NCERT solutions for class 11 physics Chapter 7 System of Particles and Rotational Motion

Q7.1 Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

Answer:

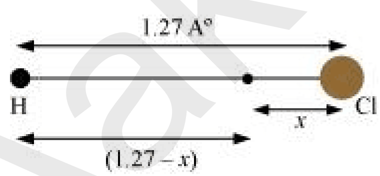
For uniform mass density, the location of the centre of mass is the same as that of the geometric centre.

No, it is not necessary that the centre of mass lies inside the body. For example in the case of a ring the centre of mass outside the body.

Q7.2 In the HCl molecule, the separation between the nuclei of the two atoms is about $1.27\mathring{A}(1\mathring{A} = 10^{-10}m)$. Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

Answer:

Let us assume that the centre of mass of the molecule is x cm away from the chlorine atom.



So, we can write

$$\frac{m\left(1.27\ -\ x\right)\ +\ 35.5mx}{m\ +\ 35.5}\ =\ 0$$

or

$$x = \frac{-1.27}{(35.5 - 1)} = -0.037\mathring{A}$$

Here negative sign indicates that the centre of mass lies leftward of the chlorine atom.

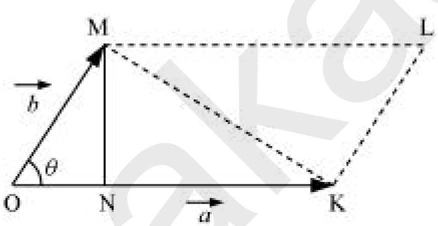
Q7.3 A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?

Answer:

Since child and trolley are considered in a system thus speed of the centre of mass change only when an external force will act on the system. When the child starts to move on the trolley the force produced is the internal force of the system so this will produce **no change** in the velocity of the CM.

Q7.4 Show that the area of the triangle contained between the vectors $a \; and \; b$ is one half of the magnitude of $a \times b$.

Answer:



Let a and b be two vectors having Θ angle between them.

Consider $\triangle$ MON,

$$\sin\Theta = \frac{MN}{MO}$$

or $\sin\Theta = \dfrac{MN}{\vec{b}}$

or $MN = b\sin\Theta$

$|a \times b| = |a|\,|b|\sin\Theta$

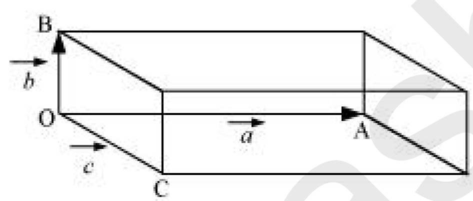
= 2 (Area of $\triangle$ MOK)

Therefore the area of $\triangle$ MOK = One half of $|a \times b|$.

Q7.5 Show that $a.(b \times c)$ is equal in magnitude to the volume of the parallelepiped formed on the three vectors , a , b and c .

Answer:

A parallelepiped is shown in the figure given below:-



Volume is given by : = abc

We can write :

$|b \times c| = |b||c|\sin\Theta\ \widehat{n}$ (The direction of $\widehat{n}$ is in the direction of vector a.)

$= |b||c|\sin 90^\circ\ \widehat{n}$

$= |b||c|\ \widehat{n}$

Now,

$a.(b \times c) \ = \ a.(bc) \ \widehat{n}$

$= \ abc\cos\Theta$

$= \ abc\cos 0^{\circ}$

$= \ abc$

This is equal to volume of parallelepiped.

Q7.6 Find the components along the x, y, z axes of the angular momentum 1 of a particle, whose position vector is r with components x, y, z and momentum is p with components p_x , p_y and p_z . Show that if the particle moves only in the $x - y$ plane the angular momentum has only a z-component.

Answer:

Linear momentum of particle is given by :

$\overrightarrow{p} \ = \ p_x\widehat{i} \ + \ p_y\widehat{j} \ + \ p_z\widehat{k}$

And the angular momentum is :

$\overrightarrow{l} \ = \overrightarrow{r} \times \overrightarrow{p}$

$$= \begin{vmatrix} i & j & k \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix}$$

When particle is confined to x-y plane then z = 0 and $p_z = 0$.

When we put the value of z and p_z in the equation of linear momentum then we observe that only the z component is non-zero.

Q7.7 Two particles, each of mass m and speed v , travel in opposite directions along parallel lines separated by a distance d . Show that the angular momentum vector of the two particle system is the same whatever be the point about which the angular momentum is taken.

Answer:

Assume two points (say A, B) separated by distance d.

So the angular momentum of a point about point A is given by :

$$= \ mv \times d \ = \ mvd$$

And about point B : $= \ mv \times d \ = \ mvd$

Now assume a point between A and B as C which is at y distance from point B.

Now the angular momentum becomes : $= \ mv \times (d - y) \ + \ mv \times y$

$$= \ mvd$$

Thus it can be seen that angular momentum is independent of the point about which it is measured.

Q7.8 A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig.7.39. The angles made by the strings with the vertical are 36.9^0 and 53.1^0 respectively. The bar is $2m$ long. Calculate the distance d of the centre of gravity of the bar from its left end.

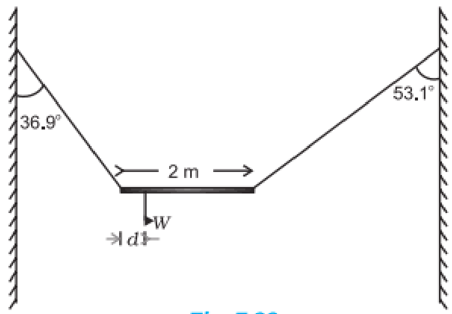
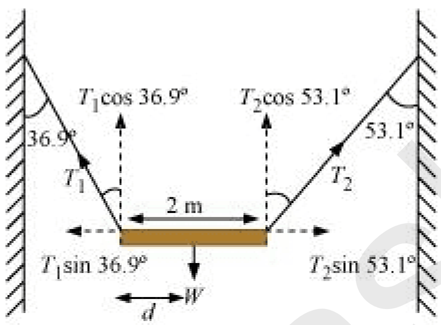


Fig. 7.39

Answer:

The FBD of the given bar is shown below :



Since the bar is in equilibrium, we can write :

$$T_1 \sin 36.9^\circ = T_2 \sin 53.1^\circ$$

or $\dfrac{T_1}{T_2} = \dfrac{0.800}{0.600} = \dfrac{4}{3}$

or $T_1 = \dfrac{4}{3} T_2$ ....(i)

For the rotational equilibrium :

$T_1 \cos 36.9° \times d \ = \ T_2 \cos 53.1° \times (2-d)$ (Use equation (i) to solve this equation)

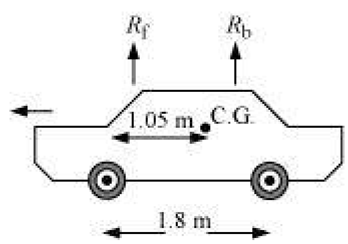
or $d \ = \ \frac{1.2}{1.67} \ = \ 0.72 \ m$

Thus the center of gravity is at 0.72 m from the left.

Q7.9 A car weighs $1800kg$. The distance between its front and back axles is $1.8m$. Its centre of gravity is $1.05m$ behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

Answer:

The FBD of the car is shown below :



We will use conditions of equilibrium here :

$R_f \ + \ R_b \ = \ mg$

$= \ 1800 \times 9.8 \ = \ 17640 \ N$ ....(i)

For rotational equilibrium :

$R_f \, (1.05) \ = \ R_b(1.8 - 1.05)$

or $1.05R_f \ = \ 0.75R_b$

$R_b \ = \ 1.4R_f$ ....(ii)

From (i) and (ii) we get :

$$R_f = \frac{17640}{2.4} = 7350 \ N$$

and $R_b = 17640 - 7350 = 10290 \ N$

Thus force exerted by the front wheel is = 3675 N

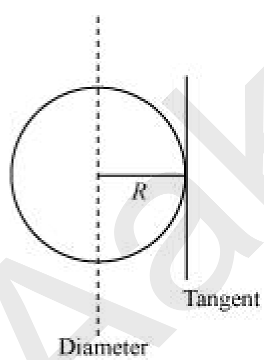
and force exerted by back wheel = 5145 N.

Q7.10 (a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.

Answer:

We know that moment of inertia of a sphere about diameter is :

$$= \frac{2}{5}MR^2$$



Using parallel axes theorem we can find MI about the tangent.

Moment of inertia of a sphere about tangent :

$$= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

Q 7.10(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

Answer:

We know that moment of inertia of a disc about its diameter is :

$$= \frac{1}{4}MR^2$$

Using perpendicular axes theorem we can write :

Moment of inertia of disc about its centre:-

$$= \frac{1}{4}MR^2 + \frac{1}{4}MR^2 = \frac{1}{2}MR^2$$

Using parallel axes theorem we can find the required MI :

Moment of inertia about an axis normal to the disc and passing through a point on its edge is given by :

$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

Q7.11 Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?

Answer:

The moment of inertia of hollow cylinder is given by $= \ mr^2$

And the moment of inertia of solid cylinder is given by :

$$= \frac{2}{5}mr^2$$

We know that : $\tau \ = \ I\alpha$

Let the torque for hollow cylinder be τ_1 and for solid cylinder let it be τ_2 .

According to the question : $\tau_1 \ = \ \tau_2$

So we can write the ratio of the angular acceleration of both the objects.

$$\frac{\alpha_2}{\alpha_1} \ = \ \frac{I_1}{I_2} \ = \ \frac{mr^2}{\frac{2}{5}mr^2} \ = \ \frac{2}{5}$$

Now for angular velocity,

$$\omega \ = \ \omega_o \ + \ \alpha t$$

Clearly, the angular velocity of the solid sphere is more than the angular velocity of the hollow sphere. (As the angular acceleration of solid sphere is greater).

Q7.12 A solid cylinder of mass $20kg$ rotates about its axis with angular speed $100 \ rad \ s^{-1}$. The radius of the cylinder is $0.25m$. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

Answer:

Firstly we will calculate moment of inertia of the solid cyliner :

$$I_c \ = \ \frac{1}{2}mr^2$$

$$= \ \frac{1}{2}(20)(0.25)^2 \ = \ 0.625 \ Kg \ m^2$$

So the kinetic energy is given by :

$$E_k \ = \ \frac{1}{2}I\omega^2$$

$$= \ \frac{1}{2} \times (6.25) \times (100)^2$$

$$= \ 3125 \ J$$

And the angular momentum is given by : $= \ \ I\omega$

$$= \ 0.625 \times 100$$

$$= \ 62.5 \ Js$$

Q7.13 (a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of $40 \ rev/min$. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to $2/5 times$ the initial value? Assume that the turntable rotates without friction.

Answer:

We are given with the initial angular speed and the relation between the moment of inertia of both the cases.

Here we can use conservation of angular momentum as no external force is acting the system.

So we can write :

$$I_1 w_1 \ = \ I_2 w_2$$

$$w_2 \ = \ \frac{I_1 w_1}{I_2}$$

$$= \ \frac{I(40)}{\frac{2}{5}I}$$

$$= \ 100 \ rev/min$$

Q7.13(b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

Answer:

The final and initial velocities are given below :

$E_f \ = \ \frac{1}{2} I_2 w_2^2$ and $E_i \ = \ \frac{1}{2} I_1 w_1^2$

Taking the ratio of both we get,

$$\frac{E_f}{E_i} \ = \ \frac{\frac{1}{2} I_2 w_2^2}{\frac{1}{2} I_1 w_1^2}$$

or $= \ \frac{2}{5} \times \frac{100 \times 100}{40 \times 40}$

or $= \ 2.5$

Thus the final energy is 2.5 times the initial energy.

The increase in energy is due to the internal energy of the boy.

Q7.14 A rope of negligible mass is wound round a hollow cylinder of mass $3kg$ and radius $40cm$. What is the angular acceleration of the cylinder if the rope is pulled with a force of $30\ N$? What is the linear acceleration of the rope ? Assume that there is no slipping.

Answer:

The moment of inertia is given by :

$I\ =\ mr^2$

or $=\ 3 \times (0.4)^2\ =\ 0.48\ Kg\ m^2$

And the torque is given by :

$\tau\ =\ r \times F$

$=\ 0.4 \times 30\ =\ 12\ Nm$

Also, $\tau\ =\ I\alpha$

So $\alpha\ =\ \frac{\tau}{I}\ =\ \frac{12}{0.48}\ =\ 25\ rad/s^{-2}$

And the linear acceleration is $a\ =\ \alpha r\ =\ 0.4 \times 25\ =\ 10\ m/s^{-2}$

Q7.15 To maintain a rotor at a uniform angular speed of $200\ rad\ s^{-1}$, an engine needs to transmit a torque of $180\ N\ m$. What is the power required by the engine?
(Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is $100^0/_0$ efficient.

Answer:

The relation between power and torque is given by :

$P = \tau\omega$

or $= 180 \times 200 = 36000\ W$

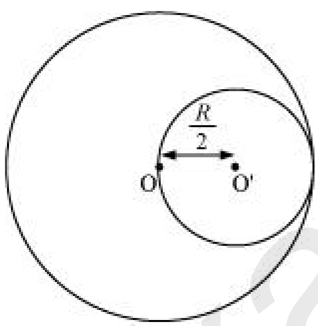
Hence the required power is 36 KW.

Q7.16 From a uniform disk of radius R , a circular hole of radius $R/2$ is cut out. The centre of the hole is at $R/2$ from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

Answer:

Let the mass per unit area of the disc be σ .

So total mass is $= \Pi r^2\sigma = m$



Mass of the smaller disc is given by :

$$= \Pi \left(\frac{r}{2}\right)^2 \sigma = \frac{m}{4}$$

Now since the disc is removed, we can assume that part to have negative mass with respect to the initial condition.

So, the centre of mass of the disc is given by the formula :

$$x = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

or $$= \frac{m \times 0 - \frac{m}{4} \times \frac{r}{2}}{m - \frac{m}{4}}$$

$$= \frac{-r}{6}$$

Hence the centre of mass is shifted $\frac{r}{6}$ leftward from point O.

Q7.17 A metre stick is balanced on a knife edge at its centre. When two coins, each of mass $5\ g$ are put one on top of the other at the $12.0cm$ mark, the stick is found to be balanced at $45.0cm$. What is the mass of the metre stick?

Answer:

The centre of mass of meter stick is at 50 cm.

Let the mass of meter stick be m.

Now according to the situation given in the question, we will use the rotational equilibrium condition at the centre point of the meter stick.

$$10g(45 - 12) - mg(50 - 45) = 0$$

or $$m = \frac{10 \times 33}{5} = 66\ g$$

Thus the mass of the meter stick is 66 g.

Q7.18 (a) A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination.

(a) Will it reach the bottom with the same speed in each case?

Answer:

Let the height of the plane is h and mass of the sphere is m.

Let the velocity at the bottom point of incline be v.

So the total energy is given by :

$$T = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

Using the law of conservation of energy :

$$\frac{1}{2}mv^2 + \frac{1}{2}Iw^2 = mgh$$

For solid sphere moment of inertia is :

$$I = \frac{2}{5}mr^2$$

So,

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)w^2 = mgh$$

Put v = wr and solve the above equation.

We obtain : $\frac{1}{2}v^2 + \frac{1}{5}v^2 = gh$

or $v = \sqrt{\frac{10}{7}gh}$

Thus the sphere will reach the bottom at the same speed since it doesn't depend upon the angle of inclination.

Q7.18 (b) A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination.

(b) Will it take longer to roll down one plane than the other?

Answer:

Let us assume the angle of inclination to be Θ_1 and Θ_2. $(\Theta_2 \;> \Theta_1)$

And the acceleration of the sphere will be $g \sin \Theta_1$ and $g \sin \Theta_2$ respectively.

Thus $a_2 > a_1$ (since $\sin \Theta_2 > \sin \Theta_1$)

Since their acceleration are different so the time taken to roll down the inclined plane will be different in both the cases.

Q7.18 (c) A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination.

(c) If so, which one and why?

Answer:

The equation of motion gives :

$v \;=\; u \;+\; at$

So $t \propto \dfrac{1}{a}$

Since we know that $a_2 > a_1$

So $t_2 < t_1$

Hence the inclined plane having a smaller angle of inclination will take more time.

Q7.19 A hoop of radius $2m$ weighs $100\ kg$. It rolls along a horizontal floor so that its centre of mass has a speed of $20cm/s$. How much work has to be done to stop it?

Answer:

Total energy of hoop is given by :

$$T = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

Also the moment inertia of hoop is given by : $I = mr^2$

We get,

$$T = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)w^2$$

And v = rw

$$T = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

So the work required is : $= mv^2 = 100(0.2)^2 = 4J$

Q7.20 The oxygen molecule has a mass of $5.30 \times 10^{-26} kg$ and a moment of inertia of $1.94 \times 10^{-46} kg\ m^2$ about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is $500m/s$ and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

Answer:

We are given the moment of inertia and the velocity of the molecule.

Let the mass of oxygen molecule be m.

So the mass of each oxygen atom is given by : $\frac{m}{2}$

Moment of inertia is :

$$I = \frac{m}{2}r^2 + \frac{m}{2}r^2 = mr^2$$

or $r = \sqrt{\frac{I}{m}}$

or $r = \sqrt{\frac{1.94 \times 10^{-46}}{5.36 \times 10^{-26}}} = 0.60 \times 10^{-10}\ m$

We are given that :

$$E_{rot} = \frac{2}{3}E_{tra}$$

or $\frac{1}{2}I\omega^2 = \frac{2}{3} \times \frac{1}{2}mv^2$

or $\omega = \sqrt{\frac{2}{3}} \times \frac{v}{r}$

or $= 6.80 \times 10^{12}\ rad/s$

Q7.21 (a) A solid cylinder rolls up an inclined plane of angle of inclination 30^0. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of $5m/s$.

(a) How far will the cylinder go up the plane?

Answer:

The rotational energy is converted into the translational energy.(Law of conservation of energy)

$$\frac{1}{2}I\omega^2 \ = \ \frac{1}{2}mv^2 \ = \ mgh$$

Since the moment of inertia for the cylinder is :

$$I \ = \ \frac{1}{2}mr^2$$

Putting the value of MI and v = wr in the above equation, we get :

$$\frac{1}{4}v^2 \ + \ \frac{1}{2}v^2 \ = \ gh$$

or $h \ = \ \frac{3}{4} \times \frac{v^2}{g}$

or $= \ \frac{3}{4} \times \frac{5 \times 5}{g} \ = \ 1.91 \ m$

Now using the geometry of the cylinder we can write :

$$\sin \Theta \ = \ \frac{h}{d}$$

or $\sin 30^\circ \ = \ \frac{h}{d}$

or $d \ = \ \frac{1.91}{0.5}$

$= \ 3.82 \ m$

Thus cylinder will travel up to 3.82 m up the incline.

Q7.21 (b) A solid cylinder rolls up an inclined plane of angle of inclination 30^0. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of $5m/s$.

b) How long will it take to return to the bottom?

Answer:

The velocity of cylinder is given by :

$$v \ = \ \left(\frac{2gh}{1 \ + \ \frac{k^2}{r^2}} \right)^{\frac{1}{2}}$$

or $$v \ = \ \left(\frac{2gd \sin \Theta}{1 \ + \ \frac{k^2}{r^2}} \right)^{\frac{1}{2}}$$

We know that for cylinder :

$$K^2 \ = \ \frac{R^2}{2}$$

Thus $$v \ = \ \left(\frac{4}{3} gd \sin \Theta \right)^{\frac{1}{2}}$$

Required time is :

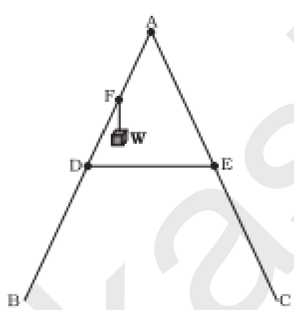
$$t \ = \frac{d}{v}$$

or $$= \ \left(\frac{11.46}{19.6} \right)^{\frac{1}{2}} \ = \ 0.764 \ s$$

Hence required time is 0.764(2) = 1.53 s.

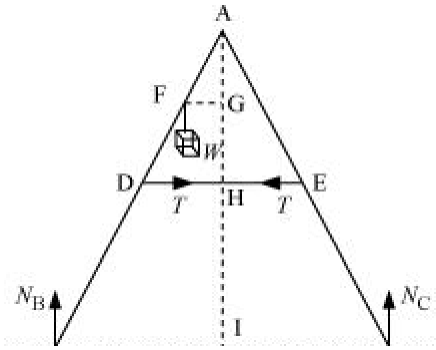
NCERT solutions for class 11 physics chapter 7 system of particles and rotational motion additional exercise

Q 7.22 As shown in Fig.7.40, the two sides of a step ladder BA and CA are $1.6m$ long and hinged at A. A rope DE , $0.5m$ is tied half way up. A weight $40kg$ is suspended from a point F , $1.2m$ from Balong the ladder BA . Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. $(Take g = 9.8m/s^2)$.
(Hint: Consider the equilibrium of each side of the ladder separately.)



Answer:

The FBD of the figure is shown below :



Consider triangle ADH,

$AH = \sqrt{(AD^2 - DH^2)}$ (AD and DH can be found using geometrical analysis.)

$= \sqrt{(0.8^2 - 0.25^2)} = 0.76\ m$

Now we will use the equilibrium conditions :

(i) For translational equilibrium :

$N_c + N_b = mg = 40 \times 9.8 = 392\ N$ ....(i)

(ii) For rotational equilibrium :

$-N_b \times BI + mg \times FG + N_c \times CI + T \times AG - T \times AG = 0$

or $(N_c - N_b) \times 0.5 = 49$

or $N_c - N_b = 98$ ....(ii)

Using (i) and (ii) we get :

$N_c = 245\ N$ and $N_b = 147\ N$

Now calculate moment about point A :

$$-N_b \times BI \ + \ mg \times FG \ + \ T \times AG \ = \ 0$$

Solve the equation : $T \ = \ 96.7 \ N$

Q 7.23 A man stands on a rotating platform, with his arms stretched horizontally holding a $5kg$ weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from $90cm$ to $20cm$. The moment of inertia of the man together with the platform may be taken to be constant and equal to $7.6kgm^2$.

(a) What is his new angular speed? (Neglect friction.)

Answer:

Moment of inertia when hands are stretched :

$$= \ 2 \times mr^2 \ = \ 2 \times (0.5)9^2$$

$$= \ 8.1 \ Kg \ m^2$$

So the moment of inertia of system (initial) = 7.6 + 8.1 = 15.7 Kg m ^{2} .

Now, the moment of inertia when hands are folded :

$$= \ 2 \times mr^2 \ = \ 2 \times 5(0.2)^2 \ = \ 0.4 \ Kg \ m^2$$

Thus net final moment of inertia is : = 7.6 + 0.4 = 8 Kg m ^{2} .

Using **conversation of angular momentum** we can write :

$$I_1\omega_1 \ = \ I_2\omega_2$$

or $\omega_2 = \frac{15.7 \times 30}{8} = 58.88\ rev/min$

Q 7.23 A man stands on a rotating platform, with his arms stretched horizontally holding a $5kg$ weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from $90cm$ to $20cm$. The moment of inertia of the man together with the platform may be taken to be constant and equal to $7.6kgm^2$.

(b) Is kinetic energy conserved in the process? If not, from where does the change come about?

Answer:

No, the kinetic is not constant. The kinetic energy increases with decrease in moment of inertia. The work done by man in folding and stretching hands is responsible for this result.

Q7.24 A bullet of mass $10g$ and speed $500m/s$ is fired into a door and gets embedded exactly at the centre of the door. The door is $1.0m$ wide and weighs $12kg$. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it.
(Hint: The moment of inertia of the door about the vertical axis at one end is $ML^2/3$.)

Answer:

The imparted angular momentum is given by :

$\alpha = mvr$

Putting all the given values in the above equation we get :

$= (10 \times 10^{-3}) \times 500 \times \frac{1}{2}$

$= 2.5\ Kg\ m^2\ s^{-1}$

Now, the moment of inertia of door is :

$I = \frac{1}{3}ML^2$

or $= \frac{1}{3}(12)1^2 = 4\ Kg\ m^2$

Also, $\alpha = I\omega$

or $\omega = \frac{\alpha}{I} = \frac{2.5}{4} = 0.625\ rad\ s^{-1}$

Q7.25 (a) Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

(a) What is the angular speed of the two-disc system?

Answer:

Let the moment of inertia of disc I and disc II be I_1 and I_2 respectively.

Similarly, the angular speed of disc I and disc II be w_1 and w_2 respectively.

So the angular momentum can be written as :

$L_1 = I_1\omega_1$ and $L_2 = I_2\omega_2$

Thus the total initial angular momentum is : $= I_1\omega_1 + I_2\omega_2$

Now when the two discs are combined the angular momentum is :

$$L_f = (I_1 + I_2)w$$

Using conservation of angular momentum :

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)w$$

Thus angular velocity is :

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

Q7.25 (b) Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

(b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take $\omega_1 \neq \omega_2$.

Answer:

The initial kinetic energy is written as :

$$K_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

or $K_i = \frac{1}{2}\left(I_1\omega_1^2 + I_2\omega_2^2\right)$

Now the final kinetic energy is :

$$K_f = \frac{1}{2}(I_1 + I_2)\,\omega^2$$

Put the value of final angular velocity from part (a).

We need to find :

Solve the above equation, we get :

$$= \frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2)$$

or $$= \frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$$

> 0 (Since none of the quantity can be negative.)

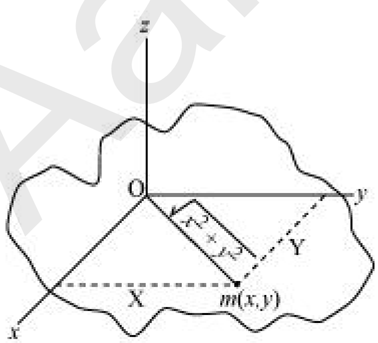
Thus initial energy is greater than the final energy. (due to frictional force).

Q7.26 (a) Prove the theorem of perpendicular axes.

(Hint: Square of the distance of a point (x, y) in the $x - y$ plane from an axis through the origin and perpendicular to the plane is $x^2 + y^2$).

Answer:

Consider the figure given below:



The moment of inertia about x axis is given by :

$$I_x = mx^2$$

And the moment of inertia about y-axis is :

$$I_y = my^2$$

Now about z-axis :

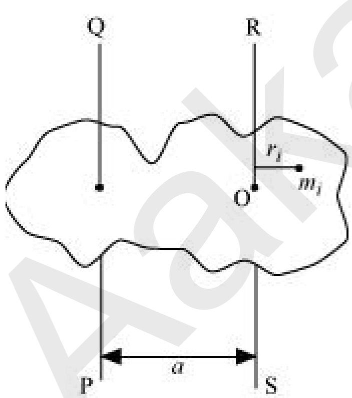
$$I_z = m\left(\sqrt{(x^2 + y^2)}\right)^2$$

or $I_z = I_x + I_y$

Q 7.26 (b) Prove the theorem of parallel axes.
(Hint : If the centre of mass of a system of n particles is chosen to be the origin $\sum m_i r_i = 0$).

Answer:

Consider the figure given below :



The moment of inertia about RS axis :-

$$I_{RS} = \sum m_i r_i^2$$

Now the moment of inertia about QP axis :-

$$I_{QP} \;=\; \sum m_i(a + r_i)^2$$

or $= \;\sum m_i(a^2 + r_i^2 + 2ar_i)$

or $= \;I_{RS} + \sum m_i a^2 + 2\sum m_i a r_i^2$

Thus $I_{QP} = \;I_{RS} + Ma^2\left(\because 2\sum m_i a r_i^2\right) \;=\; 0$

Hence proved.

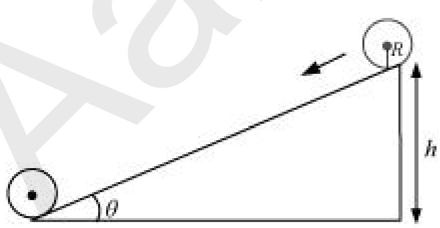
Q7.27 Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by

$$v^2 = \frac{2gh}{(1 + k^2/R^{2)}}$$

using dynamical consideration (i.e. by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

Answer:

Consider the given situation :



The total energy when the object is at the top (potential energy) = mgh.

Energy when the object is at the bottom of the plane :

$$E_b = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

Put $I = mk^2$ and $v = \omega r$, we get :

$$E_b = \frac{1}{2}mv^2\left(1 + \frac{k^2}{r^2}\right)$$

By the law of conservation of energy we can write :

$$mgh = \frac{1}{2}mv^2\left(1 + \frac{k^2}{r^2}\right)$$

or

$$v = \frac{2gh}{\left(1 + \frac{k^2}{r^2}\right)}.$$

Q7.28 A disc rotating about its axis with angular speed ω_0 is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is R . What are the linear velocities of the points A , B and C on the disc shown in Fig. 7.41? Will the disc roll in the direction indicated?

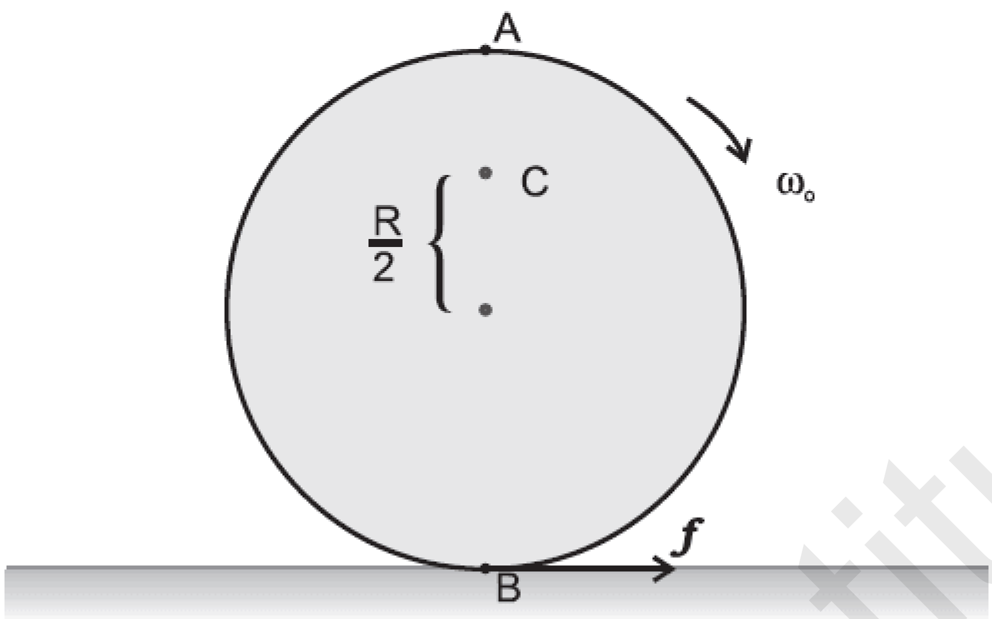


Fig. 7.41

Answer:

Let the angular speed of the disc is ω .

So the linear velocity can be written as $v = \omega r$

(a) Point A:-

The magnitude of linear velocity is ωr and it is tangentially rightward.

(b) Point B:-

The magnitude linear velocity is ωr and its direction is tangentially leftward.

(c) Point C:-

The magnitude linear velocity is $\frac{\omega r}{2}$ and its direction is rightward.

The disc cannot roll as the table is frictionless.

Q7.29 (a) Explain why friction is necessary to make the disc in Fig. 7.41 roll in the direction indicated

(a) Give the direction of frictional force at B , and the sense of frictional torque, before perfect rolling begins.

Answer:

Since the velocity at point B is tangentially leftward so the frictional force will act in the rightward direction.

The sense of frictional torque is perpendicular (outward) to the plane of the disc.

Q7.29 (b) Explain why friction is necessary to make the disc in Fig. 7.41 roll in the direction indicated.

(b) What is the force of friction after perfect rolling begins?

Answer:

Perfect rolling will occur when the velocity of the bottom point (B) will be zero. Thus the frictional force acting will be zero.

Q7.30 A solid disc and a ring, both of radius $10cm$ are placed on a horizontal table simultaneously, with initial angular speed equal to $10\Pi rad\ s^{-1}$. Which of the two will start to roll earlier ? The co-efficient of kinetic friction is $\mu_k = 0.2$.

Answer:

Friction is the cause for motion here.

So using Newton's law of motion we can write :

$f = ma$

$\mu_k mg = ma$

or $a = \mu_k g$

Now by the equation of motion, we can write :

$v = u + at$

or $v = \mu_k gt$

The torque is given by :

$\tau = I\alpha$

or $r \times f = -I\alpha$

or $\mu_k mgr = -I\alpha$

or $\alpha = -\frac{\mu_k mgr}{I}$

Now using the equation of rotational motion we can write :

$\omega = \omega_o + \alpha t$

or $\omega = \omega_o + -\frac{\mu_k mgr}{I}t$

Condition for rolling is $v = \omega r$

So we can write :

$$v = r\left(\omega_o + -\frac{\mu_k mgr}{I}t\right)$$

For ring the moment of inertia is : mr^2

So we have :

$$t = \frac{r\omega_o}{2\mu_k g}$$

or $= \dfrac{0.1 \times 10 \times 3.14}{2 \times 0.2 \times 10} = 0.80\ s$

Now in case of the disc, the moment of inertia is :

$$I = \frac{1}{2}mr^2$$

Thus $t = \dfrac{r\omega_o}{3\mu_k g}$

or $= \dfrac{0.1 \times 10 \times 3.14}{3 \times 0.2 \times 9.8} = 0.53\ s$

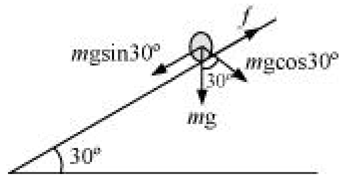
Hence disc will start rolling first.

Q7.31 (a) A cylinder of mass $10kg$ and radius $15cm$ is rolling perfectly on a plane of inclination 30^0 . The coefficient of static friction $\mu_s = 0.25$.

(a) How much is the force of friction acting on the cylinder?

Answer:

Consider the following figure :



Moment of inertia of cylinder is :

$$I = \frac{1}{2}mr^2$$

Thus acceleration is given by :

$$a = \frac{mg\sin\Theta}{m + \frac{1}{2} \times \frac{mr^2}{r^2}}$$

or $= \frac{2}{3}g\sin 30^\circ = 3.27\ m/s^2$

Now using Newton's law of motion :

$$F = ma$$

or $mg\sin 30^\circ - f = ma$

or $f = mg\sin 30^\circ - ma$

or $= 49 - 32.7 = 16.3\ N$

Hence frictional force is 16.3 N.

Q7.31 (b) A cylinder of mass $10kg$ and radius $15cm$ is rolling perfectly on a plane of inclination 30^0 . The coefficient of static friction $\mu_s = 0.25$.

(b) What is the work done against friction during rolling ?

Answer:

We know that the bottommost point of body (which is in contact with surface) is at rest during rolling. Thus work done against the frictional force is zero.

Q7.31 (c) A cylinder of mass $10kg$ and radius $15cm$ is rolling perfectly on a plane of inclination 30^0. The coefficient of static friction $\mu_s = 0.25$.

(c) If the inclination Θ of the plane is increased, at what value of Θ does the cylinder begin to skid, and not roll perfectly?

Answer:

In case of rolling without any skidding is given by :

$$\mu \ = \ \frac{1}{3}\tan\Theta$$

Thus $\tan\Theta \ = \ 3\mu$

or $= \ 3\times(0.25) \ = \ 0.75$

or $\Theta \ = \ 37.87^{\circ}$

Q7.32 (a) Read each statement below carefully, and state, with reasons, if it is true or false;

(a) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.

Answer:

False . Friction also opposes the relative motion between the contacted surfaces. In the case of rolling, the cm is moving in backward direction thus the frictional force is directed in the forward direction.

Q7.32 (b) Read each statement below carefully, and state, with reasons, if it is true or false;

(b) The instantaneous speed of the point of contact during rolling is zero.

Answer:

True. This is because the translational speed is balanced by rotational speed.

Q7.32 (c) Read each statement below carefully, and state, with reasons, if it is true or false;

(c) The instantaneous acceleration of the point of contact during rolling is zero.

Answer:

False. The value of acceleration at contact has some value, it is not zero as the frictional force is zero but the force applied will give some acceleration.

Q7.32 (d) Read each statement below carefully, and state, with reasons, if it is true or false;

(d) For perfect rolling motion, work done against friction is zero.

Answer:

True. As the frictional force at the bottommost point is zero, so the work done against it is also zero.

Q7.32 (e) Read each statement below carefully, and state, with reasons, if it is true or false;

(e) A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion.

Answer:

True. Since it is a frictionless plane so frictional force is zero thus torque is not generated. This results in slipping not rolling.

Q7.33 (a) Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass :

(a) Show $P = P_i^{'} + m_i V$

where P_i is the momentum of the ith particle (of mass m_i)and $P_t^{'} = m_t V_t$ Note $V_t^{'}$ is the velocity

of the ith particle relative to the centre of mass.Also, prove using the definition of the centre of mass $\sum P_t' = 0$

Answer:

The momentum of i th particle is given by : $p_i \ = \ m_i v_i$

The velocity of the centre of mass is V.

Then the velocity of ith particle with respect to the center of mass will be : $v_i' \ = \ v_i \ - \ v$

Now multiply the mass of the particle to both the sides, we get :

$mv_i' \ = \ mv_i \ - \ mv$

or $p_i' \ = \ p_i \ - \ p$ (Here p' $_i$ is the momentum of ith particle with respect to center of mass.)

or $p_i \ = \ p_i' \ + \ p$

Now consider p'_i :

$$\sum p'_i = \sum m_i v'_i = \sum m_i \frac{dr_i}{dt}$$

But as per the definition of centre of mass, we know that :

$$\sum m_i r'_i = 0$$

Thus $\sum p_i = 0$

Q7.33 (b) Separation of Motion of a system of particles into the motion of the centre of mass and motion about the centre of mass :

(b) show $K = K' + 1/2MV^2$ where K is the total kinetic energy of the system of particles, K' is the total kinetic energy of the system when the particle velocities are taken with respect to the centre of mass and $MV^2/2$ is the kinetic energy of the translation of the system as a whole (i.e. of the centre of mass motion of the system). The result has been used in Sec. 7.14.

Answer:

From the first part we can write :

$$\sum mv'_i = \sum mv_i - \sum mv$$

or $\sum mv_i = \sum mv'_i + \sum mv$

Squaring both sides (in vector form taking dot products with itself), we get :

$$\sum mv_i . \sum mv_i = \sum m(v'_i + v) . \sum m(v'_i + v)$$

or $M^2 \sum v_i^2 = M^2 \sum v_i'^2 + M^2 v^2$

or $\frac{1}{2}M\sum v_i^2 \;=\; \frac{1}{2}M\sum v_i'^2 \;+\; \frac{1}{2}Mv^2$

Hence $K \;=\; K' \;+\; \frac{1}{2}Mv^2$

Q7.33 (c) Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass :

(c) Show $L = L^{'} + R \times MV$
where $L = \sum r_i^{'} \times P_i^{'}$
is the angular momentum of the system about the centre of mass with velocities taken relative to the centre of mass. Remember $r_i^{'} = r_i - R$; rest of the notation is the standard notation used in the chapter. Note $L^{'}$ and $MR \times V$ can be said to be angular momenta, respectively, about and of the centre of mass of the system of particles.

Answer:

The position vector of the ith particle (with respect to the center of mass) is given by :

$r_i' \;=\; r_i \;-\; R$

or $r_i \;=\; r_i' \;+\; R$

From the first case we can write :

$p_i \;=\; p_i' \;+\; p$

Taking cross product with position vector we get ;

$\sum r_i \times p_i \;=\; \sum r_i \times p_i' \;+\; \sum r_i \times p$

or $L \;=\; L' \;+\; \sum R \times p_i' \;+\; \sum r_i \times m_i v \;+\; \sum R \times m_i v$

or $L = L^{'} + R \times MV$

7.33 (d) Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass :

(d) Show $\dfrac{dL^{'}}{dt} = \sum r_i^{'} \times \dfrac{dp^{'}}{dt}$

Further, show that

$$\frac{dL^{'}}{dt} = \tau_{ext}^{'}$$

where $\tau_{ext}^{'}$ is the sum of all external torques acting on the system about the centre of mass.

(Hint: Use the definition of centre of mass and third law of motion. Assume the internal forces between any two particles act along the line joining the particles.)

Answer:

Since we know that :

$$L' = \sum r_i' \times p_i'$$

Differentiating the equation with respect to time, we obtain :

$$\frac{dL'}{dt} = d\frac{\left(\sum r_i' \times p_i'\right)}{dt}$$

or $= \dfrac{d\left(\sum m_i r_i'\right)}{dt} \times v_i' + \sum r_i' \times \dfrac{d}{dt} p_i'$

or $\dfrac{dL'}{dt} = \sum r_i' \times m_i \dfrac{d}{dt} v_i' \left(\because \sum m_i r_i = 0\right)$

Now using Newton's law of motion we can write :

$$\sum r_i' \times m_i \frac{d}{dt} v_i' = \tau_{ext}'$$

Thus $\dfrac{dL'}{dt} = \tau_{ext}'$