

## NCERT solutions for class 9 maths chapter 12 Heron's Formula

### Exercise: 12.1

**Q1** A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side '  $a$  '. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

**Answer:**

Given the perimeter of an equilateral triangle is 180cm.

So,  $3a = 180 \text{ cm}$  or  $a = 60 \text{ cm}$ .

Hence, the length of the side is 60cm.

Now,

Calculating the area of the signal board by the Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Where,  $s$  is the half-perimeter of the triangle and  $a$ ,  $b$  and  $c$  are the sides of the triangle.

Therefore,

$$s = \frac{1}{2} \text{Perimeter} = \frac{1}{2} 180 \text{ cm} = 90 \text{ cm}$$

$a = b = c = 60 \text{ cm}$  as it is an equilateral triangle.

Substituting the values in the Heron's formula, we obtain

$$\Rightarrow A = \sqrt{90(90-60)(90-60)(90-60)} = 900\sqrt{3} \text{ cm}^2.$$

**Q2** The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig. 12.9). The advertisements yield an earning of Rs. 5000 per  $m^2$  per year. A company hired one of its walls for 3 months. How much rent did it pay?

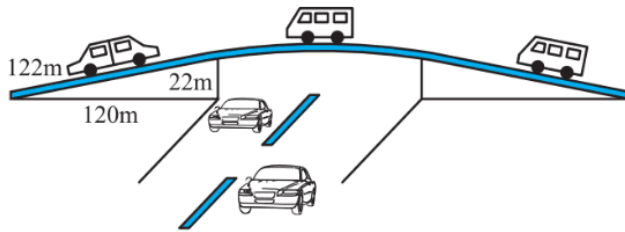


Fig. 12.9

**Answer:**

From the figure,

The sides of the triangle are:

$$a = 122m, b = 120m \text{ and } c = 22m$$

The semi perimeter,  $s$  will be

$$s = \frac{a + b + c}{2} = \frac{122 + 120 + 22}{2} = \frac{264}{2} = 132m$$

Therefore, the area of the triangular side wall will be calculated by the Heron's Formula,

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

$$= \sqrt{132(132 - 122)(132 - 120)(132 - 22)} \text{ } m^2$$

$$= \sqrt{132(10)(12)(110)} \text{ m}^2$$

$$= \sqrt{(12 \times 11)(10)(12)(11 \times 10)} \text{ m}^2 = 1320 \text{ m}^2$$

Given the rent for 1 year (i.e., 12 months) per meter square is Rs. 5000.

Rent for 3 months per meter square will be:

$$\text{Rs. } 5000 \times \frac{3}{12}$$

Therefore, for 3 months for  $1320 \text{ m}^2$ :

$$\text{Rs. } 5000 \times \frac{3}{12} \times 1320 = \text{Rs. } 16,50,000.$$

**Q3** There is a slide in a park. One of its side walls has been painted in some colour with a message “KEEP THE PARK GREEN AND CLEAN” (see Fig. 12.10 ). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

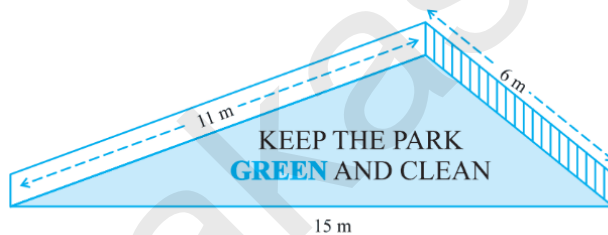


Fig. 12.10

**Answer:**

Given the sides of the triangle are:

$a = 15m$ ,  $b = 11m$  and  $c = 6m$ .

So, the semi perimeter of the triangle will be:

$$s = \frac{a + b + c}{2} = \frac{15 + 11 + 6}{2} = \frac{32}{2} = 16m$$

Therefore, Heron's formula will be:

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{16(16-15)(16-11)(16-6)} \\ &= \sqrt{16(1)(5)(10)} \\ &= \sqrt{(4 \times 4)(1)(5)(5 \times 2)} \\ &= 4 \times 5\sqrt{2} = 20\sqrt{2} \text{ m}^2 \end{aligned}$$

Hence, the area painted in colour is  $20\sqrt{2} \text{ m}^2$ .

**Q4** Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

**Answer:**

Given the perimeter of the triangle is 42cm and the sides length  $a = 18cm$  and  $b = 10cm$

So,  $a + b + c = 42cm$

Or,  $c = 42 - 18 - 10 = 14cm$

So, the semi perimeter of the triangle will be:

$$s = \frac{P}{2} = \frac{42cm}{2} = 21cm$$

Therefore, the area given by the Heron's Formula will be,

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-18)(21-10)(21-14)} \\ &= \sqrt{(7 \times 3)(3)(11)(7)} \\ &= 21\sqrt{11} \text{ cm}^2 \end{aligned}$$

Hence, the area of the triangle is  $21\sqrt{11} \text{ cm}^2$ .

**Q5** Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.

**Answer:**

Given the sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm

Let us consider the length of one side of the triangle be  $a = 12x$

Then, the remaining two sides are  $b = 17x$  and  $c = 25x$ .

So, by the given perimeter, we can find the value of x:

$$\text{Perimeter} = a + b + c = 12x + 17x + 25x = 540cm$$

$$\Rightarrow 54x = 540cm$$

$$\Rightarrow x = 10$$

So, the sides of the triangle are:

$$a = 12 \times 10 = 120cm$$

$$b = 17 \times 10 = 170cm$$

$$c = 25 \times 10 = 250cm$$

So, the semi perimeter of the triangle is given by

$$s = \frac{540cm}{2} = 270cm$$

Therefore, using Heron's Formula, the area of the triangle is given by

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{270(270-120)(270-170)(270-250)} \\ &= \sqrt{270(150)(100)(20)} \\ &= \sqrt{81000000} = 9000cm^2 \end{aligned}$$

**Hence, the area of the triangle is  $9000cm^2$ .**

**Q6** An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm.  
Find the area of the triangle.

**Answer:**

The perimeter of an isosceles triangle is 30 cm (Given).

The length of the sides which are equal is 12 cm.

Let the third side length be 'a cm'.

Then,  $Perimeter = a + b + c$

$$\Rightarrow 30 = a + 12 + 12$$

$$\Rightarrow a = 6cm$$

So, the semi-perimeter of the triangle is given by,

$$s = \frac{1}{2} Perimeter = \frac{1}{2} \times 30cm = 15cm$$

Therefore, using Herons' Formula, calculating the area of the triangle

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15(15-6)(15-12)(15-12)}$$

$$= \sqrt{15(9)(3)(3)}$$

$$= 9\sqrt{15} \text{ cm}^2$$

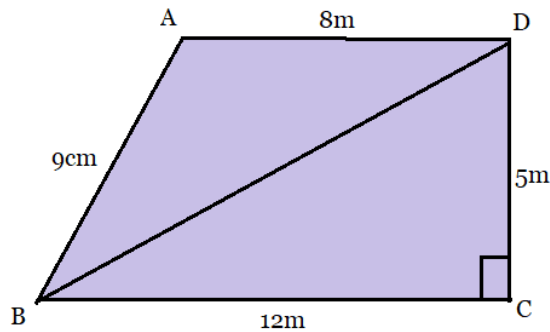
Hence, the area of the triangle is  $9\sqrt{15}cm^2$ .

### Excercise: 12.2

**Q1** A park, in the shape of a quadrilateral ABCD, has  $\angle C = 90^\circ$ ,  $AB = 9m$ ,  $BC = 12m$ ,  $CD = 5m$  and  $AD = 8m$ . How much area does it occupy?

**Answer:**

From the figure:



We have joined the BD to form two triangles so that the calculation of the area will be easy.

In triangle BCD, by Pythagoras theorem

$$BD^2 = BC^2 + CD^2$$

$$\Rightarrow BD^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$\Rightarrow BD = 13 \text{ cm}$$

The area of triangle BCD can be calculated by,

$$Area_{(BCD)} = \frac{1}{2} \times BC \times DC = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$$

and the area of the triangle DAB can be calculated by Heron's Formula:

So, the semi-perimeter of the triangle DAB,

$$s = \frac{a + b + c}{2} = \frac{9 + 8 + 13}{2} = \frac{30}{2} = 15 \text{ cm}$$

Therefore, the area will be:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where,  $a = 9\text{cm}$ ,  $b = 8\text{cm}$  and  $c = 13\text{cm}$ .

$$= \sqrt{15(15-9)(15-8)(15-13)}$$

$$= \sqrt{12(6)(7)(2)} = \sqrt{1260} = 35.5 \text{ cm}^2 \text{ (Approximately)}$$

Then, the total park area will be:

$$= \text{Area of triangle } BCD + \text{Area of triangle } DAB$$

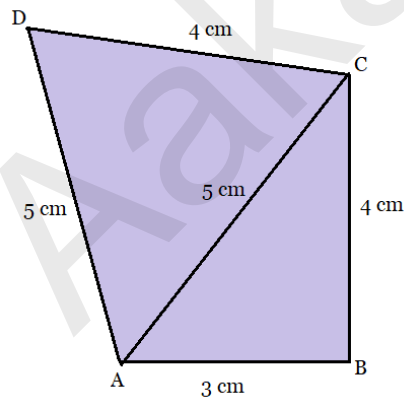
$$\Rightarrow \text{Total area of Park} = 30 + 35.35 = 65.5 \text{ cm}^2$$

**Hence, the total area of the park is  $65.5 \text{ cm}^2$ .**

**Q2** Find the area of a quadrilateral ABCD in which  $AB = 3 \text{ cm}$ ,  $BC = 4 \text{ cm}$ ,  $CD = 4 \text{ cm}$ ,  $DA = 5 \text{ cm}$  and  $AC = 5 \text{ cm}$ .

**Answer:**

From the figure:



We have joined the AC to form two triangles so that the calculation of the area will be easy.

The area of the triangle ABC can be calculated by Heron's formula:

So, the semi-perimeter, where  $a = 3cm$ ,  $b = 4cm$  and  $c = 5cm$ .

$$s = \frac{a + b + c}{2} = \frac{3 + 4 + 5}{2} = 6cm$$

Heron's Formula for calculating the area:

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6(3)(2)(1)} = \sqrt{36} = 6 \text{ cm}^2 \end{aligned}$$

And the sides of the triangle ACD are  $a' = 4cm$ ,  $b' = 5cm$  and  $c' = 5cm$ .

So, the semi-perimeter of the triangle:

$$s' = \frac{a' + b' + c'}{2} = \frac{4 + 5 + 5}{2} = \frac{14}{2} = 7cm$$

Therefore, the area will be given by, Heron's formula

$$\begin{aligned} A &= \sqrt{s'(s'-a')(s'-b')(s'-c')} \\ &= \sqrt{7(7-4)(7-5)(7-5)} \\ &= \sqrt{7(3)(2)(2)} = 2\sqrt{21} = 9.2 \text{ cm}^2 \quad (\text{Approx.}) \end{aligned}$$

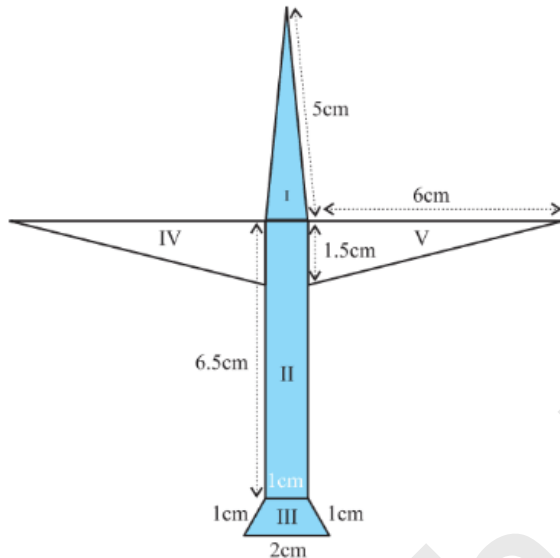
Then, the total area of the quadrilateral will be:

$$= \text{Area of triangle ABC} + \text{Area of triangle ACD}$$

$$\Rightarrow \text{Total area of quadrilateral } ABCD = 6 + 9.2 = 15.2 \text{ cm}^2$$

Hence, the area of the quadrilateral ABCD is  $15.2 \text{ cm}^2$ .

**Q3** Radha made a picture of an aeroplane with coloured paper as shown in Fig 12.15.  
Find the total area of the paper used.



**Fig. 12.15**

**Answer:**

The total area of the paper used will be the sum of the area of the sections I, II, III, IV, and V. i.e.,  $\text{Total area} = I + II + III + IV + V$

**For section I:**

Here, the sides are  $a = 1 \text{ cm}$  and  $b = c = 5 \text{ cm}$ .

So, the Semi-perimeter will be:

$$s = \frac{a + b + c}{2} = \frac{5 + 5 + 1}{2} = 5.5 \text{ cm}$$

Therefore, the area of section I will be given by Heron's Formula,

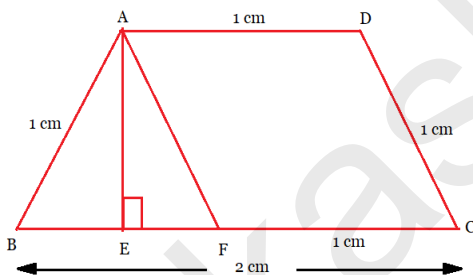
$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{5.5(5.5-1)(5.5-5)(5.5-5)} \\ &= \sqrt{5.5(4.5)(0.5)(0.5)} = \sqrt{6.1875} = 2.5 \text{ cm}^2 \quad (\text{Approx.}) \end{aligned}$$

**For section II:**

Here the sides of the rectangle are  $l = 6.5 \text{ cm}$  and  $b = 1 \text{ cm}$ .

Therefore, the area of the rectangle is  $= l \times b = 6.5 \times 1 = 6.5 \text{ cm}^2$ .

**For section III:**



From the figure:

Drawing the parallel line AF to DC and a perpendicular line AE to BC.

We have the quadrilateral ADCF,

$AF \parallel DC$  .....by construction.

$AD \parallel FC$  .....[  $\because$  ABCD is a trapezium]

So, ADCF is a parallelogram.

Therefore,  $AF = DC = 1 \text{ cm}$  and  $AD = FC = 1 \text{ cm}$

[ $\because$  Opposite sides of a parallelogram]

Therefore,  $BF = BC - FC = 2 - 1 = 1 \text{ cm}$ .

$\Rightarrow$  ABF is an equilateral triangle. [ $\because AB = BF = AF = 1 \text{ cm}$ ]

Then, the area of the equilateral triangle ABF is given by:

$$\begin{aligned}\Rightarrow \frac{\sqrt{3}}{4}a^2 &= \frac{\sqrt{3}}{4}1^2 = \frac{\sqrt{3}}{4} \\&= \frac{1}{2} \times BF \times AE \\&= \frac{1}{2} \times 1 \text{ cm} \times AE = \frac{\sqrt{3}}{4} \\ \Rightarrow AE &= \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.866 \approx 0.9\end{aligned}$$

Hence, the area of trapezium ABCD will be:

$$= \frac{1}{2} \times (AD + BC) \times AE$$

$$= \frac{1}{2} \times (1 + 2) \times 0.9$$

$$= 1.35 = 1.4 \text{ cm}^2 \quad (\text{Approx.})$$

**For Section IV:**

Here, the base is 1.5 cm and the height is 6 cm.

Therefore, the area of the triangle is :

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 1.5 \times 6 = 4.5 \text{ cm}^2$$

**For section V:**

The base length = 1.5cm and the height is 6cm.

Therefore, the area of the triangle will be:

$$= \frac{1}{2} \times 1.5 \times 6 = 4.5 \text{ cm}^2$$

Hence, the total area of the paper used will be:

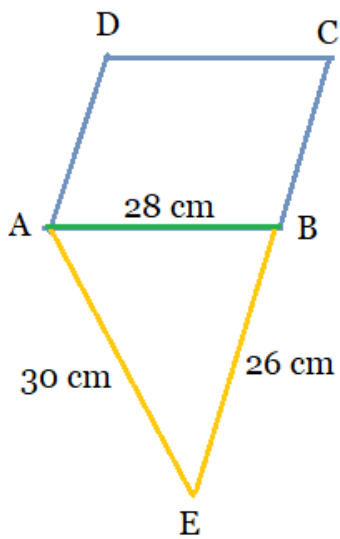
$$\text{Total area} = I + II + III + IV + V$$

$$= 2.5 + 6.5 + 1.4 + 4.5 + 4.5 = 19.4 \text{ cm}^2$$

**Q4** A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

**Answer:**

From the figure:



The sides of the triangle are  $a = 26 \text{ cm}$ ,  $b = 28 \text{ cm}$  and  $c = 30 \text{ cm}$ .

Then, calculating the area of the triangle:

So, the semi-perimeter of triangle ABE,

$$s = \frac{a + b + c}{2} = \frac{28 + 26 + 30}{2} = 42 \text{ cm}.$$

Therefore, its area will be given by the Heron's formula:

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{42(42-28)(42-26)(42-30)} \\ &= \sqrt{42(14)(16)(12)} = \sqrt{112896} = 336 \text{ cm}^2 \end{aligned}$$

Given that the area of the parallelogram is equal to the area of the triangle:

$$\text{Area of Parallelogram} = \text{Area of Triangle}$$

$$\Rightarrow \text{base} \times \text{corresponding height} = 336 \text{ cm}^2$$

$$\Rightarrow 28 \times \text{corresponding height} = 336 \text{ cm}^2$$

$$\Rightarrow \text{height} = \frac{336}{28} = 12 \text{ cm}.$$

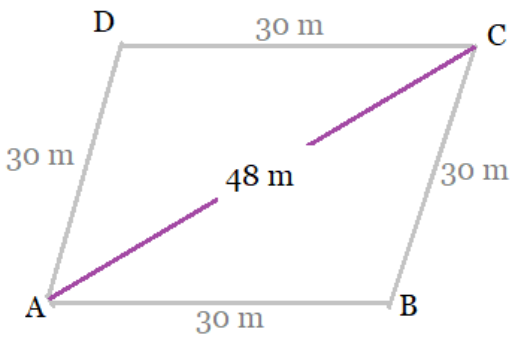
**Hence, the height of the parallelogram is 12 cm.**

**Q5** A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

**Answer:**

To find the area of the rhombus:

We first join the diagonal AC of quadrilateral ABCD. (See figure)



Here, the sides of triangle ABC are,

$$a = 30 \text{ m}, b = 30 \text{ m and } c = 48 \text{ m}.$$

So, the semi-perimeter of the triangle will be:

$$s = \frac{a + b + c}{2} = \frac{30 + 30 + 48}{2} = \frac{108}{2} = 54 \text{ m}$$

Therefore, the area of the triangle given by the Heron's formula,

$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{54(54-30)(54-30)(54-48)} \\ &= \sqrt{54(24)(24)(6)} = \sqrt{186624} = 432 \text{ m}^2 \end{aligned}$$

Hence, the area of the quadrilateral will be:

$$= 2 \times 432 \text{ m}^2 = 864 \text{ m}^2$$

Therefore, the area grazed by each cow will be given by,

$$= \frac{\text{Total area}}{\text{Number of cows}} = \frac{864}{18} = 48 \text{ m}^2.$$

**Q6** An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?



**Answer:**

The sides of the triangle are:

$a = 20 \text{ cm}$ ,  $b = 50 \text{ cm}$  and  $c = 50 \text{ cm}$ .

So, the semi-perimeter of the triangle is given by,

$$s = \frac{a + b + c}{2} = \frac{20 + 50 + 50}{2} = \frac{120}{2} = 60 \text{ cm}.$$

Therefore, the area of the triangle can be found by using Heron's formula:

$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{60(60-20)(60-50)(60-50)} \\ &= \sqrt{60(40)(10)(10)} = 200\sqrt{6} \text{ cm}^2 \end{aligned}$$

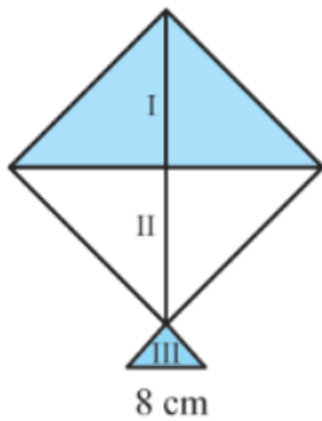
Now, for the 10 triangular pieces of cloths, the area will be,

$$= 10 \times 200\sqrt{6} = 2000\sqrt{6} \text{ cm}^2$$

**Hence, the area of cloths of each colour will be:**

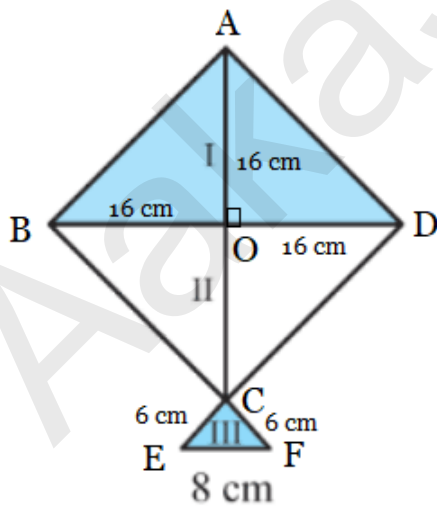
$$= \frac{2000\sqrt{6}}{2} = 1000\sqrt{6} \text{ cm}^2.$$

**Q7** A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Figure How much paper of each shade has been used in it?



**Answer:**

From the figure:



Calculation of the area for each shade:

**The shade I:** Triangle ABD

Here, base  $BD = 32 \text{ cm}$  and the height  $AO = 16 \text{ cm}$ .

Therefore, the area of triangle ABD will be:

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 32 \times 16$$

$$= 256 \text{ cm}^2$$

Hence, the area of paper used in shade I is  $256 \text{ cm}^2$ .

**Shade II:** Triangle CBD

Here, base  $BD = 32 \text{ cm}$  and height  $CO = 16 \text{ cm}$ .

Therefore, the area of triangle CBD will be:

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 32 \times 16$$

$$= 256 \text{ cm}^2$$

Hence, the area of paper used in shade II is  $256 \text{ cm}^2$ .

**Shade III:** Triangle CEF

Here, the sides are of lengths,  $a = 6 \text{ cm}$ ,  $b = 6 \text{ cm}$  and  $c = 8 \text{ cm}$ .

So, the semi-perimeter of the triangle:

$$s = \frac{a + b + c}{2} = \frac{6 + 6 + 8}{2} = \frac{20}{2} = 10 \text{ cm}.$$

Therefore, the area of the triangle can be found by using Heron's formula:

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

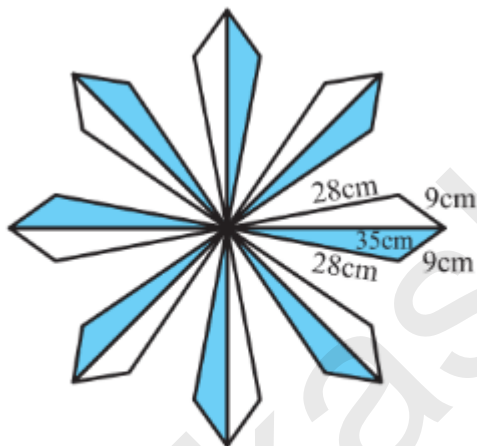
$$= \sqrt{10(10-6)(10-6)(10-8)}$$

$$= \sqrt{10(4)(6)(2)}$$

$$= 8\sqrt{5} \text{ cm}^2$$

Hence, the area of the paper used in shade III is  $8\sqrt{5} \text{ cm}^2$ .

**Q8** A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Fig. 12.18). Find the cost of polishing the tiles at the rate of 50p per  $\text{cm}^2$ .



**Fig. 12.18**

**Answer:**

Given the sides of the triangle are:

$a = 9 \text{ cm}$ ,  $b = 28 \text{ cm}$  and  $c = 35 \text{ cm}$ .

So, its semi-perimeter will be:

$$s = \frac{a + b + c}{2} = \frac{9 + 28 + 35}{2} = 36 \text{ cm}$$

Therefore, the area of the triangle using Heron's formula is given by,

$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{36(36-9)(36-28)(36-35)} = \sqrt{36(27)(8)(1)} \\ &= \sqrt{7776} \approx 88.2 \text{ cm}^2 \end{aligned}$$

So, we have the area of each triangle tile which is  $88.2 \text{ cm}^2$ .

Therefore, the area of each triangular 16 tiles will be:

$$= 16 \times 88.2 \text{ cm}^2 = 1411.2 \text{ cm}^2$$

Hence, the cost of polishing the tiles at the rate of 50 paise per  $\text{cm}^2$  will be:

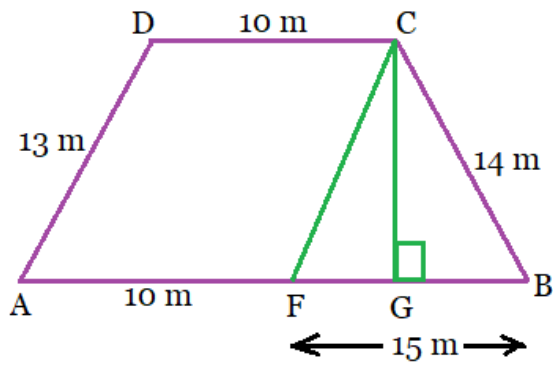
$$= \text{Rs. } 0.50 \times 1411.2 \text{ cm}^2 = \text{Rs. } 705.60$$

**Q9** A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

**Answer:**

The trapezium field is shown below in figure:

Drawing line CF parallel to AD and a line perpendicular to AB, we obtain



Then in quadrilateral ADCF,

$CF \parallel AD$  ..... [ $\because$  by construction]

$CD \parallel AF$  ..... [ $\because ABCD$  is a trapezium]

Therefore, ADCF is a parallelogram.

So,  $AD = CF = 13\text{ m}$  and  $CD = AF = 10\text{ m}$

( $\because$  Opposite sides of a parallelogram)

Therefore,  $BF = AB - AF = 25 - 10 = 15\text{ m}$

Now, the sides of the triangle;

$a = 13\text{ m}$ ,  $b = 14\text{ m}$  and  $c = 15\text{ m}$ .

So, the semi-perimeter of the triangle will be:

$$s = \frac{a + b + c}{2} = \frac{13 + 14 + 15}{2} = \frac{42}{2} = 21\text{ m}$$

Therefore, the area of the triangle can be found by using Heron's Formula:

$$\begin{aligned}
 \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{21(21-13)(21-14)(21-15)} \\
 &= \sqrt{21(8)(7)(6)} \\
 &= \sqrt{7056} = 84 \text{ m}^2.
 \end{aligned}$$

Also, the area of the triangle is given by,

$$\begin{aligned}
 \text{Area} &= \frac{1}{2} \times BF \times CG \\
 \Rightarrow \frac{1}{2} \times BF \times CG &= 84 \text{ m}^2 \\
 \Rightarrow \frac{1}{2} \times 15 \times CG &= 84 \text{ m}^2
 \end{aligned}$$

Or,

$$\Rightarrow CG = \frac{84 \times 2}{15} = 11.2 \text{ m}$$

Therefore, the area of the trapezium ABCD is:

$$\begin{aligned}
 &= \frac{1}{2} \times (AB + CD) \times CG \\
 &= \frac{1}{2} \times (25 + 10) \times 11.2 \\
 &= 35 \times 5.6 \\
 &= 196 \text{ m}^2
 \end{aligned}$$

**Hence, the area of the trapezium field is  $196 \text{ m}^2$ .**