NCERT solutions for class 11 maths chapter 14 mathematical reasoning

Question:1.(i) Which of the following sentences are statements? Give reasons for your answer.

There are 35 days in a month.

Answer:

There are 30 or 31 days in a month (And 28 or 29 in some cases). The given sentence is false. Hence it is a statement.

Question:1.(ii) Which of the following sentences are statements? Give reasons for your answer.

Mathematics is difficult.

Answer:

Mathematics can be difficult for some and easy for others. So it is neither true nor false. Hence it is not a statement.

Question:1.(iii) Which of the following sentences are statements? Give reasons for your answer.

The sum of 5 and 7 is greater than 10.

Answer:

Sum of 5 and 7 = 5 + 7 = 12 > 10. Hence the sentence is true. So it is a statement.

Question:1.(iv) Which of the following sentences are statements? Give reasons for your answer.

The square of a number is an even number.

Answer:

$2^2 = 4$, which is even, and $3^2 = 9$, which is odd. So the square of a number may be even or may be odd. Hence it is not a statement.

Question:1.(v) Which of the following sentences are statements? Give reasons for your answer.

The sides of a quadrilateral have equal length.

Answer:

The sentence is true for square and rhombus but not true for the rectangle. Hence it is not a statement.

Question:1.(vi) Which of the following sentences are statements? Give reasons for your answer.

Answer this question.

Answer:

The give sentence is an order. Hence it is not a statement.

Question:1.(vii) Which of the following sentences are statements? Give reasons for your answer.

The product of (–1) and 8 is 8.

Answer:

The product of (-1) and 8 = -1 x 8 = -8. Hence the given sentence is false. So it is a statement.

Question:1.(viii) Which of the following sentences are statements? Give reasons for your answer.

The sum of all interior angles of a triangle is 180 °.

Answer:

The sum of all interior angles of a triangle is 180 °. This sentence is true always. Hence this is a statement.

Question:1.(ix) Which of the following sentences are statements? Give reasons for your answer.

Today is a windy day.

Answer:

This may be true or false. Hence this is not a statement.

Question:1.(x) Which of the following sentences are statements? Give reasons for your answer.

All real numbers are complex numbers.

Answer:

All real numbers can be written in the form of a + i(0) (when a is a real number). This shows that all real numbers are complex numbers. Hence the sentence is always true. So it is a statement.

Question:2 Give three examples of sentences which are not statements. Give reasons for the answers.

Answer:

Following are three examples of sentences which are not statements.

How beautiful!

- This is an exclamation. Hence not a statement.

Open the door.

- This is an order. Hence not a statement.

Where are you going?

- This is a question. Hence not a statement.

NCERT solutions for class 11 maths chapter 14 mathematical reasoning-Exercise: 14.2

Question:1(i) Write the negation of the following statements:

Chennai is the capital of Tamil Nadu.

Answer:

Chennai is not the capital of Tamil Nadu.

Or

It is false to say that Chennai is the capital of Tamil Nadu.

Or

It is not the case that Chennai is the capital of Tamil Nadu.

Question:1.(ii) Write the negation of the following statements:

$\sqrt{2}$ is not a complex number

Answer:

$\sqrt{2}$ is a complex number.

Or

It is false to say that $\sqrt{2}$ is not a complex number.

Or

It is not the case that $\sqrt{2}$ is not a complex number.

Question:1.(iii) Write the negation of the following statements:

All triangles are not equilateral triangle.

Answer:

All triangles are equilateral triangle.

Or

It is false to say that all triangles are not equilateral triangle.

Or

It is not the case that all triangles are not equilateral triangle.

Question:1.(iv) Write the negation of the following statements:

The number 2 is greater than 7.

Answer:

The number 2 is not greater than 7.

Or

It is false to say that the number 2 is greater than 7.

Or

It is not the case that the number 2 is greater than 7.

Question:1.(v) Write the negation of the following statements:

Every natural number is an integer.

Answer:

Every natural number is not an integer.

Or

It is false to say that every natural number is an integer.

Or

It is not the case that every natural number is an integer.

Question:2.(i) Are the following pairs of statements negations of each other:

The number x is not a rational number.

The number x is not an irrational number.

Answer:

p: The number x is not a rational number.

r: The number x is not an irrational number.

The negation of p is: The number x is a rational number, which is the same as statement r.

The negation of r is: The number x is an irrational number, which is the same as statement p.

Hence the pairs of statements are negations of each other.

Question:2(ii) Are the following pairs of statements negations of each other:

The number x is a rational number.

The number x is an irrational number.

Answer:

p: The number x is a rational number.

r: The number x is an irrational number.

The negation of p is: The number x is not a rational number, which is the same as statement r.

The negation of r is: The number x is not an irrational number, which is the same as statement p.

Hence the pairs of statements are negations of each other.

Question:3.(i) Find the component statements of the following compound statements and check whether they are true or false.
Number 3 is prime or it is odd.

Answer:

The component statements are

p: Number 3 is prime.

r: Number 3 is odd.

Both statements are true. Here the connecting word is 'or'.

Question:3.(ii) Find the component statements of the following compound statements and check whether they are true or false.

All integers are positive or negative.

Answer:

The component statements are:

p: All integers are positive.

r: All integers are negative.

Both the components statements are false. Here the connecting word is 'or'.

Question:3.(iii) Find the component statements of the following compound statements and check whether they are true or false.

100 is divisible by 3, 11 and 5.

Answer:

The component statements are:

p: 100 is divisible by 3.

q: 100 is divisible by 11.

r: 100 is divisible by 5.

First two statements are false and the last statement is true. Here the connecting word is 'and'.

NCERT solutions for class 11 maths chapter 14 mathematical reasoning-Exercise: 14.3

Question:1.(i) For the following compound statements first identify the connecting words and then break it into component statements.

All rational numbers are real and all real numbers are not complex.

Answer:

The connecting word here is 'and'.

The component statements are:

p: All rational numbers are real.

q: All real numbers are not complex.

Question:1.(ii) For the following compound statement first identify the connecting words and then break it into component statements.

Square of an integer is positive or negative.

Answer:

The connecting word here is 'Or'.

The component statements are:

p: Square of an integer is positive.

q: Square of an integer is negative.

Question:1.(iii) For the following compound statement first identify the connecting words and then break it into component statements.

The sand heats up quickly in the Sun and does not cool down fast at night.

Answer:

The connecting word here is 'and'.

The component statements are:

p: The sand heats up quickly in the Sun.

q: The sand does not cool down fast at night.

Question:1.(iv) For the following compound statement first identify the connecting words and then break it into component statements.

$x = 2$ and $x = 3$ are the roots of the equation $3x^2 - x - 10 = 0$.

Answer:

The connecting word here is 'and'.

The component statements are:

p: x = 2 is a root of the equation $3x^2 - x - 10 = 0$.

q: x = 3 is a root of the equation $3x^2 - x - 10 = 0$.

Question:2.(i) Identify the quantifier in the following statement and write the negation of the statement.

There exists a number which is equal to its square.

Answer:

Given, p: There exists a number which is equal to its square.

Quantifier is "There exists".

Negation is, p': There does not exist a number which is equal to its square.

Question:2.(ii) Identify the quantifier in the following statement and write the negation of the statement.

For every real number x , x is less than $x + 1$.

Answer:

Given, p: For every real number x , x is less than $x + 1$.

Quantifier is "For Every".

Negation is, p': There exists a real number x such that x is not less than x + 1.

Question:2.(iii) Identify the quantifier in the following statement and write the negation of the statement.

There exists a capital for every state in India.

Answer:

Given, p: There exists a capital for every state in India.

Quantifier is "There exists".

Negation is, p': There does not exist a capital for every state in India. Or, There exists a state in India which does not have a capital.

Question:3. Check whether the following pair of statements are negation of each other. Give reasons for your answer.

(i) $x + y = y + x$ is true for every real numbers x and y .

(ii) There exists real numbers x and y for which $x + y = y + x$.

Answer:

p: $x + y = y + x$ is true for every real numbers x and y.

q: There exists real numbers x and y for which $x + y = y + x$.

The negation of p is:

There exists no real numbers x and y for which $x + y = y + x$

which is not equal to q.

Hence the given pair of statements are not negation of each other.

Question:4.(i) State whether the "Or" used in the following statement is "exclusive "or" inclusive. Give reasons for your answer.

Sun rises or Moon sets.

Answer:

It is not possible for the Sun to rise and the moon to set simultaneously.

Here 'Or' is exclusive

Question:4.(ii) State whether the "Or" used in the following statement is "exclusive "or" inclusive. Give reasons for your answer.

To apply for a driving licence, you should have a ration card or a passport.

Answer:

A person can have both ration card or a passport to apply for a driving license.

Here 'Or' is inclusive.

Question:4.(iii) State whether the "Or" used in the following statement is "exclusive "or" inclusive. Give reasons for your answer.

All integers are positive or negative.

Answer:

All integers are either positive or negative but cannot be both.

Here 'Or' is exclusive.

NCERT solutions for class 11 maths chapter 14 mathematical reasoning-Exercise: 14.4

Question:1. Rewrite the following statement with "if-then" in five different ways conveying the same meaning.
If a natural number is odd, then its square is also odd.

Answer:

a.) If the square of a natural number is odd, then the natural number is odd.

b.) A natural number is not odd only if its square is not odd.

c.) For a natural number to be odd it is necessary that its square is odd.

d.) For the square of a natural number to be odd, it is sufficient that the number is odd

e.) If the square of a natural number is not odd, then the natural number is not odd.

Question:2.(i) Write the contrapositive and converse of the following statement.

If x is a prime number, then *x* is odd.

Answer:

The contrapositive is :

If a number x is not odd, then x is not a prime number.

The converse is :

If a number x in odd, then it is a prime number.

Question:2.(ii) Write the contrapositive and converse of the following statement.

If the two lines are parallel, then they do not intersect in the same plane.

Answer:

The contrapositive is:

If two lines intersect in the same plane, then they are not parallel.

The converse is:

If two lines do not intersect in the same plane, then they are parallel.

Question:2.(iii) Write the contrapositive and converse of the following statement.

Something is cold implies that it has low temperature.

Answer:

The contrapositive is:

If something is not at low temperature, then it is not cold.

The converse is:

If something is at low temperature, then it is cold .

Question:2.(iv) Write the contrapositive and converse of the following statement.

You cannot comprehend geometry if you do not know how to reason deductively.

Answer:

The contrapositive is:

If you know how to reason deductively, then you can comprehend geometry.

The converse is:

If you do not know how to reason deductively, then you cannot comprehend geometry.

Question:2.(v) Write the contrapositive and converse of the following statement.

x is an even number implies that x is divisible by 4.

Answer:

First, we convert the given sentence into the "if-then" statement:

If x is an even number, then x is divisible by 4.

The contrapositive is:

If x is not divisible by 4, then x is not an even number.

The converse is:

If x is divisible by 4, then x is an even number.

Question:3.(i) Write the following statement in the form "if-then"

You get a job implies that your credentials are good.

Answer:

The given statement in the form "if-then" is :

If you get a job, then your credentials are good.

Question:3.(ii) Write the following statement in the form "if-then"

The Banana tree will bloom if it stays warm for a month.

Answer:

The given statement in the form "if-then" is :

If the Banana tree stays warm for a month, then it will bloom.

Question:3.(iii) Write the following statement in the form "if-then"

A quadrilateral is a parallelogram if its diagonals bisect each other.

Answer:

The given statement in the form "if-then" is :

If diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Question:3.(iv) Write the following statement in the form "if-then"

To get an A + in the class, it is necessary that you do all the exercises of the book.

Answer:

The given statement in the form "if-then" is :

(iv) If you get A+ in the class, then you have done all the exercises in the book.

Question:4.(a) Given statements in (a). Identify the statements given below as contrapositive or converse of each other.

If you live in Delhi, then you have winter clothes.
(i) If you do not have winter clothes, then you do not live in Delhi.
(ii) If you have winter clothes, then you live in Delhi.

Answer:

If <u>you live in Delhi</u> , then <u>you have winter clothes</u> . : (if p then q)

The Contrapositive is (~q, then ~p)

Hence (i) is the Contrapositive statement.

The Converse is (q, then p)

Hence (ii) is the Converse statement.

Question:4.(b) Given statements in (b). Identify the statements given below as contrapositive or converse of each other.

If a quadrilateral is a parallelogram, then its diagonals bisect each other.
(i) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.
(ii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Answer:

If <u>a quadrilateral is a parallelogram</u>, then <u>its diagonals bisect each other</u>. (if p then q)

The Contrapositive is (~q, then ~p)

Hence (i) is the Contrapositive statement.

The Converse is (q, then p)

Hence (ii) is the Converse statement.

NCERT solutions for class 11 maths chapter 14 mathematical reasoning- Exercise: 14.5

Question:1.(i) Show that the statement
p: "If x is a real number such that $x^3 + 4x = 0$, then x is 0" is true by direct method,

Answer:

If x is a real number such that $x^3 + 4x = 0$, then x is 0 : (if p then q)

p: x is a real number such that $x^3 + 4x = 0$.

q: x is 0.

In order to prove the statement "if p then q"

Direct Method: By assuming that p is true, prove that q must be true.

So,

p is true:There exists a real number x such that $x^3 + 4x = 0 \implies x(x^2 + 4) = 0$

$\implies x = 0 \; or \; (x^2 + 4) = 0$

$\implies x = 0 \; or \; x^2 = -4 \; (not \; possible)$

Hence, x = 0

Therefore q is true.

Question:1.(ii) Show that the statement

p: "If x is a real number such that $x^3 + 4x = 0$, then x is 0" is true by

method of contradiction

Answer:

If x is a real number such that $x^3 + 4x = 0$, then x is 0 : (if p then q)

p: x is a real number such that $x^3 + 4x = 0$.

q: x is 0.

In order to prove the statement "if p then q"

Contradiction: By assuming that p is true and q is false.

So,

p is true: There exists a real number x such that $x^3 + 4x = 0$

q is false: $x \neq 0$

Now, $x^3 + 4x = 0 \implies x(x^2 + 4) = 0$

$\implies x = 0 \ or \ (x^2 + 4) = 0$

$\implies x = 0 \ or \ x^2 = -4 \ (not \ possible)$

Hence, x = 0

But we assumed $x \neq 0$. This contradicts our assumption.

Therefore q is true.

Question:1(iii) Show that the statement p: "If x is a real number such that $x^3 + 4x = 0$, then x is 0" is true by method of contrapositive

Answer:

If x is a real number such that $x^3 + 4x = 0$, then x is 0 : (if p then q)

p: x is a real number such that $x^3 + 4x = 0$.

q: x is 0.

In order to prove the statement "if p then q"

Contrapositive Method: By assuming that q is false, prove that p must be false.

So,

q is false: $x \neq 0$

$\implies$ x.(Positive number) $\neq$ 0.(Positive number)

$\implies x(x^2 + 4) \neq 0(x^2 + 4)$

$\implies x(x^2 + 4) \neq 0 \implies x^3 + 4x \neq 0$

Therefore p is false.

Question:2 Show that the statement "For any real numbers a and b, $a^2 = b^2$ implies that $a = b$" is not true by giving a counter-example.

Answer:

Given,

For any real numbers a and b, $a^2 = b^2$ implies that $a = b$.

Let a = 1 & b = -1

Now,

$a^2 = (1)^2$ = 1

$b^2 = (-1)^2$ = 1

$$\implies a^2 = 1 = b^2$$

But a ≠ b

Hence $a^2 = b^2$ does not imply that $a = b$.

Hence the given statement is not true.

Question:3 Show that the following statement is true by the method of contrapositive.

p: *If x is an integer and x^2 is even, then x is also even.*

Answer:

Given, If x is an integer and x^2 is even, then x is also even.

Let, p : x is an integer and x^2 is even

q: x is even

In order to prove the statement "if p then q"

Contrapositive Method: By assuming that q is false, prove that p must be false.

So,

q is false: x is not even $\implies$ x is odd $\implies$ x = 2n+1 (n is a natural number)

$$\therefore x^2 = (2n+1)^2$$
$$\implies x^2 = 4n^2 + 4n + 1$$
$$\implies x^2 = 2.2(n^2 + n) + 1 = 2m + 1$$

Hence x^2 is odd $\implies x^2$ is not even

Hence p is false.

Hence the given statement is true.

Question:4.(i) By giving a counter example, show that the following statement is not true.

p : If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.

Answer:

We know, Sum of all the angles of a triangle = $180°$

If all the three angles are equal, then each angle is $60°$

But $60°$ is not an obtuse angle, and hence none of the angles of the triangle is obtuse.

Hence the triangle is not an obtuse-angled triangle.

Hence the given statement is not true.

Question:4.(ii) By giving a counter example, show that the following statement is not true.

q: The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

Answer:

Given,

The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

Let x = 1

$$\therefore (1)^2 - 1 = 1 - 1 = 0$$

Hence 1 is a root of the equation $x^2 - 1 = 0$.

But 1 lies between 0 and 2.

Hence the given statement is not true.

Question:5.(i) Is the following statement true or false? Give a valid reason for saying so.

p : Each radius of a circle is a chord of the circle

Answer:

The statement is False.

By definition, A chord is a line segment intersecting the circle in two points. But a radius is a line segment joining any point on circle to its centre.

Question:5.(ii) Is the following statement true or false? Give a valid reason for saying so.

q : The centre of a circle bisects each chord of the circle.

Answer:

The statement is False.

A chord is a line segment intersecting the circle in two points. But it is not necessary for a chord to pass through the centre.

Question:5.(iii) Is the following statement true or false? Give a valid reason for saying so.

r : Circle is a particular case of an ellipse.

Answer:

The statement is True.

In the equation of an ellipse if we put a = b, then it is a circle.

Question:5.(iv) Is the following statement true or false? Give a valid reason for saying so.

s : If x and y are integers such that $x > y$, then $-x < -y$.

Answer:

The statement is True.

Give, x>y

Multiplying -1 both sides

(-1)x<(-1)y $\implies$ -x < -y

(When -1 is multiplied to both L.H.S & R.H.S, sign of inequality changes)

By the rule of inequality.

Question:5.(v) Is the following statement true or false? Give a valid reason for saying so.

$t : \sqrt{11}$ is a rational number.

Answer:

The statement is False.

Since 11 is a prime number, therefore $\sqrt{11}$ is irrational.

NCERT solutions for class 11 maths chapter 14 mathematical reasoning-Miscellaneous Exercise

Question:1.(i) Write the negation of the following statement:

p : For every positive real number x , the number $x - 1$ is also positive.

Answer:

The negation of the statement is:

There exists a positive real number x such that x–1 is not positive.

Question: 1.(ii) Write the negation of the following statement:

q : All cats scratch.

Answer:

The negation of the statement is:

It is false that all cats scratch.

Or

There exists a cat which does not scratch.

Question:1.(iii) Write the negation of the following statement:

r: For every real number x , either $x > 1$ or $x < 1$.

Answer:

The negation of the statement is:

There exists a real number x such that neither x > 1 nor x < 1.

Question:1.(iv) Write the negation of the following statement:

s : There exists a number x such that $0 < x < 1$.

Answer:

The negation of the statement is:

There does not exist a number x such that 0 < x < 1.

Question:2.(i) State the converse and contrapositive of the following statement:

p : A positive integer is prime only if it has no divisors other than 1 and itself.

Answer:

The given statement as "if-then" statement is: If a positive integer is prime, then it has no divisors other than 1 and itself.

The converse of the statement is:

If a positive integer has no divisors other than 1 and itself, then it is a prime.

The contrapositive of the statement is:

If positive integer has divisors other than 1 and itself then it is not prime.

Question:2.(ii) State the converse and contrapositive of the following statement:

q : I go to a beach whenever it is a sunny day.

Answer:

The given statement as "if-then" statement is: If it is a sunny day, then I go to a beach.

The converse of the statement is:

If I go to the beach, then it is a sunny day.

The contrapositive of the statement is:

If I don't go to the beach, then it is not a sunny day.

Question:2.(iii) State the converse and contrapositive of the following statement:

r: If it is hot outside, then you feel thirsty.

Answer:

The given statement is in the form "if p then q".

The converse of the statement is:

If you feel thirsty, then it is hot outside.

The contrapositive of the statement is:

If you don't feel thirsty, then it is not hot outside.

Question:3.(i) Write the statement in the form "if p, then q"

p : It is necessary to have a password to log on to the server.

Answer:

The statement in the form "if p, then q" is :

If you log on to the server, then you have a password.

Question:3.(ii) Write the statement in the form "if p, then q"

q: There is traffic jam whenever it rains.

Answer:

The statement in the form "if p, then q" is :

If it rains, then there is a traffic jam.

Question:3.(iii) Write the statement in the form "if p, then q"

r: You can access the website only if you pay a subsciption fee.

Answer:

The statement in the form "if p, then q" is :

If you can access the website, then you pay a subscription fee.

Question:4.(i) Rewrite the following statement in the form "p if and only if q"

p: If you watch television, then your mind is free and if your mind is free, then you watch television.

Answer:

The statement in the form "p if and only if q" is :

You watch television if and only if your mind is free.

Question:4.(ii) Rewrite the following statement in the form "p if and only if q"

q: For you to get an A grade, it is necessary and sufficient that you do all the homework regularly.

Answer:

The statement in the form "p if and only if q" is :

You get an A grade if and only if you do all the homework regularly.

Question:4.(iii) Rewrite the following statement in the form "p if and only if q"

r: If a quadrilateral is equiangular, then it is a rectangle and if a quadrilateral is a rectangle, then it is equiangular.

Answer:

The statement in the form "p if and only if q" is :

A quadrilateral is equiangular if and only if it is a rectangle.

Question: 5 Given below are two statements

p : 25 is a multiple of 5.

q : 25 is a multiple of 8.

Write the compound statements connecting these two statements with "And" and "Or".

In both cases check the validity of the compound statement.

Answer:

Given,

p: 25 is a multiple of 5.

q: 25 is a multiple of 8.

p is true while q is false.

The compound statement with 'And' is: 25 is a multiple of 5 and 8.

This is a false statement.

The compound statement with 'Or' is: 25 is a multiple of 5 or 8.

This is a true statement.

Question:6.(i) Check the validity of the statement given below by the method given against it

p: The sum of an irrational number and a rational number is irrational (by contradiction method).

Answer:

Assume that the given statement p is false.

The statement becomes: The sum of an irrational number and a rational number is rational.

Let $\sqrt{p} + \dfrac{s}{t} = \dfrac{q}{r}$

Where $\sqrt{p}$ is irrational number and $\dfrac{q}{r}$ and $\dfrac{s}{t}$ are rational numbers.

$\therefore \dfrac{q}{r} - \dfrac{s}{t}$ is a rational number and $\sqrt{p}$ is an irrational number, which is not possible.

This is a contradiction.

Hence our assumption is wrong.

Thus, the given statement p is true.

Question:6.(ii) Check the validity of the statements given below by the method given against it.

q: If n is a real number with $n < 3$, then $n^2 < 9$ (by contradiction method).

Answer:

Assume that the given statement q is false.

The statement becomes: If n is a real number with n > 3, then $n^2 < 9$.

Therefore n>3 and n is a real number.

$\therefore n^2 > 3^2$
$\implies n^2 > 9$

This is a contradiction.

Therefore our assumption is wrong.

Thus, the given statement q is true.

Question:7 Write the following statement in five different ways, conveying the same meaning.
p: If a triangle is equiangular, then it is an obtuse angled triangle.

Answer:

a.) A triangle is equiangular implies it is an obtuse angled triangle.

b.) Knowing that a triangle is equiangular is sufficient to conclude that it is an obtuse angled triangle.

c.) A triangle is equiangular only if it is an obtuse angled triangle.

d.) When a triangle is equiangular, it is necessarily an obtuse angled triangle.

e.) If a triangle is not an obtuse-angled triangle, it is not equiangular.