08/01/2020
Evening

Aakash
Medical | IIT-JEE | Foundations
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Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2020 (Phase-1)

(Physics, Chemistry and Mathematics)

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **75** questions**.** The maximum marks are **300.**
3. There are **three** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 25 questions in each part of equal weightage. Each part has two sections.

   (i) **Section-I :** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

   (ii) **Section-II :** This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is

(1) 6.25×10^{-4} J

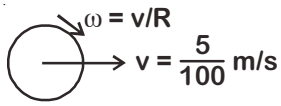
(2) 1.13×10^{-3} J

(3) 8.75×10^{-4} J

(4) 8.75×10^{-3} J

Answer (3)

Sol.



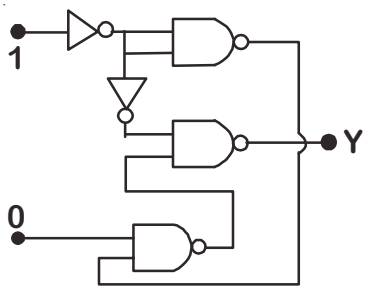
$$KE = \frac{1}{2}\left(\frac{2}{5}mR^2 + mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow KE = \frac{1}{2}mR^2 \times \frac{7}{5} \times \frac{v^2}{R^2} = \frac{7}{10} \times \frac{1}{2} \times \frac{25}{10^4}$$

$$KE = \frac{35}{4} \times 10^{-4} \text{ Joule}$$

$$KE = 8.75 \times 10^{-4} \text{ Joule}$$

2. In the given circuit, value of Y is



(1) Toggles between 0 and 1

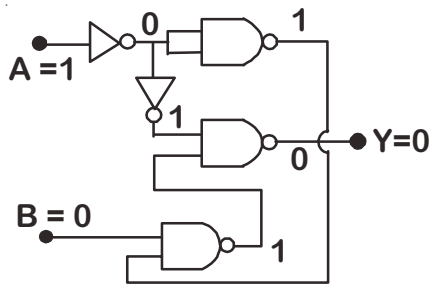
(2) 0

(3) 1

(4) Will not execute

Answer (2)

Sol.



$$Y = \overline{\overline{AB}.A} = \overline{\overline{AB}} + \overline{A} = AB + \overline{A}$$

Y = 0 + 0 = 0

3. Consider a mixture of n moles of helium gas and 2n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_p/C_v value will be

(1) 40/27 (2) 19/13

(3) 67/45 (4) 23/15

Answer (2)

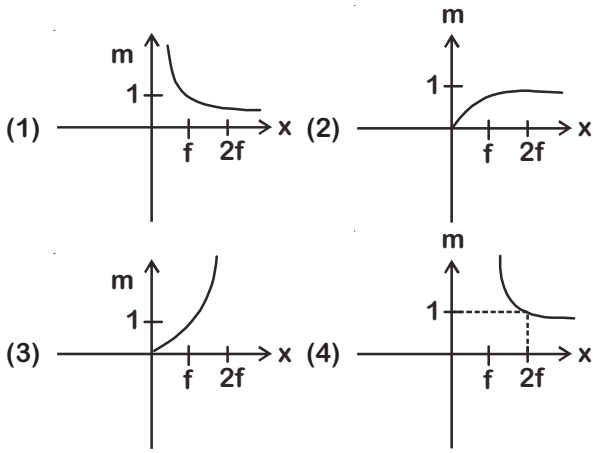
Sol. $$\gamma = \frac{N_1C_{P_1} + N_2C_{P_2}}{N_1C_{v_1} + N_2C_{v_2}}$$

$$\gamma = \frac{n.\frac{5}{2}R + 2n.\frac{7}{2}R}{n.\frac{3}{2}R + 2n.\frac{5}{2}R} = \frac{19nR \times 2}{2(13nR)}$$

$$\gamma = \left(\frac{C_p}{C_v}\right)_{mixture} = \frac{19}{13}$$

4. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by

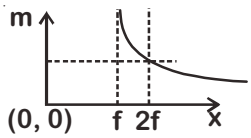
(Graphs are drawn schematically and are not to scale)



Answer (4)

Sol. At focus magnification is ∞

And at x = 2f, magnification is 1.



5. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T, the velocity changed to v/2. The value of T is close to

(1) 30.5×10^4 N (2) 2.50×10^4 N

(3) 5.15×10^3 N (4) 10.2×10^2 N

Answer (3)

Sol. $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \quad \Rightarrow \quad \frac{v}{v} \times 2 = \sqrt{\frac{2.06 \times 10^4}{T_2}}$$

$$\Rightarrow T_2 = \frac{2.06 \times 10^4}{4} = 0.515 \times 10^4 N$$

$$\Rightarrow T_2 = 5.15 \times 10^3 N$$

6. A galvanometer having a coil resistance 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V?

(1) 10 kΩ (2) 9.9 kΩ

(3) 8.9 kΩ (4) 7.9 kΩ

Answer (2)

Sol. V = 10 volt

When 1 mA current will be flowing

$\therefore$ $10 = 1 \times 10^{-3} (100 + R_0)$

$\Rightarrow$ $10000 - 100 = 9900\ \Omega = R_0$

$\Rightarrow$ $R_0 = 9.9$ kΩ

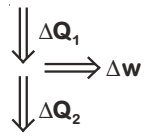
7. A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is :

(1) 90 J (2) 1 J

(3) 99 J (4) 100 J

Answer (1)

Sol.



$$\frac{\Delta Q_1 - \Delta Q_2}{\Delta Q_1} = \frac{1}{10} = \frac{\Delta w}{\Delta Q_1}$$

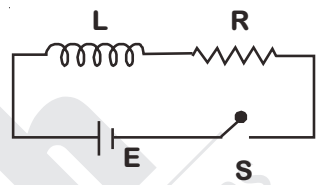
$$\Rightarrow \frac{1}{10} = \frac{\Delta w}{\Delta Q_1}$$

$\therefore$ $\Delta Q_1 = \Delta w \times 10 = 100$ J

So, $\Delta Q_1 - \Delta Q_2 = \Delta w$

$\Rightarrow$ $100 - 10 = \Delta Q_2 = 90$ J

8.



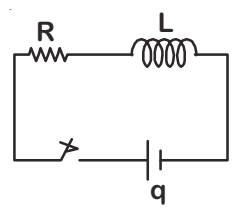
As shown in the figure, a battery of emf E is connected to an inductor L and resistance R in series. The switch is closed at t = 0. The total charge that flows from the battery, between t = 0 and t = t_C (t_C is the time constant of the circuit) is :

(1) $\frac{EL}{R^2}$ (2) $\frac{ER}{eL^2}$

(3) $\frac{EL}{R^2}\left(1 - \frac{1}{e}\right)$ (4) $\frac{EL}{eR^2}$

Answer (4)

Sol. $i = \frac{E}{R}(1 - e^{-t/t_c})$



$$\because \quad t_C = \frac{L}{R}$$

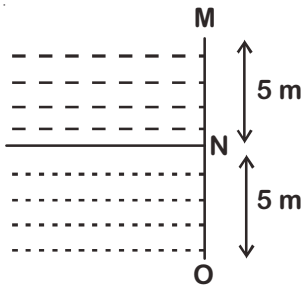
$$\Rightarrow \int dq = \int \frac{E}{R}(1 - e^{-t/t_c})dt$$

$$\Rightarrow q = \frac{E}{R}\left[t + t_C e^{-t/t_c}\right]_0^{t_c}$$

$$\Rightarrow q = \frac{E}{R}\left[t_C + \frac{t_C}{e} - t_C\right]$$

$$\Rightarrow q = \frac{E}{R}\frac{L}{Re}$$

$$\therefore \quad q = \frac{EL}{R^2 e}$$

9. 

Two liquids of densities ρ_1 and ρ_2 ($\rho_2 = 2\rho_1$) are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing)

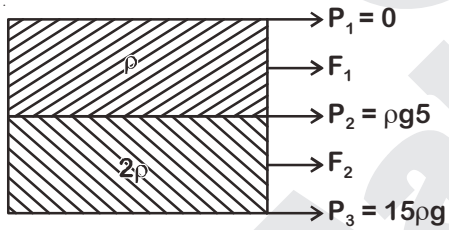
(1) 1/4

(2) 1/2

(3) 2/3

(4) 1/3

Answer (1)

Sol. 

$$F_1 = \frac{(P_1 + P_2)}{2}A$$

$$F_2 = \frac{(P_2 + P_3)}{2}A$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{5\rho g}{20\rho g} = \frac{5}{20} = \frac{1}{4}$$

10. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is

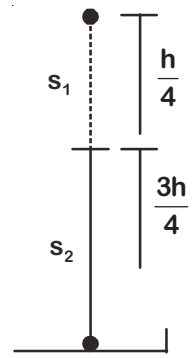
(1) $\sqrt{\frac{1}{2}}$ (2) $\sqrt{\frac{3}{4}}$

(3) $\frac{1}{2}$ (4) $\sqrt{\frac{3}{2}}$

Answer (4)

Sol.

Time of collision



$$\Rightarrow t_0 = \frac{h}{\sqrt{2gh}} = \sqrt{\frac{h}{2g}}$$

$$\therefore s_1 = \frac{1}{2}gt_0^2 = \frac{1}{2}g \cdot \frac{h}{2g} = \frac{h}{4}$$

$$\therefore s_2 = \frac{3h}{4}$$

Speed of (A) just before collision $v_1 \downarrow$

$$= gt_0 = \sqrt{\frac{gh}{2}}$$

And speed of (B) just before collision $v_2 \uparrow$

$$= \sqrt{2gh} - \sqrt{\frac{gh}{2}}$$

After collision velocity of centres of mass

$$V_{cm} = \frac{m\left(\sqrt{2gh} - \sqrt{\frac{gh}{2}}\right) - m\sqrt{\frac{gh}{2}}}{2m} = 0$$

So from there, time of fall 't'

$$\Rightarrow \frac{3h}{4} = \frac{1}{2}gt^2$$

$$\Rightarrow t = \sqrt{\frac{3}{2}\frac{h}{g}}$$

11. A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stopwatch with 1 s resolution measures the time taken for 40 oscillation to be 50 s. The accuracy in g is

(1) 3.40% (2) 2.40%

(3) 5.40% (4) 4.40%

Answer (4)

Sol. l = 25.0 cm

Time of 40 oscillation is 50 sec

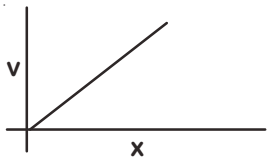
$$\therefore \quad g = \frac{4\pi^2 l}{T^2} \Rightarrow \frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$$

$$\Rightarrow \frac{\Delta g}{g} = \left(\frac{0.1}{25.0}\right) + 2\left(\frac{1}{50}\right)$$

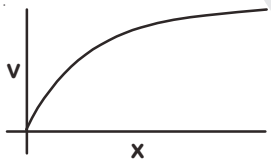
$$\Rightarrow \left(\frac{\Delta g}{g} \times 100\right) = 4.4\%$$

12. A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale)

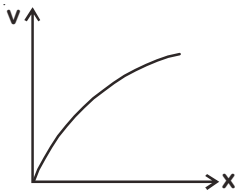
(1) 

(2) 

(3) 

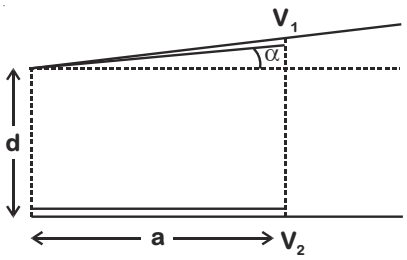
(4) 

Answer (4)

Sol. 

$$v^2 = \frac{2qE}{m} \cdot x$$

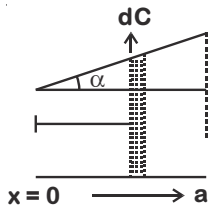
13. A capacitor is made of two square plates each of side 'a' making a very small angle α between them, as shown in figure. The capacitance will be close to



(1) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$ (2) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$

(3) $\frac{\varepsilon_0 a^2}{d}\left(1+\frac{\alpha a}{d}\right)$ (4) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$

Answer (2)

Sol. 

$y = \tan \alpha x$

$$\therefore \quad dC = \frac{adx\ 60}{d + x\tan\alpha}$$

$$\therefore \quad C_{eq} = \int dc = a\varepsilon_0 \int_{x=0}^{x=a} \frac{dx}{x\tan\alpha + d}$$

[By Binomial expansion]

$$\Rightarrow \quad C_{eq} = \frac{a\varepsilon_0}{d}\int_0^a \left(1-\frac{x\tan\alpha}{d}\right)dx = \frac{a\varepsilon_0}{d}\left(x-\frac{x^2\tan\alpha}{d\,2}\right)_0^a$$

$$\Rightarrow \quad C_{eq} = \frac{a^2\varepsilon_0}{d}\left(1-\frac{a\tan\alpha}{2d}\right) = \frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$$

14. Consider two charged metallic spheres S_1 and S_2 of radii R_1 and R_2, respectively. The electric fields E_1 (on S_1) and E_2 (on S_2) on their surfaces are such that $E_1/E_2 = R_1/R_2$. Then the ratio V_1(on S_1)/V_2(on S_2) of the electrostatic potentials on each sphere is

(1) $(R_1/R_2)^2$ (2) (R_2/R_1)

(3) $\left(\frac{R_1}{R_2}\right)^3$ (4) R_1/R_2

Answer (1)

Sol. Let ρ_1 and ρ_2 are the charge densities of two sphere. Then,

$$E_1 = \frac{\rho_1 R_1}{3\varepsilon_0} \text{ and } E_0 = \frac{\rho_2 R_2}{3\varepsilon_0}$$

$$\because \quad \frac{E_1}{E_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{R_1}{R_2}$$

$$\therefore \quad \rho_1 = \rho_2 = \rho$$

$$\text{So, } V_1 = \frac{\rho R_1^2}{3\varepsilon_0} \text{ and } V_2 = \frac{\rho R_2^2}{3\varepsilon_0}$$

$$\therefore \quad \frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^2$$

15. A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\vec{B} = 5\times10^{-8}\hat{j}\ T$. The corresponding electric field $\vec{E}$ is (speed of light c = 3 × 10^8 ms^{-1})

(1) $-1.66\times10^{-16}\hat{i}$ V/m

(2) $1.66\times10^{-16}\hat{i}$ V/m

(3) $-15\hat{i}$ V/m

(4) $15\hat{i}$ V/m

Answer (4)

Sol. $\vec{E} = \vec{B}\times\vec{C}$

$\therefore$ |E| = |B| · C

$\Rightarrow$ $|\vec{E}| = 5\times10^{-8}\times3\times10^{8} = 15$ N/C

$\vec{E} = 15\hat{i}$ V/m

16. In a double-slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is

(1) 0.568

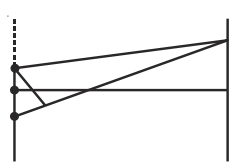
(2) 0.760

(3) 0.853

(4) 0.672

Answer (3)

Sol. $\Delta x = \lambda/8$

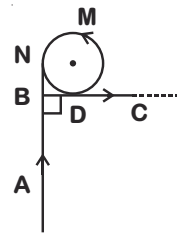


$\therefore$ Phase difference $\Rightarrow \Delta\phi = \frac{\pi}{4}$

$$I = I_0\cos^2\left(\frac{\Delta\phi}{2}\right) = I_0\cos^2\left(\frac{\pi}{8}\right)$$

$$\therefore \frac{I}{I_0} = 0.853$$

17. A very long wire ABDMNDC is shown in figure carrying current I. AB and BC parts are straight, long and at right angle. At D wire forms a circular turm DMND of radius R. AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is



(1) $\frac{\mu_0 I}{2\pi R}(\pi+1)$

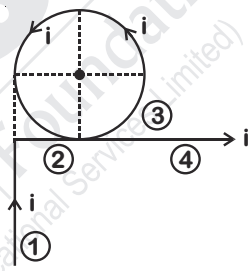
(2) $\frac{\mu_0 I}{2R}$

(3) $\frac{\mu_0 I}{2\pi R}\left(\pi+\frac{1}{\sqrt{2}}\right)$

(4) $\frac{\mu_0 I}{2\pi R}\left(\pi-\frac{1}{\sqrt{2}}\right)$

Answer (3)

Sol.

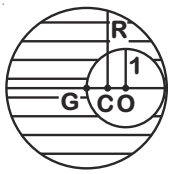


$$B_0 = (\overrightarrow{B_0})_1^{\otimes} + (\overrightarrow{B_0})_2^{\odot} + (\overrightarrow{B_0})_3^{\odot} + (\overrightarrow{B_0})_4^{\odot}$$

$$= -\frac{\mu_0 I}{4\pi R}\left(1-\frac{1}{\sqrt{2}}\right) + \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$\overrightarrow{B_0^{\odot}} = \frac{\mu_0 I}{2\pi R}\left(\pi+\frac{1}{\sqrt{2}}\right)^{\odot}$$

18. As shown in fig. when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e. on the surface of the cavity. R can be determined by the equation



(1) $(R^2 + R + 1)(2 - R) = 1$

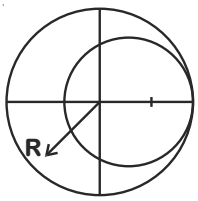
(2) $(R^2 - R + 1)(2 - R) = 1$

(3) $(R^2 - R - 1)(2 - R) = 1$

(4) $(R^2 + R - 1)(2 - R) = 1$

Answer (1)

Sol.



$$M_0 = \frac{4}{3}\pi R^3 \rho$$

$$M_{cavity} = \frac{4}{3}\pi (1)^3 \rho$$

$$M_{(Remaining)} = \frac{4}{3}\pi R^3 \rho - \frac{4}{3}\pi (1)^3 \rho$$

$$\therefore \left(\frac{4}{3}\pi R^3 \rho - \frac{4}{3}\pi (1)^3 \rho\right) \times 2 + \frac{4}{3}\pi (1)^3 \rho = \frac{4}{3}\pi R^3 \rho \cdot R$$

$\Rightarrow$ $R^4 - 2R^3 + 1 = 0$ $\quad \because \quad R \neq 1$

$\therefore$ $R^3 - R^2 - R - 1 = 0 \Rightarrow (R^2 + R + 1)(2 - R) = 1$

19. A particle moves such that its position vector $\vec{r}(t) = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle

(1) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

(2) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

(3) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$

(4) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

Answer (2)

Sol. $\vec{r} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \omega(-\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j})$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$$

$\vec{a} = -\omega^2\vec{r}$ $\quad \therefore$ $\vec{a}$ is antiparallel to $\vec{r}$

Also $\vec{v}\cdot\vec{r} = 0$ $\qquad \therefore \quad \vec{v} \perp \vec{r}$

Actually particle is in state of uniform circular motion.

20. An electron (mass m) with initial velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$ is in an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wavelength at time t is given by

(1) $\dfrac{\lambda_0}{\sqrt{1 + \dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$ $\qquad$ (2) $\dfrac{\lambda_0\sqrt{2}}{\sqrt{1 + \dfrac{e^2E^2t^2}{m^2v_0^2}}}$

(3) $\dfrac{\lambda_0}{\sqrt{1 + \dfrac{e^2E^2t^2}{2m^2v_0^2}}}$ $\qquad$ (4) $\dfrac{\lambda_0}{\sqrt{2 + \dfrac{e^2E^2t^2}{m^2v_0^2}}}$

Answer (3)

Sol. $v = v_0\hat{i} + v_0\hat{j} + \dfrac{eE_0}{m}t\,\hat{k}$

$$\therefore \quad |\vec{v}| = \sqrt{2v_0^2 + \left(\frac{eE_0t}{m}\right)^2}$$

Initialy, $\lambda_0 = \dfrac{h}{mv_0\sqrt{2}}$

Now, $\lambda = \dfrac{h}{m\sqrt{2v_0^2 + \left(\dfrac{eE_0t}{m}\right)^2}}$

$$\frac{\lambda}{\lambda_0} = \frac{1}{\sqrt{1 + \left(\dfrac{eE_0t}{\sqrt{2}mv_0}\right)^2}}$$

$$\lambda = \frac{\lambda_0}{\sqrt{1 + \dfrac{e^2E_0^2t^2}{2m^2v_0^2}}}$$

SECTION - II

Numerical Value Type Questions: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and there is no negative marking for wrong answer.

21. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in liters) are taken from each container and mixed (assume no loss of heat during the process)

C_1	C_2	C_3	T
1ℓ	2ℓ	–	60°C
–	1ℓ	2ℓ	30°C
2ℓ	–	1ℓ	60°C
1ℓ	1ℓ	1ℓ	θ

The value of θ (in °C to the nearest integer) is _____.

Answer (50.00)

Sol. Let C_1 is at θ_1 ; C_2 is at θ_2 and C_3 is at θ_3 then

$ms(\theta_1 - 60) = 2ms(60 - \theta_2)$

$\Rightarrow \theta_1 - 60 = 120 - 2\theta_2$

$\Rightarrow \theta_1 = 180 - 2\theta_2$ ...(i)

and $ms(\theta_2 - 30) = 2ms(30 - \theta_3)$

$\Rightarrow \theta_2 = 90 - 2\theta_3$ ...(ii)

and $2ms(\theta_1 - 60) = ms(60 - \theta_3)$

$\Rightarrow 2\theta_1 - 120 = 60 - \theta_3$

$\Rightarrow 2\theta_1 + \theta_3 = 180$ ...(iii)

Adding them together $3(\theta_1 + \theta_2 + \theta_3) = 9\theta$

$\Rightarrow \theta = 50°C$

22. An asteroid is moving directly towards the centre of the earth. When at a distance of 10 R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s)? Give your answer to the nearest integer in kilometer/s _____.

Answer (16.00)

Sol. $$-\frac{GM_Em}{10R} + \frac{1}{2}mV_0^2 = -\frac{GM_Em}{R} + \frac{1}{2}mV^2$$

$$\Rightarrow \frac{GM_E}{R}\left(1 - \frac{1}{10}\right) + \frac{V_0^2}{2} = \frac{V^2}{2}$$

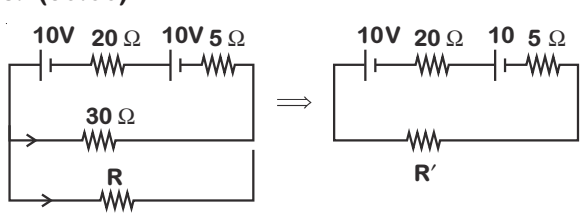
$$V^2 = V_0^2 + \frac{9}{5}gR$$

$$\Rightarrow V = \sqrt{V_0^2 + \frac{9}{5}gR} \approx 16 \text{ km/s}$$

23. The series combination of two batteries, both of the same emf 10 V, but different internal resistance of 20 Ω and 5 Ω, is connected to the parallel combination of two resistors 30 Ω and R Ω. The voltage difference across the battery of internal resistance 20 Ω is zero, the value of R (in Ω) is _____.

Answer (30.00)

Sol.



$$R' = \frac{30R}{30+R} \qquad \therefore \quad I = \frac{20}{R'+25}$$

Also $10 = \frac{20 \times 20}{R' + 25}$ $\Rightarrow R' + 25 = 40$

$\Rightarrow R' = 15 = \frac{30R}{30+R}$ $\Rightarrow 30 + R = 2R$

$\Rightarrow R = 30\ \Omega$

24. The first member of the Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is _____.

Answer (486.00)

Sol. $$\Delta E = \frac{hC}{\lambda} = \Delta E_0\left[\frac{1}{4} - \frac{1}{9}\right] \quad ...(i)$$

So, $$\frac{hC}{\lambda'} = \Delta E_0\left[\frac{1}{4} - \frac{1}{16}\right] \quad ...(ii)$$

$$\Rightarrow \frac{\lambda'}{\lambda} = \frac{5 \times 16}{9 \times 4 \times 3}$$

$$\Rightarrow \lambda' = \frac{5 \times 4 \times 656.1}{9 \times 3}(\text{nm}) = 486 \text{ nm}$$

25. A ball is dropped from the to of a 100 m high tower on a planet. In the last $\frac{1}{2}$ s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is _____.

Answer (08.00)

Sol. $h = \frac{1}{2}gt^2$

$\Rightarrow 200 = gt^2$ ...(i)

$\Rightarrow t = \sqrt{\frac{200}{g}}$

Also $\frac{1}{2}g\left(t - \frac{1}{2}\right)^2 = 81$

$\therefore g\left(t - \frac{1}{2}\right)^2 = 81 \times 2$ ...(ii)

$$\left(t - \frac{1}{2}\right) = \sqrt{\frac{81 \times 2}{g}}$$

$\therefore \frac{1}{2} = \frac{1}{\sqrt{g}}(\sqrt{200} - \sqrt{81 \times 2}) \Rightarrow \sqrt{g} = 2(10\sqrt{2} - 9\sqrt{2})$

$\Rightarrow \sqrt{g} = 2\sqrt{2}$

$\therefore g = 8\ m/s^2$

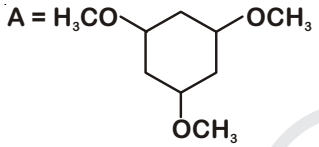
CHEMISTRY

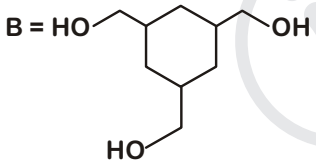
SECTION - I

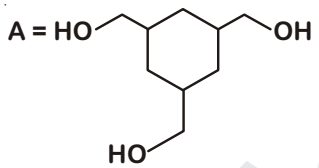
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

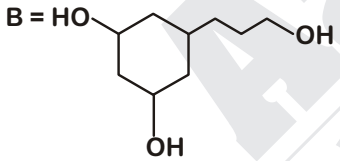
Choose the correct answer :

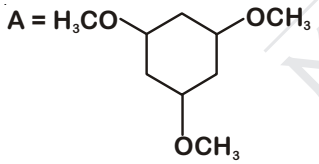
1. Among the compounds A and B with molecular formula $C_9H_{18}O_3$, A is having higher boiling point the B. The possible structures of A and B are

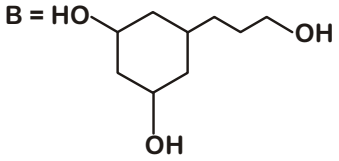
(1) 

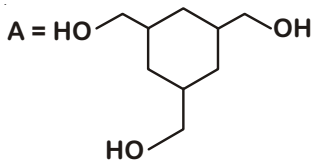


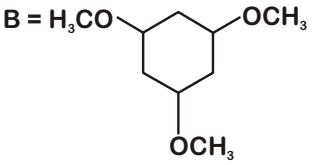
(2) 



(3) 



(4) 



Answer (4)

Sol. In (A), Intermolecular H-bonding is possible while in (B) there is no inter-molecular H-bonding. So A is having higher boiling point than B.

2. Which of the following compounds is likely to show both Frenkel and Schottky defects in its crystalline form ?

(1) ZnS (2) CsCl

(3) AgBr (4) KBr

Answer (3)

Sol. AgBr shows both, Frenkel as well as Schottky defects.

3. Among the reactions (a) - (d), the reaction(s) that does/do not occur in the blast furnace during the extraction of iron is/are

(a) $CaO + SiO_2 \rightarrow CaSiO_3$

(b) $3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$

(c) $FeO + SiO_2 \rightarrow FeSiO_3$

(d) $FeO \rightarrow Fe + \frac{1}{2}O_2$

(1) (c) and (d)

(2) (d)

(3) (a)

(4) (a) and (d)

Answer (1)

Sol. (c) $FeO + SiO_2 \rightarrow FeSiO_3$

(d) $FeO \rightarrow Fe + \frac{1}{2}O_2$

Reactions (c) and (d) do not occur in the blast furnace in the metallurgy of iron.

4. The increasing order of the atomic radii of the following elements is

(a) C (b) O (c) F

(d) Cl (e) Br

(1) (d) < (c) < (b) < (a) < (e)

(2) (b) < (c) < (d) < (a) < (e)

(3) (c) < (b) < (a) < (d) < (e)

(4) (a) < (b) < (c) < (d) < (e)

Answer (3)

Sol. C → N → O → F (Size decreases)

F → Cl → Br → I (Size increases)

Correct increasing order of atomic radii is
F < O < C < Cl < Br

5. The radius of the second Bohr orbit, in terms of the Bohr radius, a_0, in Li^{2+} is

(1) $\frac{4a_0}{3}$ (2) $\frac{4a_0}{9}$

(3) $\frac{2a_0}{3}$ (4) $\frac{2a_0}{9}$

Answer (1)

Sol. $r = 0.529\frac{n^2}{Z}$ Å

Bohr's radius for hydrogen atom (a_0) = 0.529 Å

Bohr's radius of Li^{+2} ion for n = 2

$$= a_0\frac{n^2}{Z}$$

$$= \frac{4a_0}{3}$$

6. The correct order of the calculated spin-only magnetic moments of complexes (A) to (D) is

(A) $Ni(CO)_4$ (B) $[Ni(H_2O)_6]Cl_2$

(C) $Na_2[Ni(CN)_4]$ (D) $PdCl_2(PPh_3)_2$

(1) (A) ≈ (C) ≈ (D) < (B)

(2) (C) ≈ (D) < (B) < (A)

(3) (A) ≈ (C) < (B) ≈ (D)

(4) (C) < (D) < (B) < (A)

Answer (1)

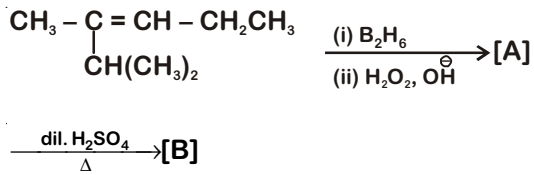
Sol.

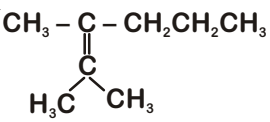
		Number of unpaired electrons	μ
(A) $Ni(CO)_4$	$Ni = 3d^8 4s^2$(SFL)	0	0
(B) $[Ni(H_2O)_6]Cl_2$	$Ni^{+2} = 3d^8$(WFL)	2	$\sqrt{8}$ BM
(C) $Na_2[Ni(CN)_4]$	$Ni^{+2} = 3d^8$(SFL)	0	0
(D) $PdCl_2(PPh_3)_2$	$Pd^{+2} = 4d^8$	0	0

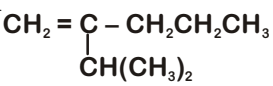
Correct order of the calculated spin only magnetic moments of complexes A to D is

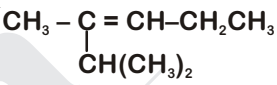
(A) ≃ (C) ≃ (D) < B

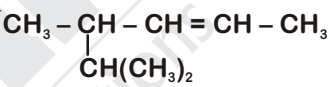
7. The major product [B] in the following sequence of reactions is



(1) 

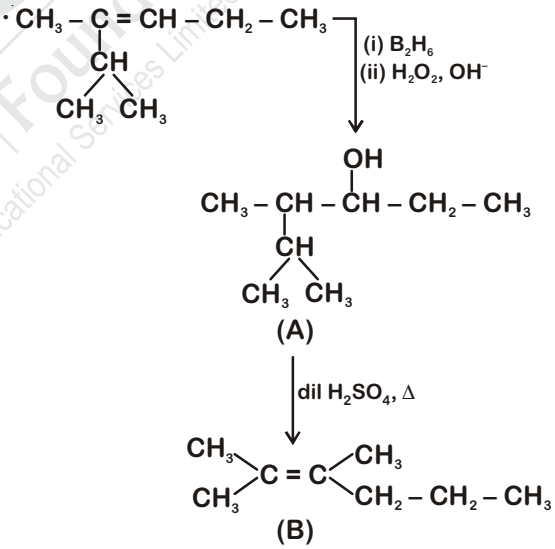
(2) 

(3) 

(4) 

Answer (1)

Sol.



8. Hydrogen has three isotopes (A), (B) and (C). If the number of neutron(s) in (A), (B) and (C) respectively, are (x), (y) and (z), the sum of (x), (y) and (z) is

(1) 4 (2) 2

(3) 3 (4) 1

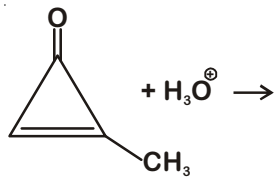
Answer (3)

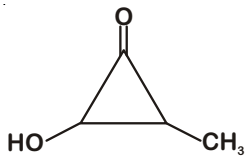
Sol. Isotopes of hydrogen and the number of neutrons present in them are

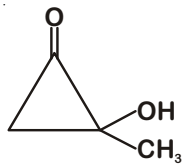
	$_1H^1$	$_1H^2$(D)	$_1H^3$(T)
Number of neutrons	0 (x)	1 (y)	2 (z)

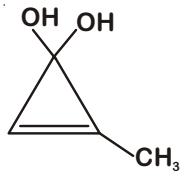
Total number of neutrons in three isotopes of hydrogen = 0 + 1 + 2 = 3

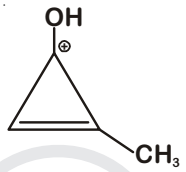
9. The major product in the following reaction is



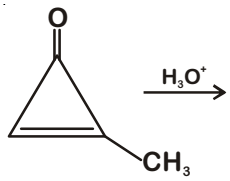
(1) 

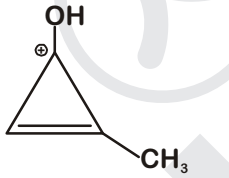
(2) 

(3) 

(4) 

Answer (4)

Sol. 

$\xrightarrow{H_3O^+}$ 

Product is aromatic

10. For the following **Assertion** and **Reason**, the correct option is

Assertion : The pH of water increases with increase in temperature.

Reason : The dissociation of water into H^+ and OH^- is an exothermic reaction.

(1) Both assertion and reason are false
(2) Assertion is not true, but reason is true
(3) Both assertion and reason are true, and the reason is the correct explanation for the assertion
(4) Both assertion and reason are true, but the reason is not the correct explanation for the assertion

Answer (1)

Sol. Both assertion and reason are incorrect

11. A metal (A) on heating in nitrogen gas gives compound B. B on treatment with H_2O gives a colourless gas which when passed through $CuSO_4$ solution gives a dark blue-violet coloured solution. A and B respectively, are

(1) Mg and Mg_3N_2 (2) Na and Na_3N
(3) Mg and $Mg(NO_3)_2$ (4) Na and $NaNO_3$

Answer (1)

Sol. $\underset{(A)}{3Mg + N_2} \xrightarrow{\Delta} \underset{(B)}{Mg_3N_2} \xrightarrow{H_2O} \underset{\text{(Colourless gas)}}{2NH_3} + 3Mg(OH)_2$

$\downarrow \frac{1}{2}CuSO_4$

$\frac{1}{2}[Cu(NH_3)_4]SO_4$ dark blue coloured

12. Preparation of Bakelite proceeds via reactions

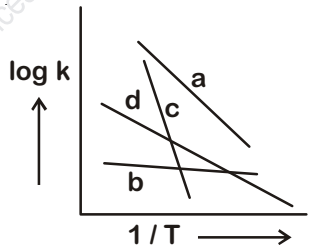
(1) Electrophilic substitution and dehydration
(2) Electrophilic addition and dehydration
(3) Nucleophilic addition and dehydration
(4) Condensation and elimination

Answer (1)

Sol. Formation of Bakelite

Electrophilic substitution reaction of phenol with formaldehyde followed by dehydration

13. Consider the following plots of rate constant versus $\frac{1}{T}$ for four different reactions. Which of the following orders is correct for the activation energies of these reactions?



(1) $E_b > E_a > E_d > E_c$ (2) $E_c > E_a > E_d > E_b$
(3) $E_a > E_c > E_d > E_b$ (4) $E_b > E_d > E_c > E_a$

Answer (2)

Sol. $\log k = \log A - \frac{E_a}{2.303\,RT}$

$\text{Slope} = -\frac{E_a}{2.303\,R}$

So correct order of activation energies $\Rightarrow E_c > E_a > E_d > E_b$

14. For the following Assertion and Reason, the correct option is

Assertion : For hydrogenation reactions, the catalytic activity increases from Group 5 to Group 11 metals with maximum activity shown by Group 7-9 elements.

Reason : The reactants are most strongly adsorbed on group 7-9 elements.

(1) Both assertion and reason are true and the reason is the correct explanation for the assertion.

(2) Both assertion and reason are false.

(3) The assertion is true, but the reason is false.

(4) Both assertion and reason are true but the reason is not the correct explanation for the assertion.

Answer (3)

Sol. The assertion is true, but the reason is false

Refer : [NCERT Page No. 132]

Reactants must not get adsorbed so strongly that they are immobilised and other reactants are left with no space on the catalyst's surface for adsorption.

15. Arrange the following bonds according to their average bond energies in descending order

C – Cl, C – Br, C – F, C – I

(1) C – Cl > C – Br > C – I > C – F

(2) C – Br > C – I > C – Cl > C – F

(3) C – F > C – Cl > C – Br > C – I

(4) C – I > C – Br > C – Cl > C – F

Answer (3)

Sol. Generally, bond energy $\propto \dfrac{1}{\text{bond length}}$

So bond energy order is

C–F > C–Cl > C–Br > C–I

16. White phosphorus on reaction with concentrated NaOH solution in an inert atmosphere of CO_2 gives phosphine and compound (X). (X) on acidification with HCl gives compound (Y). The basicity of compound (Y) is

(1) 3 (2) 2

(3) 4 (4) 1

Answer (4)

Sol. $P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$ (X)

$\downarrow$ acidification with HCl

H_3PO_2 (Y)

Basicity of $\underset{(Y)}{H_3PO_2} = 1$

17. Two monomers in maltose are

(1) α-D-glucose and α-D-glucose

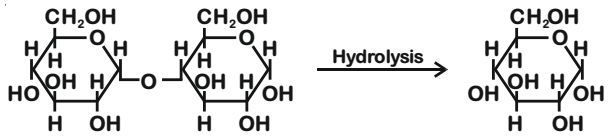
(2) α-D-glucose and β-D-glucose

(3) α-D-glucose and α-D-galactose

(4) α-D-glucose and α-D-Fructose

Answer (1)

Sol. Maltose $\xrightarrow{\text{Hydrolysis}}$ α-D-glucose + α-D-glucose



(Maltose) α-D-glucose (2 moles)

18. Kjeldahl's method cannot be used to estimate nitrogen for which of the following compounds?

(1) $CH_3CH_2–C\equiv N$ (2) $NH_2–\overset{\overset{O}{\|}}{C}–NH_2$

(3) $C_6H_5NO_2$ (4) $C_6H_5NH_2$

Answer (3)

Sol. Kjeldahl's method is not applicable to compounds containing nitrogen in nitro, azo groups and nitrogen present in ring (pyridine).

19. Among (a) – (d), the complexes that can display geometrical isomerism are

(a) $[Pt(NH_3)_3Cl]^+$

(b) $[Pt(NH_3)Cl_5]^-$

(c) $[Pt(NH_3)_2Cl(NO_2)]$

(d) $[Pt(NH_3)_4ClBr]^{2+}$

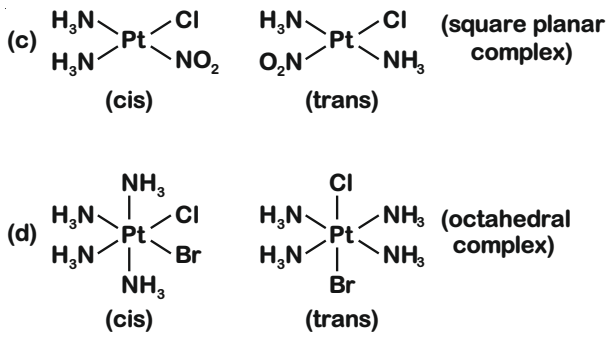
(1) (c) and (d) (2) (a) and (b)

(3) (b) and (c) (4) (d) and (a)

Answer (1)

Sol. $\underset{(c)}{[Pt(NH_3)_2Cl(NO_2)]}$ and $\underset{(d)}{[Pt(NH_3)_4ClBr]^{+2}}$

can display geometrical isomerism

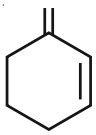


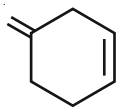
20. An unsaturated hydrocarbon X absorbs two hydrogen molecules on catalytic hydrogenation, and also gives following reaction

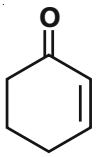
$$X \xrightarrow[Zn/H_2O]{O_3} A \xrightarrow{[Ag(NH_3)_2]^+}$$

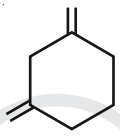
B(3-oxo-hexanedicarboxylic acid)

X will be

(1) 

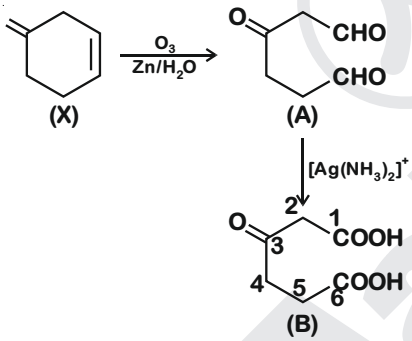
(2) 

(3) 

(4) 

Answer (2)

Sol.



SECTION - II

Numerical Value Type Questions: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and there is no negative marking for wrong answer.

21. At constant volume, 4 mol of an ideal gas when heated from 300 K to 500 K changes its internal energy by 5000 J. The molar heat capacity at constant volume is _____.

Answer (6.25)

Sol. $\Delta U = nC_v\Delta T$

$5000 = 4 \times C_v(500 - 300)$

$C_v = 6.25\ JK^{-1}mol^{-1}$

22. For an electrochemical cell

$Sn(s)|Sn^{2+}(aq, 1M)||Pb^{2+}(aq, 1M)|Pb(s)$

the ratio $\frac{[Sn^{2+}]}{[Pb^{2+}]}$ when this cell attains equilibrium is _____.

$$\left(\text{Given}: E^0_{Sn^{2+}|Sn} = -0.14V,\ E^0_{Pb^{2+}|Pb} = -0.13V,\ \frac{2.303RT}{F} = 0.06\right)$$

Answer (2.15)

Sol. At equilibrium state $E_{cell} = 0$ $\quad E^o_{cell} = 0.01\,V$

$$Sn(s) + Pb^{+2}(aq.) \longrightarrow Sn^{+2}(aq.) + Pb(s)$$

$$E = E^o_{cell} - \frac{0.06}{n}\log\frac{[P]}{[R]}$$

$$0 = 0.01 - \frac{0.06}{2}\log\frac{[Sn^{+2}]}{[Pb^{+2}]}$$

$$0.01 = \frac{0.06}{2}\log\frac{[Sn^{+2}]}{[Pb^{+2}]}$$

$$\frac{1}{3} = \log\frac{[Sn^{+2}]}{[Pb^{+2}]}$$

$$\frac{[Sn^{+2}]}{[Pb^{+2}]} = 10^{\frac{1}{3}} = 2.1544 = 2.15$$

23. $NaClO_3$ is used, even in spacecrafts, to produce O_2. The daily consumption of pure O_2 by a person is 492 L at 1 atm, 300 K. How much amount of $NaClO_3$, in grams, is required to produce O_2 for the daily consumption of a person at 1 atm, 300 K? _____.

$NaClO_3(s) + Fe(s) \rightarrow O_2(g) + NaCl(s) + FeO(s)$

$R = 0.082\ L\ atm\ mol^{-1}\ K^{-1}$

Answer (2130.00)

Sol. $NaClO_3(s) + Fe(s) \longrightarrow NaCl(s) + FeO(s) + O_2(g)$

moles of $NaClO_3$ = moles of O_2

$$\text{moles of } O_2 = \frac{PV}{RT} = \frac{1 \times 492}{0.082 \times 300} = 20\ mol$$

mass of $NaClO_3$ = 20 × 106.5 = 2130 g

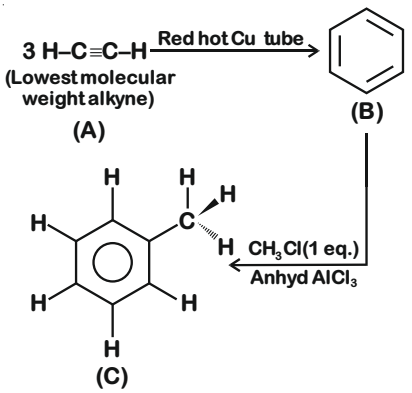
= 2130.00

24. In the following sequence of reactions the maximum number of atoms present in molecule 'C' in one plane is _____.

$$A \xrightarrow[\text{Cu tube}]{\text{Red hot}} B \xrightarrow[\text{Anhydrous } AlCl_3]{CH_3Cl\ (1.\ eq.)} C$$

(A is a lowest molecular weight alkyne)

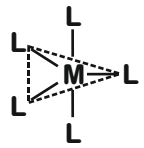
Answer (13.00)

Sol. 

Number of atoms in one plane = 13

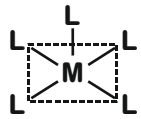
25. Complexes (ML_5) of metals Ni and Fe have ideal square pyramidal and trigonal bipyramidal geometries, respectively. The sum of the 90°, 120° and 180° L-M-L angles in the two complexes is _____.

Answer (20.00)

Sol. 

(Trigonal bipyramidal)

$\angle 120° = 3$, $\angle 90° = 6$, $\angle 180° = 1$ Total = 10



(Square pyramidal)

$\angle 90° = 8$, $\angle 180° = 2 \Rightarrow$ Total = 10

Total number of 180°, 90° and 120° L-M-L bond angles = 10 + 10 = 20

❑ ❑ ❑

MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $A=\begin{pmatrix}2 & 2\\ 9 & 4\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, then $10A^{-1}$ is equal to

(1) 6I – A

(2) 4I – A

(3) A – 4I

(4) A – 6I

Answer (4)

Sol. $A=\begin{bmatrix}2 & 2\\ 9 & 4\end{bmatrix}$

$\because$ Every square matrix satisfying characteristic equation

$\Rightarrow$ $|A-\lambda I| = 0$

$\because$ $\begin{bmatrix}2-\lambda & 2\\ 9 & 4-\lambda\end{bmatrix}=0$

$\Rightarrow$ $(\lambda-4)(\lambda-2) = 18$

$\Rightarrow$ $\lambda^2-6\lambda-10=0$

$\therefore$ $A^2-6A-10I=0$

(By Pre multiplication of A^{-1} both sides)

$\Rightarrow$ $\boxed{A-6I=10A^{-1}}$

2. The differential equation of the family of curves, $x^2=4b(y+b), b\in R$, is

(1) $x(y')^2=x-2yy'$ (2) $x(y')^2=2yy'-x$

(3) $x(y')^2=x+2yy'$ (4) $xy''=y'$

Answer (3)

Sol. $x^2 = 4b(y + b)$ ...(i)

$\Rightarrow$ $2x=4b\left(\frac{dy}{dx}\right)$ $\Rightarrow$ $x=2b\frac{dy}{dx}$

$\Rightarrow$ $b=\frac{x}{2\left(\frac{dy}{dx}\right)}$ ...(ii)

Put b from (ii) in (i)

$\Rightarrow$ $x^2=\frac{4\times x}{2\times\frac{dy}{dx}}\left(y+\frac{x}{2\left(\frac{dy}{dx}\right)}\right)$

$\Rightarrow$ $x\frac{dy}{dx}=\frac{\left(2y\frac{dy}{dx}+x\right)\times 2}{2\left(\frac{dy}{dx}\right)}$ $\Rightarrow$ $x\left(\frac{dy}{dx}\right)^2=2y\frac{dy}{dx}+x$

3. The area (in sq. units) of the region $\{(x, y)\in R^2 : x^2\le y\le 3-2x\}$, is

(1) $\frac{31}{3}$

(2) $\frac{29}{3}$

(3) $\frac{34}{3}$

(4) $\frac{32}{3}$

Answer (4)

Sol. $\because$ $x^2-y\le 0$ and $2x+y-3\le 0$

For Point of intersection we have

$x^2+2x-3=0$ $\Rightarrow$ $x = 1, x = -3$

$\therefore$ P(1, 1) and Q(–3, 9) are point of intersection

$\therefore$ Required area $=\int_{-3}^{1}(3-2x-x^2)dx$

$=12-(x^2)_{-3}^{1}-\frac{1}{3}(x^3)_{-3}^{1}$

$=12-(1-9)-\frac{1}{3}[1+27]$

$=20-\frac{28}{3}=11-\frac{1}{3}=\frac{32}{3}$

4. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}.\vec{a}=0$, then $\vec{c}.\vec{b}$ is equal to

(1) $-\frac{1}{2}$ (2) $-\frac{3}{2}$

(3) –1 (4) $\frac{1}{2}$

Answer (1)

Sol. $\vec{a}=\hat{i}-2\hat{j}+\hat{k};\vec{b}=\hat{i}-\hat{j}+\hat{k}$

$\because$ $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ $\Rightarrow$ $\vec{a}\times(\vec{b}\times\vec{c})=\vec{a}\times(\vec{b}\times\vec{a})$

$\Rightarrow$ $(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=(\vec{a}.\vec{a})\vec{b}-(\vec{a}.\vec{b})\vec{a}$

$\because$ $\vec{a}.\vec{c}=0$

$\Rightarrow$ $\vec{c}=-\frac{1}{2}(\hat{i}+\hat{j}+\hat{k})$

$\therefore$ $\boxed{\vec{b}.\vec{c}=\frac{-1}{2}}$

5. Let S be the set of all functions $f:[0,1]\to R$, which are continuous on [0, 1] and differentiable on (0, 1). Then for every f in S, there exists a c $\in$ (0, 1), depending on f, such that

(1) |f(c) – f(1) < |f'(c)|

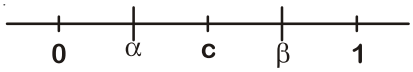
(2) $\frac{f(1)-f(c)}{1-c}f'(c)$

(3) |f(c) – f(1) < (1 – c)|f'(c)|

(4) |f(c) + f(1) < (1 + c)|f'(c)|

Answer (Bonus)

Sol. Case – 1 If f(x) is non constant



By L.M.V.T in x $\in$ (0, c) $f'(\alpha)=\frac{f(c)-f(0)}{c}$

$\Rightarrow$ $cf'(\alpha)$ = f(c) – f(0) ...(i)

By L.M.V.T in x $\in$ (c, 1) $f'(\beta)=\frac{f(1)-f(c)}{1-c}$

$\Rightarrow$ $f'(\beta)$ (1 – c) = f(1) – f(c) ...(ii)

By (i) + (ii)

$\Rightarrow$ $cf'\alpha$ + (1 – c) $f'(\beta)$ = f(1) – f(0) ...(iii)

By L.M.V.T in (0, 1), f′(c) = f(1) – f(0) ...(iv)

From eqution (iii) and (iv)

f′(c) = $cf'(\alpha)$ + (1 – c) $f'(\beta)$

$\Rightarrow$ |f′(c)| = |$cf'(\alpha)$ + (1 – c) $f'(\beta)$| ...(v)

From eqution (ii)

$\Rightarrow$ |f′(c)| = |$cf'(\alpha)$ + f(1) – f(c)|

$\therefore$ |f′(c)| > |f(1) – f(c)|

Case 2 – If f(x) is taken as constant then options(2) will satisfy which is contradictory. So it can be given as BONUS

6. The mirror image of the point (1, 2, 3) in a plane is $\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$. Which of the following points lies on this plane?

(1) (–1, –1, –1)

(2) (1, 1, 1)

(3) (–1, –1, 1)

(4) (1, –1, 1)

Answer (4)

Sol. Let A(1, 2, 3), $B\left(\frac{-7}{3},\frac{-4}{3},\frac{-1}{3}\right)$

Mid-point of $AB\equiv\left(\frac{-2}{3},\frac{1}{3},\frac{4}{3}\right)$

Let equation of plane is

$a\left(x+\frac{2}{3}\right)+b\left(y-\frac{1}{3}\right)+c\left(z-\frac{4}{3}\right)=0$ ...(i)

dr's of $AB\equiv\frac{10}{3},\frac{10}{3},\frac{10}{3}$

$\therefore$ Equation of plane is

$x+\frac{2}{3}+y-\frac{1}{3}+z-\frac{4}{3}=0$

$\Rightarrow$ $\boxed{x+y+z=1}$

$\therefore$ (1, –1, +1) lies on the plane

7. Which of the following statements is a tautology?

(1) $\sim(p \wedge \sim q) \rightarrow p \vee q$

(2) $p \vee (\sim q) \rightarrow p \wedge q$

(3) $\sim(p \vee \sim q) \rightarrow p \vee q$

(4) $\sim(p \vee \sim q) \rightarrow p \wedge q$

Answer (3)

Sol. $\because$ $p \rightarrow q \equiv q \vee \sim P$

$\therefore$ Checking option 3 $\sim (p \vee \sim q) \rightarrow (p \vee q)$ is equivalent to

$\Rightarrow$ $(p \vee q) \vee (p \vee \sim q)$

$\Rightarrow$ $p \vee T \equiv T$

8. The system of linear equations

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has

(1) infinitely many solutions when $\lambda = 2$

(2) no solution when $\lambda = 8$

(3) a unique solution when $\lambda = -8$

(4) no solution when $\lambda = 2$

Answer (4)

Sol. $\Delta = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$

$= \lambda(18 - 5\lambda) - 2(12\lambda - 20) + 2(2\lambda^2 - 12)$

$\Rightarrow$ $\Delta = -\lambda^2 - 6\lambda + 16$

$\Rightarrow$ $\Delta = (\lambda + 8)(2 - \lambda)$

$\Rightarrow$ $\Delta = 0$ for $\lambda = 2$ or $\lambda = -8$

$\Rightarrow$ for $\lambda = 2$ $\Delta_x = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$

$\Rightarrow$ $\Delta_x = 5(8) - 2(-2) + 2(-14)$

$\Delta_x = 40 + 4 - 28 = 16 \neq 0$

$\therefore$ System has no solution.

9. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2, 2) is

(1) $2\sqrt{2}$

(2) $\sqrt{2}$

(3) $4\sqrt{2}$

(4) 2

Answer (1)

Sol. Given curve $x^2 + 3xy - xy - 3y^2 = 0$

$\Rightarrow$ $(x - y)(x + 3y) = 0 \Rightarrow y = x$ and $\boxed{y = -\frac{x}{3}}$

$\therefore$ Normal pass through (2, 2) and is perpendicular to line $x - y = 0$

Let normal is $x + y + \lambda = 0 \Rightarrow \boxed{\lambda = -4}$

$\therefore$ Perpendicular distance = $\left|\frac{-4}{\sqrt{2}}\right| = 2\sqrt{2}$

10. If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then

(1) $\frac{1}{9} < I^2 < \frac{1}{8}$

(2) $\frac{1}{8} < I^2 < \frac{1}{4}$

(3) $\frac{1}{6} < I^2 < \frac{1}{2}$

(4) $\frac{1}{16} < I^2 < \frac{1}{9}$

Answer (1)

Sol. Let $f(x) = 2x^3 - 9x^2 + 12x + 4$

$\Rightarrow$ $f'(x) = 6(x^2 - 3x + 2)$

$\therefore$ f(x) decreases in (1, 2), f(1) = 9

f(2) = 8

$\Rightarrow$ $\frac{1}{3} < I < \frac{1}{\sqrt{8}}$

$\Rightarrow$ $\boxed{\frac{1}{9} < I^2 < \frac{1}{8}}$

11. If a hyperbola passes through the point P(10, 16) and it has vertices at ($\pm$6, 0), then the equation of the normal to it at P is

(1) x + 2y = 42 (2) 2x + 5y = 100

(3) x + 3y = 58 (4) 3x + 4y = 94

Answer (2)

Sol. Let hyperbola = $\frac{x^2}{36}-\frac{y^2}{b^2}=1$ $\therefore$ (10, 16) lies on it

$\Rightarrow \frac{100}{36}-\frac{256}{b^2}=1 \Rightarrow \frac{64}{36}=\frac{256}{b^2}$

$\Rightarrow \boxed{b^2=144} \quad \Rightarrow \boxed{b=12}$

$\therefore$ Equation of normal $\frac{x-10}{\frac{10}{36}}=\frac{y-16}{(-)\frac{16}{144}}$

$\Rightarrow$ 2x – 20 = –5y + 80

$\Rightarrow \boxed{2x+5y=100}$

12. $\lim_{x\to 0}\frac{\int_0^x t\sin(10t)dt}{x}$ is equal to

(1) 0 (2) $\frac{1}{10}$

(3) $-\frac{1}{5}$ (4) $-\frac{1}{10}$

Answer (1)

Sol. $\lim_{x\to 0}\frac{\int_0^x t\sin(10t)dt}{x}$

by L′ hospital rule

$\lim_{x\to 0}\frac{x\sin(10x)}{1}=\boxed{0}$

13. If the 10th term of an A.P. is $\frac{1}{20}$ and its 20th term is $\frac{1}{10}$, then the sum of its first 200 terms is

(1) $50\frac{1}{4}$ (2) 50

(3) 100 (4) $100\frac{1}{2}$

Answer (4)

Sol. $\therefore$ $a+9d=\frac{1}{20}$ ...(i)

$a+19d=\frac{1}{100}$ ...(ii)

$\therefore$ By (ii) – (i) $10d=\frac{1}{10}-\frac{1}{20}=\frac{10}{200}=\frac{1}{20}$

$\Rightarrow \boxed{d=\frac{1}{200}} \Rightarrow a=\frac{1}{200}$ from (i)

$\therefore$ $S_{200} = 100\left[2\times\frac{1}{200}+\frac{199}{200}\right]$

$= 1+\frac{199}{2}=\frac{201}{2}$

$= 100\frac{1}{2}$

14. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S

(1) Contains at least four elements

(2) Is a singleton

(3) Contains exactly two elements

(4) Is an empty set

Answer (2)

Sol. $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$

Case I: $0 < 3^x < 1 \Rightarrow -\infty < x < 0$

$\Rightarrow (3^x)^2 - 3^x + 2 = 1 - 3^x + 2 - 3^x$

$\Rightarrow (3^x)^2 + 3^x - 1 = 0 \Rightarrow 3^x=\frac{-1+\sqrt{5}}{2}<1$

$\Rightarrow$ One real solution

Case II: $1 < 3^x < 2 \Rightarrow 0 < x < \log_2 3$

$\Rightarrow (3^x)^2 - 3^x + 2 = 3^x - 1 + 2 - 3^x$

$\Rightarrow (3^x)^2 - 3^x + 1 = 0$

$\Rightarrow$ No solution $\therefore$ Discriminant is negative

Case III: $2 < 3^x < \infty$

$\Rightarrow (3^x)^2 - 3^x + 2 = 2.3^x - 3 \Rightarrow (3^x)^2 - 3.(3^x) + 5 = 0$

$\Rightarrow$ No solution $\therefore$ Discriminant is negative

15. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is

(1) 3.98 (2) 4.02

(3) 3.99 (4) 4.01

Answer (3)

Sol. $\sum x_i = 200$ and $\sum x_i^2 = 2080$

Now Actual Mean = $\frac{200+11-9}{20} = \frac{202}{20}$

$\therefore$ Actual variance = $\frac{2080-81+121}{20} - \left(\frac{202}{20}\right)^2$

$106 - (10.1)^2 = 106 - 102.01 = 3.99$

16. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6$, then

(1) $\alpha - \beta = 60$ (2) $\alpha + \beta = 60$

(3) $\alpha - \beta = -132$ (4) $\alpha + \beta = -30$

Answer (3)

Sol. $\because$ $\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6 = 2$

$$\left[{}^6C_0x^6 + {}^6C_2x^4\left(x^2-1\right) + {}^6C_4x^2\left(x^2-1\right)^2 + {}^6C_6\left(x^2-1\right)^3\right].$$

$= 2[32x^6 - 48x^4 + 18x^2 - 1]$

$\therefore$ α = coefficient of $x^4 = -96$

β = coefficient of $x^2 = 36$

$\Rightarrow$ $\alpha - \beta = -96 - 36 = -132$

17. If a line, $y = mx + c$ is a tangent to the circle, $(x-3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then

(1) $c^2 + 6c + 7 = 0$ (2) $c^2 - 7c + 6 = 0$

(3) $c^2 + 7c + 6 = 0$ (4) $c^2 - 6c + 7 = 0$

Answer (1)

Sol. Tangent at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ on circle $x^2 + y^2 = 1$ is

$\boxed{x + y = \sqrt{2}}$

$\therefore$ Slope of tangent m = 1 for circle $(x-3)^2 + y^2 = 1$

$\because$ Any tangent of circle $(x-3)^2 + y^2 = 1$ is

$y = mx - 3m \pm \sqrt{1+m^2}$

$\therefore$ $c + 3m = \pm\sqrt{1+m^2}$ $\because$ m = 1

$\Rightarrow$ $\boxed{c^2 + 6c + 7 = 0}$

18. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b = \sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation

(1) $x^2 - 101x + 100 = 0$

(2) $x^2 - 102x + 101 = 0$

(3) $x^2 + 101x + 100 = 0$

(4) $x^2 + 102x + 101 = 0$

Answer (2)

Sol. $\because$ α = w (complet non real cube of unity)

$\Rightarrow$ $a = (1 + w)(1 + w^2 + w^4 + w^6 + \ldots\ldots w^{200})$

$\Rightarrow$ $a = (1+w)\frac{\left(1-\left(w^2\right)^{101}\right)}{\left(1-w^2\right)} = \frac{(1-w)(1+w)}{1-w^2} = 1$

and $b = 1 + w^3 + w^6 + \ldots\ldots w^{300} = 101$

Equation $x^2 - (102)x + 101 = 0$.

19. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is

(1) 0.01 (2) 0.20

(3) 0.02 (4) 0.10

Answer (4)

Sol. Given $P(A\cap\bar{B})+P(\bar{A}\cap B)=\frac{2}{5}$

$\Rightarrow\ P(A)+P(B)-2P(A\cap B)=\frac{2}{5}$

$\Rightarrow\ P(A\cup B)-P(A\cap B)=\frac{2}{5}$

$\Rightarrow\ \frac{1}{2}-\frac{2}{5}=P(A\cap B)$

$\Rightarrow\ \boxed{P(A\cap B)=\frac{1}{10}}$

20. Let f : (1, 3) → R be a function defined by f(x) = $\frac{x[x]}{1+x^2}$, where [x] denotes the greatest integer ≤ x. Then the range of f is

(1) $\left(\frac{2}{5},\frac{3}{5}\right]\cup\left(\frac{3}{4},\frac{4}{5}\right)$ (2) $\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{5},\frac{4}{5}\right]$

(3) $\left(\frac{2}{5},\frac{4}{5}\right]$ (4) $\left(\frac{3}{5},\frac{4}{5}\right)$

Answer (2)

Sol. $f(x)=\frac{x[x]}{x^2+1};1<x<3$

$$\Rightarrow\ f(x)=\begin{cases}\dfrac{1}{\left(x+\frac{1}{x}\right)} & 1<x<2\\[2ex] \dfrac{2}{\left(x+\frac{1}{x}\right)} & 2\le x<3\end{cases}$$

$$\Rightarrow\ f'(x)=\begin{cases}\dfrac{\left(1-x^2\right)}{x^2\left(x+\frac{1}{x}\right)^2} & 1<x<2\\[2ex] \dfrac{2\left(1-x^2\right)}{x^2\left(x+\frac{1}{x}\right)^2} & 2\le x<3\end{cases}$$

∴ f(x) is decreasing function.

∴ Range is $\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{5},\frac{4}{5}\right]$

SECTION - II

Numerical Value Type Questions: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and there is no negative marking for wrong answer.

21. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$, $\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$, then tan(α + 2β) is equal to _____.

Answer (01)

Sol. $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\,|\cos\alpha|}=\frac{1}{7}\ \Rightarrow\ \tan\alpha=\frac{1}{7}\quad\because\ \alpha\in\left(0,\frac{\pi}{2}\right)$

∴ |cosα| = cosα.

Now, $|\sin\beta|=\frac{1}{\sqrt{10}}\ \Rightarrow\ \sin\beta=\frac{1}{\sqrt{10}}\quad\because\ \beta\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow\ \boxed{\tan\beta=\frac{1}{3}}$

$$\Rightarrow\ \tan(2\beta)=\frac{2\tan\beta}{1-\tan^2\beta}=\frac{\frac{2}{3}}{1-\frac{1}{9}}=\frac{2}{3}\times\frac{9}{8}=\frac{3}{4}.$$

$$\therefore\ \tan(\alpha+2\beta)=\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}=\frac{25}{25}=1$$

22. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _____.

Answer (2454)

Sol. EXAMINATION has letter distribution as follows

$$\overset{8}{2A},\ \overset{7}{2N},\ \overset{6}{2I},\ \overset{5}{E},\ \overset{4}{X},\ \overset{3}{M},\ \overset{2}{T},\ \overset{1}{O}$$

Case-I, When all letters are different

$\Rightarrow\ {}^8C_4\times\lfloor 4=1680$

Case-II, Two are same and two are different

$$\Rightarrow {}^3C_1 \times {}^7C_2 \times \frac{\lfloor 4}{\lfloor 2}$$

Case-III, Two same of one kind and two same of other kind

$$\Rightarrow {}^3C_2 \times \frac{\lfloor 4}{\lfloor 2 \times \lfloor 2}$$

$\therefore$ Total ways = 1680 + 756 + 18

= 2454

23. Let f(x) be a polynomial of degree 3 such that f(–1) = 10, f(1) = –6, f(x) has a critical point at x = –1, and f'(x) has a critical point x = 1. Then f(x) has a local minima at x = _____.

Answer (03)

Sol. Let f′(x) = a(x + 1)(x – 3)

$$\Rightarrow \int_{-1}^{x} f'(x) = \int_{-1}^{x} a(x^2 - 2x - 3)dx$$

$$\Rightarrow f(x) - f(-1) = a\left(\frac{x^3}{3} - x^2 - 3x\right)_{-1}^{x}$$

$$\Rightarrow f(x) = 10 + a\left[\left(\frac{x^3}{3} - x^2 - 3x\right) - \left(-\frac{1}{3} - 1 + 3\right)\right]$$

$\Rightarrow$ 3f(x) = 30 + a[(x³ – 3x² – 9x) – (–1 – 3 + 9)]

$\Rightarrow$ 3f(x) = 30 + a[x³ – 3x² – 9x – 5]

$\because$ f(1) = –6

$\Rightarrow$ (–)18 = 30 + a[–16] $\Rightarrow$ 16a = 48 $\Rightarrow$ a > 0

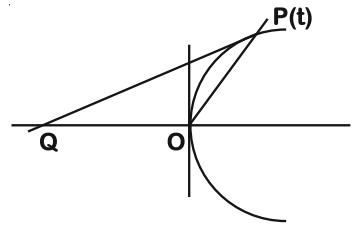
($\because$ a = 3)

$\therefore$ Minima occurs at x = 3

24. Let a line y = mx (m > 0) intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area (ΔOPQ) = 4 sq. units, then m is equal to_____.

Answer (.50)

Sol.



Tangent at Point P(t) is

$$yt = x + \frac{1}{4}t^2$$

$\therefore$ Point Q is $\left(-\frac{t^2}{4}, 0\right)$

$$P\left(\frac{t^2}{4}, \frac{t}{2}\right)$$

$\therefore$ Area of $\Delta OPQ = \frac{1}{2}\left|\frac{t^3}{8}\right| = 4$

$\Rightarrow |t^3| = 64 \Rightarrow \boxed{t = 4}$

Now, $m = \frac{t \times 4}{2 \times t^2} = \frac{2}{t} = \frac{2}{4}$

$\boxed{= \frac{1}{2}} = \boxed{.50}$

25. The sum, $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to

Answer (504)

Sol. $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4} = \frac{1}{4}\sum_{n=1}^{7}(n^2+n)(2n+1)$

$$= \frac{1}{4}\left[\sum_{n=1}^{7} 2n^3 + 3n^2 + n\right]$$

$$= \frac{1}{4}\left[2 \times (28)^2 + \frac{3 \times 7 \times 8 \times 15}{6} + \frac{7 \times 8}{2}\right]$$

= 7[56 + 15 + 1] = 7 × 72

$= \boxed{504}$

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