NCERT solutions for class 12 maths chapter 8 applications of integrals

Exercise: 8.1

Question:1 Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1, x = 4$ and the x -axis in the first quadrant.

Answer:

Area of the region bounded by the curve $y^2 = x$ and the lines $x = 1, x = 4$ and the x -axis in the first quadrant

Area = $\int_1^4 ydy = \int_1^4 \sqrt{x}dx$

$= \frac{2}{3}[8-1]$

= 14/3 units

Question:2 Find the area of the region bounded by $y^2 = 9x, x = 2, x = 4$ and the x -axis in the first quadrant.

Answer:

Area of the region bounded by the curve $y^2 = 9x, x = 2, x = 4$ and the x -axis in the first quadrant

Area = $\int_2^4 ydy = \int_2^4 \sqrt{9x}dx = 3\int_2^4 \sqrt{x}dx$

$= 2\left[8 - 2\sqrt{2}\right]$

$= \left[16 - 4\sqrt{2}\right]$ units

Question:3 Find the area of the region bounded by $x^2 = 4y, y = 2, y = 4$ and the y -axis in the first quadrant.

Answer:

The area bounded by the curves $x^2 = 4y, y = 2, y = 4$ and the y -axis in the first quadrant is ABCD.

$$= \int_2^4 x dy$$

$$= \int_2^4 2\sqrt{y} dy$$

$$= 2\int_2^4 \sqrt{y} dy$$

$$= 2\left\{\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right\}_2^4$$

$$= \frac{4}{3}\left\{(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right\}$$

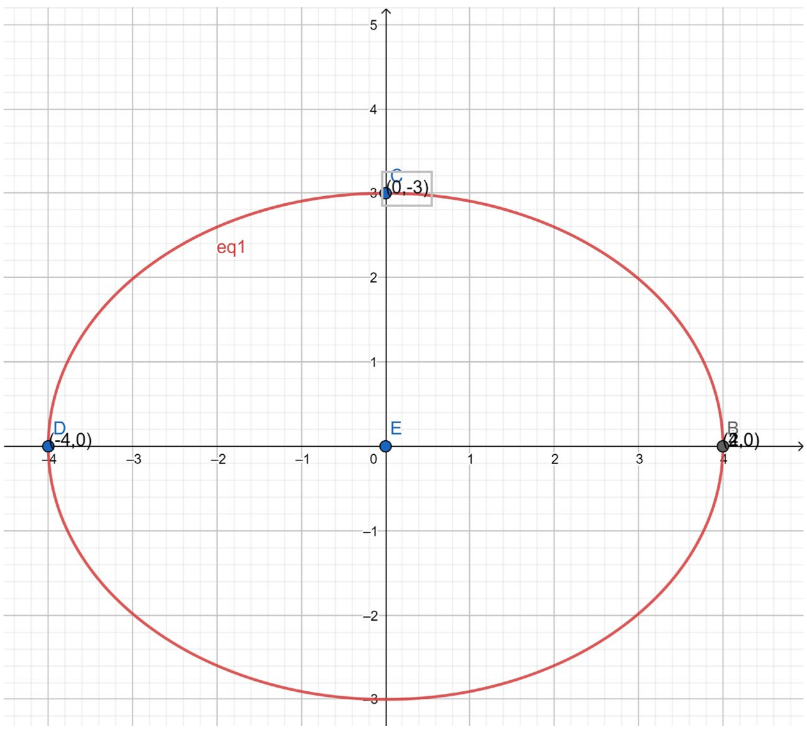
$$= \frac{4}{3}\left\{8 - 2\sqrt{2}\right\}$$

$$= \left\{\frac{32 - 8\sqrt{2}}{3}\right\} \ units.$$

Question:4 Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1.$

Answer:

The area bounded by the ellipse : $\frac{x^2}{16}+\frac{y^2}{9}=1.$



Area will be 4 times the area of EAB.

Therefore, $Area\ of\ EAB=\int_0^4 ydx$

$=\int_0^4 3\sqrt{1-\frac{x^2}{16}}dx$

$=\frac{3}{4}\int_0^4 \sqrt{16-x^2}dx$

$$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

$$= \frac{3}{4}\left[2\sqrt{16-16} + 8\sin^{-1}(1) - 0 - 8\sin^{-1}(0)\right]$$

$$= \frac{3}{4}\left[\frac{8\pi}{2}\right]$$

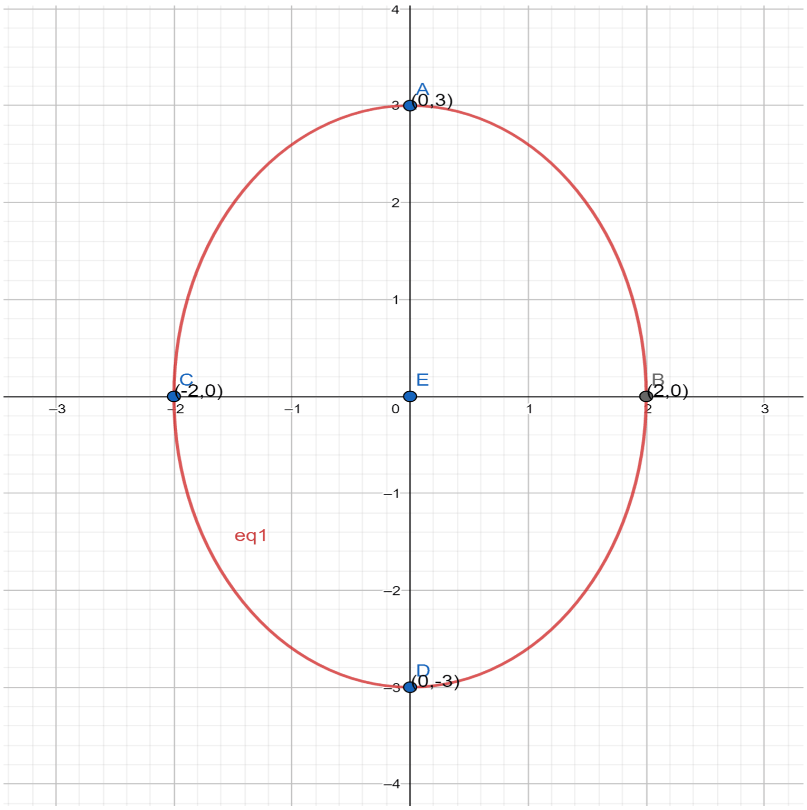
$$= \frac{3}{4}[4\pi] = 3\pi$$

Therefore the area bounded by the ellipse will be $= 4 \times 3\pi = 12\pi\ units.$

Question: 5 Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Answer:

The area bounded by the ellipse : $\frac{x^2}{4} + \frac{y^2}{9} = 1$



The area will be 4 times the area of EAB.

Therefore, $Area\ of\ EAB = \int_0^2 y dx$

$$= \int_0^2 3\sqrt{1 - \frac{x^2}{4}} dx$$

$$= \frac{3}{2} \int_0^2 \sqrt{4 - x^2} dx$$

$$= \frac{3}{2} \left[\frac{x}{2}\sqrt{4} - x^2 + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2$$

$$= \frac{3}{2} \left[\frac{2\pi}{2} \right]$$

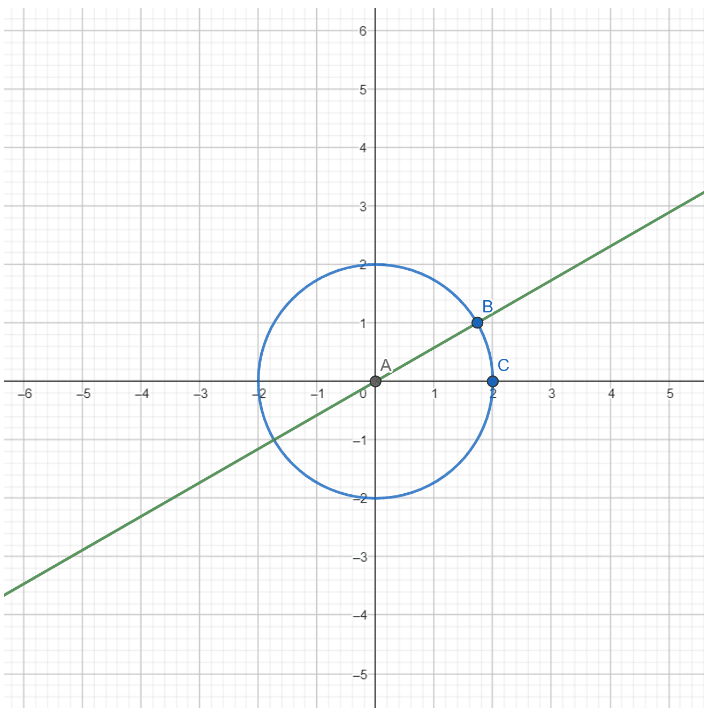
$$= \frac{3\pi}{2}$$

Therefore the area bounded by the ellipse will be $= 4 \times \frac{3\pi}{2} = 6\pi \ units.$

Question: 6 Find the area of the region in the first quadrant enclosed by x -axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$

Answer:

The area of the region bounded by $x = \sqrt{3}y$ and $x^2 + y^2 = 4$ is ABC shown:



The point B of the intersection of the line and the circle in the first quadrant is $(\sqrt{3}, 1)$.

Area ABC = Area ABM + Area BMC where, M is point in x-axis perpendicular drawn from the line.

Now,area of $ABM = \frac{1}{2} \times AM \times BM = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$ **............(1)**

and Area of $BMC = \int_{\sqrt{3}}^{2} y dx$

$$= \int_{\sqrt{3}}^{2} \sqrt{4-x^2}dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$= \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2}\sqrt{4-3} - 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \left[\pi - \frac{\sqrt{3}\pi}{2} - 2\frac{\pi}{3}\right]$$

$$= \left[\pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right]$$

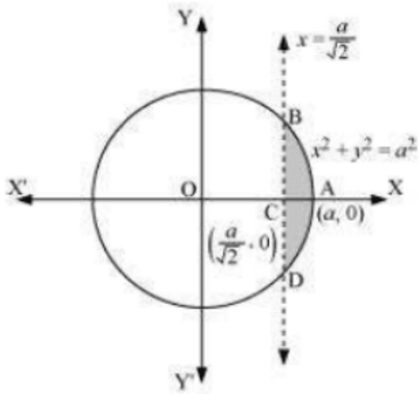
$$= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right]$$ **..................................(2)**

then adding the area (1) and (2), we have then

The Area under ABC $= \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{2} = \frac{\pi}{3}\ units.$

Question: 7 Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

Answer:



we need to find the area of smaller part of the circle

Now,

Area of ABCD = 2 X Area of ABC

Area of ABC =

$$= \left[0 + \frac{a^2}{2}\frac{\pi}{2} - \frac{a^2}{4} - \frac{a^2}{2}\frac{\pi}{4}\right]$$
$$= \frac{a^2}{4}\left(\frac{\pi}{2} - 1\right)$$

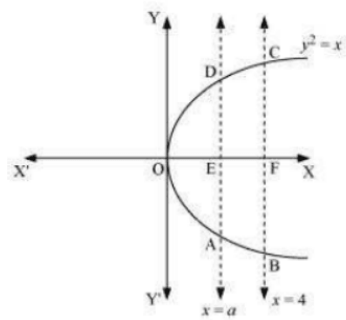
Area of ABCD = 2 X Area of ABC

$$= 2 \times \frac{a^2}{4}\left(\frac{\pi}{2} - 1\right) = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$$

Therefore, the area of the smaller part of the circle is $\frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$

Question:8 The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a .

Answer:



we can clearly see that given area is symmetrical about x - axis

It is given that

Area of OED = Area of EFCD

Area of OED =

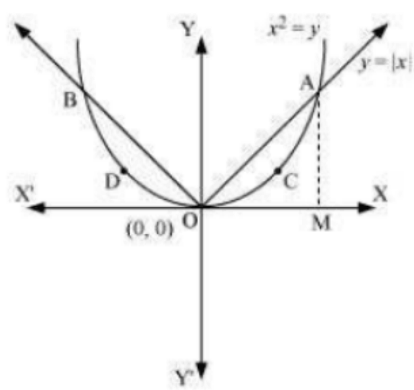
Area of EFCD =

Area of OED = Area of EFCD

Therefore, the value of a is $a = (4)^{\frac{2}{3}}$

Question:9 Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

Answer:



We can clearly see that given area is symmetrical about y-axis

Therefore,

Area of OCAO = Area of OBDO

Point of intersection of $y = x^2 \ and \ y = |x|$ is (1 , 1) and (-1 , 1)

Now,

Area od OCAO = Area OAM - Area of OCMO

Area of OAM = $\frac{1}{2}.OM.AM = \frac{1}{2}.1.1 = \frac{1}{2}$

Area of OCMO = $\int_0^1 y dx = \int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$

Therefore,

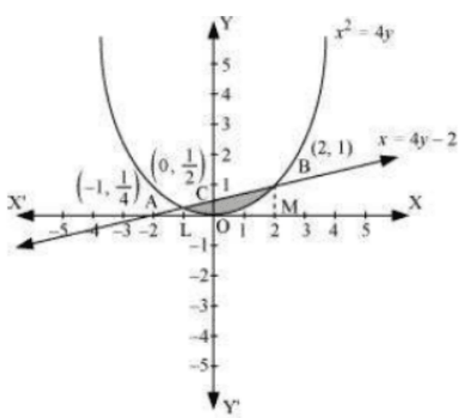
Area od OCAO $= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Now,

Area of the region bounded by the parabola $y = x^2$ and $y = |x|$ is = 2 X Area od OCAO $= 2 \times \frac{1}{6} = \frac{1}{3}$Units

Question: 10 Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

Answer:



Points of intersections of $y = x^2 \ and \ x = 4y - 2$ is
$A\left(-1, \frac{1}{4}\right) \ and \ B(2, 1)$

Now,

Area of OBAO = Area of OBCO + Area of OCAO

Area of OBCO = Area of OMBCO- Area of OMBO

Area of OMBCO =

Area of OMBO =

Area of OBCO = Area of OMBCO- Area of OMBO

$$= \frac{3}{2} - \frac{2}{3} = \frac{5}{6}$$

Similarly,

Area of OCAO = Area of OCALO - Area of OALO

Area of OCALO =

Area of OALO =

Area of OCAO = Area of OCALO - Area of OALO

$$= \frac{3}{8} - \frac{1}{12} = \frac{9-2}{24} = \frac{7}{24}$$

Now,

Area of OBAO = Area of OBCO + Area of OCAO

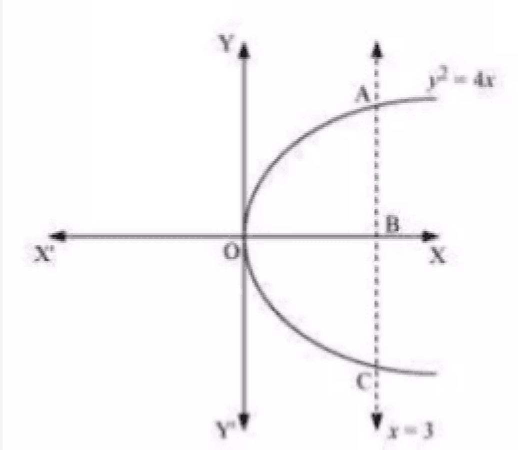
$$= \frac{5}{6} + \frac{7}{24} = \frac{20+7}{24} = \frac{27}{24} = \frac{9}{8}$$

Therefore, area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$ is $\frac{9}{8}\ units$

Question: 11 Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

Answer:

The combined figure of the curve $y^2 = 4x$ and $x = 3$



The required are is OABCO, and it is symmetrical about the horizontal axis.

Therefore, Area of OABCO = 2 × Area of OAB

$$= 2[\int_0^3 ydx]$$
$$= 2\int_0^3 2\sqrt{x}dx$$
$$= 4[\frac{x^{3/2}}{3/2}]_0^3$$
$$= 8\sqrt{3}$$

therefore the required area is $8\sqrt{3}$ units.

Question: 12 Choose the correct answer in the following

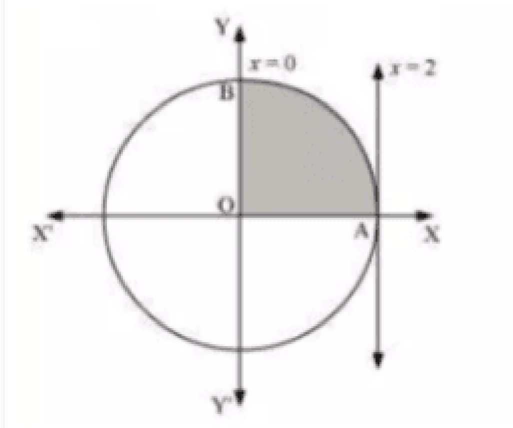
Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

$(A)\ \pi\ (B)\ \frac{\pi}{2}\ (C)\ \frac{\pi}{3}\ (D)\ \frac{\pi}{4}$

Answer:

The correct answer is A

The area bounded by circle C(0,0,4) and the line x=2 is



The required area = area of OAB

$$\int_0^2 y dx = \int_0^2 \sqrt{4 - x^2} dx$$

$$= [\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}]_0^2$$

$$= 2(\pi/2)$$

$$= \pi$$

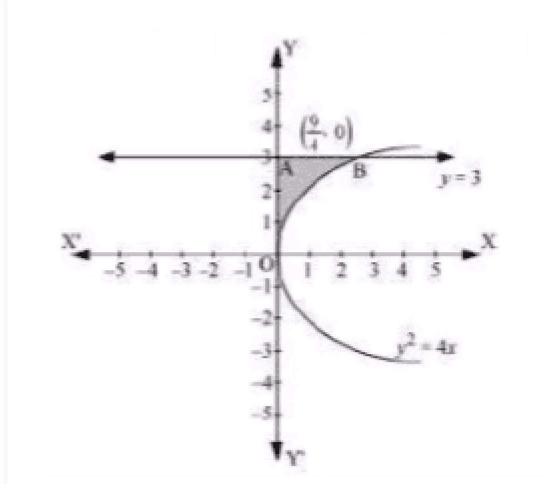
Question: 13 Choose the correct answer in the following.

Area of the region bounded by the curve $y^2 = 4x$, y -axis and the line $y = 3$ is

(A) 2 (B) $\frac{9}{4}$ (C) $\frac{9}{3}$ (D) $\frac{9}{2}$

Answer:

The area bounded by the curve $y^2 = 4x$ and y =3



the required area = OAB =
$$\int_0^3 x dy$$
$$= \int_0^3 \frac{y^2}{4} dy$$
$$= \frac{1}{4}.[\frac{y^3}{3}]_0^3$$
$$= \frac{9}{4}$$

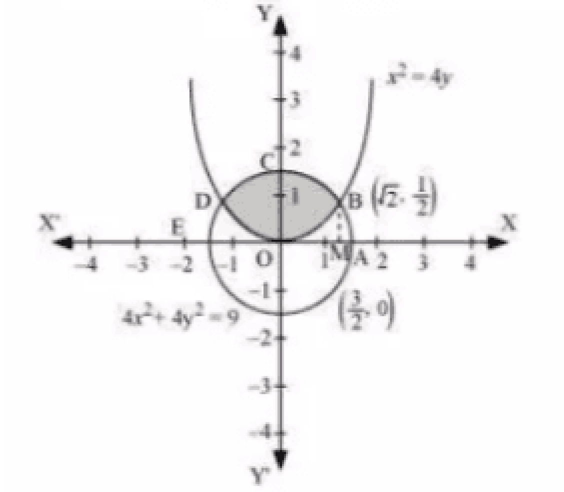
NCERT solutions for class 12 maths chapter 8 application of integrals

Exercise: 8.2

Question: 1 Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

Answer:

The area bounded by the circle $4x^2 + 4y^2 = 9$ and the parabola $x^2 = 4y$.



By solving the equation we get the intersecting point $D(-\sqrt{2}, \frac{1}{2})$ and $B(\sqrt{2}, \frac{1}{2})$

So, the required area (OBCDO)=2 times the area of (OBCO)

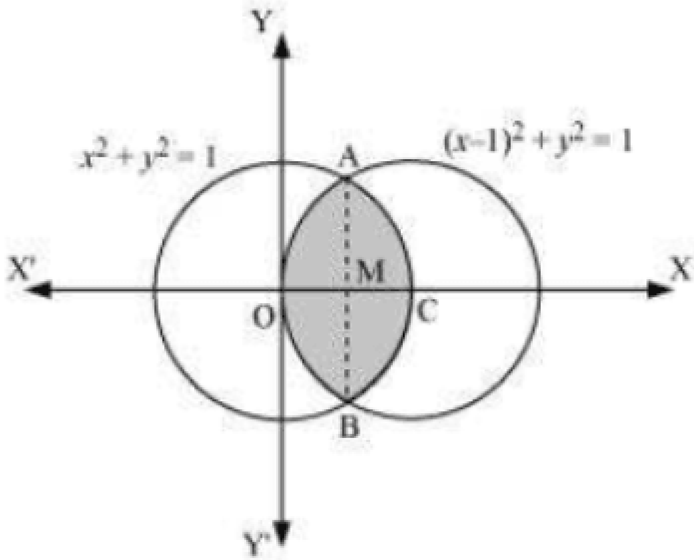
Draw a normal on the x-axis (M = $\sqrt{2}, 0$)

Thus the area of OBCO = Area of OMBCO - Area of OMBO

S0, total area =

Question:2 Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

Answer:



Given curves are $(x-1)^2+y^2=1$ and $x^2+y^2=1$

Point of intersection of these two curves are

$A=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and $B=\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$

We can clearly see that the required area is symmetrical about the x-axis

Therefore,

Area of OBCAO = 2 $\times$ Area of OCAO

Now, join AB such that it intersects the x-axis at M and AM is perpendicular to OC

Coordinates of M = $\left(\frac{1}{2},0\right)$

Now,

Area OCAO = Area OMAO + Area CMAC

$$= \left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}dx + \int_{\frac{1}{2}}^1 \sqrt{1-x^2}dx\right]$$

$$= \left[-\frac{\sqrt{3}}{8} + \frac{\pi}{6}\right] + \left[\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right]$$
$$= 2\left[-\frac{\sqrt{3}}{8} + \frac{\pi}{6}\right]$$

Now,

Area of OBCAO = 2 × Area of OCAO

$$= 2 \times 2\left[-\frac{\sqrt{3}}{8} + \frac{\pi}{6}\right]$$
$$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

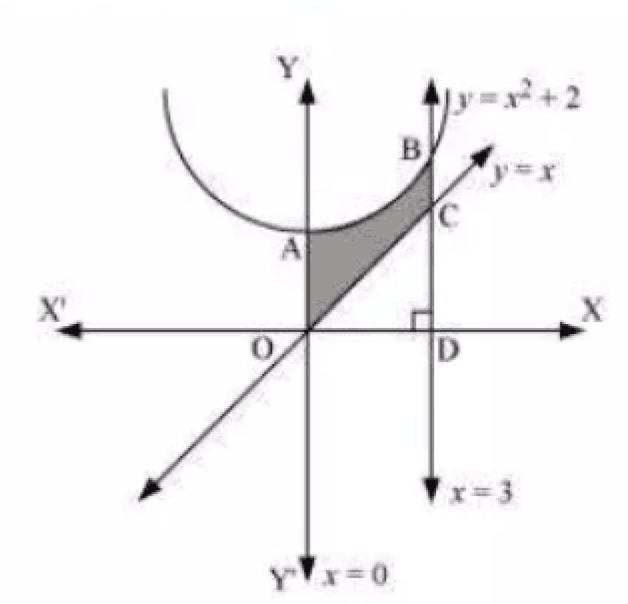
Therefore, the answer is $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

Question: 3 Find the area of the region bounded by the curves $y = x^2 + 2, y = x, x = 0$ and $x = 3$.

Answer:

The area of the region bounded by the curves,

$y=x^2+2, y=x, x=0$ and $x=3$ is represented by the shaded area OCBAO as



Then, Area OCBAO will be = Area of ODBAO - Area of ODCO

which is equal to

$$\int_0^3 (x^2+2)dx - \int_0^3 x dx$$

$$= \left(\frac{x^3}{3}+2x\right)_0^3 - \left(\frac{x^3}{2}\right)_0^3$$

$$= [9+6] - \left[\frac{9}{2}\right] = 15 - \frac{9}{2} = \frac{21}{2} units.$$

Question: 4 Using integration find the area of region bounded by the triangle whose vertices are $(-1,0), (1,3)$ and $(3,2)$.

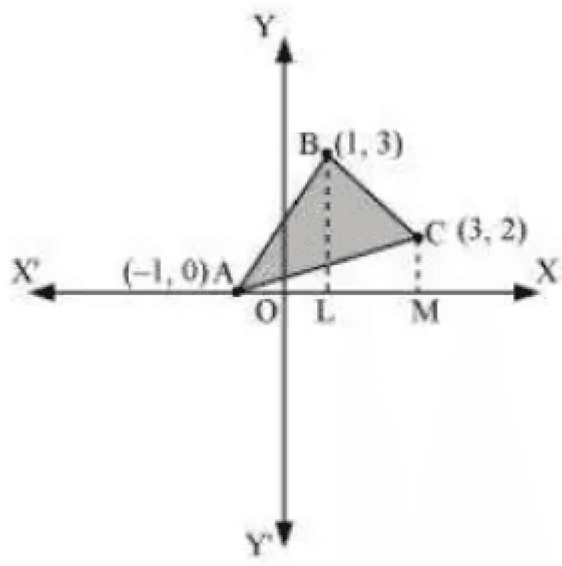
Answer:

So, we draw BL and CM perpendicular to x-axis.

Then it can be observed in the following figure that,

$Area(\triangle ACB) = Area(ALBA) + Area(BLMCB) - Area(AMCA)$

We have the graph as follows:



Equation of the line segment AB is:

$y - 0 = \frac{3-0}{1+1}(x+1)$ or $y = \frac{3}{2}(x+1)$

Therefore we have Area of $ALBA$

$= \int_{-1}^{1} \frac{3}{2}(x+1)dx = \frac{3}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{1}$

$= \frac{3}{2}\left[\frac{1}{2} + 1 - \frac{1}{2} + 1\right] = 3units.$

So, the equation of line segment BC is

$y-3=\frac{2-3}{3-1}(x-1)$ or $y=\frac{1}{2}(-x+7)$

Therefore the area of BLMCB will be,

$$=\int_{1}^{3}\frac{1}{2}(-x+7)dx=\frac{1}{2}\left[-\frac{x^2}{2}+7x\right]_1^3$$

$$=\frac{1}{2}\left[-\frac{9}{2}+21+\frac{1}{2}-7\right]=5units.$$

Equation of the line segment AC is,

$y-0=\frac{2-0}{3+1}(x+1)$ or $y=\frac{1}{2}(x+1)$

Therefore the area of AMCA will be,

$$=\frac{1}{2}\int_{-1}^{3}(x+1)dx=\frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$=\frac{1}{2}\left(\frac{9}{2}+3-\frac{1}{2}+1\right)=4units.$$

Therefore, from equations (1), we get

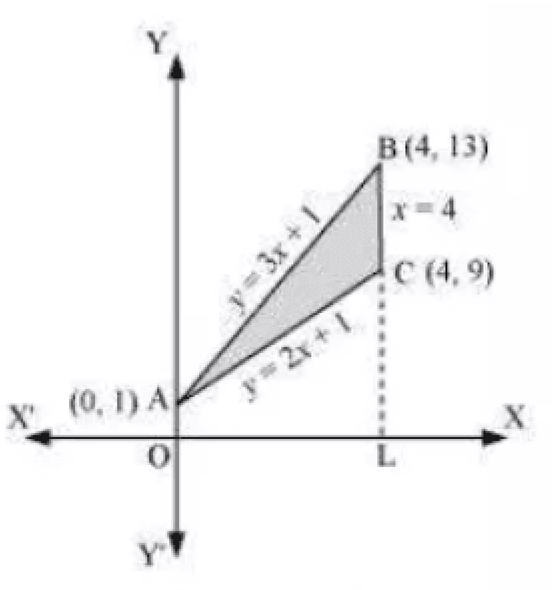
The area of the triangle $\triangle ABC=3+5-4=4units.$

Question:5 Using integration find the area of the triangular region whose sides have the equations $y=2x+1, y=3x+1$ and $x=4$.

Answer:

The equations of sides of the triangle are $y=2x+1, y=3x+1,\ and\ x=4$.

ON solving these equations, we will get the vertices of the triangle as $A(0,1), B(4,13),\ and\ C(4,9)$



Thus it can be seen that,

$Area(\triangle ACB) = Area(OLBAO) - Area(OLCAO)$

$= \int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx$

$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left[\frac{2x^2}{2} + x\right]_0^4$

$= (24+4) - (16+4) = 28 - 20 = 8units.$

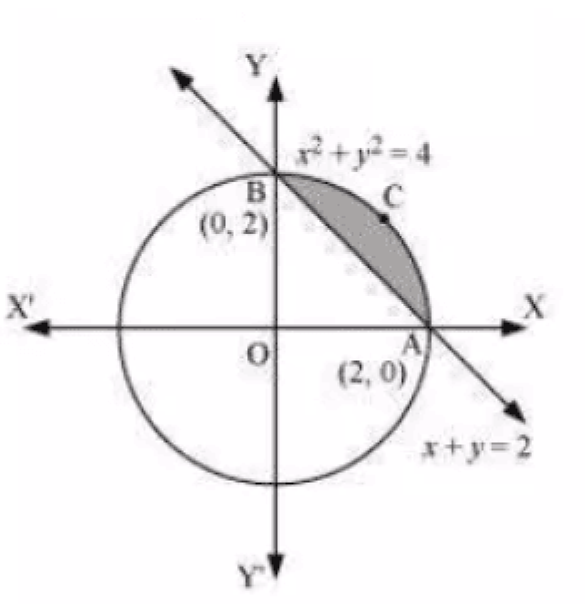
Question:6 Choose the correct answer.

Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

(A) $2(\pi - 2)$ (B) $\pi - 2$ (C) $2\pi - 1$ (D) $2(\pi + 2)$

Answer:

So, the smaller area enclosed by the circle, $x^2 + y^2 = 4$, and the line, $x + y = 2$, is represented by the shaded area ACBA as



Thus it can be observed that,

Area of ACBA = Area OACBO - Area of $(\triangle OAB)$

$$= \int_0^2 \sqrt{4 - x^2} dx - \int_0^2 (2 - x) dx$$

$$= \left[2.\frac{\pi}{2}\right] - [4 - 2]$$

$$= (\pi - 2) units.$$

Thus, the correct answer is B.

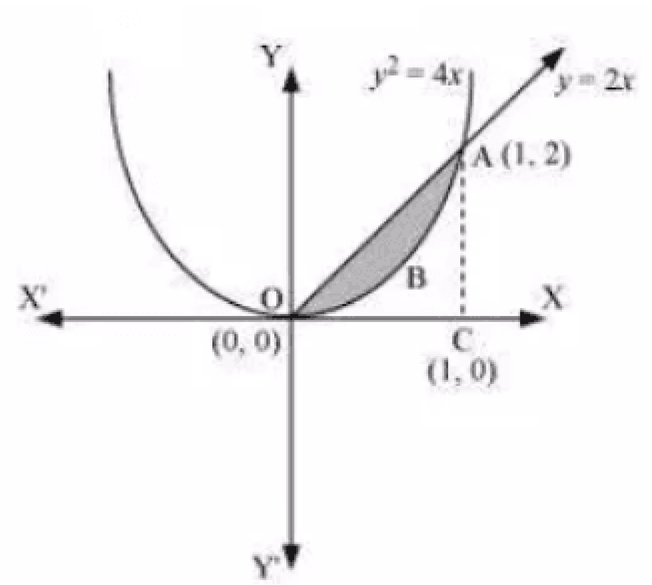
Question:7 Choose the correct answer.

Area lying between the curves $y^2 = 4x$ and $y = 2x$ is

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Answer:

The area lying between the curve, $y^2 = 4x$ and $y = 2x$ is represented by the shaded area OBAO as



The points of intersection of these curves are $O(0,0)$ and $A(1,2)$.

So, we draw AC perpendicular to x-axis such that the coordinates of C are (1,0).

Therefore the Area OBAO = $Area(\triangle OCA) - Area(OCABO)$

$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$$

$$= \left|1 - \frac{4}{3}\right| = \left|-\frac{1}{3}\right| = \frac{1}{3} units.$$

Thus the correct answer is B.

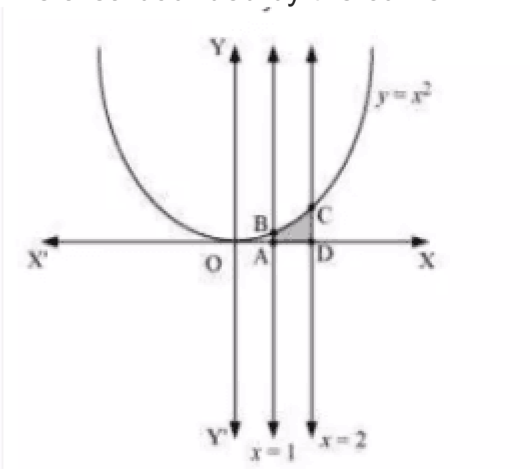
NCERT solutions for class 12 maths chapter 8 application of integrals Miscellaneous: Exercise

Question:1 Find the area under the given curves and given lines:

(i) $y = x^2, x = 1, x = 2$ and x -axis

Answer:

The area bounded by the curve $y = x^2, x = 1, x = 2$ and x -axis



The area of the required region = area of ABCD

$= \int_1^2 y dx$

$= \int_1^2 x^2 dx$

$= [\frac{x^3}{3}]_1^2$

$= \frac{7}{3}$

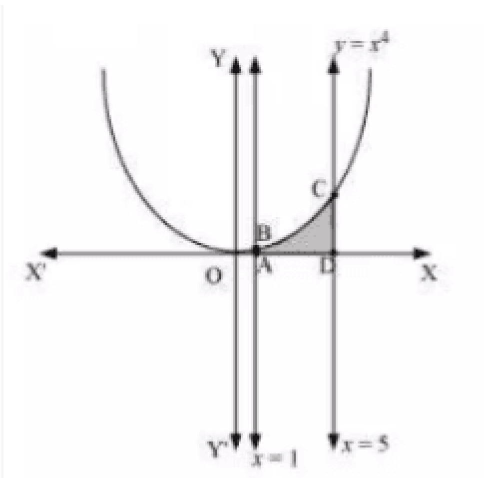
Hence the area of shaded region is 7/3 units

Question:1 Find the area under the given curves and given lines:

(ii) $y = x^4, x = 1, x = 5$ and x -axis

Answer:

The area bounded by the curev $y = x^4, x = 1, x = 5$ and x -axis



The area of the required region = area of ABCD

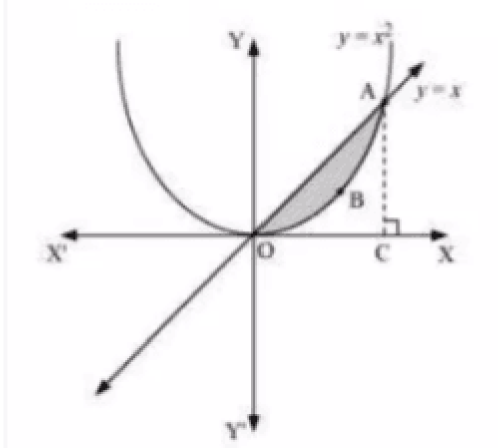
$$= \int_1^5 y dx$$
$$= \int_1^2 x^4 dx$$
$$= [\frac{x^5}{5}]_1^2$$
$$= 625 - \frac{1}{5}$$
$$= 624.8$$

Hence the area of the shaded region is 624.8 units

Question:2 Find the area between the curves $y = x$ and $y = x^2$.

Answer:

the area between the curves $y = x$ and $y = x^2$.



The curves intersect at A(1,1)

Draw a normal to AC to OC(x-axis)

therefore, the required area (OBAO)= area of (OCAO) - area of (OCABO)

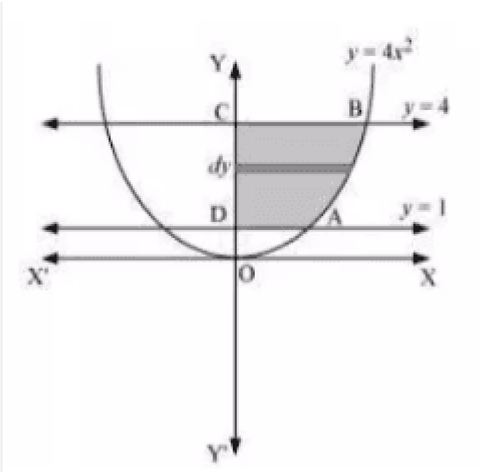
$$= \int_0^1 x dx - \int_0^1 x^2 dx$$
$$= [\frac{x^2}{2}]_0^1 - [\frac{x^3}{3}]_0^1$$
$$= 1/2 - 1/3$$
$$= \frac{1}{6}$$

Thus the area of shaded region is 1/6 units

Question:3 Find the area of the region lying in the first quadrant and bounded by $y = 4x^2, x = 0, y = 1$and $y = 4$.

Answer:

the area of the region lying in the first quadrant and bounded by $y = 4x^2, x = 0, y = 1$ and $y = 4$.



The required area (ABCD) =

The area of the shaded region is 7/3 units

Question:4 Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|dx.$

Answer:

y=|x+3|

the given modulus function can be written as

x+3>0

x>-3

for x>-3

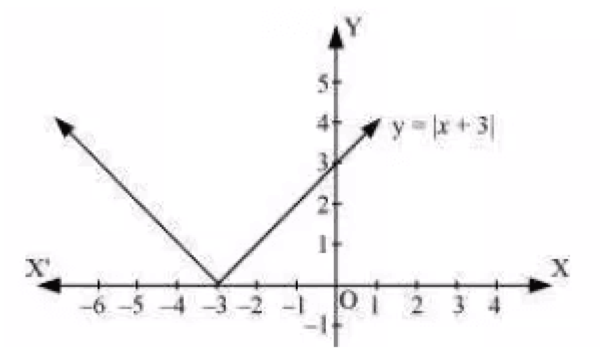
y=|x+3|=x+3

x+3<0

x<-3

For x<-3

y=|x+3|=-(x+3)

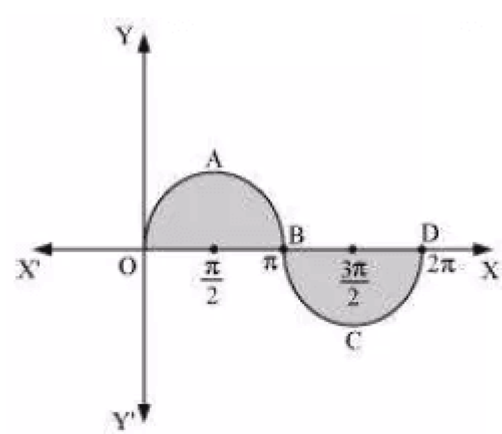


Integral to be evaluated is

Question:5 Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Answer:

The graph of y=sinx is as follows



We need to find the area of the shaded region

ar(OAB)+ar(BCD)

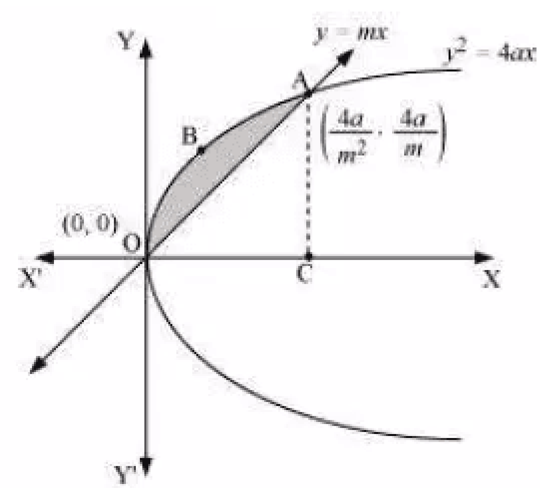
=2ar(OAB)

$$= 2 \times \int_0^{\pi} sinx dx$$
$$= 2 \times [-cosx]_0^{\pi}$$
$$= 2 \times [-(-1) - (-1)]$$
$$= 4$$

The bounded area is 4 units.

Question:6 Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$.

Answer:



We have to find the area of the shaded region OBA

The curves y=mx and y ^{2} =4ax intersect at the following points

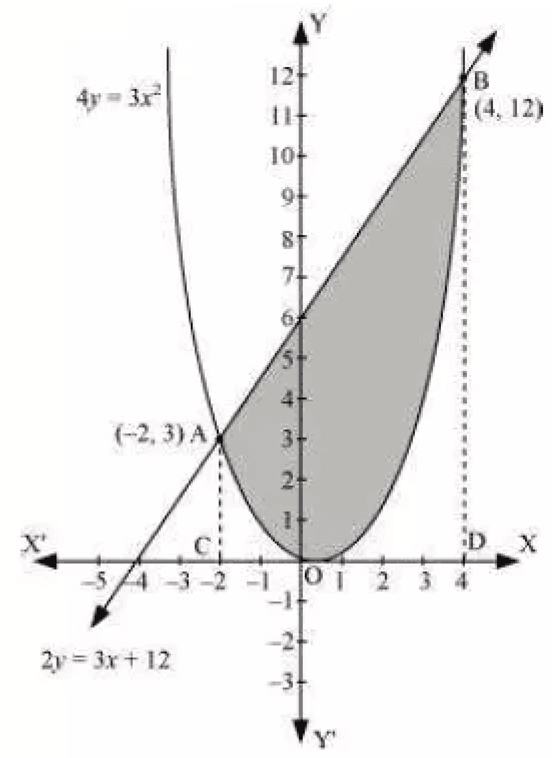
$$(0, 0)\ and \left(\frac{4a}{m^2}, \frac{4a}{m}\right)$$

$$y^2 = 4ax$$
$$\Rightarrow y = 2\sqrt{ax}$$

The required area is

Question:7 Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Answer:



We have to find the area of the shaded region COB

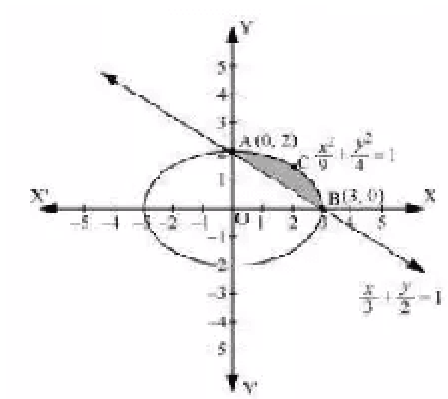
$$2y = 3x + 12$$
$$\Rightarrow y = \frac{3}{2}x + 6$$
$$4y = 3x^2$$
$$\Rightarrow y = \frac{3x^2}{4}$$

The two curves intersect at points (2,3) and (4,12)

Required area is

Question:8 Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

Answer:



We have to find the area of the shaded region

The given ellipse and the given line intersect at following points

$$(0,2)\ and\ (3,0)$$

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$
$$y=\frac{2}{3}\sqrt{9-x^2}$$

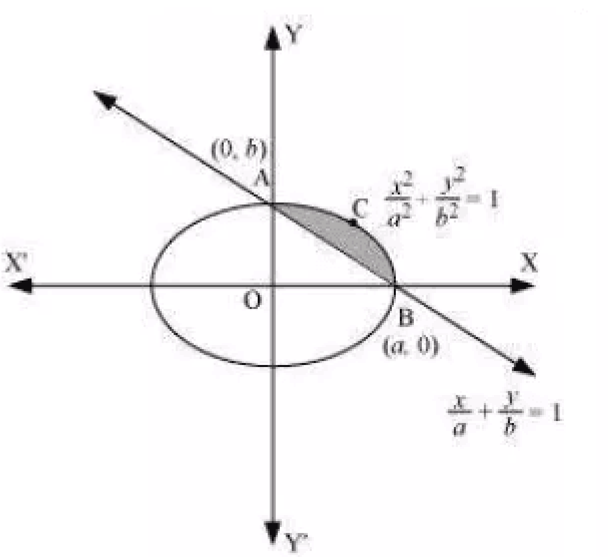
Since the shaded region lies above x axis we take y to be positive

$$\frac{x}{3}+\frac{y}{2}=1$$
$$y=\frac{2}{3}(3-x)$$

The required area is

Question:9 Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.

Answer:



The area of the shaded region ACB is to be found

The given ellipse and the line intersect at following points

$(0, b)\, and\, (a, 0)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$
$$\Rightarrow y=\frac{b}{a}\sqrt{a^2-x^2}$$

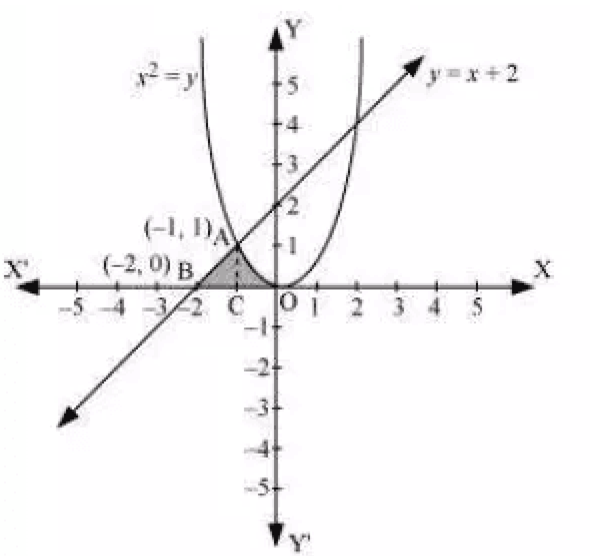
Y will always be positive since the shaded region lies above x axis

$$\frac{x}{a}+\frac{y}{b}=1$$
$$\Rightarrow y=\frac{b}{a}(a-x)$$

The required area is

Question:10 Find the area of the region enclosed by the parabola $x^2=y,$ the line $y=x+2$ and the x -axis.

Answer:



We have to find the area of the shaded region BAOB

O is(0,0)

The line and the parabola intersect in the second quadrant at (-1,1)

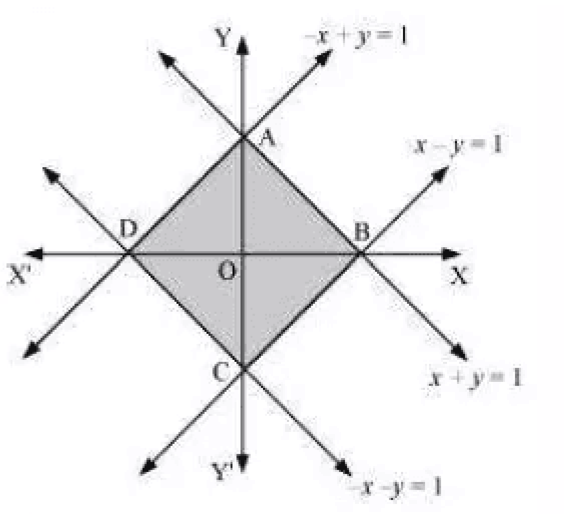
The line y=x+2 intersects the x axis at (-2,0)

The area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the x -axis is 5/6 units.

Question:11 Using the method of integration find the area bounded by the curve $|x| + |y| = 1$.

[**Hint:** The required region is bounded by lines $x + y = 1, x - y = 1, -x + y = 1$ and $-x - y = 1$]

Answer:



We need to find the area of the shaded region ABCD

ar(ABCD)=4ar(AOB)

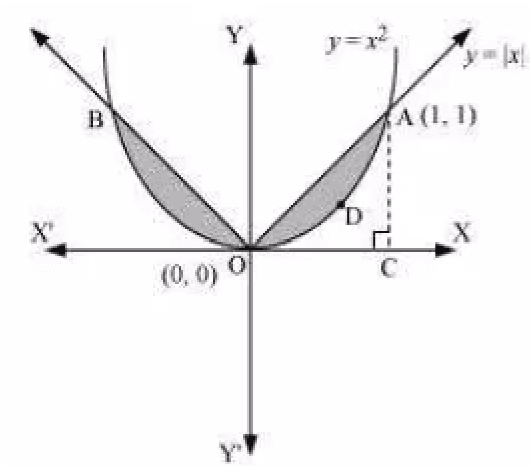
Coordinates of points A and B are (0,1) and (1,0)

Equation of line through A and B is y=1-x

The area bounded by the curve $|x|+|y|=1$ is 2 units.

Question:12 Find the area bounded by curves $\{(x, y) ; y \geq x^{2} \; and \; y=|x|\}$.

Answer:



We have to find the area of the shaded region

In the first quadrant

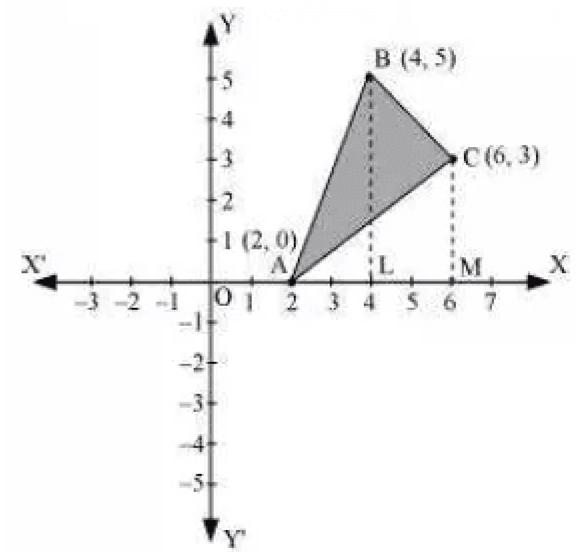
y=|x|=x

Area of the shaded region=2ar(OADO)

The area bounded by the curves is 1/3 units.

Question:13 Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(2,0), B(4,5) \; and \; C(6,3)$.

Answer:



Equation of line joining A and B is

$$\frac{y-0}{x-2}=\frac{5-0}{4-2}$$
$$y=\frac{5x}{2}-5$$

Equation of line joining B and C is

$$\frac{y-5}{x-4}=\frac{5-3}{4-6}$$
$$y=9-x$$

Equation of line joining A and C is

$$\frac{y-0}{x-2}=\frac{3-0}{6-2}$$
$$y=\frac{3x}{4}-\frac{3}{2}$$

ar(ABC)=ar(ABL)+ar(LBCM)-ar(ACM)

$$ar(ABL) = \int_2^4 (\frac{5x}{2} - 5)dx$$
$$= [\frac{5x^2}{4} - 5x]_2^4$$
$$= (20 - 20) - (5 - 10)$$
$$= 5\ units$$

$$ar(LBCM) = \int_4^6 (9 - x)dx$$
$$= [9x - \frac{x^2}{2}]_4^6$$
$$= (54 - 18) - (36 - 8)$$
$$= 8\ units$$

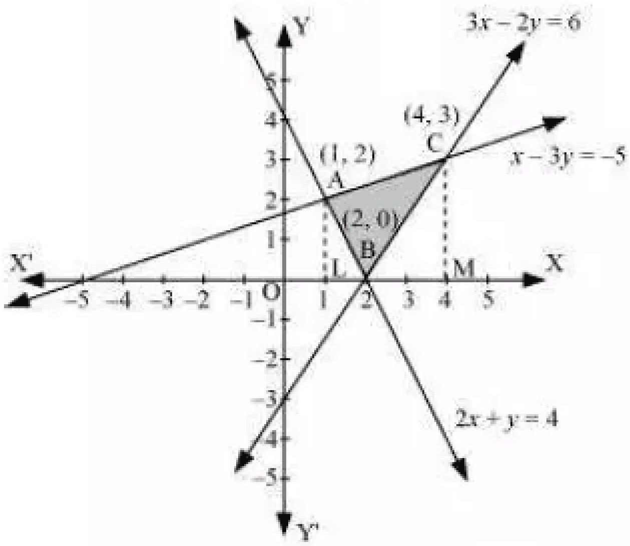
ar(ABC)=8+5-6=7

Therefore the area of the triangle ABC is 7 units.

Question:14 Using the method of integration find the area of the region bounded by lines:

$2x + y = 4, 3x - 2y = 6\ and\ x - 3y + 5 = 0.$

Answer:



We have to find the area of the shaded region ABC

ar(ABC)=ar(ACLM)-ar(ALB)-ar(BMC)

The lines intersect at points (1,2), (4,3) and (2,0)

$$x - 3y = -5$$
$$y = \frac{x}{3} + \frac{5}{3}$$

$$2x + y = 4$$
$$y = 4 - 2x$$

$$ar(ALB) = \int_1^2 (4 - 2x)dx$$
$$= [4x - x^2]_1^2$$
$$= (8 - 4) - (4 - 1)$$
$$= 1 \; unit$$

$$3x - 2y = 6$$
$$y = \frac{3x}{2} - 3$$

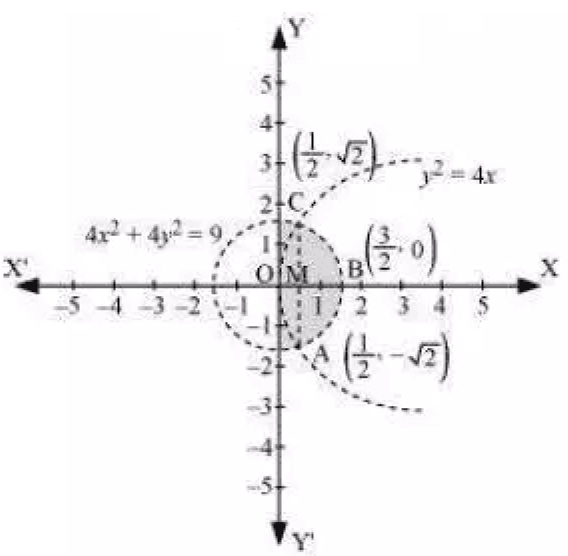
$$ar(BMC) = \int_2^4 (\frac{3x}{2} - 3)dx$$
$$= [\frac{3x^2}{4} - 3x]_2^4$$
$$= (12 - 12) - (3 - 6)$$
$$= 3\ units$$

$$ar(ABC) = \frac{15}{2} - 1 - 3$$
$$= \frac{7}{2}\ units$$

Area of the region bounded by the lines is 3.5 units

Question:15 Find the area of the region $\{(x, y); y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$.

Answer:



We have to find the area of the shaded region OCBAO

Ar(OCBAO)=2ar(OCBO)

For the fist quadrant

$$4x^2 + 4y^2 = 9$$
$$y = \sqrt{\frac{9}{4} - x^2}$$

$$y^2 = 4x$$
$$y = 2\sqrt{x}$$

In the first quadrant, the curves intersect at a point $\left(\frac{1}{2}, \sqrt{2}\right)$

Area of the unshaded region in the first quadrant is

The total area of the shaded region is-

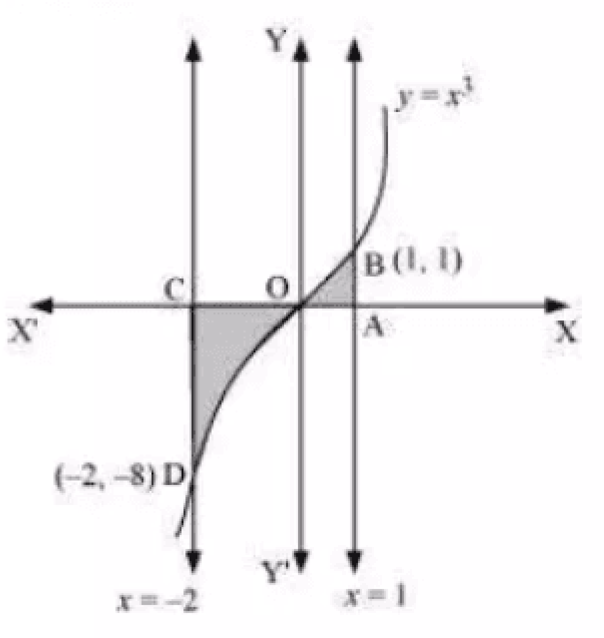
= Area of half circle - area of the shaded region in the first quadrant

Question:16 Choose the correct answer.

Area bounded by the curve $y = x^3$, the x -axis and the ordinates $x = -2$ and $x = 1$ is

(A) -9 (B) $\frac{-15}{4}$ (C) $\frac{15}{4}$ (D) $\frac{17}{4}$

Answer:



Hence the required area

$$= \int_{-2}^{1} y dx$$

$$= \int_{-2}^{1} x^3 dx = \left[\frac{x^4}{4}\right]_{-2}^{1}$$

$$= \left[\frac{x^4}{4}\right]_{-2}^{0} + \left[\frac{x^4}{4}\right]_{0}^{1}$$

$$= \left[0 - \frac{(-2)^4}{4}\right] + \left[\frac{1}{4} - 0\right]$$

$$= -4 + \frac{1}{4} = \frac{-15}{4}$$

Therefore the correct answer is B.

Question:17 Choose the correct answer.

T he area bounded by the curve $y = x|x|$, x -axis and the ordinates $x = -1$ and $x = 1$ is given by

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

[**Hint :** $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$.]

Answer:

The required area is

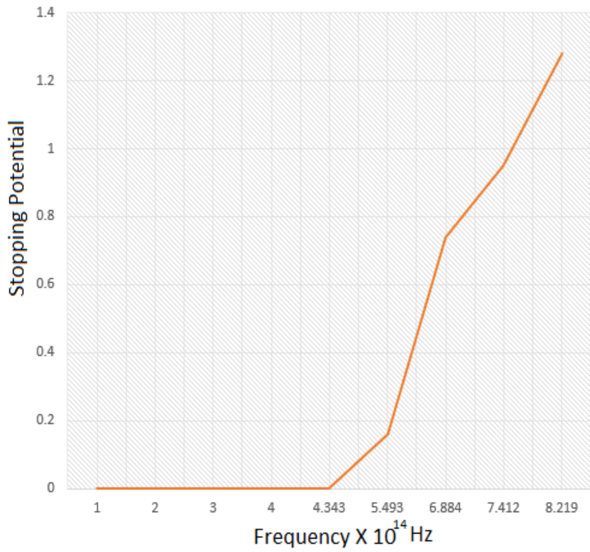
$$2\int_0^1 x^2 dx$$
$$= 2\left[\frac{x^3}{3}\right]_0^1$$
$$= \frac{2}{3}\ units$$

Question:18 Choose the correct answer.

The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

(A) $\frac{4}{3}(4\pi - \sqrt{3})$ (B) $\frac{4}{3}(4\pi + \sqrt{3})$ (C) $\frac{4}{3}(8\pi - \sqrt{3})$ (D) $\frac{4}{3}(8\pi + \sqrt{3})$

Answer:



The area of the shaded region is to be found.

Required area =ar(DOC)+ar(DOA)

The region to the left of the y-axis is half of the circle with radius 4 units and centre origin.

Area of the shaded region to the left of y axis is ar(1) = $\frac{\pi 4^2}{2} = 8\pi \ units$

For the region to the right of y-axis and above x axis

$$x^2 + y^2 = 16$$
$$y = \sqrt{16 - x^2}$$

$$y^2 = 6x$$
$$y = \sqrt{6x}$$

The parabola and the circle in the first quadrant intersect at point

$\left(2, 2\sqrt{3}\right)$

Remaining area is 2ar(2) is

Total area of shaded region is

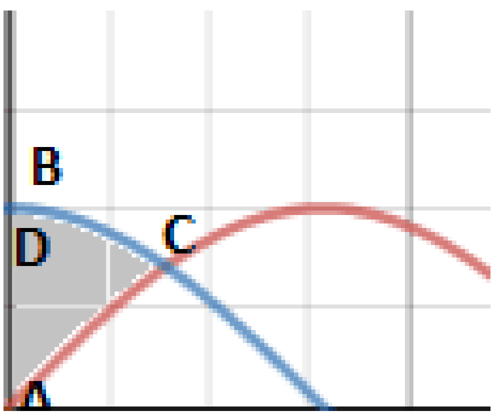
$$ar(1) + 2ar(2)$$
$$= 8\pi + \frac{8\pi}{3} - \frac{4\sqrt{3}}{3}$$
$$= \frac{4}{3}(8\pi - \sqrt{3})\ units$$

Question:19 Choose the correct answer The area bounded by the y - axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$ is

(A) $2(\sqrt{2}-1)$ (B) $\sqrt{2}-1$ (C) $\sqrt{2}+1$ (D) $\sqrt{2}$

Answer:

Given : $y = \cos x$ and $y = \sin x$



Area of shaded region = area of BCDB + are of ADCA

$$= \int_{\frac{1}{\sqrt{2}}}^{1} xdy + \int_{1}^{\frac{1}{\sqrt{2}}} xdy$$

$$= \int_{\frac{1}{\sqrt{2}}}^{1} cos^{-1}y.dy + \int_{1}^{\frac{1}{\sqrt{2}}} sin^{-1}xdy$$

$$= [y.cos^{-1}y - \sqrt{1-y^2}]_{\frac{1}{\sqrt{2}}}^{1} + [x.sin^{-1}x + \sqrt{1-x^2}]_{1}^{\frac{1}{\sqrt{2}}}$$

$$= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

$$= \frac{2}{\sqrt{2}} - 1$$

$$= \sqrt{2} - 1$$

Hence, the correct answer is B.