

Date: 06/02/2021

Test Booklet Code

44



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# Answers & Solutions

Time : 1 hr

*for*

Max. Marks : 120

## International Olympiad Qualifiers (Part I) Astronomy (IOQA) 2020-21

### INSTRUCTIONS TO CANDIDATES

- (1) There are 32 objective type questions. Out of 32 questions, 24 questions in **Part A1** and 8 questions in **Part A2**. All questions are compulsory.
- (2) In **Part A1** each question has four alternatives out of which one is correct.
- (3) In **Part A2** each question has four alternatives out of which any number of alternative(s) (1,2,3 or 4) may be correct.
- (4) For **Part A1**, each correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer.
- (5) For **Part A2**, each correct answer carries 6 marks if all correct answers are marked and no incorrect. No negative marking for this part.

**PART-A1**

1. In the Period-Luminosity relationship of Cepheid variables, type I Cepheid has 4 times the luminosity than that of type II Cepheid for the same period. The distance to Andromeda Galaxy was determined by assuming a Cepheid located in it as type I. If it had been assumed to be a type II Cepheid, the distance would
- Increase by a factor of 4
  - Increase by a factor of 2
  - Decrease by a factor of 4
  - Decrease by a factor of 2

**Answer (d)**

**Sol.**  $l_1 = 4l_2 \Rightarrow L_1 = 4L_2$

$$M_1 = 4.77 - 2.5 \log \left( \frac{L_1}{L_{\text{sun}}} \right) = m - 5 \log d_1$$

$$\text{and } M_2 = 4.77 - 2.5 \log \left( \frac{L_2}{L_{\text{sun}}} \right) = m - 5 \log d_2$$

$$\Rightarrow -2.5 \log \left( \frac{L_1}{L_2} \right) = -5 \log \left( \frac{d_1}{d_2} \right)$$

$$\Rightarrow \log \left( \frac{L_1}{L_2} \right) = 2 \log \left( \frac{d_1}{d_2} \right)$$

$$\Rightarrow \frac{L_1}{L_2} = \left( \frac{d_1}{d_2} \right)^2 \Rightarrow 4 = \left( \frac{d_1}{d_2} \right)^2$$

$$\Rightarrow d_2 = \frac{d_1}{2}$$

2. The energy emitted by the Sun is due to fusion reaction  $4_1\text{H}^1 \rightarrow 2_2\text{He}^4 + 2_1\beta^0 + 2\nu_e$  ( $\nu_e$  are neutrinos) with a release of 27 MeV. Assume the solar constant i.e., the energy received per second per meter<sup>2</sup> on the surface of Earth is  $1.4 \text{ k W m}^{-2}$ . The neutrino flux on the Earth (in  $\text{m}^{-2} \text{ s}^{-1}$ ) is approximately (assume that the neutrinos do not interact with anything on their way to the Earth)
- $6.5 \times 10^{14}$
  - $6.5 \times 10^{10}$
  - $6.5 \times 10^{18}$
  - $6.5 \times 10^{22}$

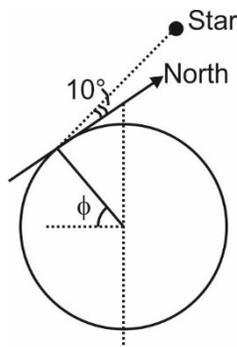
**Answer (a)**

**Sol.** Neutrino flux =  $\frac{1.4 \times 10^3}{\frac{27}{2} \times 10^6 \times 1.6 \times 10^{-19}} \text{ m}^{-2} \text{ s}^{-1}$

$$= 6.48 \times 10^{14} \text{ m}^{-2} \text{ s}^{-1}$$



**Sol.** A circumpolar star is one, which is continually visible above horizon.



$$\text{Latitude } \phi = (90 - 10)^\circ\text{N} = 80^\circ\text{N}$$

6. The Moon makes hour angle of  $40^\circ$  at 6 pm on a particular day when the Sun is about to set. The hour angle of the Moon at the same time next day is approximately

- (a)  $70^\circ$  (b)  $40^\circ$   
(c)  $-40^\circ$  (d)  $27^\circ$

**Answer (d)**

**Sol.** Hour angle of Moon changes by  $13^\circ$  in 24 hrs.

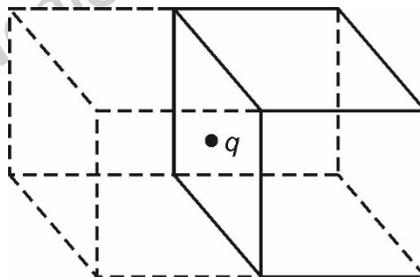
$$\text{Hour angle of the Moon at the same time next day} = 40^\circ - 13^\circ = 27^\circ$$

7. Consider a cube with one of its 6 faces open. A charge  $q$  is placed at the center of the open face. The total electric flux through the rest of the 5 faces will be nearly equal to

- (a) Zero (b)  $\frac{q}{\epsilon_0}$   
(c)  $\frac{q}{2\epsilon_0}$  (d)  $\frac{5q}{6\epsilon_0}$

**Answer (c)**

**Sol.** Consider another identical cube that encloses the charge symmetrically inside the cuboid as shown in figure.



$$\phi_1 = \phi_2 \quad \dots(1)$$

$$\phi_1 + \phi_2 = \frac{q}{\epsilon_0} \quad \dots(2)$$

$$\Rightarrow \phi_1 = \phi_2 = \frac{q}{2\epsilon_0}$$



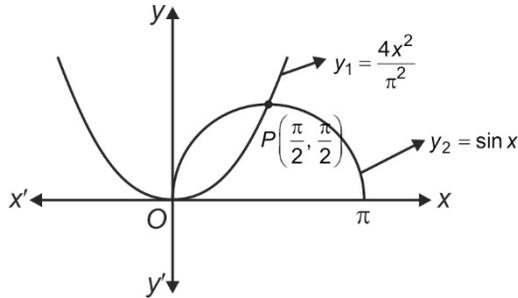


14. Consider the two curves  $y_1 = \frac{4x^2}{\pi^2}$  and  $y_2 = \sin x$  in the region  $0 < x < \pi$ . The angle made by the curves at the point of intersection is

- (a)  $\tan^{-1}\left(\frac{1}{\pi}\right)$  (b)  $\tan^{-1}\left(\frac{4}{\pi}\right)$   
 (c)  $\tan^{-1}\infty$  (d)  $\tan^{-1}0$

**Answer (b)**

**Sol.**



The point of intersection of two given curve  $y_1 = \frac{4x^2}{\pi^2}$  and  $y_2 = \sin x$  is  $P\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ .

$$\therefore \text{Slope of tangent to } y_1 = \frac{4x^2}{\pi^2} \text{ at } \left(\frac{\pi}{2}, \frac{\pi}{2}\right) = \left(\frac{8x}{\pi^2}\right)_{\text{at } \left(\frac{\pi}{2}, \frac{\pi}{2}\right)} = \frac{4}{\pi} = m_1$$

$$\text{Slope of tangent to } y_2 = \sin x \text{ at } \left(\frac{\pi}{2}, \frac{\pi}{2}\right) = (\cos x)_{\text{at } \left(\frac{\pi}{2}, \frac{\pi}{2}\right)} = 0 = m_2$$

$$\therefore \text{Angle between curves} = \theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right) = \tan^{-1}\left(\frac{4}{\pi}\right)$$

15. The three points  $(2, 3, -4)$ ,  $(1, -2, 3)$  and  $(3, 8, r)$  are collinear. The value of  $r$  is
- (a) 0 (b) -10  
 (c) -11 (d) 10

**Answer (c)**

**Sol.**  $A(2, 3, -4)$   $B(1, -2, 3)$   $C(3, 8, r)$

$$\therefore \overline{AB} = \hat{i} - 5\hat{j} + 7\hat{k} \text{ and } \overline{AC} = \hat{i} + 5\hat{j} + (r+4)\hat{k}$$

If  $A, B, C$  are collinear, then  $\overline{AB} \times \overline{AC} = 0$ .

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -5 & 7 \\ 1 & 5 & r+4 \end{vmatrix} = 0$$

$$0\hat{i} + (r+11)\hat{j} + 0\hat{k} = 0$$

$$\therefore r + 11 = 0$$

$$\Rightarrow r = -11$$

16. Time period of a simple pendulum is theoretically  $\pi$  seconds. In an experiment to measure this time period, a stop clock having least count of one-hundredth of a second is used and the time taken for 'n' oscillations is measured. The percentage error in the calculation of the time period will be

- (a)  $(n\pi)^{-1}\%$  (b)  $\pi\%$   
(c) 2% (d)  $2(n\pi)^{-1}\%$

**Answer (a)**

**Sol.** % error =  $\frac{\Delta t}{T} \times 100$

$$\Delta t = \frac{1}{100}, T = n\pi$$

$$\Rightarrow \% \text{ error} = \frac{1}{n\pi} = (n\pi)^{-1}$$

17. A large sphere A of 20 kg being accelerated at  $5.0 \text{ ms}^{-2}$  strikes another sphere B of mass 8.0 kg. At the moment of impact, the acceleration of B is  $12 \text{ ms}^{-2}$ . The force on B at the moment of impact is

- (a) 40N (b) 96N  
(c) 240N (d) 100N

**Answer (b)**

**Sol.**  $F = ma$

$$= 8.0 \times 12$$

$$= 96.0 \text{ N}$$

18. Consider that in space with no air resistance and negligible gravity, a ball of mass 250 g, initially at rest, is projected with a force of 30 N giving it a speed of  $20 \text{ ms}^{-1}$ . The ball travels a distance of 1000 m before it strikes a space ship. The original energy, E (in joule) with which the ball was projected is

- (a) 30,000 (b) 50  
(c)  $50 < E < 30,000$  (d) None of these

**Answer (b)**

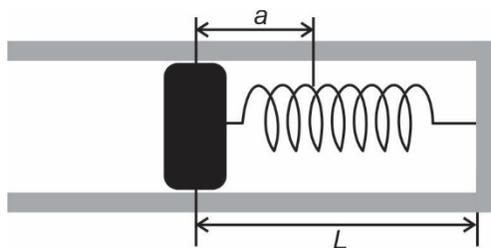
**Sol.** E = Energy of projected ball

$$= \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times \left(\frac{1}{4}\right) \times (20)^2$$

$$= 50 \text{ J}$$

19. The diagram shows a mass  $m$  free to slide inside a long frictionless tube along the  $x$ -axis. It is attached to a spring of spring constant  $k$  whose unscratched length is  $L$ . Initially the mass is pushed to a position  $x = a (< L)$  and released from rest. Let there be a certain position  $x = x_m$  at which the maximum power,  $P_{\max}$ , is generated due to kinetic energy. The values  $x_m$  and  $P_{\max}$  are respectively



- (a)  $\frac{a}{2}$  &  $\left(\frac{3ka^2}{2}\sqrt{\frac{k}{m}}\right)$                       (b)  $0$  &  $\left(ka^2\sqrt{\frac{k}{m}}\right)$
- (c)  $\frac{a}{\sqrt{2}}$  &  $\left(\frac{ka^2}{2}\sqrt{\frac{k}{m}}\right)$                       (d)  $\frac{3a}{4}$  &  $\left(2ka^2\sqrt{\frac{k}{m}}\right)$

**Answer (c)**

**Sol.**  $x = a \cos \omega t$

$$v = v_0 \sin \omega t$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 \sin^2 \omega t$$

$$\begin{aligned} \frac{d(KE)}{dt} = P &= \frac{1}{2}mv_0^2 \times 2 \sin \omega t \cos \omega t \times \omega \\ &= \frac{1}{2}mv_0^2 \times \omega \times \sin(2\omega t) \end{aligned}$$

$$\text{For } P \text{ to be max, } 2\omega t = \frac{\pi}{2}$$

$$\therefore x = a \cos\left(\frac{\pi}{4}\right) = \frac{a}{\sqrt{2}}$$

$$\text{and, } P_{\max} = \left(\frac{1}{2}ka^2\right) \times \sqrt{\frac{k}{m}}$$

20. The value of  $\tan^{-1}\left[\frac{\cos x}{1 - \sin x}\right]$  is equal to

- (a)  $\frac{x}{2}$     (b)  $\frac{x}{2} - \frac{\pi}{2}$
- (c)  $\frac{x}{2} - \frac{\pi}{4}$                                         (d)  $\frac{x}{2} + \frac{\pi}{4}$

**Answer (d)**

$$\begin{aligned} \text{Sol. } \tan^{-1} \left[ \frac{\cos x}{1 - \sin x} \right] &= \tan^{-1} \left[ \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\left( \cos \frac{x}{2} - \sin \frac{x}{2} \right)^2} \right] \\ &= \tan^{-1} \left( \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \right) = \tan^{-1} \left( \frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}} \right) = \tan^{-1} \left( \frac{\tan \frac{\pi}{4} + \tan \frac{x}{2}}{1 - \tan \frac{\pi}{4} \cdot \tan \frac{x}{2}} \right) \\ &= \tan^{-1} \left( \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) \right) = \frac{\pi}{4} + \frac{x}{2} \end{aligned}$$

21. The determinant  $\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & x \end{vmatrix} = 0$ . The value of  $x$  is

(a)  $x = 0$  or  $1$

(b)  $x = \pm 1$

(c)  $x = -1$

(d)  $x = 1$

**Answer (c)**

$$\text{Sol. } \begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & x \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2$$

$$\begin{vmatrix} 0 & 2 & 0 \\ -1 & 1 & 1 \\ 1 & 1 & x \end{vmatrix} = 0$$

$$\Rightarrow -2(-x - 1) = 0$$

$$\therefore x = -1.$$

22. The value of the given integral  $\int_{\pi/3}^{\pi/4} \frac{dx}{\sin^2 x \cdot \cos^2 x}$  is

(a)  $\frac{\pi}{4} - \frac{\pi}{3}$

(b)  $0$

(c)  $\frac{1}{\sqrt{3}}$

(d)  $-\frac{2}{\sqrt{3}}$

**Answer (d)**

**Sol.** 
$$\int_{\pi/3}^{\pi/4} \frac{dx}{\sin^2 x \cdot \cos^2 x}$$

$$= \int_{\pi/3}^{\pi/4} \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} dx$$

$$= \int_{\pi/3}^{\pi/4} (\sec^2 x + \operatorname{cosec}^2 x) dx$$

$$= [\tan x - \cot x]_{\pi/3}^{\pi/4}$$

$$= (1-1) - \left( \sqrt{3} - \frac{1}{\sqrt{3}} \right) = -\frac{2}{\sqrt{3}}$$

23. Consider the four points  $A, B, C, D$  forming a regular tetrahedron with sides each of length  $L$ . The coordinates  $(x, y, z)$  of  $A, B$  and  $C$  are  $A(0, 0, 0), B(L, 0, 0)$  and  $C\left(\frac{L}{2}, \frac{L\sqrt{3}}{2}, 0\right)$ . The possible coordinates of  $D$  are

- (a)  $\left(\frac{L}{2}, \frac{L}{2}, \pm \frac{L}{\sqrt{2}}\right)$                       (b)  $\left(\frac{L}{2}, \frac{L}{2\sqrt{3}}, \pm \frac{L\sqrt{2}}{\sqrt{3}}\right)$
- (c)  $\left(\frac{L}{\sqrt{2}}, \frac{L}{\sqrt{3}}, \pm \frac{L}{\sqrt{3}}\right)$                       (d)  $\left(\frac{L}{\sqrt{2}}, \frac{L}{\sqrt{3}}, \pm \frac{L\sqrt{5}}{\sqrt{12}}\right)$

**Answer (b)**

**Sol.** Let point  $D(\alpha, \beta, \gamma)$

$$\therefore AD = BD = CD = L$$

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha^2 - L)^2 + \beta^2 + \gamma^2 = \left(\alpha - \frac{L}{2}\right)^2 + \left(\beta - \frac{\sqrt{3}L}{2}\right)^2 + \gamma^2 = L^2$$

$$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = \alpha^2 + \beta^2 + \gamma^2 + L^2 - 2\alpha L = \alpha^2 + \beta^2 + \gamma^2 + L^2 - \alpha L - \sqrt{3}\beta L = L^2$$

$$\Rightarrow \alpha = \frac{L}{2}, \beta = \frac{L}{2\sqrt{3}} \text{ and } \gamma = \pm \frac{\sqrt{2}}{\sqrt{3}}L$$

24. The argument of the complex number  $z = \frac{1+i}{1-i\sqrt{3}}$  is

- (a)  $\pi$     (b)  $\frac{7\pi}{12}$
- (c)  $-\frac{5\pi}{12}$                                         (d)  $\frac{5\pi}{12}$

**Answer (b)**

**Sol.**  $z = \frac{1+i}{1-i\sqrt{3}}$

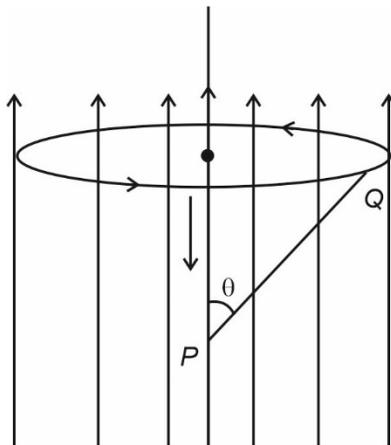
$$\therefore 1+i = \sqrt{2} e^{i\pi/4} \text{ and } 1-i\sqrt{3} = 2e^{-i\pi/3}$$

$$\text{So, } z = \frac{\sqrt{2}e^{i\pi/4}}{2 \cdot e^{-i\pi/3}} = \frac{1}{\sqrt{2}} e^{i\left(\frac{\pi}{4} + \frac{\pi}{3}\right)} = \frac{1}{\sqrt{2}} e^{i\left(\frac{7\pi}{12}\right)}$$

$$\arg(z) = \frac{7\pi}{12}$$

**PART-A2**

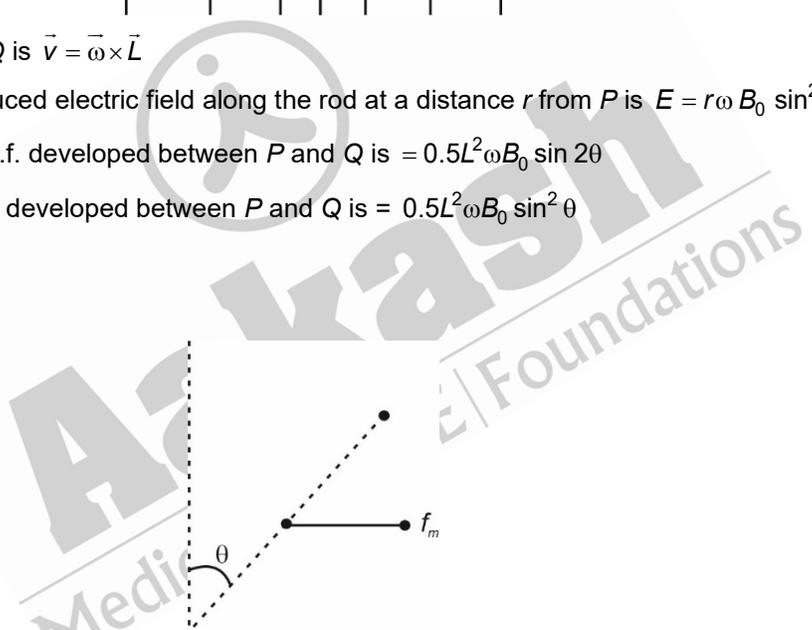
25. A uniform magnetic field  $\vec{B} = B_0 \hat{k}$  exists over a certain region of space as shown in figure. A metal rod  $PQ$  of length  $L$  is fixed at  $P$  and  $PQ$  makes a constant angle of  $\theta$  with  $\hat{k}$  as it rotates about  $\hat{k}$  with a constant angular velocity  $\omega$ .



- (a) The velocity of point  $Q$  is  $\vec{v} = \vec{\omega} \times \vec{L}$
- (b) The magnitude of induced electric field along the rod at a distance  $r$  from  $P$  is  $E = r\omega B_0 \sin^2 \theta$
- (c) The magnitude of e.m.f. developed between  $P$  and  $Q$  is  $= 0.5L^2\omega B_0 \sin 2\theta$
- (d) The magnitude of emf developed between  $P$  and  $Q$  is  $= 0.5L^2\omega B_0 \sin^2 \theta$

**Answer (a, b, d)**

**Sol.**  $v_Q = \vec{\omega} \times \vec{L}$



Net force on charge particle in conductor is zero.

$$f_e = f_m \sin \theta$$

$$E = r\omega B_0 \sin^2 \theta$$

$$\text{Now, } v_{PQ} = -\int E \cdot dr = \frac{\omega B_0 L^2}{2} \sin^2 \theta$$

26. Spectroscopic analysis of light from stars gives us information about

- (a) The abundance of elements in the stars
- (b) Parallax of stars
- (c) The radial velocity of stars
- (d) Proper motion of stars

**Answer (a, b, c)**

**Sol.** From spectral lines we can determine not only element but the temperature and density of that element in star. Width of line can tell us how fast material is moving.

27. Which of the following terms refer to a variable star?

- (a) White dwarf (b) RR Lyrae  
 (c) Black hole (d) Eclipsing binary

**Answer (a, b, d)**

**Sol.** A variable star is a star whose brightness as seen from earth fluctuates. White dwarf, RR Lyrae, Eclipsing binary comes under the category of variable stars.

28. Which of the statement(s) about Globular cluster(s) is/are true?

- (a) They are in the outer regions of the Milky Way  
 (b) They comprise of variable stars  
 (c) They comprise of young stars  
 (d) They are receding from us at very high speed

**Answer (a, b, d)**

**Sol.** A globular cluster is a spherical collection of stars that orbits a galactic case. Globular clusters are tightly bound by gravity and receding away from us at high speeds.

29. If  $a$  and  $b$  are real numbers, the equation  $a(x + 3)^2 + b(y + 4)^2 = 1$  represents

- (a) An ellipse or a circle if  $a > 0$  and  $b > 0$   
 (b) An ellipse or a circle if  $a < 0$  and  $b < 0$   
 (c) A hyperbola if  $a > 0$  and  $b < 0$  or  $a < 0$  and  $b > 0$   
 (d) A parabola if  $a > 0$  and  $b = 0$

**Answer (a, c)**

**Sol.**  $a(x + 3)^2 + b(y + 4)^2 = 1$

- (a) If  $a$  and  $b$  are positive, the curve is an ellipse or a circle  
 (b) If  $a < 0$  and  $b < 0$ , there is no such curve  
 (c) If  $ab < 0$  then the curve is hyperbola  
 (d) If any one of  $a$  or  $b$  is zero, then it is a straight line

30. If  $\tan \theta = \cot\left(\frac{\pi}{3}\right)$  then  $\theta$  can be

- (a)  $\frac{\pi}{6}$  (b)  $\frac{7\pi}{6}$   
 (c)  $\frac{13\pi}{6}$  (d)  $\frac{5\pi}{6}$

**Answer (a, b, c)**

**Sol.**  $\tan \theta = \cot\left(\frac{\pi}{3}\right)$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} = \tan \frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{6}, n \in \mathbb{N}$$

So, the possible values of  $\theta$  are  $\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$ .

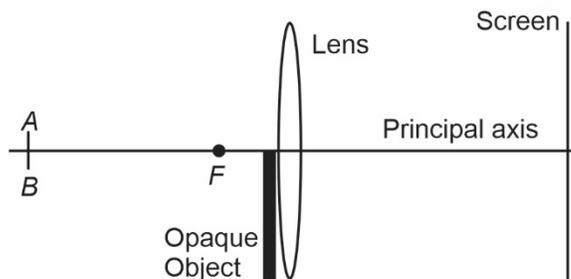
31. The critical velocity of a flowing liquid depends on

- (a) Coefficient of viscosity (b) Reynold number  
(c) Density of the liquid (d) Diameter of the tube

**Answer (a, b, c, d)**

**Sol.**  $R = \frac{\rho v D}{\eta}$

32. Consider the situation as shown in the diagram where a symmetric biconvex lens is half covered with an opaque object. Given that the object shown  $AB$  is symmetric about the principal axis, which of the following is true about the image seen on the screen?



- (a) Full image of  $AB$  is formed  
(b) Intensity of the image is reduced  
(c) Full image of  $AB$  is formed and the intensity is reduced  
(d) Only half of  $AB$  is visible

**Answer (a, b, c)**

**Sol.** Image will be formed by upper part of lens only.

Intensity will be reduced but full image of  $AB$  is formed.

