Date: 22/05/2022

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Answers & Solutions

Time: 3 hrs. **Max. Marks: 100**

for

KVPY (SA) 2021-22

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you open the question booklet.

The question paper consists of two parts (both contain only multiple choice questions) for 100 marks. There will be four sections in Part I (each section containing 15 questions) and four sections in Part II (each section containing 5 questions)

Part-I

(i) There are 60 objective type questions. 15 questions from each subject (Mathematics, Physics, Chemistry & Biology). All questions are compulsory.

(ii) Each correct answer gets **1 mark** and for each incorrect answer **0.25 mark** will be deducted.

Part-II

(i) There are 20 objective type questions. 5 questions from each subject (Mathematics, Physics, Chemistry & Biology). All questions are compulsory.

(ii) Each correct answer gets **2 marks** and for each incorrect answer 0**.5 mark** will be deducted.

PART-I : MATHEMATICS

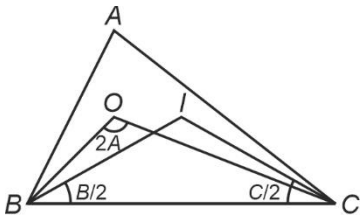
1. Let ABC be a scalene triangle with incentre I and circumcentre O. Suppose, B, C, I, O are concyclic points. Then $\angle B + \angle C$ is

(A) 60° (B) 105°

(C) 120° (D) 135°

Answer (C)

Sol.



$$\angle BIC = \pi - \left(\frac{B}{2} + \frac{C}{2}\right)$$

$\angle BOC = 2A$

As B, O, I, C are concyclic

$$2A = \pi - \left(\frac{B}{2} + \frac{C}{2}\right)$$

$$2A = \pi - \left(\frac{\pi - A}{2}\right)$$

$$2A = \pi - \frac{\pi}{2} + \frac{A}{2}$$

$$\frac{3A}{2} = \frac{\pi}{2}$$

$\Rightarrow$ $A = 60°$

So, $\angle B + \angle C = 120°$

2. Suppose $ABCD$ ($AB \parallel CD$) is a trapezium such that the diagonals AC, BD bisect the angles $\angle DAB$, $\angle CBA$ respectively. Then

(A) no two sides of the trapezium are equal (B) exactly two sides of the trapezium are equal

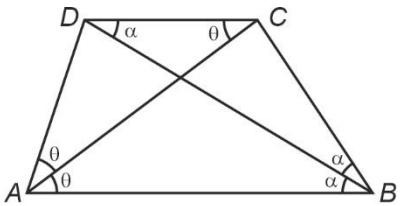
(C) exactly three sides of the trapezium are equal (D) none of the options above can be concluded

Answer (C)

Sol. As AC and BD are angle bisectors,

$\angle DAC = \angle CAB = \theta$ (let)

$\angle ABD = \angle DBC = \alpha$ (let)



As $AB \parallel CD$,

$\angle DAC = \angle DCA = \theta$

So, $DC = AD$ (in ΔADC)

Similarly,

$CD = BC$

Therefore, 3 sides are equal.

So, option (C) is correct.

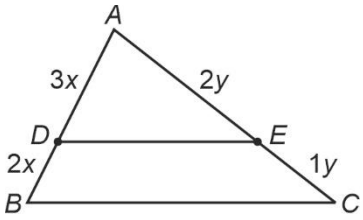
3. Suppose ABC is a triangle and D, E are points on the sides AB and AC respectively. If $AD : AB = 3 : 5$ and $AE : AC = 2 : 3$, then the ratio of the areas of the triangles ABC and ADE lies in the interval

(A) $(1, 2]$ (B) $\left(2, \frac{5}{2}\right]$

(C) $\left(\frac{5}{2}, 3\right]$ (D) $\left(3, \frac{7}{2}\right]$

Answer (B)

Sol.



Area of $ADE = \frac{1}{2} 3x \cdot 2y \sin A$

Area of $ABC = \frac{1}{2} 5x \cdot 3y \sin A$

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta ADE)} = \frac{5}{3} \times \frac{3}{2} = \frac{5}{2}$$

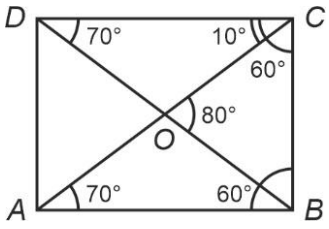
4. Let $ABCD$ be a convex quadrilateral in which $AC = BD$, $AB = CD$, $\angle BAC = 70°$ and $\angle BCD = 60°$. The acute angle between AC and BD is

(A) 70° (B) 75°

(C) 80° (D) 85°

Answer (C)

Sol.



In ΔABC and ΔDCB,

$AC = BD$

$AB = CD$

$BC = BC$ (Common)

$\therefore$ $\Delta ABC \cong \Delta DCB$

$\angle BAC = 70° = \angle BDC$

$\angle BCD = \angle CBA = 60°$

$\therefore$ $\angle ACB = 180° - (70° + 60°) = 50°$

$\therefore$ $\angle DCO = 60° - 50° = 10°$

$\therefore$ $\angle COB = 70° + 10° = 80°$

5. Integers 1, 2, 3, …, n, ($n \geq 3$) are written on a black board and an integer k ($1 < k < n$) is erased. The average of the remaining numbers is 16. Then $n + k$ is

(A) 31 (B) 40

(C) 47 (D) 50

Answer (C)

Sol. Sum of n integers (from 1 to n) = $\frac{n(n+1)}{2}$

$$\frac{\frac{n(n+1)}{2} - k}{n-1} = 16 \Rightarrow n^2 - 31n + (32 - 2k) = 0$$

It is a quadratic in n which has integer roots (as n is integer).

For integer roots, D must be perfect square.

$D = 961 - 4(32 - 2k)$

$= 833 + 8k$

For $k = 16 \Rightarrow D = (31)^2$

Now, for $k = 16 \Rightarrow n^2 - 31n = 0$

$\therefore$ $n = 31 \Rightarrow n + k = 47$

6. Let $p = 99$ and $q = 101$. Define $p_1 = \log\left(\frac{p+q}{2}\right)$ and $q_1 = \frac{1}{2}(\log p + \log q)$ and $p_2 = \log\left(\frac{p_1+q_1}{2}\right)$, $q_2 = \frac{1}{2}(\log p_1 + \log q_1)$, where all logarithms have base 10. Then

(A) $\log p_1 > p_2 > q_2 > \log q_1$ (B) $\log p_1 > q_2 > p_2 > \log q_1$

(C) $\log q_1 > p_2 > q_2 > \log p_1$ (D) $\log q_1 > q_2 > p_2 > \log p_1$

Answer (A)

Sol. $p_1 = \log\left(\frac{p+q}{2}\right), q_1 = \frac{1}{2}(\log p + \log q) = \log\sqrt{pq}$

$\because$ $\frac{p+q}{2} > \sqrt{pq}$ as A.M. > G.M. as $p \neq q$

$\Rightarrow$ $p_1 > q_1 \Rightarrow \log p_1 > \log q_1$ …(i)

Again, $p_2 = \log\left(\frac{p_1+q_1}{2}\right)$ and $q_2 = \frac{1}{2}(\log p_1 + \log q_1) = \log\sqrt{p_1 q_1}$

$\because$ $\frac{p_1+q_1}{2} > \sqrt{p_1 q_1} \Rightarrow p_2 > q_2$ …(ii)

Only option (A) satisfies (i) and (ii).

7. Let a be the largest real root and b be the smallest real root of the polynomial equation $x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1 = 0$. Then $\frac{a^2+b^2}{a+b+1}$ is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{5}{4}$ (D) $\frac{13}{7}$

Answer (B)

Sol. Given equation :

$x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1 = 0$

$\Rightarrow (x-1)^6 = 0$

Hence, $x = 1$ is a root of given equation with multiplicity 6.

Hence, $a = b = 1$

Hence, $\frac{a^2+b^2}{a+b+1} = \frac{2}{3}$

8. The number of ordered pairs (a, b) of integers such that $1 \le a, b \le 2021$ and the equations $x^2 - ax + b = 0$ and $x^3 - ax^2 + bx + a - b = 0$ have a common real root is

(A) 2017 (B) 2018

(C) 2019 (D) 2021

Answer (B)

Sol. Let α be the common root.

So, $\alpha^2 - a\alpha + b = 0$ and $\alpha(\alpha^2 - a\alpha + b) + a - b = 0$

$\Rightarrow a - b = 0 \Rightarrow a = b$

Also, α is real.

So, $a^2 \ge 4b \Rightarrow a(a-4) \ge 0 \Rightarrow a \ge 4$

So, $a = 4, 5, 6, \ldots, 2021$

Number of ordered pairs of $(a, b) = 2018$

9. The number of positive integers x satisfying the equation $\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} = \frac{13}{12}$ is

(A) 0 (B) 1

(C) 2 (D) more than 2

Answer (B)

Sol. Given equation :

$\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} = \frac{13}{12}$

$\Rightarrow \frac{1}{x} + \frac{1}{x+1} = \frac{13}{12} - \frac{1}{x+2}$

$\Rightarrow 12(2x+1)(x+2) = (13x+14)x \cdot (x+1)$

$\Rightarrow 13x^3 + 3x^2 - 46x - 24 = 0$

$\Rightarrow (x-2)(13x^2 + 29x + 12) = 0$

$x = 2$ or $13x^2 + 29x + 12 = 0$

$x = 2$ is the only positive integral solution.

10. A contractor has two teams of workers, team *A* and team *B*. Team *A* can complete a project *P* in 12 days and team *B* can complete *P* in 36 days. Team *A* starts working on *P* and team *B* joins team *A* after four days. Team *A* is withdrawn after another two days and team *B* is asked to double its efficiency. The number of additional days required for team *B* to complete *P* is

(A) 6 (B) 8

(C) 15 (D) 16

Answer (B)

Sol. Team *A* completes $\frac{1}{12}$ of project per day.

Team *B* completes $\frac{1}{36}$ of project per day.

Team *A* completes $\frac{4}{12}$ of project in 4 days.

Team (*A* + *B*) completes $\left(\frac{1}{36}+\frac{1}{12}\right)\times 2$ of the project in 2 days.

Let *B* work on remaining project for *x* days at double efficiency.

Part of project completed by *B* in *x* days = $\frac{2}{36}\times x$

Now, we should have : $\frac{4}{12}+2.\left(\frac{1}{36}+\frac{1}{12}\right)+\frac{2x}{36}=1$

$\Rightarrow$ x = 8 days

11. The number of positive integers n such that $n + 3$ divides $n^3 - 3$ is

(A) 3 (B) 4

(C) 5 (D) 8

Answer (C)

Sol. $n + 3 \mid n^3 - 3$

$n + 3 \mid (n^3 + 27) - 30$

$\Rightarrow$ $n + 3 \mid 30$

So, $n + 3$ is a factor of 30

$n + 3$ = 5, 6, 10, 15 and 30

$\Rightarrow$ n = 2, 3, 7, 12 and 27

12. Suppose we have an arithmetic progression $a_1, a_2, \ldots, a_n, \ldots$ with $a_1 = 1$, $a_2 - a_1 = 5$. The median of the finite sequence $a_1, a_2, \ldots, a_k$, where $a_k \le 2021$ and $a_{k+1} > 2021$, is

(A) 1011 (B) 1011.5

(C) 1013.5 (D) 1016

Answer (A)

Sol. By given data,

A.P has 1st term = 1 = a_1

and common difference = 5 = d

k^{th} term $a_k = a_1 + (k-1)d$

i.e. $1 + (k-1)5 \le 2021$ & $1 + k(5) > 2021$

$\Rightarrow$ $k - 1 \le 404$ and $k > 404$

$\Rightarrow$ $k \le 405$

Hence, $k = 405$

So, given sequence $a_1, a_2, \ldots, a_{405}$

$\therefore$ Median is $a_{203} = 1 + (202)5 = 1011$

13. The value of the fifth root of $10^{10^{10}}$ is

(A) $10^{2\times10^9}$ (B) $10^{20\times10^9}$

(C) 10^{10^2} (D) $10^{2^{10}}$

Answer (A)

Sol. For fifth root, power shall be $\frac{1}{5}$

i.e. $\left(10^{10^{10}}\right)^{\frac{1}{5}} = 10^{\left(10^{10}\times\frac{1}{5}\right)}$

$= 10^{\left(10\times10^9\times\frac{1}{5}\right)}$

$= 10^{2\times10^9}$

14. Let A denote the set of all 2-digit numbers in base 10 that are equal to four times the sum of the factorial of their digits. The sum of the numbers in A is

(A) 12 (B) 34

(C) 44 (D) 54

Answer (C)

Sol. Let the number be $\overline{ab} = 10a + b$

$\therefore$ $4(a! + b!) = 10a + b$

As ab is a four-digit number $a, b \in \{0, 1, 2, 3, 4\}$

If $a = 0$, we get $4(1 + b!) = b$

$\Rightarrow$ $4 + 4.b! = b \Rightarrow b \in \phi$

If $a = 1$, we get $4(1 + b!) = 10 + b$

$\Rightarrow$ $4.b! = 6 + b \Rightarrow b = 2$

If $a = 2$, we get $4(2 + b!) = 20 + b$

$\Rightarrow$ $4.b! = 8 + b \Rightarrow b \in \phi$

If $a = 3$, we get $4(6 + b!) = 30 + b$

$\Rightarrow$ $4.b! = 6 + b \Rightarrow b = 2$

If $a = 4$, we get $4(24 + b!) = 40 + b$

$\Rightarrow \quad 56 + 4b! = b \Rightarrow b \in \phi$

Here, possible numbers are 12 & 32.

Hence, sum = 44

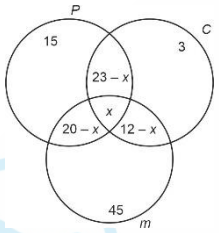
15. In a class of 100 students, 15 students chose only physics (but not mathematics and chemistry), 3 chose only chemistry (but not mathematics and physics), and 45 chose only mathematics (but not physics and chemistry). Of the remaining students, it is found that 23 have taken physics and chemistry, 20 have taken physics and mathematics, and 12 have taken mathematics and chemistry. The number of students who chose all the three subjects is

(A) 6 (B) 9

(C) 12 (D) 15

Answer (B)

Sol. Making Venn diagram and let x is number of students who opted all 3 subjects, we have



$\Rightarrow \quad 15 + 23 - x + 3 + 12 - x + x + 20 - x + 45 = 100$

$\Rightarrow \quad 2x = 18$

$\Rightarrow \quad x = 9$

Number of students who opted all 3 subjects is 9.

PART-I : PHYSICS

16. You are holding a shallow circular container of radius R, filled with water to a height h($h << R$). When you walk with speed v, it is seen that water starts spilling over. This happens due to the resonance of the periodic impulse given to the container (due to walking) with the oscillation of the water in the container. If the time period of water oscillating in the container is inversely proportional to $\sqrt{h}$, then v is proportional to

(A) R (B) $\sqrt{R}$

(C) $\frac{1}{\sqrt{R}}$ (D) $\frac{1}{R}$

Answer (D)

Sol. $t \Rightarrow$ time period of oscillation

$t \propto h^a \quad t \propto g^b \quad t \propto R^C$

where $a = \frac{-1}{2}$

$[t] \propto \left[L^{b+c+a}\right]\left[T^{-2b}\right]$

$\Rightarrow -2b = 1 \Rightarrow b = -\frac{1}{2}$

$\Rightarrow b + c + a = 0$ or $c = 1$

$\Rightarrow t \propto \frac{R}{\sqrt{gh}}$

$\frac{l}{v} \propto \frac{R}{\sqrt{gh}}$

Or $v \propto \frac{1}{R}$

17. A lens placed 10 cm away from a wall casts a sharp inverted image of a candle on it. It again casts a sharp image when the lens is moved 20 cm further away from the wall. Now, the candle and the lens are moved such that a sharp inverted image with unit magnification is formed on the wall. To achieve this configuration, the candle was moved

(A) 20 cm towards the wall
(B) 20 cm away from the wall
(C) 10 cm away from the wall
(D) 10 cm towards the wall

Answer (D)

Sol. From the data available

If candle is at 10 cm from lens then its image is 30 cm from lens on the other side

Using lens formula with $v = 30$ cm & $u = -10$ cm

$f = \frac{vu}{v-u} = 7.5$ cm

To obtain the unit magnification both candle and wall should be at a distance of $2f$ on either side of lens

Object will be at a distance of $4f$ or 30 cm from wall

It will have to be moved 10 cm towards the wall

18. An electrical circuit consists of ten 100 Ω resistors. Out of these 10 resistors, a group of n_1 resistors are connected in parallel and another group of n_2 resistors are separately connected in parallel. These two groups are then connected in series and this combination is connected to a voltage source of 100 V. If the net current through the circuit is 2.5 A, the values of n_1 and n_2 are

(A) 6, 4
(B) 5, 5
(C) 2, 8
(D) 3, 7

Answer (B)

Sol. According to the question

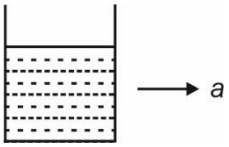
$$\frac{100V}{\frac{100}{n_1}+\frac{100}{10-n_1}} = \frac{10}{4} \text{ A}$$

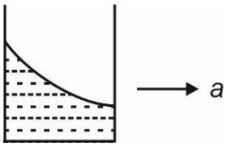
$$\Rightarrow \frac{400}{1000} = \left(\frac{1}{n_1}+\frac{1}{10-n_1}\right)$$

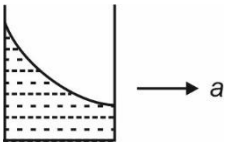
$$\Rightarrow 0.4 = \frac{1}{n_1}+\frac{1}{10-n_1}$$

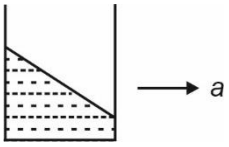
From given options $n_1 = n_2 = 5$ would satisfy the equation.

19. A rectangular box has water in it. It is being pulled to the right with an acceleration *a*. Which of the following options shows the correct shape of water surface on it?

(A) 

(B) 

(C) 

(D) 

Answer (D)

Sol. As the rectangular box is accelerated with constant acceleration *a* its surface would make a constant slope with horizontal with value of $\theta = \frac{a}{g}$. So the surface of liquid would be a plane surface making angle θ with horizontal. So profile given in option (D) is a correct presentation of shape of water surface.

20. On fission a U^{235} nucleus releases 3×10^{-11} J of energy. In a 1 GW nuclear reactor 4.2% of this energy is converted to useful energy. The U^{235} consumed (in grams) in half an hour is closest to (Avogadro number $N_A = 6.023 \times 10^{23}$)

(A) 5 (B) 50

(C) 500 (D) 1000

Answer (C)

Sol. Total energy needed to be produced per unit time = $\frac{10^9 \times 100}{4.2}$ W

$\Rightarrow$ Number of fissions required per second

$$= \left(\frac{1}{4.2} \times 10^{11}\right) \times \frac{1}{3 \times 10^{-11}} = \frac{10^{22}}{3 \times 4.2}$$

$\Rightarrow$ Number of fissions required in half an hour

$$= \frac{10^{22} \times 1800}{3 \times 4.2} = \frac{10^{25}}{7}$$

$\Rightarrow$ Mass consumed = $\frac{10^{25} \times 235}{7 \times 6.023 \times 10^{23}}$ gm $\cong$ 500 gm

21. The International Avogadro Coordination project created the world's most perfect sphere using Silicon in its crystalline form. The diameter of the sphere is 9.4 cm with an uncertainty of 0.2 nm. The atoms in the crystals are packed in cubes of side *a*. The side is measured with a relative error of 2×10^{-9}, and each cube has 8 atoms in it. Then, the relative error in the mass of the sphere is closest to (assume molar mass of Silicon and Avogadro's number to be known precisely)

(A) 6.4×10^{-9} (B) 4.0×10^{-10}

(C) 1.2×10^{-8} (D) 5.0×10^{-8}

Answer (C)

Sol. Total number of atoms $N = \dfrac{8\times\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}{a^3}$

Total mass of ball $M = N \times M_0$

$$M = \frac{4}{3}\pi M_0 D^3 a^{-3}$$

So, $$\frac{dM}{M} = 3\frac{d(D)}{D} + 3\frac{da}{a}$$

$$= 3\left[\frac{0.2\times10^{-9}}{9.4\times10^{-2}} + 2\times10^{-9}\right]$$

$$= 1.2 \times 10^{-8}$$

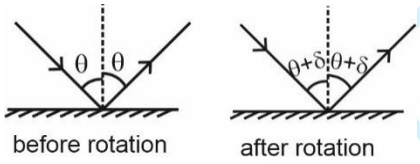
22. A laser beam is incident on a flat/plane mirror at some angle and results in a reflected beam. The mirror is now rotated by an angle δ while the direction of incident laser beam is kept the same. The angle between the new reflected beam and the reflected beam before the mirror was rotated is

(A) 2δ (B) 0

(C) δ (D) $\delta/2$

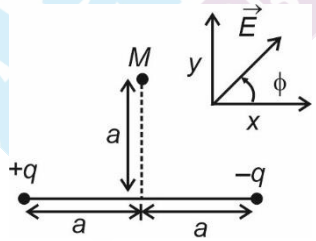
Answer (A)

Sol.



Normal will rotate by δ so new angle of incidence will be $\theta + \delta$ so angle between new reflected ray and old reflected ray is 2δ.

23. Consider two charges, $+q$ and $-q$, ($q > 0$) placed at a distance $2a$ from each other. At the point M (see figure below), the electric field makes an angle ϕ from the x axis. The correct value of ϕ is

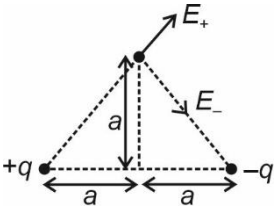


(A) 0° (B) 90°

(C) 180° (D) 270°

Answer (A)

Sol. Due to symmetry $|E_+| = |E_-|$



So their resultant on the bisector line

So angle $\phi = 0°$

24. A particle starts from rest at $x = 0$ m with an acceleration of 1 m/s^2. At $t = 5$ s it receives an additional acceleration in the same direction as its motion. At $t = 10$ s its speed and position are v and x, respectively. Had the additional acceleration not been provided, its speed and position would have been v_0 and x_0, respectively. It is found that $x - x_0$ is 12.5 m. Then one can conclude that $v - v_0$ is

(A) 5 m/s (B) 10 m/s

(C) 15 m/s (D) 20 m/s

Answer (A)

Sol. Let time starts from $t = 5$ s

$$x = 5 \times 5 + \frac{1}{2}(1 + a)5^2$$

$$x_0 = 5 \times 5 + \frac{1}{2} \times 1 \times 5^2$$

$$\Rightarrow x - x_0 = 12.5a = 12.5$$

$$\Rightarrow a = 1 \text{ m/s}^2$$

So $v = 5 + (1 + a)5$

and $v_0 = 5 + 5 = 10$

So $v - v_0 = 5a = 5$ m/s

25. The heat required to change 1 kg of ice at –8°C into water at 20°C, at 1 atm of pressure, is closest to (Assume that ice has a specific heat capacity 2.1 kJ/kg/K, water has a specific heat capacity 4.2 kJ/kg/K, and latent heat of fusion of ice is 333 kJ/kg)

(A) 414 kJ (B) 424 kJ

(C) 434 kJ (D) 444 kJ

Answer (C)

Sol. $Q = mS_{ice}\,\Delta T + mL + mS_w\,\Delta T'$

$= 1 \times 2.1 \times 8 + 1 \times 333 + 1 \times 4.2 \times 20$

$= 433.8$ kJ

$\simeq 434$ kJ

26. A button battery is rated 3 V and 225 mAh. A cricket ball (mass = 0.163 kg) having energy equal to that stored in the battery will have speed closest to

(A) 20 m/s (B) 70 m/s

(C) 90 m/s (D) 170 m/s

Answer (D)

Sol. Energy stored in the battery $= Q \cdot V$

$= (225 \text{ mAh}) \cdot (3\text{V})$

$= 2430$ J

Kinetic energy of cricket ball $= \frac{1}{2}mv^2$

$$\Rightarrow \frac{1}{2}mv^2 = 2430$$

$$\Rightarrow v \simeq 172 \cdot 67 \text{ m/s}$$

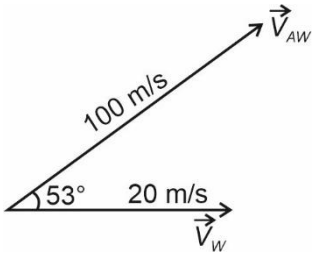
27. An airplane airspeed indicator reads 100 m/s and its compass shows that it is heading 37° *east of north*. The meteorological information provided to the navigator is that the wind velocity is 20 m/s towards east. The speed of the airplane relative to the ground is closest to

(A) 111 m/s (B) 113 m/s

(C) 115 m/s (D) 120 m/s

Answer (B)

Sol.



$\vec{V}_{AW} + \vec{V}_W =$ Velocity of aeroplane w.r.t. ground $\left(\vec{V}_A\right)$

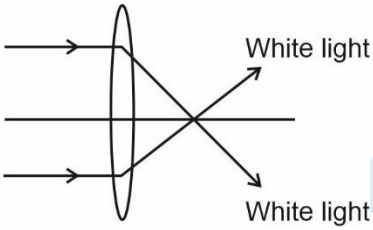
$\Rightarrow \quad \vec{V}_A = (20 + 100\cos 53°)\hat{i} + (100\sin 53°)\hat{j}$

$= 80\hat{i} + 80\hat{j}$

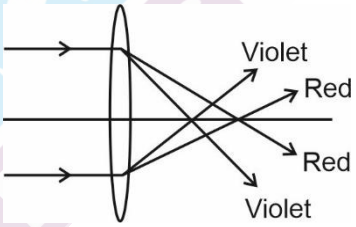
$\Rightarrow \quad \left|\vec{V}_A\right| \simeq 113 \cdot 14 \text{ m/s}$

28. A white light is falling on a bi-convex lens. Which of the following options represents the correct qualitative behaviour of the focussing of this light?

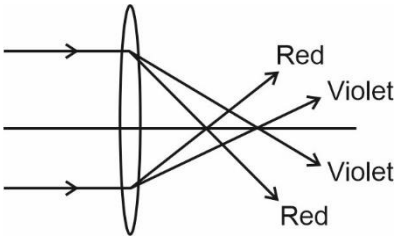
(A)



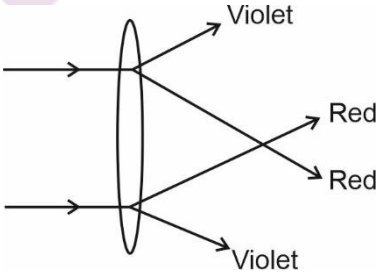
(B)



(C)



(D)



Answer (B)

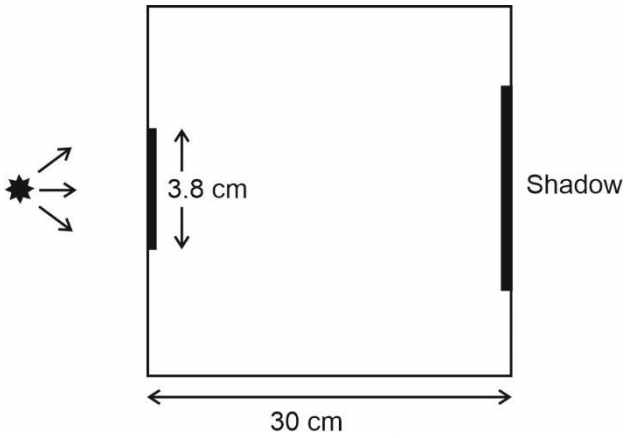
Sol. We know that $\mu_V > \mu_R$

$\Rightarrow \quad \frac{1}{f_V} > \frac{1}{f_R} \qquad \left[\because \ \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]$

$\Rightarrow \quad f_V < f_R$

$\Rightarrow$ Option (B) is correct.

29. Shown in the figure is a transparent tank of length 30 cm. A black strip of width 3.8 cm is stuck on its left wall. When a source of light is kept to the left of it, a shadow of width 7.6 cm is formed on the right wall.



Now, the tank is filled with a liquid of refractive index n, and the width of the shadow reduces to 6.4 cm. The value of n is closest to

(A) 1.20 (B) 1.35

(C) 1.45 (D) 1.55

Answer (C)

Sol. We know that

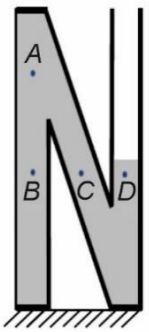
$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

$\Rightarrow \quad 1 \cdot \sin\theta_1 = n \cdot \sin\theta_2$

$\Rightarrow \quad 1 \times \frac{(1.9)}{30} \simeq n \cdot \frac{(1.3)}{30}$

$\Rightarrow \quad n \simeq \frac{19}{13} \simeq 1.46$

30. Consider a mercury-filled tube as shown in the figure below.



Which of the following options about the pressures at the lettered locations (A, B, C, D) is true?

(A) $P_B > P_A > P_C > P_D$ (B) $P_B = P_C = P_D > P_A$

(C) $P_B = P_C = P_D < P_A$ (D) $P_A = P_B = P_C = P_D$

Answer (B)

Sol. We know that pressure at same depth remains same.

$\Rightarrow \quad P_B = P_C = P_D$

Also, pressure reduces with height

$\Rightarrow \quad P_A < P_B$

$\Rightarrow \quad P_B = P_C = P_D > P_A$

PART-I : CHEMISTRY

31. Consider the reaction, $P(aq) \rightleftharpoons Q(aq)$ with an equilibrium constant K = 1.5. The reaction is started in a vessel with a concentration of [P] of 2 M and concentration of [Q] = 0. When the equilibrium is established, half the amount of P is removed, and the reaction is allowed to re-equilibrate. The concentration of Q in the vessel (in M) is closest to

(A) 0.64 (B) 0.96

(C) 0.24 (D) 1.20

Answer (B)

Sol. $K = \frac{[Q]}{[P]}$

$$\begin{array}{ccc} & P & \rightleftharpoons & Q \\ \text{At eq} & 2-x & & x \end{array}$$

$$1.5 = \frac{x}{2-x}$$

$[P]_{eq} = 0.8$ M, $[Q]_{eq} = 1.2$ M

When half of [P] is removed, $[P]_{left} = 0.4$ M

$Q_C > K_C$; Reaction moves backwards

$$\begin{array}{ccc} & P & \rightleftharpoons & Q \\ \text{New eq} & 0.4+x' & & 1.2-x' \end{array}$$

$x' = 0.24$ M

$[Q] = 1.2 - 0.24 = 0.96$ M

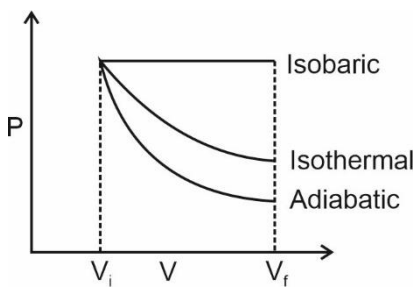
32. A gas is reversibly expanded from the same initial state to the same final volume using isobaric, isothermal and adiabatic processes. The correct order of the work done by the system on the surroundings in the three different methods is

(A) Isobaric > isothermal > adiabatic (B) Isobaric > adiabatic > isothermal

(C) Adiabatic > isothermal > isobaric (D) Isothermal > isobaric > adiabatic

Answer (A)

Sol.



Since, work done is equal to area under PV curve.

Hence, isobaric > isothermal > adiabatic

33. When 22.4 liters of H_2 (g) is mixed with 5.6 liters of Cl_2 (g), each at S.T.P., the moles of HCl (g) formed after completion of the reaction is closest to

(A) 1.0 (B) 0.75

(C) 0.5 (D) 0.25

Answer (C)

Sol. $H_2(g) + Cl_2(g) \rightarrow 2HCl(g)$

$(\text{Moles})_{H_2} = 1\,\text{mole}$

$(\text{Moles})_{Cl_2} = 0.25$ moles

Hence Cl_2 is limiting reagent.

$(\text{Moles})_{HCl} = 0.5$ moles

34. A bulb emits monochromatic yellow light of the wavelength 0.57 micron. If the rate of emission of quanta per second of the bulb is 14.33×10^{19}, the power of the bulb (in Watt) is

(A) 25 (B) 50

(C) 75 (D) 100

Answer (B)

Sol. $\lambda = 0.57 \times 10^{-6}$ m

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{0.57 \times 10^{-6}}$$

$E = 34.87 \times 10^{-20}$ J

Power = Energy × Rate of emission of quanta per second

$= 34.87 \times 10^{-20} \times 14.33 \times 10^{19}$

$= 50\ \text{J s}^{-1}$ or 50 watt

35. An isolated chamber is divided into two halves by a partition with an ideal gas in one half. By making a hole in the partition, the gas is allowed to expand to the full chamber. Among the following, the parameter with changes in the process is

(A) Internal energy (B) Heat

(C) Temperature (D) Pressure

Answer (D)

Sol. When an ideal gas expands from one half of an isolated chamber till the chamber is completely filled, the pressure of gas changes. Other thermodynamics properties of the system like internal energy, heat and temperature remain unchanged.

36. Iodination of a hydrocarbon (C – H $\rightarrow$ C – I) with molecular iodine is a slow and reversible reaction. However, it can be carried out in the presence of an oxidizing agent such as

(A) H_3BO_3 (B) HIO_3

(C) H_3PO_4 (D) CH_3CO_2H

Answer (B)

Sol. Iodination of hydrocarbon with I_2 is a slow and reversible reaction.

The equilibrium shifts in the forward direction in the presence of an oxidising agent such as HIO_3.

$$HIO_3 + 5HI \rightarrow 3I_2 + 3H_2O$$

37. A mixture of 1 mole of benzene and 1 mol of nitrobenzene is reacted with 1 mol of acetyl chloride in the presence of $AlCl_3$. The major product(s) is/are

(A) acetophenone

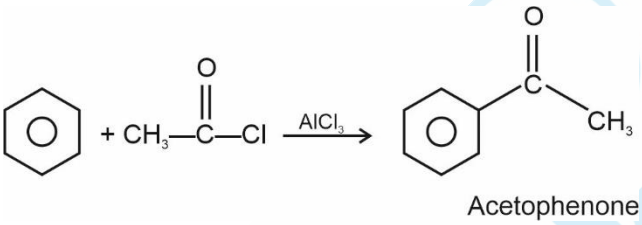
(B) 3-nitroacetophenone

(C) 1 : 1 mixture of acetophenone and 3-nitroacetophenone

(D) 1, 3-diacetyl benzene

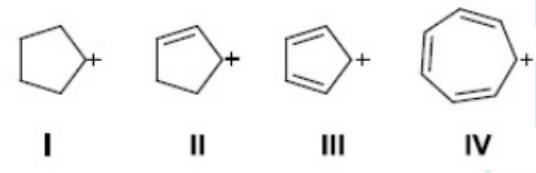
Answer (A)

Sol. Benzene is more electron rich as compared to nitrobenzene. Hence benzene will be more reactive towards electrophilic aromatic substitution reaction. Nitrobenzene does not undergo Friedel-Craft reaction.



Acetophenone

38. The stability of the carbocations



follows the order

(A) I > II > III > IV

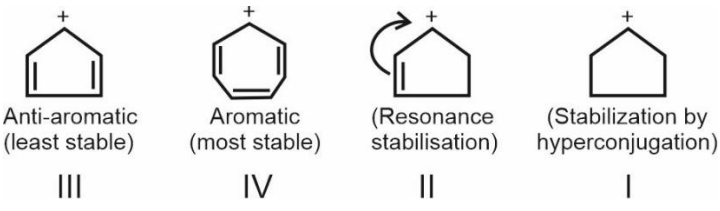
(B) III > II > IV > I

(C) IV > II > I > III

(D) IV > I > III > II

Answer (C)

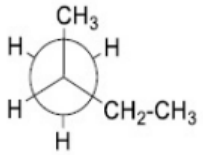
Sol.

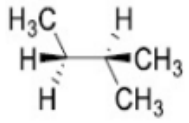


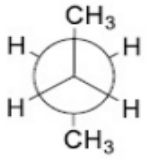
Correct order of stability is

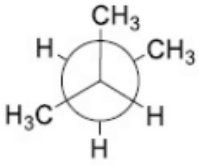
IV > II > I > III

39. Among the following, the structure which does **NOT** represent 2-methyl butane is

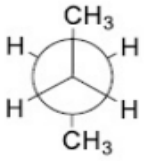
(A) 

(B) 

(C) 

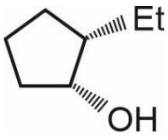
(D) 

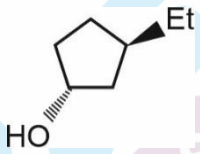
Answer (C)

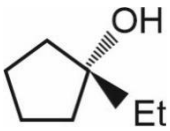
Sol. 

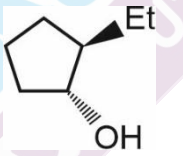
This structure represents n-butane.

40. The reaction of 1-ethylcyclopentene with BH_3/THF followed by treatment with H_2O_2/NaOH produces

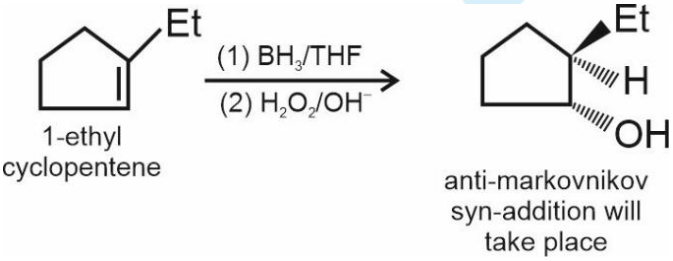
(A) 

(B) 

(C) 

(D) 

Answer (D)

Sol. 

41. The similar chemical properties of lithium and magnesium arise due to their similar

(A) Electron affinities

(B) Ionic sizes

(C) Ionization potential

(D) Hydration enthalpy

Answer (B)

Sol. The similar chemical properties of lithium and magnesium arise due to their similar ionic sizes (Ionic radii : Li^+ = 76 pm; Mg^{2+} = 72 pm)

42. The **INCORRECT** statement about the dissolution of an alkali metal in liquid ammonia is

(A) It produces a blue coloration of the solution

(B) The blue coloration occurs due to ammoniated electrons that absorbs in visible region of light

(C) On standing, the blue solution liberates hydrogen gas

(D) The blue solution is diamagnetic

Answer (D)

Sol. Dissolution of an alkali metal in liquid ammonia produces a blue colouration of the solution. The blue colour is due to ammoniated electrons. On standing blue colour disappears with the evolution of H_2 gas. The blue solution is paramagnetic due to the presence of ammoniated electrons

$$M + (x+y)\,NH_3(l) \longrightarrow \underset{(Blue)}{M^+(NH_3)_x + e^-(NH_3)_y}$$

43. Among the following, the correct statement for thionyl tetrafluoride is

(A) The geometry of thionyl tetrafluoride is trigonal bipyramidal having the sulphur-oxygen bond on the trigonal plane

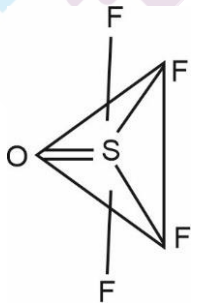
(B) The geometry of thionyl tetrafluoride is trigonal bipyramidal having the sulphur-oxygen bond perpendicular to the trigonal plane

(C) The geometry of thionyl tetrafluoride is square pyramidal having the sulphur-oxygen bond on the square plane

(D) The geometry of thionyl tetrafluoride is square pyramidal having the sulphur-oxygen bond perpendicular to the square plane

Answer (A)

Sol. Geometry of SOF_4 is trigonal bipyramidal having S = O on trigonal plane. According to Bent's rule double bonds on central atom occupy more space.



44. The thermal stability of the hydrides of group-16 elements follows the order

(A) $H_2Te < H_2S < H_2Se < H_2O$

(B) $H_2O < H_2Se < H_2S < H_2Te$

(C) $H_2Te < H_2Se < H_2O < H_2S$

(D) $H_2Te < H_2Se < H_2S < H_2O$

Answer (D)

Sol. Down the group thermal stability of the hydrides of group 16 elements decreases. As the size of the central element increases bond dissociation energy decreases.

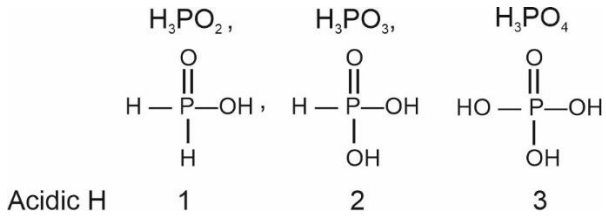
45. The number of acidic protons present in H_3PO_2, H_3PO_3 and H_3PO_4, respectively, are

(A) 1, 2, and 3 (B) 2, 3, and 3

(C) 1, 2 and 2 (D) 3, 3 and 3

Answer (A)

Sol.



Hydrogen attached to oxygen is acidic.

PART-I : BIOLOGY

46. Which ONE of the following biomolecules is an end product of amylolysis?

(A) Amino acids (B) Fatty acids

(C) Monosaccharides (D) Nucleotides

Answer (C)

Sol. Option (C) is correct because monosaccharides are the breakdown products of starch which is formed by two components- amylose and amylopectin.

Option (A) is incorrect because amino acids are the breakdown products of proteins.

Option (B) is incorrect because fatty acids are end products of lipids.

Option (D) is incorrect because nucleotides are end products of nucleic acids.

47. Which ONE of the following is **NOT** used in constructing phylogenetic trees?

(A) Nuclear DNA (B) Mitochondrial DNA

(C) Anatomical features (D) Habitat similarity

Answer (D)

Sol. Option (D) is correct because habitat similarity is not used in constructing phylogenetic tree because animals belonging to same taxon can occupy different habitats.

Nuclear DNA represents certain similarities in all eukaryotic organisms but differ from prokaryotes, so they are used to establish phylogenetic relationship.

Mitochondrial DNA is responsible for maternal inheritance and used in constructing phylogenetic tree.

Phylogenetic trees are made on the basis of anatomical studies which explains homology, analogy and vestigial organs.

48. Caecum is located between

(A) Ileum and ascending colon (B) Oesophagus and pharynx

(C) Rectum and descending colon (D) Stomach and duodenum

Answer (A)

Sol. Option (A) is correct because caecum is located between ileum (last part of small intestine) and ascending colon (first part of colon).

Option (B) is incorrect because pharynx leads to oesophagus.

Option (C) is incorrect because sigmoid colon is present between rectum and descending colon.

Option (D) is incorrect because stomach opens into duodenum via an opening guarded by pyloric sphincter.

49. Which ONE of the following plants is an invasive species that has spread to many parts of the Indian subcontinent?

(A) *Prosopis juliflora* (B) *Ficus religiosa*

(C) *Cocos nucifera* (D) *Lotus corniculatus*

Answer (A)

Sol. *Prosopis juliflora* is a shrub native to South America. It is an invasive species in India.

50. Which ONE of the following processes maximally facilitates the ascent of sap?

(A) Guttation (B) Photosynthesis

(C) Photorespiration (D) Transpiration

Answer (D)

Sol. Transpiration majorly provides the force for ascent of sap.

51. Which ONE of the following biomolecules is **NOT** present in healthy colostrum?

(A) Antibodies (B) Lysozyme

(C) Carbohydrates (D) Haemoglobin

Answer (D)

Sol. Option (D) is correct because haemoglobin is a respiratory pigment present in RBCs which are not found in a healthy colostrum.

Antibodies, lysozyme and carbohydrates such as lactose are present in colostrum.

52. You have made a mixed vegetarian curry with potato, cauliflower, radish, and tomato, and spiced it with mustard, cinnamon and clove. The final product of this culinary adventure consists of

(A) Root, flower, fruit, bark, leaf (B) Stem, flower, root, fruit, bark

(C) Stem, meristem, root, fruit, seed, bark, bud (D) Stem, meristem, root, fruit, seed, bud, leaf

Answer (C)

Sol. Potato – Underground stem

Cauliflower – Apical bud having meristem

Radish – Root

Tomato – Fruit

Mustard – Seed

Cinamon – Bark

Clove – Bud

53. Which ONE of the following processes would be an immediate effect on a plant if there is a sudden and large increase in soil salinity?

(A) Plasmolysis of root cells (B) Closure of stomata

(C) Increase in transpiration (D) Increase in root turgidity

Answer (A)

Sol. If concentration of salt suddenly increases in soil the immediate effect will be plasmolysis of root cells.

54. High blood glucose in diabetic patients is known to induce cataract. This is because high glucose

(A) Crystalises in the lens and blocks light

(B) Causes osmotic changes in aqueous humor promoting lens impairment

(C) Is polymerised into starch and is deposited in the lens

(D) Reflects light from the lens thereby impairing vision

Answer (B)

Sol. Option (B) is correct because high levels in aqueous humor increases the osmotic pressure which leads to lens impairment.

Option (A) is incorrect because cataract is caused by protein aggregation, not crystallization of glucose.

Option (C) is incorrect because glucose is not polymerized into starch inside lens.

Option (D) is incorrect because protein aggregates scatter the incoming light thereby impairing vision.

55. Which one of the following is most likely to occur because of climate change-driven temperature increase?

(A) Fish shift their ranges to shallower waters

(B) Mammals shift their ranges towards lower latitudes

(C) Frogs shift their ranges towards the equator

(D) Birds shift their ranges to higher elevations

Answer (D)

Sol. Climate change-driven temperature *i.e.,* global warming will push tropics into temperate and temperate areas towards pole and higher altitudes in mountains. Therefore, birds shift their ranges to higher elevations.

56. Which ONE of the following cell types contains Nissl's granules?

(A) Eosinophils

(B) Hepatocytes

(C) Cardiomyocyte

(D) Neurons

Answer (D)

Sol.
- Option (D) is correct because Nissl's granules are exclusive feature of neurons, present in cyton and dendrites.
- Option (A) is incorrect because eosinophils are formed elements of blood.
- Option (B) is incorrect because hepatocytes are liver cells.
- Option (C) is incorrect because cardiomyocytes are present in heart.
- All these cells do not contain Nissl's granules.

57. Within the kingdom Animalia, which ONE of the following features is primarily used for classification of the organisms?

(A) Body symmetry

(B) Habitat

(C) Mode of nutrition

(D) Locomotory organs

Answer (A)

Sol.
- Option (A) is correct because all animals are classified into two categories – Radiata and Bilateria on the basis of symmetry. So body symmetry is primary basis for the classification of organisms.

  Animals having radial symmetry are placed in Radiata while animals having bilateral symmetry are placed in Bilateria.
- Option (B), (C) and (D) are incorrect because habitat, mode of nutrition and locomotory organs may be different in same taxon so it cannot be the basis of classification.

58. In ureotelic animals, urea is produced through

(A) Cori cycle

(B) Kreb's cycle

(C) Ornithine cycle

(D) Pentose phosphate pathway

Answer (C)

Sol.
- Option (C) is correct because urea is produced through ornithine cycle, also called Kreb- Henseleit cycle and it occurs in liver.
- Option (A) is incorrect because Cori cycle occurs in muscles for lactic acid metabolism.
- Option (B) is incorrect because Kreb's cycle occurs in all living eukaryotic cells to produce ATP.
- Option (D) is incorrect because pentose phosphate pathway is a metabolic pathway parallel to glycolysis.

59. Which of the following is **INCORRECT** about pollen grains?

(A) Angiosperm species can be identified from their pollen morphology

(B) Pollen movement is facilitated by flagella in angiosperms

(C) Pollen outnumbers ovules in angiosperm flowers

(D) Pollen is found only in angiosperms

Answer (B, D)

Sol.
- In angiosperm pollen grains are not motile. They do not have flagella.
- Pollens are found in both angiosperms and gymnosperms. So, both (B) and (D) are incorrect.

60. Food chains seldom exceed four or five trophic levels because

(A) Only 10% of energy in all levels is available for decomposers to convert into nutrients for the entire ecosystem

(B) Only 10% of energy at all heterotroph levels is available for conversion of biomass by autotrophs

(C) Almost 90% of energy at the autotrophs level are converted to biomass by heterotrophs in all trophic levels.

(D) Almost 90% of energy in each heterotroph level is not converted into biomass at the next level

Answer (D)

Sol. There is transfer of only 10% of the total energy from one trophic level to the next and 90% of the energy is consumed in different biological process of the organisms and maintenance of body heat. This is the reason that food chain in an ecosystem does not exceed four or five trophic levels.

PART-II : MATHEMATICS

61. The sum of the sides of a right-angled triangle is 42, and the difference between the median and altitude drawn from the vertex at the right angle is 2. The area of the triangle is

(A) 42 (B) 51

(C) 63 (D) $9\sqrt{51}$

Answer (C)

Sol. $\frac{c}{2} = h + 2 - (1)$

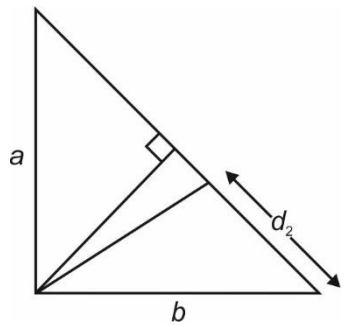
Also, $\Delta = \frac{c}{2}h = \frac{1}{2}b \Rightarrow h = \frac{ab}{c}$

Also, $a + b + c = 42$

$a^2 + b^2 = c^2$

$\Rightarrow (a + b)^2 = (42 - c)^2$

$\Rightarrow 2ab = 42^2 - 84c$



$2ch = 1764 - 84c$

$2c\left(\frac{c}{2} - 2\right) = 1764 - 84c$

$\Rightarrow$ $c^2 + 80c - 1764 = 0$

$\Rightarrow$ $c = 18 \Rightarrow a + b = 24$

$h = \frac{c}{2} - c$

$= 7$

$ab = ch \Rightarrow ab = ch$

$= 18 \times 7$

Area = $\frac{1}{2} \times 18 \times 7 = 63$

62. The number of ordered pairs (a, b) of integers such that $a - b$ is a root of $x^2 + ax + b = 0$ is

(A) 3

(B) 4

(C) 5

(D) 6

Answer (B)

Sol. If $(a - b)$ is a root then

$(a - b)^2 + a(a - b) + b = 0$

$\Rightarrow$ $2a^2 - 3ab + b^2 + b = 0$

$\Rightarrow$ $a = \frac{3b \pm \sqrt{9b^2 - 8\left(b^2 + b\right)}}{4}$

$= \frac{3b \pm \sqrt{(b-4)^2 - 16}}{4}$

$\left.\begin{array}{l} i.e.,\ b = 8 \ \text{ gives } \ a = 6 \\ \text{and } \ b = 9 \ \text{ gives } \ a = 6 \\ \text{and } \ b = -1 \ \text{ gives } \ a = 0 \\ \ \ \& \ \ b = 0 \ \text{ gives } \ a = 0 \end{array}\right\}$ *i.e.*, 4 ordered pairs

63. Let a, b, c, d be positive integers. Consider the following statements

I. If 9 divides $a^3 + b^3 + c^3$, then 3 divides abc.

II. If 9 divides $a^3 + b^3 + c^3 + d^3$, then 3 divides $abcd$.

Then

(A) Both I and II are true

(B) I is true but II is false

(C) I is false but II is true

(D) Both I and II are false

Answer (B)

Sol. If $a \in N$ then $a^3 \equiv 0, 1, -1 \pmod 9$

If $a^3 + b^3 + c^3 \equiv 0 \pmod 9$ then atleast one of a^3, b^3 or c^3 is divisible by 9 .

If $a^3 + b^3 + c^3 + d^3 \equiv 0 \pmod 9$ then it is not necessary that atleast one is divisible by 9.

64. Let λ by the positive root of the equation $x^2 - x - 1 = 0$, and set $a_n = \frac{1}{\sqrt{5}}\left(\lambda^n - (1-\lambda)^n\right)$ for $n \in \mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers. Consider the sets

$A = \{n \in \mathbb{N} : a_n$ is a rational number, but not an integer$\}$, and

$B = \{n \in \mathbb{N} : a_n$ is an irrational number$\}$.

Then

(A) both the sets A and B are empty

(B) the set A is empty but the set B is non-empty

(C) the set A is non-empty and set B is empty

(D) both the sets A and B are non-empty

Answer (A)

Sol. $x^2 - x - 1 = 0$

$$\lambda = \frac{1+\sqrt{5}}{2}$$

$$\Rightarrow 1 - \lambda = \frac{1-\sqrt{5}}{2}$$

$$\Rightarrow \lambda^n - (1-\lambda)^n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n = \frac{2\sqrt{5}}{2^n}\left\{{}^nC_1 + 5.{}^nC_3 + 5^2.{}^nC_5 + ...\right\}$$

$$\therefore \frac{\lambda^n - (1-\lambda)^n}{\sqrt{5}} = \frac{1}{2^{n-1}}\left\{{}^nC_1 + 5.{}^nC_3 + 5^2.{}^nC_5 .....\right\}$$

which is integer for $n = 1, 2, 3, 4$…….

Hence both A & B sets are empty

65. The number of integers q, $1 \le q \le 2021$, such that $\sqrt{q}$ is rational, and $\frac{1}{q}$ has a terminating decimal expansion, is

(A) 1 (B) 11

(C) 22 (D) 44

Answer (B)

Sol. $1 \le q \le 2021$

As $\sqrt{q}$ is rational and $\frac{1}{q}$ has a terminating decimal.

So, q should be multiple of 2 and 5.

$q = 2^a \cdot 5^b$

Also $\sqrt{q}$ is rational,

So a, b should be even integers.

$q = 1, 2^2, 2^4, 2^6, 2^8, 2^{10}, 5^2, 5^4, 2^2 \times 5^2, 2^4 \times 5^2, 2^6 \times 5^2$

PART-II : PHYSICS

66. A student of mass M is 1.5 m tall and has her centre of mass 1 m above ground when standing straight. She wants to jump up vertically. To do so, she bends her knees so that her centre of mass is lowered by 0.2 m and then pushes the ground by a constant force F. As a result, she jumps up such that the maximum height of her feet is 0.3 m above ground. The ratio F/Mg is

(A) 1.5 (B) 2.5

(C) 3.5 (D) 4.5

Answer (B)

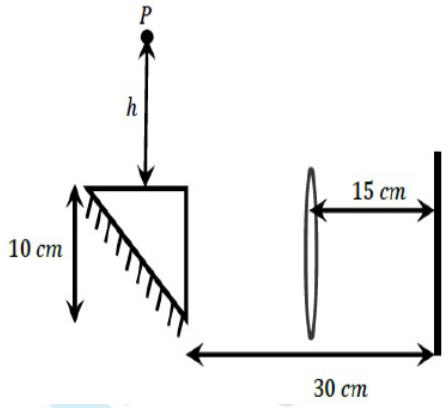
Sol. Work done = $F \times (0.2)$

Gain in $GPE = (Mg) \times (0.2 + 0.3)$

$\Rightarrow F \times (0.2) = Mg \times 0.5$

$\Rightarrow \frac{F}{Mg} = \frac{0.5}{0.2} = 2.5$

67. The 45° – 45° – 90° prism of height 10 cm (see image below) has a refractive index 2, with a silvered- hypotenuse surface.



A convex lens of focal length 10 cm placed 15 cm in front of the wall produces a sharp image of P on it. The value of h (in cm) is closest to

(A) 20 (B) 15

(C) 10 (D) 5

Answer (C)

Sol. From traceability of ray:

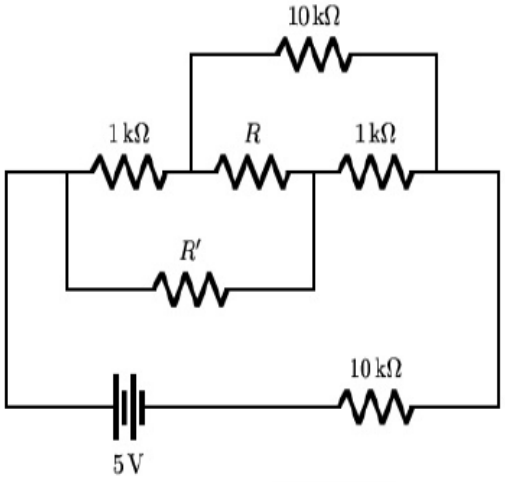
$v_1 = \frac{15 \times 10}{(15-10)} = \frac{15 \times 10}{5} = 30$ cm

Apparent height of P is $h \times \mu$

$\therefore \frac{(2h+5)+5}{2} + 15 = 30$

$\Rightarrow h = 10$ cm.

68. When the resistance R (indicated in the figure below) is changed from 1 kΩ to 10 kΩ, the current flowing through the resistance R' does not change. What is the value of the resistor R'?



(A) 5 kΩ
(B) 100 Ω
(C) 10 kΩ
(D) 1 kΩ

Answer (B)

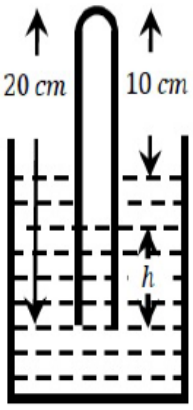
Sol. If the bridge is balanced Wheatstone, then current through R is zero always.

$$\therefore \frac{1000}{R'} = \frac{10,000}{1000}$$

$R' = 100\ \Omega$

$\therefore$ From, uniqueness theorem, $R' = 100\ \Omega$.

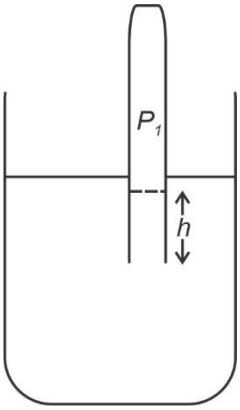
69. A 20 cm long tube is closed at one end. It is held vertically, and its open end is dipped in water until only half of it is outside the water surface. Consequently, water rises in it by height h as shown in the figure. The value of h is closest to (assume that the temperature remains constant, $P_{atmosphere} = 10^5$ N/m^2, density of water = 10^3 kg/m^3, and acceleration due to gravity g = 10 m/s^2)



(A) 2 cm
(B) 1 cm
(C) 0.4 cm
(D) 0.2 cm

Answer (D)

Sol.



$P_0 \times (A \times 20) = P_1 \times A \times (20 - h)$

$\Rightarrow P_0 \times 20 = P_1(20 - h)$ …(i)

$P_1 + \rho gh = P_0 + \rho g \times (10)$

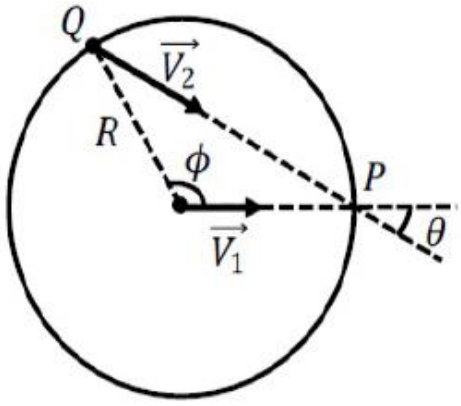
$$\frac{P_0 \times 20}{(20-h)} - P_0 = \rho g(10 - h)$$

$$\Rightarrow P_0\left[\frac{20 - 20 + h}{20 - h}\right] = \rho g\,(10 - h)$$

$$\Rightarrow \frac{10^5 \times h}{(20-h)} = 10^3 \times 10\,(10 - h) \times 10^{-2}$$

$\Rightarrow h = 0.2$ cm

70. Two particles, one at the centre of a circle of radius R, and another at a point Q on the circle, start moving towards a point P on the circle at the same time (see figure below). Both are at rest initially and move with uniform velocities $\vec{V_1}$ and $\vec{V_2}$ respectively. They also reach the point P at the same time. If the angle between the velocities is θ and the angle subtended by P and Q at the centre is ϕ (as shown in the figure), then



(A) $\tan\frac{\phi}{2} = \cot\theta$

(B) $\tan\phi = \cot\theta$

(C) $\cot\frac{\phi}{2} = \cot\theta$

(D) $\tan\frac{\phi}{2} = \cot\frac{\theta}{2}$

Answer (A)

Sol. From geometry,

$\phi + 2\theta = 180°$

$\Rightarrow \theta = 90° - \frac{\phi}{2}$

$\Rightarrow \cot\theta = \cot\left(90° - \frac{\phi}{2}\right)$

$\Rightarrow \cot\theta = \tan\frac{\phi}{2}$

PART-II : CHEMISTRY

71. A hydrocarbon having molecular formula C_5H_{10} produced a tertiary alcohol upon treatment with a few drops of conc. sulphuric acid and water. The same hydrocarbon when reacted with acidic potassium permanganate produced a ketone and a carboxylic acid. The hydrocarbon is:

(A) cyclopentane

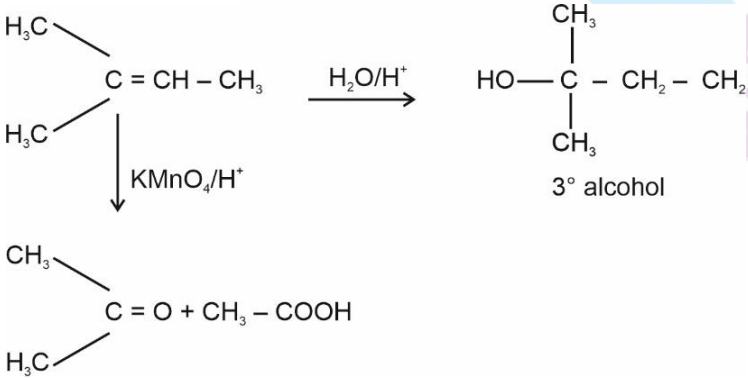
(B) 1-pentene

(C) 2-methyl-2-butene

(D) 2-pentene

Answer (C)

Sol.



72. Nitrogen present in an unknown organic compound was estimated by Dumas method to be 17.7% by weight. The compound is very likely to be:

(A) nitrobenzene

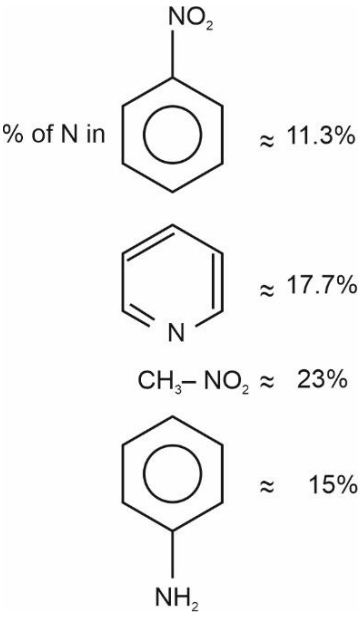
(B) pyridine

(C) nitromethane

(D) aniline

Answer (B)

Sol.



73. A pink coloured aqueous solution of $Co(NO_3)_2$ turns blue on addition of HCl gradually. This colour change happens due to the formation of

(A) $[CoCl_4]^{2-}$

(B) $[CoCl_6]^{4-}$

(C) $[Co(H_2O)_4Cl_2]$

(D) $[Co(H_2O)_2Cl_4]^{2-}$

Answer (A)

Sol. $\underset{\text{Pink}}{Co(NO_3)_2} + HCl \longrightarrow \underset{\text{Blue}}{[CoCl_4]^{2-}}$

Colour change is due to formation of $[CoCl_4]^{2-}$

74. 50 ml of 0.1 M of a weak acid HA is titrated with 0.1 M of NaOH. The ionization constant of HA (K_a) is 1.8×10^{-5}. Using the given information and from the options shown below, the best indicator for the titration of HA with NaOH is

(A) Methyl orange (changes colour from red to yellow as the pH changes from 3.2 to 4.4)

(B) Methyl red (changes colour from red to yellow as the pH changes from 4 to 6.3)

(C) Phenolphthalein (changes from colourless to pink to as the pH changes from 8.3 to 11)

(D) Sodium salt of Alizarin yellow (changes colour from yellow to red as the pH changes from 10 to 12)

Answer (C)

Sol. At equivalence point,

millimoles of HA = millimoles of NaOH

$5 = 0.1 \times V$

$V = 50$ mL

$\therefore$ Concentration of salt = $\frac{5}{100}$ = 0.05 M

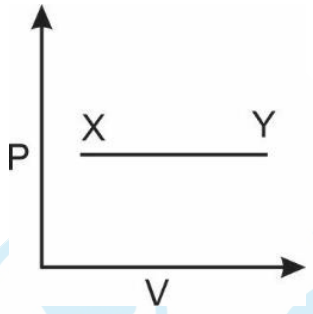
(pH) equivalence point = $7 + \frac{1}{2}(pK_a + \log C)$

$= 7 + \frac{1}{2}(-0.26 + 5 - 1.3)$

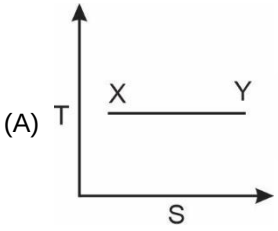
$= 7 + \frac{1}{2}(3.44) = 7 + 1.72 = 8.72$

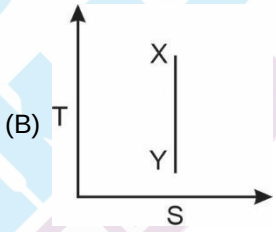
So, the most suitable indicator is (C)

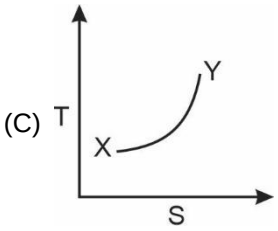
75. Consider the P-V (pressure-volume) diagram given below where an ideal gas is reversibly converted from state A to state B

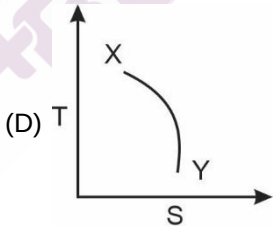


Among the following, the correct T-S (temperature-entropy) diagram, which corresponds to this process is

(A) 

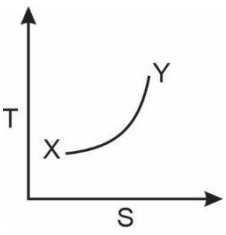
(B) 

(C) 

(D) 

Answer (C)

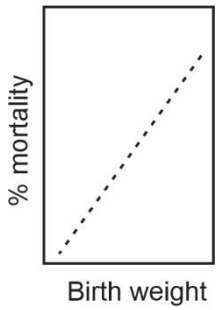
Sol. Temperature entropy curve for isobaric process will be :

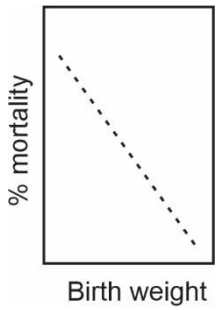


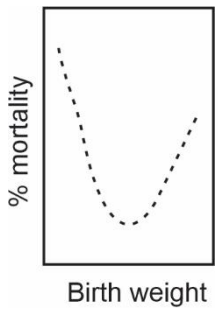
$\left[\text{For ideal gas } \Delta S = nC_{pm} \ln \frac{T_2}{T_1}\right]$

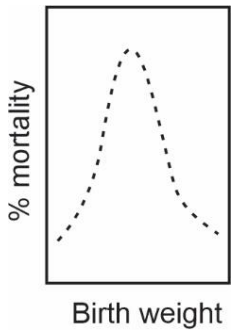
PART-II : BIOLOGY

76. Which one of the following plots would best describe the relationship between human infant mortality and birth weight (1-10 kgs)?

(A)


(B)


(C)


(D)


Answer (C)

Sol. The correct answer is option (C) as stabilizing natural selection is seen w.r.t. the new born's birth weight. If the new born's birth weight is low or high than the mean value, then the chances of mortality increases. This means the lower the weight, the higher the human infant mortality and the higher the weight, the higher the infant mortality.

77. A genetic form of a locus would be called an allele only when

(A) its frequency in a population is > 0.01 and it is heritable

(B) its frequency in a population is > 0.01 irrespective of its heritability

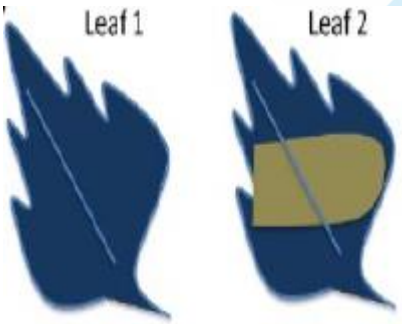
(C) it is heritable irrespective of its frequency

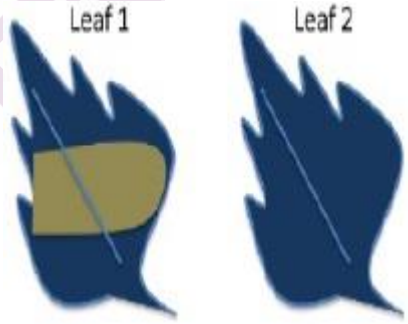
(D) it is a tandem repeat irrespective of its frequency

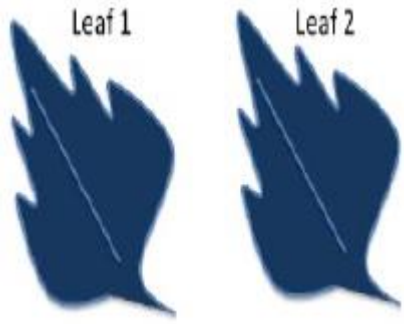
Answer (A)

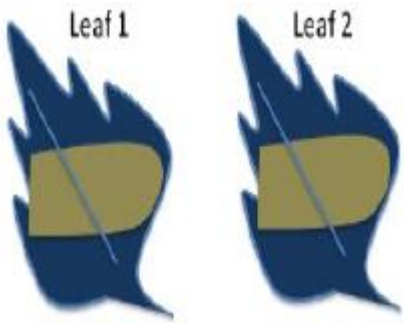
Sol. Allelic sequence variation has traditionally been described as a DNA polymorphism if more than one allele at a locus occurs in population with frequency > 0.01*, i.e.*, inheritable.

78. A student conducted an experiment to determine the role of sunlight in photosynthesis. Two plants were used, while plant 1 was kept in the dark for 48 hours before the experiment, plant 2 was kept in the sunlight. The student covered one leaf from each plant with a black paper, as shown in the figure. Then, both the plants were kept in the sunlight for a few hours and the levels of starch was immediately examined in the leaves (leaf 1 from plant 1 and leaf 2 from plant 2). Which ONE of the following figures **CORRECTLY** represent the results of this experiment?

(A) 

(B) 

(C) 

(D) 

Answer (D)

Sol. When the plant is kept in dark for 48 hours, the stored starch in the leaves will be used up. If the part of leaf of both the plants is covered with black paper and kept in sun, this part will not synthesize starch and thus will show negative test for starch.

79. In the exponential population growth model, population growth rate is given by dN/dt = rN, where r is a measure of the population's intrinsic rate of increase and N is population size. The parameter 'r' is determined by

(A) birth rate and density

(B) death rate only

(C) birth rate only

(D) birth rate and death rate

Answer (D)

Sol. In the population growth equation dN/dt = rN, r is the biotic potential. It is important to parameter to assess impact of factors on population growth.

Here, r = per capita birth rate - per capita death rate.

80. Match the fibres in **Column I** with the primary constituents in **Column II** given below.

Column I	**Column II**
P. Cobweb	i. Fibroin
Q. Silk	ii. Sericin
R. Cotton	iii. Keratin
S. Hair	iv. Cellulose

Choose the **CORRECT** combination.

(A) P-i; Q-i, ii; R-iv; S-iii

(B) P-iiii; Q-ii; R-iv; S-i

(C) P-i, ii; Q-ii; R-iv; S-iii

(D) P-i, ii; Q-i, ii; R-iii; S-i

Answer (A)

Sol. The correct answer is option (A) as cobweb or spider silk contains fibroin; Silk is made up of two primary proteins, fibrous protein known as fibroin and sticky protein known as sericin; Cotton thread is made up of homopolymer cellulose; Hair has the protein keratin.

❑ ❑ ❑