NCERT Solutions For Class 11 Maths Chapter 9 Sequences and Series

Question:1 Write the first five terms of each of the sequences in Exercises 1 to 6 whose nth terms are:

$$a_n = n(n+2)$$

Answer:

Given : $a_n = n(n+2)$

$$a_1 = 1(1+2) = 3$$

$$a_2 = 2(2+2) = 8$$

$$a_3 = 3(3+2) = 15$$

$$a_4 = 4(4+2) = 24$$

$$a_5 = 5(5+2) = 35$$

Therefore, the required number of terms =3, 8, 15, 24, 35

Question:2 Write the first five terms of each of the sequences in Exercises 1 to 6 whose nthterms are:

$$a_n = \frac{n}{n+1}$$

Answer:

Given : $a_n = \frac{n}{n+1}$

$$a_1 = \frac{1}{1+1} = \frac{1}{2}$$

$$a_2 = \frac{2}{2+1} = \frac{2}{3}$$

$$a_3 = \frac{3}{3+1} = \frac{3}{4}$$

$$a_4 = \frac{4}{4+1} = \frac{4}{5}$$

$$a_5 = \frac{5}{5+1} = \frac{5}{6}$$

Therefore, the required number of terms $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$

Question:3 Write the first five terms of each of the sequences in Exercises 1 to 6 whose nthterms are:

$a_n = 2^n$

Answer:

Given : $a_n = 2^n$

$a_1 = 2^1 = 2$

$a_2 = 2^2 = 4$

$a_3 = 2^3 = 8$

$a_4 = 2^4 = 16$

$a_5 = 2^5 = 32$

Therefore, required number of terms $= 2, 4, 8, 16, 32.$

Question:4 Write the first five terms of each of the sequences in Exercises 1 to 6 whose nth terms are:

$$a_n = \frac{2n-3}{6}$$

Answer:

Given : $a_n = \frac{2n-3}{6}$

$$a_1 = \frac{2\times 1-3}{6} = \frac{-1}{6}$$

$$a_2 = \frac{2\times 2-3}{6} = \frac{1}{6}$$

$$a_3 = \frac{2\times 3-3}{6} = \frac{3}{6} = \frac{1}{2}$$

$$a_4 = \frac{2\times 4-3}{6} = \frac{5}{6}$$

$$a_5 = \frac{2\times 5-3}{6} = \frac{7}{6}$$

Therefore, the required number of terms $= \frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}$

Question:5 Write the first five terms of each of the sequences in Exercises 1 to 6 whose nth terms are:

$$a_n = (-1)^{n-1}5^{n+1}$$

Answer:

Given : $a_n = (-1)^{n-1}5^{n+1}$

$$a_1=(-1)^{1-1}5^{1+1}=(-1)^0.5^2=25$$

$$a_2=(-1)^{2-1}5^{2+1}=(-1)^1.5^3=-125$$

$$a_3=(-1)^{3-1}5^{3+1}=(-1)^2.5^4=625$$

$$a_4=(-1)^{4-1}5^{4+1}=(-1)^3.5^5=-3125$$

$$a_5=(-1)^{5-1}5^{5+1}=(-1)^4.5^6=15625$$

Therefore, the required number of terms $=25,-125,625,-3125,15625$

Question:6 Write the first five terms of each of the sequences in Exercises 1 to 6 whose nth terms are:

$$a_n=n\frac{n^2+5}{4}$$

Answer:

Given : $a_n=n\frac{n^2+5}{4}$

$$a_1=1.\frac{1^2+5}{4}=\frac{6}{4}=\frac{3}{2}$$

$$a_2=2.\frac{2^2+5}{4}=\frac{18}{4}=\frac{9}{2}$$

$$a_3=3.\frac{3^2+5}{4}=\frac{42}{4}=\frac{21}{2}$$

$$a_4=4.\frac{4^2+5}{4}=\frac{84}{4}=21$$

$$a_5=5.\frac{5^2+5}{4}=\frac{150}{4}=\frac{75}{2}$$

Therefore, the required number of terms $= \frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21, \frac{75}{2}$

Question:7 Find the indicated terms in each of the sequences in Exercises 7 to 10 whose nth terms are:

$a_n = 4n - 3; a_{17}, a_{24}$

Answer:

$a_n = 4n - 3$

Put $n = 17,$

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Put n=24,

$a_{24} = 4(24) - 3 = 96 - 3 = 93$

Hence, we have $a_{17} = 65 \ and \ a_{24} = 93$

Question:8 Find the indicated terms in each of the sequences in Exercises 7 to 10 whose nth terms are:

$a_n = \frac{n^2}{2^n}; a_7$

Answer:

Given : $a_n = \frac{n^2}{2^n}$

Put n=7,

$$a_7 = \frac{7^2}{2^7} = \frac{49}{128}$$

Heence, we have $a_7 = \frac{49}{128}$

Question:9 Find the indicated terms in each of the sequences in Exercises 7 to 10 whose nth terms are:

$$a_n = (-1)^{n-1}n^3, a_9$$

Answer:

Given : $a_n = (-1)^{n-1}n^3$

Put n =9,

$$a_9 = (-1)^{9-1}9^3 = (1).(729) = 729$$

The value of $a_9 = 729$

Question:10 Find the indicated terms in each of the sequences in Exercises 7 to 10 whose nth terms are:

$$a_n = \frac{n(n-2)}{n+3}; a_{20}$$

Answer:

Given : $a_n = \frac{n(n-2)}{n+3}$

Put n=20,

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{360}{23}$$

Hence, value of $a_{20} = \frac{360}{23}$

Question:11 Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:

$a_1 = 3, a_n = 3a_{n-1} + 2 \ \ for \ \ all \ \ n > 1$

Answer:

Given : $a_1 = 3, a_n = 3a_{n-1} + 2 \ \ for \ \ all \ \ n > 1$

$a_2 = 3a_{2-1} + 2 = 3a_1 + 2 = 3(3) + 2 = 11$

$a_3 = 3a_{3-1} + 2 = 3a_2 + 2 = 3(11) + 2 = 35$

$a_4 = 3a_{4-1} + 2 = 3a_3 + 2 = 3(35) + 2 = 107$

$a_5 = 3a_{5-1} + 2 = 3a_4 + 2 = 3(107) + 2 = 323$

Hence, five terms of series are $3, 11, 35, 107, 323$

Series $= 3 + 11 + 35 + 107 + 323 + \ldots\ldots\ldots\ldots\ldots$

Question:12 Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:

$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

Answer:

Given : $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

$$a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = \frac{-1}{2}$$

$$a_3 = \frac{a_{3-1}}{3} = \frac{a_2}{3} = \frac{-1}{6}$$

$$a_4 = \frac{a_{4-1}}{4} = \frac{a_3}{4} = \frac{-1}{24}$$

$$a_5 = \frac{a_{5-1}}{5} = \frac{a_4}{5} = \frac{-1}{120}$$

Hence, five terms of series are $-1, \frac{-1}{2}, \frac{-1}{-6}, \frac{-1}{24}, \frac{-1}{120}$

Series

$$= -1 + \frac{-1}{2} + \frac{-1}{-6} + \frac{-1}{24} + \frac{-1}{120} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Question:13 Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series: $a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

Answer:

Given : $a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

$$a_3 = a_{3-1} - 1 = a_2 - 1 = 2 - 1 = 1$$

$$a_4 = a_{4-1} - 1 = a_3 - 1 = 1 - 1 = 0$$

$$a_5 = a_{5-1} - 1 = a_4 - 1 = 0 - 1 = -1$$

Hence, five terms of series are $2, 2, 1, 0, -1$

Series $= 2 + 2 + 1 + 0 + (-1) + \ldots\ldots\ldots\ldots\ldots\ldots$

Question:14 The Fibonacci sequence is defined by $1 = a_1 = a_2 \ \ and \ \ a_n = a_{n-1} + a_{n-2}, n > 2$

Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5

Answer:

Given : The Fibonacci sequence is defined by $1 = a_1 = a_2 \ \ and \ \ a_n = a_{n-1} + a_{n-2}, n > 2$

$a_3 = a_{3-1} + a_{3-2} = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_{4-1} + a_{4-2} = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_{5-1} + a_{5-2} = a_4 + a_3 = 3 + 2 = 5$

$a_6 = a_{6-1} + a_{6-2} = a_5 + a_4 = 5 + 3 = 8$

$For \ n = 1, \frac{a_{n+1}}{a_n} = \frac{a_{1+1}}{a_1} = \frac{a_2}{a_1} = \frac{1}{1} = 1$

$For \ n = 2, \frac{a_{n+1}}{a_n} = \frac{a_{2+1}}{a_2} = \frac{a_3}{a_2} = \frac{2}{1} = 2$

$For \ n = 3, \frac{a_{n+1}}{a_n} = \frac{a_{3+1}}{a_3} = \frac{a_4}{a_3} = \frac{3}{2}$

$For \ n = 4, \frac{a_{n+1}}{a_n} = \frac{a_{4+1}}{a_4} = \frac{a_5}{a_4} = \frac{5}{3}$

$For \ n = 5, \frac{a_{n+1}}{a_n} = \frac{a_{5+1}}{a_5} = \frac{a_6}{a_5} = \frac{8}{5}$

NCERT solutions for class 11 maths chapter 9 sequences and series-Exercise: 9.2

Question:1 Find the sum of odd integers from 1 to 2001.

Answer:

Odd integers from 1 to 2001 are $1, 3, 5, 7..........2001.$

This sequence is an A.P.

Here , first term =a =1

common difference = 2.

We know , $a_n = a + (n-1)d$

$2001 = 1 + (n-1)2$

$\Rightarrow \; 2000 = (n-1)2$

$\Rightarrow \; 1000 = (n-1)$

$\Rightarrow \; n = 1000 + 1 = 1001$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{1001}{2}[2(1) + (1001-1)2]$

$= \frac{1001}{2}[2002]$

$= 1001 \times 1001$

$= 1002001$

The , sum of odd integers from 1 to 2001 is 1002001.

Question:2 Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Answer:

Numbers divisible by 5 from 100 to 1000 are $105, 110, ...995$

This sequence is an A.P.

Here , first term =a =105

common difference = 5.

We know , $a_n = a + (n-1)d$

$995 = 105 + (n-1)5$

$\Rightarrow \; 890 = (n-1)5$

$\Rightarrow \; 178 = (n-1)$

$\Rightarrow \; n = 178 + 1 = 179$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$= \frac{179}{2}[2(105) + (179-1)5]$

$= \frac{179}{2}[2(105) + 178(5)]$

$= 179 \times 550$

$= 98450$

The sum of numbers divisible by 5 from 100 to 1000 is 98450.

Question:3 In an A.P., the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is –112.

Answer:

First term =a=2

Let the series be $2, 2+d, 2+2d, 2+3d,$ ....

Sum of first five terms $= 10 + 10d$

Sum of next five terms $= 10 + 35d$

Given : The sum of the first five terms is one-fourth of the next five terms.

$10 + 10d = \frac{1}{4}(10 + 35d)$

$\Rightarrow \; 40 + 40d = 10 + 35d$

$\Rightarrow \; 40 - 10 = 35d - 40d$

$\Rightarrow \; 30 = -5d$

$\Rightarrow \; d = -6$

To prove : $a_{20} = -112$

L.H.S : $a_{20} = a + (20-1)d = 2 + (19)(-6) = 2 - 114 = -112 = R.H.S$

Hence, 20th term is –112.

Question: 4 How many terms of the A.P. $-6, -11/2, -5...$ are needed to give the sum –25?

Answer:

Given : A.P. = $-6, -11/2, -5...$

$a = -6$

$d = \frac{-11}{2} + 6 = \frac{1}{2}$

Given : sum = -25

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \; -25 = \frac{n}{2}[2(-6) + (n-1)\frac{1}{2}]$

$\Rightarrow \; -50 = n[-12 + (n-1)\frac{1}{2}]$

$\Rightarrow \; -50 = -12n + \frac{n^2}{2} - \frac{n}{2}$

$\Rightarrow \; -100 = -24n + n^2 - n$

$\Rightarrow \; n^2 - 25n + 100 = 0$

$$\Rightarrow \quad n^2 - 5n - 20n + 100 = 0$$

$$\Rightarrow \quad n(n-5) - 20(n-5) = 0$$

$$\Rightarrow \quad (n-5)(n-20) = 0$$

$$\Rightarrow \quad n = 5 \; or \; 20.$$

Question:5 In an A.P., if pth term is 1/q and qth term is 1/p , prove that the sum of first pq terms is 1/2 (pq +1), where $p \neq q$

Answer:

Given : In an A.P., if pth term is 1/q and qth term is 1/p

$$a_p = a + (p-1)d = \frac{1}{q}\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$a_q = a + (q-1)d = \frac{1}{p}\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Subtracting (2) from (1), we get

$$\Rightarrow \; a_p - a_q$$

$$\Rightarrow \; (p-1)d - (q-1)d = \frac{1}{q} - \frac{1}{p}$$

$$\Rightarrow \; pd - d - qd + d = \frac{p-q}{pq}$$

$$\Rightarrow \; (p-q)d = \frac{p-q}{pq}$$

$$\Rightarrow \; d = \frac{1}{pq}$$

Putting value of d in equation (1),we get

$$a+(p-1)\frac{1}{pq}=\frac{1}{q}$$

$$\Rightarrow a+\frac{1}{q}-\frac{1}{pq}=\frac{1}{q}$$

$$\Rightarrow a=\frac{1}{pq}$$

$$\therefore\ S_{pq}=\frac{pq}{2}[2.\frac{1}{pq}+(pq-1).\frac{1}{pq}]$$

$$\Rightarrow\ S_{pq}=\frac{1}{2}[2+(pq-1)]$$

$$\Rightarrow\ S_{pq}=\frac{1}{2}[pq+1]$$

Hence,the sum of first pq terms is 1/2 (pq +1), where $p \neq q$.

Question:6 If the sum of a certain number of terms of the A.P. 25, 22, 19, … is 116. Find the last term.

Answer:

Given : A.P. 25, 22, 19, ……..

$$S_n=116$$

a=25 , d = -3

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow\ 116=\frac{n}{2}[2(25)+(n-1)(-3)]$$

$$\Rightarrow \ 232 = n[50 - 3n + 3]$$

$$\Rightarrow \ 232 = n[53 - 3n]$$

$$\Rightarrow \ 3n^2 - 53n + 232 = 0$$

$$\Rightarrow \ 3n^2 - 24n - 29n + 232 = 0$$

$$\Rightarrow \ 3n(n-8) - 29(n-8) = 0$$

$$\Rightarrow \ (3n - 29)(n - 8) = 0$$

$$\Rightarrow \ n = 8 \ or \ \ n = \frac{29}{3}$$

n could not be $\frac{29}{3}$ so n=8.

Last term $= a_8 = a + (n-1)d$

$= 25 + (8-1)(-3)$

$= 25 - 21 = 4$

The, last term of A.P. is 4.

Question:7 Find the sum to n terms of the A.P., whose k^{th} term is 5k + 1.

Answer:

Given : $a_k = 5k + 1$

$$\Rightarrow \ a + (k-1)d = 5k + 1$$

$$\Rightarrow \ a + kd - d = 5k + 1$$

Comparing LHS and RHS , we have

$a-d=1$ and $d=5$

Putting value of d,

$a=1+5=6$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$S_n=\frac{n}{2}[2(6)+(n-1)5]$

$S_n=\frac{n}{2}[12+5n-5]$

$S_n=\frac{n}{2}[7+5n]$

Question:8 If the sum of n terms of an A.P. is $(pn+qn^2)$, where p and q are constants, find the common difference

Answer:

If the sum of n terms of an A.P. is $(pn+qn^2)$,

$S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \ \frac{n}{2}[2a+(n-1)d]=pn+qn^2$

$\Rightarrow \ \frac{n}{2}[2a+nd-d]=pn+qn^2$

$\Rightarrow \ an+\frac{n^2}{2}d-\frac{nd}{2}=pn+qn^2$

Comparing coefficients of n^2 on both side , we get

$$\frac{d}{2} = q$$

$$\Rightarrow \ d = 2q$$

The common difference of AP is 2q.

Question:9 The sums of n terms of two arithmetic progressions are in the ratio $5n+4:9n+6$. Find the ratio of their 18th terms.

Answer:

Given: The sums of n terms of two arithmetic progressions are in the ratio. $5n+4:9n+6$

There are two AP's with first terms = a_1, a_2 and common difference = d_1, d_2

$$\Rightarrow \ \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \ \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{5n+4}{9n+6}$$

Substituting n=35,we get

$$\Rightarrow \ \frac{2a_1+(35-1)d_1}{2a_2+(35-1)d_2} = \frac{5(35)+4}{9(35)+6}$$

$$\Rightarrow \frac{2a_1+34d_1}{2a_2+34d_2} = \frac{5(35)+4}{9(35)+6}$$

$$\Rightarrow \ \frac{a_1+17d_1}{a_2+17d_2} = \frac{179}{321}$$

Thus, the ratio of the 18th term of AP's is $179 : 321$

Question:10 If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first (p + q) terms.

Answer:

Let first term of AP = a and common difference = d.

Then,

$$S_p = \frac{p}{2}[2a + (p-1)d]$$

$$S_q = \frac{q}{2}[2a + (q-1)d]$$

Given : $S_p = S_q$

$$\Rightarrow \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$$

$$\Rightarrow p[2a + (p-1)d] = q[2a + (q-1)d]$$

$$\Rightarrow 2ap + p^2d - pd = 2aq + q^2d - qd$$

$$\Rightarrow 2ap + p^2d - pd - 2aq - q^2d + qd = 0$$

$$\Rightarrow 2a(p-q) + d(p^2 - p - q^2 + q) = 0$$

$$\Rightarrow 2a(p-q) + d((p-q)(p+q) - (p-q)) = 0$$

$$\Rightarrow 2a(p-q) + d[(p-q)(p+q-1)] = 0$$

$$\Rightarrow (p-q)[2a + d(p+q-1)] = 0$$

$\Rightarrow 2a + d(p+q-1) = 0$

$\Rightarrow d(p+q-1) = -2a$

$\Rightarrow d = \frac{-2a}{p+q-1}$

Now, $S_{(p+q)} = \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2}[2a + (p+q-1)\frac{-2a}{p+q-1}]$

$= \frac{p+q}{2}[2a + (-2a)]$

$= \frac{p+q}{2}[0] = 0$

Thus, sum of p+q terms of AP is 0.

Question:11 Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that

$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

Answer:

To prove : $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

Let a_1 and d be the first term and the common difference of AP, respectively.

According to the given information, we have

$$S_p = \frac{p}{2}[2a_1 + (p-1)d] = a$$

$$\Rightarrow [2a_1 + (p-1)d] = \frac{2a}{p} \text{...........}(1)$$

$$S_q = \frac{q}{2}[2a_1 + (q-1)d] = b$$

$$\Rightarrow [2a_1 + (q-1)d] = \frac{2b}{q} \text{...........}(2)$$

$$S_r = \frac{r}{2}[2a_1 + (r-1)d] = c$$

$$\Rightarrow [2a_1 + (r-1)d] = \frac{2c}{r} \text{...........}(3)$$

Subtracting equation (2) from (1), we have

$$\Rightarrow (p-1)d - (q-1)d = \frac{2a}{p} - \frac{2b}{q}$$

$$\Rightarrow d(p-q-1+1) = \frac{2(aq-bp)}{pq}$$

$$\Rightarrow d(p-q) = \frac{2(aq-bp)}{pq}$$

$$\Rightarrow d = \frac{2(aq-bp)}{pq(p-q)}$$

Subtracting equation (3) from (2), we have

$$\Rightarrow (q-1)d-(r-1)d=\frac{2b}{q}-\frac{2c}{r}$$

$$\Rightarrow d(q-r-1+1)=\frac{2(br-cq)}{qr}$$

$$\Rightarrow d(q-r)=\frac{2(br-qc)}{qr}$$

$$\Rightarrow d=\frac{2(br-qc)}{qr(q-r)}$$

Equating values of d, we have

$$\Rightarrow d=\frac{2(aq-bp)}{pq(p-q)}=\frac{2(br-qc)}{qr(q-r)}$$

$$\Rightarrow \frac{2(aq-bp)}{pq(p-q)}=\frac{2(br-qc)}{qr(q-r)}$$

$$\Rightarrow \ (aq-bp)qr(q-r)=(br-qc)pq(p-q)$$

$$\Rightarrow \ (aq-bp)r(q-r)=(br-qc)p(p-q)$$

$$\Rightarrow \ (aqr-bpr)(q-r)=(bpr-pqc)(p-q)$$

Dividing both sides from pqr, we get

$$\Rightarrow \ (\frac{a}{p}-\frac{b}{q})(q-r)=(\frac{b}{q}-\frac{c}{r})(p-q)$$

$$\Rightarrow \ \frac{a}{p}(q-r)-\frac{b}{q}(q-r+p-q)+\frac{c}{r}(p-q)=0$$

$$\Rightarrow \ \frac{a}{p}(q-r)-\frac{b}{q}(p-r)+\frac{c}{r}(p-q)=0$$

$$\Rightarrow \ \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

Hence, the given result is proved.

Question:12 The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of mth and nth term is $(2m-1):(2n-1)$.

Answer:

Let a and b be the first term and common difference of a AP ,respectively.

Given : The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$.

To prove : the ratio of mth and nth term is $(2m-1):(2n-1)$.

$$\therefore \ \frac{sum\ of\ m\ terms}{sum\ of\ n\ terms} = \frac{m^2}{n^2}$$

$$\Rightarrow \ \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \ \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

Put $m = 2m-1 \ and \ n = 2n-1$, we get

$$\Rightarrow \ \frac{2a+(2m-2)d}{2a+(2n-2)d} = \frac{2m-1}{2n-1}$$

$$\Rightarrow \ \frac{a+(m-1)d}{a+(n-1)d} = \frac{2m-1}{2n-1}.........1$$

From equation (1) ,we get

Hence proved.

Question:13 If the sum of n terms of an A.P. is $3n^2+5n$ and its m^{th} term is 164, find the value of m.

Answer:

Given : If the sum of n terms of an A.P. is $3n^2+5n$ and its m^{th} term is 164

Let a and d be first term and common difference of a AP ,respectively.

Sum of n terms = $3n^2+5n$

$\Rightarrow \ \frac{n}{2}[2a+(n-1)d]=3n^2+5n$

$\Rightarrow \ 2a+(n-1)d=6n+10$

$\Rightarrow \ 2a+nd-d=6n+10$

Comparing the coefficients of n on both side , we have

$\Rightarrow \ d=6$

Also , $2a-d=10$

$\Rightarrow \ 2a-6=10$

$\Rightarrow \ 2a=10+6$

$\Rightarrow \ 2a=16$

$\Rightarrow \ a=8$

m th term is 164.

$\Rightarrow \ a+(m-1)d=164$

$\Rightarrow \ 8+(m-1)6=164$

$\Rightarrow \ (m-1)6=156$

$\Rightarrow \ m-1=26$

$\Rightarrow \ m=26+1=27$

Hence, the value of m is 27.

Question:14 Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

Answer:

Let five numbers be A,B,C,D,E.

Then $AP=8,A,B,C,D,E,26$

Here we have,

$a=8,a_7=26,n=7$

$\Rightarrow \ a+(n-1)d=a_n$

$\Rightarrow \ 8+(7-1)d=26$

$\Rightarrow \ 6d=18$

$$\Rightarrow \ d = \frac{18}{6} = 3$$

Thus, we have $A = a + d = 8 + 3 = 11$

$$B = a + 2d = 8 + (2)3 = 8 + 6 = 14$$

$$C = a + 3d = 8 + (3)3 = 8 + 9 = 17$$

$$D = a + 4d = 8 + (4)3 = 8 + 12 = 20$$

$$E = a + 5d = 8 + (5)3 = 8 + 15 = 23$$

Thus, the five numbers are 11,14,17,20,23.

Question:15 If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then find the value of n.

Answer:

Given : $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b.

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a + b}{2}$$

$$\Rightarrow \ 2(a^n + b^n) = (a + b)(a^{n-1} + b^{n-1})$$

$$\Rightarrow \ 2a^n + 2b^n = a^n + a.b^{n-1} + b.a^{n-1} + b^n$$

$$\Rightarrow \ 2a^n + 2b^n - a^n - b^n = a.b^{n-1} + b.a^{n-1}$$

$$\Rightarrow \ a^n + b^n = a.b^{n-1} + b.a^{n-1}$$

$$\Rightarrow \ a^n - b.a^{n-1} = a.b^{n-1} - b^n$$

$\Rightarrow \ a^{n-1}(a-b) = b^{n-1}(a-b)$

$\Rightarrow \ a^{n-1} = b^{n-1}$

$\Rightarrow \ \left[\frac{a}{b}\right]^{n-1} = 1$

$\Rightarrow \ n-1 = 0$

$\Rightarrow \ n = 1$

Thus, value of n is 1.

Question:16 Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of 7^{th} and $(m-1)^{th}$ numbers is 5 : 9. Find the value of m .

Answer:

Let A,B,C.........M be m numbers.

Then, $AP = 1, A, B, C..........M, 31$

Here we have,

$a = 1, a_{m+2} = 31, n = m+2$

$\Rightarrow \ a + (n-1)d = a_n$

$\Rightarrow \ 1 + (m+2-1)d = 31$

$\Rightarrow \ (m+1)d = 30$

$$\Rightarrow \ d = \frac{30}{m+1}$$

Given : the ratio of 7^{th} and $(m-1)^{th}$ numbers is 5 : 9.

$$\Rightarrow \ \frac{a+(7)d}{a+(m-1)d} = \frac{5}{9}$$

$$\Rightarrow \ \frac{1+7d}{1+(m-1)d} = \frac{5}{9}$$

$$\Rightarrow \ 9(1+7d) = 5(1+(m-1)d)$$

$$\Rightarrow \ 9+63d = 5+5md-5d$$

Putting value of d from above,

$$\Rightarrow \ 9+63(\frac{30}{m+1}) = 5+5m\left(\frac{30}{m+1}\right)-5\left(\frac{30}{m+1}\right)$$

$$\Rightarrow \ 9(m+1)+1890 = 5(m+1)+150m-150$$

$$\Rightarrow \ 9m+9+1890 = 5m+5+150m-150$$

$$\Rightarrow \ 1890+9-5+150 = 155m-9m$$

$$\Rightarrow \ 2044 = 146m$$

$$\Rightarrow \ m = 14$$

Thus, value of m is 14.

Question:17 A man starts repaying a loan as first instalment of Rs. 100. If he increases the instalment by Rs 5 every month, what amount he will pay in the 30th instalment?

Answer:

The first instalment is of Rs. 100.

If the instalment increase by Rs 5 every month, second instalment is Rs.105.

Then , it forms an AP.

$AP = 100, 105, 110, 115, \ldots\ldots\ldots\ldots\ldots$

We have , $a = 100\ and\ \ d = 5$

$a_n = a + (n-1)d$

$a_{30} = 100 + (30-1)5$

$a_{30} = 100 + (29)5$

$a_{30} = 100 + 145$

$a_{30} = 245$

Thus, he will pay Rs. 245 in the 30th instalment.

Question:18 The difference between any two consecutive interior angles of a polygon is 5° . If the smallest angle is 120° , find the number of the sides of the polygon.

Answer:

The angles of polygon forms AP with common difference of 5° and first term as 120° .

We know that sum of angles of polygon with n sides is $180(n-2)$

$\therefore S_n = 180(n-2)$

$\Rightarrow \frac{n}{2}[2a+(n-1)d] = 180(n-2)$

$\Rightarrow \frac{n}{2}[2(120)+(n-1)5] = 180(n-2)$

$\Rightarrow n[240+5n-5] = 360n-720$

$\Rightarrow 235n+5n^2 = 360n-720$

$\Rightarrow 5n^2-125n+720 = 0$

$\Rightarrow n^2-25n+144 = 0$

$\Rightarrow n^2-16n-9n+144 = 0$

$\Rightarrow n(n-16)-9(n-16) = 0$

$\Rightarrow (n-16)(n-9) = 0$

$\Rightarrow n = 9, 16$

Sides of polygon are 9 or 16.

NCERT solutions for class 11 maths chapter 9 sequences and series-Exercise: 9.3

Question:1 Find the 20^{th} and n^{th} terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, ...$

Answer:

G.P :

$$\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots.$$

first term = a

$$a = \frac{5}{2}$$

common ratio =r

$$r = \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$$

$$a_n = a.r^{n-1}$$

$$a_{20} = \frac{5}{2}.(\frac{1}{2})^{20-1}$$

$$a_{20} = \frac{5}{2}.(\frac{1}{2^{19}})$$

$$a_{20} = \frac{5}{2^{20}}$$

$$a_n = a.r^{n-1}$$

$$a_n = \frac{5}{2}.\left(\frac{1}{2}\right)^{n-1}$$

$$a_n = \frac{5}{2}.\frac{1}{2^{n-1}}$$

$a_n = \frac{5}{2^n}$ the nth term of G.P

Question:2 Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2.

Answer:

First term = a

common ratio =r=2

8^{th} term is 192

$a_n = a.r^{n-1}$

$a_8 = a.(2)^{8-1}$

$192 = a.(2)^7$

$a = \frac{2^6.3}{2^7}$

$a = \frac{3}{2}$

$a_n = a.r^{n-1}$

$a_{12} = \frac{3}{2}.(2)^{12-1}$

$a_{12} = \frac{3}{2}.(2)^{11}$

$a_{12} = 3.(2)^{10}$

$a_{12} = 3072$ is the 12^{th} term of a G.P.

Question:3 The $5^{th}, 8^{th}\ \ and\ \ 11^{th}$ terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$

Answer:

To prove : $q^2 = ps$

Let first term=a and common ratio = r

$a_5 = a.r^4 = p..................(1)$

$a_8 = a.r^7 = q..................(2)$

$a_{11} = a.r^{10} = s..................(3)$

Dividing equation 2 by 1, we have

$$\frac{a.r^7}{a.r^4} = \frac{q}{p}$$

$$\Rightarrow r^3 = \frac{q}{p}$$

Dividing equation 3 by 2, we have

$$\frac{a.r^{10}}{a.r^7} = \frac{s}{q}$$

$$\Rightarrow r^3 = \frac{s}{q}$$

Equating values of r^3 , we have

$$\frac{q}{p} = \frac{s}{q}$$

$$\Rightarrow q^2 = ps$$

Hence proved

Question:4 The 4^{th} term of a G.P. is square of its second term, and the first term is -3. Determine its 7^{th} term.

Answer:

First term =a= -3

4^{th} term of a G.P. is square of its second term

$\Rightarrow a_4 = (a_2)^2$

$\Rightarrow a.r^{4-1} = (a.r^{2-1})^2$

$\Rightarrow a.r^3 = a^2.r^2$

$\Rightarrow r = a = -3$

$a_7 = a.r^{7-1}$

$\Rightarrow a_7 = (-3).(-3)^6$

$\Rightarrow a_7 = (-3)^7 = -2187$

Thus, seventh term is -2187.

Question:5(a) Which term of the following sequences: $2, 2\sqrt{2}, 4., ...is \ \ 128?$

Answer:

Given : $GP = 2, 2\sqrt{2}, 4., ...$

$a = 2 \quad and \quad r = \frac{2\sqrt{2}}{2} = \sqrt{2}$

n th term is given as 128.

$a_n = a.r^{n-1}$

$\Rightarrow 128 = 2.(\sqrt{2})^{n-1}$

$\Rightarrow 64 = (\sqrt{2})^{n-1}$

$\Rightarrow 2^6 = (\sqrt{2})^{n-1}$

$\Rightarrow \sqrt{2}^{12} = (\sqrt{2})^{n-1}$

$\Rightarrow n - 1 = 12$

$\Rightarrow n = 12 + 1 = 13$

The, 13 th term is 128.

Question:5(b) Which term of the following sequences: $\sqrt{3}, 3, 3\sqrt{3}, ...is\ \ 729?$

Answer:

Given : $GP = \sqrt{3}, 3, 3\sqrt{3}, ...$

$a = \sqrt{3} \quad and \quad r = \frac{3}{\sqrt{3}} = \sqrt{3}$

n th term is given as 729.

$a_n = a.r^{n-1}$

$\Rightarrow 729 = \sqrt{3}.(\sqrt{3})^{n-1}$

$\Rightarrow 729 = (\sqrt{3})^n$

$\Rightarrow (\sqrt{3})^{12} = (\sqrt{3})^n$

$\Rightarrow n = 12$

The, 12 th term is 729.

Question:5(c) Which term of the following sequences: $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, ...is \ \frac{1}{19683}?$

Answer:

Given : $GP = \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, ...$

n th term is given as $\frac{1}{19683}$

$a_n = a.r^{n-1}$

$\Rightarrow \frac{1}{19683} = \frac{1}{3}.(\frac{1}{3})^{n-1}$

$\Rightarrow \frac{1}{19683} = \frac{1}{3^n}$

$\Rightarrow \frac{1}{3^9} = \frac{1}{3^n}$

$\Rightarrow n = 9$

Thus, n=9.

Question:6 For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.?

Answer:

$$GP = -\frac{2}{7}, x, -\frac{7}{2}$$

Common ratio=r.

$$r = \frac{x}{\frac{-2}{7}} = \frac{\frac{-7}{2}}{x}$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

Thus, for $x = \pm 1$,given numbers will be in GP.

Question:7 Find the sum to indicated number of terms in each of the geometric progressions in 0.15, 0.015, 0.0015, ... 20 terms.

Answer:

geometric progressions is 0.15, 0.015, 0.0015, ...

a=0.15 , r = 0.1 , n=20

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{20} = \frac{0.15(1-(0.1)^{20})}{1-0.1}$$

$$S_{20} = \frac{0.15(1-(0.1)^{20})}{0.9}$$

$$S_{20} = \frac{0.15}{0.9}(1-(0.1)^{20})$$

$$S_{20} = \frac{15}{90}(1-(0.1)^{20})$$

$$S_{20} = \frac{1}{6}(1 - 0.1^{20})$$

Question:8 Find the sum to indicated number of terms in each of the geometric progressions in $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, ...n\ terms$

Answer:

$$GP = \sqrt{7}, \sqrt{21}, 3\sqrt{7}, ...$$

$$a = \sqrt{7} \ \ and \ \ \ r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_n = \frac{\sqrt{7}(1 - \sqrt{3}^n)}{1 - \sqrt{3}}$$

$$S_n = \frac{\sqrt{7}(1 - \sqrt{3}^n)}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$

$$S_n = \frac{\sqrt{7}(1 - \sqrt{3}^n)}{1 - 3}(1 + \sqrt{3})$$

$$S_n = \frac{\sqrt{7}(1 - \sqrt{3}^n)}{-2}(1 + \sqrt{3})$$

$$S_n = \frac{\sqrt{7}(1 + \sqrt{3})}{2}(\sqrt{3}^n - 1)$$

Question:9 Find the sum to indicated number of terms in each of the geometric progressions in $-a, a^2, -a^3, ...nterms(if a \neq -1)$

Answer:

The sum to the indicated number of terms in each of the geometric progressions is:

$GP = 1, -a, a^2, -a^3, ...$

$a = 1 \ \ and \ \ r = -a$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{1(1-(-a)^n)}{1-(-a)}$$

$$S_n = \frac{1(1-(-a)^n)}{1+a}$$

$$S_n = \frac{1-(-a)^n}{1+a}$$

Question:10 Find the sum to indicated number of terms in each of the geometric progressions in $x^3, x^5, x^7 ... n \ \ terms(if \ \ x \neq \pm 1)$

Answer:

$GP = x^3, x^5, x^7 ....$

$a = x^3 \ \ and \ r = \frac{x^5}{x^3} = x^2$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{x^3(1-(x^2)^n)}{1-x^2}$$

$$S_n = \frac{x^3(1-x^{2n})}{1-x^2}$$

Question:11 Evaluate $\sum_{k=1}^{11}(2+3^k)$

Answer:

Given :

$$\sum_{k=1}^{11}(2+3^k)$$

$$\sum_{k=1}^{11}(2+3^k)=\sum_{k=1}^{11}2+\sum_{k=1}^{11}3^k$$

$$=22+\sum_{k=1}^{11}3^k..............(1)$$

$$\sum_{k=1}^{11}3^k=3^1+3^2+3^3+...................3^{11}$$

These terms form GP with a=3 and r=3.

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$S_n=\frac{3(1-3^{11})}{1-3}$$

$$S_n=\frac{3(1-3^{11})}{-2}$$

$$S_n=\frac{3(3^{11}-1)}{2}=\sum_{k=1}^{11}3^k$$

$$\sum_{k=1}^{11}(2+3^k)=22+\frac{3(3^{11}-1)}{2}$$

Question:12 The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Answer:

Given : The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1.

Let three terms be $\frac{a}{r}, a, ar$.

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_3 = \frac{a(1-r^3)}{1-r} = \frac{39}{10}$$

$$\frac{a}{r} + a + ar = \frac{39}{10} ....1$$

Product of 3 terms is 1.

$$\frac{a}{r} \times a \times ar = 1$$

$$\Rightarrow a^3 = 1$$

$$\Rightarrow a = 1$$

Put value of a in equation 1,

$$\frac{1}{r} + 1 + r = \frac{39}{10}$$

$$10(1 + r + r^2) = 39(r)$$

$$\Rightarrow 10r^2 - 29r + 10 = 0$$

$$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$$

$$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$$

$\Rightarrow (2r-5)(5r-2)=0$

$\Rightarrow r=\frac{5}{2}, r=\frac{2}{5}$

The three terms of AP are $\frac{5}{2}, 1, \frac{2}{5}$.

Question:13 How many terms of G.P. $3, 3^2, 3^3$, … are needed to give the sum 120?

Answer:

G.P.= $3, 3^2, 3^3$, …............

Sum =120

These terms are GP with a=3 and r=3.

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$120=\frac{3(1-3^n)}{1-3}$$

$$120\times\frac{-2}{3}=(1-3^n)$$

$$-80=(1-3^n)$$

$$\Rightarrow 3^n=1+80=81$$

$$\Rightarrow 3^n=81$$

$$\Rightarrow 3^n=3^4$$

$$\Rightarrow n=4$$

Hence, we have value of n as 4 to get sum of 120.

Question:14 The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

Answer:

Let GP be $a, ar, ar^2, ar^3, ar^4, ar^5, ar^6$................................

Given : The sum of first three terms of a G.P. is 16

$a + ar + ar^2 = 16$

$\Rightarrow a(1 + r + r^2) = 16$..............................(1)

Given : the sum of the next three terms is128.

$ar^3 + ar^4 + ar^5 = 128$

$\Rightarrow ar^3(1 + r + r^2) = 128$..............................(2)

Dividing equation (2) by (1), we have

$$\Rightarrow \frac{ar^3(1 + r + r^2)}{a(1 + r + r^2)} = \frac{128}{16}$$

$\Rightarrow r^3 = 8$

$\Rightarrow r^3 = 2^3$

$\Rightarrow r = 2$

Putting value of r =2 in equation 1,we have

$$\Rightarrow a(1+2+2^2)=16$$

$$\Rightarrow a(7)=16$$

$$\Rightarrow a=\frac{16}{7}$$

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$S_n=\frac{\frac{16}{7}(1-2^n)}{1-2}$$

$$S_n=\frac{16}{7}(2^n-1)$$

Question:15 Given a G.P. with a = 729 and 7^{th} term 64, determine s_7

Answer:

Given a G.P. with a = 729 and 7^{th} term 64.

$$a_n=a.r^{n-1}$$

$$\Rightarrow 64=729.r^{7-1}$$

$$\Rightarrow r^6=\frac{64}{729}$$

$$\Rightarrow r^6=\left(\frac{2}{3}\right)^6$$

$$\Rightarrow r=\frac{2}{3}$$

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$S_7 = \frac{729(1 - \left(\frac{2}{3}\right)^7)}{1 - \frac{2}{3}}$$

$$S_7 = \frac{729(1 - \left(\frac{2}{3}\right)^7)}{\frac{1}{3}}$$

$$S_7 = 3 \times 729 \left(\frac{3^7 - 2^7}{3^7}\right)$$

$$S_7 = \left(3^7 - 2^7\right)$$

$$S_7 = 2187 - 128$$

$S_7 = 2059$ **(Answer)**

Question:16 Find a G.P. for which sum of the first two terms is – 4 and the fifth term is 4 times the third term

Answer:

Given : sum of the first two terms is – 4 and the fifth term is 4 times the third term

Let first term be a and common ratio be r

$$a_5 = 4.a_3$$

$$\Rightarrow a.r^{5-1} = 4.a.r^{3-1}$$

$$\Rightarrow a.r^4 = 4.a.r^2$$

$$\Rightarrow r^2 = 4$$

$$\Rightarrow r = \pm 2$$

If r=2, then

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow \frac{a(1-2^2)}{1-2} = -4$$

$$\Rightarrow \frac{a(1-4)}{-1} = -4$$

$$\Rightarrow a(-3) = 4$$

$$\Rightarrow a = \frac{-4}{3}$$

If r= - 2, then

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow \frac{a(1-(-2)^2)}{1-(-2)} = -4$$

$$\Rightarrow \frac{a(1-4)}{3} = -4$$

$$\Rightarrow a(-3) = -12$$

$$\Rightarrow a = \frac{-12}{-3} = 4$$

Thus, required GP is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots\ldots\ldots$ or $4, -8, -16, -32, \ldots\ldots\ldots\ldots$

Question:17 If the $4^{th}, 10^{th}, 16^{th}$ terms of a G.P. are x, y and z, respectively. Prove that x,y, z are in G.P.

Answer:

Let x,y, z are in G.P.

Let first term=a and common ratio = r

$a_4 = a.r^3 = x..................(1)$

$a_{10} = a.r^9 = y..................(2)$

$a_{16} = a.r^{15} = z..................(3)$

Dividing equation 2 by 1, we have

$$\frac{a.r^9}{a.r^3} = \frac{y}{x}$$

$$\Rightarrow r^4 = \frac{y}{x}$$

Dividing equation 3 by 2, we have

$$\frac{a.r^{15}}{a.r^9} = \frac{z}{y}$$

$$\Rightarrow r^4 = \frac{z}{y}$$

Equating values of r^4 , we have

$$\frac{y}{x} = \frac{z}{y}$$

Thus, x,y,z are in GP

Question:18 Find the sum to n terms of the sequence, 8, 88, 888, 8888… .

Answer:

8, 88, 888, 8888… is not a GP.

It can be changed in GP by writing terms as

$S_n = 8 + 88 + 888 + 8888 + \ldots\ldots\ldots\ldots$ to n terms

$$S_n = \frac{8}{9}[9 + 99 + 999 + 9999 + \ldots\ldots\ldots\ldots\ldots]$$

$$S_n = \frac{8}{9}[(10-1) + (10^2-1) + (10^3-1) + (10^4-1) + \ldots\ldots\ldots\ldots\ldots]$$

$$S_n = \frac{8}{9}[(10 + 10^2 + 10^3 + \ldots\ldots) - (1+1+1\ldots\ldots\ldots\ldots\ldots\ldots)]$$

$$S_n = \frac{8}{9}[\frac{10(10^n-1)}{10-1} - (n)]$$

$$S_n = \frac{8}{9}[\frac{10(10^n-1)}{9} - (n)]$$

$$S_n = \frac{80}{81}(10^n-1) - \frac{8n}{9}$$

Question:19 Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2,1/2

Answer:

$$Required\ sum = 2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2}$$

$$Required\ sum = 64\left[4 + 2 + 1 + \frac{1}{2} + \frac{1}{2^2}\right]$$

Here, $4, 2, 1, \frac{1}{2}, \frac{1}{2^2}$ is a GP.

first term =a=4

common ratio =r

$$r=\frac{1}{2}$$

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$S_5=\frac{4(1-\left(\frac{1}{2}\right)^5)}{1-\frac{1}{2}}$$

$$S_5=\frac{4(1-\left(\frac{1}{2}\right)^5)}{\frac{1}{2}}$$

$$S_5=8(1-\left(\frac{1}{32}\right))$$

$$S_5=8(\frac{31}{32})$$

$$S_5=\frac{31}{4}$$

$$Required\ sum=64\left[\frac{31}{4}\right]$$

$$Required\ sum=16\times31=496$$

Question:20 Show that the products of the corresponding terms of the sequences $a, ar, ar^2, ...ar^{n-1}\ \ and\ \ A, AR, AR^2....AR^{n-1}$ form a G.P, and find the common ratio.

Answer:

To prove : $aA, arAR, ar^2AR^2, ...$ is a GP.

$$\frac{second\ term}{first\ term} = \frac{arAR}{aA} = rR$$

$$\frac{third\ term}{second\ term} = \frac{ar^2AR^2}{arAR} = rR$$

Thus, the above sequence is a GP with common ratio of rR.

Question:21 Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4^{th} by 18.

Answer:

Let first term be a and common ratio be r.

$a_1 = a, a_2 = ar, a_3 = ar^2, a_4 = ar^3$

Given : the third term is greater than the first term by 9, and the second term is greater than the $4^{th}$by 18.

$a_3 = a_1 + 9$

$\Rightarrow ar^2 = a + 9$

$\Rightarrow a(r^2 - 1) = 9................1$

$a_2 = a_4 + 18$

$\Rightarrow ar = ar^3 + 18$

$\Rightarrow ar(1 - r^2) = 18.....................2$

Dividing equation 2 by 1 , we get

$$\frac{ar(1-r^2)}{-a(1-r^2)} = \frac{18}{9}$$

$$\Rightarrow r = -2$$

Putting value of r , we get

$$4a = a + 9$$

$$\Rightarrow 4a - a = 9$$

$$\Rightarrow 3a = 9$$

$$\Rightarrow a = 3$$

Thus, four terms of GP are $3, -6, 12, -24.$

Question:22 If the p^{th}, q^{th}, r^{th} terms of a G.P. are a, b and c, respectively. Prove that $a^{q-r}b^{r-p}C^{p-q} = 1$

Answer:

To prove : $a^{q-r}b^{r-p}C^{p-q} = 1$

Let A be the first term and R be common ratio.

According to the given information, we have

$$a_p = A.R^{p-1} = a$$

$$a_q = A.R^{q-1} = b$$

$$a_r = A.R^{r-1} = c$$

L.H.S : $a^{q-r}b^{r-p}C^{p-q}$

$= A^{q-r}.R^{(q-r)(p-1)}.A^{r-p}.R^{(r-p)(q-1)}.A^{p-q}.R^{(p-q)(r-1)}$

$= A^{q-r+r-p+p-q}.R^{(qp-rp-q+r)+(rq-pq+p-r)+(pr-p-qr+q)}$

$= A^0.R^0 = 1$ =RHS

Thus, LHS = RHS.

Hence proved.

Question:23 If the first and the nth term of a G.P. are a and b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Answer:

Given : First term =a and n th term = b.

Common ratio = r.

To prove : $P^2 = (ab)^n$

Then , $GP = a, ar, ar^2, ar^3, ar^4, ...$

$a_n = a.r^{n-1} = b..............................1$

P = product of n terms

$P = (a).(ar).(ar^2).(ar^3).............(ar^{n-1})$

$P = (a.a.a..............a)((1).(r).(r^2).(r^3).............(r^{n-1}))$

$P = (a^n)(r^{1+2+.........(n-1)})$.......................................2

Here, $1 + 2 + ...(n-1)$ is a AP.

$\therefore \quad sum = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n-1}{2}[2(1) + (n-1-1)1]$

$= \frac{n-1}{2}[2 + n - 2]$

$= \frac{n-1}{2}[n]$

$= \frac{n(n-1)}{2}$

Put in equation (2),

$P = (a^n)(r^{\frac{n(n-1)}{2}})$

$P^2 = (a^{2n})(r^{n(n-1)})$

$P^2 = (a.a.r^{(n-1)})^n$

$P^2 = (a.b)^n$

Hence proved .

Question:24 Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th} \ to \ (2n)^{th}$ term is $\frac{1}{r^n}$

Answer:

Let first term =a and common ratio = r.

$$sum\ of\ n\ terms = \frac{a(1-r^n)}{1-r}$$

Since there are n terms from (n+1) to 2n term.

Sum of terms from (n+1) to 2n.

$$S_n = \frac{a_{(n\ +1)}\ (1-r^n)}{1-r}$$

$$a_{(n}+_{1)} = a.r^{n+1-1} = ar^n$$

Thus, the required ratio = $\dfrac{a(1-r^n)}{1-r} \times \dfrac{1-r}{ar^n(1-r^n)}$

$$= \frac{1}{r^n}$$

Thus, the common ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}\ \ to\ \ (2n)^{th}$ term is $\dfrac{1}{r^n}$.

Question:25 If a, b, c and d are in G.P. show that $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2.$

Answer:

If a, b, c and d are in G.P.

$bc = ad$....................(1)

$b^2 = ac$....................(2)

$c^2 = bd$....................(3)

To prove : $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2.$

RHS : $(ab + bc + cd)^2.$

$= (ab + ad + cd)^2.$

$= (ab + d(a + c))^2.$

$= a^2b^2 + d^2(a + c)^2 + 2(ab)(d(a + c))$

$= a^2b^2 + d^2(a^2 + c^2 + 2ac) + 2a^2bd + 2bcd$

Using equation (1) and (2),

$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2.b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2.c^2 + d^2c^2$

$= a^2(b^2 + c^2 + d^2) + b^2(b^2 + c^2 + d^2) + c^2(b^2 + c^2 + d^2)$

$= (b^2 + c^2 + d^2)(a^2 + b^2 + c^2)$ = LHS

Hence proved

Question:26 Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Answer:

Let A, B be two numbers between 3 and 81 such that series 3, A, B,81 forms a GP.

Let a=first term and common ratio =r.

$\therefore a_4 = a.r^{4-1}$

$81 = 3.r^3$

$27 = r^3$

$r = 3$

For $r = 3$,

$A = ar = (3)(3) = 9$

$B = ar^2 = (3)(3)^2 = 27$

The, required numbers are 9,27.

Question:27 Find the value of n so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Answer:

M of a and b is $\sqrt{ab}$.

Given :

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$$

Squaring both sides ,

$$\left(\frac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)^2 = ab$$

$$\left(a^{n+1}+b^{n+1}\right)^2=\left(a^n+b^n\right)^2 ab$$

$$\Rightarrow \left(a^{2n+2}+b^{2n+2}+2.a^{n+1}.b^{n+1}\right)=\left(a^{2n}+b^{2n}+2.a^n.b^n\right) ab$$

$$\Rightarrow \left(a^{2n+2}+b^{2n+2}+2.a^{n+1}.b^{n+1}\right)=\left(a^{2n+1}.b+a.b^{2n+1}+2.a^{n+1}.b^{n+1}\right)$$

$$\Rightarrow \left(a^{2n+2}+b^{2n+2}\right)=\left(a^{2n+1}.b+a.b^{2n+1}\right)$$

$$\Rightarrow a^{2n+2}-a^{2n+1}.b=a.b^{2n+1}-b^{2n+2}$$

$$\Rightarrow a^{2n+1}(a-b)=b^{2n+1}(a-b)$$

$$\Rightarrow a^{2n+1}=b^{2n+1}$$

$$\Rightarrow \left(\frac{a}{b}\right)^{2n+1}=1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{2n+1}=1=\left(\frac{a}{b}\right)^0$$

$$\Rightarrow 2n+1=0$$

$$\Rightarrow 2n=-1$$

$$\Rightarrow n=\frac{-1}{2}$$

Question:28 The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3+2\sqrt{2}):(3-2\sqrt{2})$

Answer:

Let there be two numbers a and b

geometric mean $=\sqrt{ab}$

According to the given condition,

$a+b=6\sqrt{ab}$

$(a+b)^2=36(ab)$ ....(1)

Also, $(a-b)^2=(a+b)^2-4ab=36ab-4ab=32ab$

$(a-b)=\sqrt{32}\sqrt{ab}$

$(a-b)=4\sqrt{2}\sqrt{ab}$ ....(2)

From (1) and (2), we get

$2a=(6+4\sqrt{2})\sqrt{ab}$

$a=(3+2\sqrt{2})\sqrt{ab}$

Putting the value of 'a' in (1),

$b=6\sqrt{ab}-(3+2\sqrt{2})\sqrt{ab}$

$b=(3-2\sqrt{2})\sqrt{ab}$

$\frac{a}{b}=\frac{(3+2\sqrt{2})\sqrt{ab}}{(3-2\sqrt{2})\sqrt{ab}}$

$\frac{a}{b}=\frac{(3+2\sqrt{2})}{(3-2\sqrt{2})}$

Thus, the ratio is $(3+2\sqrt{2}):(3-2\sqrt{2})$

Question:29 If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A\pm\sqrt{(A+G)(A-G)}$

Answer:

If A and G be A.M. and G.M., respectively between two positive numbers,

Two numbers be a and b.

$$AM = A = \frac{a+b}{2}$$

$\Rightarrow a + b = 2A$ 1

$$GM = G = \sqrt{ab}$$

$\Rightarrow ab = G^2$ 2

We know $(a-b)^2 = (a+b)^2 - 4ab$

Put values from equation 1 and 2,

$$(a-b)^2 = 4A^2 - 4G^2$$

$$(a-b)^2 = 4(A^2 - G^2)$$

$$(a-b)^2 = 4(A+G)(A-G)$$

$(a-b) = 4\sqrt{(A+G)(A-G)}$ 3

From 1 and 3 , we have

$$2a = 2A + 2\sqrt{(A+G)(A-G)}$$

$$\Rightarrow a = A + \sqrt{(A+G)(A-G)}$$

Put value of a in equation 1, we get

$b=2A-A-\sqrt{(A+G)(A-G)}$

$\Rightarrow b=A-\sqrt{(A+G)(A-G)}$

Thus, numbers are $A \pm \sqrt{(A+G)(A-G)}$

Question:30 The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and nth hour ?

Answer:

The number of bacteria in a certain culture doubles every hour.It forms GP.

Given : a=30 and r=2.

$a_3=a.r^{3-1}=30(2)^2=120$

$a_5=a.r^{5-1}=30(2)^4=480$

$a_n+_1=a.r^{n+1-1}=30(2)^n$

Thus, bacteria present at the end of the 2nd hour, 4th hour and nth hour are 120,480 and $30(2)^n$respectively.

Question:31 What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

Answer:

Given: Bank pays an annual interest rate of 10% compounded annually.

Rs 500 amounts are deposited in the bank.

At the end of the first year, the amount

$$= 500\left(1+\frac{1}{10}\right) = 500(1.1)$$

At the end of the second year, the amount $= 500(1.1)(1.1)$

At the end of the third year, the amount $= 500(1.1)(1.1)(1.1)$

At the end of 10 years, the amount $= 500(1.1)(1.1)(1.1)........(10 times)$

$$= 500(1.1)^{10}$$

Thus, at the end of 10 years, amount $= Rs.500(1.1)^{10}$

Question:32 If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation

Answer:

Let roots of the quadratic equation be a and b.

According to given condition,

$$AM = \frac{a+b}{2} = 8$$

$$\Rightarrow (a+b) = 16$$

$$GM = \sqrt{ab} = 5$$

$$\Rightarrow ab = 25$$

We know that $x^2 - x(sum\,of\,roots) + (product\,of\,roots) = 0$

$x^2 - x(16) + (25) = 0$

$x^2 - 16x + 25 = 0$

Thus, the quadratic equation = $x^2 - 16x + 25 = 0$

NCERT solutions for class 11 maths chapter 9 sequences and series- Exercise: 9.4

Question:1 Find the sum to n terms of each of the series in $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

Answer:

the series = $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

n th term = $n(n+1) = a_n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k(k+1)$$

$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left(\frac{(2n+1)}{3} + 1\right)$$

$$= \frac{n(n+1)}{2}\left(\frac{(2n+1+3)}{3}\right)$$

$$= \frac{n(n+1)}{2}\left(\frac{(2n+4)}{3}\right)$$

$$= n(n+1)\left(\frac{(n+2)}{3}\right)$$

$$= \frac{n(n+1)(n+2)}{3}$$

Question:2 Find the sum to n terms of each of the series in $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + ...$

Answer:

the series = $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + ...$

n th term = $n(n+1)(n+2) = a_n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k(k+1)(k+2)$$

$$= \sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{3.n(n+1)(2n+1)}{6} + \frac{2.n(n+1)}{2}$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{2} + n(n+1)$$

$$= \left[\frac{n(n+1)}{2}\right]\left(\frac{n(n+1)}{2} + (2n+1) + 2\right)$$

$$= \left[\frac{n(n+1)}{2}\right]\left(\frac{n^2+n+4n+2+4}{2}\right)$$

$$= \left[\frac{n(n+1)}{2}\right]\left(\frac{n^2+5n+6}{2}\right)$$

$$= \left[\frac{n(n+1)}{4}\right]\left(n^2+5n+6\right)$$

$$= \left[\frac{n(n+1)}{4}\right]\left(n^2+2n+3n+6\right)$$

$$= \left[\frac{n(n+1)}{4}\right](n(n+2)+3(n+2))$$

$$= \left[\frac{n(n+1)}{4}\right]((n+2)(n+3))$$

$$= \left[\frac{n(n+1)(n+2)(n+3)}{4}\right]$$

Thus, sum is

$$= \left[\frac{n(n+1)(n+2)(n+3)}{4}\right]$$

Question:3 Find the sum to n terms of each of the series $3 \times 1^2 + 5 \times 2^2 + 7 \times + ... + 20^2$

Answer:

the series $3 \times 1^2 + 5 \times 2^2 + 7 \times + ... + 20^2$

nth term = $(2n+1)(n^2) = 2n^3 + n^2 = a_n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} 2k^3 + k^2$$

$$= 2\sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} k^2$$

$$= 2\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \left[\frac{n^2(n+1)^2}{2}\right] + \frac{n(n+1)(2n+1)}{6}$$

$$= \left[\frac{n(n+1)}{2}\right](n(n+1) + \frac{(2n+1)}{3})$$

$$= \left[\frac{n(n+1)}{2}\right]\frac{(3n^2+3n+2n+1)}{3}$$

$$= \left[\frac{n(n+1)}{2}\right]\frac{(3n^2+5n+1)}{3}$$

$$= \frac{n(n+1)(3n^2+5n+1)}{6}$$

Thus, the sum is

$$= \frac{n(n+1)(3n^2+5n+1)}{6}$$

Question:4 Find the sum to n terms of each of the series in $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots$

Answer:

Series =

$$\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots$$

$$n^{th}\, term = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$a_1 = \frac{1}{1} - \frac{1}{2}$$

$$a_2 = \frac{1}{2} - \frac{1}{3}$$

$$a_3 = \frac{1}{3} - \frac{1}{4}$$ ....

$$a_n = \frac{1}{n} - \frac{1}{n+1}$$

$$a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots\ldots\ldots a_n = \left[\frac{1}{1}\right] - \left[\frac{1}{n+1}\right]$$

$$S_n = \frac{n+1-1}{n+1}$$

$$S_n = \frac{n}{n+1}$$

Hence, the sum is

$$S_n = \frac{n}{n+1}$$

Question:5 Find the sum to n terms of each of the series in $5^2 + 6^2 + 7^2 + ... + 20^2$

Answer:

series = $5^2 + 6^2 + 7^2 + ... + 20^2$

n th term = $(n+4)^2 = n^2 + 8n + 16 = a_n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} (k+4)^2$$

$$= \sum_{k=1}^{n} k^2 + 8 \sum_{k=1}^{n} k + \sum_{k=1}^{n} 16$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{8.n(n+1)}{2} + 16n$$

16th term is $(16+4)^2 = 20^2$

$$S_{16} = \frac{16(16+1)(2(16)+1)}{6} + \frac{8.(16)(16+1)}{2} + 16(16)$$

$$S_{16} = \frac{16(17)(33)}{6} + \frac{8.(16)(17)}{2} + 16(16)$$

$$S_{16} = 1496 + 1088 + 256$$

$$S_{16} = 2840$$

Hence, the sum of the series $5^2 + 6^2 + 7^2 + ... + 20^2$ is 2840.

Question:6 Find the sum to n terms of each of the series $3 \times 8 + 6 \times 11 + 9 \times 14 + ...$

Answer:

series = $3 \times 8 + 6 \times 11 + 9 \times 14 + ...$

=(n th term of 3,6,9,...........) $\times$ (nth terms of 8,11,14,..........)

n th term = $3n(3n+5) = a_n = 9n^2 + 15n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} 3k(3k+5)$$

$$= 9 \sum_{k=1}^{n} k^2 + 15 \sum_{k=1}^{n} k$$

$$= \frac{9.n(n+1)(2n+1)}{6} + \frac{15.n(n+1)}{2}$$

$$= \frac{3.n(n+1)(2n+1)}{2} + \frac{15.n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left(3(2n+1)+15\right)$$

$$= \frac{3.n(n+1)}{2}\left(2n+1+5\right)$$

$$= \frac{3.n(n+1)}{2}\left(2n+6\right)$$

$$= \frac{3.n(n+1)}{2}.2.\left(n+3\right)$$

$$= 3.n(n+1)\left(n+3\right)$$

Hence, sum is $= 3.n(n+1)\left(n+3\right)$

Question:7 Find the sum to n terms of each of the series in $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2)...$

Answer:

series = $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2)...$

n th term = $a_n = 1^2 + 2^2 + 3^2 + ...n^2 = \frac{n(n+1)(2n+1)}{6}$

$$= \frac{n(2n^2+3n+1)}{6} = \frac{2n^3+3n^2+n}{6}$$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \frac{2k^3+3k^2+k}{6}$$

$$= \frac{1}{3}\sum_{k=1}^{n} k^3 + \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{1}{6}\sum_{k=1}^{n} k$$

$$= \frac{1}{3}\left[\frac{n(n+1)}{2}\right]^2 + \frac{1}{2}.\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\frac{n(n+1)}{2}$$

$$= \left[\frac{n(n+1)}{6}\right](\frac{n(n+1)}{2} + \frac{2n+1}{2} + \frac{1}{2})$$

$$= \left[\frac{n(n+1)}{6}\right](\frac{n^2+n+2n+1+1}{2})$$

$$= \left[\frac{n(n+1)}{6}\right](\frac{n^2+n+2n+2}{2})$$

$$= \left[\frac{n(n+1)}{6}\right](\frac{n(n+1)+2(n+1)}{2})$$

$$= \left[\frac{n(n+1)}{6}\right](\frac{(n+1)(n+2)}{2})$$

$$= \left[\frac{n(n+1)^2(n+2)}{12}\right]$$

Question:8 Find the sum to n terms of the series in Exercises 8 to 10 whose nth terms is given by $n(n+1)(n+4)$

Answer:

nth terms is given by $n(n+1)(n+4)$

$$a_n = n(n+1)(n+4) = n(n^2+5n+4) = n^3+5n^2+4n$$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k(k+1)(k+4)$$

$$= \sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{5.n(n+1)(2n+1)}{6} + \frac{4.n(n+1)}{2}$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{5.n(n+1)(2n+1)}{6} + 2.n(n+1)$$

$$= \left[\frac{n(n+1)}{2}\right] (\frac{n(n+1)}{2} + \frac{5(2n+1)}{3} + 4)$$

$$= \left[\frac{n(n+1)}{2}\right] \left(\frac{3n^2 + 3n + 20n + 10 + 24}{6}\right)$$

$$= \left[\frac{n(n+1)}{2}\right] \left(\frac{3n^2 + 23n + 34}{6}\right)$$

$$= \left[\frac{n(n+1)}{24}\right] \left(3n^2 + 23n + 34\right)$$

Question:9 Find the sum to n terms of the series in Exercises 8 to 10 whose nth terms is given by

Answer:

nth terms are given by $n^2 + 2^n$

$$a_n = n^2 + 2^n$$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 2^k$$

This term is a GP with first term =a =2 and common ratio =r =2.

$$\sum_{k=1}^{n} 2^k$$

$$S_n = \sum_{k=1}^{n} k^2 + 2(2^n - 1)$$

$$S_n = \frac{n(n+1)(2n+1)}{6} + 2(2^n - 1)$$

Thus, the sum is

$$S_n = \frac{n(n+1)(2n+1)}{6} + 2(2^n - 1)$$

Question:10 Find the sum to n terms of the series in Exercises 8 to 10 whose nth terms is given by $(2n-1)^2$

Answer:

nth terms is given by $(2n-1)^2$.

$$a_n = (2n-1)^2 = 4n^2 + 1 - 4n$$

$$= 4\sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= \frac{4.n(n+1)(2n+1)}{6} - \frac{4.n(n+1)}{2} + n$$

$$= \frac{2.n(n+1)(2n+1)}{3} - 2.n(n+1) + n$$

$$= n[\frac{2(n+1)(2n+1)}{3} - 2(n+1) + 1]$$

$$= n(\frac{4n^2 + 6n + 2 - 6n - 6 + 3}{3})$$

$$= n(\frac{4n^2 - 1}{3})$$

$$= n(\frac{(2n+1)(2n-1)}{3})$$

NCERT solutions for class 11 maths chapter 9 sequences and series-Miscellaneous Exercise

Question:1 Show that the sum of and $(m-n)^{th}$ terms of an A.P. is equal to twice the m^{th}term.

Answer:

Let a be first term and d be common difference of AP.

Kth term of a AP is given by,

$a_k = a + (k-1)d$

$\therefore a_{m+n} = a + (m+n-1)d$

$\therefore a_{m-n} = a + (m-n-1)d$

$a_m = a + (m-1)d$

$a_{m+n} + a_{m-n} = a + (m+n-1)d + a + (m-n-1)d$

$= 2a + (2m - 2)d$

$= 2(a + (m - 1)d)$

$= 2.a_m$

Hence, the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an A.P. is equal to twice the m^{th} term.

Question:2 If the sum of three numbers in A.P., is 24 and their product is 440, find the numbers.

Answer:

Let three numbers of AP are a-d, a, a+d.

According to given information ,

$a - d + a + a + d = 24$

$3a = 24$

$\Rightarrow a = 8$

$(a - d)a(a + d) = 440$

$\Rightarrow (8 - d)(8 + d) = 55$

$\Rightarrow (8^2 - d^2) = 55$

$\Rightarrow (64 - d^2) = 55$

$\Rightarrow d^2 = 64 - 55 = 9$

$\Rightarrow d = \pm 3$

When d=3, AP= 5,8,11 also if d=-3 ,AP =11,8,5.

Thus, three numbers are 5,8,11.

Question:3 Let the sum of n, 2n, 3n terms of an A.P. be S_1, S_2, S_3 , respectively, show that $S_3 = 3(S_2 - S_1)$

Answer:

Let a be first term and d be common difference of AP.

$S_n = \frac{n}{2}[2a + (n-1)d] = S_1$.................................1

$S_2n = \frac{2n}{2}[2a + (2n-1)d] = S_2$.................................2

$S_{2n} = \frac{3n}{2}[2a + (3n-1)d] = S_3$.................................3

Subtract equation 1 from 2,

$S_2 - S_1 = \frac{2n}{2}[2a + (2n-1)d] - \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[4a + 4nd - 2d - 2a - nd + d]$

$= \frac{n}{2}[2a + 3nd - d]$

$= \frac{n}{2}[2a + (3n-1)d]$

$$\therefore 3(S_2 - S_1) = \frac{3n}{2}[2a + (3n-1)d] = S_3$$

Hence, the result is proved.

Question:4 Find the sum of all numbers between 200 and 400 which are divisible by 7.

Answer:

Numbers divisible by 7 from 200 to 400 are $203, 210, ...399$

This sequence is an A.P.

Here , first term =a =203

common difference = 7.

We know , $a_n = a + (n-1)d$

$$399 = 203 + (n-1)7$$

$$\Rightarrow \; 196 = (n-1)7$$

$$\Rightarrow \; 28 = (n-1)$$

$$\Rightarrow \; n = 28 + 1 = 29$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{29}{2}[2(203) + (29-1)7]$$

$$= 29 \times 301$$

$= 8729$

The sum of numbers divisible by 7from 200 to 400 is 8729.

Question:5 Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

Answer:

Numbers divisible by 2 from 1 to 100 are $2, 4, 6................100$

This sequence is an A.P.

Here , first term =a =2

common difference = 2.

We know , $a_n = a + (n-1)d$

$100 = 2 + (n-1)2$

$\Rightarrow\ 98 = (n-1)2$

$\Rightarrow\ 49 = (n-1)$

$\Rightarrow\ n = 49 + 1 = 50$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{50}{2}[2(2) + (50-1)2]$

$= \frac{50}{2}[2(2) + 49(2)]$

$= 25 \times 102$

$= 2550$

Numbers divisible by 5 from 1 to 100 are $5, 10, 15 \ldots\ldots\ldots\ldots\ldots 100$

This sequence is an A.P.

Here , first term =a =5

common difference = 5.

We know , $a_n = a + (n-1)d$

$100 = 5 + (n-1)5$

$\Rightarrow \; 95 = (n-1)5$

$\Rightarrow \; 19 = (n-1)$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}[2(5) + 19(5)]$

Numbers divisible by both 2 and 5 from 1 to 100 are

This sequence is an A.P.

Here , first term =a =10

common difference = 10

We know , $a_n = a + (n-1)d$

$100 = 10 + (n-1)10$

$\Rightarrow\ \ 90 = (n-1)10$

$\Rightarrow\ \ 9 = (n-1)$

$\Rightarrow\ \ n = 9 + 1 = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{10}{2}[2(10) + (10-1)10]$

$= \frac{10}{2}[2(10) + 9(10)]$

$= 5 \times 110 = 550$

$\therefore\ Required\ sum = 2550 + 1050 - 550 = 3050$

Thus, the sum of integers from 1 to 100 that are divisible by 2 or 5 is 3050.

Question:6 Find the sum of all two digit numbers which when divided by 4, yields 1 as remainder.

Answer:

Numbers divisible by 4, yield remainder as 1 from 10 to 100 are $13, 17, ...97$

This sequence is an A.P.

Here , first term =a =13

common difference = 4.

We know , $a_n = a + (n-1)d$

$97 = 13 + (n-1)4$

$\Rightarrow \ 84 = (n-1)4$

$\Rightarrow \ 21 = (n-1)$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{22}{2}[2(13) + (22-1)4]$

$= \frac{22}{2}[2(13) + 21(4)]$

$= 11 \times 110$

$= 1210$

The sum of numbers divisible by 4 yield 1 as remainder from 10 to 100 is 1210.

Question:7 If f is a function satisfying f (x +y) = f(x) f(y) for all x, y ϵ N such that f(1) = 3 and

$$\sum_{x=1}^{n} f(x) = 120$$

, find the value of n.

Answer:

Given : f (x +y) = f(x) f(y) for all x, y ϵ N such that f(1) = 3

$f(1) = 3$

Taking $x = y = 1$, we have

$f(1+1) = f(2) = f(1) * f(1) = 3 * 3 = 9$

$f(1+1+1) = f(1+2) = f(1) * f(2) = 3 * 9 = 27$

is $3, 9, 27, 81, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ forms a GP with first term=3 and common ratio = 3.

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$120 = \frac{3(1-3^n)}{1-3}$$

$$-80 = (1-3^n)$$

$$-80-1 = (-3^n)$$

Therefore, $n = 4$

Thus, value of n is 4.

Question:8 The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.

Answer:

Let the sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$315 = \frac{5(1-2^n)}{1-2}$$

$-63=(1-2^n)$

$-63-1=(-2^n)$

$-64=(-2^n)$

$2^n=64$

Therefore, $n=6$

Thus, the value of n is 6.

Last term of GP=6th term $=a.r^{n-1}=5.2^5=5*32=160$

The last term of GP =160

Question:9 The first term of a G.P. is 1. The sum of the third term and fifth term is 90. Find the common ratio of G.P.

Answer:

Given: The first term of a G.P. is 1. The sum of the third term and fifth term is 90.

$a=1$

$a_3=a.r^2=r^2\, a_5=a.r^4=r^4$

$\therefore\ r^2+r^4=90$

$\therefore\ r^4+r^2-90=0$

$\Rightarrow\ r^2=\dfrac{-1\pm\sqrt{1+360}}{2}$

$r^2 = \frac{-1 \pm \sqrt{361}}{2}$

$r^2 = -10\, or\, 9$

$r = \pm 3$

Thus, the common ratio of GP is ± 3 .

Question:10 The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Answer:

Let three terms of GP be $a, ar, ar^2.$

Then, we have $a + ar + ar^2 = 56$

$a(1 + r + r^2) = 56$ 1

$a-1, ar-7, ar^2-21$ from an AP.

$\therefore ar - 7 - (a-1) = ar^2 - 21 - (ar - 7)$

$ar - 7 - a + 1 = ar^2 - 21 - ar + 7$

$\Rightarrow ar^2 - 2ar + a = 8$

$\Rightarrow ar^2 - ar - ar + a = 8$

$\Rightarrow a(r^2 - 2r + 1) = 8$

$\Rightarrow a(r^2-1)^2 = 8$ 2

From equation 1 and 2, we get

$\Rightarrow 7(r^2-2r+1) = 1+r+r^2$

$\Rightarrow 7r^2-14r+7-1-r-r^2 = 0$

$\Rightarrow 6r^2-15r+6 = 0$

$\Rightarrow 2r^2-5r+2 = 0$

$\Rightarrow 2r^2-4r-r+2 = 0$

$\Rightarrow 2r(r-2)-1(r-2) = 0$

$\Rightarrow (r-2)(2r-1) = 0$

$\Rightarrow r = 2, r = \dfrac{1}{2}$

If r=2, GP = 8,16,32

If r=0.2, GP= 32,16,8.

Thus, the numbers required are 8,16,32.

Question:11 A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Answer:

Let GP be $A_1, A_2, A_3, \ldots\ldots\ldots\ldots\ldots\ldots A_{2n}$

Number of terms = 2n

According to the given condition,

$$(A_1, A_2, A_3, \ldots\ldots\ldots\ldots\ldots\ldots A_{2n}) = 5(A_1, A_3, \ldots\ldots\ldots\ldots\ldots\ldots A_{2n-1})$$

$$\Rightarrow (A_1, A_2, A_3, \ldots\ldots\ldots\ldots\ldots\ldots A_{2n}) - 5(A_1, A_3, \ldots\ldots\ldots\ldots\ldots\ldots A_{2n-1}) = 0$$

$$\Rightarrow (A_2, A_4, A_6, \ldots\ldots\ldots\ldots\ldots\ldots A_{2n}) = 4(A_1, A_3, \ldots\ldots\ldots\ldots\ldots\ldots A_{2n-1})$$

Let the be GP as $a, ar, ar^2, \ldots\ldots\ldots\ldots\ldots\ldots$

$$\Rightarrow \frac{ar(r^n - 1)}{r-1} = \frac{4.a(r^n - 1)}{r-1}$$

$$\Rightarrow ar = 4a$$

$$\Rightarrow r = 4$$

Thus, the common ratio is 4.

Question:12 The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

Answer:

Given : first term =a=11

Let AP be $11, 11+d, 11+2d, 11+3d, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 11+(n-1)d$

Given: The sum of the first four terms of an A.P. is 56.

$$11 + 11 + d + 11 + 2d + 11 + 3d = 56$$

$\Rightarrow 44+6d=56$

$\Rightarrow 6d=56-44=12$

$\Rightarrow 6d=12$

$\Rightarrow d=2$

Also, The sum of the last four terms is 112.

$11+(n-4)d+11+(n-3)d+11+(n-2)d+11+(n-1)d=112$

$\Rightarrow 44+(n-4)2+(n-3)2+(n-2)2+(n-1)2=112$

$\Rightarrow 44+2n-8+2n-6+2n-4+2n-2=112$

$\Rightarrow 44+8n-20=112$

$\Rightarrow 24+8n=112$

$\Rightarrow 8n=112-24$

$\Rightarrow 8n=88$

$\Rightarrow n=11$

Thus, the number of terms of AP is 11.

Question:13 If then show that a, b, c and d are in G.P.

Answer:

Given :

$$\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}(x\neq 0)$$

Taking ,

$$\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}$$

$$\Rightarrow (a+bx)(b-cx)=(b+cx)(a-bx)$$

$$\Rightarrow ab+b^2x-bcx^2-acx)=ba-b^2x+acx-bcx^2$$

$$\Rightarrow 2b^2x=2acx$$

$$\Rightarrow b^2=ac$$

$$\Rightarrow \frac{b}{a}=\frac{c}{b}..................1$$

Taking,

$$\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}$$

$$\Rightarrow (b+cx)(c-dx)=(c+dx)(b-cx)$$

$$\Rightarrow bc-bdx+c^2x-cdx^2=bc-c^2x+bdx-cdx^2$$

$$\Rightarrow 2bdx=2c^2x$$

$$\Rightarrow bd=c^2$$

$$\Rightarrow \frac{d}{c}=\frac{c}{b}..............................2$$

From equation 1 and 2 , we have

$$\Rightarrow \frac{d}{c}=\frac{c}{b}=\frac{b}{a}$$

Thus, a,b,c,d are in GP.

Question:14 Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n=S^n$

Answer:

Ler there be a GP $=a, ar, ar^2, ar^3, ...$

According to given information,

$$S=\frac{a(r^n-1)}{r-1}$$

$$P=a^n\times r^{(1+2+..................n-1)}$$

$$P=a^n\times r^{\frac{n(n-1)}{2}}$$

$$R=\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+..................\frac{1}{ar^{n-1}}$$

$$R=\frac{r^{n-1}+r^{n-2}+r^{n-3}+..............r+1}{a.r^{n-1}}$$

$$R=\frac{1}{a.r^{n-1}}\times\frac{1(r^n-1)}{r-1}$$

To prove : $P^2R^n=S^n$

LHS : P^2R^n

$$= a^{2n}.r^{n(n-1)}\frac{(r^n-1)^n}{a^n.r^{n(n-1)}.(r-1)^n}$$

$$= a^n\frac{(r^n-1)^n}{(r-1)^n}$$

$$= \left(\frac{a(r^n-1)}{(r-1)}\right)^n$$

$$= S^n = RHS$$

Hence proved

Question:15 The pth, qth and rth terms of an A.P. are a, b, c, respectively. Show that

$(q-r)a+(r-p)b+(p-q)c=0$

Answer:

Given: The pth, qth and rth terms of an A.P. are a, b, c, respectively.

To prove : $(q-r)a+(r-p)b+(p-q)c=0$

Let the first term of AP be 't' and common difference be d

$a_p = t+(p-1)d = a$..............................1

$a_q = t+(q-1)d = b$..............................2

$a_r = t+(r-1)d = c$..............................3

Subtracting equation 2 from 1, we get

$(p-1-q+1)d = a-b$

$$\Rightarrow (p-q)d = a-b$$

$$\Rightarrow d = \frac{a-b}{p-q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

Subtracting equation 3 from 2, we get

$$(q-1-r+1)d = b-c$$

$$\Rightarrow (q-r)d = b-c$$

$$\Rightarrow d = \frac{b-c}{q-r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 5$$

Equating values of d, from equation 4 and 5, we have

$$d = \frac{a-b}{p-q} = \frac{b-c}{q-r}.$$

$$\Rightarrow \frac{a-b}{p-q} = \frac{b-c}{q-r}.$$

$$\Rightarrow (a-b)(q-r) = (b-c)(p-q)$$

$$\Rightarrow aq - ar - bq + br = bp - bq - cp + cq$$

$$\Rightarrow aq - ar + br = bp - cp + cq$$

$$\Rightarrow aq - ar + br - bp + cp - cq = 0$$

$$\Rightarrow a(q-r) + b(r-p) + c(p-q) = 0$$

Hence proved.

Question:16 If $a(\frac{1}{b}+\frac{1}{c}), b(\frac{1}{c}+\frac{1}{a}), c(\frac{1}{a}+\frac{1}{b})$ are in A.P., prove that a, b, c are in A.P.

Answer:

Given: $a(\frac{1}{b}+\frac{1}{c}), b(\frac{1}{c}+\frac{1}{a}), c(\frac{1}{a}+\frac{1}{b})$ are in A.P.

$$\therefore \quad (\frac{ab^2+b^2c-a^2c-a^2b}{abc}) = (\frac{c^2b+c^2a-b^2a-b^2c}{abc})$$

$$\Rightarrow \quad ab^2+b^2c-a^2c-a^2b = c^2b+c^2a-b^2a-b^2c$$

$$\Rightarrow \quad ab(b-a)+c(b^2-a^2) = a(c^2-b^2)+bc(c-b)$$

$$\Rightarrow \quad ab(b-a)+c(b-a)(b+a) = a(c-b)(c+b)+bc(c-b)$$

$$\Rightarrow \quad (b-a)(ab+c(b+a)) = (c-b)(a(c+b)+bc)$$

$$\Rightarrow \quad (b-a)(ab+cb+ac) = (c-b)(ac+ab+bc)$$

$$\Rightarrow \quad (b-a) = (c-b)$$

Thus, a,b,c are in AP.

Question:17 If a, b, c, d are in G.P, prove that $(a^n+b^n), (b^n+c^n), (c^n+d^n)$ are in G.P.

Answer:

Given: a, b, c, d are in G.P.

To prove: $(a^n+b^n), (b^n+c^n), (c^n+d^n)$ are in G.P.

Then we can write,

$b^2 = ac$..............................1

$c^2 = bd$..............................2

$ad = bc$..............................3

Let $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ be in GP

$(b^n + c^n)^2 = (a^n + b^n)(c^n + d^n)$

LHS: $(b^n + c^n)^2$

$(b^n + c^n)^2 = b^{2n} + c^{2n} + 2b^n c^n$

$(b^n + c^n)^2 = (b^2)^n + (c^2)^n + 2b^n c^n$

$= (ac)^n + (bd)^n + 2b^n c^n$

$= a^n c^n + b^n c^n + a^n d^n + b^n d^n$

$= c^n(a^n + b^n) + d^n(a^n + b^n)$

$= (a^n + b^n)(c^n + d^n) = RHS$

Hence proved

Thus, $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in GP

Question:18 If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a G.P. Prove that (q + p) : (q – p) = 17:15.

Answer:

Given: a and b are the roots of $x^2 - 3x + p = 0$

Then, $a + b = 3 \;\; and \;\; ab = p$......................1

Also, c, d are roots of $x^2 - 12x + q = 0$

$c + d = 12 \;\; and \;\; cd = q$......................2

Given: a, b, c, d form a G.P

Let, $a = x, b = xr, c = xr^2, d = xr^3$

From 1 and 2, we get

$x + xr = 3$ and $xr^2 + xr^3 = 12$

$\Rightarrow x(1 + r) = 3 \; xr^2(1 + r) = 12$

On dividing them,

$$\frac{xr^2(1 + r)}{x(1 + r)} = \frac{12}{3}$$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

When , r=2 ,

When , r=-2,

$$x = \frac{3}{1-2} = -3$$

CASE (1) when r=2 and x=1,

$$ab = x^2 r = 2 \;\; and \;\;\; cd = x^2 r^5 = 32$$

$$\therefore \frac{q+p}{q-p} = \frac{32+2}{32-2} = \frac{34}{30} = \frac{17}{15}$$

i.e. (q + p) : (q – p) = 17:15.

CASE (2) when r=-2 and x=-3,

$$ab = x^2 r = -18 \;\; and \;\;\; cd = x^2 r^5 = -288$$

$$\therefore \frac{q+p}{q-p} = \frac{-288-18}{-288+18} = \frac{-305}{-270} = \frac{17}{15}$$

i.e. (q + p) : (q – p) = 17:15.

Question:19 The ratio of the A.M. and G.M. of two positive numbers a and b, is m : n. Show that $a:b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$

Answer:

Let two numbers be a and b.

$$AM = \frac{a+b}{2} \;\; and \;\;\; GM = \sqrt{ab}$$

According to the given condition,

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

$$\Rightarrow \frac{(a+b)^2}{4ab} = \frac{m^2}{n^2}$$

$$\Rightarrow (a+b)^2 = \frac{4ab.m^2}{n^2}$$

$$\Rightarrow (a+b) = \frac{2\sqrt{ab}.m}{n}$$ 1

$$(a-b)^2 = (a+b)^2 - 4ab$$

We get,

$$(a-b)^2 = \left(\frac{4abm^2}{n^2}\right) - 4ab$$

$$(a-b)^2 = \left(\frac{4abm^2 - 4abn^2}{n^2}\right)$$

$$\Rightarrow (a-b)^2 = \left(\frac{4ab(m^2-n^2)}{n^2}\right)$$

$$\Rightarrow (a-b) = \left(\frac{2\sqrt{ab}\sqrt{(m^2-n^2)}}{n}\right)$$ 2

From 1 and 2, we get

$$2a = \left(\frac{2\sqrt{ab}}{n}\right)\left(m + \sqrt{(m^2-n^2)}\right)$$

$$a = \left(\frac{\sqrt{ab}}{n}\right)\left(m + \sqrt{(m^2-n^2)}\right)$$

Putting the value of a in equation 1, we have

$$= \left(\frac{\sqrt{ab}}{n}\right)\left(m - \sqrt{(m^2 - n^2)}\right)$$

$$= \frac{\left(m + \sqrt{(m^2 - n^2)}\right)}{\left(m - \sqrt{(m^2 - n^2)}\right)}$$

$$a : b = \left(m + \sqrt{(m^2 - n^2)}\right) : \left(m - \sqrt{(m^2 - n^2)}\right)$$

Question:20 If a, b, c are in A.P.; b, c, d are in G.P. and 1/c , 1/d , 1/e are in A.P. prove that a, c, e are in G.P.

Answer:

Given: a, b, c are in A.P

$$b - a = c - b \text{..............................} 1$$

Also, b, c, d are in G.P.

$$c^2 = bd \text{..............................} 2$$

Also, 1/c, 1/d, 1/e are in A.P

$$\frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d} \text{...........................} 3$$

To prove: a, c, e are in G.P. i.e. $c^2 = ae$

From 1, we get $2b = a + c$

$$b = \frac{a+c}{2}$$

From 2, we get

$$d = \frac{c^2}{b}$$

Putting values of b and d, we get

$$\frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d}$$

$$\frac{2}{d} = \frac{1}{c} + \frac{1}{e}$$

$$\Rightarrow \frac{2b}{c^2} = \frac{1}{c} + \frac{1}{e}$$

$$\Rightarrow \frac{2(a+c)}{2c^2} = \frac{1}{c} + \frac{1}{e}$$

$$\Rightarrow \frac{(a+c)}{c^2} = \frac{e+c}{ce}$$

$$\Rightarrow \frac{(a+c)}{c} = \frac{e+c}{e}$$

$$\Rightarrow e(a+c) = c(e+c)$$

$$\Rightarrow ea + ec = ec + c^2$$

$$\Rightarrow ea = c^2$$

Thus, a, c, e are in G.P.

Question:21(i) Find the sum of the following series up to n terms: $5+55+555+....$

Answer:

$5+55+555+....$ is not a GP.

It can be changed in GP by writing terms as

$S_n = 5+55+555+....$ to n terms

$$S_n = \frac{5}{9}[9+99+999+9999+...............]$$

$$S_n = \frac{5}{9}[(10+10^2+10^3+........)-(1+1+1....................)]$$

$$S_n = \frac{5}{9}[\frac{10(10^n-1)}{10-1}-(n)]$$

$$S_n = \frac{5}{9}[\frac{10(10^n-1)}{9}-(n)]$$

$$S_n = \frac{50}{81}[(10^n-1)]-\frac{5n}{9}$$

Thus, the sum is

$$S_n = \frac{50}{81}[(10^n-1)]-\frac{5n}{9}$$

Question:21(ii) Find the sum of the following series up to n terms: .6 +. 66 +. 666+…

Answer:

Sum of 0.6 +0. 66 + 0. 666+…................

It can be written as

$S_n = 0.6 + 0.66 + 0.666 + ...$ to n terms

$$S_n = 6[0.1 + 0.11 + 0.111 + 0.1111 + ...]$$

$$S_n = \frac{6}{9}[0.9 + 0.99 + 0.999 + 0.9999 + ...]$$

$$S_n = \frac{6}{9}[(1 - \frac{1}{10}) + (1 - \frac{1}{10^2}) + (1 - \frac{1}{10^3}) + (1 - \frac{1}{10^4}) + ...]$$

$$S_n = \frac{2}{3}[(1+1+1....................n\,terms) - \frac{1}{10}(1 + \frac{1}{10} + \frac{1}{10^2} + ...n\,terms)]$$

$$S_n = \frac{2}{3}[n - \frac{\frac{1}{10}(\frac{1}{10}^n - 1)}{\frac{1}{10} - 1}]$$

$$S_n = \frac{2n}{3} - \frac{2}{30}[\frac{10(1 - 10^{-n})}{9}]$$

$$S_n = \frac{2n}{3} - \frac{2}{27}(1 - 10^{-n})$$

Question:22 Find the 20th term of the series $2 \times 4 + 4 \times 6 + \times 6 \times 8 + ... + n$ terms.

Answer:

the series = $2 \times 4 + 4 \times 6 + \times 6 \times 8 + ... + n$

$$\therefore n^{th}\,term = a_n = 2n(2n+2) = 4n^2 + 4n$$

$$\therefore a_{20} = 2(20)[2(20) + 2]$$

$$= 40[40 + 2]$$

$= 40[42]$

$= 1680$

Thus, the 20th term of series is 1680

Question:23 Find the sum of the first n terms of the series: 3+ 7 +13 +21 +31 +…

Answer:

The series: 3+ 7 +13 +21 +31 +…..............

n th term = $n^2 + n + 1 = a_n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^2 + k + 1$$

$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$$

$$= n\left(\frac{(n+1)(2n+1)}{6} + \frac{n+1}{2} + 1\right)$$

$$= n\left(\frac{(n+1)(2n+1) + 3(n+1) + 6}{6}\right)$$

$$= n\left(\frac{2n^2 + n + 2n + 1 + 3n + 3 + 6}{6}\right)$$

$$= n\left(\frac{2n^2 + 6n + 10}{6}\right)$$

$$= n\left(\frac{n^2 + 3n + 5}{3}\right)$$

Question:24 If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively, show that $9S_2^2 = S_3(1 + 8S_1)$

Answer:

To prove : $9S_2^2 = S_3(1 + 8S_1)$

From given information,

$$S_1 = \frac{n(n+1)}{2}$$

$$S_3 = \frac{n^2(n+1)^2}{4}$$

Here , $RHS = S_3(1 + 8S_1)$

$$= \frac{n^2(n+1)^2}{4}(1 + 4.n(n+1))$$

$$= \frac{n^2(n+1)^2}{4}\left(1 + 4n^2 + 4n\right)$$

$$= \frac{n^2(n+1)^2}{4}(2n+1)^2$$

$$= \frac{n^2(n+1)^2(2n+1)^2}{4} \text{...} 1$$

Also, $RHS = 9S_2^2$

$$\Rightarrow 9S_2^2 = \frac{9\left[n(n+2)(2n+1)\right]^2}{6^2}$$

$$= \frac{9\left[n(n+2)(2n+1)\right]^2}{36}$$

$$= \frac{[n(n+2)(2n+1)]^2}{4} \text{...} 2$$

From equation 1 and 2 , we have

$$9S_2^2 = S_3(1+8S_1) = \frac{[n(n+2)(2n+1)]^2}{4}$$

Hence proved .

Question:25 Find the sum of the following series up to n terms: $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + ...$

Answer:

n term of series :

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \text{........} = \frac{1^3+2^3+3^3+\text{..........}n^3}{1+3+5+\text{...........}(2n-1)}$$

$$= \frac{\left[\frac{n(n+1)}{2}\right]^2}{1+3+5+\text{...........}(2n-1)}$$

Here, $1, 3, 5\text{............}(2n-1)$ are in AP with first term =a=1 , last term = 2n-1, number of terms =n

$$1+3+5\text{............}(2n-1) = \frac{n}{2}\left[2(1)+(n-1)2\right]$$

$$= \frac{n}{2}\left[2+2n-2\right] = n^2$$

$$a_n = \frac{n^2(n+1)^2}{4n^2}$$

$$= \frac{(n+1)^2}{4}$$

$$= \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}$$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \frac{(k+1)^2}{4}$$

$$= \frac{1}{4}\sum_{k=1}^{n} k^2 + \frac{1}{2}\sum_{k=1}^{n} k + \sum_{k=1}^{n} \frac{1}{4}$$

$$= \frac{1}{4}\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\frac{n(n+1)}{2} + \frac{n}{4}$$

$$= n\left(\frac{(n+1)(2n+1)}{24} + \frac{n+1}{4} + \frac{1}{4}\right)$$

$$= n\left(\frac{(n+1)(2n+1) + 6(n+1) + 6}{24}\right)$$

$$= n\left(\frac{2n^2 + n + 2n + 1 + 6n + 6 + 6}{24}\right)$$

$$= n\left(\frac{2n^2 + 9n + 13}{24}\right)$$

Question:26 Show that

Answer:

To prove :

the nth term of numerator $= n(n+1)^2 = n^3 + 2n^2 + n$

nth term of the denominator $= n^2(n+1) = n^3 + n^2$

Numerator :

$$S_n = \sum_{k=1}^{n} k^3 + 2\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{2.n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}$$

$$= \left[\frac{n(n+1)}{2}\right]\left(\frac{3n^2+3n+8n+4+6}{6}\right)$$

$$= \left[\frac{n(n+1)}{2}\right]\left(\frac{3n^2+11n+10}{6}\right)$$

$$= \left[\frac{n(n+1)}{12}\right]\left(3n^2+11n+10\right)$$

$$= \left[\frac{n(n+1)}{12}\right]\left(3n^2+6n+5n+10\right)$$

$$= \left[\frac{n(n+1)}{12}\right]((n+2)(3n+5))$$

$$= \frac{n(n+1)(n+2)(3n+5)}{12} \text{..} 2$$

Denominator :

$$S_n = \sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} k^2$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \left[\frac{n(n+1)}{2}\right] (\frac{n(n+1)}{2} + \frac{2n+1}{3})$$

$$= \left[\frac{n(n+1)}{12}\right] \left(3n^2 + 7n + 2\right)$$

$$= \left[\frac{n(n+1)}{12}\right] \left(3n^2 + 6n + n + 2\right)$$

$$= \left[\frac{n(n+1)}{12}\right] (3n(n+2) + 1(n+2))$$

$$= \left[\frac{n(n+1)}{12}\right] ((n+2)(3n+1))$$

$$= \left[\frac{n(n+1)(n+2)(3n+1)}{12}\right] \text{......................................} 3$$

From equation 1,2,3,we have

$$= \frac{\frac{n(n+1)(n+2)(3n+5)}{12}}{\frac{n(n+1)(n+2)(3n+1)}{12}}$$

$$= \frac{3n+5}{3n+1}$$

Hence, the above expression is proved.

Question:27 A farmer buys a used tractor for Rs 12000. He pays Rs 6000 cash and agrees to pay the balance in annual instalments of Rs 500 plus 12% interest on the unpaid amount. How much will the tractor cost him?

Answer:

Given : Farmer pays Rs 6000 cash.

Therefore , unpaid amount = 12000-6000=Rs. 6000

According to given condition, interest paid annually is

12% of 6000,12% of 5500,12% of 5000,......................12% of 500.

Thus, total interest to be paid

$$= 12\% of\ 6000 + 12\% of\ 5500 + ...12\% of\ 500$$

$$= 12\% of\ (500 + 1000 + ... + 6000)$$

Here, $500, 1000, ...5500, 6000$ is a AP with first term =a=500 and common difference =d = 500

We know that $a_n = a + (n-1)d$

$\Rightarrow 6000 = 500 + (n-1)500$

$\Rightarrow 5500 = (n-1)500$

$\Rightarrow 11 = (n-1)$

$\Rightarrow n = 11 + 1 = 12$

Sum of AP:

$$S_{12} = \frac{12}{2}\left[2(500) + (12-1)500\right]$$

$$S_{12} = 6\left[1000 + 5500\right]$$

Thus, interest to be paid :

$$= 12\% of\left(500 + 1000 + ... + 6000\right)$$

$$= Rs. 4680$$

Thus, cost of tractor = Rs. 12000+ Rs. 4680 = Rs. 16680

Question:28 Shamshad Ali buys a scooter for Rs 22000. He pays Rs 4000 cash and agrees to pay the balance in annual instalment of Rs 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Answer:

Given: Shamshad Ali buys a scooter for Rs 22000.

Therefore , unpaid amount = 22000-4000=Rs. 18000

According to the given condition, interest paid annually is

10% of 18000,10% of 17000,10% of 16000,......................10% of 1000.

Thus, total interest to be paid

$= 10\% of\ 18000 + 10\% of\ 17000 + ...10\% of\ 1000$

$= 10\% of\ (18000 + 17000 + ... + 1000)$

$= 10\% of\ (1000 + 2000 + ... + 18000)$

Here, $1000, 2000, ...17000, 18000$ is a AP with first term =a=1000 and common difference =d = 1000

We know that $a_n = a + (n-1)d$

$\Rightarrow 18000 = 1000 + (n-1)1000$

$\Rightarrow 17000 = (n-1)1000$

$\Rightarrow 17 = (n-1)$

$\Rightarrow n = 17 + 1 = 18$

Sum of AP:

$S_{18} = \frac{18}{2}\left[2(1000) + (18-1)1000\right]$

$= 171000$

Thus, interest to be paid :

$= 10\% of\,(1000 + 2000 + \ldots\ldots\ldots\ldots + 18000)$

$= 10\% of\,(171000)$

$= Rs.17100$

Thus, cost of tractor = Rs. 22000+ Rs. 17100 = Rs. 39100

Question:29 A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8th set of letter is mailed.

Answer:

The numbers of letters mailed forms a GP : $4, 4^2, 4^3, \ldots\ldots\ldots\ldots 4^8$

first term = a=4

common ratio=r=4

number of terms = 8

We know that the sum of GP is

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{4(4^8 - 1)}{4 - 1}$$

$$= \frac{4(65536 - 1)}{3}$$

$$= \frac{4(65535)}{3}$$

$$= 87380$$

costs to mail one letter are 50 paise.

Cost of mailing 87380 letters

$$= Rs. 87380 \times \frac{50}{100}$$

Thus, the amount spent when the 8th set of the letter is mailed is Rs. 43690.

Question:30 A man deposited Rs 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15th year since he deposited the amount and also calculate the total amount after 20 years.

Answer:

Given : A man deposited Rs 10000 in a bank at the rate of 5% simple interest annually.

$\therefore$ Interest in fifteen year 10000+ 14 times Rs. 500

$\therefore$ Amount in 15 th year $= Rs. 10000 + 14 \times 500$

$= Rs. 10000 + 7000$

$= Rs. 17000$

$\therefore$ Amount in 20 th year $= Rs. 10000 + 20 \times 500$

$= Rs. 10000 + 10000$

$= Rs. 20000$

Question:31 A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Answer:

Cost of machine = Rs. 15625

Machine depreciate each year by 20%.

Therefore, its value every year is 80% of the original cost i.e. $\frac{4}{5}$ of the original cost.

$\therefore$ Value at the end of 5 years

$$= 15625 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$$

$$= 5120$$

Thus, the value of the machine at the end of 5 years is Rs. 5120

Question:32 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on.

Answer:

Let x be the number of days in which 150 workers finish the work.

According to the given information, we have

Series $150x = 150 + 146 + 142 + ...(x+8) terms$ is a AP

first term=a=150

common difference= -4

number of terms = x+8

$$\Rightarrow 150x = \frac{x+8}{2}\left[2(150) + (x+8-1)(-4)\right]$$

$$\Rightarrow 300x = x + 8\left[300 - 4x - 28\right]$$

$$\Rightarrow 300x = 300x - 4x^2 - 28x + 2400 - 32x + 224$$

$$\Rightarrow x^2 + 15x - 544 = 0$$

$$\Rightarrow x^2 + 32x - 17x - 544 = 0$$

$$\Rightarrow x(x + 32) - 17(x + 32) = 0$$

$$\Rightarrow (x + 32)(x - 17) = 0$$

$$\Rightarrow x = -32, 17$$

Since x cannot be negative so x=17.

Thus, in 17 days 150 workers finish the work.

Thus, the required number of days = 17+8=25 days.