NCERT solutions for class 9 maths chapter 10 Circles

Excercise: 10.1

Fill in the blanks:

Q1 (i) The centre of a circle lies in _____ of the circle. (exterior/ interior)

Answer:

The centre of a circle lies in the interior of the circle.

Fill in the blanks:

Q1 (ii) A point, whose distance from the centre of a circle is greater than its radius lies in_____ of the circle. (exterior/ interior)

Answer:

A point, whose distance from the centre of a circle is greater than its radius lies in exterior of the circle.

Fill in the blanks:

Q1 (iii) The longest chord of a circle is a _____ of the circle.

Answer:

The longest chord of a circle is a diameter of the circle.

Fill in the blanks:

Q1 (iv) An arc is a _____ when its ends are the ends of a diameter.

Answer:

An arc is a semi- circle when its ends are the ends of a diameter.

Fill in the blanks:

Q1 (v) Segment of a circle is the region between an arc and _____ of the circle.

Answer:

Segment of a circle is the region between an arc and chord of the circle.

Fill in the blanks:

Q1 (vi) A circle divides the plane, on which it lies, in _____ parts.

Answer:

A circle divides the plane, on which it lies, in two parts.

Write True or False: Give reasons for your answers.

Q2 (i) Line segment joining the centre to any point on the circle is a radius of the circle.

Answer:

True. As line segment joining the centre to any point on the circle is a radius of the circle.

Write True or False: Give reasons for your answers.

Q2 (ii) A circle has only finite number of equal chords.

Answer:

False . As a circle has infinite number of equal chords.

Write True or False: Give reasons for your answers.

Q2 (iii) If a circle is divided into three equal arcs, each is a major arc.

Answer:

False. If a circle is divided into three equal arcs, each arc makes angle of 120 degrees whereas major arc makes angle greater than 180 degree at centre.

Write True or False: Give reasons for your answers.

Q2 (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

Answer:

True.A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

Write True or False: Give reasons for your answers.

Q2 (v) Sector is the region between the chord and its corresponding arc.

Answer:

False. As the sector is the region between the radii and arc.

Write True or False: Give reasons for your answers.

Q2 (vi) A circle is a plane figure.

Answer:

True. A circle is a plane figure.

Excercise: 10.2

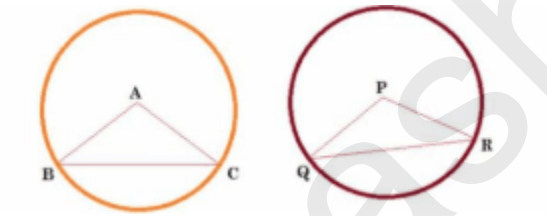
Q1 Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Answer:

Given: The two circles are congruent if they have the same radii.

To prove: The equal chords of congruent circles subtend equal angles at their centres
i.e. $\angle$ BAC= $\angle$QPR

Proof :



In $\triangle$ ABC and $\triangle$ PQR,

BC = QR (Given)

AB = PQ (Radii of congruent circle)

AC = PR (Radii of congruent circle)

Thus, $\triangle$ ABC $\cong$ $\triangle$ PQR (By SSS rule)

$\angle$ BAC= $\angle$ QPR (CPCT)

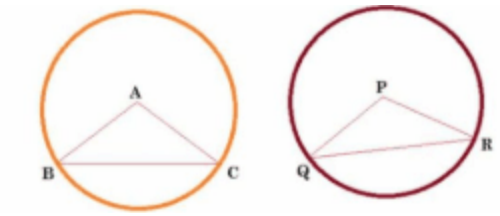
Q2 Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Answer:

Given : chords of congruent circles subtend equal angles at their centres,

To prove : BC = QR

Proof :



In $\triangle$ ABC and $\triangle$ PQR,

$\angle$ BAC= $\angle$ QPR (Given)

AB = PQ (Radii of congruent circle)

AC = PR (Radii of congruent circle)

Thus, $\triangle$ ABC $\cong$ $\triangle$ PQR (By SAS rule)

BC = QR (CPCT)

Excercise: 10.3

Q1 Draw different pairs of circles. How many points does each pair have in common? What ii the maximum number of common points?

Answer:

In (i) we do not have any common point.

In (ii) we have 1 common point.

In (iii) we have 1 common point.

In (iv) we have 2 common points.

The maximum number of common points is 2.

Q2 Suppose you are given a circle. Give a construction to find its centre.

Answer:

Given : Points P,Q,R lies on circle.

Construction :

1. Join PR and QR

2. Draw perpendicular bisector of PR and QR which intersects at point O.

3. Taking O as centre and OP as radius draw a circle.

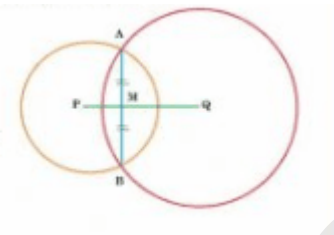
4. The circle obtained is required.

Q3 If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Answer:

Given: Two circles intersect at two points.

To prove: their centres lie on the perpendicular bisector of the common chord.



Construction: Joinpoint P and Q to midpoint M of chord AB.

Proof: AB is a chord of circle C(Q,r) and QM is the bisector of chord AB.

$\therefore PM \perp AB$

Hence, $\angle PMA = 90^\circ$

Similarly, AB is a chord of circle(Q,r') and QM is the bisector of chord AB.

$\therefore QM \perp AB$

Hence, $\angle QMA = 90^\circ$

Now, $\angle QMA + \angle PMA = 90^\circ + 90^\circ = 180^\circ$

$\angle$ PMA and $\angle$ QMA are forming linear pairs so PMQ is a straight line.

Hence, P and Q lie on the perpendicular bisector of common chord AB.

Excercise: 10.4

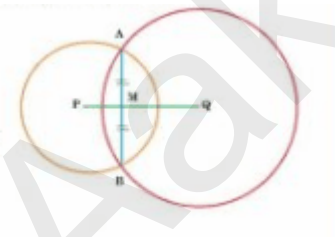
Q1 Two circles of radii $5\ cm$ and $3\ cm$ intersect at two points and the distance between their centres is $4\ cm$. Find the length of the common chord.

Answer:

Given: Two circles of radii $5\ cm$ and $3\ cm$ intersect at two points and the distance between their centres is $4\ cm$.

To find the length of the common chord.

Construction: Join OP and draw $OM \perp AB\ and\ ON \perp CD.$



Proof: AB is a chord of circle C(P,3) and PM is the bisector of chord AB.

$\therefore PM \perp AB$

$\angle PMA = 90^\circ$

Let, PM = x , so QM=4-x

In $\triangle$ APM, using Pythagoras theorem

$AM^2 = AP^2 - PM^2$ 1

Also,

In $\triangle$ AQM, using Pythagoras theorem

$AM^2 = AQ^2 - MQ^2$ 2

From 1 and 2, we get

$AP^2 - PM^2 = AQ^2 - MQ^2$

$\Rightarrow 3^2 - x^2 = 5^2 - (4-x)^2$

$\Rightarrow 9 - x^2 = 25 - 16 - x^2 + 8x$

$\Rightarrow 9 = 9 + 8x$

$\Rightarrow 8x = 0$

$\Rightarrow x = 0$

Put,x=0 in equation 1

$AM^2 = 3^2 - 0^2 = 9$

$\Rightarrow AM = 3$

$\Rightarrow AB = 2AM = 6$

Q2 If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

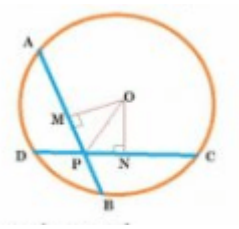
Answer:

Given: two equal chords of a circle intersect within the circle

To prove: Segments of one chord are equal to corresponding segments of the other chord i.e. AP = CP and BP=DP.

Construction : Join OP and draw $OM \perp AB \ \ and \ \ ON \perp CD.$

Proof :



In $\triangle$ OMP and $\triangle$ ONP,

AP = AP (Common)

OM = ON (Equal chords of a circle are equidistant from the centre)

$\angle$ OMP = $\angle$ ONP (Both are right angled)

Thus, $\triangle$ OMP $\cong \triangle$ ONP (By SAS rule)

PM = PN.....1 (CPCT)

AB = CD .....2(Given)

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow AM = CN$ 3

Adding 1 and 3, we have

AM + PM = CN + PN

$\Rightarrow AP = CP$

Subtract 4 from 2, we get

AB-AP = CD - CP

$\Rightarrow PB = PD$

Q3 If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

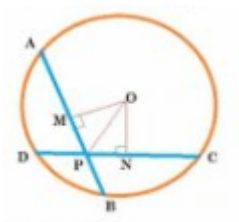
Answer:

Given: two equal chords of a circle intersect within the circle.

To prove: the line joining the point of intersection to the centre makes equal angles with the chords.

i.e. $\angle$ OPM= $\angle$ OPN

Proof :



Construction: Join OP and draw $OM \perp AB \;\; and \;\; ON \perp CD.$

In $\triangle$ OMP and $\triangle$ ONP,

AP = AP (Common)

OM = ON (Equal chords of a circle are equidistant from the centre)

$\angle$ OMP = $\angle$ ONP (Both are right-angled)

Thus, $\triangle$ OMP $\cong$ $\triangle$ ONP (By RHS rule)

$\angle$ OPM= $\angle$ OPN (CPCT)

Q4 If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$ (see Fig. 10.25).

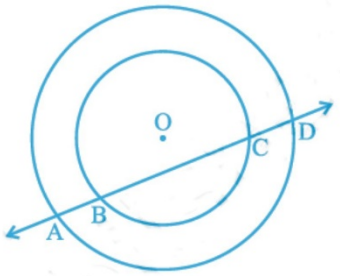


Fig. 10.25

Answer:

Given: a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D.

To prove : AB = CD

Construction: Draw $OM \perp AD$

Proof :

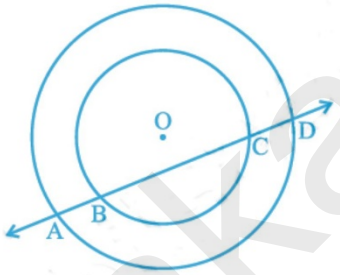


Fig. 10.25

BC is a chord of the inner circle and $OM \perp BC$

So, BM = CM 1

(Perpendicular OM bisect BC)

Similarly,

AD is a chord of the outer circle and $OM \perp AD$

So, AM = DM 2

(Perpendicular OM bisect AD)

Subtracting 1 from 2, we get

AM-BM = DM - CM

$\Rightarrow AB = CD$

Q5 Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius $5m$ drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is $6m$ each, what is the distance between Reshma and Mandip?

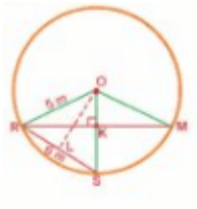
Answer:

Given: From the figure, R, S, M are the position of Reshma, Salma, Mandip respectively.

So, RS = SM = 6 cm

Construction : Join OR,OS,RS,RM and OM.Draw $OL \perp RS$.

Proof:



In $\triangle$ ORS,

OS = OR and $OL \perp RS$ (by construction)

So, RL = LS = 3cm (RS = 6 cm)

In $\triangle$ OLS, by pytagoras theorem,

$$OL^2 = OS^2 - SL^2$$

$$\Rightarrow OL^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$$\Rightarrow OL = 4$$

In $\triangle$ ORK and $\triangle$ OMK,

OR = OM (Radii)

$\angle$ ROK = $\angle$ MOK (Equal chords subtend equal angle at centre)

OK = OK (Common)

$\triangle$ ORK $\cong$ $\triangle$ OMK (By SAS)

RK = MK (CPCT)

Thus, $OK \perp RM$

area of $\triangle$ ORS = $\frac{1}{2} \times RS \times OL$ 1

area of $\triangle$ ORS = $\frac{1}{2} \times OS \times KR$ 2

From 1 and 2, we get

$$\frac{1}{2} \times RS \times OL = \frac{1}{2} \times OS \times KR$$

$$\Rightarrow RS \times OL = OS \times KR$$

$$\Rightarrow 6 \times 4 = 5 \times KR$$

$$\Rightarrow KR = 4.8cm$$

Thus, $RM = 2KR = 2 \times 4.8cm = 9.6cm$

Q6 A circular park of radius $20m$ is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Answer:

Given: In the figure, A, S, D are positioned Ankur, Syed and David respectively.

So, AS = SD = AD

Radius of circular park = 20 m

so, AO=SO=DO=20 m

Construction: AP $\perp$ SD

Proof :

Let AS = SD = AD = 2x cm

In $\triangle$ ASD,

AS = AD and AP $\perp$ SD

So, SP = PD = x cm

In $\triangle$ OPD, by Pythagoras,

$$OP^2 = OD^2 - PD^2$$

$$\Rightarrow OP^2 = 20^2 - x^2 = 400 - x^2$$

$$\Rightarrow OP = \sqrt{400 - x^2}$$

In $\triangle$ APD, by Pythagoras,

$$AP^2 = AD^2 - PD^2$$

$$\Rightarrow (AO + OP)^2 + x^2 = (2x)^2$$

$\Rightarrow (20+\sqrt{400-x^2})^2+x^2=4x^2$

$\Rightarrow 400+400-x^2+40\sqrt{400-x^2}+x^2=4x^2$

$\Rightarrow 800+40\sqrt{400-x^2}=4x^2$

$\Rightarrow 200+10\sqrt{400-x^2}=x^2$

$\Rightarrow 10\sqrt{400-x^2}=x^2-200$

Squaring both sides,

$\Rightarrow 100(400-x^2)=(x^2-200)^2$

$\Rightarrow 40000-100x^2=x^4-40000-400x^2$

$\Rightarrow x^4-300x^2=0$

$\Rightarrow x^2(x^2-300)=0$

$\Rightarrow x^2=300$

$\Rightarrow x=10\sqrt{3}$

Hence, length of string of each phone $=2x=20\sqrt{3}$ m

Excercise: 10.5

Q1 In Fig. 10.36 , A,B and C are three points on a circle with centre O such that $\angle BOC=30^\circ$ and $\angle AOB=60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.

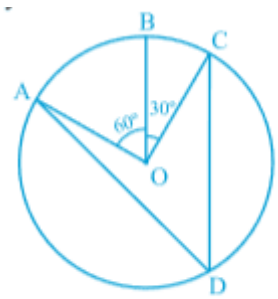


Fig. 10.36

Answer:

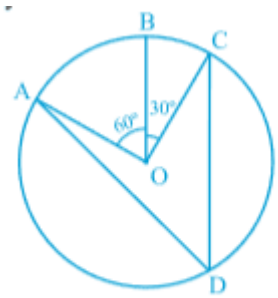


Fig. 10.36

$\angle$ AOC = $\angle$ AOB + $\angle$ BOC= $60^\circ + 30^\circ = 90^\circ$

$\angle$ AOC = 2 $\angle$ ADC (angle subtended by an arc at the centre is double the angle subtended by it at any)

$$\angle ADC = \frac{1}{2}\angle AOC$$

$$\Rightarrow \angle ADC = \frac{1}{2}90^\circ = 45^\circ$$

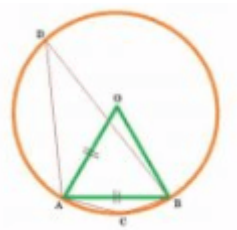
Q2 A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Answer:

Given: A chord of a circle is equal to the radius of the circle i.e. OA=OB.

To find: ADB and $\angle$ ACB.

Solution :



In $\triangle$ OAB,

OA = AB (Given)

OA = OB (Radii of circle)

So, OA=OB=AB

$\Rightarrow$ ABC is a equilateral triangle.

So, $\angle$ AOB = 60°

$\angle$ AOB = 2 $\angle$ ADB

$$\Rightarrow \angle ADB = \frac{1}{2}\angle AOB$$

$$\Rightarrow \angle ADB = \frac{1}{2}60^\circ = 30$$

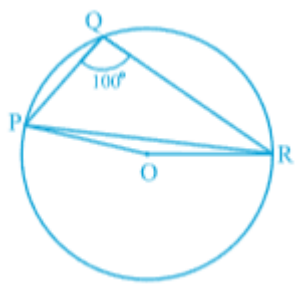
ACBD is a cyclic quadrilateral .

So, $\angle$ ACB+ $\angle$ ADB = 180°

$\Rightarrow \angle ACB + 30^\circ = 180^\circ$

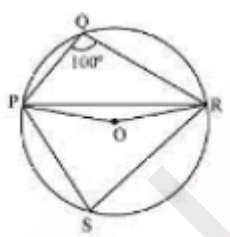
$\Rightarrow \angle ACB = 180^\circ - 30^\circ = 150^\circ$

Q3 In Fig. 10.37 , $\angle PQR = 100^\circ$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.



Answer:

Construction: Join PS and RS.



PQRS is a cyclic quadrilateral.

So, $\angle$ PSR + $\angle$ PQR = 180°

$\Rightarrow \angle PSR + 100^\circ = 180^\circ$

$\Rightarrow \angle PSR = 180^\circ - 100^\circ = 80^\circ$

Here, $\angle$ POR = 2 $\angle$ PSR

$\Rightarrow \angle POR = 2 \times 80^\circ = 160^\circ$

In $\triangle$ OPR ,

OP=OR (Radii)

$\angle$ ORP = $\angle$ OPR (the angles opposite to equal sides)

In $\triangle$ OPR ,

$\angle$ OPR+ $\angle$ ORP+ $\angle$ POR= 180°

$\Rightarrow 2\angle OPR + 160^\circ = 180^\circ$

$\Rightarrow 2\angle OPR = 180^\circ - 160^\circ$

$\Rightarrow 2\angle OPR = 20^\circ$

$\Rightarrow \angle OPR = 10^\circ$

Q4 In Fig. 10.38 , $\angle ABC = 69^\circ, \angle ACB = 31^\circ$, find $\angle BDC$

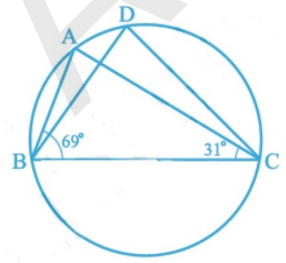


Fig. 10.38

Answer:

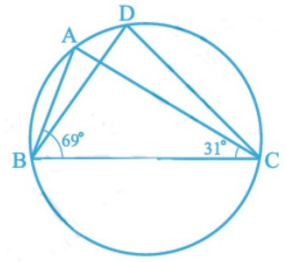


Fig. 10.38

In $\triangle$ ABC,

$\angle$ A+ $\angle$ ABC+ $\angle$ ACB= 180°

$\Rightarrow \angle A + 69^\circ + 31^\circ = 180^\circ$

$\Rightarrow \angle A + 100^\circ = 180^\circ$

$\Rightarrow \angle A = 180^\circ - 100^\circ$

$\Rightarrow \angle A = 80^\circ$

$\angle$ A = $\angle$ BDC = 80° (Angles in same segment)

Q5 In Fig. 10.39 , A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$

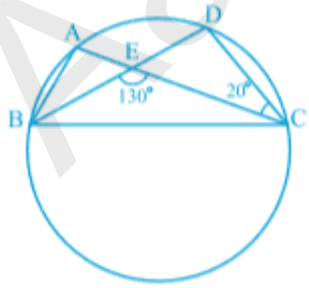


Fig. 10.39

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Answer:

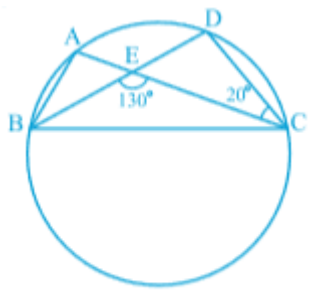


Fig. 10.39

$\angle$ DEC+ $\angle$ BEC = 180° (linear pairs)

$\Rightarrow \angle$ DEC+ $130^\circ = 180^\circ$ ($\angle$ BEC = 130°)

$\Rightarrow \angle$ DEC = 180° - 130°

$\Rightarrow \angle$ DEC = 50°

In $\triangle$ DEC,

$\angle$ D+ $\angle$ DEC+ $\angle$ DCE = 180°

$\Rightarrow \angle D + 50^\circ + 20^\circ = 180^\circ$

$\Rightarrow \angle D + 70^\circ = 180^\circ$

$\Rightarrow \angle D = 180^\circ - 70^\circ = 110^\circ$

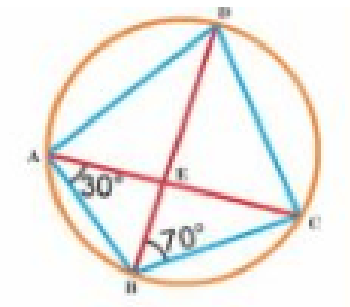
$\angle$ D = $\angle$ BAC (angles in same segment are equal)

$\angle$ BAC = 110°

Q6 ABCD is a cyclic quadrilateral whose diagonals intersect at a point E.

If $\angle DBC = 70^\circ$, $\angle BAC$ is 30°, find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Answer:



$\angle BDC = \angle BAC$ (angles in the same segment are equal)

$\angle BDC = 30^\circ$

In $\triangle BDC$,

$\angle BCD + \angle BDC + \angle DBC = 180^\circ$

$\Rightarrow \angle BCD + 30^\circ + 70^\circ = 180^\circ$

$\Rightarrow \angle BCD + 100^\circ = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 100^\circ = 80^\circ$

If AB = BC ,then

$\angle BCA = \angle BAC$

$\Rightarrow \angle BCA = 30^\circ$

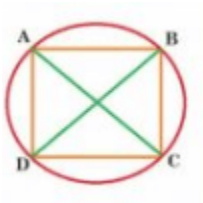
Here, $\angle ECD + \angle BCE = \angle BCD$

$\Rightarrow \angle ECD + 30^\circ = 80^\circ$

$\Rightarrow \angle ECD = 80^\circ - 30^\circ = 50^\circ$

Q7 If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Answer:



AC is the diameter of the circle.

Thus, $\angle ADC = 90^\circ$ and $\angle ABC = 90^\circ$ .....1(Angle in a semi-circle is right angle)

Similarly, BD is the diameter of the circle.

Thus, $\angle BAD = 90^\circ$ and $\angle BCD = 90^\circ$ .....2(Angle in a semi-circle is right angle)

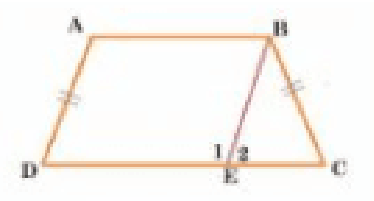
From 1 and 2, we get

$\angle BCD = \angle ADC = \angle ABC = \angle BAD = 90^\circ$

Hence, ABCD is a rectangle.

Q8 If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Answer:



Given: ABCD is a trapezium.

Construction: Draw AD || BE.

Proof: In quadrilateral ABED,

AB || DE (Given)

AD || BE (By construction)

Thus, ABED is a parallelogram.

AD = BE (Opposite sides of parallelogram)

AD = BC (Given)

so, BE = BC

In $\triangle$ EBC,

BE = BC (Proved above)

Thus, $\angle C = \angle 2$ .....1(angles opposite to equal sides)

$\angle A = \angle 1$ .....2(Opposite angles of the parallelogram)

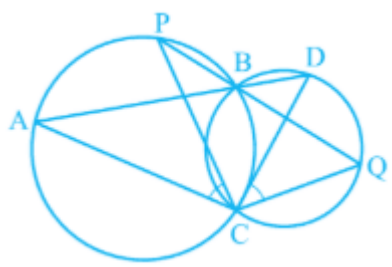
From 1 and 2, we get

$\angle 1 + \angle 2 = 180^\circ$ (linear pair)

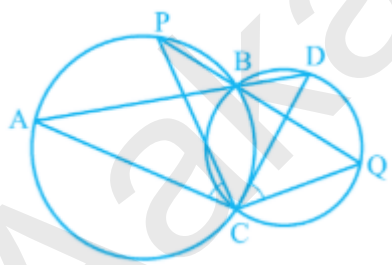
$\Rightarrow \angle A + \angle C = 180^\circ$

Thus, ABED is a cyclic quadrilateral.

Q9 Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 10.40). Prove that $\angle ACP = \angle QCD$.



Answer:



$\angle ABP = \angle QBD$ .....1(vertically opposite angles)

$\angle ACP = \angle ABP$ .....2(Angles in the same segment are equal)

$\angle QBD = \angle QCD$ .....3(angles in the same segment are equal)

From 1,2,3 ,we get

$\angle ACP = \angle QCD$

Q10 If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Answer:

Given: circles are drawn taking two sides of a triangle as diameters.

Construction: Join AD.

Proof: AB is the diameter of the circle and $\angle$ ADB is formed in a semi-circle.

$\angle$ ADB = 90° .....1(angle in a semi-circle)

Similarly,

AC is the diameter of the circle and $\angle$ ADC is formed in a semi-circle.

$\angle$ ADC = 90° .....2(angle in a semi-circle)

From 1 and 2, we have

$\angle$ ADB+ $\angle$ ADC= $90^\circ + 90^\circ = 180^\circ$

$\angle$ ADB and $\angle$ ADC are forming a linear pair. So, BDC is a straight line.

Hence, point D lies on this side.

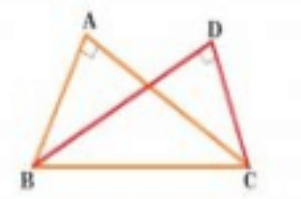
Q11 ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Answer:

Given: ABC and ADC are two right triangles with common hypotenuse AC.

To prove : $\angle CAD = \angle CBD$

Proof :



Triangle ABC and ADC are on common base BC and $\angle$ BAC = $\angle$ BDC.

Thus, point A,B,C,D lie in the same circle.

(If a line segment joining two points subtends equal angles at two other points lying on the same side of line containing line segment, four points lie on the circle.)

$\angle$ CAD = $\angle$ CBD (Angles in same segment are equal)

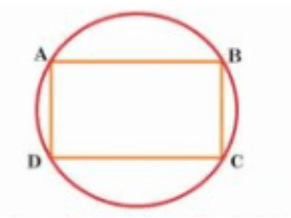
Q12 Prove that a cyclic parallelogram is a rectangle.

Answer:

Given: ABCD is a cyclic quadrilateral.

To prove: ABCD is a rectangle.

Proof :



In cyclic quadrilateral ABCD.

$\angle A + \angle C = 180^\circ$ .....1(sum of either pair of opposite angles of a cyclic quadrilateral)

$\angle A = \angle C$ .....2(opposite angles of a parallelogram are equal)

From 1 and 2,

$\angle A + \angle A = 180^\circ$

$\Rightarrow 2\angle A = 180^\circ$

$\Rightarrow \angle A = 90^\circ$

We know that a parallelogram with one angle right angle is a rectangle.

Hence, ABCD is a rectangle.

Excercise: 10.6

Q1 Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Answer:

Given: Circle C(P,r) and circle C(Q,r') intersect each other at A and B.

To prove : $\angle$ PAQ = $\angle$ PBQ

Proof : In $\triangle$ APQ and $\triangle$ BPQ,

PA = PB (radii of same circle)

PQ = PQ (Common)

QA = QB (radii of same circle)

So, $\triangle$ APQ $\cong$ $\triangle$ BPQ (By SSS)

$\angle$ PAQ = $\angle$ PBQ (CPCT)

Q2 Two chords AB and CD of lengths $5\ cm$ and $11\ cm$ respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is $6\ cm$, find the radius of the circle.

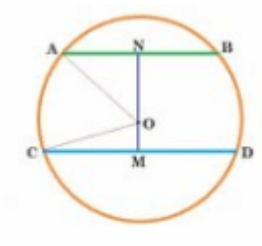
Answer:

Given : AB = 5 cm, CD = 11 cm and AB || CD.

To find Radius (OA).

Construction: Draw $OM \perp CD \text{ and } ON \perp AB$

Proof :



Proof: CD is a chord of circle and $OM \perp CD$

Thus, CM = MD = 5.5 cm (perpendicular from centre bisects chord)

and AN = NB = 2.5 cm

Let OM be x.

So, ON = 6 - x (MN = 6 cm)

In $\triangle$ OCM , using Pythagoras,

$OC^2 = CM^2 + OM^2$.....1

and

In $\triangle$ OAN , using Pythagoras,

$OA^2 = AN^2 + ON^2$.....2

From 1 and 2,

$CM^2 + OM^2 = AN^2 + ON^2$ (OC=OA =radii)

$5.5^2 + x^2 = 2.5^2 + (6 - x)^2$

$\Rightarrow 30.25 + x^2 = 6.25 + 36 + x^2 - 12x$

$\Rightarrow 30.25 - 42.25 = -12x$

$\Rightarrow -12 = -12x$

$\Rightarrow x = 1$

From 2, we get

$OC^2 = 5.5^2 + 1^2 = 30.25 + 1 = 31.25$

$\Rightarrow OC = \frac{5}{2}\sqrt{5}cm$

OA = OC

Thus, the radius of the circle is $\frac{5}{2}\sqrt{5}cm$

Q3 The lengths of two parallel chords of a circle are $6\ cm$ and $8\ cm$. If the smaller chord is at distance $4\ cm$ from the centre, what is the distance of the other chord from the centre?

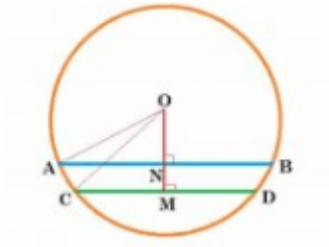
Answer:

Given : AB = 8 cm, CD = 6 cm , OM = 4 cm and AB || CD.

To find: Length of ON

Construction: Draw $OM \perp CD\ and\ \ ON \perp AB$

Proof :



Proof: CD is a chord of circle and $OM \perp CD$

Thus, CM = MD = 3 cm (perpendicular from centre bisects chord)

and AN = NB = 4 cm

Let MN be x.

So, ON = 4 - x (MN = 4 cm)

In $\triangle$ OCM , using Pythagoras,

$OC^2 = CM^2 + OM^2$ 1

and

In $\triangle$ OAN , using Pythagoras,

$OA^2 = AN^2 + ON^2$ 2

From 1 and 2,

$CM^2 + OM^2 = AN^2 + ON^2$ (OC=OA =radii)

$\Rightarrow 3^2 + 4^2 = 4^2 + (4 - x)^2$

$\Rightarrow 9 + 16 = 16 + 16 + x^2 - 8x$

$\Rightarrow 9 = 16 + x^2 - 8x$

$\Rightarrow x^2 - 8x + 7 = 0$

$\Rightarrow x^2 - 7x - x + 7 = 0$

$\Rightarrow x(x-7) - 1(x-7) = 0$

$\Rightarrow (x-1)(x-7) = 0$

$\Rightarrow x = 1, 7$

So, x=1 (since $x \neq 7 > OM$)

ON =4-x =4-1=3 cm

Hence, second chord is 3 cm away from centre.

Q4 Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

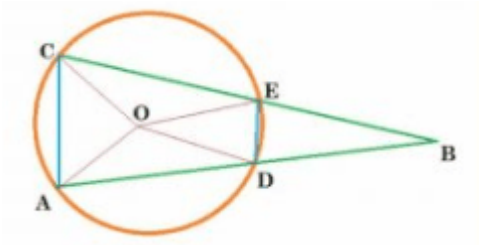
Answer:

Given : AD = CE

To prove : $\angle ABC = \frac{1}{2}(\angle AOC - \angle DOE)$

Construction: Join AC and DE.

Proof :



Let $\angle$ ADC = x , $\angle$ DOE = y and $\angle$ AOD = z

So, $\angle$ EOC = z (each chord subtends equal angle at centre)

$\angle$ AOC + $\angle$ DOE + $\angle$ AOD + $\angle$ EOC = 360°

$\Rightarrow x + y + z + z = 360^\circ$

$\Rightarrow x + y + 2z = 360^\circ$ 1

In $\triangle$ OAD ,

OA = OD (Radii of the circle)

$\angle$ OAD = $\angle$ ODA (angles opposite to equal sides)

$\angle$ OAD + $\angle$ ODA + $\angle$ AOD = 180°

$\Rightarrow 2\angle OAD + z = 180^\circ$

$\Rightarrow 2\angle OAD = 180^\circ - z$

$\Rightarrow \angle OAD = \dfrac{180^\circ - z}{2}$

$\Rightarrow \angle OAD = 90^\circ - \dfrac{z}{2}$ 2

Similarly,

$\Rightarrow \angle OCE = 90^\circ - \frac{x}{2}$ 3

$\Rightarrow \angle OED = 90^\circ - \frac{y}{2}$ 4

$\angle$ ODB is exterior of triangle OAD . So,

$\angle$ ODB = $\angle$ OAD + $\angle$ ODA

$\Rightarrow \angle ODB = 90^\circ - \frac{z}{2} + z$ (from 2)

$\Rightarrow \angle ODB = 90^\circ + \frac{z}{2}$ 5

similarly,

$\angle$ OBE is exterior of triangle OCE . So,

$\angle$ OBE = $\angle$ OCE + $\angle$ OEC

$\Rightarrow \angle OEB = 90^\circ - \frac{z}{2} + z$ (from 3)

$\Rightarrow \angle OEB = 90^\circ + \frac{z}{2}$ 6

From 4,5,6 ;we get

$\angle$ BDE = $\angle$ BED = $\angle$ OEB - $\angle$ OED

$\Rightarrow \angle BDE = \angle BED = 90^\circ + \frac{z}{2} - (90 - \frac{y}{2}) = \frac{y+z}{2}$

$\Rightarrow \angle BDE + \angle BED = y + z$ 7

In $\triangle$ BDE ,

$\angle$ DBE + $\angle$ BDE + $\angle$ BED = 180°

$\Rightarrow \angle DBE + y + z = 180^\circ$

$\Rightarrow \angle DBE = 180^\circ - (y + z)$

$\Rightarrow \angle ABC = 180^\circ - (y + z)$ 8

Here, from equation 1,

$$\frac{x-y}{2} = \frac{360^\circ - y - 2x - y}{2}$$

$$\Rightarrow \frac{x-y}{2} = \frac{360^\circ - 2y - 2x}{2}$$

$$\Rightarrow \frac{x-y}{2} = 180^\circ - y - x$$ 9

From 8 and 9,we have

$$\angle ABC = \frac{x-y}{2} = \frac{1}{2}(\angle AOC - \angle DOE)$$

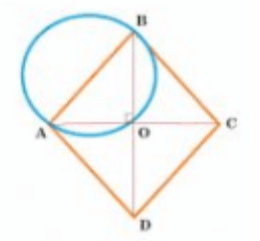
Q5 Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Answer:

Given : ABCD is rhombus.

To prove : the circle drawn with AB as diameter, passes through the point O.

Proof :



ABCD is rhombus.

Thus, $\angle AOC = 90^\circ$ (diagonals of a rhombus bisect each other at 90°)

So, a circle drawn AB as diameter will pass through point O.

Thus, the circle is drawn with any side of a rhombus as diameter passes through the point of intersection of its diagonals.

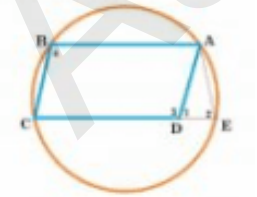
Q6 ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that $AE = AD$.

Answer:

Given: ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E.

To prove : AE = AD

Proof :



$\angle$ ADC = $\angle$ 3 , $\angle$ ABC = $\angle$ 4, $\angle$ ADE = $\angle$ 1 and $\angle$ AED = $\angle$ 2

$\angle 3 + \angle 1 = 180^\circ$ .....1(linear pair)

$\angle 2 + \angle 4 = 180^\circ$ .....2(sum of opposite angles of cyclic quadrilateral)

$\angle 3 = \angle 4$ (oppsoite angles of parallelogram)

From 1 and 2,

$\angle 3 + \angle 1 = \angle 2 + \angle 4$

From 3, $\angle 1 = \angle 2$

From 4, $\triangle$ AQB, $\angle 1 = \angle 2$

Therefore, AE = AD (In an isosceles triangle ,angles oppsoite to equal sides are equal)

Q7 (i) AC and BD are chords of a circle which bisect each other. Prove that AC and BD are diameters

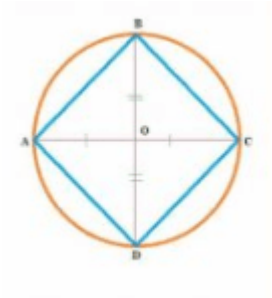
Answer:

Given: AC and BD are chords of a circle which bisect each other.

To prove: AC and BD are diameters.

Construction : Join AB,BC,CD,DA.

Proof :



In $\triangle$ ABD and $\triangle$ CDO,

AO = OC (Given)

$\angle$ AOB = $\angle$ COD (Vertically opposite angles)

BO = DO (Given)

So, $\triangle$ ABD $\cong$ $\triangle$ CDO (By SAS)

$\angle$ BAO = $\angle$ DCO (CPCT)

$\angle$ BAO and $\angle$ DCO are alternate angle and are equal .

So, AB || DC 1

Also AD || BC 2

From 1 and 2,

$\angle A + \angle C = 180^\circ$ .....3(sum of opposite angles)

$\angle$ A = $\angle$ C .....4(Opposite angles of the parallelogram)

From 3 and 4,

$\angle A + \angle A = 180^\circ$

$\Rightarrow 2\angle A = 180^\circ$

$\Rightarrow \angle A = 90^\circ$

BD is a diameter of the circle.

Similarly, AC is a diameter.

Q7 (ii) AC and BD are chords of a circle which bisect each other. Prove that ABCD is a rectangle.

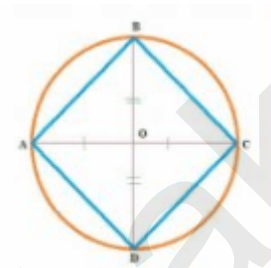
Answer:

Given: AC and BD are chords of a circle which bisect each other.

To prove: ABCD is a rectangle.

Construction : Join AB,BC,CD,DA.

Proof :



ABCD is a parallelogram. (proved in (i))

$\angle A = 90^\circ$ (proved in (i))

A parallelogram with one angle 90° , is a rectangle)

Thus, ABCD is rectangle.

Q8 Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90^\circ - \frac{1}{2}C$, $90^\circ - \frac{1}{2}B$ and $90^\circ - \frac{1}{2}A$

Answer:

Given : Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively.

To prove : the angles of the triangle DEF are $90^\circ - \frac{1}{2}C$, $90^\circ - \frac{1}{2}B$ and $90^\circ - \frac{1}{2}A$

Proof :

$\angle 1$ and $\angle 3$ are angles in same segment.therefore,

$\angle 1 = \angle 3$ .....1(angles in same segment are equal)

and $\angle 2 = \angle 4$ 2

Adding 1 and 2,we have

$\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow \angle D = \frac{1}{2}\angle B + \frac{1}{2}\angle C$,

$\Rightarrow \angle D = \frac{1}{2}(\angle B + \angle C)$

$\Rightarrow \angle D = \frac{1}{2}(180^\circ + \angle C)$

and $\Rightarrow \angle D = \frac{1}{2}(180^\circ - \angle A)$

$\Rightarrow \angle D = 90^\circ - \frac{1}{2}\angle A$

Similarly, $\Rightarrow \angle E = 90^\circ - \frac{1}{2}\angle B$ and $\angle F = 90^\circ - \frac{1}{2}\angle C$

Q9 Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that $BP = BQ$.

Answer:

Given: Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles.

To prove : BP = BQ

Proof :

AB is a common chord in both congruent circles.

$\therefore \angle APB = \angle AQB$

In $\triangle BPQ$,

$\angle APB = \angle AQB$

$\therefore BQ = BP$ (Sides opposite to equal of the triangle are equal)

Q10 In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

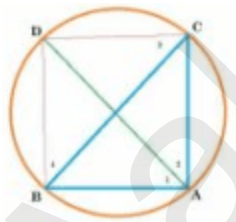
Answer:

Given :In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect.

To prove : D lies on perpendicular bisector BC.

Construction: Join BD and DC.

Proof :



Let $\angle$ ABD = $\angle$ 1 , $\angle$ ADC = $\angle$ 2 , $\angle$ DCB = $\angle$ 3 , $\angle$ CBD = $\angle$ 4

$\angle$ 1 and $\angle$ 3 lies in same segment.So,

$\angle$ 1 = $\angle$ 3 .....1(angles in same segment)

similarly, $\angle 2 = \angle 4$ 2

also, $\angle 1 = \angle 2$ .....3(given)

From 1,2,3 , we get

$\angle 3 = \angle 4$

Hence, BD = DC (angles opposite to equal sides are equal)

All points lying on perpendicular bisector BC will be equidistant from B and C.

Thus, point D also lies on perpendicular bisector BC.