

## NCERT solutions for class 12 maths chapter 11 three dimensional geometry-Exercise: 11.1

**Question:1** If a line makes angles  $90^\circ, 135^\circ, 45^\circ$  with the x, y and z-axes respectively, find its direction cosines.

**Answer:**

Let the direction cosines of the line be **l, m, and n**.

So, we have

$$l = \cos 90^\circ = 0$$

$$m = \cos 135^\circ = -\frac{1}{\sqrt{2}}$$

$$n = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Therefore the direction cosines of the lines are  $0, -\frac{1}{\sqrt{2}}, \text{ and } \frac{1}{\sqrt{2}}$ .

**Question:2** Find the direction cosines of a line which makes equal angles with the coordinate axes.

**Answer:**

If the line is making equal angle with the coordinate axes. Then,

Let the common angle made is  $\alpha$  with each coordinate axes.

Therefore, we can write;

$$l = \cos \alpha, \quad m = \cos \alpha, \text{ and } n = \cos \alpha$$

And as we know the relation;  $l^2 + m^2 + n^2 = 1$

$$\Rightarrow \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{3}$$

$$\text{or } \cos \alpha = \pm \frac{1}{\sqrt{3}}$$

Thus the direction cosines of the line are  $\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \text{ and } \pm \frac{1}{\sqrt{3}}$

**Question:3** If a line has the direction ratios  $-18, 12, -4$ , then what are its direction cosines ?

**Answer:**

Given a line has direction ratios of  $-18, 12, -4$  then its direction cosines are;

Line having direction ratio **-18** has direction cosine:

$$\frac{-18}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}} = \frac{-18}{22} = \frac{-9}{11}$$

Line having direction ratio **12** has direction cosine:

$$\frac{12}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}} = \frac{12}{22} = \frac{6}{11}$$

Line having direction ratio **-4** has direction cosine:

$$\frac{-4}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}} = \frac{-4}{22} = \frac{-2}{11}$$

Thus, the direction cosines are  $\frac{-9}{11}$ ,  $\frac{6}{11}$ ,  $\frac{-2}{11}$ .

**Question:4** Show that the points  $(2, 3, 4)$ ,  $(-1, -2, 1)$ ,  $(5, 8, 7)$  are collinear.

**Answer:**

We have the points, A  $(2, 3, 4)$ , B  $(-1, -2, 1)$ , C  $(5, 8, 7)$ ;

And as we can find the direction ratios of the line joining the points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is given by  $x_2 - x_1, y_2 - y_1$ , and  $z_2 - z_1$ .

The direction ratios of AB

are  $(-1 - 2), (-2 - 3)$ , and  $(1 - 4)$  i.e.,  $-3, -5$ , and  $-3$

The direction ratios of BC

are  $(5 - (-1)), (8 - (-2))$ , and  $(7 - 1)$  i.e.,  $6, 10$ , and  $6$ .

We can see that the direction ratios of AB and BC are proportional to each other and is - 2 times.

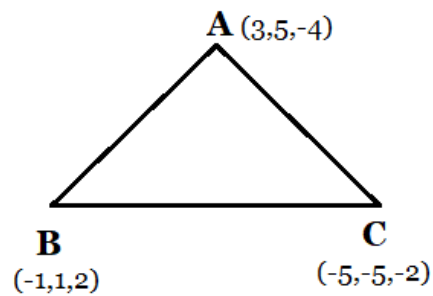
$\therefore$  AB is parallel to BC. and as point B is common to both AB and BC,

**Hence the points A, B and C are collinear.**

**Question:5** Find the direction cosines of the sides of the triangle whose vertices are  $(3, 5, -4)$ ,  $(-1, 1, 2)$  and  $(-5, -5, -2)$ .

**Answer:**

Given vertices of the triangle  $\triangle ABC$   $(3, 5, -4)$ ,  $(-1, 1, 2)$  and  $(-5, -5, -2)$ .



Finding each side direction ratios;

$\Rightarrow$  Direction ratios of side AB are  $(-1 - 3), (1 - 5),$  and  $(2 - (-4))$  i.e.,

$-4, -4,$  and  $6.$

Therefore its direction cosines values are;

Similarly for side BC;

$\Rightarrow$  Direction ratios of side BC are  $(-5 - (-1)), (-5 - 1),$  and  $(-2 - 2)$  i.e.,

$-4, -6,$  and  $-4.$

Therefore its direction cosines values are;

$\Rightarrow$  Direction ratios of side CA are  $(-5 - 3), (-5 - 5),$  and  $(-2 - (-4))$  i.e.,

$-8, -10,$  and  $2.$

Therefore its direction cosines values are;

## NCERT solutions for class 12 maths chapter 11 three dimensional geometry-Exercise: 11.2

**Question:1** Show that the three lines with direction cosines  
are mutually perpendicular.

**Answer:**

Given direction cosines of the three lines;

$$L_1 \left( \frac{12}{13}, \frac{-3}{13}, \frac{-4}{13} \right) L_2 \left( \frac{4}{13}, \frac{12}{13}, \frac{3}{13} \right) L_3 \left( \frac{3}{13}, \frac{-4}{13}, \frac{12}{13} \right)$$

And we know that two lines with direction cosines  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$  are perpendicular to each other, if  $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

Hence we will check each pair of lines:

**Lines  $L_1$  and  $L_2$  ;**

$$= \left[ \frac{48}{169} \right] - \left[ \frac{36}{169} \right] - \left[ \frac{12}{169} \right] = 0$$

$\therefore$  the lines  $L_1$  and  $L_2$  are perpendicular.

**Lines  $L_2$  and  $L_3$  ;**

$$= \left[ \frac{12}{169} \right] - \left[ \frac{48}{169} \right] + \left[ \frac{36}{169} \right] = 0$$

$\therefore$  the lines  $L_2$  and  $L_3$  are perpendicular.

**Lines  $L_3$  and  $L_1$  ;**

$$= \left[ \frac{36}{169} \right] + \left[ \frac{12}{169} \right] - \left[ \frac{48}{169} \right] = 0$$

$\therefore$  the lines  $L_3$  and  $L_1$  are perpendicular.

Thus, we have all lines are mutually perpendicular to each other.

**Question:2** Show that the line through the points  $(1, -1, 2)$ ,  $(3, 4, -2)$  is perpendicular to the line through the points  $(0, 3, 2)$  and  $(3, 5, 6)$ .

**Answer:**

We have given points where the line is passing through it;

Consider the line joining the points  $(1, -1, 2)$  and  $(3, 4, -2)$  is AB and line joining the points  $(0, 3, 2)$  and  $(3, 5, 6)$  is CD.

So, we will find the direction ratios of the lines AB and CD;

Direction ratios of AB are  $a_1, b_1, c_1$

$(3 - 1), (4 - (-1)), \text{ and } (-2 - 2)$  or  $2, 5, \text{ and } -4$

Direction ratios of CD are  $a_2, b_2, c_2$

$(3 - 0), (5 - 3), \text{ and } (6 - 2)$  or  $3, 2, \text{ and } 4$ .

Now, lines AB and CD will be perpendicular to each other if  $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$a_1a_2 + b_1b_2 + c_1c_2 = (2 \times 3) + (5 \times 2) + (-4 \times 4)$$

$$= 6 + 10 - 16 = 0$$

**Therefore, AB and CD are perpendicular to each other.**

**Question:3** Show that the line through the points  $(4, 7, 8), (2, 3, 4)$  is parallel to the line through the points  $(-1, -2, 1), (1, 2, 5)$ .

**Answer:**

We have given points where the line is passing through it;

Consider the line joining the points  $(4, 7, 8)$  and  $(2, 3, 4)$  is AB and line joining the points  $(-1, -2, 1)$  and  $(1, 2, 5)$ ..is CD.

So, we will find the direction ratios of the lines AB and CD;

Direction ratios of AB are  $a_1, b_1, c_1$

$(2 - 4), (3 - 7), \text{ and } (4 - 8)$  or  $-2, -4, \text{ and } -4$

Direction ratios of CD are  $a_2, b_2, c_2$

$(1 - (-1)), (2 - (-2)), \text{ and } (5 - 1)$  or  $2, 4, \text{ and } 4$ .

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Now, lines AB and CD will be parallel to each other if

Therefore we have now;

$$\frac{a_1}{a_2} = \frac{-2}{2} = -1 \frac{b_1}{b_2} = \frac{-4}{4} = -1 \frac{c_1}{c_2} = \frac{-4}{4} = -1$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence we can say that AB is parallel to CD.

**Question:4** Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector  $3\hat{i} + 2\hat{j} - 2\hat{k}$ .

**Answer:**

It is given that the line is passing through A (1, 2, 3) and is parallel to the vector  $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

We can easily find the equation of the line which passes through the point A and is parallel to the vector  $\vec{b}$  by the known relation;

$$\vec{r} = \vec{a} + \lambda \vec{b}, \text{ where } \lambda \text{ is a constant.}$$

So, we have now,

$$\Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$$

Thus the required equation of the line.

**Question:5** Find the equation of the line in vector and in cartesian form that passes through the point with position vector  $2\hat{i} - \hat{j} + 4\hat{k}$  and is in the direction  $\hat{i} + 2\hat{j} - \hat{k}$ .

**Answer:**

Given that the line is passing through the point with position vector  $2\hat{i} - \hat{j} + 4\hat{k}$  and is in the direction of the line  $\hat{i} + 2\hat{j} - \hat{k}$ .

And we know the equation of the line which passes through the point with the position vector  $\vec{a}$  and parallel to the vector  $\vec{b}$  is given by the equation,

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\Rightarrow \vec{r} = 2\hat{i} - \hat{j} + 4\hat{k} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$$

**So, this is the required equation of the line in the vector form.**

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 2)\hat{i} + (2\lambda - 1)\hat{j} + (-\lambda + 4)\hat{k}$$

Eliminating  $\lambda$ , from the above equation we obtain the equation in the Cartesian form :

$$\frac{x - 2}{1} = \frac{y + 1}{2} = \frac{z - 4}{-1}$$

**Hence this is the required equation of the line in Cartesian form.**

**Question:6** Find the cartesian equation of the line which passes through the point  $(-2, 4, -5)$  and parallel to the line given by  $\frac{x + 3}{3} = \frac{y - 4}{5} = \frac{z + 8}{6}$ .

**Answer:**

Given a line which passes through the point  $(-2, 4, -5)$  and is parallel to the line given by the  $\frac{x + 3}{3} = \frac{y - 4}{5} = \frac{z + 8}{6}$  ;

The direction ratios of the line,  $\frac{x + 3}{3} = \frac{y - 4}{5} = \frac{z + 8}{6}$  are **3,5 and 6**.

So, the required line is parallel to the above line.

Therefore we can take direction ratios of the required line as **3k** , **5k** , and **6k** , where k is a non-zero constant.

And we know that the equation of line passing through the point  $(x_1, y_1, z_1)$  and with direction ratios a, b, c is written by:  $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$  .

Therefore we have the equation of the required line:

$$\frac{x + 2}{3k} = \frac{y - 4}{5k} = \frac{z + 5}{6k}$$

$$\text{or } \frac{x + 2}{3} = \frac{y - 4}{5} = \frac{z + 5}{6} = k$$

**The required line equation.**

**Question:7** The cartesian equation of a line is  $\frac{x - 5}{3} = \frac{y + 4}{7} = \frac{z - 6}{7}$  . Write its vector form .

**Answer:**

Given the Cartesian equation of the line;

$$\frac{x - 5}{3} = \frac{y + 4}{7} = \frac{z - 6}{7}$$

Here the given line is passing through the point  $(5, -4, 6)$  .

So, we can write the position vector of this point as;

$$\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$$

And the direction ratios of the line are **3** , **7** , and **2**.

This implies that the given line is in the direction of the vector,  $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$  .

Now, we can easily find the required equation of line:

As we know that the line passing through the position vector  $\vec{a}$  and in the direction of the vector  $\vec{b}$  is given by the relation,

$$\vec{r} = \vec{a} + \lambda \vec{b}, \lambda \in R$$

So, we get the equation.

$$\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k}), \lambda \in R$$

**This is the required equation of the line in the vector form.**

**Question:8** Find the vector and the cartesian equations of the lines that passes through the origin and  $(5, -2, 3)$ .

**Answer:**

Given that the line is passing through the  $(0, 0, 0)$  and  $(5, -2, 3)$

Thus the required line passes through the origin.

$\therefore$  its position vector is given by,

$$\vec{a} = \vec{0}$$

So, the direction ratios of the line through  $(0, 0, 0)$  and  $(5, -2, 3)$  are,

$$(5 - 0) = 5, (-2 - 0) = -2, (3 - 0) = 3$$

The line is parallel to the vector given by the equation,  $\vec{b} = 5\hat{i} - 2\hat{j} + 3\hat{k}$

Therefore the equation of the line passing through the point with position vector  $\vec{a}$  and parallel to  $\vec{b}$  is given by;

$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \in \mathbb{R}$$

$$\Rightarrow \vec{r} = 0 + \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\Rightarrow \vec{r} = \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})$$

Now, the equation of the line through the point  $(x_1, y_1, z_1)$  and the direction ratios a, b, c is given by;

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Therefore the equation of the required line in the Cartesian form will be;

$$\frac{x - 0}{5} = \frac{y - 0}{-2} = \frac{z - 0}{3}$$

$$\text{OR } \frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$$

**Question:9** Find the vector and the cartesian equations of the line that passes through the points  $(3, -2, -5)$ ,  $(3, -2, 6)$ .

**Answer:**

Let the line passing through the points  $A(3, -2, -5)$  and  $B(3, -2, 6)$  is AB;

Then as AB passes through through A so, we can write its position vector as;

$$\vec{a} = 3\hat{i} - 2\hat{j} - 5\hat{k}$$

Then direction ratios of PQ are given by,

$$(3 - 3) = 0, (-2 + 2) = 0, (6 + 5) = 11$$

Therefore the equation of the vector in the direction of AB is given by,

$$\vec{b} = 0\hat{i} - 0\hat{j} + 11\hat{k} = 11\hat{k}$$

We have then the equation of line AB in vector form is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \in \mathbb{R}$$

$$\Rightarrow \vec{r} = (3\hat{i} - 2\hat{j} - 5\hat{k}) + 11\lambda\hat{k}$$

So, the equation of AB in Cartesian form is;

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\text{or } \frac{x - 3}{0} = \frac{y + 2}{0} = \frac{z + 5}{11}$$

**Question:10** Find the angle between the following pairs of lines:

$$(i) \vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k}) \text{ and } \vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

**Answer:**

To find the angle A between the pair of lines  $\vec{b}_1$  and  $\vec{b}_2$  we have the formula;

$$\cos A = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \right|$$

We have two lines :

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k}) \text{ and}$$

$$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

The given lines are parallel to the vectors  $\vec{b}_1$  and  $\vec{b}_2$ ;

where  $\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$  and  $\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$  respectively,

Then we have

$$\vec{b}_1 \cdot \vec{b}_2 = (3\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})$$

$$= 3 + 4 + 12 = 19$$

$$\text{and } |\vec{b}_1| = \sqrt{3^2 + 2^2 + 6^2} = 7$$

$$|\vec{b}_2| = \sqrt{1^2 + 2^2 + 2^2} = 3$$

Therefore we have;

$$\cos A = \left| \frac{19}{7 \times 3} \right| = \frac{19}{21}$$

$$\text{or } A = \cos^{-1} \left( \frac{19}{21} \right)$$

**Question:10** Find the angle between the following pairs of lines:

$$(ii) \vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda(\hat{i} - \hat{j} - 2\hat{k}) \text{ and } \vec{r} = 2\hat{i} - \hat{j} - 5\hat{k} + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$$

**Answer:**

To find the angle A between the pair of lines  $\vec{b}_1$  and  $\vec{b}_2$  we have the formula;

$$\cos A = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \right|$$

We have two lines :

$$\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda(\hat{i} - \hat{j} - 2\hat{k}) \text{ and}$$

$$\vec{r} = 2\hat{i} - \hat{j} - 5\hat{k} + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$$

The given lines are parallel to the vectors  $\vec{b}_1$  and  $\vec{b}_2$ ;

where  $\vec{b}_1 = \hat{i} - \hat{j} - 2\hat{k}$  and  $\vec{b}_2 = 3\hat{i} - 5\hat{j} - 4\hat{k}$  respectively,

Then we have

$$\vec{b}_1 \cdot \vec{b}_2 = (\hat{i} - \hat{j} - 2\hat{k}) \cdot (3\hat{i} - 5\hat{j} - 4\hat{k})$$

$$= 3 + 5 + 8 = 16$$

$$\text{and } |\vec{b}_1| = \sqrt{1^2 + (-1)^2 + (-2)^2} = \sqrt{6}$$

$$|\vec{b}_2| = \sqrt{3^2 + (-5)^2 + (-4)^2} = \sqrt{50} = 5\sqrt{2}$$

Therefore we have;

$$\cos A = \left| \frac{16}{\sqrt{6} \times 5\sqrt{2}} \right| = \frac{16}{10\sqrt{3}}$$

$$\text{or } A = \cos^{-1} \left( \frac{8}{5\sqrt{3}} \right)$$

**Question:11** Find the angle between the following pair of lines:

$$(i) \frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$

**Answer:**

Given lines are;

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$

So, we two vectors  $\vec{b}_1$  and  $\vec{b}_2$  which are parallel to the pair of above lines respectively.

$$\vec{b}_1 = 2\hat{i} + 5\hat{j} - 3\hat{k} \text{ and } \vec{b}_2 = -\hat{i} + 8\hat{j} + 4\hat{k}$$

To find the angle A between the pair of lines  $\vec{b}_1$  and  $\vec{b}_2$  we have the formula;

$$\cos A = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \right|$$

Then we have

$$\vec{b}_1 \cdot \vec{b}_2 = (2\hat{i} + 5\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 8\hat{j} + 4\hat{k})$$

$$= -2 + 40 - 12 = 26$$

$$\text{and } |\vec{b}_1| = \sqrt{2^2 + 5^2 + (-3)^2} = \sqrt{38}$$

$$|\vec{b}_2| = \sqrt{(-1)^2 + (8)^2 + (4)^2} = \sqrt{81} = 9$$

Therefore we have;

$$\cos A = \left| \frac{26}{\sqrt{38} \times 9} \right| = \frac{26}{9\sqrt{38}}$$

$$\text{or } A = \cos^{-1} \left( \frac{26}{9\sqrt{38}} \right)$$

**Question:11** Find the angle between the following pair of lines:

$$(ii) \frac{x}{2} = \frac{y}{2} = \frac{z}{1} \text{ and } \frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$$

**Answer:**

Given lines are;

$$\frac{x}{2} = \frac{y}{2} = \frac{z}{1} \text{ and } \frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$$

So, we two vectors  $\vec{b}_1$  and  $\vec{b}_2$  which are parallel to the pair of above lines respectively.

$$\vec{b}_1 = 2\hat{i} + 2\hat{j} + \hat{k} \text{ and } \vec{b}_2 = 4\hat{i} + \hat{j} + 8\hat{k}$$

To find the angle A between the pair of lines  $\vec{b}_1$  and  $\vec{b}_2$  we have the formula;

$$\cos A = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \right|$$

Then we have

$$\vec{b}_1 \cdot \vec{b}_2 = (2\hat{i} + 2\hat{j} + \hat{k}) \cdot (4\hat{i} + \hat{j} + 8\hat{k})$$

$$= 8 + 2 + 8 = 18$$

$$\text{and } |\vec{b}_1| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

$$|\vec{b}_2| = \sqrt{(4)^2 + (1)^2 + (8)^2} = \sqrt{81} = 9$$

Therefore we have;

$$\cos A = \left| \frac{18}{3 \times 9} \right| = \frac{2}{3}$$

$$\text{or } A = \cos^{-1} \left( \frac{2}{3} \right)$$

**Question:12** Find the values of p so that the

lines  $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$  and  $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$  are at right angles.

**Answer:**

First we have to write the given equation of lines in the standard form;

$$\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2} \quad \text{and} \quad \frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

Then we have the direction ratios of the above lines as;

$$-3, \frac{2p}{7}, 2 \quad \text{and} \quad \frac{-3p}{7}, 1, -5 \quad \text{respectively..}$$

Two lines with direction ratios  $a_1, b_1, c_1$  and  $a_2, b_2, c_2$  are perpendicular to each other if,  $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$$\therefore (-3) \cdot \left( \frac{-3p}{7} \right) + \left( \frac{2p}{7} \right) \cdot (1) + 2 \cdot (-5) = 0$$

$$\Rightarrow \frac{9p}{7} + \frac{2p}{7} = 10$$

$$\Rightarrow 11p = 70$$

$$\Rightarrow p = \frac{70}{11}$$

Thus, the value of p is  $\frac{70}{11}$ .

**Question:13** Show that the lines  $\frac{x-5}{7} = \frac{y+2}{-3} = \frac{z}{1}$  and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  are perpendicular to each other.

**Answer:**

First, we have to write the given equation of lines in the standard form;

$$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1} \text{ and } \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

Then we have the direction ratios of the above lines as;

7, -5, 1 and 1, 2, 3 respectively..

Two lines with direction ratios  $a_1, b_1, c_1$  and  $a_2, b_2, c_2$  are perpendicular to each other if,  $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$$\therefore 7(1) + (-5)(2) + 1(3) = 7 - 10 + 3 = 0$$

Therefore the two lines are perpendicular to each other.

**Question:14** Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

**Answer:**

So given equation of lines;

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$  and  $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$  in the vector form.

Now, we can find the shortest distance between the lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ , is given by the formula,

Now comparing the values from the equation, we obtain

$$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k} \quad \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k} \quad \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$$

Then calculating

$$\vec{b}_1 \times \vec{b}_2 = (-2 - 1)\hat{i} - (2 - 2)\hat{j} + (1 + 2)\hat{k} = -3\hat{i} + 3\hat{k}$$

So, substituting the values now in the formula above we get;

$$d = \left| \frac{(-3\hat{i} + 3\hat{k}) \cdot (\hat{i} - 3\hat{j} - 2\hat{k})}{3\sqrt{2}} \right|$$

$$\Rightarrow d = \left| \frac{-3.1 + 3(-2)}{3\sqrt{2}} \right|$$

$$d = \left| \frac{-9}{3\sqrt{2}} \right| = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

Therefore, the shortest distance between the two lines is  $\frac{3\sqrt{2}}{2}$  units.

**Question:15** Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

**Answer:**

We have given two lines:

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Calculating the shortest distance between the two lines,

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

by the formula

Now, comparing the given equations, we obtain

$$x_1 = -1, y_1 = -1, z_1 = -1$$

$$a_1 = 7, b_1 = -6, c_1 = 1$$

$$x_2 = 3, y_2 = 5, z_2 = 7$$

$$a_2 = 1, b_2 = -2, c_2 = 1$$

Then calculating determinant

$$= 4(-6 + 2) - 6(7 - 1) + 8(-14 + 6)$$

$$= -16 - 36 - 64$$

$$= -116$$

Now calculating the denominator,

$$= \sqrt{16 + 36 + 64}$$

$$= \sqrt{116} = 2\sqrt{29}$$

So, we will substitute all the values in the formula above to obtain,

$$d = \frac{-116}{2\sqrt{29}} = \frac{-58}{\sqrt{29}} = \frac{-2 \times 29}{\sqrt{29}} = -2\sqrt{29}$$

Since distance is always non-negative, the distance between the given lines is

$$2\sqrt{29} \text{ units.}$$

**Question:16** Find the shortest distance between the lines whose vector equations are  $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$  and

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

**Answer:**

Given two equations of line

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}) \quad \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k}) \text{ in the vector form.}$$

So, we will apply the distance formula for knowing the distance between two lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$

After comparing the given equations, we obtain

$$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k} \quad \vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k} \quad \vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\begin{aligned} \vec{a}_2 - \vec{a}_1 &= (4\hat{i} + 5\hat{j} + 6\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) \\ &= 3\hat{i} + 3\hat{j} + 3\hat{k} \end{aligned}$$

Then calculating the determinant value numerator.

$$= (-3 - 6)\hat{i} - (1 - 4)\hat{j} + (3 + 6)\hat{k} = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

That implies,  $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-9)^2 + (3)^2 + (9)^2}$

$$= \sqrt{81 + 9 + 81} = \sqrt{171} = 3\sqrt{19}$$

$$= (-9 \times 3) + (3 \times 3) + (9 \times 3) = 9$$

Now, after substituting the value in the above formula we get,

$$d = \left| \frac{9}{3\sqrt{19}} \right| = \frac{3}{\sqrt{19}}$$

Therefore,  $\frac{3}{\sqrt{19}}$  is the shortest distance between the two given lines.

**Question:17** Find the shortest distance between the lines whose vector equations are

$$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k} \text{ and } \vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$$

**Answer:**

Given two equations of the line

$$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k} \quad \vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k} \text{ in the vector form.}$$

So, we will apply the distance formula for knowing the distance between two lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$

After comparing the given equations, we obtain

$$\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k} \quad \vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k} \quad \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$$

Then calculating the determinant value numerator.

$$= (-2 + 4)\hat{i} - (2 + 2)\hat{j} + (-2 - 1)\hat{k} = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

That implies,

$$\begin{aligned} \left| \vec{b}_1 \times \vec{b}_2 \right| &= \sqrt{(2)^2 + (-4)^2 + (-3)^2} \\ &= \sqrt{4 + 16 + 9} = \sqrt{29} \end{aligned}$$

Now, after substituting the value in the above formula we get,

$$d = \left| \frac{8}{\sqrt{29}} \right| = \frac{8}{\sqrt{29}}$$

Therefore,  $\frac{8}{\sqrt{29}}$  units are the shortest distance between the two given lines.

### NCERT solutions for class 12 maths chapter 11 three dimensional geometry-Exercise: 11.3

**Question:1(a)** In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

$$z = 2$$

**Answer:**

Equation of plane  $Z=2$ , i.e.  $0x + 0y + z = 2$

The direction ratio of normal is 0,0,1

$$\therefore \sqrt{0^2 + 0^2 + 1^2} = 1$$

Divide equation  $0x + 0y + z = 2$  by 1 from both side

We get,  $0x + 0y + z = 2$

Hence, direction cosins are 0,0,1.

The distance of the plane from the origin is 2.

**Question:1(b)** In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

$$x + y + z = 1$$

**Answer:**

Given the equation of the plane is  $x + y + z = 1$  or we can write  $1x + 1y + 1z = 1$

So, the direction ratios of normal from the above equation are, 1, 1, and 1 .

$$\text{Therefore } \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Then dividing both sides of the plane equation by  $\sqrt{3}$  , we get

$$\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

So, this is the form of  $lx + my + nz = d$  the plane, where  $l, m, n$  are the direction cosines of normal to the plane and d is the distance of the perpendicular drawn from the origin.

$\therefore$  The direction cosines of the given line are  $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$  and the distance of the plane from the origin is  $\frac{1}{\sqrt{3}}$  units.

**Question:1(c)** In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

$$2x + 3y - z = 5$$

**Answer:**

Given the equation of plane is  $2x + 3y - z = 5$

So, the direction ratios of normal from the above equation are, 2, 3, and -1 .

$$\text{Therefore } \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$$

Then dividing both sides of the plane equation by  $\sqrt{14}$  , we get

$$\frac{2x}{\sqrt{14}} + \frac{3y}{\sqrt{14}} - \frac{z}{\sqrt{14}} = \frac{5}{\sqrt{14}}$$

So, this is the form of  $lx + my + nz = d$  the plane, where  $l, m, n$  are the direction cosines of normal to the plane and d is the distance of the perpendicular drawn from the origin.

$\therefore$  The direction cosines of the given line are  $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}$  and the distance of the plane from the origin is  $\frac{5}{\sqrt{14}}$  units.

**Question:1(d)** In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

$$5y + 8 = 0$$

**Answer:**

Given the equation of plane is  $5y + 8 = 0$  or we can write  $0x - 5y + 0z = 8$

So, the direction ratios of normal from the above equation are,  $0, -5, \text{ and } 0$  .

Therefore  $\sqrt{0^2 + (-5)^2 + 0^2} = 5$

Then dividing both sides of the plane equation by  $5$  , we get

$$-y = \frac{8}{5}$$

So, this is the form of  $lx + my + nz = d$  the plane, where  $l, m, n$  are the direction cosines of normal to the plane and  $d$  is the distance of the perpendicular drawn from the origin.

$\therefore$  The direction cosines of the given line are  $0, -1, \text{ and } 0$  and the distance of the plane from the origin is  $\frac{8}{5}$  units.

**Question:2** Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector  $3\hat{i} + 5\hat{j} - 6\hat{k}$  .

**Answer:**

We have given the distance between the plane and origin equal to 7 units and normal to the vector  $3\hat{i} + 5\hat{j} - 6\hat{k}$  .

So, it is known that the equation of the plane with position vector  $\vec{r}$  is given by, the relation,

$\vec{r} \cdot \hat{n} = d$  , where  $d$  is the distance of the plane from the origin.

Calculating  $\hat{n}$  ;

$$\vec{r} \cdot \left( \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}} \right) = 7 \quad \text{is the vector equation of the required plane.}$$

**Question:3(a)** Find the Cartesian equation of the following planes:

$$\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$$

**Answer:**

Given the equation of the plane  $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$

So we have to find the Cartesian equation,

Any point  $A(x, y, z)$  on this plane will satisfy the equation and its position vector given by,

$$\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$$

Hence we have,

$$(x\hat{i} + y\hat{j} - z\hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$$

$$\text{Or, } x + y - z = 2$$

**Therefore this is the required Cartesian equation of the plane.**

**Question:3(b)** Find the Cartesian equation of the following planes:

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$$

**Answer:**

Given the equation of plane  $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$

So we have to find the Cartesian equation,

Any point  $A(x, y, z)$  on this plane will satisfy the equation and its position vector given by,

$$\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$$

Hence we have,

$$(x\hat{i} + y\hat{j} - z\hat{k}) \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$$

$$\text{Or, } 2x + 3y - 4z = 1$$

**Therefore this is the required Cartesian equation of the plane.**

**Question:3(c)** Find the Cartesian equation of the following planes:

$$\vec{r} \cdot [(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k}] = 15$$

**Answer:**

$$\text{Given the equation of plane } \vec{r} \cdot [(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k}] = 15$$

So we have to find the Cartesian equation,

Any point  $A(x, y, z)$  on this plane will satisfy the equation and its position vector given by,  $\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$

Hence we have,

$$\text{Or, } (s - 2t)x + (3 - t)y + (2s + t)z = 15$$

**Therefore this is the required Cartesian equation of the plane.**

**Question:4(a)** In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

$$2x + 3y + 4z - 12 = 0$$

**Answer:**

Let the coordinates of the foot of perpendicular P from the origin to the plane be  $(x_1, y_1, z_1)$

Given a plane equation  $2x + 3y + 4z - 12 = 0$ ,

$$\text{Or, } 2x + 3y + 4z = 12$$

The direction ratios of the normal of the plane are 2, 3, and 4.

$$\text{Therefore } \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{29}$$

So, now dividing both sides of the equation by  $\sqrt{29}$  we will obtain,

$$\frac{2}{\sqrt{29}}x + \frac{3}{\sqrt{29}}y + \frac{4}{\sqrt{29}}z = \frac{12}{\sqrt{29}}$$

This equation is similar to  $lx + my + nz = d$  where,  $l, m, n$  are the directions cosines of normal to the plane and  $d$  is the distance of normal from the origin.

Then finding the coordinates of the foot of the perpendicular are given by  $(ld, md, nd)$ .

$\therefore$  The coordinates of the foot of the perpendicular are;

$$\text{or } \left[ \frac{24}{29}, \frac{36}{49}, \frac{48}{29} \right]$$

**Question:4(b)** In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

$$3y + 4z - 6 = 0$$

**Answer:**

Let the coordinates of the foot of perpendicular P from the origin to the plane be  $(x_1, y_1, z_1)$

Given a plane equation  $3y + 4z - 6 = 0$ ,

$$\text{Or, } 0x + 3y + 4z = 6$$

The direction ratios of the normal of the plane are 0, 3, and 4.

$$\text{Therefore } \sqrt{(0)^2 + (3)^2 + (4)^2} = 5$$

So, now dividing both sides of the equation by 5 we will obtain,

$$0x + \frac{3}{5}y + \frac{4}{5}z = \frac{6}{5}$$

This equation is similar to  $lx + my + nz = d$  where,  $l, m, n$  are the directions cosines of normal to the plane and d is the distance of normal from the origin.

Then finding the coordinates of the foot of the perpendicular are given by  $(ld, md, nd)$ .

$\therefore$  The coordinates of the foot of the perpendicular are;

$$\left(0, \frac{3}{5}, \frac{4}{5}\right) \text{ or } \left(0, \frac{18}{25}, \frac{24}{25}\right)$$

**Question:4(c)** In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

$$x + y + z = 1$$

**Answer:**

Let the coordinates of the foot of perpendicular P from the origin to the plane be  $(x_1, y_1, z_1)$

Given plane equation  $x + y + z = 1$  .

The direction ratios of the normal of the plane are 1, 1, and 1 .

$$\text{Therefore } \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

So, now dividing both sides of the equation by  $\sqrt{3}$  we will obtain,

$$\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

This equation is similar to  $lx + my + nz = d$  where,  $l, m, n$  are the directions cosines of normal to the plane and d is the distance of normal from the origin.

Then finding the coordinates of the foot of the perpendicular are given by  $(ld, md, nd)$  .

$\therefore$  The coordinates of the foot of the perpendicular are;

$$\text{or } \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) ..$$

**Question: 4(d)** In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

$$5y + 8 = 0$$

**Answer:**

Let the coordinates of the foot of perpendicular P from the origin to the plane be  $(x_1, y_1, z_1)$

Given plane equation  $5y + 8 = 0$  .

or written as  $0x - 5y + 0z = 8$

The direction ratios of the normal of the plane are  $0, -5, \text{ and } 0$  .

Therefore  $\sqrt{(0)^2 + (-5)^2 + (0)^2} = 5$

So, now dividing both sides of the equation by 5 we will obtain,

$$-y = \frac{8}{5}$$

This equation is similar to  $lx + my + nz = d$  where,  $l, m, n$  are the directions cosines of normal to the plane and d is the distance of normal from the origin.

Then finding the coordinates of the foot of the perpendicular are given by  $(ld, md, nd)$  .

$\therefore$  The coordinates of the foot of the perpendicular are;

$$\left(0, -1\left(\frac{8}{5}\right), 0\right) \text{ or } \left(0, \frac{-8}{5}, 0\right) .$$

**Question:5(a)** Find the vector and cartesian equations of the planes (a) that passes through the point  $(1, 0, -2)$  and the normal to the plane is  $\hat{i} + \hat{j} - \hat{k}$ .

**Answer:**

Given the point  $A(1, 0, -2)$  and the normal vector  $\hat{n}$  which is perpendicular to the plane is  $\hat{n} = \hat{i} + \hat{j} - \hat{k}$

The position vector of point A is  $\vec{a} = \hat{i} - 2\hat{k}$

So, the vector equation of the plane would be given by,

$$(\vec{r} - \vec{a}) \cdot \hat{n} = 0$$

$$\text{Or } [\vec{r} - (\hat{i} - 2\hat{k})] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

where  $\vec{r}$  is the position vector of any arbitrary point  $A(x, y, z)$  in the plane.

$$\therefore \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Therefore, the equation we get,

$$[(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 2\hat{k})] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$\Rightarrow [(x-1)\hat{i} + y\hat{j} + (z+2)\hat{k}] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$\Rightarrow (x-1) + y - (z+2) = 0$$

$$\Rightarrow x + y - z - 3 = 0 \text{ or } x + y - z = 3$$

**So, this is the required Cartesian equation of the plane.**

**Question:5(b)** Find the vector and cartesian equations of the planes

that passes through the point (1,4, 6) and the normal vector to the plane is  $\hat{i} - 2\hat{j} + \hat{k}$ .

**Answer:**

Given the point  $A(1, 4, 6)$  and the normal vector  $\hat{n}$  which is perpendicular to the plane is  $\hat{n} = \hat{i} - 2\hat{j} + \hat{k}$

The position vector of point A is  $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

So, the vector equation of the plane would be given by,

$$(\vec{r} - \vec{a}) \cdot \hat{n} = 0$$

$$\text{Or } \left[ \vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k}) \right] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$$

where  $\vec{r}$  is the position vector of any arbitrary point  $A(x, y, z)$  in the plane.

$$\therefore \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Therefore, the equation we get,

$$\Rightarrow \left[ (x - 1)\hat{i} + (y - 4)\hat{j} + (z - 6)\hat{k} \right] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$(x - 1) - 2(y - 4) + (z - 6) = 0$$

$$\Rightarrow x - 2y + z + 1 = 0$$

**So, this is the required Cartesian equation of the plane.**

**Question:6(a)** Find the equations of the planes that passes through three points.

$(1, 1, -1), (6, 4, -5), (-4, -2, 3)$

**Answer:**

The equation of the plane which passes through the three points  $A(1, 1, -1)$ ,  $B(6, 4, -5)$ , and  $C(-4, -2, 3)$  is given by;

Determinant method,

$$\begin{vmatrix} 1 & 1 & -1 \\ 6 & 4 & -5 \\ -4 & -2 & 3 \end{vmatrix} = (12 - 10) - (18 - 20) - (-12 + 16)$$

$$\text{Or, } = 2 + 2 - 4 = 0$$

Here, these three points A, B, C are collinear points.

**Hence there will be an infinite number of planes possible which passing through the given points.**

**Question:6(b)** Find the equations of the planes that passes through three points.

$(1, 1, 0), (1, 2, 1), (-2, 2, -1)$

**Answer:**

The equation of the plane which passes through the three points  $A(1, 1, 0)$ ,  $B(1, 2, 1)$ , and  $C(-2, 2, -1)$  is given by;

Determinant method,

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix} = (-2 - 2) - (2 + 2) = -8 \neq 0$$

As determinant value is not equal to zero hence there must be a plane that passes through the points A, B, and C.

Finding the equation of the plane through the points,  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$

After substituting the values in the determinant we get,

$$\begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(-1-1) - (y-1)(0+3) + z(0+3) = 0$$

$$\Rightarrow -2x + 2 - 3y + 3 + 3z = 0$$

$$2x + 3y - 3z = 5$$

**So, this is the required Cartesian equation of the plane.**

**Question:7** Find the intercepts cut off by the plane  $2x + y - z = 5$ .

**Answer:**

Given plane  $2x + y - z = 5$

We have to find the intercepts that this plane would make so,

Making it look like intercept form first:

By dividing both sides of the equation by 5 (as we have to make the R.H.S =1) , we get then,

$$\frac{2}{5}x + \frac{y}{5} - \frac{z}{5} = 1$$

$$\Rightarrow \frac{x}{\frac{5}{2}} + \frac{y}{5} + \frac{z}{-5} = 1$$

So, as we know that from the equation of a plane in intercept

form,  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  where a,b,c are the intercepts cut off by the plane at x,y, and z-axes respectively.

Therefore after comparison, we get the values of a,b, and c.

$$a = \frac{5}{2}, b = 5, \text{ and } c = -5$$

Hence the intercepts are  $\frac{5}{2}$ , 5, and -5 .

**Question:8** Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

**Answer:**

Given that the plane is parallel to the ZOX plane.

So, we have the equation of plane ZOX as  $y = 0$  .

And an intercept of 3 on the y-axis  $\Rightarrow b = 3$

Intercept form of a plane given by;

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

So, here the plane would be parallel to the x and z-axes both.

we have any plane parallel to it is of the form,  $y = a$  .

Equation of the plane required is  $y = 3$  .

**Question:9** Find the equation of the plane through the intersection of the planes  $3x - y + 2z - 4 = 0$  and  $x + y + z - 2 = 0$  and the point  $(2, 2, 1)$ .

**Answer:**

The equation of any plane through the intersection of the planes,

$$3x - y + 2z - 4 = 0 \text{ and } x + y + z - 2 = 0$$

Can be written in the form of;  $(3x - y + 2z - 4) + \alpha(x + y + z - 2) = 0$  ,

where  $\alpha \in R$

So, the plane passes through the point  $(2, 2, 1)$  , will satisfy the above equation.

$$(3 \times 2 - 2 + 2 \times 1 - 4) + \alpha(2 + 2 + 1 - 2) = 0$$

That implies  $2 + 3\alpha = 0$

$$\alpha = \frac{-2}{3}$$

Now, substituting the value of  $\alpha$  in the equation above we get the final equation of the plane;

$$(3x - y + 2z - 4) + \alpha(x + y + z - 2) = 0$$

$$(3x - y + 2z - 4) + \frac{-2}{3}(x + y + z - 2) = 0$$

$$\Rightarrow 9x - 3y + 6z - 12 - 2x - 2y - 2z + 4 = 0$$

$$\Rightarrow 7x - 5y + 4z - 8 = 0 \text{ is the required equation of the plane.}$$

**Question:10** Find the vector equation of the plane passing through the intersection of the planes  $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$ ,  $\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$  and through the point (2, 1, 3).

**Answer:**

$$\text{Here } \vec{n}_1 = 2\hat{i} + 2\hat{j} - 3\hat{k} \text{ and } \vec{n}_2 = 2\hat{i} + 5\hat{j} + 3\hat{k}$$

$$\text{and } d_1 = 7 \text{ and } d_2 = 9$$

Hence, using the relation  $\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2$ , we get

$$\vec{r} \cdot [2\hat{i} + 2\hat{j} - 3\hat{k} + \lambda(2\hat{i} + 5\hat{j} + 3\hat{k})] = 7 + 9\lambda$$

$$\text{or } \vec{r} \cdot [(2 + 2\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (3\lambda - 3)\hat{k}] = 7 + 9\lambda \dots\dots\dots(1)$$

where,  $\lambda$  is some real number.

$$\text{Taking } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \text{ we get}$$

$$\text{or } x(2 + 2\lambda) + y(2 + 5\lambda) + z(3\lambda - 3) = 7 + 9\lambda$$

$$\text{or } 2x + 2y - 3z - 7 + \lambda(2x + 5y + 3z - 9) = 0 \dots\dots\dots(2)$$

Given that the plane passes through the point (2, 1, 3), it must satisfy (2), i.e.,

$$(4 + 2 - 9 - 7) + \lambda(4 + 5 + 9 - 9) = 0$$

$$\text{or } \lambda = \frac{10}{9}$$

Putting the values of  $\lambda$  in (1), we get

$$\text{or } \vec{r} \cdot \left( \frac{38}{9}\hat{i} + \frac{68}{9}\hat{j} + \frac{1}{3}\hat{k} \right) = 17$$

$$\text{or } \vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$$

which is the required vector equation of the plane.

**Question:11** Find the equation of the plane through the line of intersection of the planes  $x + y + z = 1$  and  $2x + 3y + 4z = 5$  which is perpendicular to the plane  $x - y + z = 0$ .

**Answer:**

The equation of the plane through the intersection of the given two planes,  $x + y + z = 1$  and  $2x + 3y + 4z = 5$  is given in Cartesian form as;

$$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$$

$$\text{or } (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z - (1 + 5\lambda) = 0 \dots\dots\dots(1)$$

So, the direction ratios of (1) plane are  $a_1, b_1, c_1$  which are  $(1 + 2\lambda), (1 + 3\lambda),$  and  $(1 + 4\lambda)$ .

Then, the plane in equation (1) is perpendicular to  $x - y + z = 0$  whose direction ratios  $a_2, b_2, c_2$  are  $1, -1,$  and  $1$ .

As planes are perpendicular then,

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

we get,

$$(1 + 2\lambda) - (1 + 3\lambda) + (1 + 4\lambda) = 0$$

$$\text{or } 1 + 3\lambda = 0$$

$$\text{or } \lambda = -\frac{1}{3}$$

Then we will substitute the values of  $\lambda$  in the equation (1), we get

$$\frac{1}{3}x - \frac{1}{3}z + \frac{2}{3} = 0$$

$$\text{or } x - z + 2 = 0$$

**This is the required equation of the plane.**

**Question:12** Find the angle between the planes whose vector equations are  $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$  and  $\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$ .

**Answer:**

Given two vector equations of plane

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5 \text{ and } \vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3.$$

$$\text{Here, } \vec{n}_1 = 2\hat{i} + 2\hat{j} - 3\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} - 3\hat{j} + 5\hat{k}$$

The formula for finding the angle between two planes,

$$\cos A = \left| \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} \right| \dots\dots\dots(1)$$

$$|\vec{n}_1| = \sqrt{(2)^2 + (2)^2 + (-3)^2} = \sqrt{17}$$

$$\text{and } |\vec{n}_2| = \sqrt{(3)^2 + (-3)^2 + (5)^2} = \sqrt{43}$$

Now, we can substitute the values in the angle formula (1) to get,

$$\cos A = \left| \frac{-15}{\sqrt{17}\sqrt{43}} \right|$$

$$\text{or } \cos A = \frac{15}{\sqrt{731}}$$

$$\text{or } A = \cos^{-1} \left( \frac{15}{\sqrt{731}} \right)$$

**Question:13(a)** In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$$7x + 5y + 6z + 30 = 0 \text{ and } 3x - y - 10z + 4 = 0$$

**Answer:**

Two planes

$L_1 : a_1x + b_1y + c_1z = 0$  whose direction ratios

are  $a_1, b_1, c_1$  and  $L_2 : a_2x + b_2y + c_2z = 0$  whose direction ratios are  $a_2, b_2, c_2$ ,

are said to **Parallel:**

$$\text{If, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

and **Perpendicular:**

$$\text{If, } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

And the angle between  $L_1$  and  $L_2$  is given by the relation,

So, given two planes  $7x + 5y + 6z + 30 = 0$  and  $3x - y - 10z + 4 = 0$

Here,

$$a_1 = 7, b_1 = 5, c_1 = 6 \text{ and } a_2 = 3, b_2 = -1, c_2 = -10$$

So, applying each condition to check:

$$\text{Parallel check: } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Clearly, the given planes are **NOT** parallel.  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\text{Perpendicular check: } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow 7(3) + 5(-1) + 6(-10) = 21 - 5 - 60 = -44 \neq 0 .$$

Clearly, the given planes are **NOT** perpendicular.

Then find the angle between them,

$$\begin{aligned}
&= \cos^{-1} \left| \frac{-44}{\sqrt{7^2 + 5^2 + 6^2} \cdot \sqrt{3^2 + (-1)^2 + (-10)^2}} \right| \\
&= \cos^{-1} \left| \frac{-44}{\sqrt{110} \cdot \sqrt{110}} \right| \\
&= \cos^{-1} \left( \frac{44}{110} \right) \\
&= \cos^{-1} \left( \frac{2}{5} \right)
\end{aligned}$$

**Question:13(b)** In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$$2x + y + 3z - 2 = 0 \text{ and } x - 2y + 5 = 0$$

**Answer:**

Two planes

$L_1 : a_1x + b_1y + c_1z = 0$  whose direction ratios

are  $a_1, b_1, c_1$  and  $L_2 : a_2x + b_2y + c_2z = 0$  whose direction ratios are  $a_2, b_2, c_2$ ,

are said to **Parallel:**

$$\text{If, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

and **Perpendicular:**

$$\text{If, } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

And the angle between  $L_1$  and  $L_2$  is given by the relation,

So, given two planes  $2x + y + 3z - 2 = 0$  and  $x - 2y + 5 = 0$

Here,

$$a_1 = 2, b_1 = 1, c_1 = 3 \text{ and } a_2 = 1, b_2 = -2, c_2 = 0$$

So, applying each condition to check:

**Perpendicular check:**  $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\Rightarrow 2(1) + 1(-2) + 3(0) = 2 - 2 + 0 = 0 .$$

**Thus, the given planes are perpendicular to each other.**

**Question:13(c)** In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$$2x - 2y + 4z + 5 = 0 \text{ and } 3x - 3y + 6z - 1 = 0$$

**Answer:**

Two planes

$L_1 : a_1x + b_1y + c_1z = 0$  whose direction ratios

are  $a_1, b_1, c_1$  and  $L_2 : a_2x + b_2y + c_2z = 0$  whose direction ratios are  $a_2, b_2, c_2$ ,

are said to **Parallel:**

$$\text{If, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

and **Perpendicular:**

$$\text{If, } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

And the angle between  $L_1$  and  $L_2$  is given by the relation,

So, given two planes  $2x - 2y + 4z + 5 = 0$  and  $3x - 3y + 6z - 1 = 0$

Here,

$$a_1 = 2, b_1 = -2, c_1 = 4 \text{ and } a_2 = 3, b_2 = -3, c_2 = 6$$

So, applying each condition to check:

$$\text{Parallel check: } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\text{Thus, the given planes are parallel as } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

**Question:13(d)** In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$$2x - y + 3z - 1 = 0 \text{ and } 2x - y + 3z + 3 = 0$$

**Answer:**

Two planes

$L_1 : a_1x + b_1y + c_1z = 0$  whose direction ratios are  $a_1, b_1, c_1$  and  $L_2 : a_2x + b_2y + c_2z = 0$  whose direction ratios are  $a_2, b_2, c_2$ ,

are said to **Parallel:**

$$\text{If, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

and **Perpendicular:**

$$\text{If, } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

And the angle between  $L_1$  and  $L_2$  is given by the relation,

So, given two planes  $2x - y + 3z - 1 = 0$  and  $2x - y + 3z + 3 = 0$

Here,

$$a_1 = 2, b_1 = -1, c_1 = 3 \text{ and } a_2 = 2, b_2 = -1, c_2 = 3$$

So, applying each condition to check:

$$\text{Parallel check: } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\text{Therefore } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

**Thus, the given planes are parallel to each other.**

**Question:13(e)** In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$$4x + 8y + z - 8 = 0 \text{ and } y + z - 4 = 0$$

**Answer:**

Two planes

$L_1 : a_1x + b_1y + c_1z = 0$  whose direction ratios

are  $a_1, b_1, c_1$  and  $L_2 : a_2x + b_2y + c_2z = 0$  whose direction ratios are  $a_2, b_2, c_2$ ,

are said to **Parallel:**

$$\text{If, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

and **Perpendicular:**

$$\text{If, } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

And the angle between  $L_1$  and  $L_2$  is given by the relation,

So, given two planes  $4x + 8y + z - 8 = 0$  and  $y + z - 4 = 0$

Here,

$$a_1 = 4, b_1 = 8, c_1 = 1 \text{ and } a_2 = 0, b_2 = 1, c_2 = 1$$

So, applying each condition to check:

$$\text{Parallel check: } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Clearly, the given planes are **NOT** parallel as  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ .

$$\text{Perpendicular check: } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow 4(0) + 8(1) + 1(1) = 0 + 8 + 1 = 9 \neq 0.$$

Clearly, the given planes are **NOT** perpendicular.

Then finding the angle between them,

$$= \cos^{-1} \left| \frac{9}{\sqrt{4^2 + 8^2 + 1^2} \cdot \sqrt{0^2 + 1^2 + 1^2}} \right|$$

$$= \cos^{-1} \left| \frac{9}{\sqrt{81} \cdot \sqrt{2}} \right|$$

$$= \cos^{-1} \left( \frac{9}{9\sqrt{2}} \right) = \cos^{-1} \left( \frac{1}{\sqrt{2}} \right)$$

$$= 45^\circ$$

**Question:14** In the following cases, find the distance of each of the given points from the corresponding given plane

POINT	PLANE
-------	-------

a. (0, 0, 0)	$3x - 4y + 12z = 3$
b. (3, -2, 1)	$2x - y + 2z + 3 = 0$
c. (2, 3, -5)	$x + 2y - 2z = 9$
d. (-6, 0, 0)	$2x - 3y + 6z - 2 = 0$

**Answer:**

We know that the distance between a point  $P(x_1, y_1, z_1)$  and a plane  $Ax + By + Cz = D$  is given by,

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}} \right| \dots\dots\dots(1)$$

So, calculating for each case;

**(a)** Point (0, 0, 0) and Plane  $3x - 4y + 12z = 3$

$$d = \left| \frac{3(0) - 4(0) + 12(0) - 3}{\sqrt{3^2 + (-4)^2 + 12^2}} \right| = \frac{3}{\sqrt{169}} = \frac{3}{13}$$

Therefore,

**(b)** Point (3, -2, 1) and Plane  $2x - y + 2z + 3 = 0$

$$d = \left| \frac{2(3) - (-2) + 2(1) + 3}{\sqrt{2^2 + (-1)^2 + 2^2}} \right| = \frac{13}{3}$$

Therefore,

(c) Point  $(2, 3, -5)$  and Plane  $x + 2y - 2z = 9$

Therefore, 
$$d = \left| \frac{2 + 2(3) - 2(-5) - 9}{\sqrt{1^2 + 2^2 + (-2)^2}} \right| = \frac{9}{3} = 3$$

(d) Point  $(-6, 0, 0)$  and Plane  $2x - 3y + 6z - 2 = 0$

Therefore,

### NCERT solutions for class 12 maths chapter 11 three dimensional geometry-Miscellaneous Exercise

**Question:1** Show that the line joining the origin to the point  $(2, 1, 1)$  is perpendicular to the line determined by the points  $(3, 5, -1)$ ,  $(4, 3, -1)$ .

**Answer:**

We can assume the line joining the origin, be OA where  $O(0, 0, 0)$  and the point  $A(2, 1, 1)$  and PQ be the line joining the points  $P(3, 5, -1)$  and  $Q(4, 3, -1)$ .

Then the direction ratios of the line OA will

be  $(2 - 0), (1 - 0),$  and  $(1 - 0) = 2, 1, 1$  and that of line PQ will be

$(4 - 3), (3 - 5),$  and  $(-1 + 1) = 1, -2, 0$

So to check whether line OA is perpendicular to line PQ then,

Applying the relation we know,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow 2(1) + 1(-2) + 1(0) = 2 - 2 + 0 = 0$$

Therefore OA is perpendicular to line PQ.

**Question:2** If  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$  are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are  $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$ .

**Answer:**

Given that  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$  are the direction cosines of two mutually perpendicular lines.

Therefore, we have the relation:

$$l_1l_2 + m_1m_2 + n_1n_2 = 0 \dots\dots\dots(1)$$

$$l_1^2 + m_1^2 + n_1^2 = 1 \text{ and } l_2^2 + m_2^2 + n_2^2 = 1 \dots\dots\dots(2)$$

Now, let us assume  $l, m, n$  be the new direction cosines of the lines which are perpendicular to the line with direction cosines.  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$

Therefore we have,  $ll_1 + mm_1 + nn_1 = 0$  and  $ll_2 + mm_2 + nn_2 = 0$

$$\text{Or, } \frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_1}$$

$$\Rightarrow \frac{l^2 + m^2 + n^2}{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2} \dots\dots(3)$$

So,  $l, m, n$  are the direction cosines of the line.

where,  $l^2 + m^2 + n^2 = 1$  .....(4)

Then we know that,

$$\begin{aligned} \Rightarrow (l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) - (l_1l_2 + m_1m_2 + n_1n_2)^2 \\ = (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2 \end{aligned}$$

So, from the equation (1) and (2) we have,

$$(1)(1) - (0) = (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2$$

$$\text{Therefore, } (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2 = 1 \text{ ..(5)}$$

Now, we will substitute the values from the equation (4) and (5) in equation (3), to get

Therefore we have the direction cosines of the required line as;

$$l = m_1n_2 - m_2n_1$$

$$m = n_1l_2 - n_2l_1$$

$$n = l_1m_2 - l_2m_1$$

**Question:3** Find the angle between the lines whose direction ratios are  $a, b, c$  and  $b - c, c - a, a - b$ .

**Answer:**

Given direction ratios  $a, b, c$  and  $b - c, c - a, a - b$ .

Thus the angle between the lines A is given by;

$$A = \left| \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}} \right|$$

$$\Rightarrow \cos A = 0$$

$$\Rightarrow A = \cos^{-1}(0) = 90^\circ$$

**Thus, the angle between the lines is  $90^\circ$**

**Question:4** Find the equation of a line parallel to x-axis and passing through the origin.

**Answer:**

Equation of a line parallel to the x-axis and passing through the origin  $(0, 0, 0)$  is itself **x-axis** .

So, let A be a point on the x-axis.

Therefore, the coordinates of A are given by  $(a, 0, 0)$  , where  $a \in R$  .

Now, the direction ratios of OA are  $(a - 0) = a, 0, 0$

So, the equation of OA is given by,

$$\frac{x-0}{a} = \frac{y-0}{0} = \frac{z-0}{0}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0} = a$$

or

Thus, the equation of the line parallel to the x-axis and passing through origin is

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$

**Question:5** If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (− 4, 3, − 6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

**Answer:**

Direction ratios of AB are  $(4 - 1), (5 - 2), (7 - 3) = 3, 3, 4$

and Direction ratios of CD are  $(2 - (-4)), (9 - 3), (2 - (-6)) = 6, 6, 8$

So, it can be noticed that,  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Therefore, AB is parallel to CD.

**Thus, we can easily say the angle between AB and CD which is either  $0^\circ$  or  $180^\circ$ .**

**Question:6** If the lines  $\frac{x - 1}{-3} = \frac{y - 2}{k} = \frac{z - 3}{2}$  and  $\frac{x - 1}{3k} = \frac{y - 1}{1} = \frac{z - 6}{-5}$  are perpendicular, find the value of k.

**Answer:**

Given both lines are perpendicular so we have the relation;  $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

For the two lines whose direction ratios are known,

$a_1, b_1, c_1$  and  $a_2, b_2, c_2$

We have the direction ratios of the

lines,  $\frac{x - 1}{-3} = \frac{y - 2}{k} = \frac{z - 3}{2}$  and  $\frac{x - 1}{3k} = \frac{y - 1}{1} = \frac{z - 6}{-5}$  are  $-3, 2k, 2$  and  $3k, 1, -5$  respectively.

Therefore applying the formula,

$$-3(3k) + 2k(1) + 2(-5) = 0$$

$$\Rightarrow -9k + 2k - 10 = 0$$

$$\Rightarrow 7k = -10 \text{ or } k = \frac{-10}{7}$$

$\therefore$  For,  $k = \frac{-10}{7}$  the lines are perpendicular.

**Question:7** Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane  $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$

**Answer:**

Given that the plane is passing through the point  $A(1, 2, 3)$  so, the position vector of the point A is  $\vec{r}_A = \hat{i} + 2\hat{j} + 3\hat{k}$  and perpendicular to the plane  $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$  whose direction ratios are 1, 2, and -5 and the normal vector is  $\vec{n} = \hat{i} + 2\hat{j} - 5\hat{k}$

So, the equation of a line passing through a point and perpendicular to the given plane is given by,

$$\vec{l} = \vec{r} + \lambda \vec{n}, \text{ where } \lambda \in \mathbb{R}$$

$$\Rightarrow \vec{l} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$$

**Question:8** Find the equation of the plane passing through (a, b, c) and parallel to the plane  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$ .

**Answer:**

Given that the plane is passing through  $(a, b, c)$  and is parallel to the plane  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$

So, we have

The position vector of the point  $A(a, b, c)$  is,  $\vec{r}_A = a\hat{i} + b\hat{j} + c\hat{k}$

and any plane which is parallel to the plane,  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$  is of the form,

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda \quad \text{.....(1)}$$

Therefore the equation we get,

$$(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda$$

$$\text{Or, } a + b + c = \lambda$$

So, now substituting the value of  $\lambda = a + b + c$  in equation (1), we get

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c \quad \text{.....(2)}$$

**So, this is the required equation of the plane .**

Now, substituting  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  in equation (2), we get

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c$$

$$\text{Or, } x + y + z = a + b + c$$

**Question:9 Find the shortest distance between**

**lines**  $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} - 2\hat{k})$  **and**  $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$  .

**Answer:**

Given lines are;

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} - 2\hat{k}) \text{ and}$$

$$\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$$

So, we can find the shortest distance between two lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  by the formula,

$$\dots\dots\dots(1)$$

Now, we have from the comparisons of the given equations of lines.

$$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k} \quad \vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = -4\hat{i} - \hat{k} \quad \vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

So,

and

$$= 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\therefore \left| \vec{b}_1 \times \vec{b}_2 \right| = \sqrt{8^2 + 8^2 + 4^2} = 12$$

Now, substituting all values in equation (3) we get,

$$d = \left| \frac{-108}{12} \right| = 9$$

**Hence the shortest distance between the two given lines is 9 units.**

**Question:10** Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane.

**Answer:**

We know that the equation of the line that passes through the points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is given by the relation;

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

and the line passing through the points,  $\frac{x - 5}{3 - 5} = \frac{y - 1}{4 - 1} = \frac{z - 6}{1 - 6}$

$$\Rightarrow \frac{x - 5}{-2} = \frac{y - 1}{3} = \frac{z - 6}{-5} = k \text{ (say)}$$

$$\Rightarrow x = 5 - 2k, y = 3k + 1, z = 6 - 5k$$

And any point on the line is of the form  $(5 - 2k, 3k + 1, 6 - 5k)$ .

So, the equation of the YZ plane is  $x = 0$

Since the line passes through YZ- plane,

we have then,

$$5 - 2k = 0$$

$$\Rightarrow k = \frac{5}{2}$$

$$\text{or } 3k + 1 = 3\left(\frac{5}{2}\right) + 1 = \frac{17}{2} \text{ and } 6 - 5k = 6 - 5\left(\frac{5}{2}\right) = \frac{-13}{2}$$

So, therefore the required point is  $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

**Question : 11** Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX-plane.

**Answer:**

We know that the equation of the line that passes through the points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is given by the relation;

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

and the line passing through the points,  $\frac{x - 5}{3 - 5} = \frac{y - 1}{4 - 1} = \frac{z - 6}{1 - 6}$

$$\Rightarrow \frac{x - 5}{-2} = \frac{y - 1}{3} = \frac{z - 6}{-5} = k \text{ (say)}$$

$$\Rightarrow x = 5 - 2k, y = 3k + 1, z = 6 - 5k$$

And any point on the line is of the form  $(5 - 2k, 3k + 1, 6 - 5k)$ .

So, the equation of ZX plane is  $y = 0$

Since the line passes through YZ- plane,

we have then,

$$3k + 1 = 0$$

$$\Rightarrow k = -\frac{1}{3}$$

$$\text{or } 5 - 2k = 5 - 2\left(-\frac{1}{3}\right) = \frac{17}{3} \text{ and } 6 - 5k = 6 - 5\left(-\frac{1}{3}\right) = \frac{23}{3}$$

So, therefore the required point is  $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

**Question:12** Find the coordinates of the point where the line through  $(3, -4, -5)$  and  $(2, -3, 1)$  crosses the plane  $2x + y + z = 7$ .

**Answer:**

We know that the equation of the line that passes through the points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is given by the relation;

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

and the line passing through the points,  $(3, -4, -5)$  and  $(2, -3, 1)$ .

$$\Rightarrow \frac{x - 3}{2 - 3} = \frac{y + 4}{-3 + 4} = \frac{z + 5}{1 + 5} = k \text{ (say)}$$

$$\Rightarrow \frac{x - 3}{-1} = \frac{y + 4}{-1} = \frac{z + 5}{6} = k \text{ (say)}$$

$$\Rightarrow x = 3 - k, y = k - 4, z = 6k - 5$$

And any point on the line is of the form.  $(3 - k, k - 4, 6k - 5)$

This point lies on the plane,  $2x + y + z = 7$

$$\therefore 2(3 - k) + (k - 4) + (6k - 5) = 7$$

$$\Rightarrow 5k - 3 = 7$$

or  $k = 2$ .

Hence, the coordinates of the required point are  $(3 - 2, 2 - 4, 6(2) - 5)$  or  $(1, -2, 7)$ .

**Question:13** Find the equation of the plane passing through the point  $(-1, 3, 2)$  and perpendicular to each of the planes  $x + 2y + 3z = 5$  and  $3x + 3y + z = 0$ .

**Answer:**

Given

two planes  $x + 2y + 3z = 5$  and  $3x + 3y + z = 0$ .

the normal vectors of these plane are

$$n_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$n_2 = 3\hat{i} + 3\hat{j} + \hat{k}$$

Since the normal vector of the required plane is perpendicular to the normal vector of given planes, the required plane's normal vector will be :

$$\vec{n} = \vec{n}_1 \times \vec{n}_2$$

$$\vec{n} = (\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} + 3\hat{j} + \hat{k})$$

$$\vec{n} = -7\hat{i} + 8\hat{j} - 3\hat{k}$$

Now, as we know

the equation of a plane in vector form is :

$$\vec{r} \cdot \vec{n} = d$$

$$\vec{r} \cdot (-7\hat{i} + 8\hat{j} - 3\hat{k}) = d$$

Now Since this plane passes through the point  $(-1, 3, 2)$

$$(-\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (-7\hat{i} + 8\hat{j} - 3\hat{k}) = d$$

$$7 + 24 - 6 = d$$

$$d = 25$$

Hence the equation of the plane is

$$\vec{r} \cdot (-7\hat{i} + 8\hat{j} - 3\hat{k}) = 25$$

**Question:14** If the points  $(1, 1, p)$  and  $(-3, 0, 1)$  be equidistant from the plane  $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$  then find the value of  $p$ .

**Answer:**

Given that the points  $A(1, 1, p)$  and  $B(-3, 0, 1)$  are equidistant from the plane

$$\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$$

So we can write the position vector through the point  $(1, 1, p)$  is  $\vec{a}_1 = \hat{i} + \hat{j} + p\hat{k}$

Similarly, the position vector through the point  $B(-3, 0, 1)$  is

$$\vec{a}_2 = -3\hat{i} + \hat{k}$$

The equation of the given plane is  $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$

and We know that the perpendicular distance between a point whose position vector is  $\vec{a}$  and the plane,  $\vec{r} \cdot \vec{n} = d$  is  $\frac{|\vec{a} \cdot \vec{n} - d|}{|\vec{n}|}$  and  $d = -13$

Therefore, the distance between the point  $A(1, 1, p)$  and the given plane is

$$D_1 = \frac{|3 + 4 - 12p + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$

$$D_1 = \frac{|20 - 12p|}{13} \quad \text{nbsp; .....(1)}$$

Similarly, the distance between the point  $B(-1, 0, 1)$  , and the given plane is

$$D_2 = \frac{|-9 - 12 + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$

$$D_2 = \frac{8}{13} \quad \text{.....(2)}$$

And it is given that the distance between the required plane and the points,  $A(1, 1, p)$  and  $B(-3, 0, 1)$  is equal.

$$\therefore D_1 = D_2$$

$$\Rightarrow \frac{|20 - 12p|}{13} = \frac{8}{13}$$

therefore we have,

$$\Rightarrow 12p = 12$$

$$\text{or } p = 1 \text{ or } p = \frac{7}{3}$$

**Question:15** Find the equation of the plane passing through the line of intersection of the planes  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$  and  $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$  and parallel to x-axis.

**Answer:**

So, the given planes are:

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 \quad \text{and} \quad \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

The equation of any plane passing through the line of intersection of these planes is

$$\vec{r} \cdot [(2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (1 - \lambda)\hat{k}] + (4\lambda + 1) = 0 \quad \text{.....(1)}$$

Its direction ratios are  $(2\lambda + 1)$ ,  $(3\lambda + 1)$ , and  $(1 - \lambda) = 0$

The required plane is parallel to the x-axis.

Therefore, its normal is perpendicular to the x-axis.

The direction ratios of the x-axis are 1, 0, and 0.

$$\therefore 1 \cdot (2\lambda + 1) + 0 \cdot (3\lambda + 1) + 0 \cdot (1 - \lambda) = 0$$

$$\implies 2\lambda + 1 = 0$$

$$\implies \lambda = -\frac{1}{2}$$

Substituting  $\lambda = -\frac{1}{2}$  in equation (1), we obtain

$$\implies \vec{r} \cdot \left[ -\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k} \right] + (-3) = 0$$

$$\implies \vec{r}(\hat{j} - 3\hat{k}) + 6 = 0$$

So, the Cartesian equation is  $y - 3z + 6 = 0$

**Question:16** If O be the origin and the coordinates of P be  $(1, 2, -3)$ , then find the equation of the plane passing through P and perpendicular to OP.

**Answer:**

We have the coordinates of the points  $O(0, 0, 0)$  and  $P(1, 2, -3)$  respectively.

Therefore, the direction ratios of OP

are  $(1 - 0) = 1$ ,  $(2 - 0) = 2$ , and  $(-3 - 0) = -3$

And we know that the equation of the plane passing through the point  $(x_1, y_1, z_1)$  is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$  where a,b,c are the direction ratios of normal.

Here, the direction ratios of normal are 1, 2, and -3 and the point P is  $(1, 2, -3)$ .

Thus, the equation of the required plane is

$$1(x - 1) + 2(y - 2) - 3(z + 3) = 0$$

$$\implies x + 2y - 3z - 14 = 0$$

**Question:17** Find the equation of the plane which contains the line of intersection of the planes and which is perpendicular to the plane  $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

**Answer:**

The equation of the plane passing through the line of intersection of the given plane in

$$\vec{r} \cdot [(2\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (3 - \lambda)\hat{k}] + (5\lambda - 4) = 0 \dots\dots\dots(1)$$

The plane in equation (1) is perpendicular to the plane,

$$\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$$

Therefore  $5(2\lambda + 1) + 3(\lambda + 2) - 6(3 - \lambda) = 0$

$$\implies 19\lambda - 7 = 0$$

$$\implies \lambda = \frac{7}{19}$$

Substituting  $\lambda = \frac{7}{19}$  in equation (1), we obtain

$$\implies \vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 = 0 \dots\dots\dots(4)$$

So, this is the vector equation of the required plane.

The Cartesian equation of this plane can be obtained by substituting  $\implies \vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$  in equation (1).

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 = 0$$

Therefore we get the answer  $33x + 45y + 50z - 41 = 0$

**Question:18** Find the distance of the point  $(-1, -5, -10)$  from the point of intersection of the line  $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ .

**Answer:**

Given,

Equation of a line :

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda (3\hat{i} + 4\hat{j} + 2\hat{k})$$

Equation of the plane

$$\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

Let's first find out the point of intersection of line and plane.

putting the value of  $\vec{r}$  into the equation of a plane from the equation from line

$$(2 + 3\lambda) - (4\lambda - 1) + (2 + 2\lambda) = 5$$

$$\lambda + 5 = 5$$

$$\lambda = 0$$

Now, from the equation, any point p in line is

$$P = (2 + 3\lambda, 4\lambda - 1, 2 + 2\lambda)$$

So the point of intersection is

$$P = (2 + 3 * 0, 4 * 0 - 1, 2 + 2 * 0) = (2, -1, 2)$$

SO, Now,

The distance between the points (-1,-5,-10) and (2,-1,2) is

$$d = \sqrt{(2 - (-1))^2 + (-1 - (-5))^2 + (2 - (-10))^2} = \sqrt{9 + 16 + 144}$$

$$d = \sqrt{169} = 13$$

Hence the required distance is 13.

**Question:19** Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes  $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$  and  $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$ .

**Answer:**

Given

A point through which line passes

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

two plane

$$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5 \quad \text{And}$$

$$\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$$

it can be seen that normals of the planes are

$$\vec{n}_1 = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{n}_2 = 3\hat{i} + \hat{j} + \hat{k}$$

since the line is parallel to both planes, its parallel vector will be perpendicular to normals of both planes.

So, a vector perpendicular to both these normal vector is

$$\vec{d} = \vec{n}_1 \times \vec{n}_2$$

$$\vec{d} = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

Now a line which passes through  $\vec{a}$  and parallels to  $\vec{d}$  is

$$L = \vec{a} + \lambda \vec{d}$$

So the required line is

$$L = \vec{a} + \lambda \vec{d}$$

$$L = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$$

$$L = (1 - 3\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (3 + 4\lambda)\hat{k}$$

**Question: 20** Find the vector equation of the line passing through the point  $(1, 2, -4)$  and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad \text{and} \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

**Answer:**

Given

Two straight lines in 3D whose direction cosines  $(3, -16, 7)$  and  $(3, 8, -5)$

Now the two vectors which are parallel to the two lines are

$$\vec{a} = 3\hat{i} - 16\hat{j} + 7\hat{k} \quad \text{and}$$

$$\vec{b} = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

As we know, a vector perpendicular to both vectors  $\vec{a}$  and  $\vec{b}$  is  $\vec{a} \times \vec{b}$ , so

$$\vec{a} \times \vec{b} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

A vector parallel to this vector is

$$\vec{d} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Now as we know the vector equation of the line which passes through point p and parallel to vector d is

$$L = \vec{p} + \lambda \vec{d}$$

Here in our question, give point p = (1,2,-4) which means position vector of this point is

$$\vec{p} = \hat{i} + 2\hat{j} - 4\hat{k}$$

So, the required line is

$$L = \vec{p} + \lambda \vec{d}$$

$$L = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$L = (2\lambda + 1)\hat{i} + (2 + 3\lambda)\hat{j} + (6\lambda - 4)\hat{k}$$

**Question:21** Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then  $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$ .

**Answer:**

The equation of plane having a, b and c intercepts with x, y and z-axis respectively is

given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

The distance p of the plane from the origin is given by

Hence proved

**Question:22** Distance between the two planes:  $2x + 3y + 4z = 4$  and  $4x + 6y + 8z = 12$  is

(A) 2 units (B) 4 units (C) 8 units (D)  $\frac{2}{\sqrt{29}}$  unit

**Answer:**

Given equations are

$$2x + 3y + 4z = 4 \quad \text{--- (i)}$$

and

$$4x + 6y + 8z = 12$$

$$2(2x + 3y + 4z) = 12$$

$$2x + 3y + 4z = 6 \quad \text{--- (ii)}$$

Now, it is clear from equation (i) and (ii) that given planes are parallel

We know that the distance between two parallel

planes  $ax + by + cz = d_1$  and  $ax + by + cz = d_2$  is given by

$$D = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

Put the values in this equation

we will get,

$$D = \left| \frac{6 - 4}{\sqrt{2^2 + 3^2 + 4^2}} \right|$$

$$D = \left| \frac{2}{\sqrt{4 + 9 + 16}} \right| = \left| \frac{2}{\sqrt{29}} \right|$$

Therefore, the correct answer is (D)

**Question:23** The planes:  $2x - y + 4z = 5$  and  $5x - 2.5y + 10z = 6$  are

(A) Perpendicular (B) Parallel (C) intersect y-axis (D) passes through  $\left(0, 0, \frac{5}{4}\right)$

**Answer:**

Given equations of planes are

$$2x - y + 4z = 5 \quad - (i)$$

and

$$5x - 2.5y + 10z = 6$$

$$2.5(2x - y + 4z) = 6$$

$$2x - y + 4z = 2.4 \quad - (ii)$$

Now, from equation (i) and (ii) it is clear that given planes are parallel to each other

Therefore, the correct answer is (B)