Olympiad-Classroom Assessment Practice Sheet
O-CAPS-03 : Pre-Regional Mathematics Olympiad (PRMO)
(For VIII, IX, X Studying Students)

Topics Covered :
Mathematics : Geometry, Trigonometry

1. In the plane of given $\triangle ABC$, if $P$ is a point satisfying geometric condition $[\triangle PAB] = [\triangle PAC]$ where $[.]$ represents area enclosed. Then the curve traced by all such positions of $P$ if divides the plane in $m$ parts then find $5m$.

2. If the ratio of areas of circles circumscribing the $\triangle ABC$ and the circle circumscribing the $\triangle$ formed by joining reflection of its orthocentre about the sides respectively is $n$ then find $15n$.

3. For $x, y, z$ being sides of $\triangle$. If the least natural $n$ satisfies:
$$\frac{x^2(y-z)+y^2(z-x)+z^2(x-y)}{xyz} < n$$
then find $10n$.

4. Three circles, $C_1, C_2, C_3$ are drawn by taking the sides of acute $\triangle ABC$ as diameters. If the common chords of $C_1$ and $C_2$, $C_2$ and $C_3$, $C_1$ and $C_3$ meet at a point $N$ inside the triangle and $BN$ extended meets $AC$ at $D$ then find $\angle BDA$ in degrees.

5. Find the largest angle (in degrees) of a triangle having mid-points of altitudes as collinear.

6. A right triangle $D$ is divided by its altitude into two triangles $D_1$ and $D_2$. If the sum of inradius of $D_1$ and $D$ is 10 units and length of altitude of $D$ is 15 units then find the (inradius of $D_2$) $\times$ 4.

7. If $\sin^2 \theta = \cos^3 \theta$, then find the value of $\cot^6 \theta - \cot^2 \theta$.

8. If $\sin^3 x + \sin^3 y + 3\sin x \sin y = 1$ for some $x, y \in R$, then find the sum of the fourth power of least and maximum value of the expression $\sin x + \sin y$.

9. If the radius of a circle is 8 units and a triangle $ABC$ is constructed by taking three points on it such that the inradius of triangle is given by $r \in N$ then find the sum of square of naturals that $r$ can take.
10. For $-1 \leq x \leq 1$, find value of (number of roots of the equation $3x - 4x^3 = x^6 + \sqrt{2}$ satisfying the given interval) + 15.

11. If $\sin^2 x - 3\sin x + 2 = 0$ then find $17\sin^3 x + 2$.

12. For $0 \leq a, b \leq 3$ and the equation $x^2 + 4 - 3\cos(ax + b) - 2x$ has at least one solution, then find the value of $\frac{99(a+b)}{\pi}$.

13. If number of ordered pairs $(0, x)$ which satisfy $\tan^2 \theta + \sec \theta = 6x - 11 - x^2$ are given by $m$ then find $m^{2018} + 10$.

14. In a $\triangle ABC$ if $D$ divides $BC$ internally such that $\frac{BD}{DC} = \frac{3}{4}$ and length of $AD = x$, $AB = 8$, $BC = 5$, $CA = 4$ then find the value of $x^2$ to nearest integer.

15. For a hexagon inscribed in a circle of radius $m$, two of its sides have unit length, two have length 2 units, and remaining two as length 3 units, then find the value of $10m^3 - 35m$.

16. For an equilateral $\triangle ABC$ with side length 6 units, if $h_1, h_2$ and $h_3$ as lengths of perpendiculars upon sides then find $(h_1 + h_2 + h_3)^2$.

17. In a square $ABCD$, $E$ is the mid-point of $CB$, $AF$ is drawn perpendicular to $DE$. If the side of the square is 29 cm. Find the length of $FB$ in cm.

18. In a scalene triangle $ABC$, the three altitudes drawn from vertices intersect at a point $H$. If $BC = a$, $CA = b$, $AB = c$, $AH = x$, $BH = y$ and $CH = z$ then find the value of $\frac{12(xyz)}{abc} \left(\frac{a}{x} + \frac{b}{y} + \frac{c}{z}\right)$.

19. A semi-circle is drawn outwardly on chord $AB$ of the circle with centre $O$ and radius $\sqrt{8}$ cm. The perpendicular from $O$ to $AB$ meets the semi-circle at $C$. Find the length of chord $AB$ so that the line segment $OC$ has the maximum length.

20. As shown in the figure, triangle $ABC$ is divided into six smaller triangles by lines drawn from the vertices through a common interior point $P$. The areas of these smaller triangles are as indicated in the figure. Then find the value of $x - y$.

21. In a scalene triangle $ABC$, with $AB = c$, $BC = a$, $CA = b$ and $a^2 = b(b + c)$, $\angle A$ and $\angle B$ is related by $3A = (p + 1)B$ then find the value of $p$.

22. Let $A$ be one of the two points of intersection of two circles with radii 2 cm and $\sqrt{3}$ cm and centres $X$ and $Y$, respectively. The angle $XAY$ is $120^\circ$ and the tangents at $A$ to these two circles meet the circles again at points $B$ and $C$. If the square of circumradius of triangle $ABC$ is $(a - b\sqrt{c})$ c.m. then value of $a + b + c$. 

---

Aakash Educational Services Limited - Regd. Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph.011-47623456 - 2 -
23. Let \(ABCD\) be a trapezium with \(AB \parallel CD\) such that

(i) Its vertices \(A, B, C\) and \(D\) lies on a circle with centre \(O\).
(ii) Its diagonals \(AC\) and \(BD\) intersect at point \(M\) and \(\angle AMD = 60^\circ\)
(iii) \(MO = 10, BM = 4\) and \(DM = 6\)

If the sum of length of \(AB\) and \(CD\) is \(a\sqrt{b}\) where ‘\(a\)’ and ‘\(b\)’ are integers such that ‘\(b\)’ is least possible.
Find \(a + b\).

24. The diagonals \(AC\) and \(BD\) of a cyclic quadrilateral \(ABCD\) meet at right angles in \(E\). If the radius of the circumscribing circle is 3 cm, find the value of \(\frac{EA^2 + EB^2 + EC^2 + ED^2}{2}\) in square cm.

25. The trapezium \(ABCD\) has area \(S\), \(AB = b, CD = a\) with \(a < b\) and \(AB \parallel CD\). Two diagonals \(AC\) and \(BC\) meet at \(O\). The area of \(\triangle BOC\) is \(\frac{2}{9} S\). Find \((a + b)\) (where \(a\) and \(b\) are co-primes).

26. For some positive integer \(p\), there is a quadrilateral \(ABCD\) with positive integer side length, perimeter \(p\), right angles at \(B\) and \(C\), \(AB = 2\) and \(CD = AD\). How many different values of \(p < 2018\) are possible?

27. \(ABCD\) is a square with side length 15 units. \(E\) is a point inside the square such that \(\angle EBC = \angle ECB = 15^\circ\). Find the greatest integer value less than or equal to sum of lengths \(AE\) and \(ED\).

28. A rectangle \(ABCD\) is inscribed in the circle with centre at \(O\). The length of side \(AB\) is greater than side \(BC\). The ratio of area of the circle to the rectangle \(ABCD\) is \(\pi : \sqrt{3}\). The line segment \(DE\) intersects \(AB\) at \(E\) such that \(\angle ODC = \angle ADE\). If the ratio \(AE : AD = \sqrt{\frac{p}{q}}\), where \(\gcd(p, q) = 1\) then find \(q - p\).

29. In the figure shown, \(ABCD\) is a square of side length \(\sqrt{85}\) units. Two semicircles have been drawn on diameters \(AB\) and \(AD\). If the area of the shaded region is \(\frac{p}{q}\), where \(\gcd(p, q) = 1\) then find \(p + q\).

30. Consider \(f(x) = |\sin x| + |\cos x|\) and \(g(x) = |\sin x| + |\cos x - 1|\) defined for all real \(x\). Let \(T_1\) and \(T_2\) be the least positive real numbers such that \(f(x + T_1) = f(x)\) and \(g(x + T_2) = g(x)\) for all real \(x\). Find the value of \(p + q\) if \(T_1 + T_2 = \frac{p\pi}{q}\), where \(\gcd(p, q) = 1\).
### ANSWERS

<table>
<thead>
<tr>
<th>Question Level</th>
<th>Question Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy (E) - 07</td>
<td>2, 3, 5, 8, 10, 14, 26</td>
</tr>
<tr>
<td>Moderate (M) - 11</td>
<td>6, 9, 11, 13, 15, 17, 19, 21, 24, 25, 27</td>
</tr>
<tr>
<td>Difficult (D) - 12</td>
<td>1, 4, 7, 12, 16, 18, 20, 22, 23, 28, 29, 30</td>
</tr>
</tbody>
</table>
ANSWERS & SOLUTIONS

1. Answer (20)

\[ \text{Area}(\triangle PBD) = \text{Area}(\triangle PCD) \]

As, \( BD = CD \) and height of both the triangles is same.

\[ \Rightarrow [\triangle PAB] = [\triangle PAC] \]

So \( P \) will lie on median \( AD \) and extended

Again by a line passing through \( A \) and \( \parallel BC \)

\[ \Rightarrow \text{For any point} \ P \]

\[ [\triangle PAB] = [\triangle PAC] \]

As \( PA \) is common base between \( \parallel \) lines.

\[ \Rightarrow m = 4 \]

2. Answer (15)

As reflections of the orthocentre lies on circumcircle of \( \triangle ABC \) hence ratio stands as 1.

3. Answer (10)

The equation inequality can be written as

\[ \frac{(x - y)(y - z)(z - x)}{xyz} < n \]

LHS:

\[ |x - y| < z \quad |y - z| < x \]
\[ |z - x| < y \quad \Rightarrow n = 1 \]

4. Answer (90)

If circle is drawn by taking \( AB \) as diameter then it meets \( AC \) and \( BC \) at \( D, E \) such that \( BD \) and \( AE \) are altitudes.
The three altitudes of $\triangle ABC$ will be common chords of circles hence $\angle BDA = 90^\circ$

5. Answer (90)

As the mid-points of altitudes lies on the sides of medial $\Delta$, hence they will be collinear if they lie on same side like $H_1, H_2$ are endpoints and $H_3$ lies between $H_1$ and $H_2$. The only possibility is right angled $\Delta$.

6. Answer (20)

As for $\triangle ABC$:

Inradius $(r) = \frac{AB + BC - CA}{2}$

Similarly $r_1$ and $r_2$:

Now, $r + r_1 + r_2 = BM$

$r = 15 - 10 = 5$

7. Answer (01)

8. Answer (17)

$\sin^3 x + \sin^3 y + (-1)^3 = -3\sin x \sin y$

Hence, either $\sin x + \sin y = 1$

as $\sin x = \sin y = -1$

Therefore sum will be $(-2)^4 + (-1)^4 = 17$.

9. Answer (30)

For any such $\triangle ABC$:

$$R \geq 2r$$

Hence, $0 < r \leq \frac{R}{2}$ $\Rightarrow$ $r = 1, 2, 3, 4$

10. Answer (15)

Let $x = \sin \theta$

$\Rightarrow \sin 3\theta = x^3 + \sqrt{2}$

Therefore LHS $\in [-1, 1]$ but RHS $> \sqrt{2}$

Hence, no root in interval.

11. Answer (19)

$(\sin x - 1)(\sin x - 2) = 0$

As $\sin x = 1$ is only valid

Hence, $17\sin^3 x + 2 = 19$

12. Answer (99)

$x^2 - 4 - 2x = -3 \cos (ax - b)$

$-1 \leq \cos (ax + b) \leq 1$

solution only possible.

if $a + b = \pi, 3\pi, 5\pi$

$a + b = \pi$

$\frac{a + b}{\pi} = 1$

13. Answer (10)

Let $\tan^2 \theta + \sec \theta = 0$

$\Rightarrow (\sec \theta + \sec \theta - 1)$

$\Rightarrow \left(\sec \theta + \frac{1}{2}\right)^2 = \frac{5}{4} \geq \frac{5}{4}$

RHS:

$-2 - (x - 3)^2 \leq -2$

Hence, $m^{2018} + 10 = 10$

14. Answer (49)

By theorem on sections using cosine rule:

$(8)^2 \times 4 + (4)^2 \times 3 = 5(4 \times 3 + x^2)$
15. Answer (15)

\[ \frac{\alpha_1 + \alpha_2}{2} = 90^\circ - \frac{\alpha_3}{2} \]
\[ \cos \left( \frac{\alpha_1 + \alpha_2}{2} \right) = \sin \frac{\alpha_3}{2} \]
\[ \sin \frac{\alpha_3}{2} = \frac{1}{2m}, \sin \frac{\alpha_2}{2} = \frac{1}{m} \]
\[ \sin \frac{\alpha_3}{2} = \frac{3}{2m}, \text{ now substituting the value of } m. \]

16. Answer (27)

As \( \frac{1}{2} \times 6(h_1 + h_2 + h_3) = \frac{\sqrt{3}}{4} \times 36 \)
\[ h_1 + h_2 + h_3 = 3\sqrt{3} \]

17. Answer (29)

Construction: Produce \( DE \) to cut \( AB \) produce at \( M \).

In \( \triangle ECD \) and \( \triangle EBM \)
\[ \angle 1 = \angle 2 \]
\[ EC = EB \]
\[ \angle ECD = \angle FBM = 90^\circ \]

By ASA congruences
\( \triangle ECD \equiv \triangle EBM \)
\[ \therefore CD = BM \]

But \( CD = AB \)
\[ \Rightarrow BM = AB \]
\[ \Rightarrow B \text{ is the mid-point of hypotenuse } AM \]
\[ \Rightarrow FB = BM = 29 \]

18. Answer (12)

\[ [ABC] = [BHC] + [CHA] + [AHB] \]
\[ \Rightarrow \frac{abc}{4R} = \frac{1}{2} yz \sin(\pi - A) + \frac{1}{2} zx \sin(\pi - B) + \frac{1}{2} xy \sin(\pi - C) \]
\[ = \frac{1}{2} yz \sin A + \frac{1}{2} zx \sin B + \frac{1}{2} xy \sin C \]
\[ = \frac{1}{2} yz \sin A + \frac{1}{2} zx \sin B + \frac{1}{2} xy \sin C \]
\[ = \frac{1}{2} yz \left[ \frac{a}{2Rx} + \frac{b}{2Ry} + \frac{c}{2Rz} \right] \]
\[ \Rightarrow \frac{abc}{4R} = \frac{xyz}{4R} \left( \frac{a}{x} + \frac{b}{y} + \frac{c}{z} \right) \]
\[ \Rightarrow \frac{xyz}{abc} \left( \frac{a}{x} + \frac{b}{y} + \frac{c}{z} \right) = 1 \]

19. Answer (04)

Let \( OD = \sqrt{a} \) cm
\[ AD = \sqrt{8 - a} \text{ cm } \]
\[ BC = DC = \sqrt{8 - a} \text{ cm } \]
\[ OC^2 = (OD + DC)^2 = (\sqrt{a} + \sqrt{8 - a})^2 \]
\[ = 8 + 2\sqrt{a(8 - a)} \]

For \( OC \) to be maximum, \( a = 8 - a \)
\[ \Rightarrow a = 4 \]
\[ \Rightarrow AD = \sqrt{8 - 4} = 2 \text{ cm } \]
\[ AB = 4 \text{ cm } \]

20. Answer (40)

\[ \frac{AF}{FB} = \frac{84 + 35 + x}{56 + 40 + y} \]
\[ \Rightarrow 2x - 3y = 50 \]
\[ \frac{BD}{DC} = \frac{40}{y} = \frac{84 + 56 + 40}{x + 35 + y} \]
\[ \Rightarrow 7y - 2x = 70 \]
\[ \Rightarrow x = 70, \ y = 30 \]
21. Answer (05)

CA is produced to D such that AD = AB

\[ x = \frac{A}{2} \]  \( \ldots \)(i)

Also, \( a^2 = b(b + c) \)

\[ \Rightarrow \frac{a}{b} = \frac{b + c}{a} \]

\[ \Rightarrow \frac{BC}{AC} = \frac{DC}{BC} \]

\[ \Rightarrow \triangle ACB \sim \triangle BCD \]

\[ \Rightarrow \angle CDB = \angle CBA = B \]  \( \ldots \)(ii)

(i) and (ii)

\[ \Rightarrow A = 2B \]

22. Answer (12)

Perpendicular bisector of side AB will pass through X and will also be parallel to AY. Similarly, perpendicular bisector of side AC will pass through Y and will also be parallel to AX. Hence, for the circumcentre P, AXPY is a parallelogram.

Circumradius,

\[ AP = \sqrt{(AX)^2 + (AY)^2 + 2(AX)(AY)\cos(XAY)} \]

\[ = \sqrt{4 + 3 + 2 \times 2 \times \sqrt{3} \times \cos 120^\circ} \]

\[ = \sqrt{7 - 2\sqrt{3}} \]

23. Answer (13)

Since ABCD is a cyclic quadrilateral, \( \angle DCA = \angle DBA \).

Since, \( AB \parallel CD \), \( \angle DCA = \angle CAB \)

So, \( \triangle AMB \) is isosceles

Similarly, \( \triangle CMD \) is isosceles.

\[ OA = OB \]

\[ \therefore \triangle AMO \cong \triangle BMO \]

\[ \Rightarrow \angle AMO = \angle BMO \]

Since, \( \angle AMB = 90^\circ \)

\[ \angle AMB = 120^\circ \]

\[ \Rightarrow \angle AMO = \angle BMO = 60^\circ \]

Hence, \( \triangle AMX \) and \( \triangle BMX \) are congruent and have angles 30°, 60° and 90°.

Similarly, \( \triangle DMY \) and \( \triangle CMY \) are congruent \( \triangle \)'s

\[ AB = 4\sqrt{3} \]

\[ DC = 6\sqrt{3} \]

24. Answer (18)

Let O be the centre of the circle

\[ EA^2 + EC^2 = (EP + PA)^2 + (PC - PE)^2 \]

\[ = EP^2 + PA^2 + PC^2 + PE^2 + 2EP.PA - 2PC.PE \]

\[ = 2(PA^2 + PE^2), \text{ (as PA = PC)} \]

Similarly, \( EB^2 + ED^2 = 2(QD^2 + EQ^2) \)

\[ \therefore \quad EA^2 + EB^2 + EC^2 + ED^2 = 2(PA^2 + PE^2) + 2(QD^2 + OQ^2) \]

\[ = 2(PA^2 + OQ^2) + 2(QD^2 + OP^2) = 2(PA^2 + OP^2) + 2(QD^2 + OQ^2) \]

\[ = 2(OA^2 + OD^2) \]

\[ = 2(3^2 + 3^2) = 36 \]
25. Answer (03)

Let areas of $\triangle DOC = S_1$ and $\triangle AOB = S_2$

$$S_1 + S_2 + \frac{2}{9} S + \frac{2}{9} S = S$$

$\therefore$ $S_1 + S_2 = S - \frac{4}{9} S$

$\Rightarrow$ $S_1 + S_2 = \frac{5}{9} S$

and $S_1 S_2 = \left(\frac{2}{9} S\right)^2$

On solving,

$S_1 = \frac{1}{9} S$ and $S_2 = \frac{4}{9} S$

$\therefore \frac{a}{b} = \sqrt{\frac{S_1}{S_2}} = \frac{1}{2}$

$\therefore a + b = 3$

26. Answer (31)

Let $BC = x$ and $CD = AD = y$

By Pythagoras theorem

$x^2 + (y - 2)^2 = y^2$

$x^2 + y^2 + 4 - 4y = y^2$

$x^2 - 4y + 4 = 0$

$x^2 = 4(y - 1)$

$p = 2 + x + 2y$

$p = 2 + 2\sqrt{y - 1} + 2y < 2018$

$y = 31^2 + 1$ satisfy

$\therefore y = n^2 + 1$

where $1 \leq n \leq 31$

$\Rightarrow$ Number of possible values of $p$ are 31.

27. Answer (30)

Draw $\triangle AFB \cong \triangle BEC$.

$\angle FBE = 90^\circ - (15^\circ + 15^\circ) = 60^\circ$ and $FB = EB$

$\Rightarrow \triangle FBE$ is equilateral

$\Rightarrow \angle FAE = \angle AEF = x$ (say)

In $\triangle ABE$, sum of angles $= 75^\circ + (15^\circ + x)$

$+ (60^\circ + x) = 180^\circ$

$\Rightarrow x = 15^\circ$

$\Rightarrow \angle AEB = 60^\circ + 15^\circ = 75^\circ = \angle ABE$

$\Rightarrow AE = AB = 15$ units.

Similarly, $DE = DC = 15$ units.

28. Answer (02)

Let $AB = a$, $BC = b$

$\angle ODC = \angle ADE = \theta$

$\pi r^2 = \pi a b$

$ab = \sqrt{3}$

$\Rightarrow \frac{a}{2r} \times \frac{b}{2r} = \frac{\sqrt{3}}{4}$

$\Rightarrow 2 \sin \theta \cos \theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30^\circ$

$\frac{AE}{AD} = \tan \theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$
1. **PQRS** is a trapezium with \( PQ \parallel RS \) and the diagonals intersect at the point \( M \). The area of \( \triangle PQM \) is 32 \( \text{cm}^2 \) and the area of \( \triangle RSM \) is 50 \( \text{cm}^2 \). If area of the trapezium \( PQRS \) is \( \lambda \) \( \text{cm}^2 \), then \( \sqrt{\frac{\lambda}{2}} \) is equal to

2. **PQR** is a right angled triangle with \( \angle PQR = 90^\circ \), \( PR = 2 \) units, \( QR = 1 \) unit and \( QS \) is perpendicular to \( PR \). If the area of rectangle with \( QS \) as one of its diagonals is \( \mu \), then \( 512 \mu^2 \) is equal to

3. A 20 cm \( \times \) 20 cm square is divided into \( n^2 \) congruent squares by equispaced lines parallel to its sides. Circles are inscribed in each of the squares. If the sum of the areas of the circles be \( \pi A \) \( \text{cm}^2 \), then \( \sqrt{A} \) is equal to

4. **AB** is a diameter of a circle of radius 24 units. **CD** is a chord, perpendicular to **AB** that cuts **AB** at **E**. If the arc **CAD** is \( \frac{2}{3} \) of the circumference of the circle, then the length of **AE** is equal to

5. In a trapezium **ABCD** with \( AB \parallel CD \), \( AB = 20 \) cm, \( CD = 3 \) cm. \( \angle ABC = 32^\circ \) and \( \angle BAD = 58^\circ \). The distance from the mid-point of **AB** to the mid-point of **CD** is \( d \), then \( 2d \) is equal to

6. An isosceles right triangle is removed from each corner (see figure) of a square piece of paper so that a rectangle remains. If the sum of the areas of the cut off pieces is 242 sq. units, then the length of the diagonal of the rectangle is

---

**Topics Covered:**

Mathematics : Geometry, Trigonometry
7. The quarter circle shown in the figure has centre $C$ and radius 10 units. If the perimeter of the rectangle $CPQR$ is 28 units. If the perimeter of the shaded region is $(m\pi + n)$ where $m$ and $n$ are natural numbers and co-prime to each other, then $(m + n)$ is

![Diagram](image)

8. $ABCD$ is a square and $P$ is a point on side $AD$ such that $\triangle PAB$ has area 62 sq. units and $\triangle PCD$ has area 10 sq. units. The length of the side of the square is $

9. $P$ and $Q$ are points on the sides $AB$ and $AC$ respectively of $\triangle ABC$ such that $BP = CQ = \lambda$, $PA = 6$ cm, $AQ = 20$ cm, $BC = 25$ cm. If $\triangle PAQ$ and the quadrilateral $BPQC$ have equal areas, then the value of $\lambda$ is

10. $\triangle ABC$ is isosceles with $AB = AC = 10$ cm and $BC = 6$ cm, $M$ is the mid-point of $AB$. Let $l$ be the line through $A$ parallel to $BC$. If $l$ intersects the circle through $A$, $C$ and $M$ at $D$, then $3(AD)$ is equal to

11. A sequence of equilateral triangles is drawn. The altitude of each is $\sqrt{3}$ times the altitude of the preceding triangle. The difference between the areas of the first triangle and sixth triangle is $968\sqrt{3}$ sq. units. The perimeter of the first triangle is

12. The value of $\frac{\tan 205^\circ - \tan 115^\circ}{\tan 245^\circ + \tan 335^\circ} = \sec x^\circ$, where $0 < x < 99$, then $x$ is equal to

13. $x_1$ and $x_2$ are the two values of $x$ in $[0, 2\pi]$ for which $\tan x = 100$, then $\left|\tan \frac{x_1}{2} \cdot \tan \frac{x_2}{2}\right|$ is equal to

14. If $\frac{\sin x}{\sin y} = \frac{1}{2} \cdot \frac{\cos x}{\cos y} = \frac{3}{2} \cdot \frac{1}{\sin x}$ where $x, y \in \left(0, \frac{\pi}{2}\right)$, then the value of $\sec^2(x + y)$ is

15. The total number of solutions of $\left|\cot x \right| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 3\pi]$, is

16. If $x, y \in [0, 2\pi]$, then the total number of ordered pairs $(x, y)$ satisfying the equation $\sin x \cdot \cos y = 1$, is equal to

17. If $ABC$ is a triangle which is not right angled, the value of $\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B}$ is equal to

18. $\sin x + \sin^2 x = 1$, and the value of $\sin x$ is $\lambda\sin(\mu^\circ)$ $\lambda \in \mathbb{N}$ and $0 < \mu < 90$, then $\lambda \mu$ is equal to

19. If $\tan 3A = 4\tan A$ ($\tan A \neq 0$), then the value of $\frac{30\sin 3A}{\sin A}$ is

20. A quadrilateral inscribed in the circle has side lengths $\sqrt{20}$, $\sqrt{99}$, $\sqrt{22}$ and $\sqrt{97}$ in the order. Taking $\pi = \frac{22}{7}$, if the area of circle is $\frac{a}{b}$ where $a$ and $b$ are co-prime then find $a - 70b$.

21. On sides $BC$ and $CD$ of a square $ABCD$, $P$ and $Q$ are points respectively, such that $AP = 8$ cm, $PQ = 6$ cm and $AQ = 10$ cm. If the side of the square is given as $\frac{p}{\sqrt{q}}$, where $p$ and $q$ are integers and $q$ is least possible then find the value of $p + q$. 

Aakash Educational Services Limited - Regd. Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph.011-47623456 - 2 -
<table>
<thead>
<tr>
<th>Question</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.</td>
<td>Triangle $ABC$ has $AB = 90$, $BC = 50$ and $CA = 70$. A circle is drawn with centre $P$ on $AB$ such that $CA$ and $CB$ are tangents to the circle and $2AP = x$. Find $\frac{x}{5}$.</td>
</tr>
<tr>
<td>23.</td>
<td>Two circles with radii 9 units and 16 units touch each other externally. Let $r$ be the radius of the circle that touches these two circles externally as well as a common tangent to the two circles. If $\sqrt{r} = \frac{p}{q}$, where $gcd(p, q) = 1$, then find the value of $(p + q)$.</td>
</tr>
<tr>
<td>24.</td>
<td>The chords $ED$ and $AB$ of the circle $AEBD$ meet at right angle at a point $F$ such that $EF = 6$, $AF = 2$ and $FD = 4$ and the radius of the circle is $r$, then find $r^2$.</td>
</tr>
<tr>
<td>25.</td>
<td>Let $f(x) = \cos\left(\sin\left(\frac{x}{2}\right)\right)$ and $T$ be a real number such that $f(x + T) = f(x)$ for all real values of $x$. If $T = \frac{k\pi}{16}$, then find the smallest positive value of $k$.</td>
</tr>
<tr>
<td>26.</td>
<td>In $\triangle ABC$, $D$ is the mid-point of side $BC$, $E$ is the mid-point of $AD$, $F$ is the mid-point of $BE$, and $G$ is the mid-point of $FC$. If the ratio of area($\triangle ABC$) to area($\triangle EFG$) is $p : q$, where $p$ and $q$ are co-primes, then find $p + q$.</td>
</tr>
<tr>
<td>27.</td>
<td>A hexagon is drawn in a plane such that all its interior angles are equal and all its sides have unequal positive integer values. Find the least perimeter of such a hexagon.</td>
</tr>
<tr>
<td>28.</td>
<td>In a triangle $ABC$, the lengths of medians through vertices $B$ and $C$ are respectively 9 units and 12 units. If $AG = BC$, where $G$ is the centroid of $\triangle ABC$, then find $\frac{BC^2}{2}$.</td>
</tr>
<tr>
<td>29.</td>
<td>The three altitudes of a triangle $ABC$ has lengths 3 units, 4 units and $\frac{12}{5}$ units. If the area of the incircle of the triangle $ABC$ is $s$ square units then find the greatest integer value less than or equal to $s$.</td>
</tr>
<tr>
<td>30.</td>
<td>In a triangle $ABC$, $\angle A = 30^\circ$, $BC^2 = 4\left(10 - 3\sqrt{3}\right)$ and $AC : AB = 1 : 3$. If the square of the area of the triangle $ABC$ is $\frac{p}{q}$, where $gcd(p, q) = 1$, then find $p + q$.</td>
</tr>
</tbody>
</table>
# Olympiad-Classroom Assessment Practice Sheet

**O-CAPS-04 : Pre-Regional Mathematics Olympiad (PRMO)**

(For VIII, IX, X Studying Students)

## ANSWERS

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Question Level</th>
<th>Easy (E) - 04</th>
<th>Moderate (M) - 13</th>
<th>Difficult (D) - 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (09) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. (54) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. (10) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. (36) (E)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. (17) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. (22) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. (21) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. (12) (E)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. (04) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. (25) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. (12) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. (50) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. (01) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14. (16) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15. (02) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16. (03) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17. (02) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18. (36) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19. (80) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20. (47) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21. (49) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22. (21) (E)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23. (19) (E)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24. (50) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25. (32) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26. (09) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27. (21) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28. (50) (M)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29. (03) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30. (10) (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Question Level:**
- **Easy (E) - 04:** 4, 8, 22, 23
- **Moderate (M) - 13:** 1, 2, 3, 5, 6, 9, 11, 12, 19, 21, 25, 26, 28
- **Difficult (D) - 13:** 7, 10, 13, 14, 15, 16, 17, 18, 20, 24, 27, 29, 30
Olympiad-Classroom Assessment Practice Sheet
O-CAPS-04 : Pre-Regional Mathematics Olympiad (PRMO)
(For VIII, IX, X Studying Students)

ANSWERS & SOLUTIONS

1. Answer (09)

\[ \triangle PQM \sim \triangle RSM \]

\[ \Rightarrow \frac{\text{ar}(\triangle PQM)}{\text{ar}(\triangle RSM)} = \frac{PQ^2}{RS^2} = \frac{AM^2}{MB^2} = k^2 \text{ (say)} \]

\[ \Rightarrow k^2 = \frac{16}{25} \]

Now,\[ \text{ar}(PQRS) = \frac{1}{2} AB(PQ + RS) \]

\[ = \frac{1}{2} (AM + MB)(PQ + RS) \]

\[ = \frac{1}{2} [k(MB) + MB](PQ + RS) \]

\[ = \frac{1}{2} [(k + 1)MB][(k + 1)RS] \]

\[ = \frac{1}{2} (k + 1)^2(MB)(RS) \]

\[ = \left( \frac{4}{5} + 1 \right)^2 (50) \]

\[ = \left( \frac{9}{5} \right)^2 50 \]

\[ = \frac{81}{25} \times 50 \]

\[ = 162 \]

\[ \Rightarrow \frac{\lambda}{2} = 81 \]

2. Answer (54)

\[ \text{ar}(\triangle PQR) = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2} \]

\[ \text{Also, } \text{ar}(\triangle PQR) = \frac{1}{2} QS \times PR \]

\[ \Rightarrow \frac{\sqrt{3}}{2} = QS \]

\[ \text{Also, } \triangle QFS \sim \triangle PQR \]

\[ \Rightarrow SF = \frac{QS}{PR} \times QF \]

\[ \Rightarrow SF = \frac{\sqrt{3}}{4}, \text{ QF} = \frac{3}{4} \]

\[ \mu = \text{ar}(\square QESF) = QF \times SF = \frac{3\sqrt{3}}{16} \]

3. Answer (10)

The diameter of each of \( n^2 \) circles is \( \frac{20}{n} \)

\[ \Rightarrow \text{Sum of the areas of } n^2 \text{ circles} \]

\[ = n^2 \times \frac{100\pi}{n^2} = 100\pi \]

\[ \Rightarrow A = 100 \]
4. Answer (36)

\[ \overline{CAD} = \frac{2}{3}(2\pi r) \]

\[ \Rightarrow \overline{AC} = \overline{CD} = \overline{DA} \]

\[ \Rightarrow \triangle ACD \text{ is an equilateral triangle.} \]

\[ O \text{ is the centroid} \]

\[ AO = 24 \Rightarrow OE = 12 \]

\[ \Rightarrow AE = 36 \text{ cm} \]

5. Answer (17)

Produce \( AD \) and \( BC \) to intersect at \( E \).

\[ \angle AEB = 90^\circ \]

\[ PQ = EP - EQ = \frac{1}{2}AB - \frac{1}{2}CD \]

\[ = 10 - \frac{3}{2} \cdot \frac{17}{2} \]

6. Answer (22)

\[ \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}x^2 + \frac{1}{2}y^2 = 242 \]

\[ \Rightarrow x^2 + y^2 = 242 \]

Also, \( (\sqrt{2}x)^2 + (\sqrt{2}y)^2 = d^2 \)

\[ 2x^2 + 2y^2 = d^2 \]

\[ \Rightarrow d = 22 \]

7. Answer (21)

\( CPQR \) is a rectangle.

\[ PR = QC = 10 \]

Radius of circle = 10

\[ AC + BC = 20 \]

\[ AP + PC + CR + RB = 20 \]

\[ \Rightarrow AP + \frac{1}{2} \text{ (Perimeter of rectangle) + } RB = 20 \]

\[ \Rightarrow AP + 14 + RB = 20 \]

\[ \Rightarrow AP + RB = 6 \]

\[ \therefore \text{ Perimeter of shaded region} \]

\[ \Rightarrow AP + PR + RB + AQB \]

\[ = 6 + 10 + \frac{1}{4}(2\pi \times 10) = (5\pi + 16) \]

8. Answer (12)

9. Answer (04)

\[ \frac{1}{2} \text{ ar}(\triangle APQ) = \text{ ar}(\square BPQC) \]

\[ \Rightarrow \frac{1}{2} \text{ ar}(\triangle ABC) = 2 \times \text{ ar}(\triangle APQ) \]

\[ \Rightarrow \frac{\text{ ar}(\triangle ABC)}{\text{ ar}(\triangle ABQ)} = \frac{2 \times \text{ ar}(\triangle APQ)}{\text{ ar}(\triangle ABQ)} \]

\[ \Rightarrow \frac{20 + \lambda}{20} = 2 \left( \frac{6}{6 + \lambda} \right) \Rightarrow (\lambda + 20)(\lambda + 6) = 240 \]

\[ \Rightarrow \lambda = 4, -30 \]

So, \( \lambda = 4 \)
10. Answer (25)

\[ \text{BC and } AB = AC \]
\[ \Rightarrow \angle DAC = \angle ACB = \angle MBC \]
\[ ADCM \text{ is a cyclic quadrilateral.} \]
\[ \therefore \angle ADC = 180^\circ - \angle AMC \]
\[ \angle ADC = \angle BMC \]
\[ \Delta ADC \sim \Delta BMC \]
\[ \Rightarrow \frac{AD}{AC} = \frac{BM}{BC} \]

11. Answer (12)

Let \( a \) be the side and \( h \) be the altitude of the first triangle.
\[ h = \frac{\sqrt{3}}{2} \]
\[ \text{and } A = \frac{\sqrt{3}}{4} a^2 \quad [A = \text{area}] \]
\[ A = \frac{\sqrt{3}}{4} \left( \frac{2h}{\sqrt{3}} \right)^2 = \frac{h^2}{\sqrt{3}} \]

According to the question,
\[ 968\sqrt{3} = \left( \frac{\sqrt{3}}{2} h \right)^2 - \frac{h^2}{\sqrt{3}} = \frac{242h^2}{\sqrt{3}} \]
\[ \Rightarrow h^2 = 12 \]
\[ \Rightarrow a = 4 \]
\[ \therefore \text{Perimeter = 12} \]

12. Answer (50)

Given expression
\[ \frac{1 + \tan^2 25^\circ}{1 - \tan^2 25^\circ} = \sec 50^\circ \]

13. Answer (01)

\[ \tan x = 100 \]
\[ \Rightarrow \frac{2\tan^2 \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} = 100 \]
\[ \Rightarrow 100 - 100\tan^2 \frac{x}{2} = 2\tan^2 \frac{x}{2} \]
\[ \Rightarrow 100\tan^2 \frac{x}{2} + 2\tan \frac{x}{2} - 100 = 0 \]
\[ \Rightarrow \tan \frac{x}{2}, \tan \frac{x}{2} = -1 \]

14. Answer (16)

\[ \tan x = \frac{1}{3} \]
\[ \text{As, } \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \]
\[ = \frac{4\tan x}{1 - 3\tan^2 x} \]

Also, \( \sin y = 2\sin x, \cos y = \frac{2}{3} \cos x \)
\[ \Rightarrow \sin^2 y + \cos^2 y = 4 \sin^3 x + \frac{4}{9} \cos^2 x \]
\[ \Rightarrow 36\tan^2 x + 4 = 9(1 + \tan^2 x) \]
\[ \Rightarrow \tan x = \frac{\sqrt{5}}{3\sqrt{3}} \]
\[ \Rightarrow \tan(x + y) = \sqrt{5} \]

15. Answer (02)

Let \( \cot x > 0 \)
\[ \Rightarrow \cot x = \cot x + \frac{1}{\sin x} \]
which is not possible

Let \( \cot x \leq 0 \)
\[ \Rightarrow -2\cot x = \frac{1}{\sin x} \]
\[ \Rightarrow \cos x = -\frac{1}{2} \]
\[ \Rightarrow x = \frac{2\pi}{3}, \frac{8\pi}{3} \]

16. Answer (03)

\( \sin x = 1, \cos y = 1 \) or \( \sin x = -1, \cos y = -1 \)
\[ \Rightarrow x = \frac{\pi}{2}, y = 0, 2\pi \text{ or } x = \frac{3\pi}{2}, y = \pi \]

\[ \therefore \text{The solutions are} \]
\[ \left( \frac{\pi}{2}, 0 \right), \left( \frac{\pi}{2}, 2\pi \right), \left( \frac{3\pi}{2}, \pi \right) \]

17. Answer (02)

\[ \sum \frac{\cos A}{\sin B \sin C} = -\sum \frac{\cos(B + C)}{\sin B \sin C} = \sum (1 - \cot B \cot C) \]
\[ = 3 - \sum \cot B \cot C \]
\[ = 3 - 1 \]
\[ = 2 \]
18. Answer (36)
\[ \sin^2 x + \sin x - 1 = 0 \]
\[ \Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{2} \]
\[ \Rightarrow \sin x = \frac{-1 + \sqrt{5}}{2} = 2 \sin 18^\circ \]

19. Answer (80)
\[ \tan 3A = 4 \Rightarrow \frac{3 - \tan^2 A}{1 - 3\tan^2 A} = 4 \]
\[ \Rightarrow 3 - \tan^2 A = 4 - 12\tan^2 A \Rightarrow \tan^2 A = \frac{1}{11} \]
Now, \[ \frac{\sin 3A}{\sin A} = \frac{3 - 4\sin^2 A}{1} = 3 - \left( \frac{4}{1 + \cot^2 A} \right) \]
\[ = 3 - \frac{4}{12} \]
\[ = \frac{8}{3} \]

20. Answer (47)
Let \( \angle D = \theta \)
\[ \angle B = 180^\circ - \theta \]
\[ \angle A = 2\theta \]
\[ AC^2 = 20 + 99 - 2(\sqrt{20})(\sqrt{99}) \cos \theta \]
\[ AB^2 = 22 + 97 - 2(\sqrt{22})(\sqrt{97}) \cos (180 - \theta) \]
\[ \Rightarrow AC^2 = 119 - 2\sqrt{20}\sqrt{99} \cos \theta \]
and \[ AB^2 = 119 + 2\sqrt{22}\sqrt{97} \cos \theta \]
\[ \therefore 119 - 2\sqrt{20}\sqrt{99} \cos \theta = 119 + 2\sqrt{22}\sqrt{97} \cos \theta \]
\[ \text{Given } 2\cos \theta (\sqrt{22}\sqrt{97} + \sqrt{20}\sqrt{99}) = 0 \]
\[ \Rightarrow \cos \theta = 0 \text{ or } \theta = 90^\circ \]
\[ \therefore AC^2 = 20 + 99 = 119 \]
\[ \therefore (2R)^2 = 119 \]
\[ \therefore R^2 = \frac{119}{4} \]
\[ \therefore A = \pi R^2 = \frac{22}{7} \times \frac{119}{4} \]
\[ = \frac{11 \times 17 \times 187}{2} = 1 \frac{187}{2} \]
\[ \Rightarrow a = 187 ; b = 2 \]
\[ \Rightarrow a - 70b = 187 - 140 = 47 \]

21. Answer (49)
\[ \angle APQ = \frac{\pi}{2} \]
\[ AB = 8 \cos \theta = PB + PC \]
\[ \Rightarrow 8 \cos \theta = 8 \sin \theta + 6 \cos \theta \]
\[ \Rightarrow \tan \theta = \frac{1}{4} \]
\[ \Rightarrow \cos \theta = \frac{4}{\sqrt{17}} \]
\[ AB = 8 \cos \theta = \frac{32}{\sqrt{17}} \]

22. Answer (21)
Let \( CA \) touch the circle at \( R \) and \( CB \) touch the circle at \( S \).
Since \( PR = PS \)
Right-angled \( \triangle PRC \) and \( PSC \) are congruent.
Hence, \( CP \) bisects \( \angle ACB \).
By angle bisector theorem
\[ \frac{AP}{AC} = \frac{7}{5} \]
\[ \Rightarrow 5AP = 7PB \]
\[ = 7(90 - AP) \]
\[ \Rightarrow 12AP = 630 \]
\[ \Rightarrow 2AP = 105 \]
\[ \Rightarrow 2AP = 105 \]
and \( x = 105 \)
\[ \Rightarrow x \frac{5}{5} = 21 \]
23. Answer (19)

\[ PR^2 = CK^2 = AC^2 - AK^2 = (AM + MC)^2 - (AP - KP)^2 \]
\[ = (AM + MC)^2 - (AM - MC)^2 = 4(AM)(MC) \]
\[ \Rightarrow PR = 2\sqrt{9r} \text{; similarly; } RQ = 2\sqrt{16r} \text{ and } \]
\[ PQ = 2\sqrt{9 \times 16} \text{ but } PR + RQ = PQ \]
\[ \Rightarrow \sqrt{r} = \frac{12}{7} \]

24. Answer (50)

Let O be the centre of circle.

\[ FD \times FE = FB \times FA \]
\[ \Rightarrow FB = 12 \]

Let K and G be points on AB and ED respectively, such that \( OB \perp AB \) and \( GO \perp ED \).

\[ ED = 6 + 4 = 10 \]
\[ \therefore GD = GE = \frac{10}{2} = 5 \]
\[ \Rightarrow OK = GF = GD - FD = 5 - 4 = 1 \]

and \( AB = AF + FB = 2 + 12 = 14 \)

So, \( AK = KB = \frac{14}{2} = 7 \)

\[ r = \sqrt{50} \]

25. Answer (32)

As \( \cos(-\theta) = \cos \theta \) so smallest \( T \) should be such

that \( \sin \left( \frac{x + T}{2} \right) = -\sin \frac{x}{2} = \sin \left( \pi + \frac{x}{2} \right) \)

\[ \Rightarrow \frac{T}{2} = \pi \]
\[ \Rightarrow T = 2\pi \]

26. Answer (09)

**Draw EC**

Since altitude of \( \triangle BEC \)

\[ = \frac{1}{2} \times \text{altitude of } \triangle BAC \]

and \( \ar(\triangle EFC) = \frac{1}{2} \ar(\triangle ABC) \)

\[ \ar(\triangle EGF) = \frac{1}{2} \ar(\triangle EFC) \]

\[ \therefore \ar(\triangle EGF) = \frac{1}{8} \ar(\triangle ABC) \]

\[ \therefore \ar(\triangle BEC) = \frac{1}{8} \ar(\triangle ABC) \]

\[ \Rightarrow \frac{p}{q} = \frac{8}{1} \]

\[ \therefore p + q = 9 \]

27. Answer (21)

Remove from the three corners of an equilateral triangle of side 9 units equilateral triangles of sides 1 unit, 2 units and 3 units respectively. The remaining portion is now an equiangular hexagon \( ABCDEF \) with sides 1, 6, 2, 4, 3, 5.
28. Answer (50)

\[ AG = BC \]

\[ \Rightarrow DG = \frac{1}{2} BC \]

\[ \Rightarrow \] Median GD from vertex G is half of opposite side BC in \( \triangle BGC \).

\[ \Rightarrow \angle BGC = 90^\circ \]

\[ \Rightarrow BC^2 = BG^2 + GC^2 = \left(9 \times \frac{2}{3}\right)^2 + \left(12 \times \frac{2}{3}\right)^2 \]

\[ = 100 \]

29. Answer (03)

\[ \Delta = \frac{1}{2} \times a \times r + \frac{1}{2} \times b \times r + \frac{1}{2} \times c \times r \]

\[ = \left(\frac{a + b + c}{2}\right) \times r \]

\[ \Rightarrow \frac{\Delta}{h_a} = \frac{a}{2} \]

Similarly, \( \frac{\Delta}{h_b} = \frac{b}{2} \) and \( \frac{\Delta}{h_c} = \frac{c}{2} \)

So,

\[ \frac{\Delta}{r} = \frac{\Delta}{h_a} + \frac{\Delta}{h_b} + \frac{\Delta}{h_c} \]

\[ \Rightarrow \frac{1}{r} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} \]

\[ \Rightarrow \frac{1}{r} = \frac{1}{3} + \frac{5}{4} + \frac{12}{1} = 1 \]

\[ \Rightarrow r = 1 \]

Area of incircle = \( \pi \times (1)^2 = \pi \)

30. Answer (10)

\[ \cos 30^\circ = \frac{x^2 + 9x^2 - a^2}{6x^2} \]

\[ \Rightarrow \frac{\sqrt{3}}{2} = \frac{5}{3} - \frac{2(10 - 3\sqrt{3})}{3x^2} \]

\[ \Rightarrow \frac{10 - 3\sqrt{3}}{x^2} = \frac{5}{3} \times \frac{3\sqrt{3}}{4} = \frac{10 - 3\sqrt{3}}{4} \]

\[ \Rightarrow x^2 = 4 \]

Area \( \triangle ABC = \frac{1}{2} x \times 3x \times \sin 30^\circ = \frac{3x^2}{4} \)

\[ \Rightarrow = 3 \]

\[ \triangle A \]

\[ \angle 30^\circ \]

\[ 3x \]

\[ B \]
29. Answer (87)

Notice that area of region (1) = area of region (3) and area of region (2) = area of region (4). Swap the shaded part 1 with unshaded part 3; and shaded part 2 with unshaded part 4; which doesn’t change the total area of shaded region or that of unshaded region.

Hence, area of shaded region = \( \frac{1}{2} \) area of square \( \frac{85}{2} \)

30. Answer (07)

\[
f(x + \frac{\pi}{2}) = |\sin(x + \frac{\pi}{2})| + |\cos(x + \frac{\pi}{2})| = |\sin x| + |\cos x| = f(x)
\]

\[
\Rightarrow T_1 = \frac{\pi}{2}
\]

\[
g(x) = 2|\sin(x/2)|\left|\cos(x/2) + \sin(x/2)\right|
\]

Period of \( \sin \frac{x}{2} \) is \( 2\pi \) and that of \( \left|\cos \frac{x}{2} + \sin \frac{x}{2}\right| \) is \( \pi \).

Hence, period of \( g(x) \) = \( \text{lcm} (\pi, 2\pi) = 2\pi \)

\[
\Rightarrow T_2 = 2\pi
\]
ALL RIGHTS RESERVED

All rights including copyright and translation rights etc. reserved and vests exclusively with AESL. No part of this publication may be reproduced, distributed, redistributed, copied or transmitted in any form or by any means-graphical, electronic or mechanical methods including photocopying, recording, taping or stored on information retrieval systems of any nature or reproduced on any disc, tape, media, information storage device, without the prior written permission of AESL. Breach of this condition is liable for legal action (civil as well as criminal) under the applicable Laws.

Edition: 2020-21

© Aakash Educational Services Limited [AESL]