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Question Paper Code

63



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# Answers & Solutions

Time : 1 hr

*for*

Max. Marks : 120

## International Olympiad Qualifier (Part I) in PHYSICS (IOQP) 2020-21

### INSTRUCTIONS TO CANDIDATES

- (1) There are 32 objective type questions. Out of 32 questions, 24 questions in **Part A1** and 8 questions in **Part A2**. All questions are compulsory.
- (2) In **Part A1** each question has four options out of which only one is correct.
- (3) For **Part A1**, each correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer.
- (4) In **Part A2** each question has four options out of which any number of option(s) (4,3, 2 or 1) may be correct.
- (5) For **Part A2**, each correct answer carries 6 marks if all correct options are marked and no wrong option. No negative marks in this part.

**PART-A1**

1. A particle is executing Simple Harmonic Motion of time period  $T = 4\pi^2$  in a straight line. Starting from rest, it travels a distance  $a$  in the first second and distance  $b$  in the next second travelling in the same direction. The amplitude of SHM is

(a)  $\frac{2a^2}{3a-b}$

(b)  $\frac{3a^2}{3a-2b}$

(c)  $\frac{2a^2}{2a-b}$

(d) None of these

**Answer (a)**

**Sol.**  $T = 4\pi^2 \Rightarrow \omega = \frac{1}{2\pi}$

$x = A \cos \omega t$  (As motion starts from rest)

$A - a = A \cos \omega$  ... (1)

$A - a - b = A \cos 2\omega$  ... (2)

$\Rightarrow 1 - \frac{a+b}{A} = 2\left(1 - \frac{a}{A}\right)^2 - 1$

$\Rightarrow A = \frac{2a^2}{3a-b}$

2. The kinetic energy of a particle moving along a circle of radius  $R$  depends upon the distance covered  $s$  as  $KE = as^2$  where  $a$  is a constant. The magnitude of the force acting on the particle as a function of  $s$  is

(a)  $\frac{2as^2}{R}$

(b)  $\frac{2as^2}{m}$

(c)  $2as$

(d)  $2as\sqrt{1 + \left(\frac{s}{R}\right)^2}$

**Answer (d)**

**Sol.**  $\frac{1}{2}mv^2 = as^2$

$\Rightarrow F_t = mv \frac{dv}{ds} = 2as$

$\Rightarrow F_c = \frac{mv^2}{R} = \frac{2as^2}{R}$

$\Rightarrow F = \sqrt{F_t^2 + F_c^2} = 2as\sqrt{1 + \left(\frac{s}{R}\right)^2}$

3. The flow of water in a horizontal pipe is stream line flow. Along the pipe, at a point, where cross-sectional area is  $10 \text{ cm}^2$ , the velocity of water flow is  $1.00 \text{ ms}^{-1}$  and the pressure is  $2000 \text{ Pa}$ . The pressure of water at another point where cross-sectional area is  $5 \text{ cm}^2$  is

(a)  $2000 \text{ Pa}$

(b)  $1500 \text{ Pa}$

(c)  $3500 \text{ Pa}$

(d)  $500 \text{ Pa}$

**Answer (d)**

**Sol.**  $A_1v_1 = A_2v_2 \Rightarrow v_2 = \frac{1 \times 10}{5} = 2 \text{ m/s}$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow P_2 = 2000 + \frac{1}{2} \cdot 10^3 \cdot (1^2 - 2^2) = 500 \text{ Pa}$$

4. Three containers A, B and C are filled with water at different temperature. When 1 litre of water from A is mixed with 2 litre of water from B, the resulting temperature of mixture is  $52^\circ\text{C}$ . When 1 litre of water from B is mixed with 2 litre of water from C, the resulting temperature of mixture is  $40^\circ\text{C}$ . Similarly when 1 litre of water from C is mixed with 2 litre of water from A, the resulting temperature of mixture is  $34^\circ\text{C}$ . Temperature of mixture when one litre of water from each container is mixed (neglect the water equivalent of container) is

- (a)  $40^\circ\text{C}$  (b)  $42^\circ\text{C}$   
 (c)  $38^\circ\text{C}$  (d)  $45^\circ\text{C}$

**Answer (b)**

**Sol.** Using  $m_A C_A (T - T_A) = m_B C_B (T_B - T)$

$$1 \cdot 1 \cdot (52 - T_A) = 2 \cdot 1 \cdot (T_B - 52) \quad \dots(1)$$

$$1 \cdot 1 \cdot (40 - T_B) = 2 \cdot 1 \cdot (T_C - 40) \quad \dots(2)$$

$$1 \cdot 1 \cdot (34 - T_C) = 2 \cdot 1 \cdot (T_A - 34) \quad \dots(3)$$

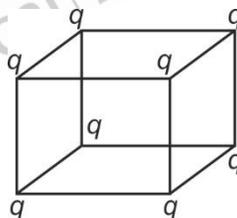
$$\Rightarrow T_A + T_B + T_C = 126^\circ\text{C}$$

When all three are mixed

$$1 \cdot 1 \cdot (T - T_A) + 1 \cdot 1 \cdot (T - T_B) + 1 \cdot 1 \cdot (T - T_C) = 0$$

$$T = \frac{T_A + T_B + T_C}{3} = 42^\circ\text{C}$$

5. Point charge  $q$  is kept at each corner of a cube of edge length  $\ell$ . The resultant force of repulsion on any one of the charges due to all others is expressed as



(a)  $\frac{\left(1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right) q^2}{\pi \epsilon_0 \ell^2}$

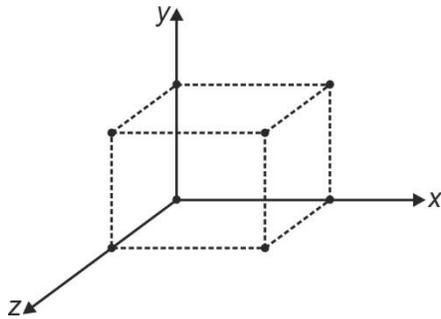
(b)  $\frac{\left(\frac{1}{2\sqrt{2}} - 1 + \frac{1}{3\sqrt{3}}\right) q^2}{\pi \epsilon_0 \ell^2}$

(c)  $\frac{(1 - 0.1775) q^2}{\pi \epsilon_0 \ell^2}$

(d) None of these

**Answer (c)**

Sol.



Force on charge placed at  $(\ell, \ell, \ell)$  will be

$$\begin{aligned}\vec{F} &= \frac{q^2}{4\pi\epsilon_0} \left[ \left( \frac{\ell\hat{i} + \ell\hat{j} + \ell\hat{k}}{3\sqrt{3}\ell^3} \right) + \left( \frac{\ell\hat{i}}{\ell^3} + \frac{\ell\hat{j}}{\ell^3} + \frac{\ell\hat{k}}{\ell^3} \right) + \left( \frac{\ell\hat{i} + \ell\hat{j}}{2\sqrt{2}\ell^3} + \frac{\ell\hat{i} + \ell\hat{k}}{2\sqrt{2}\ell^3} + \frac{\ell\hat{j} + \ell\hat{k}}{2\sqrt{2}\ell^3} \right) \right] \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{\ell^2} \left( 1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}} \right) (\hat{i} + \hat{j} + \hat{k}) \\ |\vec{F}| &= \frac{q^2}{\pi\epsilon_0\ell^2} \left( \frac{\sqrt{3}}{4} + \frac{1}{4}\sqrt{\frac{3}{2}} + \frac{1}{12} \right)\end{aligned}$$

6. If speed of light  $c$ , Planck's constant  $h$  and gravitational constant  $G$  are chosen as fundamental quantities, dimensions of time in this system of units is

- (a)  $ch^{3/2}G^{-3/2}$  (b)  $c^{-2}G^{1/2}h$   
(c)  $c^2G^{1/2}h^{5/2}$  (d)  $c^{-5/2}G^{1/2}h^{1/2}$

Answer (d)

Sol.  $[T] = [c]^a [h]^b [G]^c$

$$\begin{aligned}&= [LT^{-1}]^a [ML^2T^{-1}]^b [M^{-1}L^3T^{-2}]^c \\ &= [M^{b-c}L^{a+2b+3c}T^{-a-b-2c}]\end{aligned}$$

On comparison

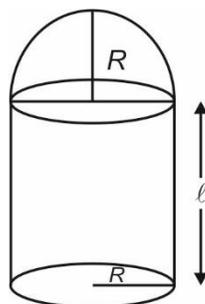
$$b - c = 0$$

$$a + 2b + 3c = 0$$

$$-a - b - 2c = 1$$

$$\Rightarrow a = -\frac{5}{2}, b = \frac{1}{2}, c = \frac{1}{2}$$

7. A solid hemisphere is cemented on the flat surface of a solid cylinder of same radius  $R$  and same material. The composite body is rotating about the axis of the cylinder of length  $\ell$  with angular speed  $\omega$ . The radius of gyration  $K$  is



(a)  $R\sqrt{\frac{2}{5}\left(\frac{15R+8\ell}{3R+2\ell}\right)}$

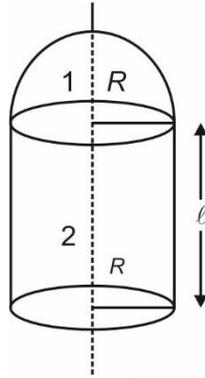
(b)  $R\sqrt{\frac{1}{10}\left(\frac{15\ell+8R}{3\ell+2R}\right)}$

(c)  $R\sqrt{\frac{3}{10}\left(\frac{15R+8\ell}{3R+2\ell}\right)}$

(d)  $R\sqrt{\frac{1}{10}\left(\frac{3\ell+2R}{15\ell+8R}\right)}$

**Answer (b)**

**Sol.**  $(m_1 + m_2)K^2 = I_1 + I_2$



$$\left(\frac{2}{3}\pi R^3\rho + \pi R^2\ell\rho\right)K^2 = \frac{2}{5}\left(\frac{2}{3}\pi R^3\rho\right)R^2 + \frac{1}{2}(\pi R^2\ell\rho)R^2$$

$$\Rightarrow K = R\sqrt{\frac{1}{10}\left(\frac{15\ell+8R}{3\ell+2R}\right)}$$

8. The shortest period of rotation of a planet (considered to be a sphere of uniform density  $\rho$ ) about its own axis, such that any mass  $m$  kept on its equator is just to fly off the surface, is

(a)  $T = \sqrt{\frac{5\pi}{\rho G}}$

(b)  $T = \sqrt{\frac{\pi}{3\rho G}}$

(c)  $T = \sqrt{\frac{3\pi}{\rho G}}$

(d)  $T = \sqrt{\frac{5\pi}{3\rho G}}$

**Answer (c)**

**Sol.**  $g' = g - R\omega^2$  at equator

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{3}{4\pi G\rho}} = \sqrt{\frac{3\pi}{\rho G}}$$

9. A body of mass 10 kg at rest explodes into two fragments of masses 3 kg and 7 kg. If the total kinetic energy of two pieces after explosion is 1680 J, the magnitude of their relative velocity in m/s after explosion is:

(a) 40

(b) 50

(c) 70

(d) 80

**Answer (a)**

**Sol.**  $\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(v_1 + v_2)^2 = 1680$

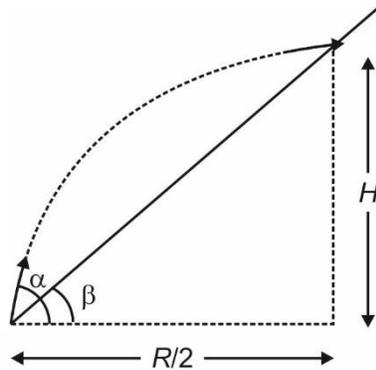
$$\Rightarrow v_1 + v_2 = \sqrt{\frac{2 \times 1680 \times 10}{21}} = 40 \text{ m/s}$$

10. A shot is fired at an angle  $\alpha$  to the horizontal up a hill (Considered to be a long straight incline plane) of inclination  $\beta$  to the horizontal. It will strike the hill horizontally if

- (a)  $\tan\alpha = 2\tan\beta$  (b)  $\sin\alpha = \sin 2\beta$   
(c)  $\sin\alpha = 2\sin\beta$  (d)  $\tan\alpha = 4\tan\beta$

**Answer (a)**

**Sol.**  $\tan\beta = \frac{H}{\frac{R}{2}} = \frac{2H}{R} = \frac{2 \frac{U^2 \sin^2 \alpha}{2g}}{U^2 \sin 2\alpha / g}$



$\Rightarrow \tan\alpha = 2\tan\beta$

11. In an experiment with potentiometer, the balancing length is 250 cm for a cell. When the cell is shunted by a resistance of  $7.5 \Omega$ , balancing point is shifted by 25 cm. If the cell is shunted by a resistance of  $20 \Omega$ , the balancing length will be nearly

- (a) 240 cm (b) 236 cm  
(c) 232 cm (d) 230 cm

**Answer (a)**

**Sol.**  $\varepsilon = kl_1 = k \cdot 2.5$  ... (1)

$\frac{\varepsilon R_1}{r + R_1} = kl_2 = k \cdot 2.25$  ... (2)

$\frac{\varepsilon R_2}{r + R_2} = kl_3$  ... (3)

Using (1), (2) & (3)

$r = \frac{5}{6} \Omega$  &  $l_3 = 240$  cm

12. The Nucleus  ${}^{23}_{10}\text{Ne}$  decays by  $\beta$  – emission through the reaction



${}^{23}_{11}\text{Na} = 22.989770$  u,  ${}^0_{-1}\beta = 0.000549$  u. The maximum kinetic energy that the emitted electron can ever have is

- (a) 4.374 MeV (b) 3.862 MeV  
(c) 2.187 MeV (d) 1.931 MeV

**Answer (a)**

**Sol.**  $Q = \left[ m({}^{23}_{10}\text{Ne}) - m({}^{23}_{11}\text{Na}) \right] c^2$

$$= [22.994466 - 22.989770] \times 931.5$$

$$= 4.374 \text{ MeV}$$

13. The distance between two slits in Young's double slits experiment is  $d = 2.5 \text{ mm}$  and the distance of the screen from the plane of slits is  $D = 120 \text{ cm}$ . The slits are illuminated with coherent beam of light of wavelength  $\lambda = 600 \text{ nm}$ . The minimum distance (from the central maximum) of a point where the intensity reduces to 25% of maximum intensity is

- (a)  $24 \mu\text{m}$  (b)  $48 \mu\text{m}$   
 (c)  $96 \mu\text{m}$  (d)  $120 \mu\text{m}$

**Answer (c)**

**Sol.**  $I = I_m \cos^2 \left( \frac{\pi}{\lambda} \cdot d \cdot \frac{Y}{D} \right)$

$$\Rightarrow \frac{\pi d Y}{\lambda D} = \frac{\pi}{3}$$

$$\Rightarrow Y = \frac{D \lambda}{3d} = 96 \mu\text{m}$$

14. What amount of heat will be generated in a coil of resistance  $R$  (ohm) due to a total charge  $Q$  (coulomb) passing through it if the current in the coil decreases down to zero halving its value every  $\Delta t$  second?

- (a)  $\frac{1}{2} \frac{Q^2 R}{\Delta t}$  (b)  $\frac{Q^2 R}{\Delta t} \ln 2$   
 (c)  $\frac{1}{2} \frac{Q^2 R}{\Delta t} \ln 2$  (d)  $\frac{1}{4} \frac{Q^2 R}{\Delta t}$

**Answer (c)**

**Sol.**  $i = i_0 e^{-\lambda t}$ , where  $\lambda = \frac{\ln 2}{\Delta t}$

$$Q = \int_0^{\infty} i dt = i_0 \int_0^{\infty} e^{-\lambda t} dt$$

$$\Rightarrow i_0 = Q \lambda$$

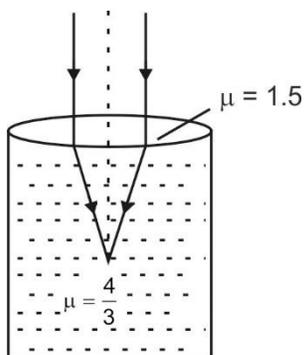
$$H = \int_0^{\infty} i^2 R dt = \lambda^2 Q^2 R \int_0^{\infty} e^{-2\lambda t} dt$$

$$= \frac{Q^2 R \lambda}{2}$$

$$= \frac{Q^2 R \ln 2}{2 \Delta t}$$



Sol.  $\frac{1}{f} = (\mu_\ell - 1) \left( \frac{2}{R} \right)$



$\Rightarrow R = f = 25 \text{ cm}$

For refraction at first surface,

$$\frac{1.5}{v'} - \frac{1}{\infty} = \frac{1.5 - 1}{25}$$

$\Rightarrow v' = +75 \text{ cm}$

For refraction at second surface,

$$\frac{4}{3} - \frac{1.5}{75} = \frac{4 - 3}{-25}$$

$\Rightarrow v = +50 \text{ cm}$

18. Even the radiation of highest wavelength in the ultraviolet region of hydrogen spectrum is just able to eject photoelectrons from a metal. The value of threshold frequency for the given metal is

(a)  $3.83 \times 10^{15} \text{ Hz}$

(b)  $4.33 \times 10^{14} \text{ Hz}$

(c)  $2.46 \times 10^{15} \text{ Hz}$

(d)  $7.83 \times 10^{14} \text{ Hz}$

**Answer (c)**

Sol. Largest wavelength in ultraviolet region of hydrogen spectrum corresponds to transition  $n = 2$  to  $n = 1$ .

$$\begin{aligned} \text{Corresponding frequency} &= \frac{\Delta E}{h} \\ &= \frac{10.2 \times 1.6 \times 10^{-19}}{6.625 \times 10^{-34}} \\ &= 2.46 \times 10^{15} \text{ Hz} \end{aligned}$$

19. One mole of a gas with  $\gamma = \frac{5}{3}$  is mixed with two moles of another non-interacting gas with  $\gamma = \frac{7}{5}$ . The ratio of

specific heats  $\gamma = \frac{C_P}{C_V}$  of mixture approximately

(a) 1.50

(b) 1.46

(c) 1.49

(d) 1.53

**Answer (b)**

**Sol.**  $\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

$$\frac{1+2}{\gamma - 1} = \frac{1}{\frac{5}{3} - 1} + \frac{2}{\frac{7}{5} - 1}$$

$$\gamma = \frac{19}{13} = 1.46$$

20. An ideal gas is expanding such that  $PT^3 = \text{constant}$ . The coefficient of volume expansion of the gas is

(a)  $\frac{1}{T}$  (b)  $\frac{2}{T}$

(c)  $\frac{3}{T}$  (d)  $\frac{4}{T}$

**Answer (d)**

**Sol.**  $PT^3 = \text{Constant}$

$$\Rightarrow \frac{T^4}{V} = \text{Constant} \Rightarrow \frac{dV}{V} = 4 \frac{dT}{T}$$

$$\gamma = \frac{dV}{VdT} = \frac{4}{T}$$

21. What is the magnetic induction  $B$  at the centre  $O$  of the semicircular arc if a current carrying wire has shape of an hair pin as shown in figure? The radius of the curved part of the wire is  $R$ , the linear parts are assumed to be very long.



(a)  $B = \frac{\mu_0 I}{4\pi R}(2 + \pi)$

(b)  $B = \frac{\mu_0 I}{4R}(2 + \pi)$

(c)  $B = \frac{3\mu_0 I}{4R}(2 + \pi)$

(d)  $B = \frac{\mu_0 2I}{4\pi R}$

**Answer (a)**

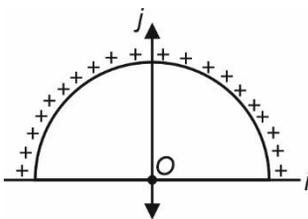
**Sol.**  $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

$$= B_1 + B_2 + B_3 \quad (\text{All in same direction})$$

$$= \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}$$

$$= \frac{\mu_0 I}{4\pi R}(2 + \pi)$$

22. A thin semi-circular metal ring of radius  $R$  has a positive charge  $q$  distributed uniformly over its curved length. The resultant electric field  $\vec{E}$  at the center  $O$  is



(a)  $-\hat{j} \frac{q}{2\pi^2 \epsilon_0 R^2}$

(b)  $+\hat{j} \frac{q}{2\pi^2 \epsilon_0 R^2}$

(c)  $+\hat{j} \frac{q}{4\pi^2 \epsilon_0 R^2}$

(d)  $-\hat{j} \frac{q}{4\pi^2 \epsilon_0 R^2}$

**Answer (a)****Sol.** Using  $E = \frac{2k\lambda}{R} \sin \frac{\theta}{2}$ For  $\theta = 180^\circ$ ,  $E = \frac{q}{2\pi^2 \epsilon_0 R^2}$  along  $-y$ -axis23. An alternating current is expressed as  $i = i_1 \cos \omega t + i_2 \sin \omega t$ . The RMS value of current is

(a)  $\sqrt{\frac{(i_1 + i_2)^2}{2}}$

(b)  $\sqrt{\frac{i_1 i_2}{2}}$

(c)  $\sqrt{\frac{(i_1^2 + i_2^2)}{2}}$

(d)  $\sqrt{\frac{(i_1 - i_2)^2}{2}}$

**Answer (c)****Sol.**  $i = i_1 \cos \omega t + i_2 \sin \omega t$ 

$$= \sqrt{i_1^2 + i_2^2} \sin \left( \omega t + \tan^{-1} \frac{i_1}{i_2} \right)$$

$$i_{\text{rms}} = \sqrt{\frac{i_1^2 + i_2^2}{2}}$$

24. A charge  $+q$  is placed at each of the points of  $x = x_0, x = 3x_0, x = 5x_0, x = 7x_0, \dots, \infty$  on the  $x$ -axis and a charge  $-q$  is placed at each of the points  $x = 2x_0, x = 4x_0, x = 6x_0, x = 8x_0, \dots, \infty$  here  $x_0$  is a positive constant.Take the electric potential at a point due to a charge  $q$  at a distance  $r$  from it to be  $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ . The electric potential at the origin due to the above system of charges is

(a) Zero

(b)  $\frac{1}{4\pi\epsilon_0} \frac{q}{x_0} \ln 2$

(c)  $\frac{1}{4\pi\epsilon_0} \frac{q}{x_0 2 \ln 2}$

(d) Infinite

**Answer (b)**

**Sol.**  $V = \frac{q}{4\pi\epsilon_0} \left( \frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4x_0} + \dots \right)$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x_0} \left( 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{x_0} \ln 2$$

**PART-A2**

25. A particle of mass  $m$  is located in a one dimensional potential field  $U(x) = U_0 (1 - \cos ax)$ ;  $U_0$  and  $a$  are constants. Which of the following statement/s is/are correct?
- (a) The particle will executed Simple Harmonic Motion for small displacements.  
 (b) The stable equilibrium condition is  $x = 0$   
 (c) The time period of small oscillations is  $\frac{2\pi}{a} \sqrt{\frac{m}{U_0}}$   
 (d) The angular frequency for small oscillations is  $\omega = a \sqrt{\frac{U_0}{m}}$

**Answer (a, b, c, d)**

**Sol.**  $U = U_0(1 - \cos ax)$

$$F = -\frac{dU}{dx} = -U_0 a \sin ax$$

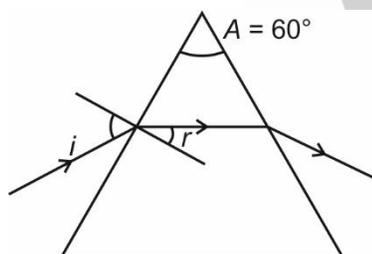
$$\text{Acceleration} = -\frac{U_0 a^2}{m} \cdot x \text{ (for small } x)$$

$$\omega^2 = \frac{U_0 a^2}{m}$$

26. A ray of light is incident on an equilateral prism made of flint glass (refractive index 1.6) placed in air.
- (a) The ray suffers a minimum deviation if it is incident at angle  $53^\circ$   
 (b) The minimum angle of deviation suffered by the ray is  $46^\circ$   
 (c) If prism is immersed in water ( $\mu = \frac{4}{3}$ ) the minimum deviation produced by the prism is  $14^\circ$   
 (d) The minimum deviation produced by the prism is  $23.6^\circ$  if it is immersed in a liquid of refractive index  $\mu = 1.2$

**Answer (a, b, c, d)**

**Sol.**



Deviation will be minimum, when ray will be parallel to base as shown

$$r = \frac{A}{2} = 30^\circ$$

$$\mu_1 \sin i = \mu_2 \sin r$$

For  $\mu_1 = 1, i = 53^\circ$

For  $\mu_1 = \frac{4}{3}, i = 37^\circ$

For  $\mu_1 = 1.2, i = 41.8^\circ$

$$\delta_{\min} = 2i - A$$

27. In a  $p$ - $n$  junction diode, the current ( $i$ ) varies with applied biasing voltage ( $V$ ) and can be expressed as  $i = i_0(e^{qV/kT} - 1)$  where  $i_0 = 5 \times 10^{-12}$  A is reverse saturation current,  $k$  is Boltzmann constant and  $q$  is the charge on the electron

At absolute temperature  $T = 300$  K

- The forward current is approximately 59.5 mA for a forward bias of 0.6 volt
- The current increases approximately by 2.75 A if the biasing voltage changes from 0.6 V to 0.7 V
- The dynamic resistance of  $p$ - $n$  junction is approximately 435 m $\Omega$  at the biasing voltage of 0.6 V
- The change in reverse bias current when biasing voltage change from -1 volt to -2 volt happens to be practically zero

**Answer (a, b, c, d)**

$$\text{Sol. } i = i_0 \left( e^{\frac{qV}{kT}} - 1 \right)$$

$$\frac{di}{dV} = \frac{i_0 q}{kT} e^{\frac{qV}{kT}}$$

$$i = 58.92 \text{ mA at } V = 0.6 \text{ volt}$$

$$i = 2810.65 \text{ mA at } V = 0.7 \text{ volt}$$

$$\Delta i = 2.75 \text{ A}$$

$$\begin{aligned} \text{Dynamic resistance} &= \frac{dV}{di} = \frac{kT}{i_0 q} e^{-\frac{qV}{kT}} \\ &\approx 439 \text{ m}\Omega \end{aligned}$$

28. A charged oil (density 880 kg m<sup>-3</sup>) drop is held stationary between two parallel horizontal metal plates 6.0 mm apart when a potential difference of  $V = 103$  is applied between the two plates. When the electric field is switched off, the drop falls. At a certain time the drop is seen to fall a distance of 2.0 mm in 35.7 s and next 1.2 mm in 21.4 s. (The upper plate in the experiment is at higher potential).

Given that the viscosity of air =  $1.80 \times 10^{-5}$  Nsm<sup>-2</sup> and density of air =  $1.29$  kg m<sup>-3</sup>

- The radius of the drop is  $a = 7.25 \times 10^{-7}$  m
- The charge on the drop is  $q = 8.0 \times 10^{-19}$  C
- The terminal velocity of the oil drop, under its free fall, is  $5.6 \times 10^{-5}$  ms<sup>-1</sup>
- The oil drop carries 5 excess electrons

**Answer (a, b, c, d)**

$$\text{Sol. } v_T = \frac{2 \times 10^{-3}}{35.7} \approx \frac{1.2 \times 10^{-3}}{21.4} \approx 5.6 \times 10^{-5} \text{ m/s}$$

$$v_T = \frac{2}{9} \frac{R^2 g}{\eta} (\rho_0 - \rho_a)$$

$$\Rightarrow R \approx 7.25 \times 10^{-7} \text{ m}$$

$$qE = mg$$

$$\Rightarrow q = \frac{mgd}{V} \approx 8.0 \times 10^{-19} \text{ C}$$

29. A parallel plate capacitor of plate area  $A$  and plate separation  $d$  is charged to potential  $V$ . Then the battery is disconnected. A slab of dielectric constant  $k$  is then inserted between the plates of the capacitor so as to fill the space between the plates completely. If  $Q$ ,  $E$  and  $W$  denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and work done on the system, in question, in the process of inserting the slab, then

(a)  $Q = k\epsilon_0 AE$

(b)  $Q = \frac{\epsilon_0 kAV}{d}$

(c)  $E = \frac{V}{kd}$

(d)  $W = \frac{\epsilon_0 AV^2}{2d} \left(1 - \frac{1}{k}\right)$

**Answer (a, c)**

**Sol.**  $Q = \frac{\epsilon_0 A}{d} V$  and  $U_i = \frac{1}{2} \frac{\epsilon_0 A}{d} V^2$

$Q$  remains unchanged as the battery is disconnected

$$C = \frac{k\epsilon_0 A}{d} \text{ (after insertion of dielectric slab)}$$

$$E = \frac{Q}{k\epsilon_0 A} = \frac{V}{kd}$$

$$W = U_f - U_i = \frac{U_i}{k} - U_i = \frac{\epsilon_0 AV^2}{2d} \left(\frac{1}{k} - 1\right)$$

30. The magnitudes of the gravitational field at distances  $r_1$  and  $r_2$  from the centre of a uniform solid sphere of radius  $R$  and mass  $M$  are  $F(r_1)$  and  $F(r_2)$  respectively. Such that

(a)  $\frac{F(r_1)}{F(r_2)} = \frac{r_1}{r_2}$  if  $r_1 \leq R$  and  $r_2 \leq R$

(b)  $\frac{F(r_1)}{F(r_2)} = \frac{r_2^2}{r_1^2}$  if  $r_1 \geq R$  and  $r_2 \geq R$

(c)  $\frac{F(r_1)}{F(r_2)} = \frac{r_1}{r_2}$  if  $r_1 \geq R$  and  $r_2 \geq R$

(d)  $\frac{F(r_1)}{F(r_2)} = \frac{r_1^2}{r_2^2}$  if  $r_1 \leq R$  and  $r_2 \leq R$

**Answer (a, b)**

**Sol.**  $F(r) = \frac{GM}{R^3} r$  ( $r < R$ )

$$= \frac{GM}{r^2} \quad (r > R)$$

31. The intensity of sound at a point  $P$  is  $I_0$ , when the sounds reach this point directly and in same phase from two identical sources  $S_1$  and  $S_2$ . The power of  $S_1$  is now reduced by 64% and the phase difference ( $\phi$ ) between  $S_1$  and  $S_2$  is varied continuously. The maximum and minimum intensities recorded at  $P$  are now  $I_{\max}$  and  $I_{\min}$  such that

(a)  $I_{\max} = 0.64 I_0$

(b)  $I_{\min} = 0.36 I_0$

(c)  $\frac{I_{\max}}{I_{\min}} = 16$

(d)  $\frac{I_{\max}}{I_{\min}} = \frac{16}{9}$

**Answer (a, c)**

**Sol.**  $l_0 = 4l_s$

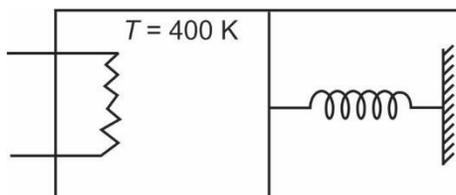
$$l = l_1 + l_2 + 2\sqrt{l_1 l_2} \cos \phi$$

$$l_1 = .36l_s \text{ \& } l_2 = l_s$$

$$l_{\max} = (\sqrt{l_1} + \sqrt{l_2})^2 = 2.56l_s = .64l_0$$

$$l_{\min} = (\sqrt{l_2} - \sqrt{l_1})^2 = .16l_s = .04l_0$$

32. An ideal monatomic gas is confined within a cylinder by a spring loaded piston of cross-sectional area  $4 \times 10^{-3} \text{ m}^2$ . Initially the gas is at 400 K and occupies a volume  $2 \times 10^{-3} \text{ m}^3$  and the spring is in its relaxed position. The gas is heated by an electric heater for some time. During this time the gas expands and the piston moves out by a distance 0.1 m. The spring connected to the rigid wall is massless and frictionless. The force constant of the spring is  $2000 \text{ Nm}^{-1}$  and atmospheric pressure is  $10^5 \text{ Nm}^{-2}$  then



- (a) The final temperature of the gas is 720 K.                      (b) The work done by gas in expanding is 50 J  
 (c) The heat supplied by heater is 190 J                              (d) The heat supplied by heater is 290 J

**Answer (a, b, d)**

**Sol.**  $PV = nRT$

$$10^5 \times 2 \times 10^{-3} = n \times 8.31 \times 400$$

$$\Rightarrow n = 0.06$$

$$P' = P + \frac{Kx_0}{A} = 1.5 \times 10^5 \text{ Pa}$$

$$V' = V + Ax_0 = 2.4 \times 10^{-3} \text{ m}^3$$

$$T' = \frac{P'V'}{nR} = 720 \text{ K}$$

$$W = \int_0^{x_0} \left( P + \frac{Kx}{A} \right) A dx$$

$$= P\Delta V + \frac{1}{2} Kx_0^2$$

$$= 10^5 \times 0.4 \times 10^{-3} + \frac{1}{2} \times 2000 \times (0.1)^2$$

$$= 50 \text{ J}$$

$$Q = \Delta U + W$$

$$= nC_V \Delta T + W$$

$$= .06 \times \frac{3}{2} R \times 320 + 50$$

$$= 290 \text{ J}$$

