_3 = 12$ ...(vii) **[½]**

From (i) and (vii), we get $x_3 = 12 - 6 = 6$

From (iii) and (vii) we get $x_1 = 12 - 8 = 4$

From (v) and (vii), we get $x_2 = 12 - 10 = 2$ **[½]**

Now, adding (ii), (iv) and (vi), we get

$20(y_1 + y_2 + y_3) = 8 + 12 + 14 = 34$

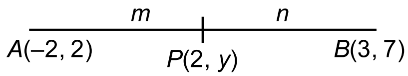
$\Rightarrow$ $y_1 + y_2 + y_3 = 17$ ...(viii) **[½]**

From (ii) and (viii), we get $y_3 = 17 - 8 = 9$

From (iv) and (viii), we get $y_1 = 17 - 12 = 5$

From (vi) and (viii), we get $y_2 = 17 - 14 = 3$ **[½]**

Hence, the vertices of ΔABC are $A(4, 5)$, $B(2, 3)$, $C(6, 9)$. **[½]**

45. 

Lets say ratio = $m : n$

$\therefore \quad (2,\ y) = \left(\dfrac{3m-2n}{m+n},\ \dfrac{2n+7m}{m+n}\right)$ **[1]**

$2 = \dfrac{3m-2n}{m+n}$

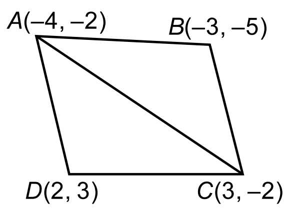
$2m + 2n = 3m - 2n$

$m : n = 4 : 1$ **[1]**

$y = \dfrac{2+7\times 4}{5}$

$y = \dfrac{30}{5}$

$y = 6$ **[1]**

46. 

Join AC

Area of Quadrilateral $ABCD$ = ar(ΔABC) + ar(ΔADC) **[½]**

Area of triangle $ABC = \dfrac{1}{2}\left|\begin{matrix}-4(-5-(-2))+(-3)\\(-2-(-2))+3(-2-(-5))\end{matrix}\right|$

$= \dfrac{1}{2}\left|\begin{matrix}-4(-5+-2)+(-3)\\(-2+2)+3(-2+5)\end{matrix}\right|$

$= \dfrac{1}{2}\left|-4(-3)-3(0)+3(3)\right|$

$= \dfrac{1}{2}\left|12-0+9\right|$

$= \dfrac{21}{2}$ square units **[1]**

Area of triangle $ADC = \dfrac{1}{2}\left|\begin{matrix}-4(3-(-2))+\\2(-2-(-2))+3(-2-3)\end{matrix}\right|$

$= \dfrac{1}{2}\left|\begin{matrix}-4(3+2)-\\3(-2+2)+3(-2-3)\end{matrix}\right|$

$= \dfrac{1}{2}\left|-4(5)-3(0)+3(-5)\right|$

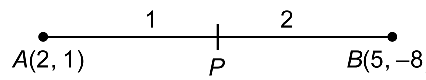
$= \dfrac{1}{2}\left|-20-0-15\right|$

$= \dfrac{1}{2}\left|-35\right| = \dfrac{35}{2}$ sq. units **[1]**

$\therefore$ Area of quadrilateral ($ABCD$) $= \dfrac{21}{2} + \dfrac{35}{2}$

= 28 sq. units

[½]

47. 

Given :

$\dfrac{AP}{AB} = \dfrac{1}{3}$

$\Rightarrow \quad \dfrac{AP}{AP+PB} = \dfrac{1}{3}$

$PB = 2AP$

$\Rightarrow AP : PB = 1 : 2$ **[1]**

By section formula

$\therefore \quad P = \left(\dfrac{2\times 2+5}{3},\ \dfrac{2-8}{3}\right)$

$P = (3, -2)$ **[1]**

Also it is given that P lies on $2x - y + k = 0$

$\therefore \quad 2(3) - (-2) + k = 0$

$\boxed{k = -8}$ **[1]**

48. Since $R(x, y)$ is a point on the line segment joining the points, $P(a, b)$ and $Q(b, a)$

$\therefore$ $P(a, b)$, $Q(b, a)$ and $R(x, y)$ are the collinear.

[½]

$\Rightarrow$ Area of $\Delta PQR = 0$ **[½]**

$\dfrac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right| = 0$

[1]

$\Rightarrow \quad \dfrac{1}{2}\left|a(a-y)+b(y-b)+x(b-a)\right| = 0$

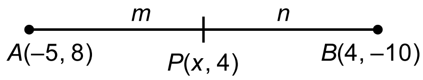
$\Rightarrow \quad a^2 - ay + by - b^2 + x(b-a) = 0$

$\Rightarrow \quad y(b-a) + x(b-a) = b^2 - a^2$

$\Rightarrow \quad (x+y)(b-a) = (b-a)(b+a)$

$\Rightarrow \quad x + y = a + b$ **[1]**

49.



Lets say ratio = $m : n$

$$P(x,\ 4) = \left(\frac{4m-5n}{m+n},\ \frac{-10m+8n}{m+n}\right)$$ **[1]**

$$4 = \frac{-10m+8m}{m+n}$$ [On equating]

$\Rightarrow$ $4m + 4n = -10m + 8n$

$\Rightarrow$ $14m = 4n$

$$\Rightarrow \frac{m}{n} = \frac{2}{7}$$ **[1]**

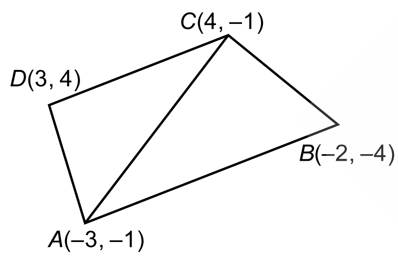
We know $x = \dfrac{4m-5n}{m+n}$

$$\Rightarrow x = \frac{4\left(\frac{m}{n}\right)-5}{\frac{m}{n}+1} = \frac{4\left(\frac{2}{7}\right)-5}{\frac{2}{7}+1}$$

$$\Rightarrow x = \frac{8-35}{9}$$

$\Rightarrow$ $x = -3$ **[1]**

50.



Area of quadrilateral $ABCD$ = ar(ΔABC) + ar(ΔADC)

We know that,

$$\text{Area of triangle} = \frac{1}{2}\left|\begin{matrix} x_2(y_2-y_3)-x_2(y_3-y_1) \\ -x_3(y_1-y_2) \end{matrix}\right|$$

[½]

Thus,

$$\text{Area of } \Delta\text{ABC} = \frac{1}{2}\left|\begin{matrix}(-3)(-4+1)+(-2) \\ (-1+1)+4(-1+4)\end{matrix}\right|$$

$$= \frac{1}{2}|9+0+12|$$

$$= \frac{21}{2} \text{ sq. units}$$ **[1]**

$$\text{Area of } \Delta ADC = \frac{1}{2}\left|\begin{matrix}(-3)(4+1)+3(-1+1) \\ +4(-1-4)\end{matrix}\right|$$

$$= \frac{1}{2}|-15+0-20|$$

$$= \frac{1}{2}|-35|$$

$$= \frac{35}{2} \text{ sq. units}$$ **[1]**

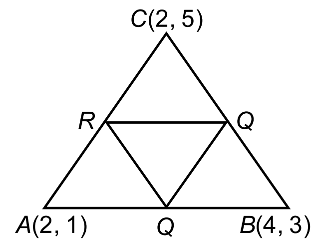
Substitute these values in equation (i), we have,

$$\text{Area of quadrilateral } ABCD = \frac{21}{2}+\frac{35}{2} = \frac{56}{2}$$

= 28 sq. units **[½]**

Hence, area of quadrilateral is 28 square units.

51.



P, Q, R are the mid-points to the sides of the ΔABC

$$P = \left(\frac{4+2}{2},\ \frac{3+1}{2}\right) = (3,\ 2)$$

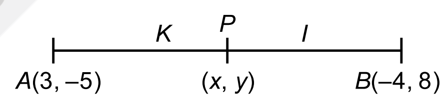
Similarly, Q = (3, 4), R = (2, 3) **[1½]**

$$\text{Area of } \Delta PQR = \frac{1}{2}\left|\begin{matrix}3(4-3)+3(3-2) \\ +2(2-4)\end{matrix}\right|$$ **[½]**

$$= \frac{1}{2}|3+3-4|$$

= 1 sq. unit **[1]**

52.



Let the co-ordinates of point P be (x, y)

By using the section formula co-ordinates of P are.

$$x = \frac{-4K+3}{K+1} \qquad y = \frac{8K-5}{K+1}$$ **[1]**

Since P lies on $x + y = 0$

$$\therefore \quad \frac{-4K+3}{K+1}+\frac{8K-5}{K+1} = 0$$

[On putting the values of x and y] **[½]**

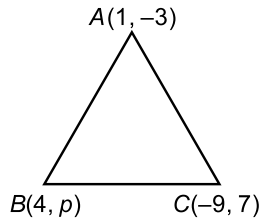
$\Rightarrow$ $4K - 2 = 0$

$$\Rightarrow K = \frac{2}{4}$$ **[½]**

$$\Rightarrow K = \frac{1}{2}$$

Hence the value of $K = \dfrac{1}{2}$ **[1]**

53.



The area of a Δ, whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| \quad \textbf{[1]}$$

Substituting the given coordinates

$$\text{Area of } \Delta = \frac{1}{2}\left|1(p-7) + 4(7+3) + (-9)(-3-p)\right|$$

[½]

$$\Rightarrow \frac{1}{2}\left|(p-7) + 40 + 27 + 9p\right| = 15 \quad \textbf{[½]}$$

$\Rightarrow$ $10p + 60 = \pm 30$

$\Rightarrow$ $10p = -30$ or $10p = -90$ **[½]**

$\Rightarrow$ $p = -3$ or $p = -9$

Hence the value of $p = -3$ or -9 **[½]**

54. Let the y-axis divide the line segment joining the points $(-4, -6)$ and $(10, 12)$ in the ratio $k : 1$ and the point of the intersection be $(0, y)$. Using section formula, we have:

$$\left(\frac{10k + -4}{k+1}, \frac{12k + -6}{k+1}\right) = 0,\ y$$

$$\therefore \frac{10k - 4}{k+1} = 0 \Rightarrow 10k - 4 = 0$$

$$\Rightarrow k = \frac{4}{10} = \frac{2}{5} \quad \textbf{[1]}$$

Thus, the y-axis divides the line segment joining the given points in the ratio 2 : 5

$$\therefore y = \frac{12k + (-6)}{k+1} = \frac{12 \times \frac{2}{5} - 6}{\frac{2}{5} + 1} = \frac{\left(\frac{24-30}{5}\right)}{\left(\frac{2+5}{5}\right)} = \frac{-6}{7}$$

[1]

Thus, the coordinates of the point of division are $\left(0, -\frac{6}{7}\right)$ **[1]**

55. The given points are $A(-2, 3)$ $B(8, 3)$ and $C(6, 7)$. Using distance formula, we have :

$AB^2 = (8 + 2)^2 + (3 - 3)^2$

$\Rightarrow$ $AB^2 = 10^2 + 0$

$\Rightarrow$ $AB^2 = 100$ **[½]**

$BC^2 = (6 - 8)^2 + (7 - 3)^2$

$\Rightarrow$ $BC^2 = (-2)^2 + 4^2$

$\Rightarrow$ $BC^2 = 4 + 16$

$\Rightarrow$ $BC^2 = 20$ **[½]**

$CA^2 = (2 - 6)^2 + (3 - 7)^2$

$\Rightarrow$ $CA^2 = (-8)^2 + (-4)^2$

$\Rightarrow$ $CA^2 = 64 + 16$

$\Rightarrow$ $CA^2 = 80$ **[½]**

It can be observed that :

$BC^2 + CA^2 = 20 + 80 = 100 = AB^2$ **[1]**

So, by the converse of Pythagoras Theorem,

ΔABC is a right triangle right angled at C. **[½]**

56. The given points are $A(0, 2)$, $B(3, p)$ and $C(p, 5)$.

It is given that A is equidistant from B and C.

$\therefore$ $AB = AC$

$\Rightarrow$ $AB^2 = AC^2$

$\Rightarrow$ $(3 - 0)^2 + (p - 2)^2 = (p - 0)^2 + (5 - 2)^2$ **[1]**

$\Rightarrow$ $9 + p^2 + 4 - 4p = p^2 + 9$

$\Rightarrow$ $4 - 4p = 0$

$\Rightarrow$ $4p = 4$

$\Rightarrow$ $p = 1$ **[1]**

Thus, the value of p is 1

$$\text{Length of } AB = \sqrt{(3-0)^2 + (1-2)^2} = \sqrt{3^2 + (-1)^2}$$

$$= \sqrt{9+1} = \sqrt{10} \text{ units.} \quad \textbf{[1]}$$

57. The given points are $A(-2, 1)$, $B(a, b)$ and $C(4, -1)$.

Since the given points are collinear, the area of the triangle ABC is 0. **[½]**

$\Rightarrow$

$$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = 0$$

Here, $x_1 = 2$, $y_1 = 1$, $x_2 = a$, $y_2 = b$, $x_3 = 4$, $y_3 = -1$

$$\therefore \frac{1}{2}\left|-2(b+1) + a(-1-1) + 4(1-b)\right| = 0 \quad \textbf{[½]}$$

$\Rightarrow$ $-2b - 2 - 2a + 4 - 4b = 0$

$\Rightarrow$ $2a + 6b = 2$

$\Rightarrow$ $a + 3b = 1$ ...(i) **[1]**

Given :

$a - b = 1$ ...(ii)

Subtracting equation (i) from (ii) we get :

$4b = 0$

$\Rightarrow\ b = 0$

Subtracting $b = 0$ in (ii), we get :

$a = 1$

Thus, the values of a and b are 1 and 0, respectively. **[1]**

58. Here, $P(x, y)$ divides line segment AB, such that

$$AP = \frac{3}{7}AB$$

$$\Rightarrow \frac{AP}{AB} = \frac{3}{7}$$

$$\Rightarrow \frac{AB}{AP} = \frac{7}{3}$$

$$\Rightarrow \frac{AB}{AP} - 1 = \frac{7}{3} - 1 \quad [½]$$

$$\Rightarrow \frac{AB - AP}{AP} = \frac{7-3}{3}$$

$$\Rightarrow \frac{BP}{AP} = \frac{4}{3}$$

$$\Rightarrow \frac{AP}{BP} = \frac{3}{4} \quad [1]$$

$\therefore$ P divides AB in the ratio 3 : 4

$$x = \frac{3\times 2 + 4(-2)}{3+4};\ y = \frac{3\times(-4)+4(-2)}{3+4} \quad [½]$$

$$x = \frac{6-8}{7};\ y = \frac{-12-8}{7}$$

$$x = \frac{-2}{7};\ y = \frac{-20}{7}$$

$\therefore$ The coordinates of P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$ **[1]**

59. $P(x, y)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$.

$\therefore\ AP = BP$

$$\Rightarrow \sqrt{[x-(a+b)]^2 + [y-(b-a)]^2}$$

$$= \sqrt{[x-(a-b)]^2 + [y-(a+b)]^2} \quad [1]$$

$$\Rightarrow [x - (a + b)]^2 + [y - (b - a)]^2$$

$$= [x - (a - b)]^2 + [y - (a + b)]^2$$

$$\Rightarrow x^2 - 2x(a + b) + (a + b)^2 + y^2 - 2y(b - a) + (b - a)^2$$

$$= x^2 - 2x(a - b) + (a - b)^2 + y^2 - 2y(a + b) + (a + b)^2 \quad [1]$$

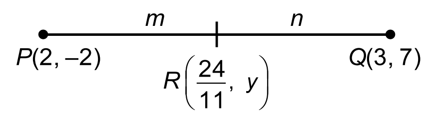
$$\Rightarrow -2x(a + b) - 2y(b - a) = -2x(a - b) - 2y(a + b)$$

$$\Rightarrow ax + bx + by - ay = ax - bx + ay + by$$

$$\Rightarrow 2bx = 2ay$$

$\therefore\ bx = ay$ ....(proved) **[1]**

60.



Lets say ratio is $m + n$

Then

$$\left(\frac{24}{11}, y\right) = \left(\frac{3m+2n}{m+n}, \frac{7m-2n}{m+n}\right) \quad [1]$$

$$\frac{24}{11} = \frac{3m+2n}{m+n},\ y = \frac{7m-2n}{m+n}$$

$\therefore\ 24(m + n) = 11(3m + 2n)$

$24m + 24n = 33m + 22n$

$2n = 9n$

$$\therefore \frac{m}{n} = \frac{2}{9} \Rightarrow \text{Ratio} = 2:9 \quad [1]$$

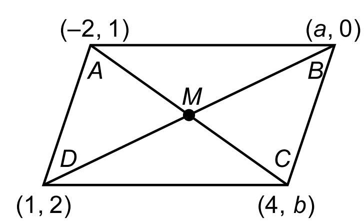
$m = 2,\ n = 9$

$$y = \frac{7\times 2 - 2\times 9}{11}$$

$$y = \frac{-4}{11} \quad [1]$$

61. M is mid-point of diagonals AC and BD

Using mid-point formula,



$$\left(\frac{-2+4}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2+0}{2}\right) \quad [1]$$

$$\left(\frac{2}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2}{2}\right)$$

$$\therefore \frac{2}{2} = \frac{a+1}{2} \Rightarrow a+1 = 2 \Rightarrow a = 1 \quad [½]$$

and $\frac{1+b}{2} = \frac{2}{2} \Rightarrow 1+b = 2 \Rightarrow b = 1$ [½]

Side $AD = BC = \sqrt{(-2-1)^2 + (1-2)^2}$

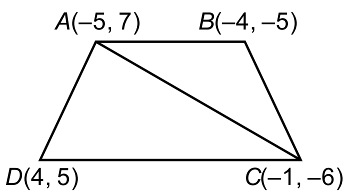
$= \sqrt{9+1} = \sqrt{10}$

Side $DC = AB = \sqrt{(1-4)^2 + (2-1)^2}$

$= \sqrt{9+1} = \sqrt{10}$ [1]

62. $\text{Ar}(\Delta ABC) = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

If $A = (x_1, y_1)$, $B = (x_2, y_2)$, $C = (x_3, y_3)$ are vertices of ΔABC.



Ar(□ABCD) = Ar(ΔABC) + Ar(ΔADC) ....(i) [½]

$\text{Ar}(\square ABC) = \frac{1}{2}\left| -5(-5+6) - 4(-6-7) - 1(7+5) \right|$

$= \frac{1}{2}|-5+52-12|$

$= \frac{1}{2}|35|$

$= \frac{35}{2}$ Sq. units [1]

$\text{Ar}(\Delta ADC) = \frac{1}{2}|-5(-5+6) - 4(-6-7) - 1(7-5)|$

$= \frac{1}{2}|-55-52-2|$

$= \frac{|-109|}{2}$

∴ Area cannot be negative.

∴ $\text{Ar}(\Delta ADC) = \frac{109}{2}$ sq. units [1]

∴ $\text{Ar}(\square ABCD) = \frac{35}{2} + \frac{109}{2} = \frac{144}{3} = 72$ sq. units

[½]

63. Let the point on y-axis be $P(0, y)$ which is equidistant from the points $A(5, -2)$ and $B(-3, 2)$.

[½]

We are given that $AP = BP$

So, $AP^2 = BP^2$ [½]

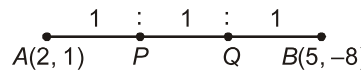
i.e., $(5-0)^2 + (-2-y)^2 = (-3-0)^2 + (2-y)^2$ [1]

$\Rightarrow$ $25 + y^2 + 4 + 4y = 9 + 4 + y^2 - 4y$

$\Rightarrow$ $8y = -16$

$\Rightarrow$ $y = -2$

Hence, the required point is (0, –2) [1]

64. 

Here, $AP : PB = 1 : 2$ [½]

∴

Coordinates of $P = \left(\frac{1\times 5 + 2\times 2}{1+2}, \frac{1\times -8 + 2\times 1}{1+2}\right)$

$\Rightarrow$ Coordinates of $P = (3, -2)$ [1]

Since, P lies on the line $2x - y + k = 0$ [½]

∴ $2(3) - (-2) + k = 0$

$\Rightarrow$ $6 + 2 + k = 0$

$\Rightarrow$ $k = -8$ [1]

65. $AD = 100 \times 1$ m

$= 100$ m

Niharika runs $\frac{1}{4}$th of $AD = \frac{100}{4} = 25$ m on 2nd line.

∴ Coordinates of green flag posted by Niharika are (2, 25)

Preet runs $\frac{1}{5}$th of $AD = \frac{100}{5} = 20$ m on 8th line.

∴ Coordinates of red flag posted by Preet are (8, 20) [1]

(i) Distance between two flags

$= \sqrt{(8-2)^2 + (20-25)^2}$

$= \sqrt{6^2 + (-5)^2}$

$= \sqrt{36+25}$

$= \sqrt{61}$ m [1]

(ii) Mid-point of line segment joining the two flags $= \left(\dfrac{8+2}{2}, \dfrac{25+20}{2}\right)$

$$= \left(5, \frac{45}{2}\right) = (5, 22.5)$$

$\therefore$ Rashmi will post a blue flag on fifth line at the distance of 22.5 m. **[1]**

66. Here, $x_1 = -5$, $y_1 = 7$, $x_2 = -4$, $y_2 = -5$, $x_3 = 4$, $y_3 = 5$ **[½]**

Area of

$\Delta PQR = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$ **[½]**

$= \frac{1}{2}\left|-5(-5-5) - 4(5-7) + 4(7-(-5))\right|$ **[½]**

$= \frac{1}{2}\left|50 + 8 + 48\right|$ **[½]**

$= \frac{1}{2}\left|106\right|$ **[½]**

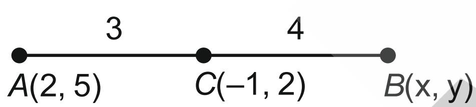
$= 53$ **[½]**

$\therefore$ Area of ΔPQR = 53 sq. units

OR

Now,

Using section formula



$\Rightarrow \quad -1 = \dfrac{(3 \times x) + (4 \times 2)}{3+4}$ **[½]**

$\Rightarrow \quad -1 = \dfrac{3x+8}{7}$

$\Rightarrow$ $3x + 8 = -7$ **[½]**

$\Rightarrow$ $3x = -15$

$\Rightarrow$ $x = -5$

Also,

$2 = \dfrac{(3 \times y) + (4 \times 5)}{3+4}$ **[½]**

$\Rightarrow \quad 2 = \dfrac{3y+20}{7}$

$\Rightarrow$ $3y + 20 = 14$

$\Rightarrow$ $3y = -6$

$\Rightarrow$ $y = -2$ **[½]**

$\therefore$ Coordinates of B are $(-5, -2)$ **[1]**

67. The given vertices are $A(x, y)$, $B(1, 2)$ and $C(2, 1)$.

It is know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$\therefore$ Area of $\Delta ABC = \dfrac{1}{2}\left|\begin{matrix} x_2(y_2 - y_3) + x_2(y_3 - y_1) \\ + x_3(y_1 - y_2) \end{matrix}\right|$ **[½]**

$= \dfrac{1}{2}\left|\begin{matrix} x(2-1) + 1 \times (1-y) \\ +2(y-2) \end{matrix}\right|$ **[½]**

$= \dfrac{1}{2}\left|x + 1 - y + 2y - 4\right|$ **[½]**

$= \dfrac{1}{2}\left|x + y - 3\right|$ **[½]**

$(x + y - 3)$ will be positive

Since the area of ΔABC is given as 6 sq. units.

$\Rightarrow \quad \dfrac{1}{2}\left|x + y - 3\right| = 6$ **[1]**

$\Rightarrow$ $x + y - 3 = 12$

$\therefore$ $x + y = 15$, Proved **[1]**

68. Let the Point $P(x, 2)$ divide the line segment joining the points $A(12, 5)$ and $B(4, -3)$ in the ratio $k : 1$

Then, the coordinates of P are

$\left(\dfrac{4k+12}{k+1}, \dfrac{-3k+5}{k+1}\right)$ **[½]**

Now, the coordinates of P are $(x, 2)$

$\therefore \quad \dfrac{4k+12}{k+1} = x$ and $\dfrac{-3k+5}{k+1} = 2$ **[1]**

$\dfrac{-3k+5}{k+1} = 2$

$\Rightarrow$ $-3k + 5 = 2k + 2$

$\Rightarrow$ $5k = 3$

$\Rightarrow \quad k = \dfrac{3}{5}$ **[1]**

Substituting $k = \dfrac{3}{5}$ in $\dfrac{4k+12}{k+1} = x$, we get

$x = \dfrac{4 \times \frac{3}{5} + 12}{\frac{3}{5} + 1}$ **[½]**

$\Rightarrow \quad x = \dfrac{12+60}{3+5}$

$\Rightarrow \quad x = \dfrac{72}{8}$

$\Rightarrow \quad x = 9$

Thus, the value of x is 9 **[½]**

Also, the point P divides the line segment joining the points $A(12, 5)$ and $(4, -3)$ in the ratio $\dfrac{3}{5} : 1$, *i.e.* 3 : 5. **[½]**

69. Take $(x_1, y_1) = (1, -1), (-4, 2k)$ and $(-k, -5)$

It is given that the area of the triangle is 24 sq. unit

Area of the triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$\Delta = \dfrac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$ **[1]**

$\Rightarrow \quad 24 = \dfrac{1}{2}\left|\begin{matrix} 1(2k-(-5)) + (-4)((-5) \\ -(-1)) + (-k)((-1) - 2k) \end{matrix}\right|$ **[1]**

$\Rightarrow \quad 48 = |(2k + 5) + 16 + (k + 2k^2)|$

$\Rightarrow \quad 2k^2 + 3k - 27 = 0$

$\Rightarrow \quad (2k + 9)(k - 3) = 0$ **[1]**

$\Rightarrow \quad k = -\dfrac{9}{2}$ or $k = 3$

The values of k are $-\dfrac{9}{2}$ and 3. **[1]**

70. $\dfrac{AD}{AB} = \dfrac{AE}{AC} = \dfrac{1}{3}$

$\therefore \quad \dfrac{AB}{AD} = \dfrac{AC}{AE} = 3$

$\therefore \quad \dfrac{AD+DB}{AD} = \dfrac{AE+EC}{AE} = 3$

$\therefore \quad 1 + \dfrac{DB}{AD} = 1 + \dfrac{EC}{AE} = 3$

$\therefore \quad \dfrac{DB}{AD} = \dfrac{EC}{AE} = 2$

$\therefore \quad \dfrac{AD}{DB} = \dfrac{AE}{EC} = \dfrac{1}{2}$

$\therefore \quad AD : DB = AE : EC = 1 : 2$ **[½]**

So, D and E divide AB and AC respectively in the ratio 1 : 2.

By using section formula

The coordinates of D is

$\left(\dfrac{1+8}{1+2}, \dfrac{5+12}{1+2}\right) = \left(3, \dfrac{17}{3}\right)$ and

Coordinates of E is

$\left(\dfrac{7+8}{1+2}, \dfrac{2+12}{1+2}\right) = \left(5, \dfrac{14}{3}\right)$ **[1]**

$$\begin{matrix} 4 & 3 & 5 & 4 \\ 6 & \dfrac{17}{3} & \dfrac{14}{3} & 6 \end{matrix}$$

Area of $\Delta ADE = \dfrac{1}{2}\left|\begin{matrix}\left(4\times\dfrac{17}{3} + 3\times\dfrac{14}{3} + 5\times 6\right) \\ -\left(3\times 6 + 5\times\dfrac{17}{3} + 4\times\dfrac{14}{3}\right)\end{matrix}\right|$

$= \dfrac{1}{2}\left|\begin{matrix}\left(\dfrac{68}{3} + 14 + 30\right) \\ -\left(18 + \dfrac{85}{3} + \dfrac{56}{3}\right)\end{matrix}\right|$

$= \dfrac{1}{2}\left|\begin{matrix}\left(\dfrac{68+42+90}{3}\right) \\ -\left(\dfrac{54+85+56}{3}\right)\end{matrix}\right|$

$= \dfrac{1}{2}\left|\left(\dfrac{200}{3}\right) - \left(\dfrac{195}{3}\right)\right|$

$= \dfrac{1}{2} \times \dfrac{5}{3}$

$= \dfrac{5}{6}$ sq. units ...(i) **[1]**

$$\begin{matrix} 4 & 1 & 7 & 4 \\ 6 & 5 & 2 & 6 \end{matrix}$$

Area of $\Delta ABC = \dfrac{1}{2}\left|\begin{matrix}(4\times 5 + 1\times 2 + 7\times 6) \\ -(1\times 6 + 7\times 5 + 4\times 2)\end{matrix}\right|$

$= \dfrac{1}{2}\left|(20 + 2 + 42) - (6 + 35 + 8)\right|$

$= \dfrac{1}{2}\left|(64) - (49)\right|$

$$= \frac{1}{2}(15)$$

$$= \frac{15}{2} \text{ sq. units} \quad ...(ii) \qquad [1]$$

From (i) and (ii)

$$\therefore \quad \frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{\frac{5}{6}}{\frac{15}{2}} = \frac{5}{6} \times \frac{2}{15} = \frac{1}{9} \qquad [½]$$

71. Given $A(k + 1, 2k)$, $B(3k, 2k + 3)$, $C(5k - 1, 5k)$ are collinear.

If three points are collinear then the area of the triangle will be zero. For any 3 points (x_1, y_1), (x_2, y_2), (x_3, y_3) Area will be

$$\Rightarrow \quad \text{Area} = \frac{1}{2}\left|\begin{matrix} x_1(y_2 - y_3) + x_2(y_3 - y_1) \\ +x_3(y_1 - y_2) \end{matrix}\right| = 0 \qquad [½]$$

$$\therefore \quad 0 = \frac{1}{2}\left|\begin{matrix} (k+1)(2k+3-5k) + 3k(5k-2k) \\ +(5k-1)(2k-2k-3) \end{matrix}\right| \qquad [½]$$

$$0 = |(k + 1)(3 - 3k) + 3k(3k) - 15k + 3|$$

$$\Rightarrow |-3k^2 + 3 + 9k^2 + 3 - 15k| = 0$$

$$\Rightarrow |6k^2 - 15k + 6| = 0 \qquad [1]$$

$$\Rightarrow 6k^2 - 15k + 6 = 0$$

$$\Rightarrow 2k^2 - 5k + 2 = 0 \qquad [½]$$

$$\Rightarrow 2k^2 - 4k - k + 2 = 0$$

$$\Rightarrow 2k(k - 2) - 1(k - 2) = 0$$

$$\Rightarrow (k - 2)(2k - 1) = 0 \qquad [½]$$

$$\Rightarrow k = 2, \frac{1}{2} \qquad [½]$$

Hence the value of k are 2 and $\frac{1}{2}$ [½]

8 : Introduction to Trigonometry

1. $\tan A = \frac{5}{12}$

$$(\sin A + \cos A)\sec A = \frac{\sin A}{\cos A} + \frac{\cos A}{\cos A} \qquad [½]$$

$$= \tan A + 1$$

$$= \frac{5}{12} + 1$$

$$= \frac{17}{12} \qquad [½]$$

2. $\sec^2\theta(1 + \sin\theta)(1 - \sin\theta) = k$

$$\Rightarrow \sec^2\theta(1 - \sin^2\theta) = k \qquad [½]$$

$$\Rightarrow \sec^2\theta \cdot \cos^2\theta = k$$

$$\Rightarrow \frac{\cos^2\theta}{\cos^2\theta} = k$$

$$\Rightarrow k = 1 \qquad [½]$$

3. Given $3x = \operatorname{cosec}\theta$

$$\frac{3}{x} = \cot\theta$$

We know that $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\Rightarrow 9x^2 - \frac{9}{x^2} = 1 \qquad [½]$$

$$\Rightarrow 9\left(x^2 - \frac{1}{x^2}\right) = 1$$

$$\Rightarrow \boxed{3\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{3}} \qquad [½]$$

4. $\cos^2 67° - \sin^2 23°$

as $\cos(90° - \theta) = \sin\theta$

Let $\theta = 23°$ [½]

$\cos^2(90° - 23°) = \sin 23°$

$\cos^2 67° = \sin 23°$

$\therefore \cos^2 67° = \sin^2 23°$

$\therefore \cos^2 67° - \sin^2 23° = 0$ [½]

5. $\tan 2A = \cot(A - 24°)$

$$\Rightarrow \cot(90° - 2A) = \cot(A - 24°) \qquad [½]$$

$$\Rightarrow 90° - 2A = A - 24°$$

$$\Rightarrow 3A = 114°$$

$$\Rightarrow A = 38° \qquad [½]$$

6. $\sin^2 33° + \sin^2 57°$

$$= \sin^2 33° + \cos^2(90° - 57°) \qquad [½]$$

$$= \sin^2 33° + \cos^2 33°$$

$$= 1 \qquad [½]$$

7. $\sin 20° \cos 70° + \sin 70° \cos 20°$

$$= \cos(90° - 20°)\cos 70° + \sin 70° \sin(90° - 20°)$$

$$= \cos 70° \cos 70° + \sin 70° \sin 70°$$

$$= \cos^2 70° + \sin^2 70°$$

$$= 1 \qquad [1]$$

8. $\tan(A+B)=\sqrt{3}$

$\Rightarrow$ $A + B = 60°$ ...(i)

Also, $\tan(A-B)=\frac{1}{\sqrt{3}}$

$\Rightarrow$ $A - B = 30°$ ...(ii) $[\because A > B]$

On adding (i) and (ii), we get

$2A = 90°$

$\Rightarrow$ $A = 45°$ **[1]**

9. $\tan\theta=\frac{3}{5}$

Now, $\frac{5\sin\theta-3\cos\theta}{4\sin\theta+3\cos\theta}=\frac{5\tan\theta-3}{4\tan\theta+3}$ **[½]**

[Dividing numerator and denominator by $\cos\theta$]

$=\frac{3-3}{\frac{12+15}{5}}$

$= 0$ **[½]**

10. $\frac{\cos(90°-10°)}{\sin 10°}+\cos 59°\operatorname{cosec}(90°-59°)$

$\Rightarrow$ $\frac{\sin 10°}{\sin 10°}+\cos 59°\cdot\sec 59°$

$\Rightarrow$ $1 + 1$

$\Rightarrow$ 2 **[1]**

11. $\sin^2\theta+\frac{1}{\sec^2\theta}=\sin^2\theta+\cos^2\theta=1$ **[1]**

(using $\sec^2\theta - \tan^2\theta = 1$)

OR

$(1+\tan^2\theta)(1-\sin^2\theta)$

$\Rightarrow$ $\sec^2\theta \times \cos^2\theta$

$\Rightarrow$ 1 **[1]**

12. Answer (d)

$\tan^2 45° - \cos 2\ 60°$

$=1^2-\left(\frac{1}{2}\right)^2=\frac{3}{4}$ **[1]**

13. Answer (c)

$\tan 90°=\frac{\sin 90°}{\cos 90°}=\frac{1}{0}=$ Not defined **[1]**

14. Answer (c)

$\therefore$ $\angle R = 180° - 90° - 45° = 45°$

$\tan P - \cos^2 R = \tan 45° - \cos^2 45°$

$=1-\left(\frac{1}{\sqrt{2}}\right)^2$

$=\frac{1}{2}$ **[1]**

15. Answer (a)

$\sec\theta=\sqrt{1+\tan^2\theta}$

$=\frac{\sqrt{13}}{3}$ $\left[\because \tan\theta=\frac{2}{3}\right]$ **[1]**

16. Answer (b)

$\sin\theta - \cos\theta = 0$

$\Rightarrow$ $\sin\theta = \cos\theta$

$\Rightarrow$ $\tan\theta = 1$

$\theta = 45°$ **[1]**

17. Answer (d)

$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=\frac{1-\sin\theta}{1-\sin^2\theta}+\frac{1+\sin\theta}{1-\sin^2\theta}$

$=\frac{1-\sin\theta}{\cos^2\theta}+\frac{1+\sin\theta}{\cos^2\theta}$

$=\frac{2}{\cos^2\theta}=2\sec^2\theta$ **[1]**

18. Answer (b)

$(1 + \tan^2 A)(1 + \sin A)(1 - \sin A)$

$= \sec^2 A \times \cos^2 A$

$= 1$ **[1]**

19. Answer (d)

$\sec^2\theta + \operatorname{cosec}^2\theta$

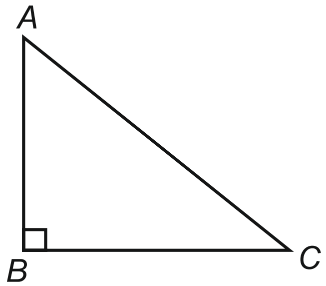
$= 1 + \tan^2\theta + 1 + \cot^2\theta$

$= 2+\frac{1}{3}+3$

$= 5\frac{1}{3}$ **[1]**

20. Answer (a)

$$\sin A = \frac{7}{25} = \frac{BC}{AC}$$



i.e., $BC = 7a$ and $AC = 25a$, where a is any non-zero positive constant.

In ΔABC,

$$\cos C = \frac{BC}{AC} = \frac{7}{25}$$ **[1]**

21. Answer (a)

$$\tan\theta = \pm\sqrt{\sec^2\theta - 1}$$

$$= \pm\sqrt{2-1}$$

$$= \pm 1$$

and $\sin\theta = \pm\sqrt{1 - \frac{1}{\sec^2\theta}}$

$$= \pm\sqrt{1-\frac{1}{2}}$$

$$= \pm\sqrt{\frac{1}{2}}$$

$$\Rightarrow \quad \frac{1+\tan\theta}{\sin\theta} = \frac{(1\pm 1)(\pm\sqrt{2})}{1}$$

$$= 2\sqrt{2} \text{ or } 0$$ **[1]**

22. Answer (c)

$\tan\theta + \cot\theta = 2$

$\Rightarrow$ $\tan^2\theta + 1 = 2\tan\theta$

$$\left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$\Rightarrow$ $(\tan\theta - 1)^2 = 0$

$\Rightarrow$ $\tan\theta - 1 = 0$

$\Rightarrow$ $\tan\theta = 1 = \tan 45°$

$\theta = 45°$

$\therefore$ $\sin^3\theta + \cos^3\theta = \sin^3 45° + \cos^3 45°$

$$\left[\because \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$ **[1]**

23. Answer (b)

$a\cot\theta + b\,\text{cosec}\theta = p$ …(i)

$b\cot\theta + a\,\text{cosec}\theta = q$ …(ii)

Squaring both the equations and subtracting,

$p^2 - q^2 = (a\cot\theta + b\,\text{cosec}\theta)^2 - (b\cot\theta + a\,\text{cosec}\theta)^2$

$= (a^2\cot^2\theta + b^2\text{cosec}^2\theta + 2ab\cot\theta\text{cosec}\theta) - (b^2\cot^2\theta + a^2\text{cosec}^2\theta + 2ab\cot\theta\text{cosec}\theta)$

$= (a^2 - b^2)(\cot^2\theta - \text{cosec}^2\theta)$

$= b^2 - a^2$ $[\because \text{cosec}^2\theta - \cot^2\theta = 1]$ **[1]**

24. Answer (b) **[1]**

$\sec\theta + \tan\theta = p$

$\Rightarrow$ $\sec\theta - \tan\theta = \dfrac{1}{p}$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$\Rightarrow$ $2\tan\theta = p - \dfrac{1}{p}$

$\Rightarrow$ $\tan\theta = \dfrac{p^2-1}{2p}$

25. $\sec 4A = \text{cosec}(A - 20)$

$\sec 4A = \sec(90 - (A - 20))$

$[\sec(90 - x) = \text{cosec}\,x]$ **[½]**

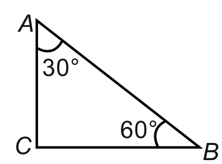
$\sec A = \sec(110 - A)$

$4A = 110 - A$ **[½]**

$5A = 110°$ **[½]**

$A = 22°$ **[½]**

26. In ΔABC, $\angle C = 90°$



$$\tan A = \frac{1}{\sqrt{3}} = \tan 30°$$

$\Rightarrow$ $A = 30°$

$\therefore$ $\angle B = 90° - 30° = 60°$ **[1]**

$\sin A\cos B + \cos A\sin B = \sin 30°\cos 60° + \cos 30°\sin 60°$

$$= \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$ **[1]**

27. $\cot\theta = \dfrac{15}{8}$ [Given]

$\dfrac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos)(2-2\cos\theta)} = \dfrac{2(1-\sin^2\theta)}{2(1-\cos^2\theta)}$ **[½]**

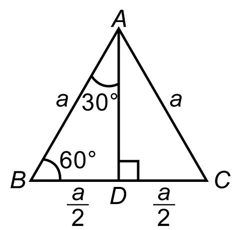
$= \dfrac{\cos^2\theta}{\sin^2\theta}$ **[½]**

$= \cot^2\theta$ **[½]**

$= \left(\dfrac{15}{8}\right)^2 = \dfrac{225}{64}$ **[½]**

28. Consider an equilateral ΔABC of side a

Draw $AD \perp BC$.



$\therefore\ \Delta ABD \cong \Delta ACD$

$\therefore\ BD = DC$

$\Rightarrow\ BD = \dfrac{1}{2}BC$

$= \dfrac{1}{2}a$

and $\angle BAD = \angle CAD = \dfrac{60°}{2} = 30°$ **[1]**

Using Pythagoras

$AD^2 = AB^2 - BD^2$

$= a^2 - \dfrac{a^2}{4}$

$= \dfrac{3a^2}{4}$

$AD = \dfrac{\sqrt{3}a}{2}$

$\therefore\ \tan 60° = \dfrac{AD}{BD} = \dfrac{\frac{\sqrt{3}a}{2}}{\frac{a}{2}} = \sqrt{3}$ **[1]**

29. $\dfrac{\sec(90°-\theta).\operatorname{cosec}\theta - \tan\theta(90°-\theta)\cot\theta + \cos^2 25° + \cos^2 65°}{3\tan 27°\cdot\tan 63°}$

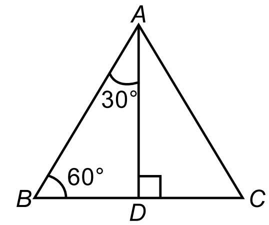
$= \dfrac{\operatorname{cosec}^2\theta - \cot^2\theta + \left(\sin(90°-25°)\right)^2 + \cos^2 65°}{3\tan 27°\cdot\tan 63°}$ **[1]**

$= \dfrac{1+\sin^2 65° + \cos^2 65°}{3\cot(90°-27°)\tan 63°}$

$= \dfrac{2}{4\cot 63°\tan 63°}$ [$\because$ $\cos^2 65° + \sin^2 65° = 1$]

$= \dfrac{2}{3}$ **[1]**

30.



$\angle A = \angle B = \angle C = 60°$

Draw $AD \perp BC$

In ΔABD and ΔACD,

$AD = AD$ (common)

$\angle ADB = \angle ADC$ (90°)

$AB = AC$ (ΔABC is equilateral Δ)

$\therefore\ \Delta ABD \cong \Delta ACD$ (RHS congruence criterion) **[1]**

$BD = DC$ (C.P.C.t)

$\angle BAD = \angle CAD$ (C.P.C.t)

$BD = \dfrac{2a}{2} = a$ *and* $\angle BAD = \dfrac{60°}{2} = 30°$

In right ΔABD,

$\sin 30° = \dfrac{BD}{AB}$ $\left(\because\ \sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}\right)$

$\Rightarrow\ \sin 30° = \dfrac{a}{2a}$

$\Rightarrow\ \sin 30° = \dfrac{1}{2} \Rightarrow \dfrac{1}{\sin 30°} = 2$

$\Rightarrow\ \boxed{\operatorname{cosec} 30° = 2}$ **[1]**

31. L.H.S.

$= \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\dfrac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$ [On rationalisation] **[½]**

$= \dfrac{1-\sin\theta}{\cos\theta}$ [$\because$ $1-\sin^2\theta = \cos^2\theta$] **[½]**

$= \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$ **[½]**

$= (\sec\theta - \tan\theta)$ **[½]**

L.H.S. = R.H.S.

OR

L.H.S.

$$= \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\csc^2\theta}$$

$[\because \sec^2\theta = 1+\tan^2\theta, \csc^2\theta = 1+\cot^2\theta]$ **[½]**

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$$ **[½]**

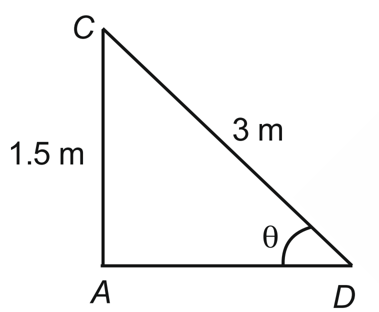
$$\left[\because \sec^2\theta = \frac{1}{\cos^2\theta}, \csc^2\theta = \frac{1}{\sin^2\theta}\right]$$

$= \sin^2\theta + \cos^2\theta$ **[½]**

$= 1$ **[½]**

L.H.S. = R.H.S.

32. $AD = \sqrt{9-2.25}$



$= \sqrt{6.75}$

$= \frac{3\sqrt{3}}{2}$ **[½]**

$\therefore \tan\theta = \frac{CA}{AD} = \frac{1.5}{3\sqrt{3}} \times \frac{2}{1} = \frac{1}{\sqrt{3}}$ **[½]**

$\sec\theta + \csc\theta = \frac{CD}{AD} + \frac{CD}{CA}$

$= 3\left[\frac{1\times 2}{3\sqrt{3}} + \frac{1}{1.5}\right]$ **[½]**

$= 3\left[\frac{2}{3\sqrt{3}} + \frac{2}{3}\right]$

$= 6\left[\frac{1+\sqrt{3}}{3\sqrt{3}}\right]$

$= \frac{2\left(\sqrt{3}+1\right)}{\sqrt{3}}$

$= \frac{2}{3}\left(3+\sqrt{3}\right)$ **[½]**

33. L.H.S. = $(1 + \cos A + \tan A)(\sin A - \cos A)$

$= \left(1 + \frac{1}{\tan A} + \tan A\right)\left(\frac{\sin A}{\cos A} - 1\right)\cos A$ **[½]**

$= \frac{(1+\tan^2 A + \tan A)(\tan A - 1)\cos A}{\tan A}$ **[½]**

$= \frac{(\tan^3 A - 1)\cos A}{\tan A}$

$[\because a^3 - b^3 = (a-b)(a^2+b^2+ab)]$ **[1]**

$= \tan^2 A \cos A - \cot A \cos A$

$= \tan A \cdot \frac{\sin A}{\cos A} \cdot \cos A - \cot A \cos A$ **[½]**

$= \sin A \tan A - \cot A \cos A$ = R.H.S.; Proved **[½]**

34. $2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left(\frac{\cos 38^\circ \csc 72^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right)$

$$\left[\begin{array}{l}\because \tan 75^\circ = \tan(90^\circ - 15^\circ) = \cot 15^\circ \\ \therefore \tan 15^\circ \tan 75^\circ = 1, \tan 60^\circ = \sqrt{3} \\ \sin 32^\circ = \cos 58^\circ,\ \cos 38^\circ = \sin 72^\circ\end{array}\right]$$ **[1]**

Substituting the above values in the given expression

$= 2\left(\frac{\sin 32^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left(\frac{\cos 38^\circ \sec 38^\circ}{\sqrt{3}}\right)$ **[1]**

$= 2 - 1$

$= 1$ **[1]**

35. $\frac{2}{3}\csc^2 58^\circ - \frac{2}{3}\cot 58^\circ \tan 32^\circ - \frac{5}{3}\tan 13^\circ$

$\tan 37^\circ \tan 45^\circ \tan 58^\circ$

$\tan 32^\circ = \tan(90^\circ - 58^\circ) = \cot 58^\circ$

$\tan 77^\circ = \tan(90^\circ - 13^\circ) = \cot 13^\circ = \frac{1}{\tan 13^\circ}$

$\tan 53^\circ = \tan(90^\circ - 37^\circ) = \cot 37^\circ = \frac{1}{\tan 37^\circ}$

$\tan 45^\circ = 1$ **[1]**

Substituting the above values in the given expression

$= \frac{2}{3}\csc^2 58^\circ - \frac{2}{3}\cot^2 58^\circ - \frac{5}{3}$

$$\left(\tan 13^\circ \tan 37^\circ \times 1 \times \frac{1}{\tan 37^\circ} \times \frac{1}{\tan 13^\circ}\right) \quad \textbf{[1]}$$

$$= \frac{2}{3}\left[\text{cosec}^2 58^\circ - \cot^2 58^\circ\right] - \frac{5}{3}(1)$$

$$= \frac{2}{3}(1) - \frac{5}{3}$$

[$\because$ $\text{cosec}^2\theta - \cot^2\theta = 1$]

$$= -\frac{3}{3} = -1 \quad \textbf{[1]}$$

36. L.H.S. $= \dfrac{\tan A}{1-\cot A} + \dfrac{\cot A}{1-\tan A}$

$$= \frac{\tan A}{\left(1 - \frac{1}{\tan A}\right)} + \frac{\cot A}{1-\tan A}$$

$$= \frac{-\tan^2 A}{1-\tan A} + \frac{\cot A}{1-\tan A} \quad \textbf{[1]}$$

$$= \frac{1}{1-\tan A}\left(-\tan^2 A + \cot A\right)$$

$$= \frac{1}{1-\tan A}\left(-\tan^2 A + \frac{1}{\tan A}\right) \quad \textbf{[1]}$$

$$= \frac{1-\tan^3 A}{\tan A(1-\tan A)}$$

$$= \frac{(1-\tan A)(1+\tan^2 A + \tan A)}{\tan A(1-\tan A)}$$

[$\because$ $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$]

$= \cot A + \tan A + 1 =$ R.H.S. **[1]**

Hence proved.

37. L.H.S. $= (\text{cosec}A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \frac{\left(1-\sin^2 A\right)\left(1-\cos^2 A\right)}{\sin A \cos A}$$

$$= \frac{\cos^2 A \sin^2 A}{\sin A \cos A}$$

$= \sin A \cdot \cos A$ ...(i) **[1]**

R.H.S. $= \dfrac{1}{\tan A + \cot A}$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}}$$

$$= \frac{\sin A \cdot \cos A}{1} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$= \sin A \cdot \cos A$ ...(ii) **[1]**

From (i) and (ii)

L.H.S. = R.H.S.; Hence Proved **[1]**

38. Given that,

$\tan\theta = \dfrac{3}{4}$ $\quad\therefore\ \tan^2\theta = \dfrac{9}{16}$ **[½]**

We know that,

$\sec^2\theta = 1 + \tan^2\theta$

$$\therefore \quad \sec^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$$

$$\Rightarrow \quad \sec\theta = \frac{5}{4} \quad \textbf{[½]}$$

Now,

$$\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right) = \left(\frac{\frac{4\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{4\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}\right) \quad \textbf{[½]}$$

$$= \frac{4\tan\theta - 1 + \sec\theta}{4\tan\theta + 1 - \sec\theta}$$

$$= \frac{3 - 1 + \frac{5}{4}}{3 + 1 - \frac{5}{4}} \quad \textbf{[½]}$$

$$= \frac{2 + \frac{5}{4}}{4 - \frac{5}{4}} \quad \textbf{[½]}$$

$$= \frac{\frac{(8+5)}{4}}{\frac{(16-5)}{4}}$$

$$= \frac{13}{11} \quad \textbf{[½]}$$

39. Given that,

$\tan 2A = \cot(A - 18^\circ)$

$\Rightarrow$ $\cot(90^\circ - 2A) = \cot(A - 18^\circ)$

[$\because$ $\tan\theta = \cot(90^\circ - \theta)$] **[1]**

$\Rightarrow$ $90^\circ - 2A = A - 18^\circ$ **[1]**

$\Rightarrow$ $3A = 108^\circ$

$$\Rightarrow \quad A = \frac{108^\circ}{3}$$

$\Rightarrow$ $A = 36^\circ$ **[1]**

40. L.H.S : $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \operatorname{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$

$\left[\because \sin\theta = \frac{1}{\operatorname{cosec}\theta} \text{ and } \cos\theta = \frac{1}{\sec\theta}\right]$ **[1]**

$= (\sin^2\theta + \cos^2\theta) + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 4$

$[\because \cos^2\theta + \sin^2\theta = 1]$

[1]

$= 1 + 1 + 1 + 4 + \tan^2\theta + \cot^2\theta$

$[\because \operatorname{cosec}^2\theta + 1 + \cot^2\theta \text{ and } \sec^2\theta = 1 + \tan^2\theta]$

[½]

$= 7 + \tan^2\theta + \cot^2\theta$ = R.H.S.

Hence Proved **[½]**

41. L.H.S : $\left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$

$= \left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$ **[½]**

$= \frac{(\sin A+\cos A)^2-(1)^2}{\sin A\cdot\cos A}$ **[½]**

$= \frac{\sin^2 A+\cos^2 A+2\sin A\cdot\cos A-1}{\sin A\cdot\cos A}$ **[½]**

$= \frac{1+2\sin A\cdot\cos A-1}{\sin A\cdot\cos A}$ $[\because \sin^2 A + \cos^2 A = 1]$

[½]

= 2 = R.H.S.

Hence Proved **[1]**

42. LHS = $(1 + \tan A - \sec A) \times (1 + \tan A + \sec A)$

$\because$ $(x - y)(x + y) = x^2 - y^2$

here $x = 1 + \tan A$

$y = \sec A$

LHS $= (1 + \tan A)^2 - (\sec A)^2$ **[1]**

$= 1 + \tan^2 A + 2\tan A - \sec^2 A$ **[1]**

$= \sec^2 A + 2\tan A - \sec^2 A$ $(1 + \tan^2 A = \sec^2 A)$

$= 2\tan A$ = RHS **[1]**

Hence, proved.

OR

LHS $= \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-1}+\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta+1}$

$= \operatorname{cosec}\theta\left(\frac{1}{\operatorname{cosec}\theta-1}+\frac{1}{\operatorname{cosec}\theta+1}\right)$

$= \operatorname{cosec}\theta\left(\frac{\operatorname{cosec}\theta+1+\operatorname{cosec}\theta-1}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)}\right)$ **[1]**

$= \operatorname{cosec}\theta\left(\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}\right)$

$= \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta}$ $\left[\begin{array}{l}\because 1+\cot^2\theta=\operatorname{cosec}^2\theta\\ \Rightarrow \operatorname{cosec}^2\theta-1=\cot^2\theta\end{array}\right]$ **[1]**

$= \frac{2\times\frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}}$ $\left[\begin{array}{l}\because \operatorname{cosec}\theta=\frac{1}{\sin\theta}\\ \cot\theta=\frac{\cos\theta}{\sin\theta}\end{array}\right]$

$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$ = RHS **[1]**

Hence, proved.

43. $\sin\theta+\cos\theta=\sqrt{3}$

On squaring both sides, we get

$\Rightarrow$ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$ **[½]**

$\Rightarrow$ $1 + 2\sin\theta\cos\theta = 3$ **[½]**

$\Rightarrow$ $\sin\theta\cos\theta = 1$ **[½]**

Now, $\tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$ **[½]**

$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$ **[½]**

$= \frac{1}{1}$

= 1 = RHS **[½]**

Hence proved.

44. L.H.S. $= \frac{\sin A-2\sin^3 A}{2\cos^3 A-\cos A}$

$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A-1)}$ **[1]**

$= \frac{\sin A}{\cos A}\left(\frac{\sin^2 A+\cos^2 A-2\sin^2 A}{2\cos^2 A-\sin^2 A-\cos^2 A}\right)$ **[1]**

$[\because \sin^2 A + \cos^2 A = 1]$

$= \tan A\left(\frac{\cos^2 A-\sin^2 A}{\cos^2 A-\sin^2 A}\right)$ **[1]**

$= \tan A$ = R.H.S.

Hence proved. **[1]**

45. LHS $=\dfrac{\sin A-\cos A+1}{\sin A+\cos A-1}$ **[½]**

$=\dfrac{\tan A-1+\sec A}{\tan A+1-\sec A}$

(Dividing numerator & denominator by cos A) **[½]**

$=\dfrac{(\tan A+\sec A)-1}{(\tan A-\sec A)+1}$ **[½]**

$=\dfrac{\{(\tan A+\sec A)-1\}(\tan A-\sec A)}{\{(\tan A-\sec A)+1\}(\tan A-\sec A)}$ **[½]**

$=\dfrac{(\tan^2 A-\sec^2 A)-(\tan A-\sec A)}{\{\tan A-\sec A+1\}(\tan A-\sec A)}$ **[½]**

$=\dfrac{-1-\tan A+\sec A}{(\tan A-\sec A+1)(\tan A-\sec A)}$ **[½]**

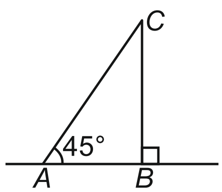
$=\dfrac{-1(\tan A-\sec A+1)}{(\tan A-\sec A+1)(\tan A-\sec A)}$ **[½]**

$=\dfrac{1}{\sec A-\tan A}=$ R.H.S. **[½]**

Hence proved.

9 : Some Applications of Trigonometry

1. Answer (C)



Given AB = 25 m

And angle of elevation of the top of the tower (BC) from A = 45°

$\because \angle BAC = 45°$

In ΔABC, $\tan 45° = \dfrac{BC}{AB}$

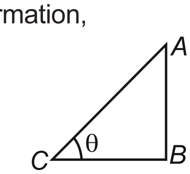
$\Rightarrow BC = 25$ m

$\therefore$ Height of the tower = 25 m

2. Answer (B)

Let AB be the tower and BC be its shadow. Let θ be the angle of elevation of the sun.

According to the given information,



$BC=\sqrt{3}\,AB \quad \ldots(1)$

In ΔABC,

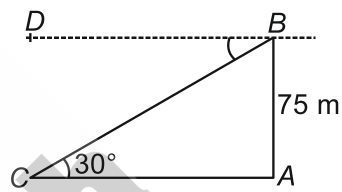
$\tan\theta=\dfrac{AB}{BC}=\dfrac{AB}{\sqrt{3}AB}=\dfrac{1}{\sqrt{3}}$ [Using (1)]

We know that $\tan 30° = \dfrac{1}{\sqrt{3}}$

$\therefore \theta = 30°$

Hence, the angle of elevation of the sun is 30°.

3. Answer (C)



Let AB be the tower of height 75 m and C be the position of the car

In ΔABC,

$\cot 30° = \dfrac{AC}{AB}$

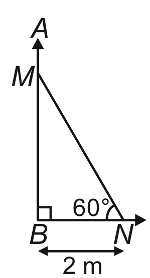
$\Rightarrow AC = AB\cot 30°$

$\Rightarrow AC = 75\text{ m}\times\sqrt{3}$

$\Rightarrow AC = 75\sqrt{3}$ m

Thus, the distance of the car from the base of the tower is $75\sqrt{3}$ m.

4. Answer (D)



In the figure, MN is the length of the ladder, which is placed against the wall AB and makes an angle of 60° with the ground.

The foot of the ladder is at N, which is 2 m away from the wall.

$\therefore BN = 2$ m

In right-angled triangle *MNB*:

$$\cos 60° = \frac{BN}{MN} = \frac{2}{MN}$$

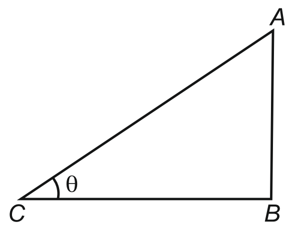
$$\Rightarrow \quad \frac{1}{2} = \frac{2}{MN}$$

$\Rightarrow$ $MN = 4$ m

Therefore, the length of the ladder is 4 m.

Hence, the correct option is D

5.



Let *AB* be the tower and *BC* be its shadow.

$AB = 20, BC = 20\sqrt{3}$

In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\tan\theta = \frac{20}{20\sqrt{3}}$$ [½]

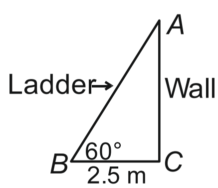
$$\tan\theta = \frac{1}{\sqrt{3}}$$

But, $\tan\theta = \frac{1}{\sqrt{3}}$

$\therefore$ $\theta = 30°$

The Sun is at an altitude of 30°. [½]

6.



Let *AB* be the ladder and *CA* be the wall.

The ladder makes an angle of 60° with the horizontal.

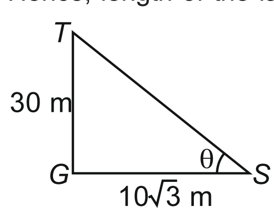
$\therefore$ ΔABC is a 30° – 60° – 90°, right triangle. [½]

Given: $BC = 2.5$ m, $\angle ABC = 60°$

$\therefore$ $AB = 5$ m

Hence, length of the ladder is $AB = 5$ m. [½]

7.



Angle of elevation of sun = $\angle GST = \theta$

Height of tower $TG = 30$ m

Length of shadow $GS = 10\sqrt{3}$ m [½]

ΔTGS is a right angled triangle

$$\therefore \quad \tan\theta = \frac{30}{10\sqrt{3}}$$

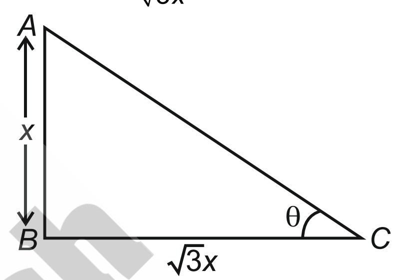
$$\tan\theta = \sqrt{3}$$ [½]

$\theta = 60°$

8. In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{x}{\sqrt{3}x}$$

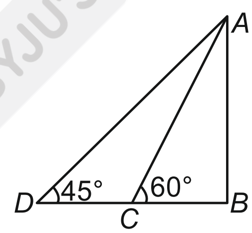


[½]

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$\therefore$ $\boxed{\theta = 30°}$ [½]

9.



Given $CD = 100$ m, $AB = ?$

In $\Delta ABC, \tan 60° = \frac{AB}{BC}$

$$BC = \frac{AB}{\sqrt{3}}$$ [1]

$BD = AB$ [$\because \tan 45° = 1$]

$BD - BC = CD$

$$AB - \frac{AB}{\sqrt{3}} = 100$$ [1]

$$AB\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 100$$

$$AB = \frac{100\sqrt{3}}{\sqrt{3}-1}$$

$AB = 236.98$

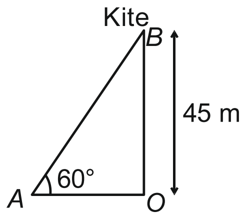
$AB = 237$ m [1]

10. Given: Position of kite is *B*.

Height of kite above ground = 45 m

Angle of inclination = 60°

Required length of string = *AB*



[1]

In right angled triangle *AOB*,

$$\sin A = \frac{OB}{AB}$$

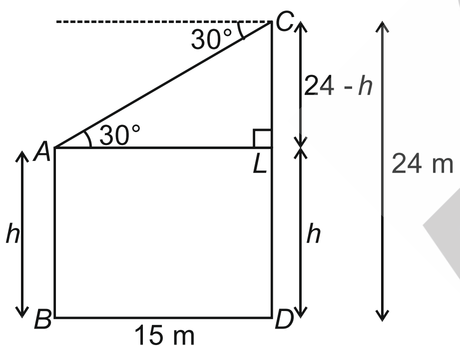
$$\Rightarrow \quad \sin 60^\circ = \frac{45}{AB}$$ [1]

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{45}{AB}$$

$$\Rightarrow \quad AB = \frac{45 \times 2}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 30\sqrt{3} \text{ m}$$

Hence, the length of the string is $30\sqrt{3}$ m. [1]

11.



Let *AB* and *CD* be the two poles, where *CD* (the second pole) = 24 m.

BD = 15 m

Let the height of pole *AB* be *h* m.

AL = *BD* = 15 m and *AB* = *LD* = *h*

So, *CL* = *CD* – *LD* = 24 – *h* [1]

In ΔACL,

$$\tan 30^\circ = \frac{CL}{AL}$$

$$\Rightarrow \quad \tan 30^\circ = \frac{24 - h}{15}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{24 - h}{15}$$ [1]

$$\Rightarrow \quad 24 - h = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

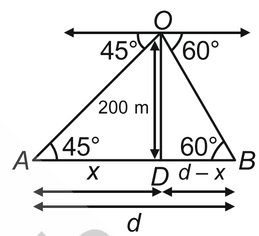
$$\Rightarrow \quad h = 24 - 5\sqrt{3}$$

$\Rightarrow$ *h* = 24 – 5 × 1.732 [Taking $\sqrt{3} = 1.732$]

$\Rightarrow$ *h* = 15.34

Thus, height of the first pole is 15.34 m. [1]

12. Let *d* be the distance between the two ships. Suppose the distance of one of the ships from the light house is *x* meters, then the distance of the other ship from the light house is (*d* – *x*) meter.



In right-angled ΔADO, we have.

$$\tan 45^\circ = \frac{OD}{AD} = \frac{200}{x}$$

$$\Rightarrow \quad 1 = \frac{200}{x}$$

$\Rightarrow$ *x* = 200 …(i) [1]

In right-angled ΔBDO, we have

$$\tan 60^\circ = \frac{OD}{BD} = \frac{200}{d - x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{200}{d - x}$$

$$\Rightarrow \quad d - x = \frac{200}{\sqrt{3}}$$ [1]

Putting *x* = 200. We have:

$$d - 200 = \frac{200}{\sqrt{3}}$$

$$d = \frac{200}{\sqrt{3}} + 200$$

$\Rightarrow$ *d* = 200 × 1.58

$\Rightarrow$ *d* = 316 m (approx.) [1]

Thus, the distance between two ships is approximately 316 m.

13. Let BC be the height at watch the aeroplane is observed from point A.

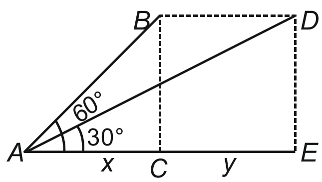
Then, $BC = 1500\sqrt{3}$

In 15 seconds, the aeroplane moves from point B to D.

B and D are the points where the angles of elevation 60° and 30° are formed respectively. **[1]**

Let $AC = x$ metres and $CE = y$ metres

$AE = x + y$



In ΔCBA,

$$\tan 60° = \frac{BC}{AC}$$

$$\sqrt{3} = \frac{1500\sqrt{3}}{x}$$ **[1]**

$\therefore$ $x = 1500$ m ...(i)

In ΔADE,

$$\tan 30° = \frac{DE}{AE}$$

$$\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{x+y}$$

$\therefore$ $x + y = 1500 \times (3) = 4500$

$\therefore$ $1500 + y = 4500$

$\therefore$ $y = 3000$ m ...(ii)

We know that, the aeroplane moves from point B to D in 15 seconds and the distance covered is 3000 metres.

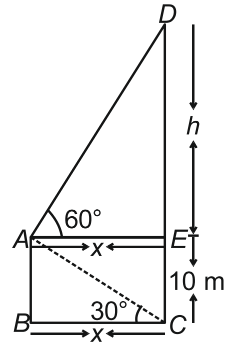
$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Speed} = \frac{3000}{15}$$

Speed 200m/s

Converting it to km/hr $= 200 \times \frac{18}{5} = 720$ km/hr **[1]**

14.



Let CD be the hill and suppose the man is standing on the deck of a ship at point A.

The angle of depression of the base C of the hill CD observed from A is 30° and the angle of elevation of the top D of the hill CD observed from A is 60°.

$\therefore$ $\Delta EAD = 60°$ and $\angle BCA = 30°$ **[1]**

In ΔAED,

$$\tan 60° = \frac{DE}{EA}$$

$$\sqrt{3} = \frac{h}{x}$$

$h = \sqrt{3}x$ ...(i)

In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{x}$$

$x = 10\sqrt{3}$ ...(ii) **[1]**

Substituting $x = 10\sqrt{3}$ in equation (i), we get

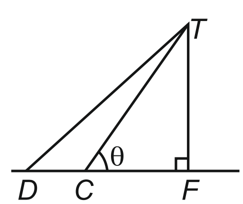
$h = \sqrt{3} \times 10\sqrt{3} = 10 \times 3 = 30$

$DE = 30$ m

$CD = CE + ED = 10 + 30 = 40$ m

Thus, the distance of the hill from the ship is $10\sqrt{3}$ m and the height of the hill is 40 m. **[1]**

15.



Given $CF = 4$ m

$DF = 16$ m

$\angle TCF + \angle TDF = 90°$

Let say $\angle TCF = \theta$ **[1]**

$\angle TDF = 90° - \theta$

In a right angled triangle TCF

$$\tan\theta = \frac{TF}{CF} = \frac{TF}{4}$$

$TF = 4\tan\theta$ ...(i)

In ΔTDF

$$\tan(90° - \theta) = \frac{TF}{16}$$ **[1]**

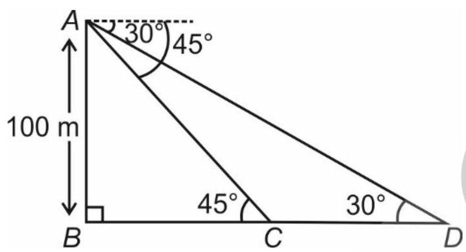
$TF = 16\cot\theta$ ...(ii)

Multiply (i) and (ii), we get

$(TF)^2 = 64 \Rightarrow TF = 8$ m

$\Rightarrow$ Height of tower = 8 m **[1]**

16.



In ΔABC,

$$\frac{AB}{BC} = \tan 45° = 1$$ **[½]**

$\Rightarrow AB = BC = 100$ m …(i) **[½]**

In ΔABD,

$$\frac{AB}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow BD = AB \times \sqrt{3}$

$= 100\sqrt{3}$ m …(ii) **[½]**

$\therefore CD = BD - BC$

$= \left(100\sqrt{3} - 100\right)$ m [From (i) and (ii)] **[½]**

$= 100\left(\sqrt{3} - 1\right)$ m

= 100 (1.73 –1) m

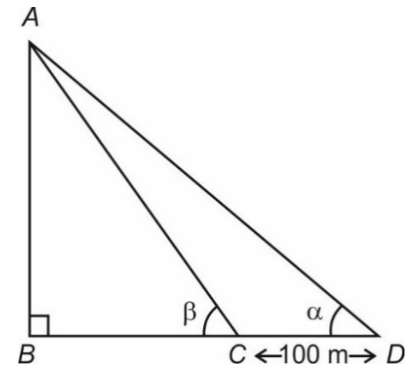
= 100 × 0.73 m **[½]**

= 73 m

$\therefore$ Ship will travel 73 m during the given time. **[½]**

OR

(b)



Let AB represents the tower. Observer is moving from D to C.

In ΔABC,

$$\tan\beta = \frac{AB}{BC} = \frac{3}{4}$$ …(i) **[½]**

and in ΔABD,

$$\tan\alpha = \frac{AB}{BD} = \frac{1}{3}$$ …(ii) **[½]**

From (i) and (ii), we get

$BC = \frac{4AB}{3}$ and $BD = 3AB$ **[½]**

$\Rightarrow CD = BD - BC$ **[½]**

$\Rightarrow 100 = 3AB - \frac{4AB}{3}$

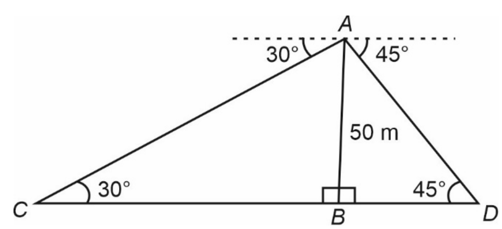
$\Rightarrow 100 = \frac{9AB - 4AB}{3}$ **[½]**

$\Rightarrow 300 = 5AB$

$\Rightarrow AB = 60$ m

$\therefore$ Height of tower is 60 m. **[½]**

17.



$\angle ACB = 30°$

and $\angle ADB = 45°$ [From figure]

Distance between two cars

$= CD = BC + BD$ [From figure] …(i) **[½]**

Now,

In ΔABC,

$$\tan 30° = \frac{AB}{BC} = \frac{50}{BC}$$

or $BC = \frac{50}{\tan 30^\circ} = 50\sqrt{3}$ m **[½]**

and In ΔABD,

$$\tan 45^\circ = \frac{AB}{BD} = \frac{50}{BD}$$

$$BD = \frac{50}{1}$$

$BD = 50$ m **[1]**

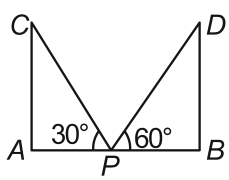
From equation (i), we get

$CD = BC + BD$

$= 50\sqrt{3} + 50$

$= 50\left(\sqrt{3}+1\right)$ m **[1]**

18. Let AC and BD be the two poles of the same height h m.



Given $AB = 80$ m

Let $AP = x$ m, therefore, $PB = (80 - x)$ m

In ΔAPC,

$$\tan 30^\circ = \frac{AC}{AP}$$ **[1]**

$$\frac{1}{\sqrt{3}} = \frac{h}{x} \qquad \text{…(i)}$$

In ΔBPD,

$$\tan 60^\circ = \frac{BD}{PB}$$

$$\sqrt{3} = \frac{h}{80-x} \qquad \text{…(ii)}$$ **[1]**

Dividing (ii) by (ii), we get

$$\frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{\frac{h}{x}}{\frac{h}{80-x}}$$

$$\Rightarrow \frac{1}{3} = \frac{80-x}{x}$$

$\Rightarrow$ $x = 240 - 3x$

$\Rightarrow$ $4x = 240$ **[1]**

$\Rightarrow$ $x = 60$ m

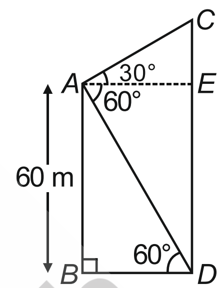
From (i),

$$\frac{1}{3} = \frac{h}{x}$$

$$\Rightarrow h = \frac{60}{\sqrt{3}} = 20\sqrt{3} \text{ m}$$

Thus, the height of both the poles is $20\sqrt{3}$ m and the distances of the point from the poles are 60 m and 20 m. **[1]**

19. Let AB be the building and CD be the tower.



In right ΔABD,

$$\frac{AB}{BD} = \tan 60^\circ$$

$$\Rightarrow \frac{60}{BD} = \sqrt{3}$$

$$\Rightarrow BD = \frac{60}{\sqrt{3}}$$ **[2]**

$$\Rightarrow BD = 20\sqrt{3}$$

In right ΔACE,

$$\frac{CE}{AE} = \tan 30^\circ$$

$$\Rightarrow \frac{CE}{AE} = \frac{1}{\sqrt{3}} \quad (\because AE = BD)$$

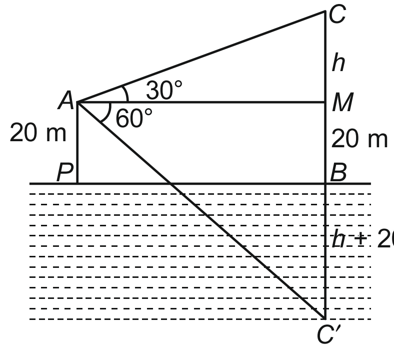
$$\Rightarrow CE = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

Height of the tower = $CE + ED = CE + AB$ = 20 m + 60 m = 80 m

Difference between the heights of the tower and the building = 80 m – 60 m = 20 m

Distance between the tower and the building $= BD = 20\sqrt{3}$ m **[2]**

20.



Let PB be the surface of the lake and A be the point of observation such that

AP = 20 metres. Let C be the position of the cloud and C' be its reflection in the lake.

Then $CB = C'B$. Let AM be perpendicular from A on CB. **[1]**

Then $m\angle CAM = 30°$ and $m\angle C'AM = 60°$

Let $CM = h$. Then, $CB = h + 20$ and $C'B = h + 20$.

In CMA we have,

$$\tan 30° = \frac{CM}{AM}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AM}$$

$$\Rightarrow AM = \sqrt{3}h \qquad \ldots(i) \qquad \textbf{[1]}$$

In $\Delta AMC'$ we have,

$$\tan 60° = \frac{C'M}{AM}$$

$$\Rightarrow \sqrt{3} = \frac{C'B + BM}{AM}$$

$$\Rightarrow \sqrt{3} = \frac{h + 20 + 20}{AM}$$

$$\Rightarrow AM = \frac{h + 20 + 20}{\sqrt{3}} \qquad \ldots(ii) \qquad \textbf{[1]}$$

From equation (i) and (ii), we get

$$\sqrt{3}h = \frac{h + 20 + 20}{\sqrt{3}}$$

$\Rightarrow$ $3h = h + 40$

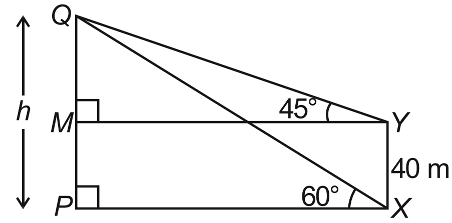
$\Rightarrow$ $2h = 40$

$\Rightarrow$ $h = 20$ m

In $\Delta CMA, \sin 30° = \dfrac{h}{CA}$ $\Rightarrow$ CA = 40 m

Hence, the distance of the cloud from the point A is 40 metres. **[1]**

21.



$MP = YX$ = 40 m

$\therefore$ $QM = h - 40$

In right angled ΔQMY,

$$\tan 45° = \frac{QM}{MY} \Rightarrow 1 = \frac{h - 40}{PX} \qquad \ldots(MY = PX) \qquad \textbf{[1]}$$

$\therefore$ $PX = h - 40$ ...(i)

In right angled ΔQPX,

$$\tan 60° = \frac{QP}{PX} \Rightarrow \sqrt{3} = \frac{QP}{PX}$$

$$PX = \frac{h}{\sqrt{3}} \qquad \ldots(ii) \qquad \textbf{[1]}$$

From (i) and (ii), we get

$$h - 40 = \frac{h}{\sqrt{3}}$$

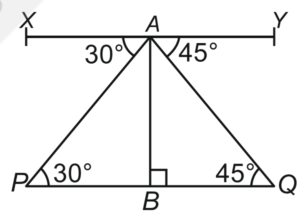
$\therefore$ $\sqrt{3}h - 40\sqrt{3} = h$

$\therefore$ $\sqrt{3}h - h = 40\sqrt{3}$ **[1]**

$\therefore$ $1.73h - h = 40(1.73) \Rightarrow h = 94.79$ m

Thus, PQ is 94.79 m and $PX = 94.79 \div 1.73$ = 54.79 m **[1]**

22.



Given aeroplane is at height of 300 m

$\therefore$ AB = 300 m and $XY \parallel PQ$

Angles of depression of the two points P and Q are 30° and 45° respectively. **[1]**

$\angle XAP = 30°$ and $\angle YAQ = 45°$

$\angle XAP = \angle APB = 30°$

[Alternate interior angles]

$\angle YAQ = \angle AQB = 45°$ **[1]**

In ΔPAB,

$$\tan 30° = \frac{AB}{PB}$$

$PB = 300\sqrt{3}$ m **[1]**

In ΔBAQ,

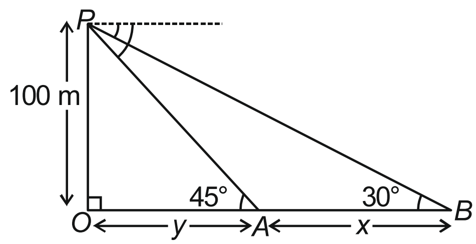
$$\tan 45° = \frac{AB}{BQ}$$

$BQ = 300$ m

$\therefore$ Width of the river = $PB + BQ$

$= 300(1+\sqrt{3})$ m **[1]**

23. Let ships are at distance x from each other.



In ΔAPO

$$\tan 45° = \frac{100}{y} = 1 \qquad \therefore\ y = 100 \text{ m} \qquad \ldots\text{(i)}$$ **[1]**

In ΔPOB

$$\tan 30° = \frac{OP}{OB} = \frac{100}{x+y} = \frac{1}{\sqrt{3}}$$ **[1]**

$$\sqrt{3} = \frac{x+y}{100}$$

$$x + y = 100\sqrt{3} \qquad \ldots\text{(ii)}$$ **[1]**

$$x = 100\sqrt{3} - y = 100\sqrt{3} - 100 = 100(\sqrt{3}-1)$$

$\therefore$ $x = 100(1.732 - 1)$

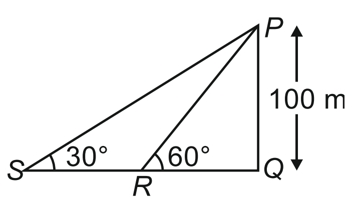
$= 100 \times 0.732$

$= 73.2$ m

$\therefore$ Ships are 73.2 meters apart. **[1]**

24. Let the light house be PQ and the boat changes its position from R to S.

Here, $PQ = 100$ m, $\angle PRQ = 60°$ and $\angle PSR = 30°$.



In ΔPQR,

$$\tan 60° = \frac{PQ}{QR} = \frac{100}{QR}$$

$$\Rightarrow \quad QR = \frac{100\sqrt{3}}{3} \text{ m} \qquad \ldots\text{(i)}$$ **[1]**

In ΔPQS,

$$\tan 30° = \frac{PQ}{QS}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{QS}$$

$$\Rightarrow \quad QS = 100\sqrt{3} \text{ m}$$ **[1]**

$\therefore$ $RS = QS - QR =$

$$100\sqrt{3} - \frac{100\sqrt{3}}{3} = \frac{200\sqrt{3}}{3}$$ **[1]**

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

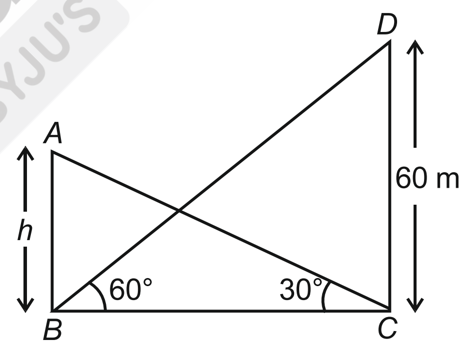
$$= \frac{200\sqrt{3}}{3 \times 2} = \frac{100\sqrt{3}}{3}$$

$= 57.73$ (approx.) (Using $\sqrt{3} = 1.732$)

$= 57.73$ m/min **[1]**

25. Let $AB = h$ m be the height of building and CD be height of tower.

$\therefore$ $CD = 60$ m



[1]

In ΔBDC, $\tan 60° = \dfrac{CD}{BC}$

$$\Rightarrow \quad BC = \frac{60}{\sqrt{3}} = 20\sqrt{3} \text{ m} \quad \ldots\text{(i)}$$ **[1]**

In ΔABC,

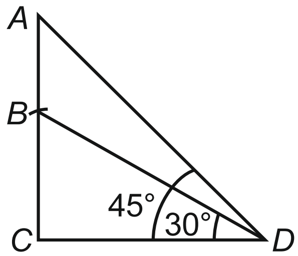
$$\tan 30° = \frac{AB}{BC}$$ **[1]**

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{20\sqrt{3}} \qquad \text{[From (i)]}$$

$\Rightarrow$ $AB = 20$ m

$\therefore$ Height of building = 20 m. **[1]**

26. AB = height of flag-staff = 6 m



Let BC = height of tower = h m **[½]**

In ΔBCD

$$\frac{BC}{CD} = \tan 30° \quad \textbf{[½]}$$

$$\Rightarrow \frac{h}{CD} = \frac{1}{\sqrt{3}} \Rightarrow CD = h\sqrt{3} \quad \ldots\text{(i)} \quad \textbf{[½]}$$

In ΔACD, $\dfrac{AC}{CD} = \tan 45°$ **[½]**

$$\Rightarrow \frac{h+6}{CD} = 1 \Rightarrow h = CD - 6$$

$$\Rightarrow h = h\sqrt{3} - 6 \quad \text{[From (i)]} \quad \textbf{[½]}$$

$$\Rightarrow h(\sqrt{3}-1) = 6$$

$$\Rightarrow h = \frac{6}{\sqrt{3}-1} \quad \textbf{[½]}$$

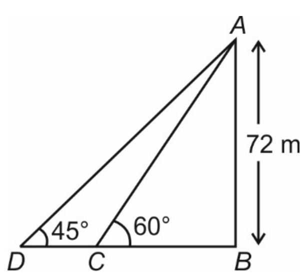
$$\Rightarrow h = 3(\sqrt{3}+1) \quad \textbf{[½]}$$

h = 3 × 2.73

h = 8.19 m **[½]**

∴ Height of the tower is 8.19 m

27. (i) Let positions of Charu and Daljeet be C and D respectively,



[1]

Charu is nearer to Qutub Minar as its angle of elevation is greater.

(ii) In ΔABC,

$$\tan 60° = \frac{AB}{BC} \quad \textbf{[½]}$$

$$\Rightarrow \sqrt{3} = \frac{72}{BC} \quad \textbf{[½]}$$

$\Rightarrow BC$ = 41.52 m **[½]**

In ΔABD,

$$\tan 45° = \frac{AB}{BD} \quad \textbf{[½]}$$

$$\Rightarrow 1 = \frac{72}{BD}$$

$\Rightarrow BD$ = 72 m **[½]**

$CD = BD - BC$

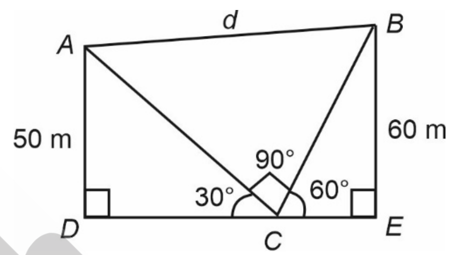
CD = (72 – 41.52) m

= 30.48 m **[½]**

28. (1) As from the figure, length of strings are AC and BC.

AD = 50 m

BE = 60 m



In ΔADC,

$$\sin 30° = \frac{AD}{AC} \quad \textbf{[½]}$$

$$\Rightarrow \frac{1}{2} = \frac{50}{AC}$$

$\Rightarrow AC$ = 100 m **[½]**

In ΔBCE,

$$\sin 60° = \frac{BE}{BC} \quad \textbf{[½]}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{BC}$$

$$\Rightarrow BC = 40\sqrt{3} \text{ m} \quad \textbf{[½]}$$

(2) As from the figure, we can see that $\angle ACB$ = 90° **[½]**

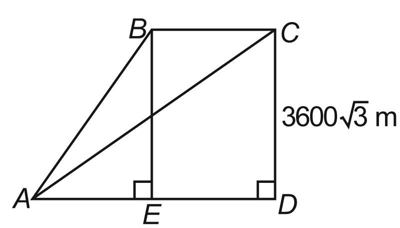
Applying Pythagoras theorem in ΔACB, we get

$$d = \sqrt{AC^2 + BC^2} \quad \textbf{[½]}$$

$$= \sqrt{(100)^2 + \left(40\sqrt{3}\right)^2} \quad \textbf{[½]}$$

$$= 20\sqrt{37} \text{ m} \quad \textbf{[½]}$$

29.



Height of aeroplane (CD) = $3600\sqrt{3}$ m = BE

$\angle BAD = 60°$ and $\angle CAD = 30°$

In ΔABE

$$\tan 60° = \frac{BE}{AE}$$ **[1]**

$$AE = \frac{BE}{\tan 60°}$$

$AE = 3600$ m $\quad [\because BE = 3600\sqrt{3}\text{ m}]$ **[1]**

In ΔACD

$$\tan 30° = \frac{CD}{AD}$$

$$AD = \frac{3600\sqrt{3}}{\frac{1}{\sqrt{3}}}$$

$AD = 10800$ m **[1]**

$\therefore \quad BC = AD - AE = 10800 - 3600$ **[1]**

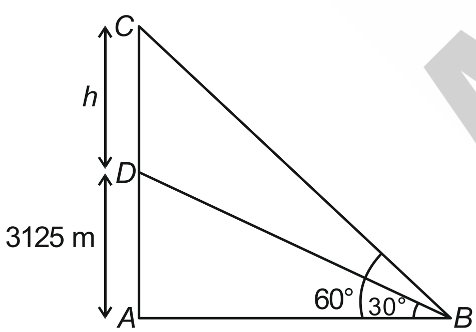
$BC = 7200$ m

Speed of aeroplane $= \dfrac{\text{distance}}{\text{time}}$ **[1]**

$$= \frac{7200}{30} = 240 \text{ m/s}$$

Speed (in km/hr) = 864 km/hour **[1]**

30.



Let the distance between the two planes be h m.

Given that: $AD = 3125$ m and

$\angle ABC = 60°$ **[1]**

$\angle ABD = 30°$

In ΔABD,

$$\tan 30° = \frac{AD}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{3125}{AB}$$

$\Rightarrow \quad AB = 3125\sqrt{3} \quad$ ...(i) **[1]**

ΔABC

$$\tan 60° = \frac{AC}{AB}$$

$$\sqrt{3} = \frac{AD + DC}{AB}$$ **[1]**

$$\sqrt{3} = \frac{3125 + h}{AB}$$

$\Rightarrow \quad AB = \dfrac{3125 + h}{\sqrt{3}} \quad$ ...(ii) **[1]**

Equating equation (i) and (ii), we have

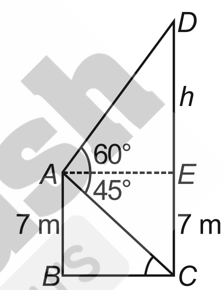
$$\frac{3125 + h}{\sqrt{3}} = 3125\sqrt{3}$$

$h = 3125 \times 3 - 3125$ **[1]**

$h = 6250$

Hence, distance between the two planes is 6250 m. **[1]**

31.



Let AB be the building and CD be the tower such that $\angle EAD = 60°$ and $\angle EAC = \angle ACB = 45°$ **[1]**

Now, in triangle ABC, $\tan 45° = 1 = AB/BC$

So, $AB = AE = 7$ m **[1]**

Again in triangle AED,

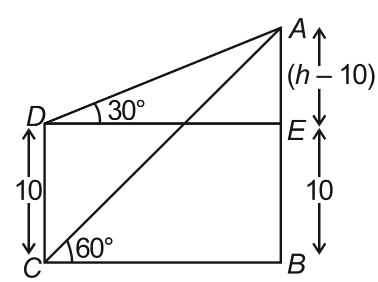
$\tan 60° = \sqrt{3} = DE/AE$ **[1]**

So, $DE = AE\sqrt{3} = 7\sqrt{3}$ **[1]**

$\Rightarrow \quad h = 7\sqrt{3}$ m **[1]**

Height of tower $= h + 7 = 7(1 + \sqrt{3})$ m **[1]**

32.



Height of the tower (AB) = h

Given CD =10 m and $BC = ED$

$BE = CD = 10$ m **[1]**

In $\Delta ABC, \tan 60^\circ = \dfrac{h}{BC}$ **[1]**

$BC = \dfrac{h}{\sqrt{3}}$ **[1]**

In ΔADE,

$\tan 30^\circ = \dfrac{h-10}{ED}$ **[1]**

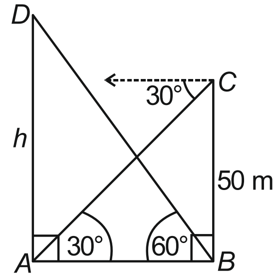
$ED = (h-10)\sqrt{3}$

$\therefore \quad \dfrac{h}{\sqrt{3}} = (h-10)\sqrt{3}$ **[1]**

$10 = \dfrac{2}{3}h$

$\boxed{h = 15 \text{ m}}$ **[1]**

33.



Let the height of hill be h.

In right triangle ABC,

$\dfrac{50}{AB} = \tan 30^\circ \Rightarrow \dfrac{50}{AB} = \dfrac{1}{\sqrt{3}} \Rightarrow AB = 50\sqrt{3}$ **[2]**

In right triangle BAD,

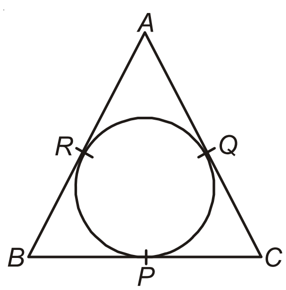
$\dfrac{h}{AB} = \tan 60^\circ \Rightarrow \dfrac{h}{AB} = \sqrt{3} \Rightarrow h = \sqrt{3}AB$ **[2]**

$\Rightarrow \quad h = \sqrt{3}(50\sqrt{3}) = 150 \text{ m}$

Hence, the height of hill is 150 m. **[2]**

10 : Circles

1.



Given BR = 3 cm, AR = 4 cm & AC = 11 cm

$BP = BR$

$AR = AQ$

$CP = CQ$

(Lengths of tangents to circle from external point will be equal)

$\therefore$ AQ = 4 cm and BP = 3 cm **[½]**

As AC = 11 cm

$QC + AQ$ = 11 cm

$\Rightarrow$ QC = 7 cm

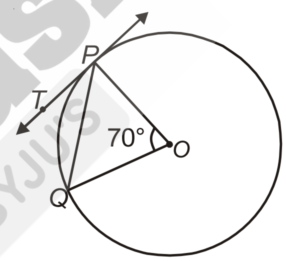
$\therefore$ PC = 7 cm

We know $BC = BP + PC$

$\therefore$ BC = 3 + 7

BC = 10 cm **[½]**

2. Answer (D)



Given $\angle POQ = 70^\circ$

In ΔPOQ, $OP = OQ$ (radii)

$\therefore$ It is an isosceles triangle

$\Rightarrow \quad \angle OPQ = \angle OQP$

In ΔPOQ,

$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$

$\angle POQ + 2\angle OPQ = 180^\circ$

$\angle OPQ = 55^\circ$ **[½]**

We know that $OP \perp PT$

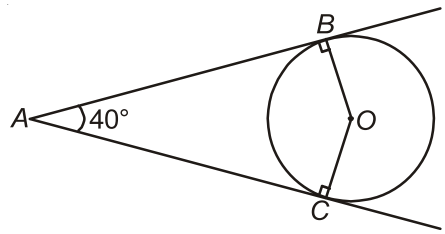
$\therefore \quad \angle OPT = 90^\circ$

$\angle OPT = \angle TPQ + \angle OPQ$

$90^\circ = \angle TPQ + 55^\circ$

$\angle TPQ = 35^\circ$ **[½]**

3. Answer (C)



AB and AC are the tangents drawn from external point A to the circle.

$\therefore$ $OB \perp AB \Rightarrow \angle OBA = 90°$

$OC \perp AC \Rightarrow \angle OCA = 90°$

$ABCD$ is a quadrilateral in which sum of opposite angles is 180°

i.e.; $\angle OBA + \angle OCA = 180°$ **[½]**

$\therefore$ $ABCD$ is a cyclic quadrilateral

$\Rightarrow$ $\angle BAC + \angle BOC = 180°$

$\angle BOC = 180° - 40°$

$\boxed{\angle BOC = 140°}$ **[½]**

4. Answer (A)

It is known that the tangents from an external point to the circle are equal.

$\therefore$ $EK = EM$, $DK = DH$ and $FM = FH$ ...(i) **[½]**

Perimeter of $\Delta EDF = ED + DF + FE$

$= (EK - DK) + (DH + HF) + (EM - FM)$

$= (EK - DH) + (DH + HF) + (EM - FH)$

[Using (i)]

$= EK + EM$

$= 2\ EK = 2\ (9\text{ cm}) = 18\text{ cm}$

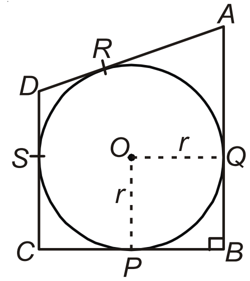
Hence, the perimeter of EDF is 18 cm. **[½]**

5. Answer (A)

Given: AB, BC, CD and AD are tangents to the circle with centre O at Q, P, S and R respectively. $AB = 29$ cm,

$AD = 23$, $DS = 5$ cm and $\angle B = 90°$

Construction: Join PQ.



We know that, the lengths of the tangents drawn from an external point to a circle are equal.

$DS = DR = 5$ cm

$\therefore$ $AR = AD - DR = 23\text{ cm} - 5\text{ cm} = 18\text{ cm}$

$AQ = AR = 18$ cm

$\therefore$ $QB = AB - AQ = 29\text{ cm} - 18\text{ cm} = 11\text{ cm}$

$QB = BP = 11$ cm

In ΔPQB,

$PQ^2 = QB^2 + BP^2 = (11\text{ cm})^2 + (11\text{ cm})^2 = 2 \times (11\text{ cm})^2$

$PQ = 11\sqrt{2}$ cm ...(i) **[½]**

In ΔOPQ,

$PQ^2 = OQ^2 + OP^2 = r^2 + r^2 = 2r^2$

$(11\sqrt{2})^2 = 2r^2$

$121 = r^2$

$r = 11$

Thus, the radius of the circle is 11 cm. **[½]**

6. Answer (B)

$AP \perp PB$ (Given)

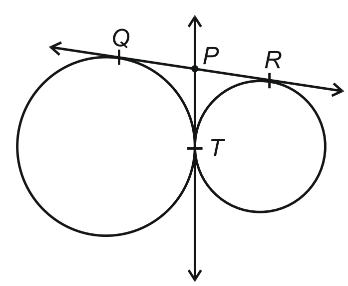
$CA \perp AP$, $CB \perp BP$ (Since radius is perpendicular to tangent)

$AC = CB$ = radius of the circle **[½]**

Therefore, $APBC$ is a square having side equal to 4 cm.

Therefore, length of each tangent is 4 cm. **[½]**

7. Answer (B)



It is known that the length of the tangents drawn from an external point to a circle is equal.

$\therefore$ $QP = PT = 3.8$ cm ...(i)

$PR = PT = 3.8$ cm ...(ii)

From equations (i) and (ii), we get :

$QP = PR = 3.8$ cm **[½]**

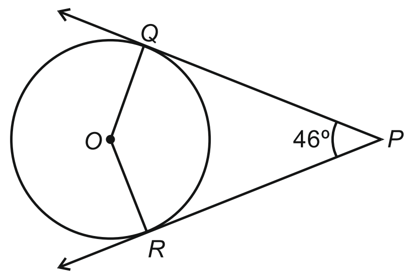
Now, $QR = QP + PR$

$= 3.8\text{ cm} + 3.8\text{ cm}$

$= 7.6$ cm

Hence, the correct option is B. **[½]**

8. Answer (B)



Given: $\angle QPR = 46°$

PQ and PR are tangents.

Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

So, we have $OQ \perp PQ$ and $OR \perp RP$.

$\Rightarrow \angle OQP = \angle ORP = 90°$ **[½]**

So, in quadrilateral $PQOR$, we have

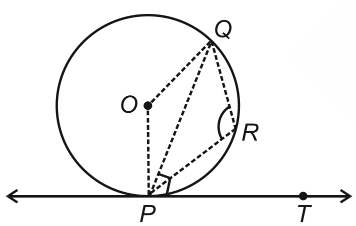
$\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360°$

$\Rightarrow 90° + 46° + 90° + \angle ROQ = 360°$

$\Rightarrow \angle ROQ = 360° - 226° = 134°$

Hence, the correct option is B. **[½]**

9.



$\angle OPT = 90°$

(radius is perpendicular to the tangent)

So, $\angle OPQ = \angle OPT - \angle QPT$

$= 90° - 60°$

$= 30°$

$\angle POQ = 180° - 2\angle QPO = 180° - 60° = 120°$

Reflex $\angle POQ = 360° - 120° = 240°$ **[½]**

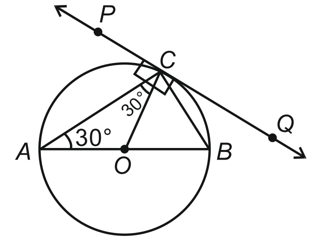
$$\angle PRQ = \frac{1}{2}\text{reflex}\angle POQ$$

$$= \frac{1}{2} \times 240°$$

$$= 120°$$

$\therefore \ PRQ = 120°$ **[½]**

10.



In ΔACO,

$OA = OC$ [Radii of the same circle]

$\therefore \ \Delta ACO$ is an isosceles triangle.

$\angle CAB = 30°$ [Given]

$\therefore \ \angle CAO = \angle ACO = 30°$ **[½]**

[angles opposite to equal sides of an isosceles triangle are equal]

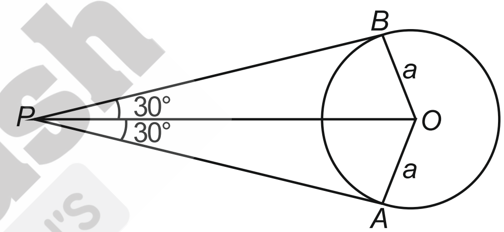
$\angle PCO = 90°$

[radius drawn at the point of contact is perpendicular to the tangent]

Now $\angle PCA = \angle PCO - \angle ACO$

$\therefore \ \angle PCA = 90° - 30° = 60°$ **[½]**

11.



Given that $\angle BPA = 60°$

$OB = OA = a$ [radii]

$PA = PB$ [length of tangents are equal]

$OP = OP$ [Common]

$\therefore \ \Delta PBO$ and ΔPAO are congruent. **[½]**

[By SSS criterion of congruency]

$$\therefore \ \angle BPO = \angle OPA = \frac{60°}{2} = 30°$$

In ΔPBO, $\sin 30° = \dfrac{a}{OP} = \dfrac{1}{2}$ ($\because$ $OB \perp BP$)

$OP = 2a$ units **[½]**

12. Answer (c)

In ΔPOT,

$(OP)^2 = (OT)^2 + (PT)^2$

$\Rightarrow OP^2 = (7)^2 + (24)^2$

$\Rightarrow OP^2 = (25)^2$

$\Rightarrow OP = 25$ cm

$\therefore PR = OP + OR = 25 + 7$

$= 32$ cm

Hence, option (c) is correct. **[1]**

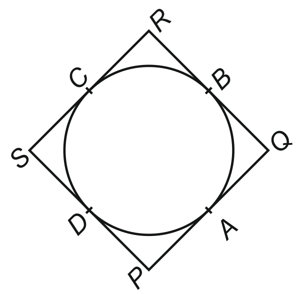
13. $BP = BQ = 3$ cm

$AR = AP = 4$ cm

$RC = AC - AR = 7$ cm

$RC = QC = 7$ cm

$\therefore \quad BC = 7 + 3 = 10$ cm **[1]**

14. 

Given a parallelogram *PQRS* in which a circle is inscribed

We know $PQ = RS$

$QR = PS$ **[½]**

$DP = PA$ ...(i)

(tangents to the circle from external point have equal length)

Similarly,

$QA = BQ$ ...(ii)

$BR = RC$ ...(iii)

$DS = CS$ ...(iv)

Adding above four equations, **[½]**

$DP + BQ + BR + DS = PA + QA + RC + CS$

$(DP + DS) + (BQ + BR) = (PA + QA) + (RC + CS)$

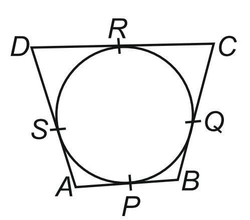
[½]

$2QR = 2(PQ)$

$\therefore \quad PQ = QR$

$\Rightarrow \quad PQ = QR = RS = QS$

$\therefore$ *PQRS* is a rhombus **[½]**

15. 

$AB = 6$ cm

$BC = 9$ cm

$CD = 8$ cm

AB, BC, CD, AD, are tangents to the circle

And $AP = AS$, $RD = DS$,

$BP = BQ$ and

$CQ = CR$ **[½]**

Also $AB = AP + BP$ ...(i)

$BC = BQ + QC$ ...(ii)

$CD = RC + DR$ ...(iii)

$AD = AS + DS$ ...(iv) **[½]**

Adding (i), (ii), (iii), (iv), we have

$6 + 9 + 8 + AD = AP + AS + BP + BQ + CQ + RC + RD + DS$ **[½]**

$23 + AD = 2(AP) + 2(BP) + 2(RC) + 2(RD)$

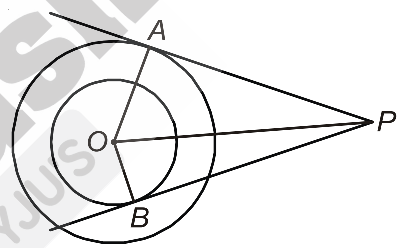
$23 + AD = 2(AB) + 2(CD)$

$\boxed{AD = 5 \text{ cm}}$ **[½]**

16. Given : Tangents *PA* and *PB* are drawn from an external point *P* to two concentric circles with centre *O* and radii $OA = 8$ cm, $OB = 5$ cm respectively. Also, $AP = 15$ cm

To find : Length of *BP*

Construction : We join the points *O* and *P*.



Solution : $OA \perp AP$; $OB \perp BP$

[Using the property that radius is perpendicular to the tangent at the point of contact of a circle]

In right angled triangle *OAP*,

$OP^2 = OA^2 + AP^2$ [Using Pythagoras Theorem]

$= (8)^2 + (15)^2 = 64 + 225 = 289$ **[½]**

$\therefore \quad OP = 17$ cm **[½]**

In right angled triangle *OBP*,

$OP^2 = OB^2 + BP^2$

$\Rightarrow \quad BP^2 = OP^2 - OB^2$

$= 17^2 - 5^2 = 289 - 25 = 264$ **[½]**

$\therefore \quad BP^2 = 264 \Rightarrow BP = 2\sqrt{66}$ cm **[½]**

17. Given : *ABC* is an isosceles triangle, where $AB = AC$, circumscribing a circle.

To prove : The point of contact *P* bisects the base *BC*.

i.e. $BP = PC$

Proof : It can be observed that

BP and *BR*; *CP* and *CQ; AR* and *AQ* are pairs of tangents drawn to the circle from the external points *B, C* and *A* respectively.

So, applying the theorem that the tangents drawn from an external point to a circle are equal, we get

$BP = BR$ …(i)

$CP = CQ$ …(ii)

$AR = AQ$ …(iii) **[½]**

Given that $AB = AC$

$\Rightarrow$ $AR + BR = AQ + CQ$ **[½]**

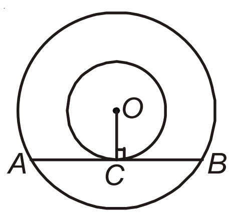
$\Rightarrow$ $BR = CQ$ [from (iii)]

$\Rightarrow$ $BP = CP$ [from (i) and (ii)] **[½]**

$\therefore$ P bisects BC.

Hence proved. **[½]**

18.



Given : *AB* is chord to larger circle and tangent to smaller circle at *C* concentric to it.

To prove : $AC = BC$

Construction : Join *OC* **[1]**

Proof : $OC \perp AB$ **[½]**

($\because$ Radius is perpendicular to tangent at point of contact)

$\Rightarrow$ $AC = BC$ **[½]**

($\because$ Perpendicular from centre bisects the chord)

19. Given : $AB = 12$ cm, $BC = 8$ cm and $AC = 10$ cm.

Let, $AD = AF = x$ cm, $BD = BE = y$ cm and $CE = CF = z$ cm

(Tangents drawn from an external point to the circle are equal in length)

$\Rightarrow$ $2(x + y + z) = AB + BC + AC = AD + DB + BE + EC + AF + FC = 30$ cm **[½]**

$\therefore$ $x + y + z = 15$ cm

$AB = AD + DB = x + y = 12$ cm **[½]**

$\therefore$ $z = CF = 15 - 12 = 3$ cm

$AC = AF + FC = x + z = 10$ cm

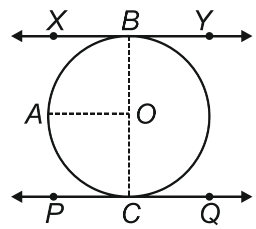
$\therefore$ $y = BE = 15 - 10 = 5$ cm **[½]**

$\therefore$ $x = AD = x + y + z - z - y = 15 - 3 - 5$
$= 7$ cm **[½]**

20. Let *XBY* and *PCQ* be two parallel tangents to a circle with centre *O*.

Construction : Join *OB* and *OC*.

Draw $OA \parallel XY$



Now, $XB \parallel AO$

$\Rightarrow$ $\angle XBO + \angle AOB = 180°$ **[½]**

(Sum of adjacent interior angles is 180°)

Now, $\angle XBO = 90°$

(A tangent to a circle is perpendicular to the radius through the point of contact)

$\Rightarrow$ $90° + \angle AOB = 180°$

$\Rightarrow$ $\angle AOB = 180° - 90° = 90°$ **[½]**

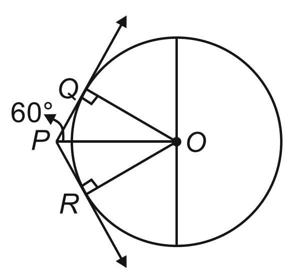
Similarly , $\angle AOC = 90°$

$\angle AOB + \angle AOC = 90° + 90° = 180°$ **[½]**

Hence, *BOC* is a straight line passing through *O*.

Thus, the line segment joining the points of contact of two parallel tangents of a circle passes through its centre. **[½]**

21. Let us draw the circle with extent point *P* and two tangents *PQ* and *PR*.



We know that the radius is perpendicular to the tangent at the point of contact.

$\therefore$ $\angle OQP = 90°$ **[½]**

We also know that the tangents drawn to a circle from an external point are equally inclined to the line joining the centre to that point.

$\therefore$ $\angle QPO = 60°$ **[½]**

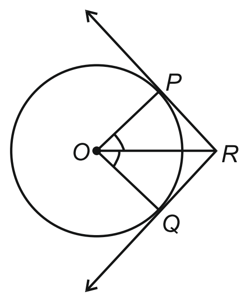
Now, in ΔQPO,

$$\cos 60^\circ = \frac{PQ}{PO} \quad [½]$$

$$\Rightarrow \frac{1}{2} = \frac{PQ}{PO}$$

$$\Rightarrow 2PQ = PO \quad [½]$$

22.



Given that $\angle PRQ = 120°$

We know that the line joining the centre and the external point is the angle bisector of angle between the tangents.

Thus,

$$\angle PRO = \angle QRO = \frac{120^\circ}{2} = 60^\circ \quad [½]$$

Also we know that lengths of tangents from an external point are equal.

Thus, $PR = RQ$.

Join OP and OQ.

Since OP and OQ are the radii from the centre O,

$OP \perp PR$ and $OQ \perp RQ$. [½]

Thus, ΔOPR and ΔOQR are right angled congruent triangles.

Hence, $\angle POR = 90° - \angle PRO = 90° - 60° = 30°$

$\angle QOR = 90° - \angle QRO = 90° - 60° = 30°$ [½]

$$\sin \angle QRO = \sin 30^\circ = \frac{1}{2}$$

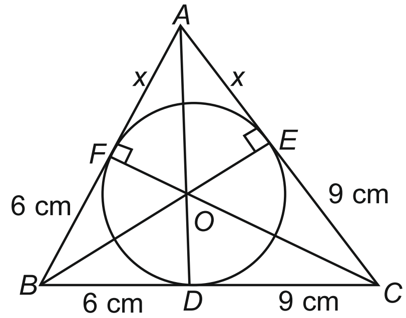
$$\frac{PR}{OR} = \frac{1}{2}$$

Thus, $\Rightarrow OR = 2PR$

$\Rightarrow OR = PR + PR$

$\Rightarrow OR = PR + QR$ [½]

23.



Let the given circle touch the sides AB and AC of the triangle at points F and E respectively and let the length of line segment AF be x.

Now, it can be observed that:

$BF = BD = 6$ cm (tangents from point B)

$CE = CD = 9$ cm (tangents from point C)

$AE = AF = x$ (tangents from point A)

$AB = AF + FB = x + 6$

$BC = BD + DC = 6 + 9 = 15$

$CA = CE + EA = 9 + x$ [½]

$2s = AB + BC + CA = x + 6 + 15 + 9 + x = 30 + 2x$

$s = 15 + x$

$s - a = 15 + x - 15 = x$

$s - b = 15 + x - (x + 9) = 6$

$s - c = 15 + x - (6 + x) = 9$

Area of $\Delta ABC = \sqrt{s(s-a)(a-b)(s-c)}$ [½]

$$54 = \sqrt{(15+x)(x)(6)(9)}$$

$$54 = 3\sqrt{6(15x+x^2)}$$

$$18 = \sqrt{6(15x+x^2)}$$

$324 = 6(15x + x^2)$

$54 = 15x + x^2$

$x^2 + 15x - 54 = 0$ [½]

$x^2 + 18x - 3x - 54 = 0$

$x(x + 18) - 3(x + 18)$

$(x + 18)(x - 3) = 0$

As distance cannot be negative, $x = 3$ cm

$AC = 3 + 9 = 12$ cm

$AB = AF + FB = 6 + x = 6 + 3 = 9$ cm [½]

24. Since tangents drawn from an exterior point to a circle are equal in length,

$AP = AS$ ...(i)

$BP = BQ$ ...(ii)

$CR = CQ$ ...(iii)

$DR = DS$ ...(iv) **[½]**

Adding equations (i), (ii), (iii) and (iv), we get

$AP + BP + CR + DR = AS + BQ + CQ + DS$ **[½]**

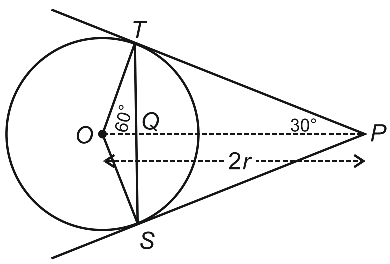
$\therefore \; (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

[½]

$\therefore \; AB + CD = AD + BC$

$\therefore \; AB + CD = BC + DA$ [Proved] **[½]**

25.



In the given figure,

$OP = 2r$ [Given]

$\angle OTP = 90°$

[radius drawn at the point of contact is perpendicular to the tangent]

In ΔOTP,

$$\sin\angle OPT = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2} = \sin 30°$$

$\angle OPT = 30°$

$\angle TOP = 60°$ **[½]**

$\therefore$ ΔOTP is a $30° - 60° - 90°$, right triangle.

In ΔOTS,

$OT = OS$ [Radii of the same circle]

$\therefore$ ΔOTS is an isosceles triangle.

$\therefore \; \angle OTS = \angle OST$ **[½]**

[Angles opposite to equal sides of an isosceles triangle are equal]

In ΔOTQ and ΔOSQ

$OS = OT$ [Radii of the same circle]

$OQ = OQ$

[side common to both triangles]

$\angle OTQ = \angle OSQ$

[angles opposite to equal sides of an isosceles triangle are equal]

$\therefore \; \Delta OTQ = \Delta OSQ$ [By S.A.S] **[½]**

$\therefore \; \angle TOQ = \angle SOQ = 60°$ [C.A.C.T]

$\therefore \; \angle TOS = 120°$

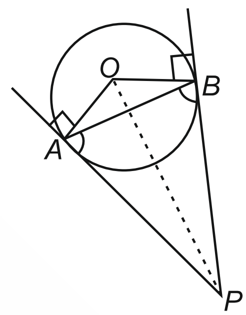
$[\angle TOS = \angle TOQ + \angle SOQ$

$= 60° + 60° = 120°]$

$\therefore \; \angle OTS + \angle OST = 180° - 120° = 60°$

$\therefore \; \angle OTS = \angle OST = 60° \div 2 = 30°$ **[½]**

26.



AB is the chord

We know that $OA = OB$ [radii]

$\angle OBP = \angle OAP = 90°$

Join OP and $OP = OP$ [Common] **[½]**

By RHS congruency

$\Delta OBP \cong \Delta OAP$ **[½]**

$\therefore$ By $CPCT$, $BP = AP$ **[½]**

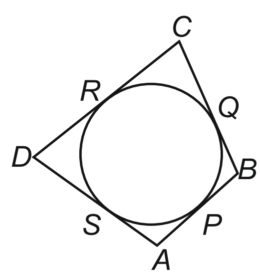
In ΔABP $BP = AP$

Angles opposite to equal sides are equal

$\therefore \; \angle BAP = \angle ABP$ **[½]**

Hence proved.

27.



$ABCD$ is the Quadrilateral

Circle touches the sides at P, Q, R, S

For the circle AS & AP are tangents

$\therefore \; AS = AP$ ...(i)

Similarly,

$BP = BQ$ ...(ii) **[½]**

$CQ = CR$ ...(iii)

$RD = DS$ ...(iv) **[½]**

Now, $AB + CD = AP + PB + CR + RD$ ...(v)

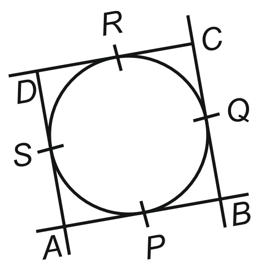
and $BC + AD = BQ + QC + DS + AS$ ...(vi) **[½]**

$BC + AD = BP + CR + RD + AP$ using (i), (i), (iii), (iv)

$\therefore$ $AB + CD = BC + AD$ [Using (v)]

Hence proved **[½]**

28. $\because$ Tangents from external point are equal in length.



$\therefore$ $AP = AS$ ...(1)

$BP = BQ$ ...(2)

$CR = CQ$ ...(3)

$DR = DS$ ...(4)

Adding equations (1 + 2 + 3 + 4)

$AP + BP + CR + DR = AS + BQ + CQ + DS$ **[1]**

$AB + CD = AD + BC$

$6 + 8 = AD + 9$

$AD = 14 - 9 = 5$ cm **[1]**

29. Join OQ.

$\angle OPQ = \angle OQP$ $\{OP = OQ\}$

$\Rightarrow$ $\angle OPQ + \angle OQP + \angle POQ = 180°$ **[½]**

{Angle sum property}

$\Rightarrow$ $2\angle OPQ = 180° - \angle POQ$ ...(i)

Also, $\angle PTQ + \angle POQ = 180°$

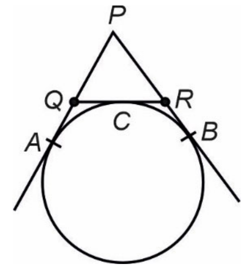
$\Rightarrow$ $\angle PTQ = 180° - \angle POQ$ ...(ii) **[½]**

From (i) and (ii), **[½]**

$\angle PTQ = 2\angle OPQ$ **[½]**

Hence Proved.

30. (a) $PA = PQ + QA$



$= PQ + QC$ …(i) [$\because$ $QA = QC$] **[½]**

and $PB = PR + BR$ **[½]**

$= PR + CR$ …(ii) [$\because$ $BR = CR$]

Adding (i) and (ii), we get

$PA + PB = PQ + QC + CR + PR$

$\Rightarrow$ $2PA = PQ + QR + PR$ **[½]**

[$\because$ $PA = PB$]

$$\Rightarrow \quad PA = \frac{\text{Perimeter of } \Delta PQR}{2}$$

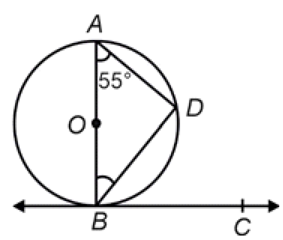
$$= \frac{20}{2}$$ **[½]**

$= 10$ cm

OR

(b) $\angle ADB = 90°$ [Angle in semi-circle] **[½]**

$\angle ABD = 90° - \angle BAD$ [Angle sum property of ΔABD]



$= 90° - 55°$

$= 35°$ **[½]**

Now, $\angle DBC = 90° - \angle ABD$ **[½]**

[$\because$ $AB \perp BC$]

$= 90° - 35°$

$= 55°$ **[½]**

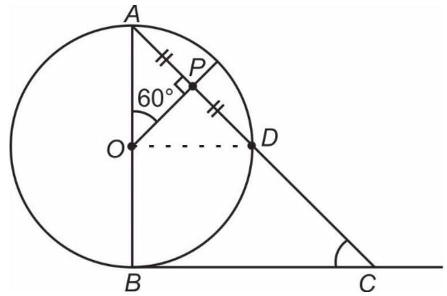
31. (a) In ΔAOP,

$\angle OPA = 90°$, as OP bisects chord AD

$\therefore$ $\angle OAP = 180° - (90° + 60°)$

$= 180° - 150°$

$\angle OAP = 30°$ **[1]**



In ΔABC,

$\angle ABC = 90°$

[$\because$ The tangent to a circle is perpendicular to the radius through the point of contact]

$\angle ABC + \angle BAC + \angle BCA = 180°$

$\Rightarrow$ $90° + 30° + \angle BCA = 180°$

[$\because$ $\angle BAC = \angle OAP = 30°$]

$\Rightarrow$ $\angle BCA = 180° - 120°$

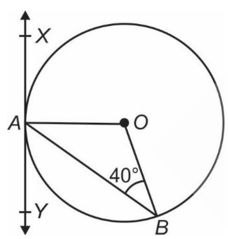
$\Rightarrow$ $m\angle C = 60°$ **[1]**

OR

(b) In ΔOAB,

$OA = OB$ [radius of circle]

$\therefore$ $\angle OAB = \angle OBA = 40°$ [$\because$ $OA = OB$]



Since, XAY is tangent to the circle.

$\therefore$ $\angle OAY = 90°$

[$\because$ The tangent to a circle is perpendicular to the radius through the point of contact]

$\therefore$ $\angle BAY + \angle OAB = 90°$

$\angle BAY = 90° - 40°$

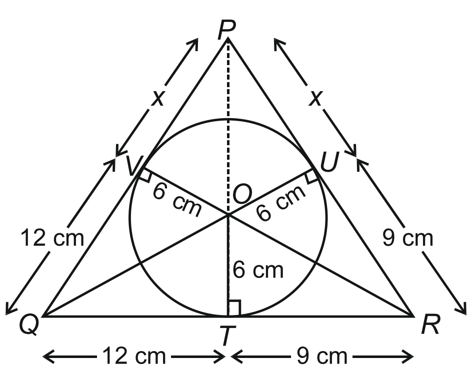
$\angle BAY = 50°$ **[1]**

Further in ΔABO,

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\angle AOB = 180° - 80° = 100°$ **[1]**

32.



$ar(\Delta PQR) = ar(\Delta POQ) + ar(\Delta QOR) + ar(\Delta POR)$

$$\Rightarrow \quad 189 = \frac{1}{2}\times OV\times PQ + \frac{1}{2}\times OT\times QR + \frac{1}{2}\times OU\times PR$$ **[½]**

$$189 = \frac{1}{2}\times 6(PQ+QR+PR) = 3(PQ+QR+PR)$$ **[½]**

($\because$ $OT = OV = OU = 6$ cm)

$\Rightarrow$ $189 = 3(x + 12 + 12 + 9 + 9 + x)$

[$\because$ $PV = PU = x$, $QT = 12$ cm and $RT = RU = 9$ cm as tangents from external point to a circle are equal] **[½]**

$\Rightarrow$ $63 = 24 + 18 + 2x$

$\Rightarrow$ $2x = 21$

$$\Rightarrow \quad x = \frac{21}{2} = PV = PU$$ **[½]**

$$\therefore \quad PQ = PV + QV = 12 + \frac{21}{2} = \frac{45}{2} \text{ cm}$$ **[½]**

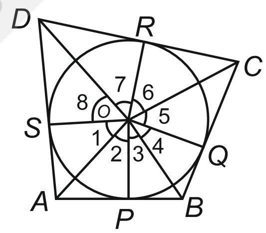
$$\text{and} \quad PR = PU + UR = 9 + \frac{21}{2} = \frac{39}{2} \text{ cm}$$ **[½]**

33. A circle with centre O touches the sides AB, BC, CD, and DA of a quadrilateral $ABCD$ at the points P, Q, R

and S respectively.

To Prove : $\angle AOB + \angle COD = 180°$

and $\angle AOD + \angle BOC = 180°$



CONSTRUCTION

Join OP, OQ, OR and OS.

Proof : Since the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore$ $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$ and $\angle 7 = \angle 8$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ **[½]**

[Sum of all the angles subtended at a point is 360°]

$\Rightarrow$ $2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$ and

$2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$ **[½]**

$\Rightarrow$ $(\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180°$ and

$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$ **[1]**

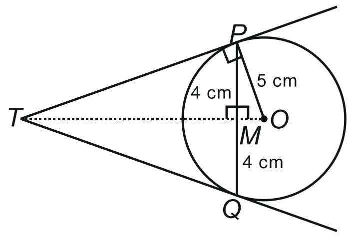
and $\angle 2 + \angle 3 = \angle AOB$, $\angle 6 + \angle 7 = \angle COD$

$\angle 1 + \angle 8 = \angle AOD$ and $\angle 4 + \angle 5 = \angle BOC$ **[½]**

$\Rightarrow$ $\angle AOB + \angle COD = 180°$ and $\angle AOD + \angle BOC = 180°$

Hence, proved **[½]**

34. Join *OT* which bisects *PQ* at *M* and perpendicular to *PQ*



In ΔOPM,

$OP^2 = PM^2 + OM^2$ [By Pythagoras Theorem] **[½]**

$\Rightarrow$ $(5)^2 = (4)^2 + OM^2$

$\Rightarrow$ $OM = 3$ cm **[½]**

In ΔOPT and ΔOPM,

$\angle MOP = \angle TOP$ [Common angles]

$\angle OMP = \angle OPT$ [Each 90°]

$\therefore$ $\Delta POT \sim \Delta MOP$ [By AA similarity] **[½]**

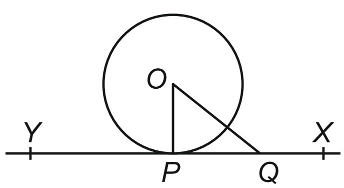
$\Rightarrow$ $\dfrac{TP}{MP} = \dfrac{OP}{OM}$ **[½]**

$\Rightarrow$ $TP = \dfrac{4 \times 5}{3}$ **[½]**

[$\because$ $OP = 5$ cm, $PM = 4$ cm, $MO = 3$ cm]

$\Rightarrow$ $TP = \dfrac{20}{3} = 6\dfrac{2}{3}$ cm **[½]**

35.



Given : A circle with centre *O* and a tangent *XY* to the circle at a point *P* **[½]**

To Prove : *OP* is perpendicular to *XY*.

Construction : Take a point *Q* on XY other than *P* and join *OQ*. **[½]**

Proof : Here the point *Q* must lie outside the circle as if it lies inside the tangent *XY* will become secant to the circle. **[½]**

Therefore, *OQ* is longer than the radius *OP* of the circle, That is, *OQ* > *OP*. **[1]**

This happens for every point on the line *XY* except the point *P*. **[½]**

So *OP* is the shortest of all the distances of the point *O* to the points on *XY*. **[½]**

And hence *OP* is perpendicular to *XY*. **[½]**

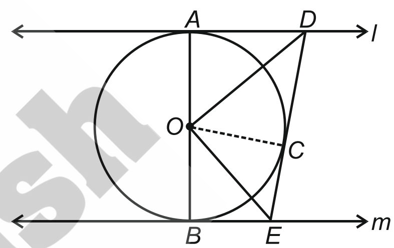
Hence, proved.

36. Given : *l* and m are two parallel tangents to the circle with centre *O* touching the circle at *A* and *B* respectively. *DE* is a tangent at the point *C*, which intersects *l* at *D* and *m* at *E*.

To prove: $\angle DOE = 90°$

Construction: Join *OC*.

Proof:



In ΔODA and ΔODC,

$OA = OC$ [Radii of the same circle]

$AD = DC$

(Length of tangents drawn from an external point to a circle are equal]

$DO = OD$ [Common side]

$\Delta ODA \cong \angle ODC$ [SSS congruence criterion] **[1]**

$\therefore$ $\angle DOA = \angle COD$ ...(i) **[½]**

Similarly, $\Delta OEB \cong \Delta OEC$ **[½]**

$\therefore$ $\angle EOB = \angle COE$ ...(ii) **[½]**

Now, *AOB* is a diameter of the circle. Hence, it is a straight line.

$\angle DOA + \angle COD + \angle COE + \angle EOB = 180°$ **[½]**

From (i) and (ii), we have:

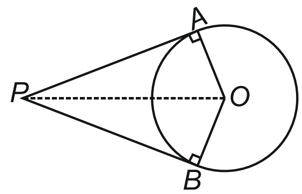
$2\angle COD + 2\angle COE = 180°$ **[½]**

$\Rightarrow$ $\angle COD + \angle COE = 90°$

$\Rightarrow$ $\angle DOE = 90°$

Hence, proved. **[½]**

37. Let AP and BP be the two tangents to the circle with centre O.



To Prove : $AP = BP$

Proof : **[½]**

In ΔAOP and ΔBOP,

$OA = OB$ [radii of the same circle]

$\angle OAP = \angle OBP = 90°$ **[1]**

[since tangent at any point of a circle is perpendicular to the radius through the point of contact]

$OP = OP$ [common]

$\therefore \Delta AOP \cong \Delta BOP$ **[1]**

[by R.H.S. congruence criterion]

$\therefore AP = BP$ **[1]**

[corresponding parts of congruent triangles]

Hence, the length of the tangents drawn from an external point to a circle are equal. **[½]**

38. In the figure, C is the midpoint of the minor arc PQ, O is the centre of the circle and

AB is tangent to the circle through point C.

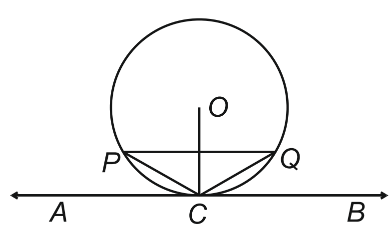
We have to show the tangent drawn at the midpoint of the arc PQ of a circle is parallel to the chord joining the end points of the arc PQ.

We will show $PQ \parallel AB$. **[½]**

It is given that C is the midpoint point of the arc PQ.

So, arc PC = arc CQ. **[½]**

$\Rightarrow PC = CQ$



This shows that ΔPQC is an isosceles triangle. **[½]**

Thus, the perpendicular bisector of the side PQ of ΔPQC passes through vertex C.

The perpendicular bisector of a chord passes through the centre of the circle. **[½]**

So the perpendicular bisector of PQ passes through the centre O of the circle. **[½]**

Thus perpendicular bisector of PQ passes through the points O and C.

$\Rightarrow PQ \perp OC$ **[½]**

AB is the tangent to the circle through the point C on the circle.

$\Rightarrow AB \perp OC$ **[½]**

The chord PQ and the tangent AB of the circle are perpendicular to the same line OC.

$\therefore PQ \parallel AB$. **[½]**

39. $AO' = O'X = XO = OC$ **[½]**

[Since the two circles are equal.]

So, $OA = AO' + O'X + XO$

$\therefore OA = 3O'A$ **[1]**

In $\angle AO'D$ and ΔAOC,

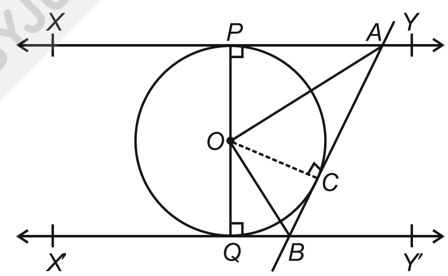
$\angle DAO' = \angle CAO$ [Common angle]

$\angle ADO' = \angle ACO$ [Both measure 90°] **[½]**

$\Delta ADO' \sim \Delta ACO$ [By AA test of similarity] **[1]**

$\frac{DO'}{CO} = \frac{O'A}{OA} = \frac{O'A}{3O'A} = \frac{1}{3}$ **[1]**

40.



To prove : $\angle AOB = 90°$

In ΔAOC and ΔAOP,

$OA = OA$ [Common]

$OP = OC$ [radii] **[½]**

$\angle ACO = \angle APO$ [right angle]

$\therefore \Delta AOC \cong \Delta AOP$ (By RHS congruency) **[½]**

By $CPCT$, $\angle AOC = \angle AOP$ ...(i) **[½]**

Similarly In ΔBOC and ΔBOQ

$OC = OQ$ [radii]

$OB = OB$ [Common] **[½]**

and $\angle BCO = \angle BQO = 90°$

By RHS congruency, $\Delta BOC \cong \Delta BOQ$ **[½]**

By CPCT, $\angle BOC = \angle BOQ$ ...(ii) **[½]**

PQ is a straight line

$\therefore \quad \angle AOP + \angle AOC + \angle BOC + \angle BOQ = 180°$

From equations (i) and (ii), we have **[½]**

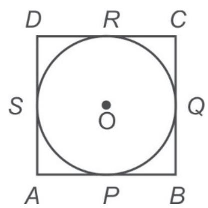
$2(\angle AOC + \angle BOC) = 180°$

$$\angle AOB = \frac{180°}{2}$$

$\therefore \quad \angle AOB = 90°$ **[½]**

41. **(a)** **Given :** A circle with centre O.

A parallelogram $ABCD$ touching the circle at Points P, Q, R and S.



To Prove: $ABCD$ is a rhombus

Proof: A rhombus is a parallelogram with all sides equal

In parallelogram $ABCD$

$AB = CD$ and $BC = AD$ **[1]**

We know that the lengths of tangents from an external point are equal

$\therefore \quad AP = AS$ …(i)

$BP = BQ$ …(ii)

$CQ = CR$ …(iii)

$DR = DS$ …(iv) **[1]**

Adding (i), (ii), (iii) and (iv), we get

$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow AB + (CR + DR) = AS + BQ + CQ + DS$

$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC$

$\Rightarrow CD + CD = BC + BC$ **[1]**

$[\because AB = CD$ and $AD = BC]$

$\Rightarrow CD = BC$

$\therefore AB = CD = BC = AD$

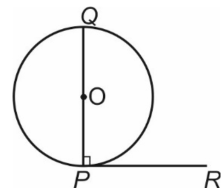
All sides are equal

$\Rightarrow$ Hence, $ABCD$ is a rhombus **[1]**

OR

(b) Let, O is the centre of the given circle. A segment PR has been drawn touching the circle at point P. **[½]**

Draw $QP \perp RP$ at point P, such that point Q lies on the circle. **[½]**



$\angle OPR = 90°$ [Radius $\perp$ Tangent] **[½]**

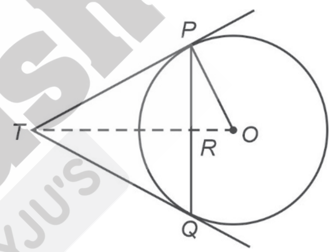
Also, $\angle QPR = 90°$ [given] **[½]**

$\therefore \angle OPR = \angle QPR$ **[½]**

Now, the above case is possible only when centre O lies on the line QP. **[1]**

Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle. **[½]**

42.



In ΔORP and ΔOPT,

$\angle ORP = \angle OPT$ [Each 90°]

$\angle POR = \angle POT$ [Common]

$\therefore \quad \Delta ORP \sim \Delta OPT$ [By AA similarity] **[1]**

$\therefore \quad \dfrac{OR}{OP} = \dfrac{PR}{PT}$ …(i) **[1]**

In ΔPOR,

$OP^2 = OR^2 + PR^2$

[By Pythagoras theorem]

$\therefore \quad (5)^2 = OR^2 + (4)^2 \quad \left[\because PR = \dfrac{PQ}{2}\right]$

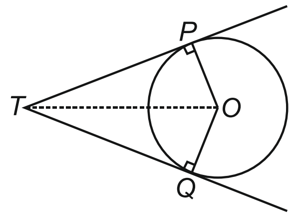
$\Rightarrow \quad OR = 3$ cm **[1]**

From (i),

$$\frac{3}{5} = \frac{4}{PT}$$

$\therefore \quad PT = \dfrac{20}{3}$ cm **[1]**

43.



Given : PT and TQ are two tangents drawn from an external point T to the circle $C(O, r)$.

To prove : $PT = TQ$

Construction: Join OT. **[½]**

Proof : We know that a tangent to circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OPT = \angle OQT = 90°$

In ΔOPT and ΔOQT,

$OT = OT$ [Common] **[½]**

$OP = OQ$ [Radius of the circle] **[½]**

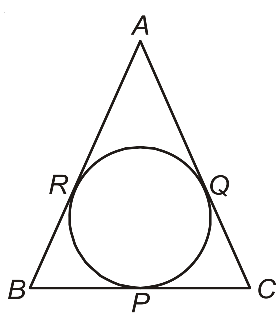
$\angle OPT = \angle OQT = 90°$

$\therefore \quad \Delta OPT \cong \Delta OQT$ [RHS congruence criterion] **[½]**

$\Rightarrow \quad PT = TQ$ [CPCT] **[½]**

$\therefore$ The lengths of the tangents drawn from an external point to a circle are equal. **[½]**

Now,



We know that the tangents drawn from an exterior point to a circle are equal in length.

$\therefore \quad AR = AQ$ (Tangents from A) ...(i) **[½]**

$BP = BR$ (Tangents from B) ...(ii)

$CQ = CP$ (Tangents from C) ...(iii) **[½]**

Now, the given triangle is isosceles ($\because$ $AB = AC$)

Subtract AR from both sides, we get

$AB - AR = AC - AR$ **[½]**

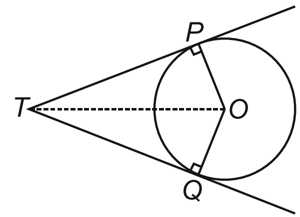
$\Rightarrow AB - AR = AC - AQ$ [Using (ii)] **[½]**

$BR = CQ$

$\Rightarrow \quad BP = CP$ (Using (ii), (iii)] **[½]**

So $BP = CP$, shows that BC is bisected at the point of contact. **[½]**

44. PT and TQ are two tangent drawn from an external pant T to the circle $C(O, r)$



To prove : $PT = TQ$

Construction : Join OT **[½]**

Proof: We know that, a tangent to circle is perpendicular to the radius through the point of contact **[½]**

$\therefore \quad \angle OPT = \angle OQT = 90°$ **[½]**

In ΔOPT and ΔOQT,

$OT = OT$ [Common]

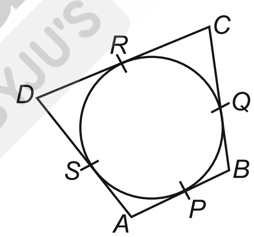
$OP = OQ$ [Radius of the circle] **[½]**

$\angle OPT = OQT = 90°$

$\therefore \Delta OPT \cong \Delta OQT$ [RHS congruence criterion] **[½]**

$\Rightarrow \quad PT = TQ$ [CPCT]

$\therefore$ The lengths of the tangents drawn from an external point to a circle are equal. **[½]**



Let AB touches the circle at P. BC touches the circle at Q. DC touches the circle at $R.AD$. touches the circle at S. **[½]**

Then, $PB = QB$ (Length of the tangents drawn from the external point are always equal)

Similarly, $QC = RC'$ **[½]**

$AP = AS$

$DS = DR$ **[½]**

Now,

$AB + CD$

$= AP + PB + DR + RC$ **[½]**

$= AS + QB + DS + CQ$ **[½]**

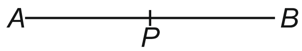
$= AS + DS + QB + CQ$

$= AD + BC$

Hence, Proved **[½]**

11 : Constructions

1. Given a line segment $AB = 7$ cm

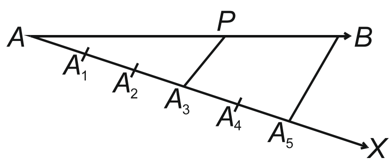


Given

$$\frac{AP}{AB} = \frac{3}{5} \Rightarrow \frac{AP}{AP+PB} = \frac{3}{5} \Rightarrow 5AP = 3AP + 3PB$$

$\Rightarrow$ $2AP = 3PB$

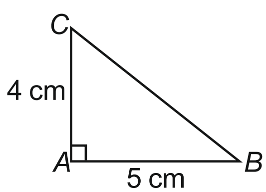
$\Rightarrow$ $\frac{AP}{PB} = \frac{3}{2}$ **[1]**



[1]

$\therefore$ The desired point is P which divides AB in 3 : 2.

2.



Steps :

1) Draw a line segment AB = 5 cm, Draw a ray SA making 90° with it.

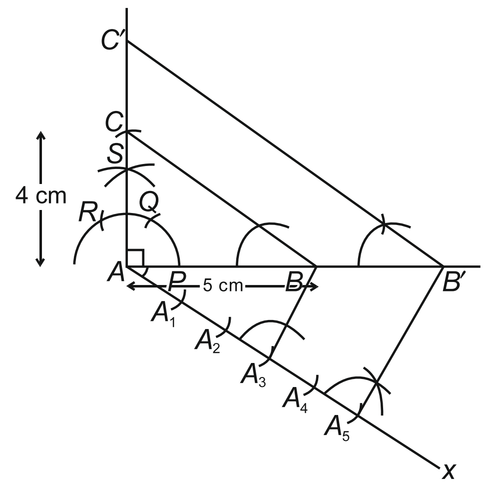
2) Draw an arc with radius 4 cm to cut ray SA at C. Join BC to form ΔABC.

3) Draw a ray AX making an acute angle with AB, opposite to vertex C.

4) Locate 5 points (as 5 is greater in 5 and 3), A_1, A_2, A_3, A_4, A_5, on line segment AX such that $AA = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$

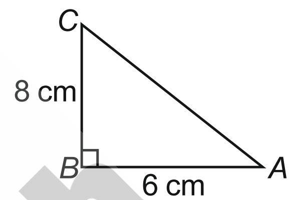
5) Join A_3B. Draw a line through A_5 parallel to A_3B intersecting line segment AB at B'.

6) Through B', draw a line parallel to BC intersecting extended line segment AC at C'. $\Delta AB'C'$ is the required triangle. **[1]**



[2]

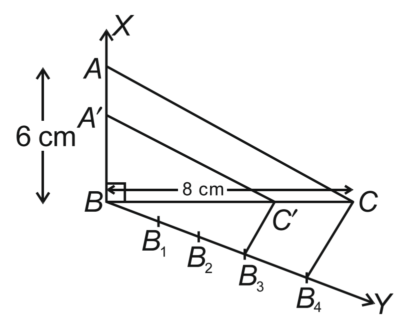
3.



Given ΔABC which is a right angled triangle $\angle B = 90°$

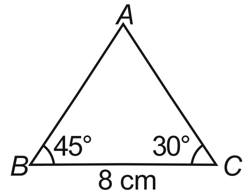
Steps :

1. Draw line segment BC = 8 cm, draw a ray BX making an angle 90° with BC
2. Draw an arc with radius 6 cm from B so that it cuts BX at A
3. Now join AC to form ΔABC



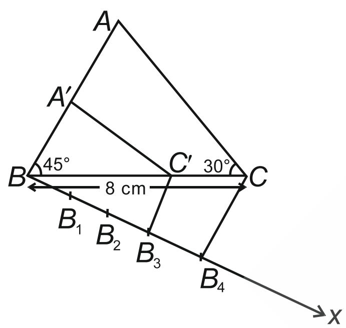
[2]

4. Draw a ray BY by making an acute angle with BC, opposite to vertex A
5. Locate 4 points B_1, B_2, B_3, B_4, on BY such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
6. Join B_4C and now draw a line from B_3 parallel to B_4C so that it cuts BC at C'
7. From C' draw a line parallel to AC and cuts AB at A'
8. $\Delta A'BC'$ is the required triangle **[1]**

4. 

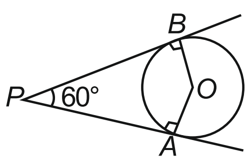
Steps :

1) Draw a ΔABC with BC = 8 cm, $\angle B$ = 45° & $\angle C$ = 30°
2) Draw a ray BX making acute angle with BC on the opposite side of vertex A
3) Mark four points B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
4) Join B_4C and draw a line parallel to B_4C from B_3 such that it cuts BC at C'.
5) Form C' draw another line parallel to AC such that it cuts AC at A'. **[1]**
6) $\Delta A'BC'$ is the required triangle.



[2]

5. Pair of a circle with radius = 3 cm inclined to each other with angle 60°



If $\angle APB$ = 60°

[As $AOBP$ is a cyclic quadrilateral]

Then $\angle AOB$ = 180 – 60°

= 120° **[½]**

Tangents can be constructed in the following manner:

Step 1

Draw a circle of radius 3 cm with center O.

Step 2

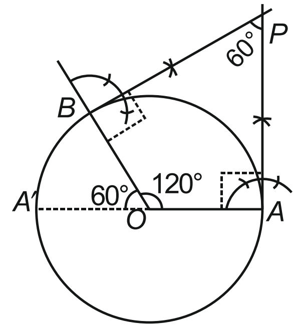
Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

Step 3

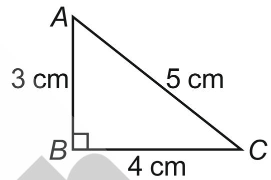
Draw a radius OB, making an angle of 120° with OA.

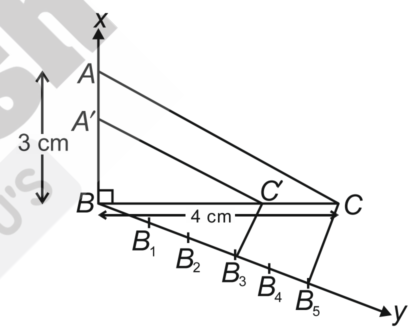
Step 4

Draw a perpendicular to OB at point B. Let both the perpendicular intersect at point P. PA and PB are the required tangents at an angle of 60°. **[1]**



[1½]

6. 



[2]

Steps :

1) Draw BC = 4 cm
2) Draw a ray BX such that $\angle XBY$ = 90°
3) Take compass with radius 3 cm and draw an arc from B cutting BX at A
4) Join A and C to from ΔABC
5) Draw a ray BY opposite side of A such that $\angle CBY$ is acute angle
6) Along BY mark 5 equidistant points B_1, B_2, B_3, B_4, B_5 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$
7) Join B_5 to C and draw a line parallel to B_5C from B_3 such that it cuts BC at C'
8) From C' draw a line parallel to AC such that it cuts AB at A' thus $\Delta A'BC'$ is the required triangle **[1]**

$$\therefore \quad \frac{A'B}{AB} = \frac{A'B}{AC} = \frac{BC'}{BC} = \frac{5}{3}$$

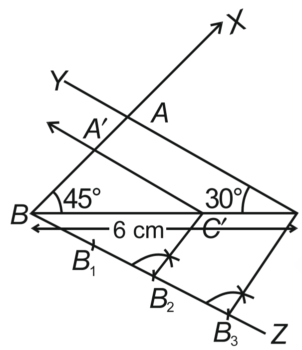
7. It is given that $\angle A = 105°$, $\angle C = 30°$.

Using angle sum property of triangle, we get, $\angle B = 45°$

The steps of construction are as follows:

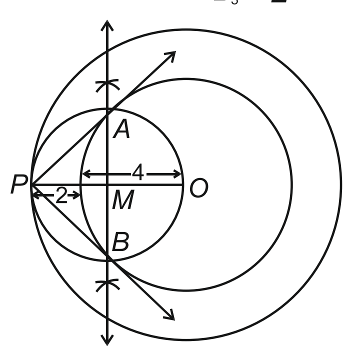
1. Draw a line segment $BC = 6$ cm.
2. At B, draw a ray BX making an angle of 45° with BC.
3. At C, draw a ray CY making an angle of 30° with BC. Let the two rays meet at point A.
4. Below BC, make an acute angle $\angle CBZ$.
5. Along BZ mark three points B_1, B_2, B_3 such that $BB_1 = B_1B_2 = B_2B_3$.
6. Join B_3C.
7. From B_2, draw $B_2C' \parallel B_3C$.
8. From C' draw $C'A' \parallel CA$, meeting BA at the point A′. **[1]**

Then $A'BC'$ is the required triangle.



[2]

8.



[2]

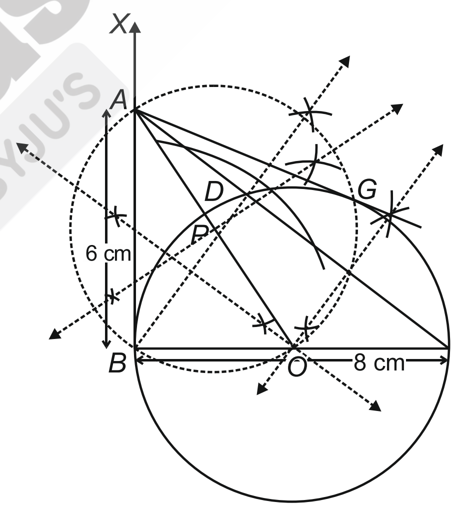
Steps of construction :

1. Draw two concentric circle with centre O and radii 4 cm and 6 cm. Take a point P on the outer circle and then join OP.
2. Draw the perpendicular bisector of OP. Let the bisector intersects OP at M.
3. With M as the centre and OM as the radius, draw a circle. Let it intersect the inner circle at A and B.
4. Join PA and PB. Therefore, PA and PB are the required tangents. **[1]**

9. Follow the given steps to construct the figure.

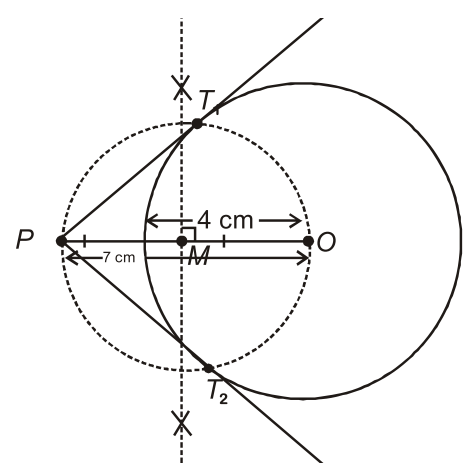
1. Draw a line BC of 8 cm length.
2. Draw BX perpendicular to BC.
3. Mark an arc at the distance of 6 cm on BX. Mark it as A.
4. Join A and C to get ΔABC.
5. With B as the centre, draw an arc on AC.
6. Draw the bisector of this arc and join it with B. Thus, BD is perpendicular to AC.
7. Now, draw the perpendicular bisector of BD and CD. Take the point of intersection of both perpendicular bisector as O.
8. With O as the centre and OB as the radius, draw a circle passing through points B, C and D.
9. Join A and O and bisect it Let P be the midpoint of AO.
10. Taking P as the centre and PO as its radius, draw a circle which will intersect the circle at point B and G. Join A and G.

Here, AB and AG are the required tangents to the circle from A. **[1]**



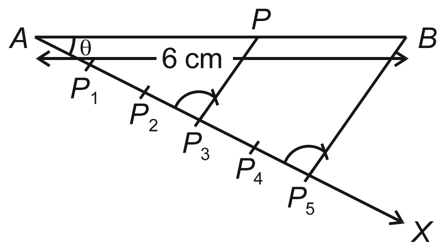
[2]

10. **[3]**



PT_1 and PT_2 are required tangents.

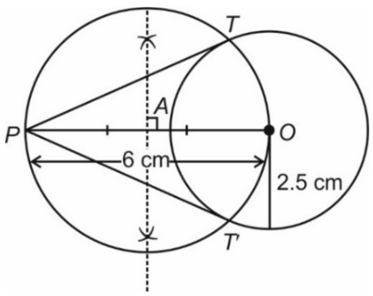
OR



[3]

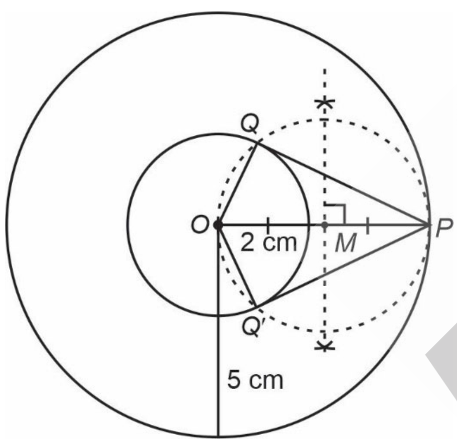
Required $AP : PB = 3 : 2$

11.



$\therefore$ PT and PT' are the required tangents. **[3]**

12.



$\therefore$ PQ and PQ' are the required tangents. **[3]**

13. 1. Construct the ΔABC as per given measurements.

2. In the half plane of $\overline{AB}$ which does not contain C, draw. $\overrightarrow{AX}$ such that $\angle BAX$ is an acute angle.

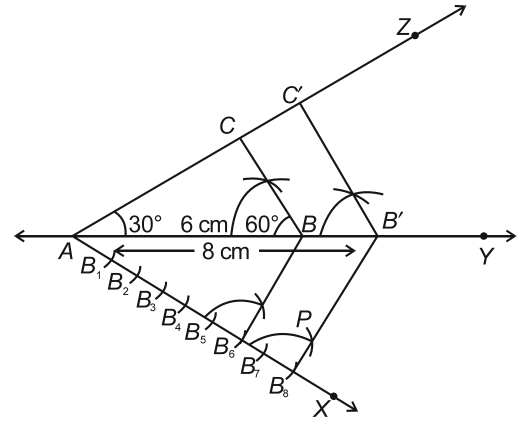
3. Along AX mark 8 equidistant points B_1, B_2 ..., B_8 such that $B_1B_2 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7 = B_7B_8$

4. Draw $\overline{B_6B}$.

5. Through B_8 draw a ray B_8B' parallel to $\overline{B_6B}$. to intersect $\overline{AY}$ at B'.

6. Through B' draw a ray $B'C'$ parallel to $\overline{BC}$ to intersect $\overline{AZ}$ at C'.

Thus, $\Delta AB'C'$ is the required triangle. **[1½]**



[2½]

14. **Steps :**

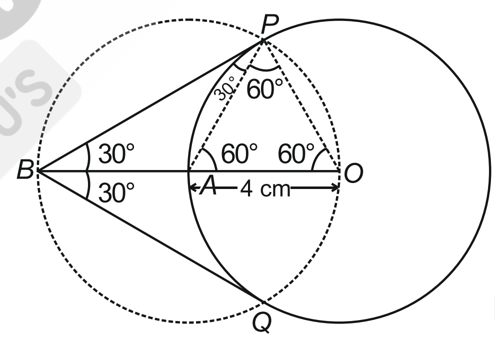
(i) Take a point O on the plane of the paper and draw a circle of radius OA = 4 cm.

(ii) Produce OA to B such that $OA = AB$ = 4 cm.

(iii) Draw a circle with centre at A and radius AB.

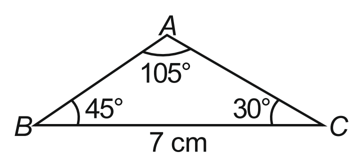
(iv) Suppose it cuts the circle drawn in step (i) at P and Q.

(v) Join BP and BQ to get the required tangents. **[2]**



[2]

15.



In the ΔABC, $\angle A + \angle B + \angle C = 180°$

$\therefore$ $\angle C = 30°$

Steps :

1. Draw $\overline{BC} = 7$ cm with help of a ruler

2. Take a protractor measure angle 45° from point B and draw a ray $\overrightarrow{BX}$

3. From point C, make angle 30° with help of protractor such that $\angle BCY = 30°$

4. Now both $\overrightarrow{BX}$ and $\overrightarrow{CY}$ intersect at a point A

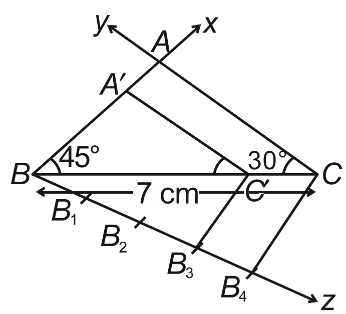
5. Draw a ray BZ making an acute angle with BC

6. Along the ray BZ mark 4 points B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

7. Now join B_4 to C and draw a line parallel to B_4C from B_3 intersecting the line BC at C'

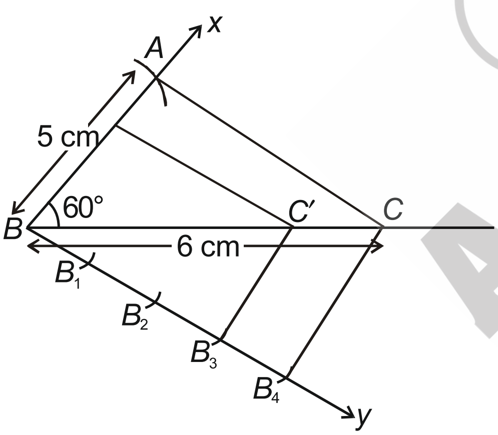
8. Draw a line through C' parallel to CA which intersects BA at A' **[1½]**

$A'BC'$ is the required triangle.



[2½]

16. **[2½]**



Steps :

(i) Draw a line segment BC = 6 cm, draw a ray BX making 60° with BC.

(ii) Draw an arc with radius 5 cm from B so that it cuts BX at A.

(iii) Now join AC to form ΔABC.

(iv) Draw a ray BY making an acute angle with BC opposite to vertex A.

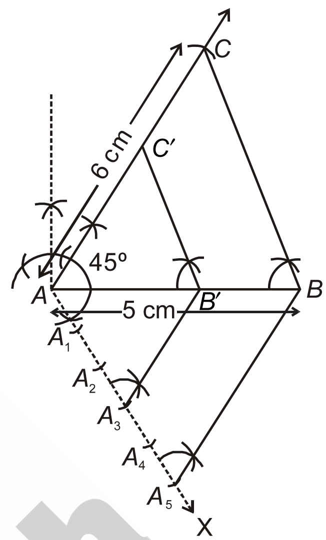
(v) Locate 4 points B_1, B_2, B_3, B_4 on BY such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(vi) Join B_4C and now draw a line from B_3 parallel to B_4C so that it cuts BC at C'.

(vii) From C' draw a line parallel to AC and cuts AB at A'.

(viii) $\Delta A'BC'$ is the required triangle. **[1½]**

17.



[2½]

Steps :

(i) Construct ΔABC such that AB = 5 cm, $\angle CAB$ = 45° and CA = 6 cm.

(ii) Draw any ray AX making an acute angle with AB on the side opposite to the vertex C.

(iii) Mark points A_1, A_2, A_3, A_4, A_5 on AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

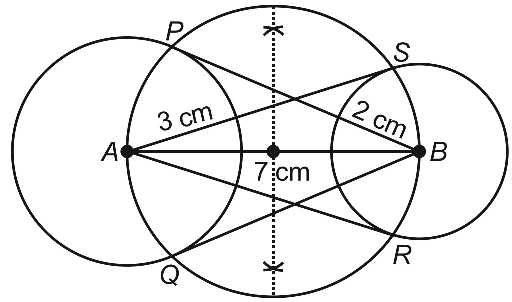
(iv) Join A_5B.

(v) Through A_3, draw a line parallel to A_5B intersecting with AB at B'.

(vi) Through B', draw a line parallel to BC intersecting with AC at C'.

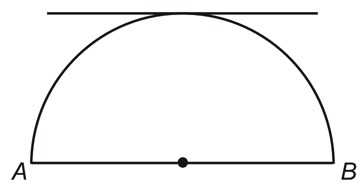
Now, $\Delta AB'C'$ is the required triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC. **[1½]**

18.



[4]

12 : Areas Related to Circles

1. 

Given diameter of semicircular protractor (AB) = 14 cm

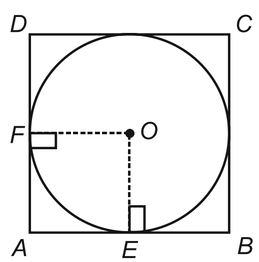
Perimeter of a semicircle $= \pi\left(\frac{d}{2}\right) + d$ **[½]**

$\therefore$ Perimeter of protractor $= \pi\left(\frac{14}{2}\right) + 14$

$= \frac{22}{7} \times \frac{14}{2} + 14$

$= 36$ cm **[½]**

2. Answer (A)



Given $OE = OF = a$

Side of the square circumscribing the circle = 2a **[½]**

- $\therefore$ Perimeter of square = 4 × 2a = 8a units. **[½]**

3. Answer (B)

Diameters of two circles are given as 10 cm and 24 cm.

Radius of one circle = r_1 = 5 cm

Radius of one circle = r_2 = 12 cm

According to the given information,

Area of the larger circle $= \pi(r_1)^2 + \pi(r_2)^2$ **[½]**

$= \pi(5)^2 + \pi(12)^2$

$= \pi(25 + 144)$

$= 169\pi$

$= \pi(13)^2$

$\therefore$ Radius of larger circle = 13 cm

Hence, the diameter of larger circle = 26 cm **[½]**

4. Answer (B)

Let r be the radius of the circle.

From the given information, we have

$2\pi r - r = 37$

$\Rightarrow r(2\pi) - 1 = 37$ cm

$\Rightarrow r\left(2\times\frac{22}{7} - 1\right) = 37$ cm **[½]**

$\Rightarrow r \times \frac{37}{7} = 37$ cm

$\Rightarrow r = 7$ cm

$\therefore$ Circumference of the circle

$= 2\pi r = 2\times\frac{22}{7}\times 7 \text{ cm} = 44 \text{ cm}$ **[½]**

5. Let radius of two circles be r_1 and r_2

$\therefore \frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{4}$

$\therefore \frac{r_1}{r_2} = \frac{3}{2}$ **[½]**

Now ratio of circumferences is $\frac{2\pi r_1}{2\pi r_2}$

$= \frac{r_1}{r_2} = \frac{3}{2}$ **[½]**

6. Answer (c)

$\frac{2\pi r \times \theta}{360°} = 22$

$\frac{2\times 22}{7} \times \frac{21\times\theta}{360°} = 22$

$\therefore \theta = 60°$ **[1]**

7. Answer (c)

$OC = \frac{AB}{2} = 14$ cm

Radius of inner-circle $= \frac{14}{2} = 7$ cm **[1]**

8. Answer (d)

Perimeter of the sector

$= 2\pi R \times \left(\frac{\theta}{360°}\right) + 2R$

$= \frac{45°}{360°} \times 2\pi R + 2R$

$= 39$ cm **[1]**

9. Answer (a)

Area of shaded region

$= \pi\left(R^2 - r^2\right)$

$= \frac{22}{7}\left(14^2 - 7^2\right)$

$= \frac{22}{7}(21)(7)$

$= 462 \text{ cm}^2$ **[1]**

10. Answer (c)

Area of quadrant = $\frac{\pi r^2}{4}$

$= \frac{22\times 28\times 28}{7\times 4}$ $[\because\ 2\pi r = 176 \text{ m}]$

$= 616 \text{ m}^2$ **[1]**

11. Answer (c)

Angle made by minute hand of a clock in 1 minute = 6°

Angle made by minute hand of the clock between 10:10 am to 10:25 am

(*i.e.* 15 minutes) = 15 × 6° = 90°

Distance covered (l) = $\frac{90^\circ}{180^\circ}\times\frac{22}{7}\times 84$

$\left[\because\ l = \frac{\theta}{180^\circ}\times\pi r\right]$

= 132 cm **[1]**

12. Answer (b)

Distance covered by a wheel in 1 revolution

$= 2\times\frac{22}{7}\times\frac{42}{2}$

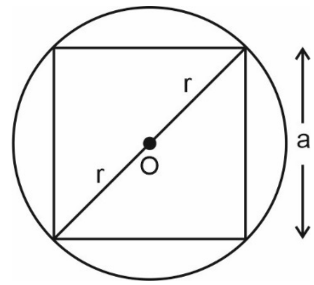
= 132 cm

Let number of revolutions taken by the car wheel to cover 132 km be N.

$N = \frac{132\times 1000\times 100}{132}$ $[\because 1 \text{ km} = 10^5 \text{ cm}]$

$= 10^5$ **[1]**

13. Answer (c)



$\pi r^2 = \frac{1408}{7}$

$\Rightarrow$ $r = 8$ cm

$\Rightarrow$ Side of square = $8\sqrt{2}$ cm

$\therefore$ Area of square = 128 cm^2 **[1]**

14. Answer (d)

Perimeter of a circle $= \frac{1}{2}$ perimeter of a square

$2\pi r = \frac{1}{2}\times 4a$

$\frac{r}{a} = \frac{1}{\pi}$ ...(i)

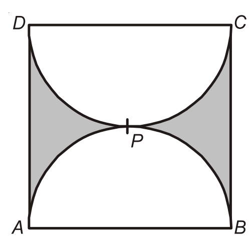
Now, $\frac{\text{area of a circle}}{\text{area of a square}} = \frac{\pi r^2}{a^2}$

$= \pi\left(\frac{r}{a}\right)^2$

$= \pi\left(\frac{1}{\pi}\right)^2$ [from eqn. (i)]

$= \frac{1}{\pi}$ **[1]**

15.



Given a square $ABCD$ with side = 14 cm

$AB = CD = BC = AD = 14$ cm

Semicircles APB and CPD with diameter = 14 cm

Perimeter of shaded region = $AD + BC$ + arc(CPD) + arc(APB) **[½]**

Length of arcCPD are $= \frac{180^\circ}{360^\circ}\times 2\times\frac{22}{7}\times\frac{14}{2} = 22$

[½]

Length of arc APB = CPD = 22 cm [½]

Perimeter of Shaded region = 14 + 14 + 22 + 22

= 72 cm [½]

16. Given, $OABC$ is a square of side 7 cm

i.e. $OA = AB = BC = OC = 7$ cm

$\therefore$ Area of square $OABC$ = (side)^2 = 7^2 = 49 sq.cm [½]

Given, $OAPC$ is a quadrant of a circle with centre O.

$\therefore$ Radius of the sector = $OA = OC = 7$ cm.

Sector angle = 90° [½]

$\therefore$ Area of quadrant $OAPC = \frac{90°}{360°}\times\pi r^2$

$= \frac{1}{4}\times\frac{22}{7}\times(7)^2$

$= \frac{77}{2}$ sq.cm

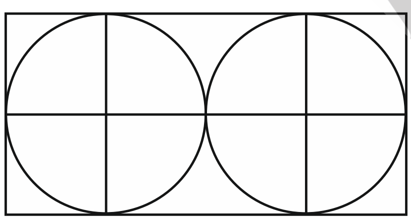
= 38.5 sq. cm [½]

$\therefore$ Area of shaded region = Area of Square ($OABC$)-Area of quadrant ($OAPC$)

= (49 – 38.5)sq. cm = 10.5 sq. cm [½]

17. Dimension of the rectangular card board = 14 cm × 7 cm.

Since, two circular pieces of equal radii and maximum area touching each other are cut from the rectangular card board, therefore, the diameter of each of each circular piece is $\frac{14}{2} = 7$ cm



Radius of each circular piece $= \frac{7}{2}$ cm

$\therefore$ Sum of area of two circular pieces

$$= 2\times\pi\left(\frac{7}{2}\right)^2 = 2\times\frac{22}{7}\times\frac{49}{4} = 77 \text{ cm}^2 \quad [1]$$

Area of the remaining card board

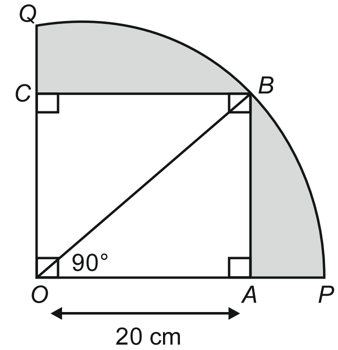
= Area of the card board - Area of two circular pieces

= 14 cm × 7 cm – 77 cm^2

= 98 cm^2 – 77 cm^2

= 21 cm^2 [1]

18. Let us join OB.



In ΔOAB : $OB^2 = OA^2 + AB^2 = (20)^2 + (20)^2$

$= 2 \times (20)^2$

$\Rightarrow \quad OB = 20\sqrt{2}$ cm

Radius of the circle, $r = 20\sqrt{2}$ cm [½]

Area of quadrant $OPBQ$

$= \frac{90°}{360°}\times\pi r^2$

$= \frac{90°}{360°}\times 3.14\times\left(20\sqrt{2}\right)^2$ cm^2

$= \frac{1}{4}\times 3.14\times 800$ cm^2

= 628 cm^2 [1]

Area of square $OABC$ = (Side)^2 = (20)^2 cm^2

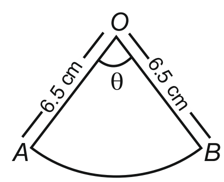
= 400 cm^2

$\therefore$ Area of the shaded region = Area of quadrant $OPBQ$ – Area of square $OABC$

= (628 – 400) cm^2

= 228 cm^2 [½]

19.



Perimeter of sector OAB = OA + OB + length of arc AB = $\left(6.5 + 6.5 + \frac{2\pi r\theta}{360°}\right)$cm

$31 = 13 + 2 \times \pi \times r\times\frac{\theta}{360°}$ [½]

$\frac{\pi r\theta}{360°} = 9$ cm [½]

Area of sector = $\frac{\pi r^2\theta}{360°}$

$= \frac{\pi r\theta}{360°}\times r = 9\times 6.5$ [½]

= 58.5 cm^2 [½]

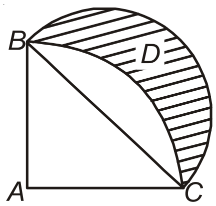
20. Length of arc = 22 cm

$\Rightarrow \dfrac{2\pi r\theta}{360°} = 22$ **[½]**

$\Rightarrow 2\times\dfrac{22}{7}\times r\times\dfrac{60°}{360°} = 22$ **[½]**

$\Rightarrow r = \dfrac{22\times7\times6}{2\times22}$ **[½]**

$\Rightarrow r$ = 21 cm **[½]**

21. 

Given $AC = AB$ = 14 cm

$BC = \sqrt{14^2+14^2} = 14\sqrt{2}$ cm

Area of shaded region = Area of semi-circle – (Area of quadrant $ABDC$ – Area of ΔABC)

$\therefore$ Area of $\Delta ABC = \dfrac{1}{2}\times14\times14 = 98 \text{ cm}^2$

Area of Quadrant $ABDC = \dfrac{1}{4}\times\dfrac{22}{7}(14)^2 = 154 \text{ cm}^2$

[1]

Area of segment BDC = ar(Quadrant $ABDC$) – ar(ΔABC)

= 154 – 98

= 56 cm^2 **[½]**

Area of semicircle with diameter BC

$= \dfrac{1}{2}\pi\left(\dfrac{BC}{2}\right)^2 = \dfrac{1}{2}\times\dfrac{22}{7}\times\dfrac{1}{4}\times14\sqrt{2}\times14\sqrt{2}$

= 154 cm^2 **[½]**

Area of shaded region = Area of semicircle of diameter BC – Area of segment BDC

= 154 – 56

= 98 cm^2 **[1]**

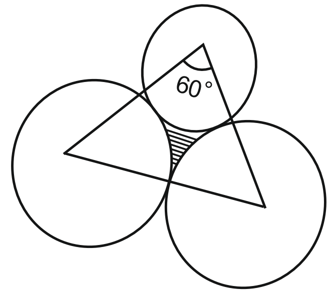
22. Let a be the side of equilateral triangle

$\dfrac{\sqrt{3}a^2}{4} = 49\sqrt{3};$

$a^2 = 49 \times 4;$

$a = 7 \times 2 = 14$ cm

Radius of circle = 14/2 = 7 cm **[1]**



Area of the first circle occupied by triangle = area of sector with angle 60°.

$= \dfrac{60°\pi r^2}{360°} = \dfrac{22}{7}\times\dfrac{1}{6}\times7\times7 = \dfrac{77}{3} \text{ cm}^3$ **[½]**

Area of all the 3 sectors = 77/3 × 3 = 77 cm^2

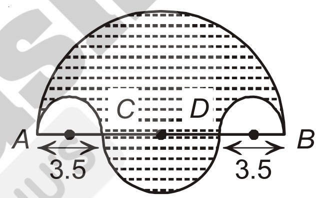
[½]

Area of triangle not included in the circle

= area of triangle- area of all the 3 sectors

$= 49\sqrt{3} - 77$ = 49(1.732) – 77

= 7.868 cm^2 **[1]**

23. 

Given AB = 14 cm and $AC = BD$ = 3.5 cm

$\Rightarrow DC$ = 7 cm **[1]**

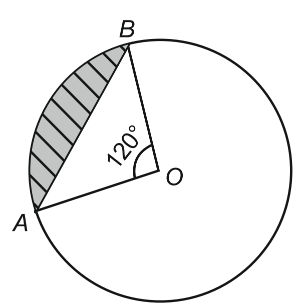
Area of shaded region = Area of semicircle AB + Area of semicircle CD –2 (Area of semicircle AC) **[1]**

$= \dfrac{\pi}{2}\left(\dfrac{14}{2}\right)^2 + \dfrac{\pi}{2}\left(\dfrac{7}{2}\right)^2 - 2\left(\dfrac{\pi}{2}\left(\dfrac{3.5}{2}\right)^2\right)$

$= \dfrac{\pi}{4}\left[\dfrac{196}{2} + \dfrac{49}{2} - \dfrac{49}{4}\right] = 86.625 \text{ cm}^2$ **[1]**

24. Area of minor segment

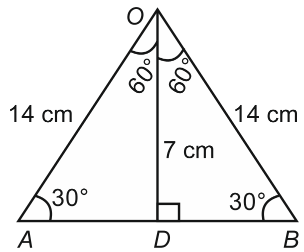
= Area of sector AOB – Area of ΔAOB



Given

$\angle AOB = 120°$

$OA = OB = 14$ cm



Area of sector $AOB = \dfrac{120°}{360°} \times \pi r^2$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times (14)^2 = \dfrac{616}{3}$ **[1]**

Draw $OD \perp AB$

In ΔODB,

$\angle O = 60°$ $\angle B = 30°$, $\angle D = 90°$

$OD = 7$ cm

$DB = 7\sqrt{3}$ cm

$\therefore$ Area of $\Delta AOB = \dfrac{1}{2} \times AB \times OD$

$= \dfrac{1}{2} \times 14\sqrt{3} \times 7$

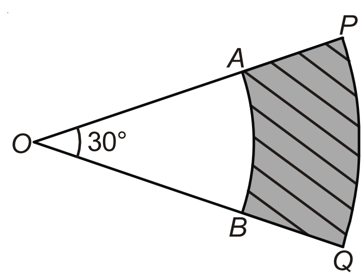
$= 49\sqrt{3}$ **[1]**

$= 84.77$ cm^2

Area of minor segment $= \dfrac{616}{3} - 84.77$ **[1]**

$= 120.56$ cm^2

25.



Area of the shaded region

= Area of sector POQ – Area of sector AOB

$= \left(\dfrac{\theta}{360}\pi R^2 - \dfrac{\theta}{360}\pi r^2\right)$ **[1]**

$= \dfrac{30}{360} \times \dfrac{22}{7} \times \left(7^2 - 3.5^2\right)$ **[1]**

$= \dfrac{77}{8}$ cm^2 **[1]**

26. The arc subtends an angle of 60° at the centre.

(i) $l = \dfrac{\theta}{360°} \times 2\pi r$ **[½]**

$= \dfrac{60°}{360°} \times 2 \times \dfrac{22}{7} \times 21$

$= 22$ cm **[1]**

(ii) Area of the sector $= \dfrac{\theta}{360°} \times \pi r^2$ **[½]**

$= \dfrac{60°}{360°} \times \dfrac{22}{7} \times 21 \times 21$

$= 231$ cm^2 **[1]**

27. AB and CD are the diameters of a circle with centre O.

$\therefore$ $OA = OB = OC = OD = 7$ cm (Radius of the circle) **[½]**

Area of the shaded region

= Area of the circle with diameter OB + (Area of the semi-circle $ACDA$ – Area of ΔACD) **[1]**

$= \pi\left(\dfrac{7}{2}\right)^2 + \left(\dfrac{1}{2} \times \pi \times 7^2 - \dfrac{1}{2} \times CD \times OA\right)$

$= \dfrac{22}{7} \times \dfrac{49}{4} + \dfrac{1}{2} \times \dfrac{22}{7} \times 49 - \dfrac{1}{2} \times 14 \times 7$ **[½]**

$= \dfrac{77}{2} + 77 - 49$

$= 66.5$ cm^2 **[1]**

28. Radius of Semicircle $PSR = \dfrac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$

[½]

Radius of Semicircle $RTQ = \dfrac{1}{2} \times 3 = 1.5$ cm **[½]**

Radius of semicircle $PAQ = \dfrac{1}{2} \times 7 \text{ cm} = 3.5$ cm **[½]**

Perimeter of the shaded region = Circumference of semicircle PSR + Circumference of semicircle RTQ + Circumference of semicircle PAQ **[½]**

$= \left[\dfrac{1}{2} \times 2\pi(5) + \dfrac{1}{2} \times 2\pi(1.5) + \dfrac{1}{2} \times 2\pi(3.5)\right]$ cm

$= \pi(5 + 1.5 + 3.5)$ cm

$= 3.14 \times 10$ cm

$= 31.4$ cm **[1]**

29. It is given that ABC is an equilateral triangle of side 12 cm.

Construction:

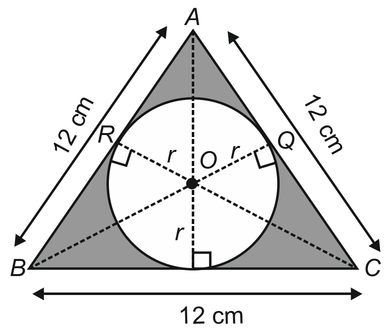
Join OA, OB and OC.

Draw.

$OP \perp BC$

$OQ \perp AC$

$OR \perp AB$ **[½]**



Let the radius of the circle be r cm.

Area of ΔAOB + Area of ΔBOC + Area of ΔAOC = Area of ΔABC **[½]**

$$\Rightarrow \frac{1}{2}\times AB\times OR+\frac{1}{2}\times BC\times OP+\frac{1}{2}\times AC\times OQ = \frac{\sqrt{3}}{4}\times(\text{side})^2$$

$$\Rightarrow \frac{1}{2}\times 12\times r+\frac{1}{2}\times 12\times r+\frac{1}{2}\times 12\times r=\frac{\sqrt{3}}{4}\times(12)^2$$

$$\Rightarrow 3\times\frac{1}{2}\times 12\times r=\frac{\sqrt{3}}{4}\times 12\times 12$$

$$\Rightarrow r=2\sqrt{3}=2\times 1.73=3.46$$ **[1]**

Therefore, the radius of the inscribed circle is 3.46 cm.

Now, area of the shaded region = Area of ΔABC – Area of the inscribed circle

$$=\left[\frac{\sqrt{3}}{4}\times(12)^2-\pi\left(2\sqrt{3}\right)^2\right]\text{cm}^2$$

$$=\left[36\sqrt{3}-12\pi\right]\text{cm}^2$$

$= [36 \times 1.73 – 12 \times 3.14]$ cm^2

$= [62.28 – 37.68]$ cm^2

$= 24.6$ cm^2 **[1]**

Therefore, the area of the shaded region is 24.6 cm^2.

30. Radius of the circle = 14 cm

Central Angle, $\theta = 60°$,

Area of the minor segment

$$=\frac{\theta}{360°}\times\pi r^2-\frac{\sqrt{3}}{4}r^2$$ **[1]**

$$=\frac{60°}{360°}\times\pi\,(14)^2-\frac{\sqrt{3}}{4}\times 14^2$$

$$=\frac{1}{6}\times\frac{22}{7}\times 14\times 14-\sqrt{3}\times(7)^2$$

$$=\frac{22\times 14}{3}-49\sqrt{3}$$

$$=\frac{22\times 14}{3}-\frac{147\sqrt{3}}{3}$$

$$=\frac{308-147\sqrt{3}}{3}\ \text{cm}^2$$ **[1]**

$\therefore$ Area of the major segment

$$=\pi(14)^2-\left(\frac{308-147\sqrt{3}}{3}\right)\text{cm}^2$$

$$=616-\frac{1}{3}\left[308-147\sqrt{3}\right]$$

$$=\left(1540+147\sqrt{3}\right)/3\ \text{cm}^2$$ **[1]**

31. Diameter, AB = 13 cm

$\therefore$ Radius of the circle, $r=\frac{13}{2}=6.5$ cm

$\because$ $\angle ACB$ is the angle in the semi-circle.

$\therefore$ $\angle ACB = 90°$ **[½]**

Now, in ΔACB, using Pythagoras theorem, we have

$AB^2 = AC^2 + BC^2$

$(13)^2 = (12)^2 + (BC)^2$

$(BC)^2 = (13)^2 – (12)^2 = 169 – 144 = 25$

$BC=\sqrt{25}=5$ cm **[1]**

Now, area of shaded region

= Area of semi-circle ABC – Area of (ΔACB) **[½]**

$$=\frac{1}{2}\pi r^2-\frac{1}{2}\times BC\times AC$$

$$=\frac{1}{2}\times 3.14\times(6.5)^2-\frac{1}{2}\times 5\times 12$$ **[½]**

= 66.3325 – 30

= 36.3325 cm^2

Thus, the area of the shaded region is 36.3325 cm^2. **[½]**

32. Area of the region $ABDC$

= Area of sector AOC – Area of sector BOD **[½]**

$$=\frac{40^\circ}{360^\circ}\times\frac{22}{7}\times14\times14-\frac{40^\circ}{360^\circ}\times\frac{22}{7}\times7\times7$$

$$=\frac{1}{9}\times22\times14\times2-\frac{1}{9}\times22\times7\times1$$

$$=\frac{22}{9}\times(28-7)$$

$$=\frac{22}{9}\times21$$

$$=\frac{154}{3}\text{ cm}^2$$ **[½]**

Area of circular ring

$$=\frac{22}{7}\times14\times14-\frac{22}{7}\times7\times7$$ **[1]**

= 22 × 14 × 2 – 22 × 7 × 1

= 22 × (28 – 7)

= 22 × 21

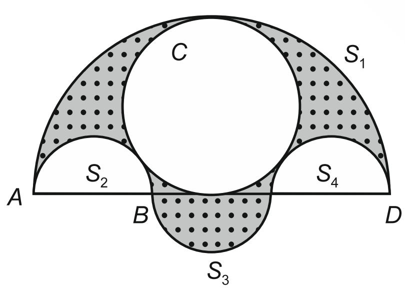
= 462 cm^2 **[½]**

∴ Area of shaded region

= Area of circular ring – Area of region $ABDC$

$$= 462-\frac{154}{3}$$

$$=\frac{1232}{3}\text{ cm}^2$$ **[½]**

33. 

Given that $AB = BC = CD = 3$ cm **[½]**

Circle C has diameter = 4.5 cm

Semicircle S_1 has diameter = 9 cm **[½]**

Area of shaded region

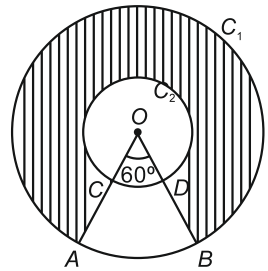
= Area of S_1 – Area of $(S_2 + S_4)$ – Area of C + Area of S_3 **[1]**

Area of shaded region

$$=\frac{\pi}{2}\left(\frac{9}{2}\right)^2-\frac{\pi}{2}\left(\frac{3}{2}\right)^2-\frac{\pi}{2}\left(\frac{3}{2}\right)^2-\pi\left(\frac{4.5}{2}\right)+\frac{\pi}{2}\left(\frac{3}{2}\right)^2$$ **[½]**

$$=\frac{\pi\times81}{16}-\frac{\pi\times9}{8}$$

= 12.375 cm^2 **[½]**

34. 

Given $OC = OD = 21$ cm

$OA = OB = 42$ cm

Area of $ACDB$ region

= Area of sector OAB – Area sector OCD **[½]**

$$=\frac{60^\circ}{360^\circ}\times\pi(42)^2-\frac{60^\circ}{360^\circ}\times\pi(21)^2$$ **[½]**

$$=\frac{1}{6}\times\frac{22}{7}\times21\times63$$

= 11 × 63 = 693 cm^2 **[½]**

Area of shaded region

= Area of c_1 – Area of c_2 – Area of $ACDB$ region **[½]**

= $\pi(42)^2 - \pi(21)^2 - 693$

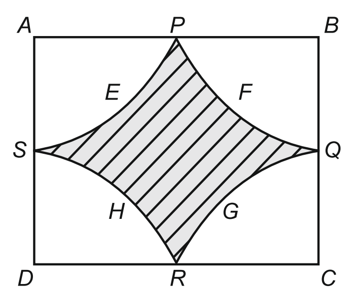
$$=\frac{22}{7}\times63\times21-693$$

= 3,465 cm^2 **[1]**

35. Given that $ABCD$ is a square and P, Q, R and S are the mid-points of AB, BC, CD and DA respectively

and $AB = 12$ cm

⇒ $AP = 6$ cm [P bisects AB]



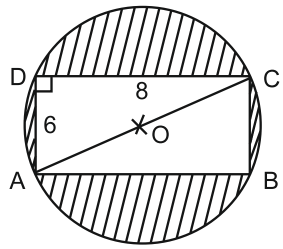
Area of the shaded region = Area of square $ABCD$ – (Area of sector $APEC$ + Area of sector $PFQB$ + Area of sector $RGQC$ + Area of sector $RHSD$) **[1]**

$$=12^2-\left(\frac{\pi(6^2)}{4}+\frac{\pi(6^2)}{4}+\frac{\pi(6^2)}{4}+\frac{\pi(6^2)}{4}\right)$$ **[1]**

= $12^2 - \pi \times 36$

= 144 – 113.04

= 30.96 cm^2 **[1]**

36. 

In right triangle ADC, $\angle D = 90°$

$AC^2 = AD^2 + DC^2$ [By Pythagoras theorem] **[½]**

$= 6^2 + 8^2 = 100$

$AC = 10$ cm **[½]**

$2(AO) = 10$

$AO = 5$ cm

$\Rightarrow$ Radius (r) = 5 cm **[½]**

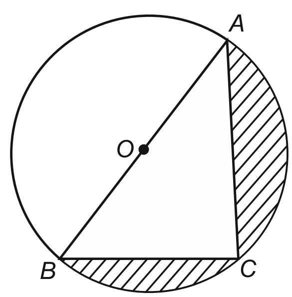
Area of the shaded region

= Area of the circle – Area of rectangle **[½]**

$= \pi r^2 - l \times b$

$= 3.14(5)^2 - 6 \times 8$ **[½]**

$= 78.5 - 48 = 30.5$ cm^2 **[½]**

37. 

$AC = 24$ cm, $BC = 10$ cm **[½]**

$AB = \sqrt{24^2 + 10^2}$

$AB = 26$ cm **[1]**

Diameter of circle = 26 cm

Area of shaded region

= Area of semicircle – Area of ΔABC **[1]**

$= \frac{\pi}{2}(13)^2 - \frac{1}{2} \times 24 \times 10$ **[½]**

$= \frac{3.14}{2} \times 169 - 120$

$= 145.33$ cm^2 **[1]**

38. $PQRS$ is a square.

So each side is equal and angle between the adjacent sides is a right angle.

Also the diagonals perpendicularly bisect each other.

In ΔPQR using pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$PR^2 = (42)^2 + (42)^2$

$PR^2 = \sqrt{2}(42)$

$OR = \frac{1}{2}PR = \frac{42}{\sqrt{2}} = OQ$ **[1]**

From the figure we can see that the radius of flower bed ORQ is OR.

Area of sector $ORQ = \frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi\left(\frac{42}{\sqrt{2}}\right)^2$

Area of the $\Delta ROQ = \frac{1}{2} \times RO \times OQ$

$= \frac{1}{2} \times \frac{42}{\sqrt{2}} \times \frac{42}{\sqrt{2}}$

$= \left(\frac{42}{2}\right)^2$ **[1]**

Area of the flower bed ORQ

= Area of sector ORQ – Area of the ROQ

$= \frac{1}{2}\pi\left(\frac{42}{\sqrt{2}}\right)^2 - \left(\frac{42}{2}\right)^2$

$= \left(\frac{42}{2}\right)^2\left[\frac{\pi}{2} - 1\right]$

= (441) [0.57]

= 251.37 cm^2 **[1]**

Area of the flower bed ORQ = Area of the flower bed OPS

= 251.37 cm^2

Total area of the two flower beds

= Area of the flower bed ORQ + Area of the flower bed OPS

= 251.37 + 251.37

= 502.74 cm^2 **[1]**

39. Perimeter of shaded region = $AB + PB$ + arc length AP ...(i) **[½]**

Arc length $AP = \frac{\theta}{360°} \times 2\pi r = \frac{\pi\theta r}{180°}$ ...(ii) **[½]**

In right angled ΔOAB,

$\tan\theta = \frac{AB}{r} \Rightarrow AB = r\tan\theta$ ...(iii) **[½]**

$\sec\theta = \frac{OB}{r} \Rightarrow OB = r\sec\theta$ **[½]**

$OB = OP + PB$

$\Rightarrow \; r\sec\theta = r + PB \qquad [\therefore\; OB = r\sec\theta]$

$\Rightarrow \; PB = r\sec\theta - r$ ...(iv) **[1]**

Substitute (ii), (iii) and (iv) in (i), we get

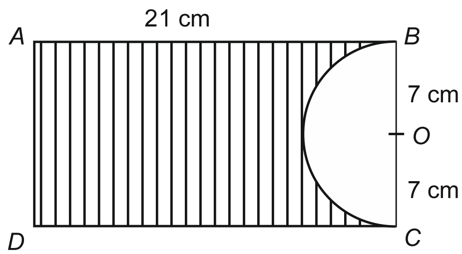
Perimeter of shaded region

$= AB + PB + \text{arc}\,(AP)$

$= r\tan\theta + r\sec\theta - r + \frac{\pi\theta r}{180^\circ}$

$= r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180^\circ} - 1\right]$ **[1]**

40.



Area of shaded region = Area of rectangle – Area of semicircle **[1]**

$= 21\times 14 - \frac{\pi(7)^2}{2}$

$= 217\ \text{cm}^2$ **[1]**

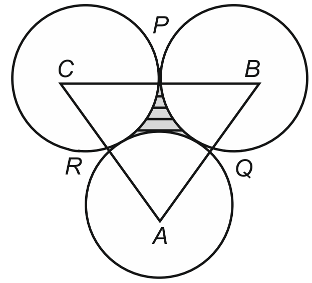
Perimeter of shaded region

$= AB + AD + CD + \text{length of arc } BC$ **[1]**

$= 21 + 14 + 21 + \frac{180^\circ}{360^\circ}\times 2\times\frac{22}{7}\times 7$

$= 78$ cm **[1]**

41.



Given that all circles have radii = 3.5 cm

$\therefore \; AB = BC = AC = 7$ cm

ΔABC is an equilateral triangle area of

$\Delta ABC = \frac{\sqrt{3}}{4}\times 49\ \text{cm}^2$ **[1]**

Area of sector BPQ $= \frac{60^\circ}{360^\circ}\times\frac{22}{7}\times(3.5)^2$ **[1]**

$= \frac{77}{12}\ \text{cm}^2$

$= \frac{60^\circ}{360^\circ}\times\frac{22}{7}\times(3.5)^2$

$= \frac{77}{12}\ \text{cm}^2$ **[1]**

Similarly areas of other sectors PCR and $RAQ = \frac{77}{12}\ \text{cm}^2$ **[1]**

Area of shaded region

$= \text{ar}(\Delta ABC) - 3\,(\text{area of } BPQ)$ **[1]**

$= \frac{49\sqrt{3}}{4} - \frac{3(77)}{12}$

$= \frac{49\sqrt{3} - 77}{4} = \frac{7}{4}\left(7\sqrt{3} - 11\right)$ **[1]**

13 : Surface Areas and Volumes

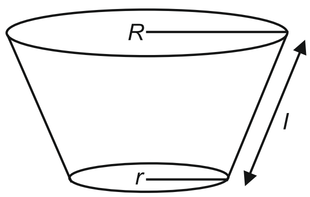
1. Surface area of sphere = 616 cm^2

$4\pi r^2 = 616$ **[½]**

$4\times\frac{22}{7}\times r^2 = 616$

$\boxed{r = 7\ \text{cm}}$ **[½]**

2.



Given slant height $(\ell) = 4$ cm

Perimeters of circular ends:

$2\pi r = 6$ cm

$2\pi R = 18$ cm **[½]**

$C.S.A = \pi(\ell)\,(r + R) = 4\times 12 = 48\ \text{cm}^2$ **[½]**

3. Answer (B)

Largest cone that can be cut from a cube has the

Diameter = side of cube **[½]**

Height = side of cube

$\therefore \;$ radius $= \frac{4.2}{2} = 2.1$ cm **[½]**

4. Answer (C)

Let the original radius and the height of the cylinder be r and h respectively.

Volume of the original cylinder = $\pi r^2 h$

Radius of the new cylinder $= \dfrac{r}{2}$

Height of the new cylinder = h

Volume of the new cylinder $= \pi\left(\dfrac{r}{2}\right)^2 h = \dfrac{\pi r^2 h}{4}$ [½]

Required ratio $= \dfrac{\text{Volume of the new cylinder}}{\text{Volume of the original cylinder}}$

$$= \frac{\frac{\pi r^2 h}{4}}{\pi r^2 h} = \frac{1}{4} = 1:4$$ [½]

5. Answer (B)

Let r and h be the radius and the height of the cylinder, respectively.

Given: Diameter of the cylinder = 4 cm

$\therefore$ Radius of the cylinder, r = 2 cm

Height of the cylinder, h = 45 cm

Volume of the solid cylinder = $\pi r^2 h = \pi \times (2)^2 \times 45$ cm^3 = 180π cm^3 [½]

Suppose the radius of each sphere be R cm.

Diameter of the sphere = 6 cm

$\therefore$ Radius of the sphere, R = 3 cm

Let n be the number of solids formed by melting the solid metallic cylinder.

• $\therefore$ n × volume of the solid spheres

= Volume of the solid cylinder

$$\Rightarrow \quad n \times \frac{4}{3}\pi R^3 = 180\pi$$

$$\Rightarrow \quad n \times \frac{4}{3}\pi R^3 = 180\pi$$

$$\Rightarrow \quad n = \frac{180 \times 3}{4 \times 27} = 5$$

Thus, the number of solid spheres that can be formed is 5. [½]

6. Let r_1, r_2 and h_1, h_2 be the radius and height of two cones resectively

According to the question,

$\dfrac{r_1}{r_2} = \dfrac{3}{1}$ and $\dfrac{h_1}{h_2} = \dfrac{1}{3}$ [½]

$$\therefore \quad \frac{\text{Volume of Cone}_1}{\text{Volume of Cone}_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$= \left(\frac{3}{1}\right)^2 \times \left(\frac{1}{3}\right)$$

$$= \frac{3}{1}$$ [½]

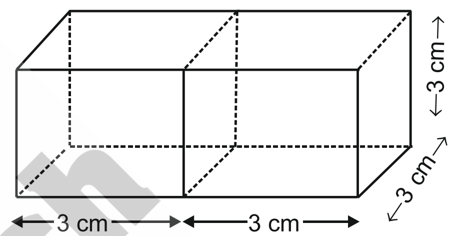
7. Volume of cube = 27 cm^3

$\therefore$ Volume of cube = (side)^3 = 27 cm^3

Side $= \sqrt[3]{27}$ cm

Side = 3 cm [½]

If two cubes are joined end to end the resulting figure is cuboid



i.e., length = l = 6 cm

breadth = b = 3 cm [½]

height = h = 3 cm

Surface area of resulting cuboid = $2(lb + bh + hl)$ [½]

= 2 × (6 × 3 + 3 × 3 + 3 × 6) cm^2

= 2 × (18 + 9 + 18)

= 2 × 45 = 90 cm^2 [½]

8. Cone: height = 20 cm

Base radius = 5 cm

Cone is reshaped into a sphere

• $\therefore$ Volume of cone = volume of sphere [1]

$$\frac{1}{3}\pi(5)^2(20) = \frac{4}{3}\pi(r)^3$$

$r^3 = 5^3$

$\Rightarrow$ r = 5 cm [1]

9. Given volume of a hemisphere $= 2425\frac{1}{2}$ cm^3

$$= \frac{4851}{2} \text{ cm}^3$$ [½]

Now, let r be the radius of the hemisphere

Volume of a hemisphere $= \dfrac{2}{3}\pi r^3$

$\therefore \quad \frac{2}{3}\pi r^3 = \frac{4851}{2}$

$\Rightarrow \quad \frac{2}{3}\times\frac{22}{7}\times r^3 = \frac{4851}{2}$

$\Rightarrow \quad r^3 = \frac{4851}{2}\times\frac{3}{2}\times\frac{7}{22} = \left(\frac{21}{2}\right)^3$ **[½]**

$\therefore \quad r = \frac{21}{2}$ cm

So, curved surface area of the hemisphere = $2\pi r^2$

$= 2\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2} = 693$ sq.cm **[1]**

10. Dimensions of cuboid are 24 cm, 8 cm, 8 cm

T.S.A of cuboid = $2(lb + bh + lh)$ **[½]**

= 2[24(8) + 8(8) + 24(8)] **[½]**

= 2[192 + 64 + 192] **[½]**

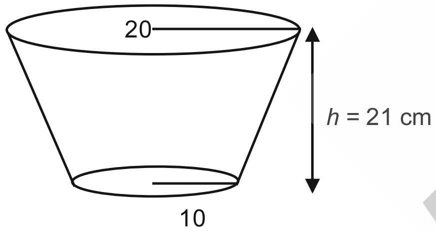
= 2[448] = 896 cm^2 **[½]**

11. Volume of cuboid = Volume of n-solid spheres

$\therefore \quad 11 \times 7 \times 7 = n\times\frac{4}{3}\pi\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}$ **[1]**

$\Rightarrow \quad n = 3$ **[1]**

12.



Volume of frustum $= \frac{\pi}{3}h\left(R^2 + r^2 + rR\right)$ **[1]**

$= \frac{22}{7\times 3}\times 21\times\left(10^2 + 20^2 + 10\times 20\right)$

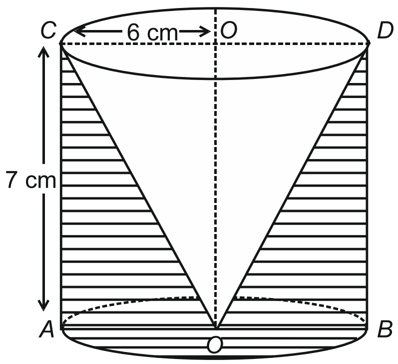
= 22(700) cm^3

= 15400 cm^3 = 15.4ℓ **[1]**

Cost of milk = 15.4 × 30

= ₹462 **[1]**

13.



Given: Radius of cylinder = radius of cone = r = 6 cm

Height of the cylinder = height of the cone = h = 7 cm **[½]**

Slant height of the cone = $l = \sqrt{7^2 + 6^2}$

$= \sqrt{85}$ cm **[½]**

Total surface area of the remaining solid =

Curved surface area of the cylinder + area of the base of the cylinder + curved surface area of the cone

$\therefore$ Total surface area of the remaining solid = $(2\pi rh + \pi r^2 + \pi rl)$ **[1]**

$= 2\times\frac{22}{7}\times 6\times 7 + \frac{22}{7}\times 6^2 + \frac{22}{7}\times 6\sqrt{85}$

$= 264 + \frac{792}{7} + \frac{132}{7}\sqrt{85}$

$= 377.1 + \frac{132}{7}\sqrt{85}\ cm^2$ **[1]**

14. Volume of the conical heap = volume of the sand emptied from the bucket.

Volume of the conical heap

$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\times 24\ cm^3$ ...(i)

(height of the cone is 24) **[1]**

Volume of the sand in the bucket = $\pi r^2 h$

= $\pi(18)^2 \times 32\ cm^3$ ...(ii) **[1]**

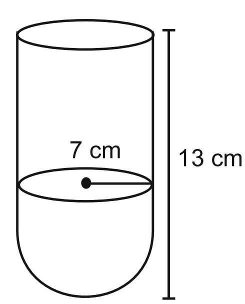
Equating (i) and (ii),

$\frac{1}{3}\pi r^2\times 24 = \pi(18)^2\times 32$ **[½]**

$\Rightarrow \quad r^2 = \frac{(18)^2\times 32\times 3}{24}$ **[½]**

$\Rightarrow \quad r = 36$ cm

15.



Let the radius and height of cylinder be r cm and h cm respectively.

Diameter of the hemispherical bowl = 14 cm

$\therefore$ Radius of the hemispherical bowl = Radius of the cylinder

$$= r = \frac{14}{2}\text{ cm} = 7\text{ cm}$$ **[1]**

Total height of the vessel = 13 cm

$\therefore$ Height of the cylinder, h = 13 cm – 7 cm = 6 cm **[1]**

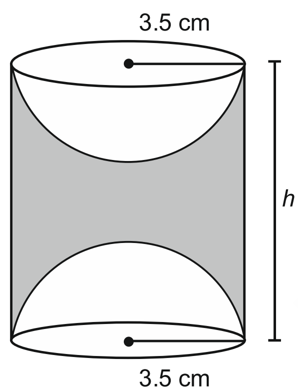
Total surface area of the vessel = 2 (curved surface area of the cylinder + curved surface area of the hemisphere) (Since, the vessel is hollow)

$= 2(2\pi rh + 2\pi r^2) = 4\pi r(h + r)$

$$= 4 \times \frac{22}{7} \times 7 \times (6+7)\text{ cm}^2$$

$= 1144\text{ cm}^2$ **[1]**

16.



Height of the cylinder, h = 10 cm

Radius of the cylinder = Radius of each hemisphere = r = 3.5 cm **[½]**

Volume of wood in the toy = Volume of the cylinder – 2 × Volume of each hemisphere

$$= \pi r^2 h - 2 \times \frac{2}{3}\pi r^3$$ **[1]**

$$= \pi r^2\left(h - \frac{4}{3}r\right)$$

$$= \frac{22}{7} \times (3.5)^2 \left(10 - \frac{4}{3} \times 3.5\right)$$

= 38.5 × (10 – 4.67) **[1]**

= 38.5 × 5.33

$= 205.205\text{ cm}^3$ **[½]**

17. For the given tank

Diameter = 10 m

Radius, R = 5 m

Depth, H = 2 m **[½]**

Internal radius of the pipe

$$= r = \frac{20}{2}\text{ cm} = 10\text{ cm} = \frac{1}{10}\text{ m}$$ **[½]**

Rate of flow of water = v = 4 km/h = 4000 m/h

Let t be the time taken to fill the tank. **[½]**

So, the volume of water flows through the pipe in t hours will equal to the volume of the tank.

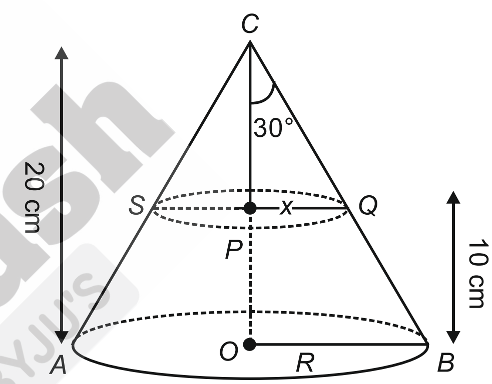
$\therefore$ $\pi r^2 \times v \times t = \pi R^2 H$ **[1]**

$$\Rightarrow \left(\frac{1}{10}\right)^2 \times 4000 \times t = (5)^2 \times 2$$

$$\Rightarrow t = \frac{25 \times 2 \times 100}{4000} = 1\frac{1}{4}$$

Hence, the time taken is $1\frac{1}{4}$ hours **[½]**

18.



Let ACB be the cone whose vertical angle $\angle ACB$ = 60°. Let R and x be the radii of the lower and upper end of the frustum.

Here, height of the cone, $OC = H$ = 20 cm

Height $CP = h$ = 10 cm **[½]**

Let us consider P as the mid-point of OC.

After cutting the cone into two parts through P.

$$OP = \frac{20}{2} = 10\text{ cm}$$ **[½]**

Also, $\angle ACO$ and $\angle OCB = \frac{1}{2} \times 60° = 30°$

After cutting cone CQS from cone CBA, the remaining solid obtained is a frustum.

Now, in triangle CPQ

$$\tan 30° = \frac{x}{10}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{10}$$

$\Rightarrow \quad x = \frac{10}{\sqrt{3}}$ cm [½]

In triangle COB

$\tan 30° = \frac{R}{CO}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{R}{20}$

$\Rightarrow \quad R = \frac{20}{\sqrt{3}}$ cm [½]

Volume of the frustum, $V = \frac{1}{3}\pi\left(R^2H - x^2h\right)$

$$\Rightarrow \quad V = \frac{1}{3}\pi\left(\left(\frac{20}{\sqrt{3}}\right)^2.20 - \left(\frac{10}{\sqrt{3}}\right)^2.10\right)$$

$$= \frac{1}{3}\pi\left(\frac{8000}{3} - \frac{1000}{3}\right)$$

$$= \frac{1}{3}\pi\left(\frac{7000}{3}\right)$$

$$= \frac{1}{9}\pi \times 7000$$

$$= \frac{7000}{9}\pi$$ [½]

The volumes of the frustum and the wire formed are equal.

$$\pi \times \left(\frac{1}{24}\right)^2 \times l = \frac{7000}{9}\pi \left[\text{Volume of wire} = \pi r^2 h\right]$$

$$\Rightarrow \quad l = \frac{7000}{9} \times 24 \times 24$$

$\Rightarrow \quad l$ = 448000 cm = 4480 m [½]

Hence, the length of the wire is 4480 m.

19. Diameter of the tent = 4.2 m

Radius of the tent, r = 2.1 m

Height of the cylindrical part of tent, h_{cylinder} = 4 m

Height of the conical part, h_{cone} = 2.8 m [½]

Slant height of the conical part, l

$$= \sqrt{h^2_{\text{cone}} + r^2}$$

$$= \sqrt{2.8^2 + 2.1^2}$$

$$= \sqrt{2.8^2 + 2.1^2}$$

= 3.5 m [½]

Curved surface area of the cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.1 \times 4$$

= 22 × 0.3 × 8 = 52.8 m^2 [½]

Curved surface area of the conical tent

$$= \pi rl = \frac{22}{7} \times 2.1 \times 3.5 = 23.1\,\text{m}^2$$ [½]

Total area of cloth required for building one tent = Curved surface area of the cylinder + Curved surface area of the conical tent

= 52.8 + 23.1

= 75.9 m^2 [½]

Cost of building one tent = 75.9 × 100 = ₹ 7590

Total cost of 100 tents = 7590 × 100

= ₹ 7,59,000

Cost to be borne by the associations

$$= \frac{759000}{2} = 3{,}79{,}500$$ [½]

It shows the helping nature, unity and cooperativeness of the associations.

20. Internal diameter of the bowl = 36 cm

Internal radius of the bowl, r = 18 cm

Volume of the liquid, $V = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \pi \times 18^3$ [½]

Let the height of the small bottle be 'h'

Diameter of a small cylindrical bottle = 6 cm

Radius of a small bottle, R = 3 cm

Volume of a single bottle = $\pi R^2 h = \pi \times 3^2 \times h$ [½]

Number of small bottles, n = 72

Volume wasted in the transfer $= \frac{10}{100} \times \frac{2}{3} \times \pi \times 18^3$

[½]

Volume of liquid to be transferred in the bottles

$$= \frac{2}{3} \times \pi \times 18^3 - \frac{10}{100} \times \frac{2}{3} \times \pi \times 18^3$$

$$= \frac{2}{3} \times \pi \times 18^3 \left(1 - \frac{10}{100}\right)$$

$$= \frac{2}{3} \times \pi \times 18^3 \times \frac{90}{100}$$ [½]

Number of small cylindrical bottles

$$= \frac{\text{Volume of the liquid to be transferred}}{\text{Volume of single bottle}}$$ [½]

$$\Rightarrow \quad 72 = \frac{\frac{2}{3}\times\pi\times 18^3\times\frac{90}{100}}{\pi\times 3^2\times h}$$

$$\Rightarrow \quad 72 = \frac{\frac{2}{3}\times 18^3\times\frac{9}{10}}{3^2\times h}$$

$$\Rightarrow \quad h = \frac{\frac{2}{3}\times 18\times 18\times 18\times\frac{9}{10}}{3^2\times 72}$$

$\therefore$ h = 5.4 cm [½]

Height of the small cylindrical bottle = 10.8 cm

21. Side of the cubical block, a = 10 cm

Largest diameter of a hemisphere = side of the cube

Since the cube is surmounted by a hemisphere,

Diameter of the hemisphere = 10 cm

Radius of the hemisphere, r = 5 cm [1]

Total surface area of the solid = Total surface area of the cube – Inner cross-section area of the hemisphere + Curved surface area of the hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2$ [1]

$= 6a^2 + \pi r^2$

$= 6 \times (10)^2 + 3.14 \times 5^2$

$= 600 + 78.5 = 678.5$ cm^2

Total surface area of the solid = 678.5 cm^2 [1]

22. Number of cones = 504

Diameter of a cone = 3.5 cm

Radius of the cone, r = 1.75 cm

Height of the cone, h = 3 cm [½]

Volume of a cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\times\pi\times\left(\frac{3.5}{2}\right)^2\times 3$$

$$= \frac{1}{3}\times\pi\times\frac{3.5}{2}\times\frac{3.5}{2}\times 3 \text{ cm}^3$$ [½]

Volume of 504 cones

$$= 504\times\frac{1}{3}\times\pi\times\frac{3.5}{2}\times\frac{3.5}{2}\times 3 \text{ cm}^3$$ [½]

Let the radius of the new sphere be 'R'.

Volume of the sphere $= \frac{4}{3}\pi R^3$

Volume of 504 cones = Volume of the sphere [½]

$$504\times\frac{1}{3}\times\pi\times\frac{3.5}{2}\times\frac{3.5}{2}\times 3 = \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad \frac{504\times 1\times\pi\times 3.5\times 3.5\times 3\times 3}{3\times 2\times 2\times 4\times\pi} = R^3$$

$$\Rightarrow \quad R^3 = \frac{504\times 3\times 49}{64}$$

$$\Rightarrow \quad R^3 = \frac{7\times 8\times 9\times 3\times 7^2}{64}$$

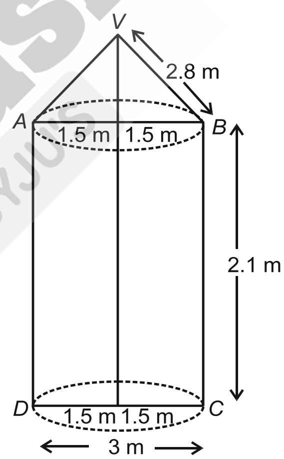
$$\Rightarrow \quad R^3 = \frac{8\times 27\times 7^3}{64}$$

$$\Rightarrow \quad R = \frac{2\times 3\times 7}{4}$$

$$\Rightarrow \quad R = \frac{21}{2} = 10.5 \text{ cm}$$ [1]

Radius of the new sphere = 10.5 cm

23.



For conical portion, we have

r = 1.5 m and l = 2.8 m

$\therefore$ S_1 = Curved surface area of conical portion

$\therefore$ $S_1 = \pi rl$

$= 4.2\pi$ m^2 [½]

For cylindrical portion, we have

r = 1.5 m and h = 2.1 m

$\therefore$ S_2 = Curved surface area of cylindrical portion

$\therefore$ $S_2 = 2\pi rh$

$= 2 \times \pi \times 1.5 \times 2.1$ [½]

$= 6.3\pi$ m^2

Area of canvas used for making the tent = $S_1 + S_2$

$= 4.2\pi + 6.3\pi$

$= 10.5\pi$

$= 10.5 \times \frac{22}{7}$ **[1]**

$= 33 \text{ m}^2$

Total cost of the canvas at the rate of ₹ 500 per m^2 = ₹(500 × 33) = ₹16500. **[1]**

24. Let the radius of the conical vessel = r_1 = 5 cm

Height of the conical vessel = h_1 = 24 cm **[½]**

Radius of the cylindrical vessel = r_2

Let the water rise upto the height of h_2 cm in the cylindrical vessel.

Now, volume of water in conical vessel = volume of water in cylindrical vessel

$$\frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$r_1^2 h_1 = 3 r_2^2 h_2$$ **[1½]**

$5 \times 5 \times 24 = 3 \times 10 \times 10 \times h_2$

$$h_2 = \frac{5 \times 5 \times 24}{3 \times 10 \times 10} = 2 \text{ cm}$$ **[1]**

Thus, the water will rise upto the height of 2 cm in the cylindrical vessel.

25. Radius of sphere = r = 6 cm

Volume of sphere

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (6)^3 = 288\pi \text{ cm}^3$$ **[½]**

Let R be the radius of cylindrical vessel.

Rise in the water level of cylindrical vessel

$$= h = 3\frac{5}{9} \text{ cm} = \frac{32}{9} \text{ cm}$$

Increase in volume of cylindrical vessel

$$= \pi R^2 h = \pi R^2 \times \frac{32}{9} = \frac{32}{9}\pi R^2$$ **[½]**

Now, volume of water displaced by the sphere is equal to volume of sphere

$$\therefore \quad \frac{32}{9}\pi R^2 = 288\pi$$ **[1]**

$$\therefore \quad R^2 = \frac{288 \times 9}{32} = 81$$ **[½]**

$\therefore$ R = 9 cm

$\therefore$ Diameter of the cylindrical vessel = 2 × R = 2 × 9 = 18 cm **[½]**

26. Given canal width = 5.4 m

Depth = 1.8 m **[½]**

Water flow speed = 25 km/hr

Distance covered by water in 40 minutes

$$= \frac{25 \times 40}{60}$$ **[½]**

$$= \frac{50}{3} \text{ km}$$

Volume of water flows through pipe

$$= \frac{50}{3} \times 5.4 \times 1.8 \times 1000$$

$= 162 \times 10^3 \text{ m}^3$ **[1]**

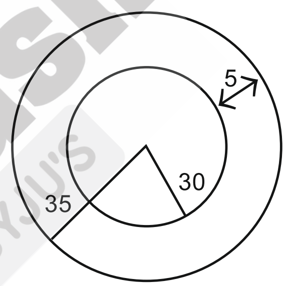
Area irrigate with 10 cm of water standing

$$= \frac{162 \times 10^3}{10 \times 10^{-2}}$$

$= 162 \times 10^4 \text{ m}^2$ **[1]**

27. Volume of cuboid = 4.4 × 2.6 × 1

= 11.44 m^3 **[½]**



Length = l

Inner radius = 30 cm **[½]**

Outer radius = 35 cm

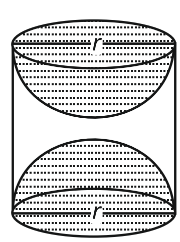
Volume of cuboid = volume of cylindrical pipe

$$11.44 = \frac{\pi \times l \times (35^2 - 30^2)}{100 \times 100 \times 100}$$ **[1]**

$l = 10.205 \times 10^4$ cm

l = 102.05 km **[1]**

28.



Let r be the radius of the base of the cylinder and h be its height. Then, Total surface area of the article = curved surface area of the cylinder + 2 (Curved surface area of a hemisphere) **[1]**

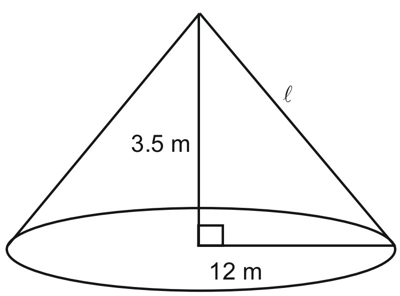
$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r(h + 2r)$ **[1]**

$= 2 \times \frac{22}{7} \times 3.5(10 + 2 \times 3.5)\ \text{cm}^2$

$= 22 \times 17\ \text{cm}^2 = 374\ \text{cm}^2$ **[1]**

29. Given



Base diameter = 24 m

Base radius = 12 m

Height = 3.5 m

$\text{Volume} = \frac{1}{3}\pi r^2 h$ **[½]**

$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5$

$= 22 \times 4 \times 12 \times 0.5$

$= 264 \times 2$ **[1]**

$= 528\ \text{m}^3$

$\therefore\ \ell^2 = 12^2 + 3.5^2 = 144 + 12.25$

$\ell^2 = 156.25$ **[½]**

$\ell = \sqrt{156.25} = 12.5\ \text{m}$

Curved surface area = $\pi r\ell$

$\frac{22}{7} \times 12 \times 12.5 = \frac{150 \times 22}{7} = 471.428\ \text{m}^2$ **[1]**

30. Width of the canal = 6 m

Depth of the canal = 1.5 m

Length of the water column formed in $\frac{1}{2}$ hr

= 5 km or 5000 m **[½]**

$\therefore$ Volume of water flowing in $\frac{1}{2}$ hr

= Volume of cuboid of length 5000 m, width 6 m and depth 1.5 m.

$= 5000 \times 6 \times 1.5 = 45000\ \text{m}^3$ **[1]**

On comparing the volumes,

Volume of water in field = Volume of water coming out from canal in 30 minutes. **[½]**

Irrigated area × standing water = 45000.

$\text{Irrigated Area} = \frac{45000}{\frac{8}{100}}$ [$\because$ 1 m = 100 cm] **[½]**

$= \frac{45000 \times 100}{8} = 5{,}62{,}500\ \text{m}^2$ **[½]**

31. Volume of cuboid = $24 \times 11 \times 7\ \text{cm}^3$

Volume of 1 cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6\ \text{cm}^3$ **[1]**

Let no. of cones formed = n

$\therefore$ Volume of n cones

$= n \times \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6\ \text{cm}^3$

Now, according of question

Volume of n cones = volume of cuboid

$\Rightarrow\ n \times \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 6 = 24 \times 11 \times 7$ **[1]**

$n = \frac{24 \times 11 \times 7 \times 3 \times 7}{22 \times 3.5 \times 3.5 \times 6} = 24$ **[1]**

$\therefore$ Number of cones formed are 24.

32. Here, $r_1 = 4$ cm

$\Delta VO'A' \sim \Delta VOA$ (AA similarity)

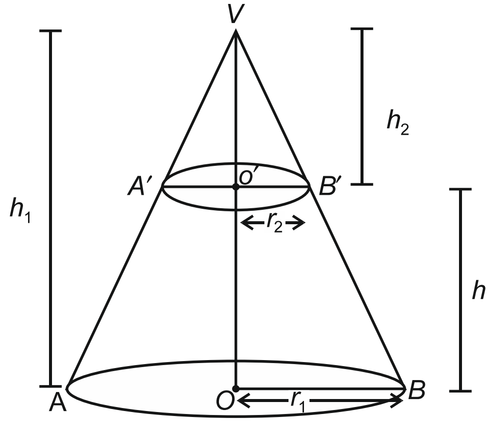
Now, $\frac{r_1}{r_2} = \frac{h_1}{h_2}$ **[½]**

Also, $h_1 = 2h_2$

$\Rightarrow \frac{r_1}{r_2} = 2$ **[½]**

$\Rightarrow r_2 = 2$ cm

Now, $\frac{\text{Volume of smaller cone } VA'B'}{\text{Volume of frustum } ABB'A'}$



[½]

$$= \frac{\frac{1}{3}\pi r_2^2 h_2}{\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)}$$ [½]

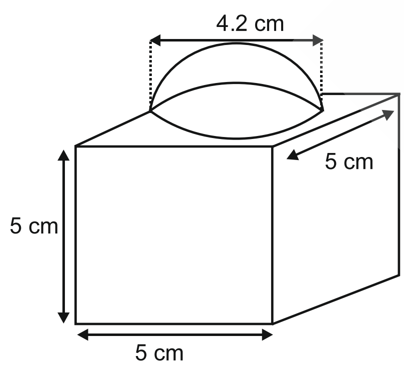
$$= \frac{r_2^2}{r_1^2 + r_2^2 + r_1 r_2}$$

$$= \frac{4}{16+4+8}$$ [$\because$ $h = h_2$] [½]

$$= \frac{4}{28}$$

= 1 : 7 [½]

33.



The total surface area of the cube = 6 × (edge)^2 = 6 × 5 × 5 cm^2 = 150 cm^2 [1]

Note that the part of the cube where the hemisphere is attached is not included in the surface area.

So, the surface area of the block = *TSA* of cube – base area of hemisphere + *CSA* of hemisphere [1]

= 150 – πr^2 + $2\pi r^2$ = (150 + πr^2) cm^2 [1]

$$= 150 \text{ cm}^2 + \left(\frac{22}{7}\times\frac{4.2}{2}\times\frac{4.2}{2}\right)\text{cm}^2$$

= (150 + 13.86) cm^2 = 163.86 cm^2 [1]

34. Diameter of circular end of pipe = 2 cm

$\therefore$ Radius r_1 of circular end of pipe

$$= \frac{2}{200}\text{ m} = 0.01\text{ m}$$ [½]

Area of cross-section

$$= \pi\times r_1^2 = \pi\times 0.01^2 = 0.0001\pi\text{m}^2$$ [½]

Speed of water = 0.4 m/s s = 0.4 × 60

= 24 metre/min

Volume of water that flows in 1 minute from pipe

= 24 × 0.0001 πm^3 = 0.0024 πm^3

Volume of water that flows in 30 minutes from pipe = 30 × 0.0024πm^3 = 0.072 πm^3 [½]

Radius (r_2) of base of cylindrical tank = 40 cm = 0.4 m [½]

Let the cylindrical tank be filled up to h m in 30 minutes.

Volume of water filled in tank in 30 minutes is equal to the volume of water flowed out in 30 minutes from the pipe [1]

$\therefore$ $\pi\times r_2^2\times h = 0.072\pi$

$\Rightarrow$ 0.4^2 × h = 0.072 [½]

$\Rightarrow$ 0.16h = 0.072

$\Rightarrow$ $h = \dfrac{0.072}{0.16}$

$\Rightarrow$ h = 0.45 m = 45 cm [½]

Therefore, the rise in level of water in the tank in half an hour is 45 cm.

35. Diameter of upper end of bucket = 30 cm

$\therefore$ Radius (r_1) of upper end of bucket = 15 cm [½]

Diameter of lower end of bucket = 10 cm

$\therefore$ Radius (r_1) of lower end of bucket = 5 cm [½]

Slant height (l) of frustum

$$= \sqrt{(r_1 - r_2)^2 + h^2}$$

$$= \sqrt{(15-5)^2 + 24^2} = \sqrt{10^2 + 24^2} = \sqrt{100+576}$$

$$= \sqrt{676} = 26\text{ cm}$$ [1]

Area of metal sheet used to make the bucket

$$= \pi(r_1 + r_2)l + \pi r_2^2$$ [1]

$= \pi(15 + 5)26 + \pi(5)^2$

$= 520\pi + 25\pi = 545\pi$ cm^2 **[½]**

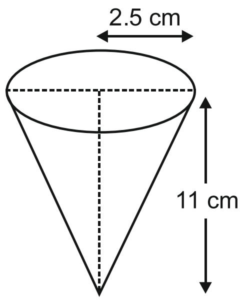
Cost of 100 cm^2 metal sheet = ₹10

Cost of 545 π cm^2 metal sheet

$= ₹\dfrac{545 \times 3.14 \times 10}{100} = ₹171.13$ **[½]**

Therefore, cost of metal sheet used to make the bucket is ₹ 171.13.

36.



Height (h) of the conical vessel = 11 cm

Radius (r_1) of the conical Vessel = 2.5 cm

Radius (r_2) of the metallic spherical balls $= \dfrac{0.5}{2} = 0.25$ cm **[½]**

Let n be the number of spherical balls = that were dropped in the vessel.

Volume of the water spilled = Volume of the spherical balls dropped **[½]**

$\dfrac{2}{5} \times$ Volume of cone = n × Volume of one spherical ball **[1]**

$\Rightarrow \quad \dfrac{2}{5} \times \dfrac{1}{3}\pi r_1^2 h = n \times \dfrac{4}{3}\pi r_2^3$ **[½]**

$\Rightarrow \quad r_1^2 h = n \times 10 r_2^3$

$\Rightarrow \quad (2.5)^2 \times 11 = n \times 10 \times (0.25)^3$

$\Rightarrow \quad 68.75 = 0.15625\, n$ **[½]**

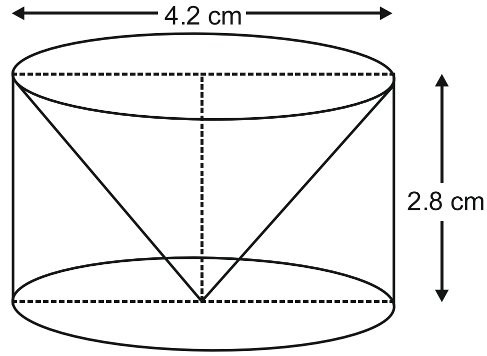
$\Rightarrow \quad n = 440$

Hence, the number of spherical balls that were dropped in the vessel is 440.

Sushant made the arrangement so that the water that flows out, irrigates the flower beds.

This shows the judicious usage of water. **[1]**

37. The following figure shows the required cylinder and the conical cavity



Given Height (b) of the conical Part = Height (h) of the cylindrical part = 2.8 cm

Diameter of the cylindrical part = Diameter of the conical part = 4.2 cm

$\therefore$ Radius $\rightarrow$ of the cylindrical part = Radius $\rightarrow$ of the conical part = 2.1 cm **[½]**

Slant height (l) of the conical part

$= \sqrt{(2.1)^2 + (2.8)^2}$ cm

$= \sqrt{4.41 + 7.81}$ cm

$= \sqrt{12.25}$ cm **[½]**

= 3.5 cm

Total surface area of the remaining solid = Curved surface area of the cylindrical part +Curved surface area of the conical part + Area of the cylindrical base

$= 2\pi rh + \pi rl + \pi r^2$ **[1]**

$= \left(2 \times \dfrac{22}{7} \times 2.1 \times 2.8 + \dfrac{22}{7} \times 2.1 \times 3.5 + \dfrac{22}{7} \times 2.1 \times 2.1\right)$ cm^2 **[1]**

= (36.96 + 23.1 + 13.86) cm^2

= 73.92 cm^2 **[½]**

Thus, the total surface area of the remaining solid is 73.92 cm^2 **[½]**

38. Height of the cylinder (h) = 10 cm

Radius of the base of the cylinder = 4.2 cm **[½]**

Volume of original cylinder = $\pi r^2 h$ **[½]**

$= \dfrac{22}{7} \times (4.2)^2 \times 10$

= 554.4 cm^3 **[½]**

Volume of hemisphere $= \dfrac{2}{3}\pi r^3$ **[½]**

$$= \frac{2}{3} \times \frac{22}{7} \times (4.2)^3$$

$= 155.232 \text{ cm}^3$ **[½]**

Volume of the remaining cylinder after scooping out hemisphere from each end

Volume of original cylinder – 2 × Volume of hemisphere

$= 554.4 - 2 \times 155.232$ **[½]**

$= 243.936 \text{ cm}^3$

The remaining cylinder is melted and converted to a new cylindrical wire of 1.4 cm thickness.

So they have same volume and radius of new cylindrical wire is 0.7 cm.

Volume of the remaining cylinder = Volume of the new cylindrical wire

$243.936 = \pi r^2 h$ **[½]**

$$243.936 = \frac{22}{7}(0.7)^2 h$$

$h = 158.4$ cm

∴ The length of the new cylindrical wire of 1.4 cm thickness is 158.4 cm **[½]**

39. Height of conical upper part = 3.5 m, and radius = 2.8 m

(Slant height of cone)^2 $= 2.1^2 + 2.8^2$

$= 4.41 + 7.84$

Slant height of cone $= \sqrt{12.25} = 3.5$ m **[½]**

The canvas used for each tent

Curved surface area of cylindrical base + curved surface area of conical upper part **[½]**

$= 2\pi rh + \pi rl$

$= \pi r(2h + l)$

$$= \frac{22}{7} \times 2.8(7 + 3.5)$$ **[½]**

$$= \frac{22}{7} \times 2.8 \times 10.5$$

$= 92.4 \text{ m}^2$ **[½]**

So, the canvas used for one tent is 92.4 m^2

Thus, the canvas used for 1500 tents

$= (92.4 \times 1500) \text{ m}^2$ **[½]**

Canvas used to make the tents cost ₹ 120 per sq. m

So, canvas used to make 1500 tents will cost ₹ 92.4 × 1500 × 120 **[½]**

The amount shared by each school to set up the tents

$$= \frac{92.4 \times 1500 \times 120}{50} = ₹\,332640$$ **[½]**

The amount shared by each school to set up the tents is ₹332640.

The value to help others in times of troubles is generated from the problem. **[½]**

40. Water from the roof drains into cylindrical tank

Volume of water from roof flows into the tank of the rainfall is x cm and given the tank is full we can write, **[½]**

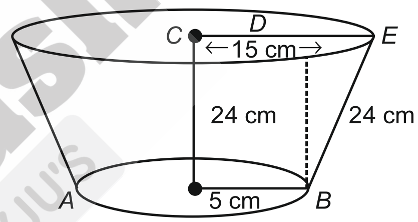
Volume of water collected on roof = volume of the tank **[1]**

$$\frac{22 \times 20 \times x}{100} = \pi\left(\frac{2}{2}\right)^2 \times 3.5$$ **[1½]**

$x = 2.5$ cm **[½]**

∴ Rainfall is of 2.5 cm **[½]**

41. Let $r_1 = 5$ cm and r_2 15 cm are radii of lower and upper circular faces.



Metal sheet required = Area of curved surface + Area of Base

$= \pi(r_1 + r_2)\ell + \pi r_1^2$ ...(i) **[½]**

Slant height of frustum = $l = \sqrt{(r_2 - r_1)^2 + h^2}$ **[½]**

$l = \sqrt{(15-5)^2 + 24^2}$

$l = \sqrt{10^2 + 24^2}$

$= \sqrt{100 + 576}$

$l = \sqrt{676}$ **[½]**

$l = 26$ cm

Metal required $= \pi(5 + 15)\,26 + \pi(5)^2$ **[½]**

$= \pi \times 20 \times 26 + \pi \times 25$

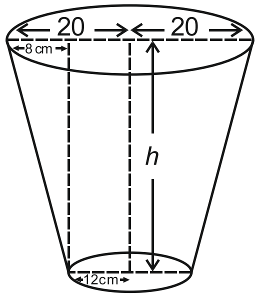
$= 5\pi(4 \times 26 + 5)$

$= 5\pi\,(109)$

$$= 5 \times \frac{22}{7} \times 109$$

$= 1712.85 \text{ cm}^2$ **[1]**

There is a chance of breakdown due to stress on ordinary plastic. **[1]**

42. 

Let the height of the bucket be h cm and slant height be l cm.

Here r_1 = 20 cm

r_2 = 12 cm **[½]**

And capacity of bucket = 12308.8 cm^3

We know that capacity of bucket

$= \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$ **[½]**

$= 3.14 \times \frac{h}{3}[400 + 144 + 240]$

$= 3.14 \times \frac{h}{3} \times 784$

So we have $\frac{h}{3} \times 3.14 \times 784 = 12308.8$ **[½]**

$h = \frac{12308.8 \times 3}{3.14 \times 784}$

= 15 cm **[½]**

Now, the slant height of the frustum,

$l = \sqrt{h^2 + (r_1 - r_2)^2}$ **[½]**

$= \sqrt{15^2 + 8^2}$

$= \sqrt{289}$ **[½]**

= 17 cm

Area of metal sheet used in making it

$= \pi r_2^2 + \pi(r_1 + r_2) l$ **[½]**

= 3.14 × [144 + (20 + 12) × 17]

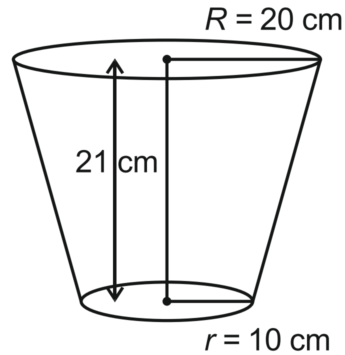
= 2160.32 cm^2 **[½]**

43. For given frustum

h = 21 cm

r = 10 cm

R = 20 cm



[1]

Volume of frustum = $\frac{1}{3}\pi(r^2 + R^2 + rR)h$ **[½]**

$= \frac{1}{3} \times \frac{22}{7}(100 + 400 + 200) \times 21$

$= \frac{1}{3} \times \frac{22}{7} \times 700 \times 21$ **[½]**

= 15400 cm^3	1 litre = 1000 cm^3
= 15.4 litre	1 cm^3 = $\frac{1}{1000}$ litre

∴ Total quantity of milk = 15.4 litre **[1]**

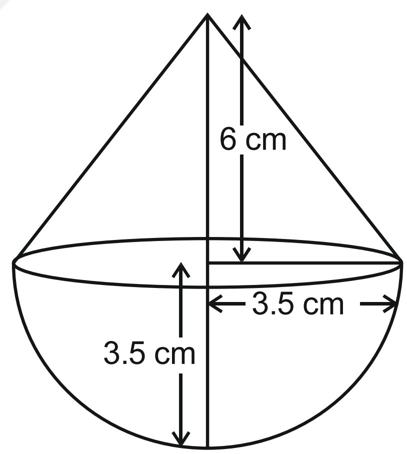
Cost of 1 litre milk = Rs. 40

∴ Cost of 15.4 litre milk = 15.4 × 40

= Rs. 616 **[1]**

OR

According to the question, we get following figure.



[1]

∴ Volume of solid = Volume of cone + volume of hemisphere

⇒ Volume $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ **[1]**

⇒ Volume $= \frac{1}{3}\pi(3.5)^2 \times 6 + \frac{2}{3}\pi(3.5)^3$

$\Rightarrow$ Volume $=\frac{1}{3}\pi(3.5)^2[6+3.5\times 2]$

$\Rightarrow$ Volume $=\frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}[6+7]$ **[1]**

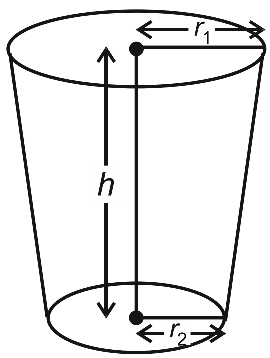
$\Rightarrow$ Volume $=\frac{1}{3}\times\frac{22}{7}\times\frac{49}{4}\times 13$

$\Rightarrow$ Volume $=\frac{1}{3}\times\frac{2002}{4}=\frac{1001}{6}$ **[½]**

$\Rightarrow$ Volume $=166\frac{5}{6}$ cm^3

$\therefore$ Volume of solid $=166\frac{5}{6}$ cm^3 **[½]**

44. Here



r_1 = 20 cm

r_2 = 10 cm **[1]**

h = 30 cm

Volume of the bucket $=\frac{1}{3}\pi h[r_1^2+r_2^2+r_1r_2]$

$=\frac{1}{3}\times\frac{22}{7}\times 30[400+100+200]$ **[1]**

$=\frac{1}{3}\times\frac{22}{7}\times 30\times 700$

= 22000 cm^3 **[1]**

= 22 litres (1000 cm^3 = 1 litre)

Cost of milk = ₹40 × 22 **[1]**

= ₹880

45. (i) Dimensions of cuboid = 10 cm × 10 cm × 8 cm
Dimensions of cone,

Radius, R = 2.1 cm

Height, H = 6 cm

Volume of wood carved out

= Volume of 5 cones = $\frac{1}{3}(\pi)R^2H\times 5$ **[1]**

= $5\times\frac{1}{3}\times\frac{22}{7}\times(2.1)^2\times 6$ = 138.6 cm^3 **[1]**

(ii) Volume of the wood in the final product = Volume of cuboid – Volume of wood carved out **[1]**

= (10 × 10 × 8 – 138.6) cm^3 **[½]**

= 661.4 cm^3 **[½]**

46. (1) For cylinder,

height, H = 9 m

radius, R = 15 m

For cone,

height, h = 8 m

radius, R = 15 m

Slant height, l = $\sqrt{8^2+15^2}=17$ m **[½]**

Area of canvas used in making the tent

= Curved surface area of cylinder

+ Curved surface area of cone **[½]**

= $2\pi RH + \pi Rl = \pi R(2H + l)$ **[½]**

= $\frac{22}{7}\times 15\,(2\times 9+17)$ **[½]**

= 1650 m^2 **[1]**

(2) Total canvas used to make tent

= Curved surface area of tent

+ Canvas wasted during stitching

= 1650 + 30 = 1680 m^2 **[½]**

Cost of canvas = ₹(1680 × 200)

= ₹3,36,000 **[½]**

47. Radius of the bigger end of the frustum (bucket) of cone = R = 20 cm **[½]**

Radius of the smaller end of the frustum (bucket) of the cone = r = 8 cm **[½]**

Height = 16 cm **[½]**

Volume = $1/3\pi rh$ $[R^2 + r^2 + R \times r]$ **[½]**

= 1/3 × 22/7 × 16 $[20^2 + 8^2 + 20 \times 8]$

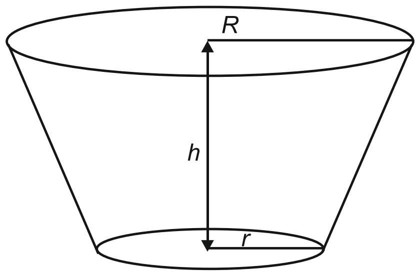
= 352/21 [400 + 64 + 160] **[½]**

= (352 × 624)/21

= 219648/21

= 10459.43 cu. cm **[½]**

Now,



Slant height of the frustum $= l = \sqrt{(R-r)^2 + h^2}$ **[½]**

$l = \sqrt{(20-8)^2 + 16^2}$

$l = \sqrt{12^2 + 16^2}$

$l = \sqrt{144 + 256}$

$l = \sqrt{400}$

l = 20 cm **[½]**

Slant height is 20 cm

Now,

Surface area = $\pi[r^2 + (R + r) \times l]$ **[1]**

$= 22/7[8^2 + (20 + 8) \times 20]$ **[½]**

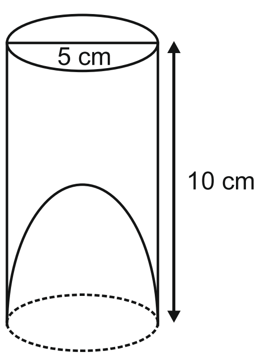
$= \frac{22}{7}[64 + 560]$

$= \frac{22}{7} \times 624$

$= \frac{13728}{7}$ **[½]**

= 1961.14 cm^2

48. Apparent capacity of the glass = Volume of cylinder **[½]**



Actual capacity of the glass = Volume of cylinder – Volume of hemisphere **[½]**

Volume of the cylindrical glass = $\pi r^2 h$ **[½]**

$= 3.14 \times (2.5)^2 \times 10$

$= 3.14 \times 2.5 \times 2.5 \times 10$

$= 3.14 \times 6.25 \times 10$ **[½]**

= 196.25 cm^3

Volume of hemisphere $= \frac{2}{3}\pi r^3$ **[½]**

$= \frac{2}{3}\pi(2.5)^3$

= 32.7 cm^3 **[½]**

Apparent capacity of the glass = Volume of cylinder = 196.25 cm^3

Actual capacity of the glass

= Total volume of cylinder – volume of hemisphere **[1]**

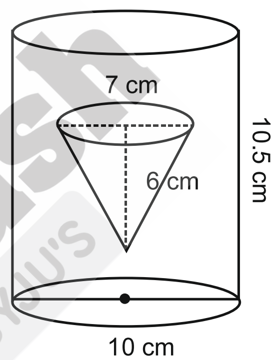
= 196.25 – 32.7 **[½]**

= 163.54 cm^3 **[½]**

Hence, apparent capacity = 196.25 cm^3 **[½]**

Actual capacity of the glass = 163.54 cm^3 **[½]**

49.



Given, internal diameter of the cylinder = 10 cm

Internal radius of the cylinder = 5 cm **[½]**

and height of the cylinder = 10.5 cm

Similarly, diameter of the cone = 7 cm **[½]**

Radius of the cone = 3.5 cm and Height of the cone = 6 cm

(i) Volume of water displaced out of cylindrical vessel = volume of cone **[1]**

$= \frac{1}{3}\pi r^2 h$ **[½]**

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 6 = 77 \text{ cm}^3$ **[1]**

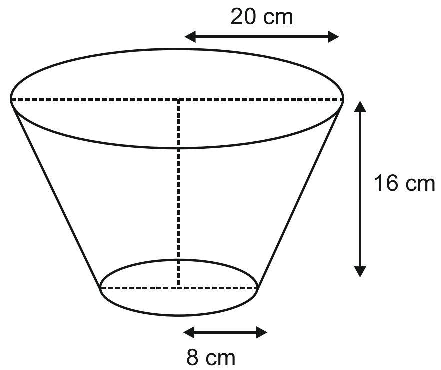
(ii) Volume of water left In the cylindrical vessel = volume of cylinder – volume of cone **[1]**

$= \pi R^2 H$ – Volume of cone **[½]**

$= \frac{22}{7} \times 5 \times 5 \times 10.5 - 77$

= 825 – 77 = 748 cm^3 **[1]**

50.



Let the radius of lower end of the frustum be r = 8 cm **[½]**

Let the radius of upper end of the frustum be R = 20 cm **[½]**

Let the height of the frustum be h cm

Volume of the frustum

$$\frac{\pi}{3}h\left(R^2+r^2+Rr\right)=10459\frac{3}{7}=\frac{73216}{7}$$ **[1]**

Therefore, substituting the value of R and r.

$$\frac{22}{7}\times\frac{1}{3}h\left(20^2+8^2+20\times8\right)=\frac{73216}{7}$$

$$h(400+64+160)=\frac{73216}{7}\times\frac{7}{22}\times3$$

h × 624 = 9984

$$h=\frac{9984}{624}=16\text{ cm}$$ **[1]**

Total surface area of the container

$$=\pi(R+r)\sqrt{(R-r)^2+h^2}+\pi r^2$$ **[1]**

$$=\frac{22}{7}(20+8)\sqrt{(20-8)^2+16^2}+\frac{22}{7}\times8^2$$ **[½]**

$$=\frac{22}{7}\times28\sqrt{12^2+16^2}+\frac{22}{7}\times64$$

$$=\frac{22}{7}\times28\sqrt{144+256}+\frac{22}{7}\times64$$

$$=\frac{22}{7}\left(28\times\sqrt{400}+64\right)=\frac{22}{7}(28\times20+64)$$

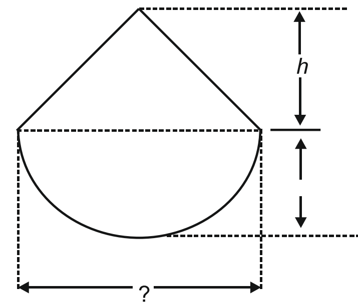
$$=\frac{22}{7}(560+64)=\frac{22}{7}\times624$$ **[½]**

Cost of 1 cm square metal sheet is 1. 40 ₹

Cost of required sheet =

$$\frac{22}{7}\times624\times1.40=2745.60\text{ ₹}$$ **[1]**

51.



Radius of base of the cone = r = 21 cm **[½]**

Let the height of the cone be h cm

Volume of the cone = 2/3 volume of the hemisphere **[½]**

$$\frac{1}{3}\pi r^2h=\frac{2}{3}\times\frac{2}{3}\pi r^3$$ **[½]**

$$\Rightarrow\quad h=\frac{4}{3}r=\frac{4}{3}\times21=28\text{ cm}$$ **[½]**

Surface area of the toy = lateral surface area of cone + curved surface area of hemisphere **[1]**

$$\pi r\sqrt{r^2+h^2}+2\pi r^2$$ **[1]**

$$=\frac{22}{7}\times21\times\sqrt{21^2+28^2}+2\times\frac{22}{7}\times21\times21$$ **[1]**

$$=66\times\sqrt{441+784}+2772$$

= 66 × 35 + 2772

= 2310 + 2772 = 5082 cm^2 **[1]**

52. Let the level of water in the pond rises by 21 cm in t hours.

Speed of water = 15 km/hr

= 15000 m/hr **[½]**

Diameter of pipe = 14 cm $=\frac{14}{100}$ m

∴ Radius of the pipe, $r=\frac{7}{100}$ m **[½]**

Volume of water flowing out of the pipe in 1 hour = $\pi r^2 h$ **[½]**

$$=\frac{22}{7}\times\left(\frac{7}{100}\text{ m}\right)^2\times15000\text{ m}$$

= 231 m^3 **[1]**

∴ Volume of water flowing out of the pipe in t hours = 231t m^3 **[½]**

Volume of water in the cuboidal pond

$$=50\text{ m}\times44\text{ m}\times\frac{21}{100}\text{ m (Volume of cuboid = lbh)}$$

= 462 m^3 **[1]**

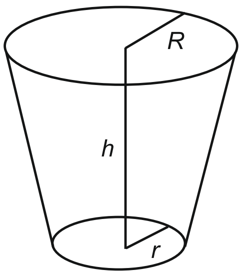
Volume of water flowing out of the pipe in t hours = Volume of water in the cuboidal pond **[1]**

$\therefore \quad 231t = 462$

$\Rightarrow \quad t = \dfrac{462}{231} = 2$ hrs

Thus, the water in the pond rise by 21 cm in 2 hours. **[1]**

53.



Here, R = 28 cm and r = 21 cm, **[1]**

Volume of frustum = 28.49 L

$= 28.49 \times 1000 \text{ cm}^3$

$= 28490 \text{ cm}^3$ **[1]**

Now, volume of frustum $= \dfrac{\pi h}{3}\left(R^2 + Rr + r^2\right)$ **[1½]**

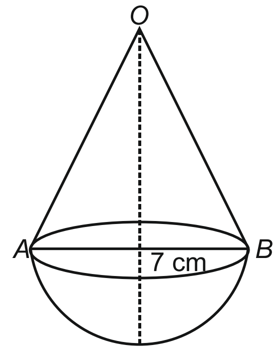
$\Rightarrow \quad \dfrac{22\text{ h}}{7 \times 3}\left(28^2 + 28 \times 21 + 21^2\right) = 28490$ **[1]**

$\Rightarrow \quad \dfrac{22}{21} h \times 1813 = 28490$ **[½]**

$\Rightarrow \quad h = \dfrac{28490 \times 21}{22 \times 1813} = 15$ cm

Hence the height of bucket is 15 cm. **[1]**

54.



Radius of hemi-sphere = 7 cm **[½]**

Radius of cone = 7 cm **[½]**

Height of cone = diameter = 14 cm **[½]**

Volume of solid = Volume of cone + Volume of hemi-sphere **[1]**

$= \dfrac{1}{3}\pi r^2 h + \dfrac{2}{3}\pi r^3$ **[1]**

$= \dfrac{1}{3}\pi r^2 (h + 2r)$ **[½]**

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 49(14 + 14)$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 49 \times 28$ **[1]**

$= \dfrac{22 \times 7 \times 28}{3} = \dfrac{4312}{3} \text{ cm}^3$ **[1]**

14 : Statistics

1.

Class	Class marks	
10 – 25	$\dfrac{10 + 25}{2} = 17.5$	[½]
35 – 55	$\dfrac{35 + 55}{2} = 45$	[½]

2. Answer (b)

3 Median – 2 Mean = Mode

$\Rightarrow$ 3 × 26 – 2 Mean = 29

$\Rightarrow$ Mean = 24.5

Hence, option (b) is correct. **[1]**

3. Mean $= \dfrac{1 + 2 + 3 + 4 \ldots + n}{n}$

$\Rightarrow \quad \dfrac{\left(\dfrac{n(n+1)}{2}\right)}{n} = 15$ **[½]**

$\Rightarrow \quad \dfrac{n+1}{2} = 15$

$\Rightarrow \quad n = 29$ **[½]**

4.

Class	Mid-value (x_i)	Frequency (f_i)	$f_i x_i$
3-5	4	5	20
5-7	6	10	60
7-9	8	10	80
9-11	10	7	70
11-13	12	8	96
Total		$\Sigma f_i = 40$	$\Sigma f_i x_i = 326$

[1]

$\therefore$ $\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{326}{40}$ [½]

$= 8.15$ [½]

OR

Here, the maximum frequency is 12 and the corresponding class is 60-80. So, 60-80 is the modal class such that $l = 60$, $h = 20$, $f_0 = 12$, $f_1 = 10$ and $f_2 = 6$. [1]

$\therefore$ $\text{Mode} = 60 + \left(\frac{12-10}{2\times12-10-6}\right)\times20$ [½]

$= 60 + \frac{2}{8}\times20$

$= 60 + 5$

$= 65$ [½]

5. $\text{Mode} = l + \frac{(f_m - f_1)}{(2f_m - f_1 - f_2)}\times h$ [½]

$\Rightarrow$ $f_m = 45$

$f_1 = 30$

$f_2 = 42$

$h = 10$ [½]

$l = 40$

$\therefore$ Mode $= 40 + \left(\frac{45-30}{90-72}\right)\times10$ [½]

$= 40 + \left(\frac{15}{18}\times10\right) = 40 + \left(\frac{150}{18}\right) = 40 + 8.33 = 48.33$ [½]

6. Mode = 55 $\Rightarrow$ Modal class is 45 – 60

$\therefore$ $l = 45$, $f_m = 15$, $f_1 = x$, $f_2 = 10$, $h = 15$

Mode $= \ell + \frac{(f_m - f_1)}{(2f_m - f_1 - f_2)}\times h$

$55 = 45 + \left(\frac{15-x}{30-x-10}\right)\times15$ [1]

$10 = \left(\frac{15-x}{20-x}\right)\times15$

$\Rightarrow$ $2(20 - x) = 3(15 - x)$

$\Rightarrow$ $40 - 2x = 45 - 3x$

$\Rightarrow$ $x = 5$ [1]

7. [1]

Class	Frequency	Cumulative frequency
5 – 10	49	49
10 – 15	133	182
15 – 20	63	245
20 – 25	15	260
25 – 30	6	266
30 – 35	7	273
35 – 40	4	277
40 – 45	2	279
45 – 50	1	280

Let N = total frequency

$\therefore$ We have $N = 280$

$\therefore$ $\frac{N}{2} = \frac{280}{2} = 140$ [½]

The cumulative frequency just greater than $\frac{N}{2}$ is 182 and the corresponding class is 10 – 15.

Thus, 10 – 15 is the median class such that $l = 10$, $f = 133$, $F = 49$ and $h = 5$ [½]

$\text{Median} = l + \left(\frac{\frac{N}{2} - F}{f}\right)\times h = 10 + \left(\frac{140-49}{133}\right)\times5$

$= 13.42$ [1]

8. [½]

Class	Frequency
0 - 10	8
10 - 20	10
20 - 30	$10 \to f_0$
30 - 40	$16 \to f_1$
40 - 50	$12 \to f_2$
50 - 60	6
60 - 70	7

Here, 30 – 40 is the modal class, and $l = 30$, $h = 10$ [½]

$\therefore$ $\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)\times h$ [1]

$= 30 + \left(\frac{16-10}{2\times16-10-12}\right)\times10$ [½]

$= 30 + \frac{6}{10}\times10 = 30 + 6 = 36$ [½]

9.

Class	Frequency (f_i)	Class Marks (x_i)	Product (f_ix_i)
10–15	4	12.5	50.00
15–20	10	17.5	175.00
20–25	5	22.5	112.50
25–30	6	27.5	165.00
30–35	5	32.5	162.50
Total	***N* = 30**		$\sum f_i x_i$ = **665.00**

Mean $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{k} f_i x_i$ [1]

$= \frac{\sum_{i=1}^{5} f_i x_i}{N} = \frac{665.0}{30}$ [1]

$= 22.17$ (approx.)

10.

Class	Frequency	c.f.
0–10	6	6
10–20	9	15
20–30	10	25
30–40	8	33
40–50	x	33+x

[½]

Median = 25

$\Rightarrow$ Median class is 20–30

$\Rightarrow$ f = 10, c.f. = 15,

$N = 33 + x$, h = 10 and l = 20 [½]

Median = $l+\left(\dfrac{\frac{N}{2}-\text{cf}}{f}\right)\times h$ [½]

$\Rightarrow$ $25=20+\left(\dfrac{\frac{33+x}{2}-15}{10}\times 10\right)$ [½]

$\Rightarrow$ $5=\dfrac{33+x-30}{2}$ [½]

$\Rightarrow$ 10 = 3 + x

$\therefore$ x = 7 [½]

11. (a) [1]

Class	Class mark (x_i)	Frequency (f_i)	f_ix_i
0 - 10	5	5	25
10 - 20	15	18	270
20 - 30	25	15	375
30 - 40	35	f	35f
40 - 50	45	6	270
Total		$\Sigma f_i = 44 + f$	$\Sigma f_ix_i = 940+35f$

$\text{Mean}(\bar{x})=\dfrac{\sum f_ix_i}{\sum f_i}=\dfrac{940+35f}{44+f}$ [1]

$\Rightarrow 25=\dfrac{940+35f}{44+f}$

$\Rightarrow f$ = 16 [1]

OR

(b) [2]

Class	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - a$	f_id_i
0 - 5	8	2.5	–10	–80
5 - 10	7	7.5	–5	–30
10 - 15	10	12.5=a	0	0
15 - 20	13	17.5	5	65
20 - 25	12	22.5	10	120
Total	N = 50			$\Sigma f_id_i = 70$

Let assumed mean be a = 12.5 and N = 50

$\therefore$ $\bar{x}=a+\dfrac{1}{N}\sum_{i=1}^{5}f_id_i$

$=12.5+\dfrac{1}{50}\times 70$

= 12.5 + 1.4 = 13.9 [1]

12. [1]

Height (in cm)	Number of Students (f_i)	Cumulative frequency
130 – 135	4	4
135 – 140	11	15
140 – 145	12	27
145 – 150	7	34
150 – 155	10	44
155 – 160	6	50

N = 50, so $\dfrac{N}{2}=25.$ So, median class lies in the class 140 – 145, then

l = 140

c.f. = 15

f = 12

h = 5

Median $=l+\dfrac{\left(\frac{N}{2}-\text{c.f.}\right)}{f}\times h$ [1]

$=140+\left(\dfrac{25-15}{12}\right)5$ [½]

= 144.166.....

Median height of students = 144.17 (approx.) [½]

13.

Class	Mid values x_i	Frequency f_i	$d_i = x_i$ –18	$u_i=\frac{x_i-18}{2}$	f_iu_i
11 – 13	12	3	–6	–3	–9
13 – 15	14	6	–4	–2	–12
15 – 17	16	9	–2	–1	–9
17 – 19	18	13	0	0	0
19 – 21	20	f	2	1	f
21 – 23	22	5	4	2	10
23 – 25	24	4	6	3	12
		$\Sigma f_i = 40 + f$			

[1]

$\Sigma f_iu_i = f - 8$

We have

h = 2; A = 18, $N = 40 + f, \Sigma f_iu_i = f - 8$, $\bar{X}=18$ [½]

$\therefore$ Mean $=A+h\left\{\dfrac{1}{N}\sum f_iu_i\right\}$ [1]

$18=18+2\left\{\dfrac{1}{40+f}(f-8)\right\}$ [½]

$\dfrac{2(f-8)}{40+f}=0$ [½]

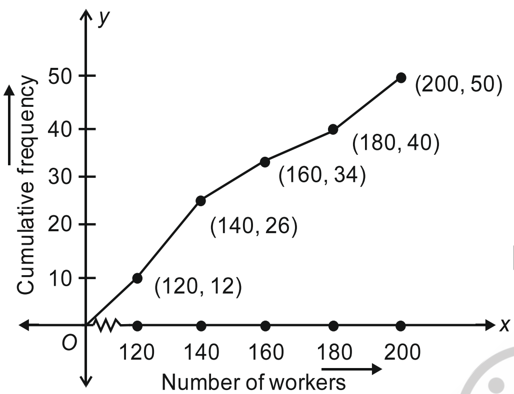
$f - 8 = 0$

$f = 8$ [½]

14. **[1]**

Daily income	Frequency	Income less than	Cumulative frequency
100 – 120	12	120	12
120 – 140	14	140	26
140 – 160	8	160	34
160 – 180	6	180	40
180 – 200	10	200	50

Using these values we plot the points (120, 12) (140, 26) (160, 34), (180, 40) (200, 50) on the axes to get less than ogive **[1]**



[2]

15. **[1]**

Class	Frequency	Cumulative Frequency
0 – 10	f_1	f_1
10 – 20	5	$5 + f_1$
20 – 30	9	$14 + f_1$
30 – 40	12	$26 + f_1$
40 – 50	f_2	$26 + f_1 + f_2$
50 – 60	3	$29 + f_1 + f_2$
60 – 70	2	$31 + f_1 + f_2$
Total = 40 = n		

$f_1 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$

$f_1 + f_2 = 40 - 31 = 9$ ...(i)

Median = 32.5 [Given]

$\therefore$ Median Class is 30 – 40

$\ell = 30$, $h = 10$, $cf = 14 + f_1$, $f = 12$ **[1]**

$$\text{Median} = \ell + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$ **[½]**

$$32.5 = 30 + \left[\frac{\frac{40}{2} - (14 + f_1)}{12}\right] \times 10$$ **[½]**

$$2.5 = \frac{10}{12}(20 - 14 - f_1)$$

$3 = 6 - f_1$

$f_1 = 3$ **[½]**

On putting in (i),

$f_1 + f_2 = 9$

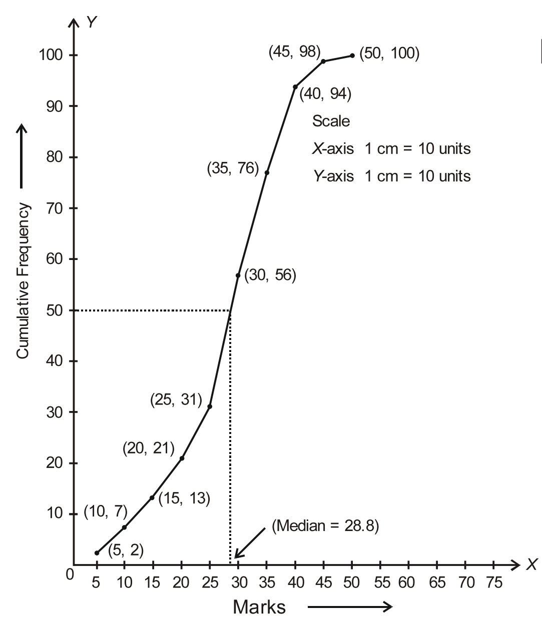
$f_2 = 9 - 3$ [$\because f_1 = 3$]

$= 6$ **[½]**

16. **[2]**

Marks	Number of students	Marks less than	Cumulative frequency
0-5	2	Less than 5	2
5-10	5	Less than 10	7
10-15	6	Less than 15	13
15-20	8	Less than 20	21
20-25	10	Less than 25	31
25-30	25	Less than 30	56
30-35	20	Less than 35	76
35-40	18	Less than 40	94
40-45	4	Less than 45	98
45-50	2	Less than 50	100

Let us now plot the points corresponding to the ordered pairs (5, 2), (10, 7), (15, 13), (20, 21), (25, 31), (30, 56), (35, 76), (40, 94), (45, 98), (50, 100). Join all the points by a smooth curve.



[1]

Locate $\frac{n}{2} = \frac{100}{2} = 50$ on Y-axis

From this point draw a line parallel to X-axis cutting the curve at a point. From this point, draw a perpendicular to X-axis. The point of intersection of perpendicular with the X-axis determines the median of the data.

Therefore median = 28.8 **[1]**

17. **[2]**

Classes	x_i	f_i	$A = 50$, $d_i = x_i - A$	$u_i = \frac{x_i - A}{h}$, $h = 20$	f_iu_i
0-20	10	20	10 – 50 = –40	–2	–40
20-40	30	35	30 – 50 = –20	–1	–35
40-60	50	52	50 – 50 = 0	0	0
60-80	70	44	70 – 50 = 20	1	44
80-100	90	38	90 – 50 = 40	2	76
100-120	110	31	110 – 50 = 60	3	93
		$\Sigma f_i = 220$			$\Sigma f_iu_i = 138$

$$\bar{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$ **[1]**

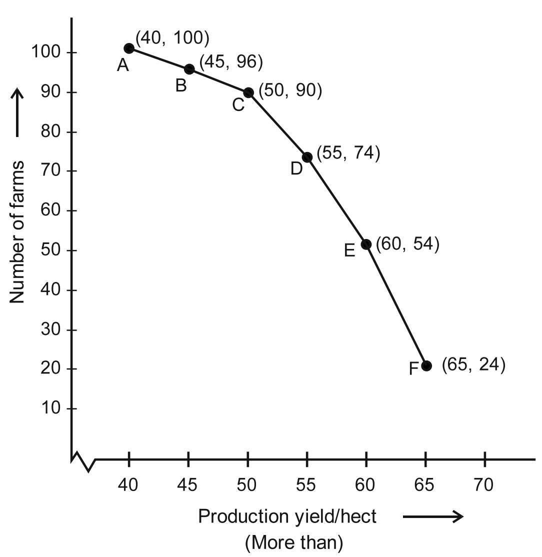
$$= 50 + \frac{138}{220} \times 20$$

$$= 50 + 12.55$$

$$= 62.55$$ **[1]**

18. **[2]**

Production yield/hect	Number of farms	Production yield more than/hect	Cumulative frequency
40–45	4	40	100
45–50	6	45	96
50–55	16	50	90
55–60	20	55	74
60–65	30	60	54
65–70	24	65	24



[2]

OR

Class	Frequency F_i	c.f.
0–100	2	2
100–200	5	7
200–300	x	7 + x
300–400	12	19 + x
400–500	17	36 + x
500–600	20	56 + x
600–700	y	56 + x + y
700–800	9	65 + x + y
800–900	7	72 + x + y
900–1000	4	76 + x + y = N

[1]

Here $N = 100$

$\Rightarrow$ $76 + x + y = 100$

$x + y = 24$ ...(i) **[½]**

Median = 525

Median class = 500 – 600

$l = 500$, $h = 100$

$f = 20$

c.f. = 36 + x **[½]**

$$\text{Median} = l + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$ **[½]**

$$\Rightarrow \quad 525 = 500 + \left[\frac{50 - 36 - x}{20}\right] \times 100$$ **[½]**

$\Rightarrow$ $25 = (14 - x)5$

$\Rightarrow$ $14 - x = 5$

$\Rightarrow$ $x = 9$ **[½]**

Now from (i)

$9 + y = 24$

$y = 15$ **[½]**

19. **[1]**

Class	Frequency	Class mark (x_i)	x_if_i
0 – 20	6	10	60
20 – 40	8	30	240
40 – 60	10	50	500
60 – 80	12	70	840
80 – 100	6	90	540
100 – 120	5	110	550
120 – 140	3	130	390
	$\Sigma f_i = 50$		$\Sigma f_ix_i = 3120$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{3120}{50}$$

$= 62.4$ **[1]**

Class	f	Less than cumulative frequency
0 – 20	6	6
20 – 40	8	14
40 – 60	10	24
60 – 80	12	36
80 – 100	6	42
100 – 120	5	47
120 – 140	3	50

$\therefore \quad n = \Sigma f_i = 50$

$$\frac{n}{2} = 25$$

$\therefore \quad$ Median class = 60 – 80 **[1]**

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times h$$

$$\text{Median} = 60 + \left(\frac{25 - 24}{12}\right) \times 20$$

Median = 61.66 **[1]**

Mode :

Maximum class frequency = 12

$\therefore \quad$ Model class = 60 – 80 **[1]**

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \left(\frac{12 - 10}{2 \times 12 - 10 - 6}\right) \times 20$$

$= 65$ **[1]**

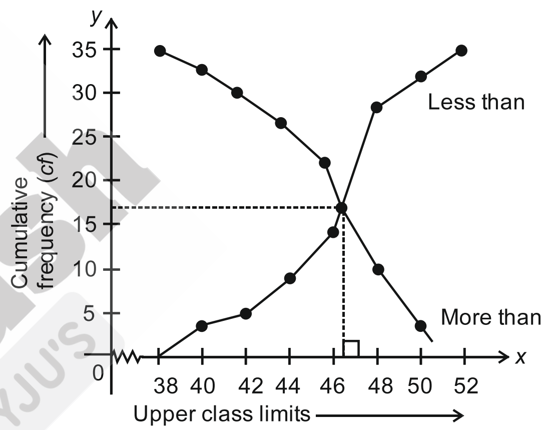
20. **[2]**

Weight	Cumulative (More than type)
More than 38	35
More than 40	32
More than 42	30
More than 44	26
More than 46	21
More than 48	7
More than 50	3
More than 52	0

Weight (in kg) Upper class limits	Number of students (Cumulative frequency)
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
More than 52	35

[2]

Taking upper class limits on x-axis and their respective cumulative frequency on y-axis its ogive give can be drawn as follows:



Here, $n = 35$

So,

$$\frac{n}{2} = 17.5$$

There is a intersection point of less than and more than ogive mark that point A whose ordinate is 17.5 and its x-coordinate is 46.5. Therefore, median of this data is 46.5. **[2]**

21. **[1]**

Class	f_i	Class mark(x_i)	$F_i x_i$
0 – 10	4	5	20
10 – 20	4	15	60
20 – 30	7	25	175
30 – 40	10	35	350
40 – 50	12	45	540
50 – 60	8	55	440
60 – 70	5	65	325
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 1910$

mean $= \frac{1910}{50} = 38.2$ **[1]**

Class	Frequency	Cumulative frequency
0 – 10	4	4
10 – 20	4	8
20 – 30	7	15
30 – 40	10	25
40 – 50	12	37
50 – 60	8	45
60 – 70	5	50
	$N = 50$	

[1]

$\frac{N}{2} = 25$

Cumulative frequency just greater than 25 is 37.

∴ Median class 40–50

$$\text{Median} = \ell + \left(\frac{\frac{N}{2} - C.f}{f}\right) \times h$$

Here $\ell = 40$

$N = 50$

$Cf = 25$, $f = 12$, $h = 10$

$$\text{Median} = 40 + \left(\frac{25-25}{12}\right)10 = 40 + 0$$

$\boxed{\text{Median} = 40}$ **[1]**

Mode :

Maximum frequency = 12 so modal class 40 – 50

$$\text{mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

Here $\ell = 40$, $h = 10$

$f_0 = 10$ $f_1 = 12$ $f_2 = 8$

$$\text{Mode} = 40 + \left(\frac{12-10}{2\times12-10-8}\right)\times10$$

Mode = 40 + 3.33

= 43.33 **[2]**

15 : Probability

1. Total possible outcomes = 6

   Outcomes which are less than 3 = 1, 2 **[½]**

   Probability $= \frac{2}{6}$

   $= \frac{1}{3}$ **[½]**

2. Two coins are tossed simultaneously

   Total possible outcomes = {HH, HT, TH, TT}

   Number of total outcomes = 4

   Favourable outcomes for getting exactly

   One head = {HT, TH} **[½]**

   Probability $= \frac{2}{4} = \frac{1}{2}$ **[½]**

3. A card is drawn from well shuffled 52 playing cards so total no of possible outcomes = 52

   Number of face cards = 12

   Number of Red face cards = 6 **[½]**

   Probability of drawing $= \frac{6}{52}$

   A red face card $= \frac{3}{26}$ **[½]**

4. Answer (C)

   Number of aces in deck of cards = 4

   Probability of drawing an ace card

   $= \frac{\text{Number of ace}}{\text{Total cards}} = \frac{4}{52}$ **[½]**

   Probability that the card is not an Ace

   $= 1 - \frac{4}{52} = \frac{12}{13}$ **[½]**

5. Answer (C)

   When two dice are thrown together, the total number of outcomes is 36.

   Favourable outcomes = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} **[½]**

   ∴ Required probability

   $= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{6}{36} = \frac{1}{6}$ **[½]**

6. Answer (A)

$S = \{1, 2, 3, 4, 5, 6\}$

Let event E be defined as 'getting an even number'.

$n(E) = \{2, 4, 6\}$ **[½]**

$$\therefore \quad P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} = \frac{3}{6}$$

$$= \frac{1}{2}$$ **[½]**

7. Answer (C)

$S = \{1, 2, 3,..90\}$

$n(S) = 90$

The prime number less than 23 are 2, 3, 5, 7, 11, 13, 17, and 19.

Let event E be defined as 'getting a prime number less than 23'. **[½]**

$n(E) = 8$

$$\therefore \quad P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$$

$$= \frac{8}{90} = \frac{4}{45}$$ **[½]**

8. Answer (D)

Possible outcomes on rolling the two dice are given below :

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} **[½]**

Total number of outcomes = 36

Favourable outcomes are given below:

{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}

Total number of favourable outcomes = 9

• $\therefore$ Probability of getting an even number on both dice

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{9}{36} = \frac{1}{4}$$ **[½]**

9. Answer (C)

Total number of possible outcomes = 30

Prime numbers from 1 to 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Total number of favourable outcomes = 10 **[½]**

$\therefore$ Probability of selecting a prime number from 1 to 30

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{10}{30} = \frac{1}{3}$$ **[½]**

10. Two dice are tossed

S = [(1, 1), (1, 2), (1, 3),(1, 4), (1, 5), (1, 6),

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)] **[½]**

Total number of outcomes when two dice are tossed = 6 × 6 = 36

Favourable events of getting product as 6 are:

(1 × 6 = 6), (6 × 1 = 6), (2 × 3 = 6), (3 × 2 = 6)

i.e. (1, 6), (6, 1), (2, 3), (3, 2)

Favourable events of getting product as 6 = 4

$$\therefore \quad P(\text{getting product as 6}) = \frac{4}{36} = \frac{1}{9}$$ **[½]**

11. There are 26 red cards including 2 red queens.

Two more queens along with 26 red cards will be 26 + 2 = 28

• $\therefore$ P(getting a red card or a queen) $= \frac{28}{52}$ **[½]**

$\therefore$ P(getting neither a red card nor a queen)

$$= 1 - \frac{28}{52} = \frac{24}{52} = \frac{6}{13}$$ **[½]**

12. Probability of selecting rotten apple

$$= \frac{\text{Number of rotten apples}}{\text{Total number of apple}}$$ **[½]**

$$\therefore \quad 0.18 = \frac{\text{Number of rotten apples}}{900}$$

Number of rotten apples = 900 × 0.18 = 162 **[½]**

13. Answer (d)

Favourable outcomes are 4, 8, 12, i.e., 3 outcomes and total number of outcomes = 15

$\therefore$ Required probability = $\frac{3}{15} = \frac{1}{5}$

Option (d) is correct. **[1]**

14. Total outcomes = 36

Favourable outcomes {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)} **[½]**

Number of favourable outcomes = 5

$P(\text{sum}\,8) = \frac{5}{36}$ **[½]**

15. $n(s)$ = Total number of alphabets in English = 26.

$n(E)$ = Total number of consonant in English alphabet = 21 **[½]**

$\therefore$ Probability (Chosen letter is a consonant)

$= \frac{n(E)}{n(s)}$

$= \frac{21}{26}$ **[½]**

16. Total number of outcomes = 6

Number of favourable outcomes = 2 **[½]**

P(getting a number less than 3) = $\frac{2}{6}$

$= \frac{1}{3}$ **[½]**

OR

Required probability

= 1 – Probability of winning a game **[½]**

= 1 – 0.07

= 0.93 **[½]**

17. Answer (b) **[1]**

Total possible outcomes = {HT, TH, HH, TT}

$\therefore$ Required probability = $\frac{1}{2}$

18. Answer (d) **[1]**

$P(\bar{E})$ or P (not E) = 1 – P(E)

= 1 – 0.65

= 0.35

19. Answer (b) **[1]**

$\text{Probability} = \frac{\text{Number of favourable events in sample space}}{\text{Total number of events in sample space}}$

$P(\text{Blue balls}) = \frac{6}{16+8+6} = \frac{6}{30} = \frac{1}{5}$

20. Answer (c) **[1]**

P(Not happening of an event)

= 1 – P (Happening of the event)

= 1 – 0.02

= 0.98

21. Answer (a) **[1]**

$x = 1$ $\quad [\because P(E) + P(\bar{E}) = 1]$

$\Rightarrow x^3 - 3 = -2$

22. Answer (a) **[1]**

P(Neither ace nor spade) = 1 – P (Ace or spade)

$= 1 - \frac{16}{52}$

$= \frac{9}{13}$

23. Answer (d) **[1]**

Probability of any event always $0 \le P(E) \le 1$.

24. Answer (d) **[1]**

(E) = Outcomes not possible are {(5, 5) (1, 5) (2, 5) (3, 5) (4, 5) (6, 5) (5, 1) (5, 2) (5, 3) (5, 4) (5, 6)}

$n(E) = 11$

Total outcomes = 36

$\therefore$ Number of possible outcomes = 36 – 11

= 25

$\therefore$ Probability $= \frac{25}{36}$

25. A ticket is drawn at random from 40 tickets

Total outcomes = 40

Out of the tickets numbered from 1 to 40 the number of tickets which is multiple of 5 = 5, 10, 15, 20, 25, 30, 35, 40

= 8 tickets

$\therefore$ Favorable outcomes = 8 **[1]**

• $\therefore$ Probability $= \frac{8}{40}$

$= \frac{1}{5}$ **[1]**

26. The total number of outcomes is 50.

Favourable outcomes = {12, 24, 36, 48} **[1]**

$\therefore$ Required probability

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{4}{50} = \frac{2}{25}$ **[1]**

27. Let E be the event that the drawn card is neither a king nor a queen.

Total number of possible outcomes = 52

Total number of kings and queens = 4 + 4 = 8

Therefore, there are 52 – 8 = 44 cards that are neither king nor queen. **[1]**

Total number of favourable outcomes = 44

$\therefore$ Required probability = $P(E)$

$$= \frac{\text{Favourable outcomes}}{\text{Total number of outcomes}} = \frac{44}{52} = \frac{11}{13}$$ **[1]**

28. Rahim tosses two coins simultaneously. The sample space of the experiment is {HH, HT, TH, and TT}.

Total number of outcomes = 4

Outcomes in favour of getting at least one tail on tossing the two coins = {HT, TH, TT} **[1]**

Number of outcomes in favour of getting at least one tail = 3

• $\therefore$ Probability of getting at least one tail on tossing the two coins

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{4}$$ **[1]**

29. Sample space = S = {(1, 1) (1, 2)...,(6, 6)}

$n(s) = 36$

(i) A = getting a doublet

A = {(1, 1), (2, 2) ...., (6, 6)}

$n(A) = 6$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$ **[1]**

(ii) B = getting sum of numbers as 10

B = {(6, 4), (4, 6), (5, 5)}

$n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$ **[1]**

30. An integer is chosen at random from 1 to 100

Therefore $n(S) = 100$

(i) Let A be the event that number chosen is divisible by 8

$\therefore$ A = {8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96}

$\therefore$ $n(A) = 12$

Now, P (that number is divisible by 8)

$$= P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{12}{100} = \frac{6}{50} = \frac{3}{25}$$ **[1]**

$$\boxed{P(A) = \frac{3}{25}}$$

(ii) Let 'A' be the event that number is not divisible by 8.

$\therefore$ $P(A') = 1 - P(A)$

$$= 1 - \frac{3}{25} \qquad \boxed{P(A') = \frac{22}{25}}$$ **[1]**

31. Total possible outcomes are (HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT) *i.e.*, 8.

The favourable outcomes to the event E 'Same result in all the tosses' are TTT, HHH. **[1]**

So, the number of favourable outcomes = 2

$$\therefore \quad P(E) = \frac{2}{8} = \frac{1}{4}$$

Hence, probability of losing the game = $1 - P(E)$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$ **[1]**

32. Total outcomes = 1, 2, 3, 4, 5, 6

Prime numbers = 2, 3, 5

Numbers lie between 2 and 6 = 3, 4, 5

(i) P (Prime Numbers) $= \frac{3}{6} = \frac{1}{2}$ **[1]**

(ii) P (Numbers lie between 2 and 6) $= \frac{3}{6} = \frac{1}{2}$ **[1]**

33. Let the number of blue balls be x.

So, total number of balls in the bag = $(x + 5)$ **[½]**

According to the question,

$$\frac{x}{x+5} = 3 \times \frac{5}{x+5}$$ **[1]**

$\Rightarrow$ $x = 15$

$\therefore$ Number of blue balls = 15 **[½]**

34. Total number of outcomes = 6 × 6 = 36 **[½]**

Favourable outcomes = {(1, 1)(1, 2)(1, 3)(2, 1)(2, 2)(3, 1)} **[½]**

Number of favourable outcomes = 6 **[½]**

$\therefore$ P(less than 5) = $\frac{6}{36} = \frac{1}{6}$ **[½]**

OR

In month of November 4 sundays are fixed.

But there are two extra days. They may be {(Sun, Mon), (Mon, Tues), (Tues, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Sat), (Sat, Sun)} **[1]**

Number of favourable outcomes = 2 **[½]**

$\therefore$ Required probability (5 sundays) = $\frac{2}{7}$ **[½]**

35. Let E be the event of getting square of a number less than or equal to 4.

S be the sample space. Then,

$S = \{-3, -2, -1, 0, 1, 2, 3\}$ **[½]**

$\Rightarrow n(S) = 7$

and, $E = \{-2, -1, 0, 1, 2\}$

$\Rightarrow n(E) = 5.$ **[½]**

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{5}{7}$ **[1]**

36. Total outcomes = 6 × 6 = 36

(i) Total outcomes when 5 comes up on either dice are (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) (6, 5) (4, 5) (3, 5) (2, 5) (1, 5)

P (5 will come up on either side) $\frac{11}{36}$ **[1]**

P (5 will not come up) $= 1 - \frac{11}{36}$

$= \frac{25}{36}$

(ii) P (5 will come at least once) $= \frac{11}{36}$ **[1]**

(iii) P (5 will come up on both dice) $= \frac{1}{36}$ **[1]**

37. Total number of cards $= \frac{35-1}{2} + 1$

= 18 **[1]**

(i) Favourable outcomes = {3, 5, 7, 11, 13}

P(prime number less than 15) $= \frac{5}{18}$ **[1]**

(ii) Favourable outcomes = {15}

P(a number divisible by 3 and 5) $= \frac{1}{18}$ **[1]**

38. Two dice are rolled once. So, total possible outcomes = 6 × 6 = 36 **[1]**

Product of outcomes will be 12 for

(2, 6), (6, 2), (3, 4) and (4, 3). **[1]**

Number of favourable cases = 4

Probability $= \frac{4}{36} = \frac{1}{9}$ **[1]**

39. A disc drawn from a box containing 80 **[1]**

Total possible outcomes = 80

Number of cases where the disc will be numbered perfect square = 8

Perfect squares less than 80 **[1]**

= 1, 4, 9, 16, 25, 36, 49, 64

Probability $= \frac{8}{80} = \frac{1}{10}$ **[1]**

40. Total number of outcomes = 52

(i) Probability of getting a red king

Here the number of favourable outcomes = 2

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{2}{52}$$

$= \frac{1}{26}$ **[1]**

(ii) Favourable outcomes = 12

Probability $= \frac{12}{52} = \frac{3}{13}$ **[1]**

(iii) Probability of queen of diamond.

Number of queens of diamond = 1, hence

Probability

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{52}$$ **[1]**

41. Here the jar contains red, blue and orange balls.

Let the number of red balls be x.

Let the number of blue balls be y.

Number of orange balls = 10

Total number of balls = $x + y + 10$

Now, let P be the probability of drawing a ball from the jar

P(a red ball) $= \frac{x}{x+y+10}$

$\Rightarrow \frac{1}{4} = \frac{x}{x+y+10}$

$\Rightarrow 4x = x + y + 10$

$\Rightarrow 3x - y = 10$ …(i) **[1]**

Next,

$P(\text{a blue ball}) = \frac{y}{x+y+10}$

$\Rightarrow \frac{1}{3} = \frac{y}{x+y+10}$

$\Rightarrow 3y = x + y + 10$

$\Rightarrow 2y - x = 10$ …(ii) **[1]**

Multiplying eq. (i) by 2 and adding to eq. (ii), we get

$$\begin{array}{rcrcr} 6x & - & 2y & = & 20 \\ -x & + & 2y & = & 10 \\ \hline & & 5x & = & 30 \end{array}$$

$\Rightarrow x = 6$

Substitute $x = 6$ in eq. (i), we get $y = 8$

Total number of balls $= x + y + 10 = 6 + 8 + 10 = 24$

Hence, total number of balls in the jar is 24. **[1]**

42. When three coins are tossed together, the possible outcomes are

HHH, *HTH*, *HHT*, *THH*, *THT*, *TTH*, *HTT*, *TTT*

$\therefore$ Total number of possible outcomes = 8

(i) Favourable outcomes of exactly two heads are *HTH*, *HHT*, *THH*

$\therefore$ Total number of favourable outcomes = 3

$\therefore P(\text{exactly two heads}) = \frac{3}{8}$ **[1]**

(ii) Favourable outcomes of at least two heads are *HHH*, *HTH*, *HHT*, *THH*

$\therefore$ Total number of favourable outcomes = 4

$\therefore P(\text{at least two heads}) = \frac{4}{8} = \frac{1}{2}$ **[1]**

(iii) Favourable outcomes of at least two tails are *THT*, *TTH*, *HTT*, *TTT*

$\therefore$ Total number of favourable outcomes = 4

$\therefore P(\text{at least two tails}) = \frac{4}{8} = \frac{1}{2}$ **[1]**

43. Bag contains 15 white balls.

Let say there be x black balls.

Probability of drawing a black ball

$P(B) = \frac{x}{15+x}$ **[1]**

Probability of drawing a white ball

$P(W) = \frac{15}{15+x}$

Given that $P(B) = 3P(W)$ **[1]**

$\therefore \frac{x}{15+x} = \frac{3\times15}{15+x}$

$x = 45$ **[1]**

Number of black balls = 45

44. The group consists of 12 persons.

$\therefore$ Total number of possible outcomes = 12

Let A denote event of selecting persons who are extremely patient.

$\therefore$ Number of outcomes favourable to A is 3. **[1]**

Let B denote event of selecting persons who are extremely kind or honest. Number of persons who are extremely honest is 6. Number of persons who are extremely kind is 12 – (6 + 3) = 3 **[1]**

$\therefore$ Number of outcomes favourable to $B = 6 + 3 = 9$.

(i) $P(A) = \frac{\text{Number of outcomes favourable to } A}{\text{Total number of possible outcomes}}$

$= \frac{3}{12} = \frac{1}{4}$ **[1]**

(ii) $P(B) = \frac{\text{Number of outcomes favourable to } B}{\text{Total number of possible outcomes}}$

$= \frac{9}{12} = \frac{3}{4}$ **[1]**

Each of the three values, patience, honesty and kindness is important in one's life.

45. Total number of cards = 49

(i) Total number of outcomes = 49

The odd numbers from 1 to 49 are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47 and 49.

Total number of favourable outcomes = 25

$\therefore$ Required probability

$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{25}{49}$ **[1]**

(ii) Total number of outcomes = 49

The number 5, 10, 15, 20, 25, 30, 35, 40 and 45 are multiples of 5.

The number of favourable outcomes = 9

$\therefore$ Required probability

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{9}{49}$$ **[1]**

(iii) Total number of outcomes = 49

The number 1, 4, 9, 16, 25, 36 and 49 are perfect squares.

Total number of favourable outcomes = 7

$\therefore$ Required probability

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{7}{49} = \frac{1}{7}$$ **[1]**

(iv) Total number of outcomes = 49

We know that there is only one even prime number which is 2.

Total number of favourable outcomes = 1

$\therefore$ Required probability

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{49}$$ **[1]**

46. Let S be the sample space of drawing a card from a well-shuffled deck.

$n(S) = 52$

(i) There are 13 spade cards and 4 ace's in a deck. As ace of spade is included in 13 spade cards, so there are 13 spade cards and 3 ace's.

A card of spade or an ace can be drawn in

= 16 ways

Probability of drawing a card of spade or an ace $= \frac{16}{52} = \frac{4}{13}$ **[1]**

(ii) There are 2 black king cards in a deck a card of black king can be drawn in = 2 ways

Probability of drawing a black king $= \frac{2}{52} = \frac{1}{26}$

[1]

(iii) There are 4 Jack and 4 King cards in a deck.

So there are 52 – 8 = 44 cards which are neither Jacks nor Kings. A card which is neither a Jack nor a King.

Can be drawn in = 44 ways

Probability of drawing a card which is neither a Jack nor a King $= \frac{44}{52} = \frac{11}{13}$ **[1]**

(iv) There are 4 King and 4 Queen cards in a deck.

So there are 4 + 4 = 8 cards which are either King or Queen.

A card which is either a King or a Queen can be drawn in = 8 ways

So, probability of drawing a card which is either a King or a Queen $= \frac{8}{52} = \frac{2}{13}$ **[1]**

47. x is selected from 1, 2, 3 and 4

1, 2, 3, 4

y is selected from 1, 4, 9 and 16

Let A = {1, 4, 9, 16, 2, 8, 18, 32, 3, 12, 27, 48, 36, 64} which consists of elements that are product of x and y. **[2]**

P(product of x and y is less than 16)

$$= \frac{\text{Number of outcomes less than 16}}{\text{Total number of outcomes}}$$ **[1]**

$$= \frac{7}{14}$$

$$= \frac{1}{2}$$ **[1]**

48. Two dice are thrown together total possible outcomes = 6 × 6 = 36

(i) Sum of outcomes is even

This can be possible when

$\Rightarrow$ Both outcomes are even

$\Rightarrow$ Both outcomes are odd

For both outcomes to be even number of cases = 3 × 3 = 9 **[1]**

Similarly,

Both outcomes odd = 9 cases

Total favourable cases = 9 + 9 =18

Probability that $= \frac{18}{36}$

Sum of the even outcomes is $\frac{1}{2}$. **[1]**

(ii) Product of outcomes is even

This is possible when

$\Rightarrow$ Both outcomes are even

$\Rightarrow$ First outcome even & the other odd

$\Rightarrow$ First outcome odd & the other even

Number of cases where both outcomes are even = 9 **[1]**

Number of cases for first outcome odd and the other even = 9

Number of cases for first outcome even and the other odd = 9

Total favourable cases = 9 + 9 + 9 = 27

$$\text{Probability} = \frac{27}{36}$$

$$= \frac{3}{4}$$ **[1]**

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