Date: 22/05/2022

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Answers & Solutions

for

Time : 3 hrs.

KVPY (SX) 2021-22

Max. Marks : 160

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you open the question booklet.

The question paper consists of two parts (both contain only multiple choice questions) for 160 marks. There will be four sections in Part I (each section containing 20 questions) and four sections in Part II (each section containing 10 questions)

Part-I

(i) There are 80 objective type questions. 20 questions from each subject (Mathematics, Physics, Chemistry & Biology), you have to attempt any **3 subjects** out of **4 subjects**.

(ii) Each correct answer gets **1 mark** and for each incorrect answer **0.25 mark** will be deducted.

Part-II

(i) There are 40 objective type questions. 10 questions from each subject (Mathematics, Physics, Chemistry & Biology), you have to attempt **2 subjects** out of **4 subjects**.

(ii) Each correct answer gets **2 marks** and for each incorrect answer **0.5 mark** will be deducted.

PART-I : MATHEMATICS

1. Consider the set of all 7-digit numbers formed by the digits 0, 1, 2, 3, 4, 5, 6, each chosen exactly once. If a number is randomly drawn from this set, the probability that it is divisible by 4 is

(A) $\frac{26}{105}$ (B) $\frac{13}{45}$

(C) $\frac{2}{7}$ (D) $\frac{1}{3}$

Answer (B)

Sol. Total number of 7-digit numbers = $6\underline{|6}$

Number will be divisible by 4 if last two digits are 04, 12, 16, 20, 24, 32, 36, 40, 52, 56, 60 or 64.

The total number of numbers divisible by $4 = 8(4\underline{|4}) + 4(\underline{|5})$

Required probability $= \frac{8(4\underline{|4}) + 4(\underline{|5})}{616} = \frac{13}{45}$

2. Let a, b, x be positive real numbers with $a \neq 1, x \neq 1, ab \neq 1$. Suppose $\log_a b = 10$, and $\frac{\log_a x \log_x \left(\frac{b}{a}\right)}{\log_x b \log_{ab} x} = \frac{p}{q}$,

where p and q are positive integers which are coprime. Then $p + q$ is

(A) 9 (B) 99

(C) 109 (D) 199

Answer (C)

Sol. Clearly $b = a^{10}$

So, $\frac{\log_a(x) \cdot \log_x(a^9)}{\log_x(a^{10}) \cdot \log_{a^{11}}(x)} = \frac{9\log_a x \cdot \log_x a}{\frac{10}{11}\log_a x \cdot \log_x a} = \frac{99}{10}$

3. Let $x, y, z \in [0, 1]$. Then the maximum value of $\sqrt{|x-y|} + \sqrt{|y-z|} + \sqrt{|z-x|}$ is

(A) $1+\sqrt{2}$ (B) $\sqrt{2}$

(C) $2\sqrt{2}$ (D) $2+\sqrt{2}$

Answer (A)

Sol. WLOG, $x \geq y \geq z$

$S = \sqrt{x-y} + \sqrt{y-z} + \sqrt{x-z}$

$\sqrt{x-y} + \sqrt{y-z}$ is maximum if $y = \frac{x+z}{2}$

$\Rightarrow S \leq 2\sqrt{\frac{x-z}{2}} + \sqrt{x-z}$

$\Rightarrow S \leq \left(\sqrt{2}+1\right)\sqrt{x-z}$ and $\sqrt{x-z} \leq 1$

$\Rightarrow S \leq \sqrt{2}+1$

Equality holds when x, y, z are the permutations of $0, \frac{1}{2}$ and 1.

4. Let $\mathbb{R}$ be the set of all real numbers and $\alpha \in \mathbb{R}$ be positive. Define a function $f : \mathbb{R} \to \mathbb{R}$ by $f(0) = 0$ and $f(x) = |x|^{\alpha} \sum_{n=0}^{\infty} (1+x^2)^{-n}$ for $x \neq 0$. Then the set of real numbers α for which f is continuous at $x = 0$ has

(A) 2 elements
(B) 3 elements
(C) 4 elements
(D) more than 4 elements

Answer (D)

Sol. $f(x) = |x|^{\alpha}\left(\dfrac{1}{1-\dfrac{1}{1+x^2}}\right) = (1+x^2)|x|^{\alpha-2}$

$\because$ $\lim_{x\to 0} f(x)$ must be zero. Then $\alpha > 2$.

Hence there exist infinitely many values of α.

5. In this question, all integers are represented in base 10. Consider the set E of positive integers n having the property that when any nonzero digit d is written to the right of n, the resulting number is divisible by d. Let N be the smallest element of E. The product of the digits of N is

(A) 20
(B) 24
(C) 30
(D) 36

Answer (A)

Sol. Let $m = 10n + d$

$\because$ $d | m \ \forall \ d = 1, 2, 3, ..., 9$

$\Rightarrow$ $d | 10n \ \forall d = 1, 2, 3, ..., 9$

So least value of $n = 9 \times 4 \times 7 = 252$

6. Let $\mathbb{R}$ be the set of all real numbers. The number of continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that for all real x $f(x) + f(2x) = 0$ is

(A) 0
(B) 1
(C) 2
(D) not finite

Answer (B)

Sol. $\because$ $f(x) = -f\left(\frac{x}{2}\right) = (-1)^2 f\left(\frac{x}{4}\right) = (-1)^3 f\left(\frac{x}{8}\right)$

$= .... = (-1)^n f\left(\frac{x}{2^n}\right)$

As $n \to \infty$ and $f(x)$ is continuous then $f(x) = f(0) = a$(say)

Put $x = 0$ in given equation, $2a = 0 \Rightarrow a = 0$

So $f(x) = 0$ is the only possible function.

7. Let E denote the set of all integers a such that the point of intersection of the parabola $y = x^2 + 2ax + 2021$ with x-axis has rational coordinates. The largest element of E is

(A) 45
(B) 1010
(C) 1011
(D) 2021

Answer (C)

Sol. If a monic polynomial has rational roots then it must have integral roots.

let the roots be α, β.

$\because \quad \alpha \cdot \beta = 2021$

and $2a = -(\alpha + \beta) = -\left(\alpha + \frac{2021}{\alpha}\right)$

where α is a negative factor of 2021

$\because \quad 2021 = 43 \times 47$

hence maximum value of $2a = -(-2021-1)$

$\Rightarrow \quad a_{max} = 1011$

8. Let m, n be real numbers such that $0 \le m \le \sqrt{3}$ and $-\sqrt{3} \le n \le 0$. The minimum possible area of the region of the plane consisting of points (x, y) satisfying the inequalities $y \ge 0, y - 3 \le mx, y - 3 \le nx$, is

(A) 0

(B) $\frac{3\sqrt{3}}{2}$

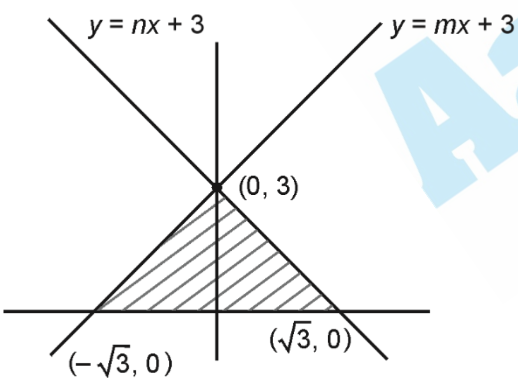
(C) $3\sqrt{3}$

(D) $6\sqrt{3}$

Answer (C)

Sol. The area is least

when $m = \sqrt{3}$ and $n = -\sqrt{3}$



Minimum possible area

$= \frac{1}{2}(2\sqrt{3})3$

$= 3\sqrt{3}$

9. Let AB be the diameter of a semicircle S. The locus of the centres of circles which are tangent to AB and to S is an arc of

(A) a circle

(B) an ellipse

(C) a parabola

(D) a cycloid

Answer (C)

Sol. Let $S : x^2 + y^2 = a^2$

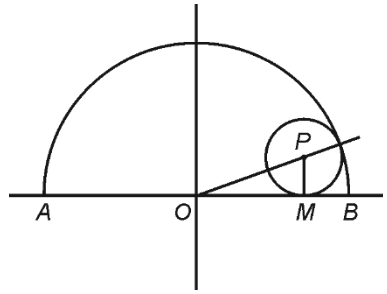
Let point P be (h, k)

$\because \ OP = a - k$

$\Rightarrow h^2 + k^2 = a^2 + k^2 - 2ak$

$\Rightarrow \ x^2 = a^2 - 2ay$

So locus is a parabola



10. Let $\theta, 0 < \theta < \pi/2$, be an angle such that the equation $x^2 + 4x\cos\theta + \cot\theta = 0$ has equal roots for x. Then θ in radians is

(A) $\frac{\pi}{6}$ only

(B) $\frac{\pi}{12}$ or $\frac{5\pi}{12}$

(C) $\frac{\pi}{6}$ or $\frac{5\pi}{12}$

(D) $\frac{\pi}{12}$ only

Answer (B)

Sol. $D = 0$

$\Rightarrow \ 16\cos^2\theta = 4\cot\theta$

$\Rightarrow \ \cos\theta = 0$ or $4\sin\theta\cos\theta = 1$

$\because \ \theta \in \left(0, \frac{\pi}{2}\right)$ so $\sin 2\theta = \frac{1}{2}$

$\Rightarrow 2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$

11. The graph of the function $f(x) = \frac{\cos x}{\cos 2x}$ in the domain $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ is

(A) increasing on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

(B) decreasing on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

(C) decreasing on $\left(-\frac{\pi}{4}, 0\right)$ and increasing on $\left(0, \frac{\pi}{4}\right)$

(D) increasing on $\left(-\frac{\pi}{4}, 0\right)$ and decreasing on $\left(0, \frac{\pi}{4}\right)$

Answer (C)

Sol. $\because \ f(x) = \frac{1}{2\cos x - \frac{1}{\cos x}}$

$\Rightarrow f'(x) = \sin x\left(\frac{2 + \sec^2 x}{(2\cos x - \sec x)^2}\right)$

So $f(x)$ increases if $\sin x > 0 \Rightarrow x \in \left(0, \frac{\pi}{4}\right)$

and $f(x)$ decreases if $\sin x < 0 \Rightarrow x \in \left(-\frac{\pi}{4}, 0\right)$

12. The number of differentiable functions $y:(-\infty,\infty)\to[0,\infty)$ satisfying $y'=2\sqrt{y},\ y(0)=0$ is

(A) 1
(B) 2
(C) finite but more than 2
(D) infinite

Answer (D)

Sol. $y'=2\sqrt{y}$

$\Rightarrow \sqrt{y}=x+c$

when $y(0)=0 \quad \Rightarrow c=0$

$\sqrt{y}=x$

So, $f(x)=\begin{cases}0 & \text{if } x<a\\ (x-a)^2 & \text{if } x\ge a\end{cases}$

where, $a\in[0,\infty)$

$\therefore$ Thus infinitely many functions are possible.

13. The number of continuous functions $f:[0,1]\to(-\infty,\infty)$ satisfying the condition

$\int_0^1 (f(x))^2dx = 2\int_0^1 f(x)dx$

is

(A) 2
(B) 3
(C) 4
(D) more than 4

Answer (D)

Sol. $\int_0^1 (f(x)-1)^2dx=1$

Let $(f(x)-1)^2=g(x)$

So, $\int_0^1 g(x)dx=1$

There are infinitely many possible functions $g(x)$ and hence $f(x)$.

For example: $g(x)=2x$ or $3x^2$ or $4x^3$ and many more.

14. The value of the definite integral

$\int_0^{\pi/2}\frac{\sin x\cos x}{1+\cos^4 x}dx$

(A) $\frac{\pi}{8}$
(B) $\frac{\pi}{4}$
(C) 1
(D) 2

Answer (A)

Sol. $I=\int_0^{\pi/2}\frac{\sin x\cdot\cos x}{1+\cos^4 x}dx.$

Let $\cos^2x=t\Rightarrow -2\sin x\cdot\cos x\,dx=dt$

$\therefore \quad I=\int_1^0\frac{-\frac{1}{2}dt}{1+t^2}$

$=\frac{1}{2}\int_0^1\frac{dt}{1+t^2}$

$=\frac{1}{2}[\tan^{-1}t]_0^1 = \frac{\pi}{8}.$

15. Let $\vec{v}$ be a vector such that $\vec{v} \times \left((\hat{i}-\hat{k}) \times \left((3\hat{i}+4\hat{j}) \times (\hat{j}+\hat{k})\right)\right) = \vec{0}$. Suppose $\vec{v} \cdot \hat{j} = -7$. Then $\vec{v} \cdot \hat{i}$ is

(A) –3
(B) –2
(C) –1
(D) 0

Answer (A)

Sol. $\vec{v} \times \left((\hat{i}-\hat{k}) \times \left((3\hat{i}+4\hat{j}) \times (\hat{j} \times \hat{k})\right)\right) = \vec{0}$

$\Rightarrow \quad \vec{v} \times \left((\hat{i}-\hat{k}) \times (4\hat{i}-3\hat{j}+3\hat{k})\right) = \vec{0}$

$\Rightarrow \quad \vec{v} \times (-3\hat{i}-7\hat{j}-3\hat{k}) = \vec{0}$

$\therefore \quad \vec{v} = \lambda(3\hat{i}+7\hat{j}+3\hat{k})$ (some scalar λ)

Now $\vec{v} \cdot \hat{j} = -7 \Rightarrow \lambda = -1$

$\therefore \quad \vec{v} = -3\hat{i}-7\hat{j}-3\hat{k}$

$\therefore \quad \vec{v} \cdot \hat{i} = -3$.

16. In a multiple-choice test consisting of 8 questions, each question has four options. For each of the questions, exactly one of the four options is the right answer. A student answers all the question by choosing one option for each question. The number of ways in which the student can get exactly 5 correct answer is

(A) 56
(B) 168
(C) 504
(D) 1512

Answer (D)

Sol. Number of ways to select 5 questions out of 8 = ${}^8C_5$ = 56.

Number of ways to give three incorrect answers = 3^3 = 27.

$\therefore$ Total number of required ways = 56 × 27 = 1512.

17. The minimum value of the expression

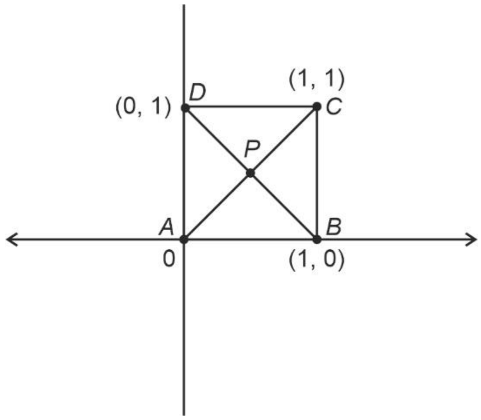
$|z| + |z - 1| + |z - 1 - i| + |z - i|$,

where z is a complex number and $i = \sqrt{-1}$, is

(A) $2+\sqrt{2}$
(B) $2\sqrt{2}$
(C) $\sqrt{2}$
(D) 2

Answer (B)

Sol. For the minimum value of $|z| + |z - 1| + |z - 1 - i| + |z - i|$



z lies on point P and it value = $PA + PB + PC + PD$

$= 2\sqrt{2}$

18. The number of real numbers x such that there exists an isosceles triangle having two of its angles measured in degrees equal to $2x + 7$ and $7x + 10$ is

(A) 0
(B) 1
(C) 2
(D) 3

Answer (D)

Sol. Two angles has measure as $2x + 7$ and $7x + 10$.

Then one of the possible triangle has angles as $2x + 7$, $2x + 7$ and $7x + 10$.

$\therefore \quad 4x + 14 + 7x + 10 = 180$

$$\therefore \quad x = \frac{156}{11} \qquad \ldots(1)$$

Other possible triangle has angles $2x + 7$, $7x + 10$ and $7x + 10$.

Then, $2x + 7 + 14x + 20 = 180$

$$x = \frac{153}{16} \qquad \ldots(2)$$

For third triangle $2x + 7 = 7x + 10$ then $x = -\frac{3}{5}$.

$\therefore$ Three real values of x are possible.

19. A disease affects two-thirds of the population of a country. A test for the disease gives the correct outcome with probability $\frac{2}{3}$. A person X tested positive for the disease. The probability that X has the disease is

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{4}{9}$
(D) $\frac{4}{5}$

Answer (D)

Sol. D = A person is affected by disease.

C = Test positive for the disease.

$$\therefore \quad P\left(\frac{D}{C}\right) = \frac{P(D)\cdot P\left(\frac{C}{D}\right)}{P(D)\cdot P\left(\frac{C}{D}\right) + P(\bar{D})\cdot P\left(\frac{C}{\bar{D}}\right)}$$

$$= \frac{\frac{2}{3}\cdot\frac{2}{3}}{\frac{2}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{1}{3}}$$

$$= \frac{4}{4+1} = \frac{4}{5}$$

20. The value of the integral

$$\int_0^\infty \frac{dx}{(1+x^2)(1+x)^2}$$

is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{3}{4}$ (D) ∞

Answer (B)

Sol. $I = \int_0^\infty \frac{dx}{(1+x^2)(1+x)^2}$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

$$= \int_0^{\pi/2} \frac{\sec^2\theta d\theta}{\sec^2\theta(1+\tan\theta)^2}$$

$$= \int_0^{\pi/2} \frac{d\theta}{(1+\tan\theta)^2}$$

$$= \int_0^{\pi/2} \frac{\cos^2\theta d\theta}{1+\sin 2\theta}$$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{2\cos^2\theta - 1 + 1}{1+\sin 2\theta} d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{\cos 2\theta}{1+\sin 2\theta} d\theta + \frac{1}{2}\int_0^{\pi/2} \frac{d\theta}{1+\sin 2\theta}$$

$$= \frac{1}{2}\cdot 0 + \frac{1}{2}\int_0^{\pi/2} \frac{\sec^2\theta d\theta}{(1+\tan\theta)^2}$$

Let, $1 + \tan\theta = t \Rightarrow \sec^2\theta d\theta = dt$

$$= \frac{1}{2}\int_1^\infty \frac{dt}{t^2}$$

$$= \frac{1}{2}\left(-\frac{1}{t}\right)_1^\infty$$

$$= \frac{1}{2}.$$

PART-I : PHYSICS

21. The cumulative number of ill patients $N(t)$ during an epidemic in a country is given by the following equation

$$N(t)=\frac{N_0\exp(t/\tau)}{1+N_0(\exp(t/\tau)-1)/N_2}$$

where N_0 is the initial population of ill patients, τ a positive constant and $N_2(>> N_0)$ is a large number. Then, which of the following statements is true?

(A) At large time $N(t)$ will approach zero.

(B) The population curve will have an inflection point when $N(t)$ is $N_s/2$.

(C) $N(t)$ will decrease monotonically.

(D) $N(t)$ will exhibit a maximum.

Answer (B)

Sol. $N(t)=\dfrac{N_0e^{t/\tau}}{1+N_0(e^{t/\tau}-1)/N_s}$

The graph to the above equation has its maximum at N_s.

At $N(t) = N_s$ graph is concave downwards and at $N(t) = 0$ the graph is concave upwards. By symmetry of graph the point of inflection occurs at $N(t)=\dfrac{N_s}{2}$

22. Kármán line is a theoretical construct that separates the earth's atmosphere from outer space. It is defined to be the height at which the lift on an aircraft flying at the speed of a polar satellite (8 km/s) is equal to its weight. Taking a fighter aircraft of wing area 30 m^2, and mass 7500 kg, the height of the Kármán line above the ground will be in the range (assume the density of air at height h above ground to be $\rho(h)$ = 1.2 $e^{-\frac{h}{10}}$ kg/m^3 where h is in km and the lift force to be $\frac{1}{2}\rho v^2 A$, where v is the speed of the aircraft and A its wing area).

(A) 25 – 50 km

(B) 75 – 100 km

(C) 125 – 150 km

(D) 175 – 200 km

Answer (B)

Sol. According to question

$\frac{1}{2}\rho v^2 A = mg$ (assuming 'g' to be constant for $h << R$)

$\frac{1}{2}(1.2\times e^{-h/10})\times 64\times 10^6\times 30 = 7500\times 9.8$

$\Rightarrow$ $e^{-h/10} = 63.8 \times 10^{-6}$

$\Rightarrow$ h = 96.6 km

which lies in the range of 75 – 100 km.

23. A particle of mass m with initial kinetic energy K approaches the origin from $x = +\infty$. Assume that a conservative force acts on it and its potential energy $V(x)$ is given by

$$V(x) = \frac{K}{\exp(3x/x_0) + \exp(-3x/x_0)}$$

where $x_0 = 1$ m. The speed of the particle at $x = 0$ is

(A) $\sqrt{K/m}$ (B) $\sqrt{2K/m}$

(C) $\sqrt{3K/m}$ (D) 0

Answer (A)

Sol. $V(\infty) = \dfrac{K}{e^{3\infty} + e^{-3\infty}} = 0$

$K(\infty) = K$

And $V(0) = \dfrac{K}{e^0 + e^0} = \dfrac{K}{2}$

From conservation of mechanical energy

$V(\infty) + K(\infty) = V(0) + K(0)$

$$\Rightarrow \quad 0 + K = \frac{K}{2} + \frac{1}{2}mv^2 \Rightarrow v = \sqrt{K/m}$$

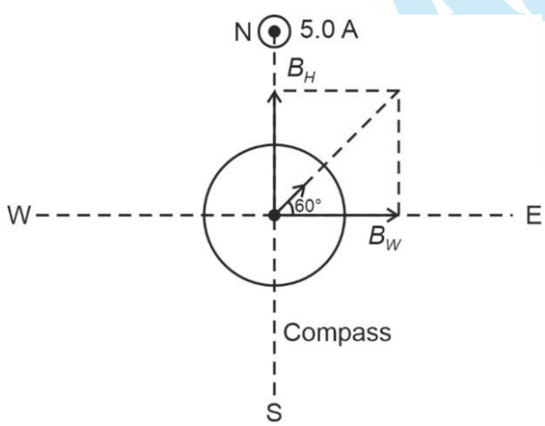
24. A long vertical wire carries a steady current of 5.0 A. A sensitive magnetic compass is placed in a plane perpendicular to the wire and 10.0 cm south of it. It registers a deflection 60° north of east. The magnitude of the horizontal component of the earth's magnetic field is (permeability of free space is $4\pi \times 10^{-7}$ N/A^2)

(A) 0.0 T (B) 0.6×10^{-5} T

(C) 1.0×10^{-5} T (D) 1.7×10^{-5} T

Answer (D)

Sol.



Due to wire:

$$B_W = \frac{\mu_0}{2\pi} \times \frac{5}{0.1} = 10^{-5}$$

From geometry; $\tan 60° = \dfrac{B_H}{B_W}$

$\Rightarrow \quad B_H = 1.7 \times 10^{-5}$ T

25. A small object is placed at a distance of 4 *m* from the objective of a telescope of focal length 2 *m*. The focal length of the eyepiece is 0.2 *m*. The final image of the object

(A) will be at infinity.

(B) will be real.

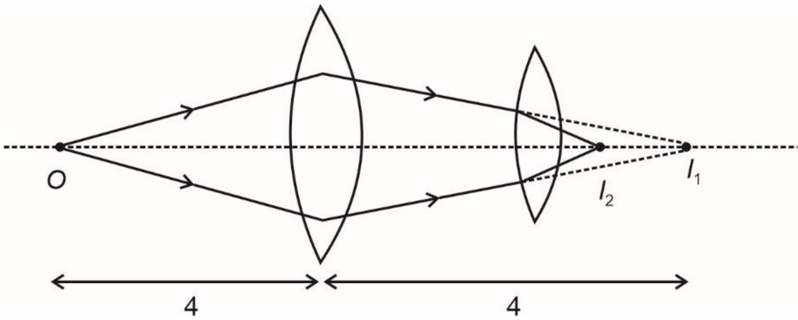
(C) will be at distance 0.18 *m* from the objective and between the objective and the eyepiece.

(D) will be at distance 4.4 *m* from the eyepiece and on the observer side.

Answer (B)

Sol. For normal vision, length of telescope = $f_0 + f_e = 2.2$ m

Since object is at $2f_0$ distance from objective, its image also forms at $2f_0$.



(I_1 is intermediate and I_2 is final image)

I_1 is 1.8 m from eyepiece

$\Rightarrow \quad \frac{1}{v} - \frac{1}{1.8} = \frac{1}{0.2}$

$\Rightarrow \quad v = +\,0.18$ m

26. New SI unit of mass 1 *kg* is defined in terms of the difference in the masses of two $^{133}Cs_{55}$ atoms. One of these atoms is in its ground state and the other in an excited state that has frequency of excitation close to 9.2×10^9 *Hz*. Number of atoms required to get 1 *kg* of mass this way is the order of (Planck's constant = 6.63×10^{-34} *Js*; mass of proton = 1.67×10^{-27} *kg*; Avogadro number = 6.02×10^{23} particles; speed of light = 3×10^8 *m/s*)

(A) 10^{24}

(B) 10^{21}

(C) 10^{40}

(D) 10^{15}

Answer (C)

Sol. $\Delta E = h\nu$

$= 6.636 \times 10^{-34} \times 9.2 \times 10^9$

$= 61 \times 10^{-25}$ J

$$m = \frac{nh\nu}{c^2} = 1\,\text{kg}$$

$$n = \frac{9 \times 10^{16}}{61 \times 10^{-25}}$$

$$\approx 1.48 \times 10^{40}$$

27. A simple pendulum consisting of a light inextensible string of length *l* attached to a heavy small bob of mass *m* is at rest. The bob is imparted a horizontal impulsive force which gives it a speed of $\sqrt{4gl}$. The speed of the bob at its highest point is (*g* is the acceleration due to gravity)

(A) 0

(B) $\sqrt{\frac{1}{3}gl}$

(C) $\sqrt{\frac{2}{3}gl}$

(D) $\sqrt{\frac{8}{27}gl}$

Answer (D)

Sol. As $v < \sqrt{5gL}$, string becomes slack at point *B*

(T = 0) at point B

Applying equation of force $mg\cos\theta = \frac{mv^2}{L}$

$\Rightarrow \quad v = \sqrt{gL\cos\theta}$

It will show projectile motion at point *B*.

Velocity at highest point = $v\cos\theta$

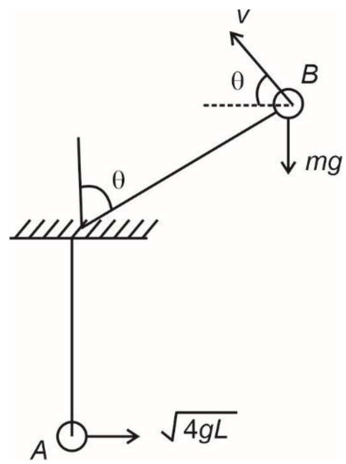
Applying conservation of ME at point *A* and *B*

$$\frac{1}{2}m(4gL) = mgL(1+\cos\theta) + \frac{1}{2}mv^2$$

$$2mgL = \frac{mgL\cos\theta}{2} + mgL(1+\cos\theta)$$

$$\cos\theta = \frac{2}{3}$$

$$v_H = v\cos\theta = \sqrt{gL\cos\theta}\cos\theta = \sqrt{\frac{8gL}{27}}$$



28. An ideal gas, initially in state (P_{12}, V_1, T_1) is expanded isobarically to (P_{12}, V_2, T_2), then adiabatically to (P_{34}, V_3, T_3). It is then contracted isobarically to (P_{34}, V_4, T_4) and finally adiabatically back to the initial state. The efficiency of this cycle is

(A) $1-\frac{T_4}{T_1}$

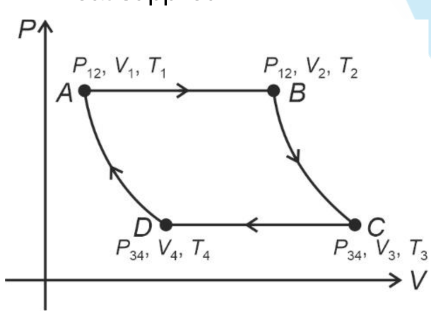
(B) $1-\frac{T_4}{T_2}$

(C) $1-\frac{T_3}{T_1}$

(D) $1-\frac{P_{34}}{P_{12}}$

Answer (A)

Sol. $\eta = \frac{W_{net}}{\text{Heat supplied}}$ *BC* & *CD*–adiabatic processes hence no heat interaction.



$$\eta = \frac{Q_S - Q_R}{Q_S} = \frac{nC_P(T_2-T_1) - nC_P(T_3-T_4)}{nC_P(T_2-T_1)}$$

$$\eta = 1 - \frac{(T_3-T_4)}{(T_2-T_1)}$$

$$\eta = 1 - \frac{T_4}{T_1}\frac{\left[\frac{T_3}{T_4}-1\right]}{\left[\frac{T_2}{T_1}-1\right]}$$

For adiabatic process, $\frac{T_2}{T_3}=\left(\frac{P_{12}}{P_{34}}\right)^{\frac{\gamma-1}{\gamma}}$

$$\Rightarrow \quad \frac{T_1}{T_4}=\left(\frac{P_{12}}{P_{34}}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_2}{T_3}=\frac{T_1}{T_2} \Rightarrow \frac{T_2}{T_1}=\frac{T_3}{T_4}$$

$$\Rightarrow \quad \frac{T_2}{T_1}-1=\frac{T_3}{T_4}-1 \Rightarrow \eta=1-\frac{T_4}{T_1}$$

29. A dipole consisting of two charges $\pm q$ separated by a distance $2a$ is placed with its centre D away from the center of a grounded sphere of radius $R(D >> a)$. When the dipole moment vector is perpendicular to the line joining the two centers (those of the dipole and the sphere) the charge induced on the sphere is

(A) $\frac{2aR}{D^2}q$

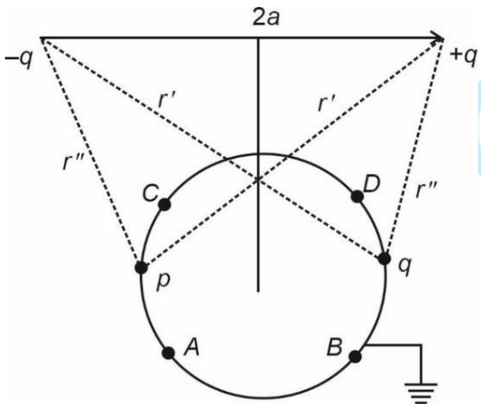
(B) $\frac{aR}{D^2}q$

(C) $\frac{aR}{2D^2}q$

(D) 0

Answer (D)

Sol.



Let induced charge on sphere be Q

So $\frac{KQ}{R}+\left[(V_C+V_D)+(V_p+V_q)+(V_A+V_B).....\right]=0$

$$\frac{KQ}{R}+\left[\frac{kq}{r'}+\left(\frac{-kq}{r''}\right)+\left(\frac{-kq}{r'}\right)+\frac{kq}{r''}....\right]=0$$

$\Rightarrow \quad Q = 0$

$\Rightarrow$ Induced charge = 0

30. Consider diffraction of light through a rectangular slit which is twice as wide as it is high. Which of the following is statements is true?

(A) A central diffraction peak is wider in the vertical direction than the horizontal direction.

(B) The central diffraction peak is wider in the horizontal direction than the vertical direction.

(C) The central diffraction peak is equally wide in both horizontal and vertical directions.

(D) Width of the central diffraction peak is independent of the wavelength of light used.

Answer (A)

Sol. Angular width of central maxima $=\frac{2\lambda}{a}$.

Angular width is inversely proportional to a. Since it is wider with respect to its height.

Therefore, central diffraction peak is wider in vertical direction than the horizontal direction.

31. The percentage of ^{235}U presently on earth is 0.72 and the rest (99.28%) may be taken to be ^{238}U. Assume that all uranium on earth was produced in a supernova explosion long ago with the initial ratio $^{235}U/^{238}U = 2.0$. How long ago did the supernova event occur? (Take the half-lives of ^{235}U and ^{238}U to be 7.1×10^8 *years* and 4.5×10^9 *years* respectively)

(A) 4×10^9 *years*.
(B) 5×10^9 *years*.
(C) 6×10^9 *years*.
(D) 7×10^9 *years*.

Answer (D)

Sol. At $t = 0$

$$\frac{u_{235}}{u_{238}} = 2$$

$$\frac{N_1}{N_2} = \frac{2N_0 e^{-\lambda t}}{N_0 e^{-\lambda_2 t}} = \frac{0.72}{99.28}$$

$$\frac{e^{\lambda_2 t}}{e^{\lambda_1 t}} = 0.003626$$

$$e^{(\lambda_2 - \lambda_1)t} = 0.003626$$

$$(\lambda_2 - \lambda_1)t = \ln|0.003626| \qquad \left[\because\ \lambda = \frac{0.693}{T_{1/2}}\right]$$

$$10^{-9} \times t(0.693)\left(\frac{1}{4.5} - \frac{1}{0.71}\right) = -5.612$$

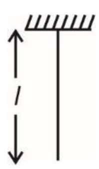
$$t = 6.9 \times 10^9 \approx 7 \times 10^9 \text{ years}$$

32. A long stiff uniform wire is suspended from one end. The time period of oscillations of this wire is *T.* If the wire is now bent into a circle and suspended from a knife edge so that it can oscillate freely in the plane of the ring, its time period will be

(A) T
(B) $\sqrt{\frac{1}{2\pi}}T$
(C) $\sqrt{\frac{1}{\pi}}T$
(D) $\sqrt{\frac{3}{2\pi}}T$

Answer (D)

Sol. Case I:

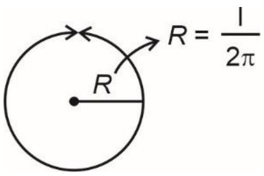


we know, $T = 2\pi\sqrt{\frac{I}{mgl}}$

$$T = 2\pi\sqrt{\frac{ml^2}{3mg\left(\frac{l}{2}\right)}}$$

$$= 2\pi\sqrt{\frac{2l}{3g}}$$

Case II:



$$T' = 2\pi\sqrt{\frac{mR^2 + mR^2}{mg\,R}}$$

$$T' = 2\pi\sqrt{\frac{2R}{g}}$$

$$T' = 2\pi\sqrt{\frac{2\,l}{2\pi g}} \qquad (\because\ 2\pi R = l)$$

Now, $\frac{T'}{T} = \frac{2\pi\sqrt{\frac{2l}{2\pi g}}}{2\pi\sqrt{\frac{2l}{3g}}}$

$\Rightarrow \quad T' = T\sqrt{\frac{3}{2\pi}}$

33. If an ideal gas is compressed isothermally, which of the following statements is true?

(A) Energy is transferred into the gas by heat.

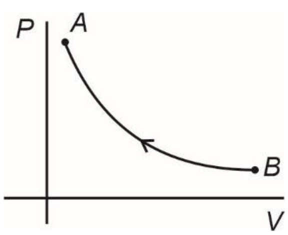
(B) Work is done by the gas.

(C) Pressure of the gas decreases.

(D) The internal energy of the gas remains constant.

Answer (D)

Sol. We know,



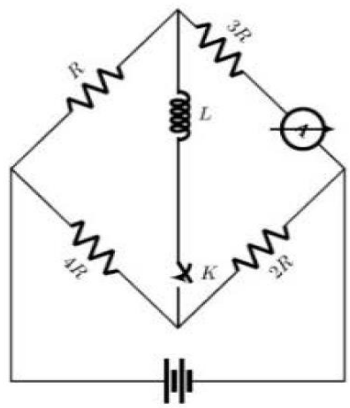
In the above process 'T' remains constant

Thus $dU = \frac{nfRdT}{2} = 0$

Hence, Internal energy of gas remains constant as dQ = dU + dW, and in isothermal compression, $w < 0$

$\therefore$ $Q < 0$, thus heat is transferred out of gas work is done on the gas and as $P \propto \frac{1}{V}$, pressure increases.

34. In the circuit shown below, a long time after the key K is closed, the reading in the ammeter is 20 mA.



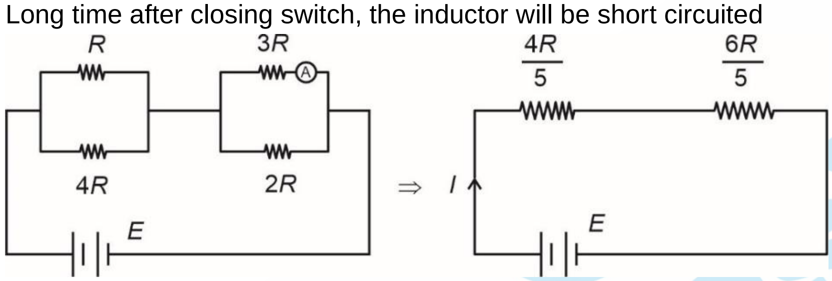
What was the reading immediately after it was closed?

(A) 0 mA (B) 16 mA

(C) 25 mA (D) 32 mA

Answer (C)

Sol. Long time after closing switch, the inductor will be short circuited



$$R_{net} = \frac{4R}{5} + \frac{6R}{5} = 2R$$

$$I = \frac{E}{2R}$$

$$\Rightarrow \quad I_{3R} = I \times \frac{2R}{2R + 3R} = \frac{2I}{5}$$

$$\therefore \quad 20 \times 10^{-3} = \frac{2I}{5}$$

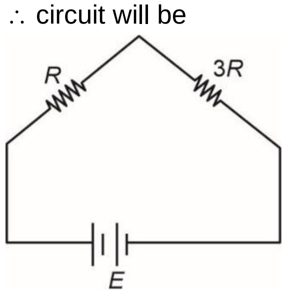
$$\Rightarrow \quad I = 50\ mA$$

$$\Rightarrow \quad \frac{E}{2R} = 50$$

$$\Rightarrow \quad \frac{E}{R} = 100\ mA$$

Immediately after circuit was closed the inductor behaves as open circuit

$\therefore$ circuit will be



$$I = \frac{E}{4R} = \frac{100}{4} mA = 25\ mA$$

35. To accommodate the view that matter is made up of 5 elements only, a scientist proposed the following hypothesis; that atoms can have a maximum principal quantum number n_{max} and no higher. Then, which of the following statements must be true?

(A) n_{max} = 1, and electrons have spin

(B) n_{max} = 2, and electrons are spinless but nevertheless obey the Pauli Exclusion principle

(C) n_{max} = 3, and electrons are spinless but nevertheless obey the Pauli Exclusion principle

(D) n_{max} = 4, and electrons have spin

Answer (B)

Sol. Here n = 2 (maximum)

	For n = 1	For n = 2
(n) Principle quantum number →	1	2
(l) Azimuthal quantum number →	0	0 and 1
(m) Magnetic quantum number →	0	0 ↓ and −1, 0, 1
	Ist Element	IInd Element; III, IV and V Element

Note: If the particle is spinless it certainly obeys Pauli's Exclusion principle

- First shell accommodates → 1 electron : $1s^1$
- Second shell accommodates → 4 electrons : $2s^1\ 2p^3$

Elements	Configuration
Ist	$1s^1$
IInd	$2s^1$
IIIrd	$2s^12p^1$
IVth	$2s^12p^2$
Vth	$2s^12p^3$

36. The speed of a satellite in a circular orbit of radius R_0 around the earth is v_0. Another satellite is in the elliptic orbit around the earth. If the minimum and maximum speeds of the second satellite are αv_0 and βv_0 respectively, then its time period is:

(A) $\dfrac{2\pi R_0}{v_0}\left(\dfrac{\alpha^2+\beta^2}{2}\right)^{\frac{3}{2}}$

(B) $\dfrac{2\pi R_0}{v_0}\left(\dfrac{\alpha+\beta}{2}\right)^{\frac{3}{2}}$

(C) $\dfrac{2\pi R_0}{v_0}(\alpha\beta)^{\frac{3}{2}}$

(D) $\dfrac{2\pi R_0}{v_0}(\alpha\beta)^{-\frac{3}{2}}$

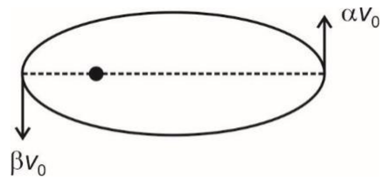
Answer (D)

Sol. From angular momentum conservation

$m\beta v_0 a(1-e) = m\alpha v_0 a(1+e)$

$\beta(1-e) = \alpha(1+e)$

$$e = \frac{\beta-\alpha}{\beta+\alpha} \qquad \ldots(1)$$



$$1-e = 1-\frac{\beta-\alpha}{\beta+\alpha} = \frac{2\alpha}{\beta+\alpha} \qquad \ldots(2)$$

From energy conservation law

$$\frac{1}{2}m\beta^2 v_0^2 - \frac{GMm}{a(1-e)} = \frac{1}{2}m\alpha^2 v_0^2 - \frac{GMm}{a(1+e)}$$

$$\frac{1}{2}\left(\beta^2-\alpha^2\right)v_0^2 = \frac{GM}{a}\left(\frac{1}{1-e}-\frac{1}{1+e}\right)$$

$$(\beta^2-\alpha^2)v_0^2 = \frac{2GM}{a}\frac{2e}{(1-e)\frac{\beta}{\alpha}(1-e)}$$

$$(\beta^2-\alpha^2)v_0^2 = \frac{4GM}{a}\frac{\alpha}{\beta}\frac{e}{(1-e)^2}$$

$$(\beta^2-\alpha^2)v_0^2 = \frac{4GM}{a}\left(\frac{\alpha}{\beta}\right)\frac{\frac{(\beta-\alpha)}{\beta+\alpha}}{\frac{4\alpha^2}{(\beta+\alpha)^2}}$$

$$(\beta^2-\alpha^2)v_0^2 = \frac{4GM}{a}\frac{\alpha}{\beta}\frac{(\beta^2-\alpha^2)}{4\alpha^2}$$

$$v_0^2 = \frac{4Gm}{a}\frac{1}{4\alpha\beta}$$

$$v_0^2 = \frac{Gm}{a(\alpha\beta)}$$

$$a = \frac{GM}{v_0^2\alpha\beta}$$

$$T = \frac{2\pi}{\sqrt{GM}}a^{3/2} = \frac{2\pi}{\sqrt{GM}}\left(\frac{GM}{v_0^2\alpha\beta}\right)^{3/2}$$

$$T = 2\pi\frac{GM}{v_0^3}(\alpha\beta)^{-3/2}$$

$$T = \frac{2\pi R_0}{v_0}(\alpha\beta)^{-3/2} \qquad \left[\because\ v_0^2 = \frac{GM}{R_0}\right]$$

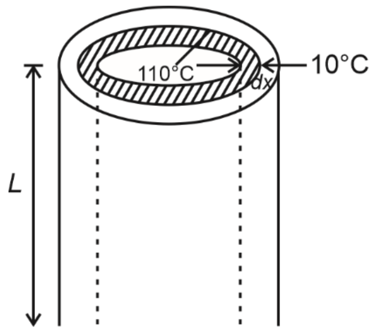
37. A copper pipe of length 10 m carries steam at temperature 110°C. The outer surface of the pipe is maintained at a temperature 10°C. The inner and outer radii of the pipe are 2 cm and 4 cm, respectively. The thermal conductivity of copper is 0.38 kW/m/°C. In the steady state, the rate at which heat flows radially outward through the pipe is closet to

(A) 3245 kW (B) 3445 kW

(C) 3645 kW (D) 3845 kW

Answer (B)

Sol. Resistance of elementary cylindrical shell



$$dR = \frac{dx}{K\,2\pi xL}$$

Total resistance,

$$R = \frac{1}{2\pi KL}\int_2^4 \frac{dx}{x}$$

$$= \frac{1}{2\pi KL}\ln 2$$

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta T}{R} = \frac{(T_1 - T_2)2\pi KL}{\ln 2}$$

$$= \frac{(110-10)(2\pi \times 380 \times 10)}{\ln 2}$$

$\simeq 3445 \times 10^3$ W

38. In the scenarios given below, a person is standing on a wooden plank. In which of the following options do they draw the most current, when simultaneously touching.

(A) the live and neutral terminals of household electric socket at 220 V.

(B) a van de Graaff generator in a science museum charged to 12000 V.

(C) the two terminals of a car battery at 12 V.

(D) the two end terminals of 10 batteries in series each 1.5 V.

Answer (A)

Sol. For current to be drawn, circuit should be completed and voltage difference should be maximum.

Here for option A, Potential difference is maximum and circuit is completed.

39. In saloons, there is always a characteristic smell due to the ammonia-based chemicals used in hair dyes and other products. Assume the initial concentration of ammonia molecules to be 1000 molecules/m^3. Due to air ventilation, the number of molecules leaving in one minute is one tenth of the molecules present at the start of that minute. How long will it take for the concentration of ammonia molecules to reach 1 molecule/m^3?

(A) 7 minutes

(B) 70 minutes

(C) 100 minutes

(D) Very long time which cannot be calculated

Answer (B)

Sol. $N = N_0 e^{-\alpha t}$

$900 = 1000\, e^{-\alpha \times 1}$

$\ln\left(\frac{9}{10}\right) = -\alpha$

$\frac{1}{1000} = e^{-\alpha t}$

$\ln 10^3 = +\, \alpha t$

$3\ln 10 = \ln\left(\frac{10}{9}\right)t$

$t = \frac{3\ \ln 10}{\ln\left(\frac{10}{9}\right)} = 65.56$

$t \approx 70$ minutes

40. Consider a metallic cube of edge length L. Its resistance. R, measured across its opposite faces is

$$R = \frac{m_e v}{ne^2 L^{2'}}$$

Where n is the number density and v is the drift speed of electrons in the cube, and e and m_e are the charge and mass of an electron respectively. Assuming the de Broglie wavelength of the electron to be L, the maximum resistance of the sample is closest to ($e = 1.60 \times 10^{-19}$ C; $m_e = 9.11 \times 10^{-31}$ kg; Planck's constant, $h = 6.63 \times 10^{-34}$ Js)

(A) $10^2\,\Omega$

(B) $10^4\,\Omega$

(C) $10^6\,\Omega$

(D) $10^8\,\Omega$

Answer (B)

Sol. $R = \frac{m_e v}{ne^2 L^2}$

No. of electrons $N = nL^3$

$\Rightarrow \quad nL^2 = \frac{N}{L}$

or $R = \frac{m_e vL}{e^2 N}$

also, $L = \frac{h}{m_e v} \Rightarrow m_e vL = h$

or $R = \frac{h}{Ne^2}$

$R_{max} = \frac{h}{(1)e^2} = 2.6 \times 10^4\ \Omega$

PART-I : CHEMISTRY

41. When copper is added to conc. H_2SO_4, $CuSO_4$ is produced along with another sulphur-containing compound X. The compound X is

(A) H_2S
(B) SO_2
(C) SO_3
(D) $H_2S_2O_3$

Answer (B)

Sol. • $Cu(s) + H_2SO_4\ (conc.) \rightarrow CuSO_4(aq) + SO_2(g) + 2H_2O$

• Sulphur containing compound $SO_2(g)$ is obtained

So, X is $SO_2(g)$

42. A metal ion M^{n+} having d^6 valence electronic configuration combines with three bidentate ligands to form a complex. Assuming crystal field splitting (Δ_0) > pairing energy, the d-orbital electronic configuration would be

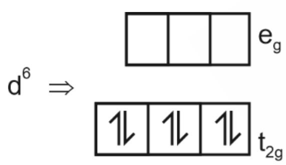
(A) $t_{2g}^{6}e_g^0$
(B) $t_{2g}^{4}e_g^2$
(C) $t_{2g}^{3}e_g^3$
(D) $t_{2g}^{5}e_g^1$

Answer (A)

Sol. • M^{+n} with three bidentate ligands form octahedral complex $[M(AA)_3]^{+n}$. (AA) is a bidentate ligands

• Δ_0 > pairing energy means low spin complex will formed

• d-orbital electronic configuration is $t_{2g}^{6}e_g^0$



43. Zeolite is hydrated sodium aluminium silicate. When treated with hard water, the sodium ions in zeolite are exchanged with

(A) Zn^{2+}
(B) Mg^{2+}
(C) Ni^{2+}
(D) Cu^{2+}

Answer (B)

Sol. • Sodium aluminium silicate ($NaAlSiO_4$) can be written as NaZ.

• When this is added in hard water, exchange reactions take place

$2NaZ(s) + M^{+2}(aq) \rightarrow MZ_2(s) + 2Na^{+}(aq)$

$[Z = AlSiO_4^-]$

$(M^{+2} = Mg^{+2}, Ca^{+2})$

44. Among the molecules

O_2, KO_2, H_2O_2, F_2O_2 and BaO_2

the pair that have the most similar oxygen-oxygen bond length is

(A) O_2 and H_2O_2
(B) KO_2 and H_2O_2
(C) O_2 and BaO_2
(D) KO_2 and F_2O_2

Answer (D)

Sol.

Molecule/Ion	Bond Length (Å) of $-O-O-$
O_2	≈ 1.2
$O_2^-(KO_2)$	≈ 1.3
H_2O_2	≈ 1.5
F_2O_2	≈ 1.2
$O_2^{-2}(BaO_2)$	≈ 1.5

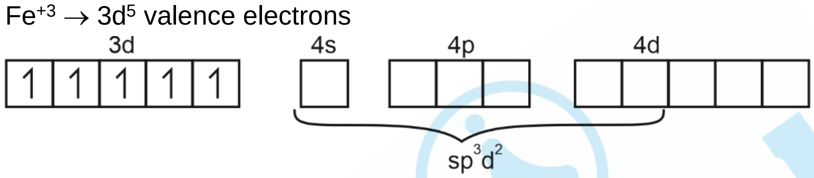
KO_2 and F_2O_2 having oxygen-oxygen bond length almost similar.

45. Hybridizations of the central atoms in $[Fe(H_2O)_6]^{3+}$ and $[Co(NH_3)_6]^{3+}$, respectively, are

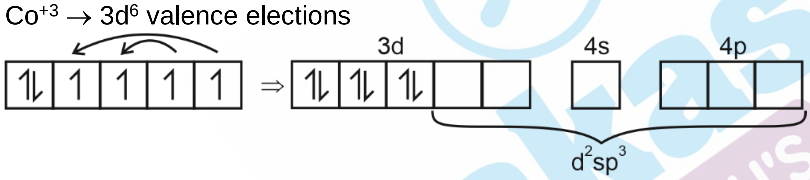
(A) sp^3d^2 and sp^3d^2
(B) sp^3d^2 and d^2sp^3
(C) d^2sp^3 and d^2sp^3
(D) d^2sp^3 and sp^3d^2

Answer (B)

Sol. • In $[Fe(H_2O)_6]^{3+}$, H_2O act as weak field ligand
So, pairing of d-orbitals electrons do not take place
$Fe^{+3} \rightarrow 3d^5$ valence electrons



• In $[Co(NH_3)_6]^{3+}$, NH^3 act as a strong field ligand, So, pairing of d-orbital electrons is possible
$Co^{+3} \rightarrow 3d^6$ valence elections



$[Fe(H_2O)_6]^{3+} \rightarrow sp^3d^2$
$[Co(NH_3)_6]^{3+} \rightarrow d^2sp^3$

46. The energy of combustion per mole of H_2, LPG and octane follows the order

(A) octane > LPG > H_2
(B) H_2 > LPG > octane
(C) LPG > octane > H_2
(D) H_2 > octane > LPG

Answer (A)

Sol.

Fuels	$H_2(g)$	LPG(g)	Octane(l)
Energy released on combustion in KJ per mole	286	2220	5511

Correct order of energy released is

Octane > LPG > H_2

47. The **INCORRECT** statement about the conductivity of electrolytic (ionic) solutions is

(A) It is independent of the solvent viscosity
(B) It depends on the size of the ions and their solvation
(C) It increases with the increase in electrolyte concentration
(D) It increases with the increase in temperature

Answer (A)

Sol. Conductivity of electrolytic solution increases on

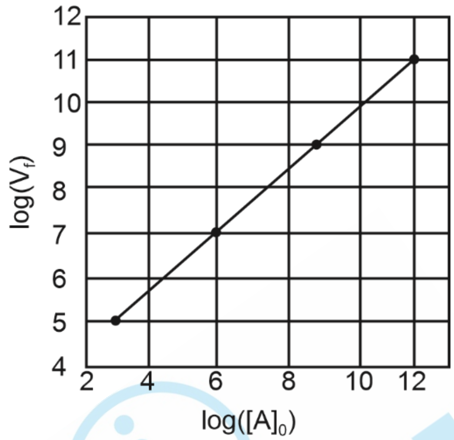
(i) decreasing viscosity of solvent

(ii) decreasing size of ions

(iii) increasing number of ions

(iv) increasing temperature

48. For the reaction A + B → C, experiments were performed in the presence of a large amount of B to measure the initial reaction rate (V_f) as a function of the initial concentration of A($[A]_0$). The data from the experiments are plotted as shown below. The order of the reaction with respect to A is



(A) 1 (B) 3

(C) $\frac{2}{3}$ (D) $\frac{3}{2}$

Answer (C)

Sol. rate = K $[A]^{\alpha}$ $[B]^{\beta}$

Since large amount of B is present,

$(V_f) = K' [A]^{\alpha}$ $\alpha \rightarrow$ order of the reaction

taking log both sides

$\log V_f = \log K' + \alpha \log [A]$

Slope = $\alpha = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-7}{9-6} = \frac{2}{3}$

$\alpha = \frac{2}{3}$

49. Among the following, the **INCORRECT** statement regarding the collision theory of chemical reactions is

(A) The reactant molecules are assumed to be hard spheres

(B) Collision frequency between reactants is one of the factors that determines the rate of the reaction

(C) The steric factor takes into account the relative orientation of the reactant molecules during collisions

(D) The theory takes into account the structural aspects of the molecules during collision

Answer (D)

Sol.
- In collision theory, reactant molecules are assumed to be hard spheres.
- Rate = P Z_{AB} $e^{-Ea/RT}$

  P → Probability factor/steric factor

  Z_{AB} → Collision frequency of reactants
- It does not account the structural aspects of molecule.

50. Among the following, the crystal system which includes end-centered is

(A) $a = b = c$ and $\alpha = \beta = \gamma = 90°$
(B) $a = b \neq c$ and $\alpha = \beta = \gamma = 90°$
(C) $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90°$
(D) $a = b = c$ and $\alpha = \beta = \gamma \neq 90°$

Answer (C)

Sol. $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90°$ represents orthorhombic crystal system in which primitive, body-centred, face-centred and end-centred variations are possible.

51. In crystalline silicon, Si atoms occupy all the ccp sites and every alternate tetrahedral voids. The value of packing efficiency is closest to

(A) 40%
(B) 28%
(C) 54%
(D) 34%

Answer (D)

Sol. • $\because$ Distance from the corner of the unit cell and tetrahedral void is one fourth of body diagonal of the unit cell.

$$\therefore \quad 2r = \frac{\sqrt{3}a}{4}$$

$$a = \frac{8r}{\sqrt{3}}$$

$$\text{Packing efficiency} = \frac{\text{Number of atoms} \times \text{Volume of atom} \times 100}{\text{Volume of unit cell}}$$

$$= \frac{8 \times \frac{4}{3}\pi r^3 \times 100}{a^3} = \frac{8 \times \frac{4}{3}\pi r^3 \times 100}{\left(\frac{8r}{\sqrt{3}}\right)^3} = 34\%$$

52. Among the following methods

(i) Addition of an electrolyte
(ii) Electrophoresis
(iii) Addition of a protective colloid
(iv) Addition of an oppositely charged sol

the coagulation of a lyophobic sol can be carried out by

(A) (i) and (iv) only
(B) (ii), (iii) and (iv) only
(C) (iii) and (iv) only
(D) (i), (ii) and (iv) only

Answer (D)

Sol. • In electrophoresis, colloidal particles move towards oppositely charged electrodes, get discharged and precipitated

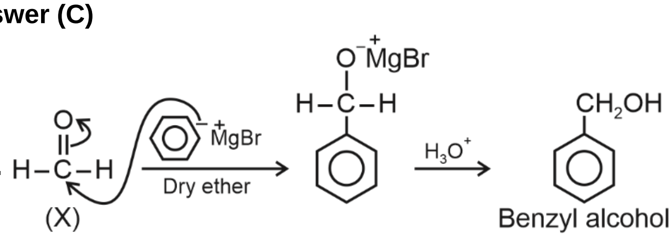
• Addition of an oppositely charged sol neutralises the charged of colloidal particles and partially or completely precipitated

• Due to addition of an electrolyte the colloidal particle interact with ions carrying charge opposite to that present on themselves, this causes neutralisation and ultimately coagulation.

53. Phenylmagnesium bromide, upon reaction with a compound **X** followed by treatment with acid gives benzyl alcohol. The compound **X** is

(A) Carbon dioxide
(B) Ethylene
(C) Formaldehyde
(D) Methanol

Answer (C)

Sol. 

54. DNA fingerprinting involves

(A) Carrying out DNA analysis from different parts of a fingerprint

(B) Identifying sequence of DNA which is unique to an individual

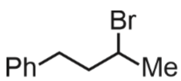
(C) Carrying out RT-PCR from a sample obtained from a fingerprint

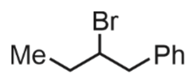
(D) Finding out the ratio of purine and pyrimidine bases from the DNA

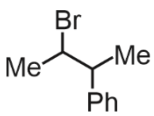
Answer (B)

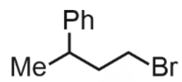
Sol. DNA fingerprinting involves identifying sequence of DNA which is unique to an individual and cannot be altered by any known treatment.

55. Among the following, the compound which undergoes the fastest solvolysis is

(A) 

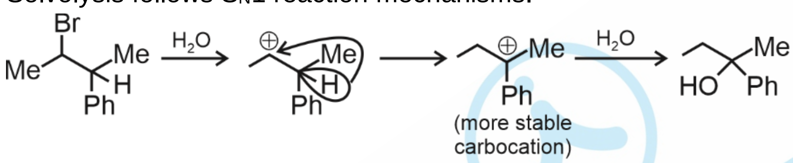
(B) 

(C) 

(D) 

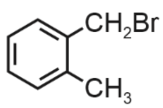
Answer (C)

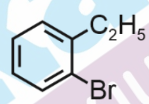
Sol. Solvolysis follows S_N1 reaction mechanisms.

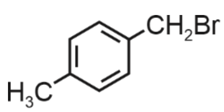


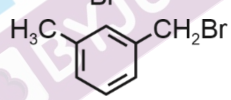
More stable is the carbocation formed faster will be solvolysis.

56. A compound **X** on heating with alcoholic $AgNO_3$ gives a white precipitate. Oxidation of **X** gives an acid with the formula $C_8H_6O_4$, which easily forms a cyclic anhydride on heating. The compound **X** is

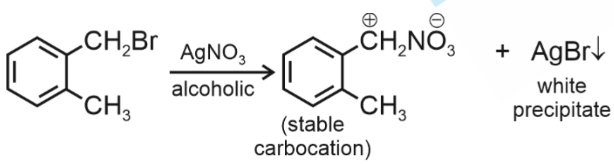
(A) 

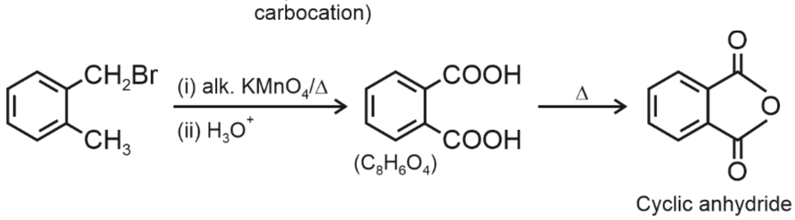
(B) 

(C) 

(D) 

Answer (A)

Sol. 



57. Among the following, the method which can be used for distinguishing aniline from ethylamine is

(A) Treatment with $CHCl_3$ and KOH

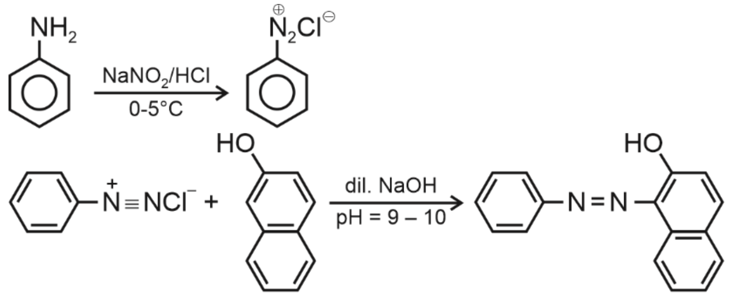
(B) Reaction with $NaNO_2$/HCl followed by treatment of 2-naphthol

(C) Reaction with benzene sulfonyl chloride

(D) Reaction with benzaldehyde

Answer (B)

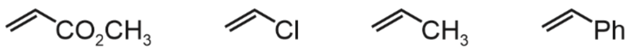
Sol. Aniline forms diazonium salt with $NaNO_2/HCl$. Which forms red dye with 2-naphthol



Red dye

No characteristic change is observed on reaction of ethylamine with $NaNO_2/HCl$ followed by treatment with 2-naphthol.

58. The expected reactivity of the monomers shown below



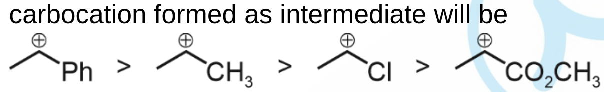
I **II** **III** **IV**

towards cationic polymerization follows the order

(A) **I** < **II** < **III** < **IV**
(B) **IV** < **III** < **II** < **I**
(C) **III** < **IV** < **II** < **I**
(D) **IV** < **III** < **I** < **II**

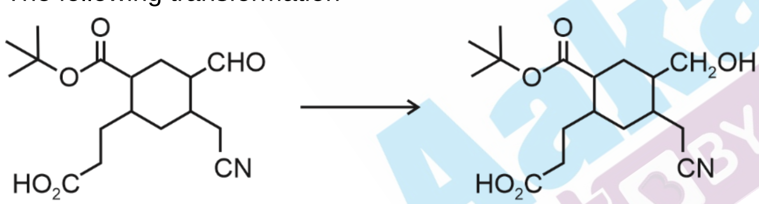
Answer (A)

Sol. More stable is the carbocation formed faster will be the cationic polymerisation. The correct order of stability of carbocation formed as intermediate will be



The expected reactivity of the given compounds towards cationic polymerisation will be IV > III > II > I.

59. The following transformation



can be best carried out using

(A) $LiAlH_4$ in THF
(B) BH_3 in THF
(C) $NaBH_4$ in EtOH
(D) DIBAL-H in hexane

Answer (C)

Sol. $NaBH_4$ in ethanol reduces aldehyde to primary alcohol. It does not reduce ester, cyanide and carboxylic acid groups.

60. The radius (in Å) of the 3rd Bohr orbit in Li^{2+} ion is closest to

[Given: Atomic number of Li = 3)

(A) 0.520
(B) 1.018
(C) 1.587
(D) 1.881

Answer (C)

Sol. $r_n = 0.529\dfrac{n^2}{Z}$ Å

$r_3 = 0.529 \times \dfrac{(3)^2}{3}$

= 1.587 Å

PART-I : BIOLOGY

61. Which ONE of the following statements about viruses is **INCORRECT**?
    (A) They contain DNA or RNA as genetic material
    (B) Their coat can contain proteins and lipids
    (C) They contain cytoplasm and nucleus
    (D) They can infect plants and animals

Answer (C)

Sol. Viruses are inert outside the living cell. They lack cytoplasm and nucleus.

62. Garreau's potometer is used to measure unequal transpiration from two surfaces of a leaf, where calcium chloride is used as a sensor. Which ONE of the following compounds can replace calcium chloride in this experiment?
    (A) Calcium acetate
    (B) Sodium chloride
    (C) Potassium chloride
    (D) Magnesium chloride

Answer (D)

Sol. In Garreau's potometer, the function of $CaCl_2$ is to absorb water. So to replace $CaCl_2$ we need an anhydrous compound. Since salts of alkaline earth metals are more anhydrous, so $MgCl_2$ can be used in place of $CaCl_2$.

63. Which of these is an example of Mullerian mimicry?
    (A) A stick insect that resembles a twig in a grassy ecosystem
    (B) Two venomous snake species that closely resemble each other
    (C) A non-poisonous species of butterfly that resembles a poisonous one
    (D) A carnivorous plant that has bright, fragrant flowers to lure insects.

Answer (B)

Sol. The correct answer is option (B) as Mullerian mimicry is when two or more inedible or unpalatable species resemble each other.

Two venomous snake species are unpalatable and they closely resemble each other would be an example of Mullerian mimicry.

Batesian mimicry is when one edible species resemble an inedible one.

64. In rocky coastal pools, a barnacle species *Balanus* occupies only the lower part of the pool and another barnacle species *Chthamalus* only occupies the upper part of the pool. Upon removing *Balanus* entirely, *Chthamalus* also occupies the lower part of the pool. The naturally observed segregation of *Balanus* and *Chthamalus* in their use of pool depths is **BEST** explained by
    (A) primary succession
    (B) predation of *Balanus* by *Chthamalus*
    (C) cooperative displacement
    (D) competitive exclusion

Answer (D)

Sol. Upon removing *Balanus* entirely, *Chathamalus* also occupies the lower part of the pool along with the upper part.
Segregation of *Balanus* and *Chathamalus* in their use of pool depths explains competitive exclusion.

65. Which ONE of the following statements is **CORRECT** regarding nitrogen fixation in plants?
    (A) Most plants directly fix atmospheric N_2
    (B) It is performed by symbiotic protozoa
    (C) It is performed by symbiotic prokaryotes
    (D) Symbiosis does not play a role in nitrogen fixation in most plants

Answer (C)

Sol. Nitrogen fixation is performed by prokaryotes only as they possess nitrogenase enzyme. They can fix N_2 symbiotically with plants.

66. The probability of extinction of a species is maximum under which ONE of the following conditions?

(A) High carrying capacity
(B) Diverse genetic pool
(C) High frequency of heterozygosity
(D) High frequency of homozygosity

Answer (D)

Sol. Probability of extinction of a species is maximum when there is high frequency of homozygosity. High carrying capacity, diverse genetic pool, and high frequency of heterozygosity increase species diversity.

67. Which ONE of the following is **NOT** driven by enzyme-catalyzed reactions?

(A) Digestion of proteins by the digestive system
(B) Exchange of gases from the blood the tissues in the lungs
(C) Breakdown of glucose through glycolysis
(D) Maintenance of pH of the blood

Answer (B)

Sol. The correct answer is option (B) as the exchange of gases from the blood to the tissues occur due to diffusion.

Option (A) is incorrect as digestion of proteins by the digestive system is mediated by enzymes like trypsin, chymotrypsin, dipeptidases etc.

Option (C) is incorrect as breakdown of glucose through glycolysis is a ten-step process which needs ten different enzymes.

Option (D) is incorrect as the H^+ concentration in blood depends on H_2CO_3 breakdown by carbonic anhydrase.

68. The DNA fragment 5′ ATG-AGA-GGC-GGA-TGA 3′ codes for a tetrapeptide. Which ONE of the following options represent the **CORRECT** order of molecular weights of the indicated molecules involved in this process?

(A) template strand > coding strand > peptide
(B) peptide > coding strand > template strand
(C) coding strand > template strand > peptide
(D) coding strand > template strand > peptide

Answer (C, D)

Sol. The coding strand has 5A, 2T, 7G and 1C. The molecular weight will be 675 + 252 + 1057 + 111 = 2095

The template strand has 2A, 5T, 1G and 7C. The molecular weight will be 270 + 630 + 151 + 777 = 1828

The tetrapeptide with Met, Arg and two Gly. will be of molecular weight about 473. Therefore, the order is coding strand > template strand > tetrapeptide.

69. If all the lysosomes in the cell are suddenly disrupted, which ONE of the following statements **CORRECTLY** describes its immediate effect?

(A) More proteins would be made
(B) Macromolecules in the cytosol would break down
(C) Mitochondrial division would be promoted
(D) Only DNA within mitochondria would break down

Answer (B)

Sol. The lysosomes in the cell are suddenly disrupted, so the enzymes from the lysosomes will move out and the macromolecules present in the cytosol would break down.

70. If the same lysozyme is present in tears, saliva, milk and duodenum of humans, where would its maximal specific activity be found?

(A) In the tears, where pH is neutral
(B) In the duodenum, where pH is acidic
(C) In milk, where the pH is basic
(D) In saliva, where the pH is basic

Answer (B)

Sol. The correct answer is option (B) as the maximum specific activity of lysozyme is found at acidic pH. When the contents from the stomach enter the duodenum they are at a low pH. The pH then gradually increases as the contents pass through the small intestine.

Hence, as the optimum pH required by lysozyme is found in the duodenum, its specific activity will be maximal in the duodenum.

71. Which ONE of the following statements is **CORRECT** about the life cycle of *Plasmodium falciparum*?

(A) Gametocytes are developed within human red blood cells (RBC) and fertilized inside mosquitos

(B) Gametocytes are developed and fertilized within human RBC

(C) The parasite reproduces sexually inside hepatocytes and the released gametocytes infect RBC

(D) Sporozoites grow inside the salivary glands of mosquitos as well as in RBC

Answer (A)

Sol. The correct answer is option (A) as mosquito is the definitive host of *Plasmodium i.e.*, fertilisation occurs inside the mosquito.

Option (B) is incorrect as gametocytes develop within the human RBCs but do not get fertilised within the humans.

Option (C) is incorrect as the parasites reproduce asexually inside hepatocytes (liver cells).

Option (D) is incorrect as sporozoites escape from the gut and migrate to the mosquito's salivary glands.

72. A newly discovered organism possesses a non-cellulosic cell wall and has a nuclear membrane. To which ONE of the following kingdoms does this organism likely belong?

(A) Monera (B) Fungi

(C) Animalia (D) Plantae

Answer (B)

Sol. The discovered organism has **non-cellulosic cell wall** and has **a nuclear membrane**.

If given organisms has nuclear membrane, then it is not a member of Monera.

If it has cell wall then it is surely not a member of kingdom Animalia.

Cell wall is non-cellulosic, so it is not a member of plantae. Hence it is a member of kingdom Fungi.

73. Assume that the sun provides 100 kJ light energy. In a three trophic level ecosystem, with plants, deer and tiger, what is the expected amount of energy represented by tiger biomass?

(A) 1 J (B) 10 J

(C) 100 J (D) 1 kJ

Answer (B)

Sol. Sun

100 kJ → 1% of it absorbed by plant

$T_1 \longrightarrow T_2 \longrightarrow T_3$

Plant	Deer	Tiger
1 kJ	0.1 kJ	0.01 kJ

Energy in tiger biomass = 0.01 kJ = 0.01 × 1000 = 10 J

74. Which ONE of the following terms **CORRECTLY** describes a cell obtained after a successful karyokinesis but a failed cytokinesis?

(A) Multipolar (B) Polyploid

(C) Syncytium (D) Aneuploid

Answer (C)

Sol. When a cell is obtained after a successful karyokinesis but a failed cytokinesis then it forms syncytium.

Polyploid is a condition when there are multiple sets of chromosomes in a single cell.

75. Which of the following vitamins is/are fat-soluble?

(A) Vitamin A, D, E and K
(B) Vitamin K only
(C) Vitamin A, B, D and E
(D) Vitamin B only

Answer (A)

Sol. The correct answer is option (A) as vitamins A, D, E and K are fat-soluble vitamins.

Vitamins are divided into two groups depending on their solubility. Vitamins C and B complex are water soluble vitamins.

76. Which ONE of the following is most likely to be affected by the complete absence of external pinna?

(A) Sense of balance of the body
(B) Resolution of the sound source along a vertical plane
(C) Resolution of the sound source along a horizontal plane
(D) Range of audible sound frequency

Answer (B)

Sol. The correct answer is option (B) as the complete absence of pinna affects the catching of sound waves and funnelling them down the ear canal to the tympanic membrane (eardrum). If the pinna is absent, the sound waves coming from a source along the horizontal plane can be caught more in comparison with the sound waves coming along a vertical plane.

Option (A) is incorrect as the sense of balance of the body is under the control of vestibular apparatus.

Option (D) is incorrect as the range of audible sound frequency is unaffected by presence or absence of pinna.

77. Which ONE of the following statements is **INCORRECT**?

(A) A protein-coding gene always produces a single species of mRNA.
(B) mRNAs with different sequences can encode a polypeptide with the same sequence.
(C) A peptide bond in a cell can never be formed outside the ribosome system.
(D) Both DNA and RNA can serve as a template in the replication of an RNA strand.

Answer (A)

Sol. Introns and exons both are transcribed from DNA into m-RNA. The m-RNA then undergo splicing where introns are removed. By choosing which introns are removed we can make different species of m-RNA, called alternate splicing.

78. In proteosome-mediated protein degradation in a cell, the ubiquitin protein is conjugated to the target protein. This conjugation can be mediated by a/an

(A) Peptide bond between lysine and glycine
(B) Peptide bond between two cysteines
(C) Isopeptide bond between lysine and glycine
(D) Isopeptide bond between two cysteines

Answer (C)

Sol. The correct answer is option (C) as in ubiquitinylation there is binding of ubiquitin to lysine residues on the protein substrate via an isopeptide bond, cysteine residues through a thioester bond, serine and threonine residues through as ester bond.

79. Which ONE of the following represents the **CORRECT** order of human pacemaker tissues based on the frequency of heartbeat generated by them?

(A) Sinoatrial node > bundle of His > atrioventricular node
(B) Atrioventricular node > sinoatrial node > bundle of His
(C) Sinoatrial node > atrioventricular node > bundle of His
(D) Bundle of His > sinoatrial node > atrioventricular node

Answer (C)

Sol. The correct answer is option (C) as SA node generates maximum number of action potentials *i.e.*, 70-75 min^{-1} and is responsible for initiating and maintaining the rhythmic contractile activity of the heart. The AV node on the other hand has the ability to generate around 40-60 action potentials per minute and Bundle of His around 20 per minute.

80. In which ONE of the following sensory processes does *cis-trans* isomerisation play a key role in the initial step?

(A) Smell
(B) Touch
(C) Hearing
(D) Vision

Answer (D)

Sol. The correct answer is option (D) as rhodopsin is a visual receptor in the retina that absorbs visible light. When light strikes rhodopsin, the cis-double bond is isomerized to a trans-double bond, a process called photo-isomerization.

PART-II : MATHEMATICS

81. The number of solutions of the equation $x^2 + y^2 = a^2 + b^2 + c^2$, where x, y, a, b, c are all prime numbers is

(A) 0
(B) 1
(C) More than 1 but finite
(D) Infinite

Answer (A)

Sol. All prime numbers are of the form of $(6n \pm 1)$ except 2 and 3.

Let x, y, a, b, c are not 2 and 3 then $(6n \pm 1)^2 = 6\lambda + 1$

LHS = $x^2 + y^2 = 6\lambda_1 + 1 + 6\lambda_2 + 1 = 6\mu + 2$ $\quad [\lambda_1 + \lambda_2 = \mu]$

RHS = $a^2 + b^2 + c^2 = 6\lambda_3 + 1 + 6\lambda_4 + 1 + 6\lambda_5 + 1$ $\quad [\lambda_3 + \lambda_4 + \lambda_5 = \delta]$

$= 6\delta + 3$

LHS $\neq$ RHS

$\Rightarrow$ 2 and 3 must be there in equation

Now let 2 is not in equation

$\Rightarrow$ All are odd

$\Rightarrow$ LHS = even and RHS = odd {Not possible}

$\Rightarrow$ 2 must be there somewhere

Case I: $x = y = 2$

$$\Rightarrow \underbrace{a^2 + b^2 + c^2}_{\text{atleast one has to be 2}} = 8$$

$\Rightarrow$ $b^2 + c^2 = 4$ (not possible)

[$\because$ b, c are prime]

Case II: $x = 2$, $y \neq 2$

$4 + y^2 = a^2 + b^2 + c^2$ [not possible]

$\Rightarrow$ No solution

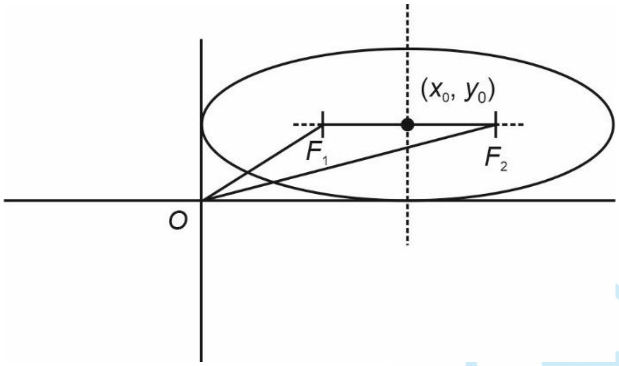
82. An ellipse $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1, a>b,$ is tangent to both x and y axes and is placed in the first quadrant. Let F_1 and F_2 be two foci of the ellipse and O be the origin with $OF_1 < OF_2$. Suppose the triangle OF_1F_2 is an isosceles triangle with $\angle OF_1F_2 = 120°$. Then the eccentricity of the ellipse is

(A) $\frac{1}{2\sqrt{3}}$ (B) $\frac{2}{3}$

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

Answer (C)

Sol.



$F_1(x_0 - ae, b/2)$

$F_2(x_0 + ae, b/2)$

$y_0 = b/2$

$\angle OF_1F_2 = 120°$ and $OF_1 = F_1F_2 = 2ae$

ΔOF_1F_2

$$\cos120° = \frac{OF_1^2 + F_1F_2^2 - OF_2^2}{2 \cdot OF_1 \cdot F_1F_2}$$

$$-\frac{1}{2} = \frac{4a^2e^2 + 4a^2e^2 - OF_1^2}{2(2ae)(2ae)}$$

$$\Rightarrow \quad OF_2^2 = 12a^2e^2$$

$$\Rightarrow \quad (x_0 + ae)^2 + \frac{b^2}{4} = 12a^2e^2 \qquad \text{...(1)}$$

$$OF_1^2 = 4a^2e^2$$

$$(x_0 - ae)^2 + \frac{b^2}{4} = 4a^2e^2 \qquad \text{...(2)}$$

(1) + (2)

$$2x_0^2 + \frac{b^2}{2} = 14a^2e^2 \qquad \text{...(3)}$$

(0, 0) will lie on director circle

Director circle : $(x - x_0)^2 + (y - y_0)^2 = a^2 + b^2$

$\Rightarrow \quad x_0^2 + y_0^2 = a^2 + b^2$

$\Rightarrow \quad x_0^2 = a^2 + \frac{3b^2}{4}$ $\qquad \left[\because \; y_0 = \frac{b}{2}\right]$

Put in (3)

$2a^2 + \frac{3b^2}{2} + \frac{b^2}{2} = 14a^2e^2$

$a^2 + b^2 = 7a^2e^2$

$1 + 1 - e^2 = 7e^2$ $\qquad [\because \; a^2(1 - e^2) = b^2]$

$2 = 8e^2$

$e^2 = \frac{1}{4}$

$e = \frac{1}{2}$

83. In a triangle the lengths of the sides are integers. Suppose that the length of one side is 1, and the longest altitude is twice the shortest altitude. Let R and r be the circumradius and inradius of the triangle, respectively. If $R : r = m : n$, where m and n are coprime positive integers, then $m + n$ is

(A) 5 (B) 7

(C) 9 (D) 11

Answer (D)

Sol. Let $a \le b \le c$ be the sides and $h_1 \ge h_2 \ge h_3$ be the respective altitudes.

$\because \quad ah_1 = ch_3$ (here $h_1 = 2h_3$ and $a = 1$)

So $c = 2$

Hence only possible value of b is 2

$\Delta = \frac{\sqrt{15}}{4}$

Now $R = \frac{abc}{4\Delta}$ and $r = \frac{\Delta}{s}$

So $\frac{R}{r} = \frac{abcs}{4\Delta^2} = \frac{4.2.2.1\left(\frac{5}{2}\right)}{15} = \frac{8}{3}$

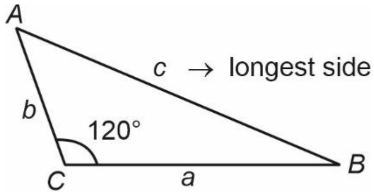
84. In a triangle ABC, $\cos3A + \cos3B + \cos3C = 1$. If the circumradius of triangle ABC is $\sqrt{3}$, then the length of its longest side is

(A) $\sqrt{3}$ (B) 2

(C) 3 (D) $2\sqrt{3}$

Answer (C)

Sol.



$\cos 3A + \cos 3B + \cos 3C = 1$

$$2\cos 3\left(\frac{A+B}{2}\right)\cos 3\left(\frac{A-B}{2}\right) = 1 - \cos 3C$$

$$-2\sin\frac{3C}{2}\cos 3\left(\frac{A-B}{2}\right) = 2\sin^2\frac{3C}{2} \qquad [\because\ A + B + C = \pi]$$

$$2\sin\frac{3C}{2}\left[\sin\frac{3C}{2} + \cos 3\left(\frac{A-B}{2}\right)\right] = 0$$

$$\Rightarrow\ \sin\frac{3C}{2} = 0 \quad \Rightarrow\ C = \frac{2\pi}{3}$$

$$R = \frac{abc}{4\Delta} = \frac{abc}{4\left[\frac{1}{2}ab\sin 120\right]} = \sqrt{3}$$

$\Rightarrow\ c = 3$

85. Suppose that the sides a, b, c of a triangle ABC satisfy $b^2 = ac$. Then the set of all possible values of $\dfrac{\sin A\cot C + \cos A}{\sin B\cot C + \cos B}$ is

(A) $(0, \infty)$

(B) $\left(0, \dfrac{\sqrt{5}+1}{2}\right)$

(C) $\left(\dfrac{\sqrt{5}-1}{2}, \dfrac{\sqrt{5}+1}{2}\right)$

(D) $\left(\dfrac{\sqrt{5}-1}{2}, \infty\right)$

Answer (C)

Sol. $\dfrac{\sin A\cot C + \cos A}{\sin B\cot C + \cos B}$

$$\Rightarrow \frac{\sin A\cos C + \cos A\sin C}{\sin B\cos C + \cos B\sin C} = \frac{\sin[A+C]}{\sin[B+C]}$$

$$\Rightarrow \frac{\sin B}{\sin \mathrm{A}} = \frac{b}{a} = r \qquad \{\text{Common ratio of GP}\}\ [r > 0]$$

Now, $\boxed{b^2 = ac} \Rightarrow a, b, c \to \text{GP}$

Let sides be a, ar, ar^2

Case (1) $a + ar > ar^2$

$\Rightarrow r^2 - r - 1 < 0$

$\Rightarrow r \in \left(\dfrac{1-\sqrt{5}}{2}, \dfrac{1+\sqrt{5}}{2}\right)$

Case (2) $a + ar^2 > ar$

$\Rightarrow r^2 - r + 1 > 0 \Rightarrow r \in R$

Case (3) $ar + ar^2 > a$

$\Rightarrow r^2 + r - 1 > 0$

$\Rightarrow r \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$

$(1) \cap (2) \cap (3) \Rightarrow r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$

86. Let $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Then the sum $\frac{1}{2^{10}} \sum_{k=0}^{10} \binom{10}{k} k^2$, lies in the interval

(A) (26, 27)
(B) (27, 28)
(C) (28, 29)
(D) (29, 30)

Answer (B)

Sol. $\sum_{k=0}^{10} {}^{10}C_k \cdot k^2 = \sum_{k=0}^{10} {}^{10}C_k \cdot k(k-1) + \sum_{k=0}^{10} {}^{10}C_k \cdot k$

$= 10 \cdot 9 \cdot \sum_{k=0}^{10} {}^{8}C_{k-2} + 10 \sum_{k=0}^{10} {}^{9}C_{k-1}$

$= 10 \cdot 9 \cdot 2^8 + 10 \cdot 2^9 = 2^8(90 + 20)$

So, $\frac{1}{2^{10}}\left(2^8.110\right) = \frac{110}{4}$

87. The number of continuous functions $f : \left[0, \frac{3}{2}\right] \to (0, \infty)$ satisfying the equation

$4\int_0^{3/2} f(x)dx + 125\int_0^{3/2} \frac{dx}{\sqrt{f(x) + x^2}} = 108$ is

(A) 0
(B) 1
(C) 2
(D) greater than 2

Answer (B)

Sol. $\because\ I = \int_0^{3/2}\left(4\left(f(x) + x^2\right) + \frac{125}{2\sqrt{f(x)+x^2}} + \frac{125}{2\sqrt{f(x)+x^2}}\right)dx = 108 + \frac{9}{2}$

Using A.M $\geq$ G.M.

$I \geq \int_0^{3/2} 75dx \Rightarrow I \geq \frac{225}{2}$

Equality holds, So $\left(f(x) + x^2\right)^{3/2} = \frac{125}{8}$

$\Rightarrow\ f(x) = \frac{25}{4} - x^2$

88. For each real number x, let $[x]$ denote the greatest integer less than or equal to x, and let $\{x\} = x - [x]$. Then the smallest positive integer M for which $\int_1^M \{x\}^{[x]} dx > 1$ is

(A) 2
(B) 3
(C) 4
(D) 5

Answer (C)

Sol. $\int_1^M (x-[x])^{[x]} dx$

If $M = 2 \Rightarrow \int_1^2 (x-1)^1 dx = \left.\frac{(x-1)^2}{2}\right|_1^2 = \frac{1}{2}$

$M = 3 \Rightarrow \frac{1}{2} + \int_2^3 (x-2)^2 dx = \frac{1}{2} + \left.\frac{(x-2)^3}{3}\right|_2^3$

$= \frac{5}{6}$

$M = 4 \Rightarrow \frac{5}{6} + \int_2^3 (x-3)^3 dx = \frac{5}{6} + \frac{1}{4} > 1$

So, $M = 4$

89. In a collection of ten tickets, there are two winning tickets. From this collection, five tickets are drawn at random. Let p_1 and p_2 be the probabilities of obtaining one and two winning tickets, respectively. Then $p_1 + p_2$ lies in the interval

(A) $\left(0, \frac{1}{2}\right]$

(B) $\left(\frac{1}{2}, \frac{3}{4}\right]$

(C) $\left(\frac{3}{4}, 1\right]$

(D) $\left(1, \frac{3}{2}\right]$

Answer (C)

Sol. $p_1 = \frac{{}^2C_1 \cdot {}^8C_4}{{}^{10}C_5}$, $p_2 = \frac{{}^2C_2 \cdot {}^8C_3}{{}^{10}C_5}$

$p_1 + p_2 = \frac{196}{{}^{10}C_5} = \frac{196}{252}$

So option (C) is correct

90. The number of real values of x at which the function $f(x) = \begin{vmatrix} 1 & |x| & x^2 \\ 1 & |x-1| & (x-1)^2 \\ 1 & |x-2| & (x-2)^2 \end{vmatrix}$ is **not** differentiable is

(A) 0

(B) 1

(C) 2

(D) 3

Answer (C)

Sol. Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & |x|-|x-1| & 1-2x \\ 0 & |x-1|-|x-2| & 3-2x \\ 1 & |x-2| & (x-2)^2 \end{vmatrix}$$

For $x \geq 2 \Rightarrow f(x) = \begin{vmatrix} 1 & 1-2x \\ 1 & 3-2x \end{vmatrix} = 2$

For $x \leq 0 \Rightarrow f(x) = -2$

For $x \in (0, 1)$

$f(x) = \begin{vmatrix} 2x-1 & 1-2x \\ -1 & 3-2x \end{vmatrix} = -(4x^2 - 6x + 2)$

For $x \in [1, 2)$

$$f(x)=\begin{vmatrix}1 & 1-2x\\ 2x-3 & 3-2x\end{vmatrix}=4x^2-10x+6$$

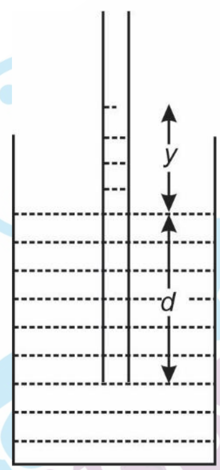
$$f(x)=\begin{cases}-2 & x<0\\ -4x^2+6x-2 & x\in(0,1)\\ 4x^2-10x+6 & x\in[1,2)\\ 2 & x\geq 2\end{cases}$$

So, graph is continuous at all points but non-differentiable at $x = 0$ and $x = 2$

as $f'(0^+)\neq f'(0^-)$ and $f'(2^-)\neq f'(2^+)$

PART-II : PHYSICS

91. A drinking straw is dipped in a pan of water to depth d from the surface (see figure below). Now water is sucked into it up to an initial height h_0 and then left to oscillate. As a result, its height y from the surface of the water varies periodically.



Ignoring damping, the equation for y is (g is the acceleration due to gravity):

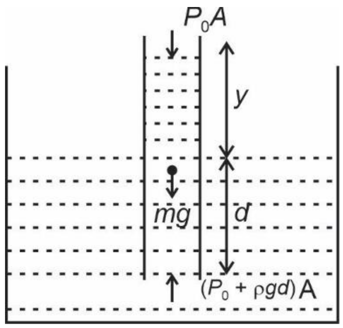
(A) $\ddot{y}+\frac{g}{d}y=0$

(B) $\ddot{y}(y+d)+\frac{g}{d}(y+d)=0$

(C) $\ddot{y}+\frac{\dot{y}^2}{d}+\frac{g}{d}(y+d)=0$

(D) $\ddot{y}(y+d)+\dot{y}^2+gy=0$

Answer (D)

Sol. 

Momentum

$p = \rho A(d + y)\dot{y}$

$\frac{dp}{dt} = \rho A\left[(\dot{y})^2 + (d + y)\ddot{y}\right]$

$= F_{net}$

$= \rho dgA - \rho(d + y)gA$

$= -\rho gyA$

$\Rightarrow \quad \ddot{y}(y + d) + (\dot{y})^2 + gy = 0$

92. A cubic metal block of mass 5 *kg* and edge length 0.1 *m* and at an initial temperature of 100°C is placed on a thermally insulating flat surface and exposed to air at 0°C. The time in *seconds* required to cool the block to a temperature of 37°C is closest to (Note: Specific heat of the metal = 500 *J*/*Kg*/°C; Heat transfer coefficient from block to air = 50 W/m^2/°C

(A) 500

(B) 1000

(C) 1500

(D) 2000

Answer (B)

Sol. $\frac{dQ}{dt} = -CA(T - T_0)$

Or $ms\frac{dT}{dt} = -CA(T - T_0)$

$\Rightarrow \quad 5 \times 500 \times \frac{dT}{dt} = -50 \times 5 \times 10^{-2}(T - T_0)$

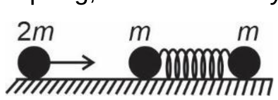
$\Rightarrow \quad 2500\frac{dT}{dt} = -250 \times 10^{-2} \times (T - T_0)$

$\Rightarrow \quad \frac{dT}{T - 0} = -10^{-3}dt$

$\Rightarrow \quad t = 100\ln\left(\frac{100}{37}\right)$

Or $\boxed{t \simeq 1000 \text{ s}}$

93. A ball of mass 2*m* and a system of two balls with equal masses *m* connected by a massless spring, are placed on a smooth horizontal surface (see figure below). Initially, the ball of mass 2*m* moves along the line passing through the centres of all the balls and the spring, whereas the system of two balls is at rest.



Assuming that the collision between the individual balls is perfectly elastic, the ratio of vibrational energy stored in the system of two connected balls to the initial kinetic energy of the ball of mass 2*m* is

(A) 1

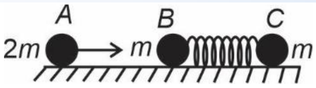
(B) $\frac{4}{9}$

(C) $\frac{1}{2}$

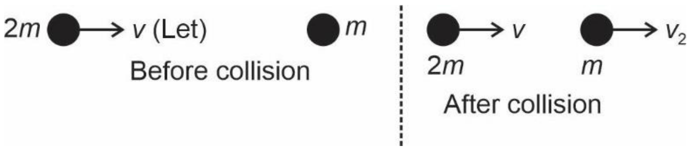
(D) $\frac{2}{3}$

Answer (B)

Sol.



For collision between *A* and *B*



Using conservation of linear momentum

$2mv = 2mv_1 + mv_2$

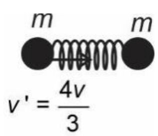
$2v = 2v_1 + v_2$ …I

As the collision is elastic, $e = 1$

$\frac{v_2 - v_1}{v} = 1 \Rightarrow v_2 - v_1 = v$ …II

On solving I and II $v_2 = \frac{4v}{3}$

Now, for *B* and *C*, just after collision



Now, when spring is in maximum compression

$mv' = 2mv_c$

$v_c = \frac{v'}{2} = \frac{2v}{3}$

Vibrational energy of system

$E_v = \frac{1}{2}mv'^2 - \frac{1}{2} \times 2m \times \frac{v'^2}{4}$

$= \frac{1}{4}mv'^2$

$= \frac{1}{4}m \cdot \frac{16v^2}{9} = \frac{4mv^2}{9}$

Hence required ratio will be $= \frac{E_v}{K_i}$

$= \frac{\left(\frac{4}{9}mv^2\right)}{\left(\frac{1}{2} \times 2mv^2\right)}$

$= \frac{4}{9}$

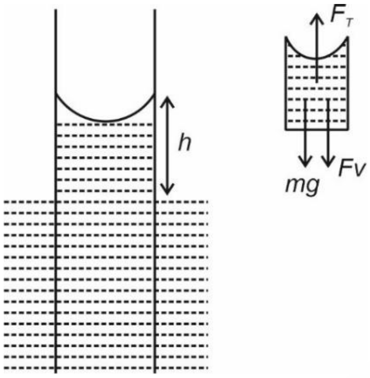
94. According to Poiseuille's law, the pressure drop per unit length required to overcome viscous forces is $\Delta P = \frac{8\eta v}{r^2}$, where r is the radius of cross section v is the fluid velocity and η is the coefficient of viscosity. A capillary tube of radius a is dipped in a liquid of density ρ, surface tension T and coefficient of viscosity η. The liquid starts rising in it so that its height $h(t)$ is a function of time t. The resulting rate of change of the momentum of liquid column in the capillary (taking vertically up to be positive direction and the contact angle to be close to 0°) is $-\pi a^2\rho gh + F$. Then F is (g is the acceleration due to gravity):

(A) $4\pi Ta + 8\pi\eta h\frac{dh}{dt}$ (B) $4\pi Ta - 8\pi\eta h\frac{dh}{dt}$

(C) $2\pi Ta - 8\pi\eta h\frac{dh}{dt}$ (D) $2\pi Ta + 8\pi\eta h\frac{dh}{dt}$

Answer (C)

Sol.



Pressure drop per unit length required to overcome viscous force

$$\Delta P = \frac{8nv}{r^2}$$

At any time, consider the section of liquid of height 'h'

$$F_{\text{net}} = \frac{dP}{dt}$$

$$= F_T - mg - F_v$$

Here, $F_v = \Delta P \cdot hA = \frac{8\eta v}{r^2} h \cdot A = 8\eta h\pi\frac{dh}{dt}$ and $F_T = 2\pi aT$ (Since angle of contact is zero)

So, $F_{\text{net}} = 2\pi aT - \rho\pi a^2 hg - 8\eta h\pi\frac{dh}{dt}$ … I

Given, $-\pi a^2\rho gh + F = F_{\text{net}}$ …II

On comparing I and II

$$F = 2\pi aT - 8\pi\eta h\frac{dh}{dt}$$

95. From a carbon nanotube of 1 μm length and 1 nm radius, 10 electrons have been removed. Assume the resulting positive charge to be distributed uniformly over the surface of the tube. The energy of an electron moving in a stable circular orbit around the axis along the length of the tube is calculated by applying the Bohr model. Accordingly, the frequency of radiation required to excite an electron from its ground state to the next level is in the range of (charge of the electron, $e = 1.60 \times 10^{-19}$ C; mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg; Planck's constant, $h = 6.63 \times 10^{-34}$ Js; Permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m)

(A) Infrared (B) Visible

(C) Ultraviolet (D) X-rays

Answer (A)

Sol. Resulting charge = $Q = 10 \times e = 1.6 \times 10^{-18}$C

Length (l) = 1 μm

Radius (r) = 1 nm

As $l >> r \Rightarrow$ the tube can be considered of infinite length.

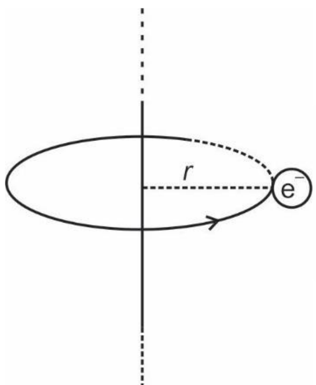
Now, λ (Linear charge density) $= \frac{Q}{l}$

$= 1.6 \times 10^{-12}$ C/m

Allowed radii(r) can be calculated using following condition : $mvr = \frac{nh}{2\pi}$

Now, $\frac{\lambda}{2\pi\varepsilon_0 r} = \frac{mv^2}{r}$

$\Rightarrow \quad v = \sqrt{\frac{\lambda}{2\pi \in_0 m}}$



Thus velocity will remain unchanged as r changes.

$\therefore$ in excitation, only potential energy will be changed:

$$\Delta E = \Delta U = \int_{\frac{h}{2\pi mv}}^{\frac{2h}{2\pi mv}} \frac{\lambda e}{2\pi\varepsilon_0 r} dr$$

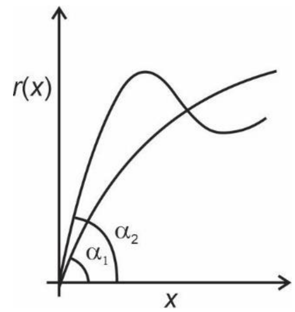
$$= \frac{\lambda e}{2\pi\varepsilon_0} \ln(2) = 0.0199 \text{ eJ}$$

Now, $h\nu = 0.199e = 0.199 \times 1.6 \times 10^{-19}$

$\simeq 4.8 \times 10^{13}$

$\lambda = \frac{c}{\nu} = 6.25\ \mu\text{m} \Rightarrow$ Infrared

96. A projectile is launched from the origin in the xy plane (x is the horizontal and y is the vertically up direction) making an angle α from the x-axis. If its distance, $r = \sqrt{x^2+y^2}$ from the origin is plotted against x, the resulting curves show different behaviours for launch angles α_1 and α_2 as shown in the figure below. For α_1, $r(x)$ keeps increasing with x while for α_2, $r(x)$ increases and reaches a maximum, then decreases and goes through a minimum before increasing again.



The switch between these two cases takes place at an angle α_c ($\alpha_1 < \alpha_c < \alpha_2$). The value of α_c is [ignore air drag and take $y(x) = x\tan\alpha - \frac{1}{2}\frac{g\sec^2\alpha}{v_0^2}x^2$, where v_0 is the initial speed of the projectile and g is the acceleration due to gravity]

(A) $\sin^{-1}\left(\frac{1}{3}\right)$

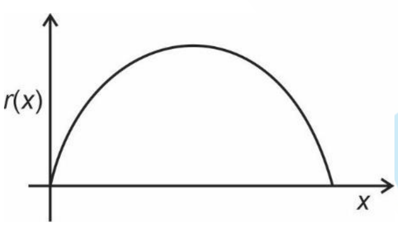
(B) $\cos^{-1}\left(\frac{1}{3}\right)$

(C) $\tan^{-1}\left(\frac{1}{3}\right)$

(D) $\tan^{-1}(3)$

Answer (B)

Sol.



$$r = \sqrt{(u\cos\alpha_c t)^2 + (u\sin\alpha_c t - gt)^2}$$

$$\vec{v}\cdot\vec{r} > 0$$

$$\vec{v} = u\cos\alpha_c\hat{i} + (u\sin\alpha_c - gt)\hat{j}$$

$$\vec{r} = u\cos\alpha_c t\hat{i} + \left(u\sin\alpha_c - \frac{1}{2}gt^2\right)\hat{j}$$

$$\vec{v}\cdot\vec{r} = u^2\cos^2\alpha_c t + u^2\sin^2\alpha_c t - u\sin\alpha\frac{1}{2}gt^2 - gt^2u\sin\alpha_c + \frac{1}{2}g^2t^3 > 0$$

$$= u^2t - \frac{3}{2}u^2\sin\alpha_c t^2 + \frac{1}{2}g^2t^3 > 0$$

$$t\left(u^2 - \frac{3}{2}u^2\sin\alpha_c t + \frac{1}{2}g^2t^2\right) > 0$$

$2u^2 - 3u^2\sin\alpha_c t + g^2t^2 > 0$

$g^2t^2 - 3u^2\sin\alpha_c t + 2u^2 > 0$

$$t = \frac{3u^2 \sin\alpha \pm \sqrt{gu^2 \sin^2 \alpha_c - 8g^2u^2}}{2g^2}$$

$t \rightarrow$ real

So, $gu^2\sin^2\alpha_c - 8g^2u^2 > 0$

$gu^2\sin^2\alpha_c > 8g^2u^2$

$$\sin^2 \alpha_c > \frac{8}{9}$$

$$\tan\alpha_c > \sqrt{8}$$

$$\sin\alpha_c > \sqrt{\frac{8}{9}}$$

$$\cos\alpha_c > \frac{1}{3}$$

97. One mole of a monoatomic ideal gas $\left(C_V = \frac{3}{2}R\right)$ undergoes a cycle where it first goes isochorically from the state $\left(\frac{3}{2}P_0, V_0\right)$ to (P_0, V_0), and then is isobarically contracted to the volume $\frac{1}{2}V_0$. It is then taken back to the initial state by a path which is a quarter ellipse on the $P - V$ diagram. The efficiency of this cycle is

(A) $\frac{1}{\pi}$

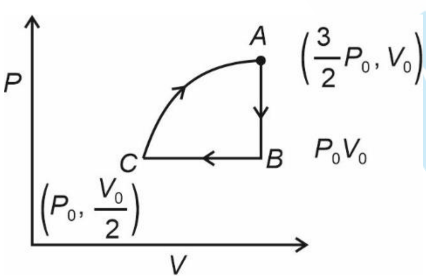
(B) $\frac{\pi}{16+\pi}$

(C) $\frac{\pi}{32+\pi}$

(D) $\frac{2\pi}{32+\pi}$

Answer (C)

Sol.



$$\text{Work}_{(\text{net})} = \frac{\pi}{4}\left(\frac{P_0}{2} \times \frac{V_0}{2}\right)$$

$$\text{Work}_{(\text{net})} = \frac{\pi}{16}P_0V_0$$

$A \rightarrow B$: Heat will be lost

$B \rightarrow C$: Heat will be lost

$C \rightarrow A$

Equation of CA: $\frac{P}{\left(\frac{P_0}{2}\right)} + \frac{V}{\left(\frac{V_0}{2}\right)} = 1$

$$\Rightarrow \frac{2P}{P_0}+\frac{2V}{V_0}=1$$

$$\Rightarrow P=\frac{P_0}{2}\left(1-\frac{2V}{V_0}\right)$$

Given $C_V=\frac{3}{2}R$ (monoatomic gas)

$$\Delta U_{C\to A}=C_V\Delta T=\frac{3}{2}R\Delta T=\frac{3}{2}\left(\frac{3}{2}P_0V_0-\frac{P_0V_0}{2}\right)$$

$$=\frac{3}{2}P_0V_0$$

$$W_{C\to A}=\frac{\pi}{16}P_0V_0+\frac{P_0V_0}{2}\simeq\frac{P_0V_0}{2}\left(1+\frac{\pi}{8}\right)$$

$$Q_{C\to A}=W_{CA}+\Delta U_{CA}=\frac{P_0V_0}{2}+\frac{\pi}{16}P_0V_0+\frac{3}{2}P_0V_0$$

$$=\left(\frac{\pi}{16}+2\right)P_0V_0$$

$$\text{Efficiency}=\frac{W_{\text{net}}}{Q_{C\to A}}=\frac{\frac{\pi}{16}P_0V_0}{\left(\frac{\pi}{16}+2\right)P_0V_0}$$

$$=\frac{\pi}{\pi+32}$$

98. A rectangular region *ABCD* contains a uniform magnetic field B_0 directed perpendicular to the plane of the rectangle. A narrow stream of charged particles moving perpendicularly to the side *AB* enters this region and is ejected through the adjacent side *BC* suffering a deflection through 30°. In order to increase this deflection to 60°, the magnetic field has to be

(A) $\frac{3}{2}B_0$

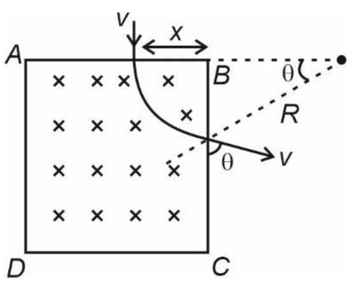
(B) $2B_0$

(C) $\left(2+\sqrt{3}\right)B_0$

(D) $\left(3+\sqrt{3}\right)B_0$

Answer (C)

Sol.



$X = R - R\cos\theta$

$X = R\,(1 - \cos\theta)$

$$X=\frac{mv}{qB}(1-\cos\theta)$$

Value of X remains same

Initially θ = 30°, $B = B°$

Finally θ = 60°, $B = B_1$

$$\frac{mv}{qB_0}(1-\cos 30°) = \frac{mv}{qB_1}(1-\cos 60°)$$

$$\frac{\left(1-\frac{\sqrt{3}}{2}\right)}{B_0} = \frac{1}{2B_1}$$

$$B_1 = \frac{B_0}{\left(2-\sqrt{3}\right)}$$

$$B_1 = B_0\left(2+\sqrt{3}\right)$$

99. The persistence of sound in a room after the source of sound is turned off is called reverberation. The measure of reverberation time is the time required for sound intensity to decrease by 60 dB.

It is given that the intensity of sound falls off as $I_0 \exp(-c_1\alpha)$ where I_0 is the initial intensity, c_1 is a dimensionless constant with value $\frac{1}{4}$. Here, α is a positive constant which depends on the speed of sound, volume of the room, reverberation time, and the effective absorbing area A_e. The value of A_e is the product of absorbing coefficient (with value between 0 and 1, 1 being a perfect absorber) and the area of the room. For a concert hall of volume 600 m^3, the value of A_e (in m^2) required to give a reverberation time of 1 s is closest to (speed of sound in air = 340 m/s)

(A) 50
(B) 100
(C) 110
(D) 67

Answer (B)

Sol. $T = \frac{0.161V}{\alpha S}$

Here $\alpha S = Ae$

$$T = \frac{0.161V}{Ae}$$

$$1 = \frac{0.161 \times 600}{Ae}$$

$Ae = 96.6$

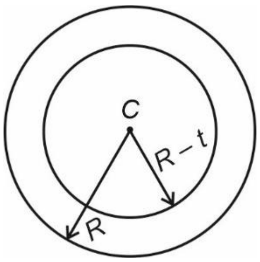
$Ae \approx 100$

100. Two uniform thin spherical shells are made from different materials. Both shells have a mass of 2 kg and outer radius of 20 cm. When they are both rolled down the same inclined plane without slipping, the times they take to cover equal distances differ by 1%. If the thickness of the thinner shell is 0.5 cm, that of the other one is closest to

(A) 0.505 cm
(B) 0.525 cm
(C) 1.0 cm
(D) 1.5 cm

Answer (D)

Sol.



Solid sphere

$$I_C = \frac{2}{5} \times \frac{4}{3}\pi R^3 \rho \times R^2$$

$$= \frac{8}{15}\rho\pi R^5$$

So, for thick shell

$$I = \frac{8}{15}\rho\pi\left\{R^5 - (R-t)^5\right\}$$

$$= \frac{8}{15}\rho\pi R^5\left\{1 - \left(1 - \frac{t}{R}\right)^5\right\}$$

$$= \frac{8}{15}\rho\pi R^5\left\{\frac{5t}{R} - \frac{10t^2}{R^2}\right\}$$

And mass $M = \rho \times \frac{4}{3}\pi\left(R^3 - (R-t)^3\right)$

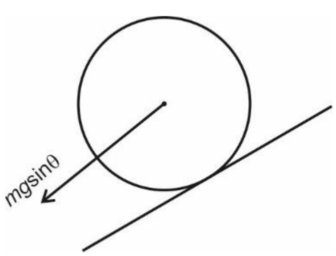
$$= \frac{4}{3}\rho\pi R^3 \times \left\{\frac{3t}{R} - \frac{3t^2}{R^2}\right\}$$

$$\frac{I}{MR^2} = \frac{8}{15} \times \frac{3}{4} \times \frac{\frac{t}{R}\left(5 - \frac{10t}{R}\right)}{\frac{t}{R}\left(3 - \frac{3t}{R}\right)}$$

$$\frac{I}{MR^2} = \frac{2}{5} \times \frac{5}{3}\frac{\left(1 - \frac{2t}{R}\right)}{\left(1 - \frac{t}{R}\right)}$$

$$= \frac{2}{3}\left[1 - \frac{2t}{R} + \frac{t}{R}\right]$$

Or $\frac{I}{MR^2} = \frac{2}{3}\left(1 - \frac{t}{R}\right)$



$mg\sin\theta R = (I + mR^2)\alpha$

$$\Rightarrow \alpha = \frac{mg\sin\theta R}{I + mR^2}$$

$$T = \sqrt{\frac{2l}{a}}$$

$$T = \sqrt{\frac{2l}{g\sin\theta}\left\{1+\frac{2}{3}\left(1-\frac{t}{R}\right)\right\}}$$

Thinner shell $\Rightarrow$ time greater

$$\frac{\Delta T}{T} = \frac{1}{100}$$

$$\left(\frac{5}{3}-\frac{2}{3}\frac{t_1}{R}\right)^{\frac{1}{2}} - \left(\frac{5}{3}-\frac{2}{3}\frac{t_2}{R}\right)^{\frac{1}{2}} = \frac{1}{100}$$

$$\Rightarrow \sqrt{\frac{5}{3}}\left\{\frac{-t_1}{5R}+\frac{t_2}{5R}\right\} = \frac{1}{100}$$

$$\Rightarrow t_2 - t_1 = \frac{5R}{100}\sqrt{\frac{3}{5}}$$

$$\Rightarrow t_2 = t_1 + \frac{R}{20}\sqrt{\frac{3}{5}}$$

$\simeq 1.27$ cm closest to 1.5 cm

PART-II : CHEMISTRY

101. The correct match of the complexes with their structure and magnetic property is

	Complex		Structure & magnetic property
(i)	$NiCl_4^{2-}$	(p)	Tetrahedral and diamagnetic
(ii)	$Ni(CO)_4$	(q)	Tetrahedral and paramagnetic
(iii)	$PtCl_4^{2-}$	(r)	Square planar and diamagnetic
(iv)	$Ni(CN)_4^{2-}$	(s)	Square planar and paramagnetic

(A) (i) $\to$ (q), (ii) $\to$ (p), (iii) $\to$ (r) and (iv) $\to$ (r)

(B) (i) $\to$ (p), (ii) $\to$ (q), (iii) $\to$ (r) and (iv) $\to$ (s)

(C) (i) $\to$ (p), (ii) $\to$ (s), (iii) $\to$ (p) and (iv) $\to$ (r)

(D) (i) $\to$ (s), (ii) $\to$ (r), (iii) $\to$ (q) and (iv) $\to$ (p)

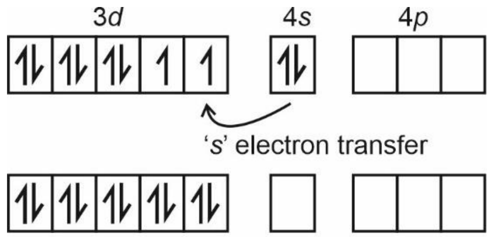
Answer (A)

Sol. (i) $[NiCl_4]^{2-} \Rightarrow Ni^{2+} \Rightarrow 3d^8 4s^0$, weak field ligand, hence no pairing

Hybridisation $\Rightarrow sp^3$

Tetrahedral, paramagnetic

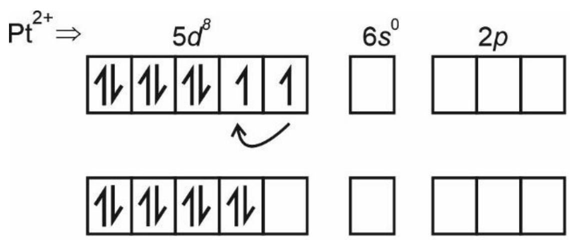
(ii) $Ni(CO)_4 \Rightarrow Ni \Rightarrow 3d^8 4s^2$,



sp^3, tetrahedral, diamagnetic

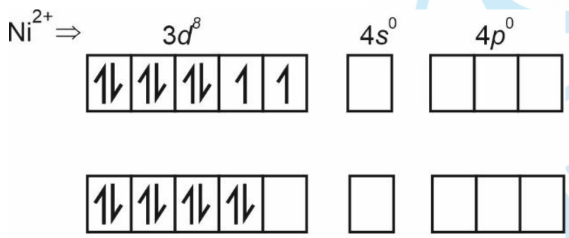
(iii) $[PtCl_4]^{2-}$

Metal Pt is from 5*d* hence Cl^- will be strong field ligand and pairing will take place



hybridisation dsp^2, square planar diamagnetic

(iv) $[Ni(CN)_4]^{2-}$ strong field ligand hence pairing will take place



dsp^2, square planar diamgnetic

102. The reaction $2NO_2(g) \rightleftharpoons N_2O_4(g)$ is at equilibrium in a closed 15 L vessel at 300 K. The total weight of the mixture of NO_2 and N_2O_4 in the vessel is 64.4 g. The equilibrium constant for the reaction is K_p = 6.67. Assuming ideal gas behaviour, the total pressure in the vessel (in atm) is

[Given: Gas constant R = 0.082 atm LK^{-1} mol^{-1}]

(A) 0.78 (B) 1.34

(C) 1.96 (D) 2.25

Answer (B)

Sol.

$$\begin{array}{lcc} & 2NO_2 & \rightleftharpoons N_2O_4 \\ t=0 & 2 & 0 \\ t=t_{eq} & 2-2x & x \end{array}$$

$$K_P = \frac{p_{N_2O_4}}{p^2_{NO_2}} = \frac{\frac{xP}{2-x}}{\left(\frac{2-2x}{2-x}\right)^2 P^2}$$

$$6.67 = \frac{xP(2-x)}{4(1-x)^2 P} \qquad \ldots(i)$$

Now applying ideal gas equation on equilibrium mixture

$$P \times 15 = \frac{64.4}{\frac{x \times 96}{2-x} + \frac{2(1-x) \times 46}{2-x}} \times R \times T$$

$$15P = \frac{64.4}{96\left(\frac{-1}{2-x}\right)}$$

$$2 - x = \frac{P}{1.1} \quad \text{...(ii)}$$

Putting this in equation (i)

$$\frac{x}{(1-x)^2} = 29.34$$

$29.34x^2 - 59.6x + 29.34 = 0$

Solving above equation will give

x = 0.83

Putting this on equation (ii)

P = (2 – 0.83) × 1.1

P = 1.34 atm

103. 651 g of ethylene glycol ($HOCH_2CH_2OH$) is dissolved in 1.5 kg of water at 363 K. The vapour pressure of pure water at 363 K is 0.7 atm. Assuming ideal solution behaviour, the vapour pressure of water over the solution (in atm) is closest to

(A) 0.57
(B) 0.62
(C) 0.65
(D) 0.68

Answer (B)

Sol. $n_{(HOCH_2CH_2OH)} = \frac{651}{62} = 10.5$

$$n_{H_2O} = \frac{1500}{18} = 83.33$$

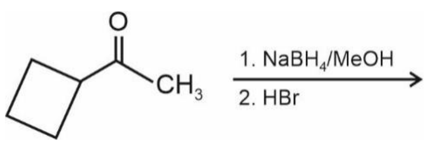
$$\frac{P^\circ - P_S}{P_S} = \frac{X_B}{X_A} = \frac{n_B}{n_A}$$

$$\frac{0.7 - P_S}{P_S} = \frac{10.5}{83.33}$$

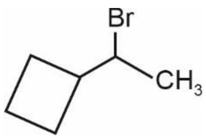
$0.7 - P_S = 0.126\ P_S$

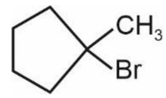
$$\Rightarrow P_S = \frac{0.7}{1.126} = 0.62 \text{ atm}$$

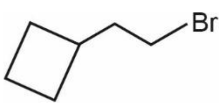
104. The major product formed in the following transformation

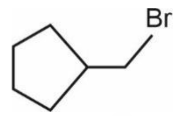


Is most likely to be

(A) 

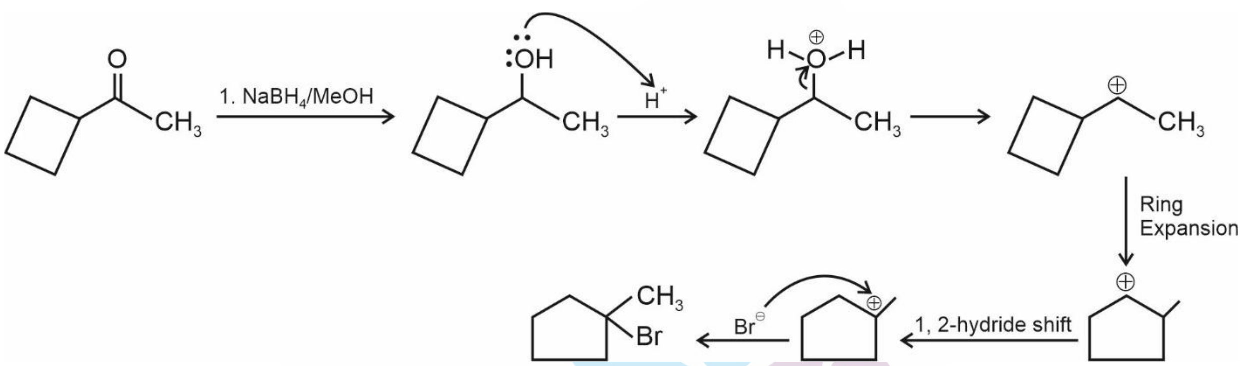
(B) 

(C) 

(D) 

Answer (B)

Sol.



105. A compound X with formula $C_4H_{11}N$ reacts with HNO_2 to liberate nitrogen and produce compound Y. The compound Y on treatment with I_2/NaOH produces sodium salt of a carboxylic acid. The compound X is

(A) $CH_3 - CH_2 - CH_2 - CH_2 - NH_2$

(B) $CH_3 - CH(CH_3) - CH_2 - NH_2$

(C) $CH_3 - CH(NH_2) - CH_2 - CH_3$

(D) $CH_3 - C(CH_3)_2 - NH_2$

Answer (C)

Sol.

- $CH_3 - CH_2 - CH(CH_3) - NH_2 \xrightarrow{HNO_2} [CH_3 - CH_2 - CH(CH_3) - \overset{\oplus}{N}_2] \xrightarrow{H_2O} CH_3 - CH_2 - CH(CH_3) - OH + N_2\uparrow$

  (X) Primary Amine; Unstable; (Y)

- $CH_3 - CH_2 - CH(CH_3) - OH \xrightarrow[\text{Iodoform Test}]{I_2/NaOH} CH_3 - CH_2 - \overset{O}{\overset{||}{C}} - O^-Na^+ + CHI_3$

  (Y); Sodium salt of propanoic acid; (Iodoform)

106. Phenol reacts with CO_2 in the presence of sodium hydroxide followed by acidification to give a steam volatile compound X. The compound X on treatment with acetic anhydride in the presence of a catalytic amount of H_2SO_4 produces Y.

Among the following

(i) Antipyretic

(ii) Anti-inflammatory

(iii) Narcotic analgesic

(iv) Antiplatelet

the properties shown by compound Y are

(A) (i), (ii), (iii) and (iv)

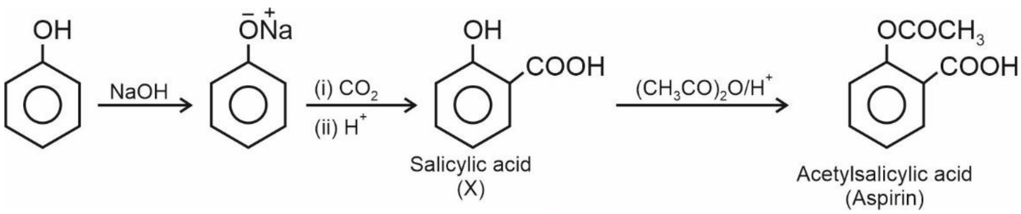
(B) (i), (ii) and (iii) only

(C) (ii), (iii) and (iv) only

(D) (i), (ii) and (iv) only

Answer (D)

Sol.



- Aspirin possesses non-narcotic analgesic, anti-inflammatory and antipyretic properties and prevent platelet coagulation

107. For the ideal gas reaction, $X + Y \rightleftharpoons Z$, a mixture with n_X = 1 mol, n_Y = 3 mol, and n_Z = 2 mol is at equilibrium at 300 K and 1 bar. If the pressure is isothermally increased to 2 bar, the number of moles of X in the new equilibrium is closest to

(A) 2.367

(B) 0.633

(C) 1.358

(D) 0.727

Answer (B)

Sol. At 300 K temperature and 1 bar pressure,

	X	+	Y	$\rightleftharpoons$	Z
At equilibrium	1 mol		3 mol		2 mol
	$\frac{1}{6}$ bar		$\frac{3}{6}$ bar		$\frac{2}{6}$ bar

$$K_P = \frac{\frac{2}{6}}{\frac{1}{6}\times\frac{3}{6}} = \frac{6\times 2}{3} = 4 \qquad \ldots(i)$$

On increasing pressure to 2 bar,

	X	+	Y	$\rightleftharpoons$	Z
At equilibrium	$(1-x)$mol		$(3-x)$mol		$(2+x)$mol
	$\frac{2(1-x)}{6-x}$ bar		$\frac{2(3-x)}{6-x}$ bar		$\frac{2(2+x)}{6-x}$ bar

$$K_P = \frac{\frac{2(2+x)}{6-x}}{\left(\frac{2(3-x)}{6-x}\right)\left(\frac{2(1-x)}{6-x}\right)} \qquad \ldots(ii)$$

Upon equating (i) and (ii)

$$\frac{\frac{2(2+x)}{6-x}}{\left(\frac{2(3-x)}{6-x}\right)\left(\frac{2(1-x)}{6-x}\right)}=4$$

$$\frac{(2+x)(6-x)}{(3-x)(1-x)}=8$$

$$\frac{12+4x-x^2}{3-4x+x^2}=8$$

$12 + 4x – x^2 = 24 – 32x + 8x^2$

$9x^2 – 36x + 12 = 0$

$3x^2 – 12x + 4 = 0$ ….(iii)

On solving equation (iii)

x = 0.367

No. of moles of X = 1 – x

= 1 – 0.367

= 0.633 mol

108. When SO_2 is bubbled into an acidic $KMnO_4$ solution, decolorization of the purple solution takes place along with the formation of a manganese compound X. Under neutral conditions, compound X reduces $KMnO_4$ in the presence of zinc oxide to give another manganese compound Y. The oxidation states of manganese in compounds X and Y, respectively are

(A) +7 and +2
(B) +2 and +4
(C) +4 and +7
(D) +2 and +2

Answer (B)

Sol. $2\overset{+7}{Mn}O_4^- + 5SO_2 + 2H_2O \longrightarrow 2\underset{(X)}{\overset{+2}{Mn}}{}^{2+} + 5SO_4^{2-} + 4H^+$

$$3Mn^{2+} + 2\overset{+7}{Mn}O_4^- + 2H_2O \xrightarrow{ZnO} 5\underset{(Y)}{\overset{+4}{Mn}}O_2 + 4H^+$$

The oxidation state of Mn is (X) and (Y) are +2 and +4 respectively.

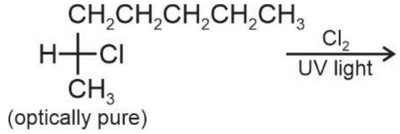
109. The molecule having sp^3d^2 hybridisation is

(A) SF_4
(B) $XeOF_4$
(C) PF_5
(D) BeF_3

Answer (B)

Sol. Molecule	**Hybridisation**
SF_4	sp^3d
$XeOF_4$	sp^3d^2
PF_5	sp^3d
BrF_3	sp^3d

110. The number of possible chiral dichloro products that can be formed when (*R*)-2-chlorohexane reacts with Cl_2 in the presence of UV irradiation, is

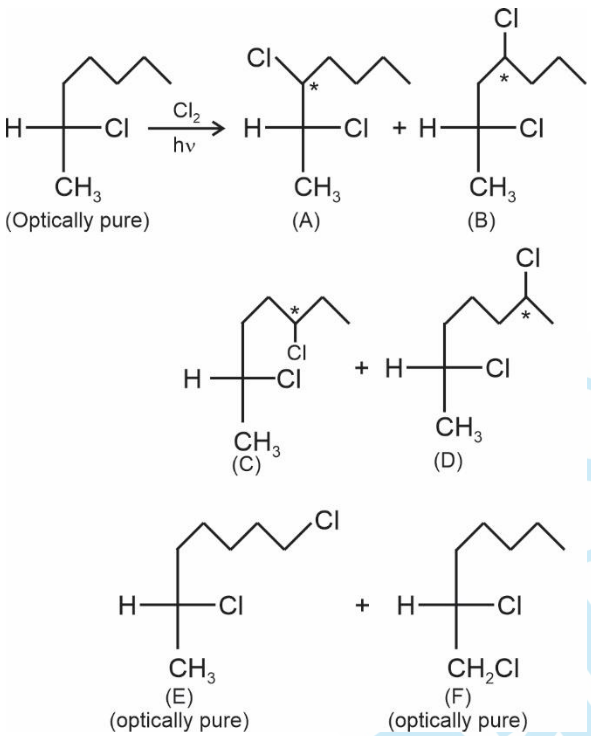


(A) 10 (B) 9

(C) 7 (D) 6

Answer (B)

Sol.



- A, B and C each will form a pair of diastereomers.
- D will give two stereoisomers in which one will be meso (achiral) and other will be optically active (chiral).
- E and F each will give one stereoisomer (chiral).
- Total number of chiral products = 2 + 2 + 2 + 1 + 1 + 1 = 9

PART-II : BIOLOGY

111. Match the molecules in column I with their functional groups in column II

Column I	**Column II**
P. Glycerol	(i) $-COO^-$ and $-OH$
Q. Cysteine	(ii) $-OH$ only
R. Citrate	(iii) $-NH_2$, $-COO^-$ and $-SH$
S. Pyruvate	(iv) $-CHO$, $-OH$ and $-NH_2$
	(v) $-CO$ and $-COO^-$

Choose the **correct** combination.

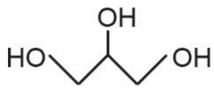
(A) P-iii, Q-iv, R-i, ii; S-v (B) P-i, Q-iii, R-v; S-iv, ii

(C) P-ii, Q-iii, R-i; S-v (D) P-i, ii Q-iv, R-v; S-i

Answer (C)

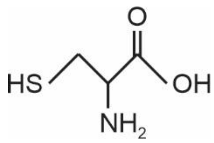
Sol. **Structure** **Functional group**

P. Glycerol



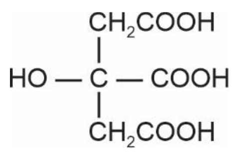
–OH only

Q. Cysteine



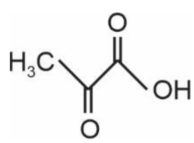
$–NH_2$, $–COO^-$ and SH

R. Citrate



$–COO^-$ and OH

S. Pyruvate

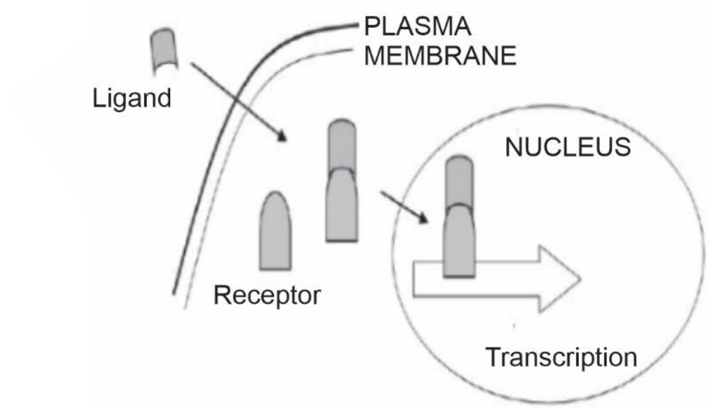


–CO and $–COO^-$

Correct combination is

P(ii), Q(iii), R(i) and S(v)

112. The diagram depicted below pertains to which one of the following signal transduction pathways?



(A) Growth factor signaling

(B) Steroid hormone signaling

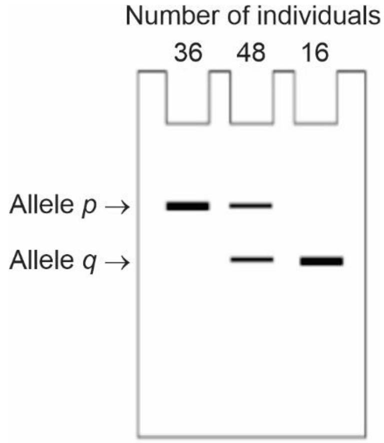
(C) Peptide hormone signaling

(D) Cytokine signaling

Answer (B)

Sol.
- Option (B) is correct because the ligand is fat soluble so it crosses plasma membrane and binds with intracellular receptor to form hormone receptor complex. Steroid hormones are fat-soluble hormones.
- Option (A) and (C) are incorrect because growth factors are proteins and proteins, peptide hormones bind to membrane-bound receptors.
- Option (D) is incorrect because cytokines do not interact via intracellular receptors.

113. The result of an electrophoretic analysis of DNA fragments of two alleles (*p* and *q*) of a locus in a population is given in the figure below. Based on this, what would be the approximate allele frequencies?



(A) $p = 0.33$; $q = 0.67$
(B) $p = 0.4$; $q = 0.6$
(C) $p = 0.5$; $q = 0.5$
(D) $p = 0.6$; $q = 0.4$

Answer (D)

Sol. The correct answer is option (D). The total individuals in the given population will be 100 as 36 + 48 + 16.

36 individuals only have allele p which means they are p^2 individuals.

$p^2 = 36$

$\therefore \quad p = \sqrt{\frac{36}{100}} = 0.6$

$p + q = 1$

$\therefore \quad q = 1 - p = 0.4$

$\therefore \quad p = 0.6$ and $q = 0.4$

$2pq = 2 \times 0.6 \times 0.4 = 0.48 = 48\%$

114. In a PCR reaction, only the reverse primer was omitted from the reaction mixture. How many single-stranded (ss) and double-stranded (ds) DNA molecules, excluding the primer molecules, will be present in the mixture at the end of 5 cycles, given 10 molecules of double-stranded DNA were used as template?

(A) 50 ss and 10 ds molecules
(B) 320 ds and ss molecules
(C) Zero ds and 160 ss molecules
(D) 10 ss and 50 ds molecules

Answer (A)

Sol. Option (A) is correct because 10 molecules of double-stranded DNA after 5 cycles will produce 10 ds DNA and 50 ss DNA molecules.

After the first step of denaturation, 20 ss DNA molecules are formed. The forward primers attach to 10 ss DNA molecules, but as reverse primers are absent the other 10 ss DNA molecules stay as such.

At the end of the first cycle of PCR, 10 ds DNA and 10 ss DNA molecules will be formed.

The same repeats for 4 more cycles and finally 10 ds DNA and 50 ss DNA molecules are formed.

115. A peptide sequence <u>M</u>YKS<u>V</u>LDSTKI forms an α-helix in a protein, Prl1 whereas it forms a β-sheet in another protein, Pro2. Based on this information, the distance between the Cα atom of the residue <u>M</u> and the Cα atom of the residue <u>V</u> would be

(A) Equal in both
(B) Less in Pro1 than in Pro2
(C) More in Pro1 than in Pro2
(D) Dependent on the nature of the flanking residues

Answer (B)

Sol. The correct answer is option (B) as the backbone of the α helix is compact while the peptide backbone of the β-sheet is highly extended. A β-turn involves four aminoacyl residues and a complete turn of α helix contains an average of 3.6 aminoacyl residues.

116. Match the following enzymes in column I with their substrates in column II and corresponding roles in column III

	Column I		Column II		Column III
P.	Triosephosphate isomerase	i.	Protein	a.	Photosynthesis
Q.	Trypsin	ii.	CO_2	b.	Digestion
R.	Carbonic anhydrase	iii.	Three-carbon sugar	c.	Blood pH
S.	Rubisco	iv.	Five-carbon sugar	d.	Glycolysis

Choose the **correct** combination.

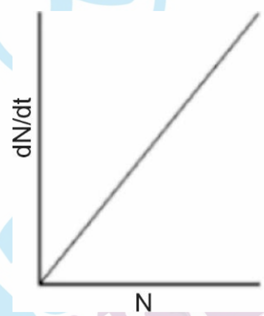
(A) P-iii-d; Q-i-b; R-ii-c; S-iv-a
(B) P-iv-a; Q-i-c; R-iii-c; S-ii-a
(C) P-i-d; Q-iii-b; R-iv-a; S-ii-c
(D) P-iii-d; Q-iv-b; R-ii-c; S-i-a

Answer (A)

Sol. Option (A) is correct because:

- Triosephosphate isomerase is a protein which acts on three-carbon sugar and play a role in glycolysis.
- Trypsin is a protein which plays an important role in digestion.
- Carbonic anhydrase is a enzyme used in transport of CO_2 and maintains blood pH.
- Rubisco is a protein which acts on a five-carbon sugar and it plays an important role in photosynthesis.

117. The graph shown below, where N denotes population size and t denotes time, represents.



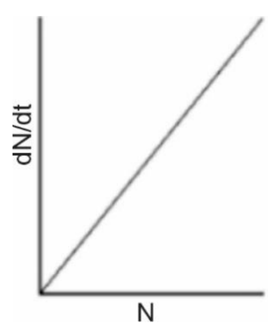
(A) A linear growth model
(B) Logistic growth model
(C) An exponential growth model
(D) An Allee effect model

Answer (C)

Sol. Exponential population growth

$$\frac{dN}{dt} = rN$$

Graph of $\frac{dN}{dt}$ versus N shows exponential growth



118. An asynchronous population of actively dividing cells are following a cell-cycle regime of 2h of M phase, 10h of G_1 phase, 6h of S phase and 6h of G_2 phase. If the population is exposed to tritiated thymidine for 30 minutes and two cells are randomly selected, then what would be the probability of the first cell being in the M-phase and second cell showing radioactive signal?

(A) 1/3
(B) 1/4
(C) 1/12
(D) 1/48

Answer (D)

Sol. Total time taken by the cell to complete the cycle = 10 + 6 + 6 + 2 = 24 h

Probability of being a cell in M phase $= \frac{2}{24} = \frac{1}{12}$

Since cell is exposed to tritiated thymidine interacts in S-phase, so the probability of cell showing radioactive signal $= \frac{6}{24}$

Therefore, overall probability $= \frac{1}{12} \times \frac{6}{24}$

$= \frac{1}{48}$

119. The kinetic parameters of three enzymes E_1, E_2 and E_3 are given below.

	k_{cat}	K_M
E_1	0.1	25
E_2	0.01	2.5
E_3	0.001	0.25

Which one of the following orders is correct regarding their specificity constants?

(A) $E_1 > E_2 > E_3$
(B) $E_1 < E_2 < E_3$
(C) $E_1 = E_2 = E_3$
(D) $E_1 > E_2 < E_3$

Answer (C)

Sol.
- Option (C) is correct because the specificity constants of E_1, E_2 and E_3 are the same, that is, 0.004.
- The ratio of K_{cat}/K_M is known as the specificity constant and is a measure of the relative rate of reaction at low substrate concentrations.

120. Choose the correct combination of net charges for the following four tetrapeptides at pH 7.0.

(i) Ala-Glu-Glu-Gly
(ii) Arg-Gly-Lys-Ser
(iii) Gly-Ser-Gly-Ala
(iv) Ala-Asp-Ala-Gly

(A) (i)–2, (ii)+2, (iii)0, (iv)–1
(B) (i)–2, (ii)+1, (iii)+1, (iv)–1
(C) (i)–1, (ii)+2, (iii)0, (iv)–2
(D) (i)–1, (ii)0, (iii)+2, (iv)+1

Answer (A)

Sol. Option (A) is correct because at pH 7.0,

- Alanine and glycine are neutral amino acids. Glutamic acid has –1 charge so total charge in (i) is –2.
- Arginine and lysine have +1 charge each so total charge in (ii) is +2.
- Glycine, serine and alanine are neutral amino acids so net charge in (iii) is 0.
- Aspartic acid has –1 charge. Alanine and glycine are neutral so net charge in (iv) is –1.

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