

Question 1:

$\sin 2x$

ANSWER:

The anti derivative of $\sin 2x$ is a function of x whose derivative is $\sin 2x$.

It is known that,

$$\begin{aligned}\frac{d}{dx}(\cos 2x) &= -2 \sin 2x \\ \Rightarrow \sin 2x &= -\frac{1}{2} \frac{d}{dx}(\cos 2x) \\ \therefore \sin 2x &= \frac{d}{dx} \left(-\frac{1}{2} \cos 2x \right)\end{aligned}$$

Therefore, the anti derivative of $\sin 2x$ is $-\frac{1}{2} \cos 2x$

Page No 299:**Question 2:**

$\cos 3x$

ANSWER:

The anti derivative of $\cos 3x$ is a function of x whose derivative is $\cos 3x$.

It is known that,

$$\begin{aligned}\frac{d}{dx}(\sin 3x) &= 3 \cos 3x \\ \Rightarrow \cos 3x &= \frac{1}{3} \frac{d}{dx}(\sin 3x) \\ \therefore \cos 3x &= \frac{d}{dx} \left(\frac{1}{3} \sin 3x \right)\end{aligned}$$

Therefore, the anti derivative of $\cos 3x$ is $\frac{1}{3} \sin 3x$.

Page No 299:

Question 3:

$$e^{2x}$$

ANSWER:

The anti derivative of e^{2x} is the function of x whose derivative is e^{2x} .

It is known that,

$$\begin{aligned}\frac{d}{dx}(e^{2x}) &= 2e^{2x} \\ \Rightarrow e^{2x} &= \frac{1}{2} \frac{d}{dx}(e^{2x}) \\ \therefore e^{2x} &= \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)\end{aligned}$$

Therefore, the anti derivative of e^{2x} is $\frac{1}{2}e^{2x}$.

Page No 299:**Question 4:**

$$(ax+b)^2$$

ANSWER:

The anti derivative of $(ax+b)^2$ is the function of x whose derivative is $(ax+b)^2$.

It is known that,

$$\begin{aligned}\frac{d}{dx}(ax+b)^3 &= 3a(ax+b)^2 \\ \Rightarrow (ax+b)^2 &= \frac{1}{3a} \frac{d}{dx}(ax+b)^3 \\ \therefore (ax+b)^2 &= \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)\end{aligned}$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

Page No 299:

Question 5:

$$\sin 2x - 4e^{3x}$$

ANSWER:

The anti derivative of $(\sin 2x - 4e^{3x})$ is the function of x whose derivative is $(\sin 2x - 4e^{3x})$.

It is known that,

$$\frac{d}{dx} \left(-\frac{1}{2} \cos 2x - \frac{4}{3} e^{3x} \right) = \sin 2x - 4e^{3x}$$

Therefore, the anti derivative of $(\sin 2x - 4e^{3x})$ is $\left(-\frac{1}{2} \cos 2x - \frac{4}{3} e^{3x} \right)$.

Page No 299:

Question 6:

$$\int (4e^{3x} + 1) dx$$

ANSWER:

$$\int (4e^{3x} + 1) dx$$

$$= 4 \int e^{3x} dx + \int 1 dx$$

$$= 4 \left(\frac{e^{3x}}{3} \right) + x + C$$

$$= \frac{4}{3} e^{3x} + x + C$$

Page No 299:

Question 7:

$$\int x^2 \left(1 - \frac{1}{x^2} \right) dx$$

ANSWER:

$$\begin{aligned} & \int x^2 \left(1 - \frac{1}{x^2} \right) dx \\ &= \int (x^2 - 1) dx \\ &= \int x^2 dx - \int 1 dx \\ &= \frac{x^3}{3} - x + C \end{aligned}$$

Page No 299:

Question 8:

$$\int (ax^2 + bx + c) dx$$

ANSWER:

$$\begin{aligned} & \int (ax^2 + bx + c) dx \\ &= a \int x^2 dx + b \int x dx + c \int 1 dx \\ &= a \left(\frac{x^3}{3} \right) + b \left(\frac{x^2}{2} \right) + cx + C \\ &= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C \end{aligned}$$

Page No 299:

Question 9:

$$\int (2x^2 + e^x) dx$$

ANSWER:

$$\int (2x^2 + e^x) dx$$

$$\begin{aligned}
&= 2 \int x^2 dx + \int e^x dx \\
&= 2 \left(\frac{x^3}{3} \right) + e^x + C \\
&= \frac{2}{3} x^3 + e^x + C
\end{aligned}$$

Page No 299:

Question 10:

$$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx$$

ANSWER:

$$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx$$

$$\begin{aligned}
&= \int \left(x + \frac{1}{x} - 2 \right) dx \\
&= \int x dx + \int \frac{1}{x} dx - 2 \int 1 dx \\
&= \frac{x^2}{2} + \log|x| - 2x + C
\end{aligned}$$

Page No 299:

Question 11:

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

ANSWER:

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

$$\begin{aligned}
&= \int (x + 5 - 4x^{-2}) dx \\
&= \int x dx + 5 \int 1 dx - 4 \int x^{-2} dx \\
&= \frac{x^2}{2} + 5x - 4 \left(\frac{x^{-1}}{-1} \right) + C \\
&= \frac{x^2}{2} + 5x + \frac{4}{x} + C
\end{aligned}$$

Page No 299:

Question 12:

$$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$

ANSWER:

$$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$

$$\begin{aligned}
&= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} \right) dx \\
&= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3 \left(x^{\frac{3}{2}} \right)}{\frac{3}{2}} + \frac{4 \left(x^{\frac{1}{2}} \right)}{\frac{1}{2}} + C \\
&= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C \\
&= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C
\end{aligned}$$

Page No 299:

Question 13:

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

ANSWER:

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

On dividing, we obtain

$$\begin{aligned} &= \int (x^2 + 1) dx \\ &= \int x^2 dx + \int 1 dx \\ &= \frac{x^3}{3} + x + C \end{aligned}$$

Page No 299:

Question 14:

$$\int (1-x)\sqrt{x} dx$$

ANSWER:

$$\begin{aligned} &\int (1-x)\sqrt{x} dx \\ &= \int \left(\sqrt{x} - x^{\frac{3}{2}} \right) dx \\ &= \int x^{\frac{1}{2}} dx - \int x^{\frac{3}{2}} dx \\ &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C \\ &= \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + C \end{aligned}$$

Page No 299:

Question 15:

$$\int \sqrt{x}(3x^2 + 2x + 3) dx$$

ANSWER:

$$\int \sqrt{x}(3x^2 + 2x + 3) dx$$

$$\begin{aligned}
&= \int \left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) dx \\
&= 3 \int x^{\frac{5}{2}} dx + 2 \int x^{\frac{3}{2}} dx + 3 \int x^{\frac{1}{2}} dx \\
&= 3 \left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}} \right) + 2 \left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right) + 3 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\
&= \frac{6}{7} x^{\frac{7}{2}} + \frac{4}{5} x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C
\end{aligned}$$

Page No 299:

Question 16:

$$\int (2x - 3 \cos x + e^x) dx$$

ANSWER:

$$\int (2x - 3 \cos x + e^x) dx$$

$$\begin{aligned}
&= 2 \int x dx - 3 \int \cos x dx + \int e^x dx \\
&= \frac{2x^2}{2} - 3(\sin x) + e^x + C \\
&= x^2 - 3 \sin x + e^x + C
\end{aligned}$$

Page No 299:

Question 17:

$$\int (2x^2 - 3 \sin x + 5\sqrt{x}) dx$$

ANSWER:

$$\int (2x^2 - 3 \sin x + 5\sqrt{x}) dx$$

$$\begin{aligned}
&= 2 \int x^2 dx - 3 \int \sin x dx + 5 \int x^{\frac{1}{2}} dx \\
&= \frac{2x^3}{3} - 3(-\cos x) + 5 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\
&= \frac{2}{3}x^3 + 3 \cos x + \frac{10}{3}x^{\frac{3}{2}} + C
\end{aligned}$$

Page No 299:

Question 18:

$$\int \sec x (\sec x + \tan x) dx$$

ANSWER:

$$\int \sec x (\sec x + \tan x) dx$$

$$\begin{aligned}
&= \int (\sec^2 x + \sec x \tan x) dx \\
&= \int \sec^2 x dx + \int \sec x \tan x dx \\
&= \tan x + \sec x + C
\end{aligned}$$

Page No 299:

Question 19:

$$\int \frac{\sec^2 x}{\cos \operatorname{csc}^2 x} dx$$

ANSWER:

$$\int \frac{\sec^2 x}{\cos \operatorname{csc}^2 x} dx$$

$$\begin{aligned}
&= \int \frac{1}{\frac{\cos^2 x}{1}} dx \\
&= \int \frac{\sin^2 x}{\cos^2 x} dx \\
&= \int \tan^2 x dx \\
&= \int (\sec^2 x - 1) dx \\
&= \int \sec^2 x dx - \int 1 dx \\
&= \tan x - x + C
\end{aligned}$$

Page No 299:

Question 20:

$$\int \frac{2 - 3 \sin x}{\cos^2 x} dx$$

ANSWER:

$$\int \frac{2 - 3 \sin x}{\cos^2 x} dx$$

$$\begin{aligned}
&= \int \left(\frac{2}{\cos^2 x} - \frac{3 \sin x}{\cos^2 x} \right) dx \\
&= \int 2 \sec^2 x dx - 3 \int \tan x \sec x dx \\
&= 2 \tan x - 3 \sec x + C
\end{aligned}$$

Page No 299:

Question 21:

The anti derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)$ equals

(A) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (B) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$

(C) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (D) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

ANSWER:

$$\begin{aligned} & \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx \\ &= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx \\ &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C \end{aligned}$$

Hence, the correct answer is C.

Page No 299:

Question 22:

If $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$, then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

ANSWER:

It is given that,

$$\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$$

\therefore Anti derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$$\begin{aligned} \therefore f(x) &= \int 4x^3 - \frac{3}{x^4} dx \\ f(x) &= 4 \int x^3 dx - 3 \int (x^{-4}) dx \\ f(x) &= 4 \left(\frac{x^4}{4} \right) - 3 \left(\frac{x^{-3}}{-3} \right) + C \\ \therefore f(x) &= x^4 + \frac{1}{x^3} + C \end{aligned}$$

Also,

$$\begin{aligned} f(2) &= 0 \\ \therefore f(2) &= (2)^4 + \frac{1}{(2)^3} + C = 0 \\ \Rightarrow 16 + \frac{1}{8} + C &= 0 \\ \Rightarrow C &= -\left(16 + \frac{1}{8}\right) \\ \Rightarrow C &= \frac{-129}{8} \\ \therefore f(x) &= x^4 + \frac{1}{x^3} - \frac{129}{8} \end{aligned}$$

Hence, the correct answer is A.

Page No 304:

Question 1:

$$\frac{2x}{1+x^2}$$

ANSWER:

$$\text{Let } 1+x^2 = t$$

$$\therefore 2x dx = dt$$

$$\Rightarrow \int \frac{2x}{1+x^2} dx = \int \frac{1}{t} dt$$

$$= \log|t| + C$$

$$= \log|1+x^2| + C$$

$$= \log(1+x^2) + C$$

Page No 304:

Question 2:

$$\frac{(\log x)^2}{x}$$

ANSWER:

Let $\log |x| = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{(\log |x|)^2}{x} dx &= \int t^2 dt \\ &= \frac{t^3}{3} + C \\ &= \frac{(\log |x|)^3}{3} + C \end{aligned}$$

Page No 304:

Question 3:

$$\frac{1}{x + x \log x}$$

ANSWER:

$$\frac{1}{x + x \log x} = \frac{1}{x(1 + \log x)}$$

Let $1 + \log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{1}{x(1 + \log x)} dx = \int \frac{1}{t} dt$$

$$= \log |t| + C$$

$$= \log |1 + \log x| + C$$

Page No 304:

Question 4:

$$\sin x \cdot \sin(\cos x)$$

ANSWER:

$$\sin x \cdot \sin(\cos x)$$

Let $\cos x = t$

$$\therefore -\sin x dx = dt$$

$$\begin{aligned} \Rightarrow \int \sin x \cdot \sin(\cos x) dx &= - \int \sin t dt \\ &= -[-\cos t] + C \\ &= \cos t + C \\ &= \cos(\cos x) + C \end{aligned}$$

Page No 304:

Question 5:

$$\sin(ax + b) \cos(ax + b)$$

ANSWER:

$$\sin(ax + b) \cos(ax + b) = \frac{2 \sin(ax + b) \cos(ax + b)}{2} = \frac{\sin 2(ax + b)}{2}$$

$$\text{Let } 2(ax+b) = t$$

$$\therefore 2adx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{\sin 2(ax+b)}{2} dx &= \frac{1}{2} \int \frac{\sin t}{2a} dt \\ &= \frac{1}{4a} [-\cos t] + C \\ &= \frac{-1}{4a} \cos 2(ax+b) + C\end{aligned}$$

Page No 304:

Question 6:

$$\sqrt{ax+b}$$

ANSWER:

$$\text{Let } ax + b = t$$

$$\Rightarrow adx = dt$$

$$\therefore dx = \frac{1}{a} dt$$

$$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a} \int t^{\frac{1}{2}} dt$$

$$\begin{aligned}&= \frac{1}{a} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\ &= \frac{2}{3a} (ax+b)^{\frac{3}{2}} + C\end{aligned}$$

Page No 304:

Question 7:

$$x\sqrt{x+2}$$

ANSWER:

$$\text{Let } (x+2) = t$$

$$\therefore dx = dt$$

$$\begin{aligned}\Rightarrow \int x\sqrt{x+2}dx &= \int (t-2)\sqrt{t} dt \\ &= \int \left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \right) dt \\ &= \int t^{\frac{3}{2}} dt - 2 \int t^{\frac{1}{2}} dt \\ &= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\ &= \frac{2}{5} t^{\frac{5}{2}} - \frac{4}{3} t^{\frac{3}{2}} + C \\ &= \frac{2}{5} (x+2)^{\frac{5}{2}} - \frac{4}{3} (x+2)^{\frac{3}{2}} + C\end{aligned}$$

Page No 304:

Question 8:

$$x\sqrt{1+2x^2}$$

ANSWER:

$$\text{Let } 1 + 2x^2 = t$$

$$\therefore 4xdx = dt$$

$$\begin{aligned}
\Rightarrow \int x\sqrt{1+2x^2} dx &= \int \frac{\sqrt{t} dt}{4} \\
&= \frac{1}{4} \int t^{\frac{1}{2}} dt \\
&= \frac{1}{4} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\
&= \frac{1}{6} (1+2x^2)^{\frac{3}{2}} + C
\end{aligned}$$

Page No 304:

Question 9:

$$(4x+2)\sqrt{x^2+x+1}$$

ANSWER:

$$\text{Let } x^2+x+1=t$$

$$\therefore (2x+1)dx = dt$$

$$\int (4x+2)\sqrt{x^2+x+1} dx$$

$$= \int 2\sqrt{t} dt$$

$$= 2 \int \sqrt{t} dt$$

$$= 2 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{4}{3} (x^2+x+1)^{\frac{3}{2}} + C$$

Page No 304:

Question 10:

$$\frac{1}{x-\sqrt{x}}$$

ANSWER:

$$\frac{1}{x - \sqrt{x}} = \frac{1}{\sqrt{x}(\sqrt{x} - 1)}$$

Let $(\sqrt{x} - 1) = t$

$$\therefore \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{x}(\sqrt{x} - 1)} dx = \int \frac{2}{t} dt$$

$$= 2 \log |t| + C$$

$$= 2 \log |\sqrt{x} - 1| + C$$

Page No 304:

Question 11:

$$\frac{x}{\sqrt{x+4}}, x > 0$$

ANSWER:

Let $I = \int \frac{x}{\sqrt{x+4}} dx$ put $x+4=t \Rightarrow dx=dt$ Now, $I = \int \frac{(t-4)t^{1/2}}{t^{1/2}} dt = \int (t - 4)t^{1/2} dt = \int (t^{3/2} - 4t^{1/2}) dt = \frac{2}{5} t^{5/2} - 4 \cdot \frac{2}{3} t^{3/2} + C = \frac{2}{5} (x+4)^{5/2} - \frac{8}{3} (x+4)^{3/2} + C$

Page No 304:

Question 12:

$$(x^3 - 1)^{1/3} x^5$$

ANSWER:

$$\text{Let } x^3 - 1 = t$$

$$\therefore 3x^2 dx = dt$$

$$\begin{aligned}\Rightarrow \int (x^3 - 1)^{\frac{1}{3}} x^5 dx &= \int (x^3 - 1)^{\frac{1}{3}} x^3 \cdot x^2 dx \\ &= \int t^{\frac{1}{3}} (t+1) \frac{dt}{3} \\ &= \frac{1}{3} \int \left(t^{\frac{4}{3}} + t^{\frac{1}{3}} \right) dt \\ &= \frac{1}{3} \left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + C \\ &= \frac{1}{3} \left[\frac{3}{7} t^{\frac{7}{3}} + \frac{3}{4} t^{\frac{4}{3}} \right] + C \\ &= \frac{1}{7} (x^3 - 1)^{\frac{7}{3}} + \frac{1}{4} (x^3 - 1)^{\frac{4}{3}} + C\end{aligned}$$

Page No 304:

Question 13:

$$\frac{x^2}{(2+3x^3)^3}$$

ANSWER:

$$\text{Let } 2 + 3x^3 = t$$

$$\therefore 9x^2 dx = dt$$

$$\begin{aligned}
 \Rightarrow \int \frac{x^2}{(2+3x^3)^3} dx &= \frac{1}{9} \int \frac{dt}{(t)^3} \\
 &= \frac{1}{9} \left[\frac{t^{-2}}{-2} \right] + C \\
 &= \frac{-1}{18} \left(\frac{1}{t^2} \right) + C \\
 &= \frac{-1}{18(2+3x^3)^2} + C
 \end{aligned}$$

Page No 304:

Question 14:

$$\frac{1}{x(\log x)^m}, x > 0$$

ANSWER:

Let $\log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{1}{x(\log x)^m} dx = \int \frac{dt}{(t)^m}$$

$$\begin{aligned}
 &= \left(\frac{t^{-m+1}}{1-m} \right) + C \\
 &= \frac{(\log x)^{1-m}}{(1-m)} + C
 \end{aligned}$$

Page No 304:

Question 15:

$$\frac{x}{9-4x^2}$$

ANSWER:

$$\text{Let } 9 - 4x^2 = t$$

$$\therefore -8x \, dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{x}{9-4x^2} dx &= \frac{-1}{8} \int \frac{1}{t} dt \\ &= \frac{-1}{8} \log|t| + C \\ &= \frac{-1}{8} \log|9-4x^2| + C \end{aligned}$$

Page No 304:

Question 16:

$$e^{2x+3}$$

ANSWER:

$$\text{Let } 2x+3 = t$$

$$\therefore 2dx = dt$$

$$\begin{aligned} \Rightarrow \int e^{2x+3} dx &= \frac{1}{2} \int e^t dt \\ &= \frac{1}{2} (e^t) + C \\ &= \frac{1}{2} e^{(2x+3)} + C \end{aligned}$$

Page No 304:

Question 17:

$$\frac{x}{e^{x^2}}$$

ANSWER:

$$\text{Let } x^2 = t$$

$$\therefore 2x dx = dt$$

$$\Rightarrow \int \frac{x}{e^{x^2}} dx = \frac{1}{2} \int \frac{1}{e^t} dt$$

$$= \frac{1}{2} \int e^{-t} dt$$

$$= \frac{1}{2} \left(\frac{e^{-t}}{-1} \right) + C$$

$$= -\frac{1}{2} e^{-x^2} + C$$

$$= \frac{-1}{2e^{x^2}} + C$$

Page No 305:

Question 18:

$$\frac{e^{\tan^{-1} x}}{1+x^2}$$

ANSWER:

$$\text{Let } \tan^{-1} x = t$$

$$\therefore \frac{1}{1+x^2} dx = dt$$

$$\Rightarrow \int \frac{e^{\tan^{-1} x}}{1+x^2} dx = \int e^t dt$$

$$= e^t + C$$

$$= e^{\tan^{-1} x} + C$$

Page No 305:

Question 19:

$$\frac{e^{2x} - 1}{e^{2x} + 1}$$

ANSWER:

$$\frac{e^{2x} - 1}{e^{2x} + 1}$$

Dividing numerator and denominator by e^x , we obtain

$$\frac{\frac{(e^{2x} - 1)}{e^x}}{\frac{(e^{2x} + 1)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let $e^x + e^{-x} = t$

$$\therefore (e^x - e^{-x}) dx = dt$$

$$\Rightarrow \int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$\begin{aligned} &= \int \frac{dt}{t} \\ &= \log|t| + C \\ &= \log|e^x + e^{-x}| + C \end{aligned}$$

Page No 305:

Question 20:

$$\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$$

ANSWER:

Let $e^{2x} + e^{-2x} = t$

$$\therefore (2e^{2x} - 2e^{-2x}) dx = dt$$

$$\Rightarrow 2(e^{2x} - e^{-2x}) dx = dt$$

$$\begin{aligned}\Rightarrow \int \left(\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} \right) dx &= \int \frac{dt}{2t} \\ &= \frac{1}{2} \int \frac{1}{t} dt \\ &= \frac{1}{2} \log|t| + C \\ &= \frac{1}{2} \log|e^{2x} + e^{-2x}| + C\end{aligned}$$