NCERT solutions for class 9 Maths Chapter 2 Polynomials

Q1 (i) Is the following expression polynomial in one variable? State reasons for your answer. $4x^2 - 3x + 7$

Answer:

YES

Given polynomial $4x^2 - 3x + 7$ has only one variable which is **x**

Q1 (ii) Is the following expression polynomial in one variable? State reasons for your answer. $y^2 + \sqrt{2}$

Answer:

YES

Given polynomial has only one variable which is **y**

Q1 (iii) Is the following expression polynomial in one variable? State reasons for your answer. $3\sqrt{t} + t\sqrt{2}$

Answer:

NO

Because we can observe that the exponent of variable t in term $3\sqrt{t}$ is $\frac{1}{2}$ which is not a whole number.

Therefore this expression is not a polynomial.

Q1 (iv) Is the following expression polynomial in one variable? State reasons for your answer. $y + \frac{2}{y}$

Answer:

NO

Because we can observe that the exponent of variable y in term $\frac{2}{y}$ is -1 which is not a whole number. Therefore this expression is not a polynomial.

Q1 (v) Is the following expression polynomial in one variable? State reasons for your answer. $x^{10} + y^3 + t^{50}$

Answer:

NO

Because in the given polynomial $x^{10} + y^3 + t^{50}$ there are 3 variables which are **x, y, t.** That's why this is polynomial in three variable not in one variable.

Q2 (i) Write the coefficients of x^2 in the following: $2 + x^2 + x$

Answer:

Coefficient of x^2 in polynomial $2 + x^2 + x$ is **1.**

Q2 (ii) Write the coefficients of x^2 in the following: $2 - x^2 + x^3$

Answer:

Coefficient of x^2 in polynomial $2 - x^2 + x^3$ is **-1.**

Q2 (iii) Write the coefficients of x^2 in the following: $\frac{\pi}{2}x^2 + x$

Answer:

Coefficient of x^2 in polynomial $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$

Q2 (iv) Write the coefficients of x^2 in the following: $\sqrt{2}x - 1$

Answer:

Coefficient of x^2 in polynomial $\sqrt{2}x - 1$ is **0**

Q3 Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

In binomial, there are two terms

Therefore, binomial of degree 35 is

Eg:- $x^{35} + 1$

In monomial, there is only one term in it.

Therefore, monomial of degree 100 can be written as y^{100}

Q4 (i) Write the degree the following polynomial: $5x^3 + 4x^2 + 7x$

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

Therefore, the degree of polynomial $5x^3 + 4x^2 + 7x$ is **3** .

Q4 (ii) Write the degree the following polynomial: $4 - y^2$

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

Therefore, the degree of polynomial $4 - y^2$ is **2.**

Q4 (iii) Write the degree the following polynomial: $5t - \sqrt{7}$

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

Therefore, the degree of polynomial $5t - \sqrt{7}$ is **1**

Q4 (iv) Write the degree the following polynomial: 3

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

In this case, only a constant value 3 is there and the degree of a constant polynomial is always **0.**

Q5 (i) Classify the following as linear, quadratic and cubic polynomial: $x^2 + x$

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively

Given polynomial is $x^2 + x$ with degree 2

Therefore, it is a quadratic polynomial.

Q5 (ii) Classify the following as linear, quadratic and cubic polynomial: $x - x^3$

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively

Given polynomial is $x - x^3$ with degree 3

Therefore, it is a cubic polynomial

Q5 (iii) Classify the following as linear, quadratic and cubic polynomial: $y + y^2 + 4$

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively

Given polynomial is $y + y^2 + 4$ with degree 2

Therefore, it is quadratic polynomial.

Q5 (iv) Classify the following as linear, quadratic and cubic polynomial: $1 + x$

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively

Given polynomial is $1 + x$ with degree 1

Therefore, it is linear polynomial

Q5 (v) Classify the following as linear, quadratic and cubic polynomial: $3t$

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively

Given polynomial is $3t$ with degree 1

Therefore, it is linear polynomial

Q5 (vi) Classify the following as linear, quadratic and cubic polynomial: r^2

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively

Given polynomial is r^2 with degree 2

Therefore, it is quadratic polynomial

Q5 (vii) Classify the following as linear, quadratic and cubic polynomial: $7x^3$

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively

Given polynomial is $7x^3$ with degree 3

Therefore, it is a cubic polynomial

NCERT solutions for class 9 maths chapter 2 Polynomials Excercise: 2.2

Q1 (i) Find the value of the polynomial $5x - 4x^2 + 3$ at $x = 0$

Answer:

Given polynomial is $5x - 4x^2 + 3$

Now, at $x = 0$ value is

$\Rightarrow 5(0) - 4(0)^2 + 3 = 0 - 0 + 3 = 3$

Therefore, value of polynomial $5x - 4x^2 + 3$ at x = 0 is **3**

Q1 (ii) Find the value of the polynomial $5x - 4x^2 + 3$ at $x = -1$

Answer:

Given polynomial is $5x - 4x^2 + 3$

Now, at $x = -1$ value is

$\Rightarrow 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$

Therefore, value of polynomial $5x - 4x^2 + 3$ at x = -1 is **-6**

Q1 (iii) Find the value of the polynomial $5x - 4x^2 + 3$ at $x = 2$

Answer:

Given polynomial is $5x - 4x^2 + 3$

Now, at $x = 2$ value is

$\Rightarrow 5(2) - 4(2)^2 + 3 = 10 - 16 + 3 = -3$

Therefore, value of polynomial $5x - 4x^2 + 3$ at x = 2 is **-3**

Q2 (i) Find *p(0)* , *p(1)* and *p(2)* for each of the following polynomials: $p(y) = y^2 - y + 1$

Answer:

Given polynomial is

$p(y) = y^2 - y + 1$

Now,

$p(0) = (0)^2 - 0 + 1 = 1$

$p(1) = (1)^2 - 1 + 1 = 1$

$p(2) = (2)^2 - 2 + 1 = 3$

Therefore, values of *p(0)* , *p(1)* and *p(2) are 1 , 1 and 3 respectively* .

Q2 (ii) Find *p(0)* , *p(1)* and *p(2)* for each of the following polynomials: $p(t) = 2 + t + 2t^2 - t^3$

Answer:

Given polynomial is

$p(t) = 2 + t + 2t^2 - t^3$

Now,

$p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$

$p(1) = 2 + 1 + 2(1)^2 - (1)^3 = 4$

$p(2) = 2 + 2 + 2(2)^2 - (2)^3 = 4$

Therefore, values of *p(0)* , *p(1)* and *p(2) are 2 , 4 and 4 respectively*

Q2 (iii) Find p(0), p(1) and p(2) for each of the following polynomials: $p(x) = x^3$

Answer:

Given polynomial is

$$p(x) = x^3$$

Now,

$$p(0) = (0)^3 = 0$$

$$p(1) = (1)^3 = 1$$

$$p(2) = (2)^3 = 8$$

Therefore, values of *p(0)* , *p(1)* and *p(2) are 0 , 1 and 8 respectively*

Q2 (iv) Find p(0), p(1) and p(2) for each of the following polynomials: $p(x) = (x - 1)(x + 1)$

Answer:

Given polynomial is

$$p(x) = (x - 1)(x + 1) = x^2 - 1$$

Now,

$$p(0) = (0)^2 - 1 = -1$$

$$p(1) = (1)^2 - 1 = 0$$

$p(2) = (2)^2 - 1 = 3$

Therefore, values of *p(0)* , *p(1)* and *p(2) are -1 , 0 and 3 respectively*

Q3 (i) Verify whether the following are zeroes of the polynomial, indicated against it. $p(x) = 3x + 1, x = -\frac{1}{3}$

Answer:

Given polynomial is $p(x) = 3x + 1$

Now, at $x = -\frac{1}{3}$ it's value is

$$p\left(-\frac{1}{3}\right) = 3 \times \left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$$

Therefore, yes $x = -\frac{1}{3}$ is a zero of polynomial $p(x) = 3x + 1$

Q3 (ii) Verify whether the following are zeroes of the polynomial, indicated against it. $p(x) = 5x - \pi, x = \frac{4}{5}$

Answer:

Given polynomial is $p(x) = 5x - \pi$

Now, at $x = \frac{4}{5}$ it's value is

$$p\left(\frac{4}{5}\right) = 5 \times \left(\frac{4}{5}\right) - \pi = 4 - \pi \neq 0$$

Therefore, no $x = \frac{4}{5}$ is not a zero of polynomial $p(x) = 5x - \pi$

Q3 (iii) Verify whether the following are zeroes of the polynomial, indicated against it. $p(x) = x^2 - 1, x = 1, -1$

Answer:

Given polynomial is $p(x) = x^2 - 1$

Now, at **x = 1** it's value is

$$p(1) = (1)^2 - 1 = 1 - 1 = 0$$

And at **x = -1**

$$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$$

Therefore, yes x = 1 , -1 are zeros of polynomial $p(x) = x^2 - 1$

Q3 (iv) Verify whether the following are zeroes of the polynomial, indicated against it. $p(x) = (x + 1)(x - 2), x = -1, 2$

Answer:

Given polynomial is $p(x) = (x + 1)(x - 2)$

Now, at **x = 2** it's value is

$p(2) = (2+1)(2-2) = 0$

And at **x = -1**

$p(-1) = (-1+1)(-1-2) = 0$

Therefore, yes x = 2 , -1 are zeros of polynomial $p(x) = (x+1)(x-2)$

Q3 (v) Verify whether the following are zeroes of the polynomial, indicated against it. $p(x) = x^2 .x = 0$

Answer:

Given polynomial is $p(x) = x^2$

Now, at **x = 0** it's value is

$p(0) = (0)^2 = 0$

Therefore, yes **x = 0** is a zeros of polynomial $p(x) = (x+1)(x-2)$

Q3 (vi) Verify whether the following are zeroes of the polynomial, indicated against it. $p(x) = lx + m,\ x = -\frac{m}{l}$

Answer:

Given polynomial is $p(x) = lx + m$

Now, at $x = -\frac{m}{l}$ it's value is

$$p\left(-\frac{m}{l}\right) = l \times \left(-\frac{m}{l}\right) + m = -m + m = 0$$

Therefore, yes $x = -\frac{m}{l}$ is a zeros of polynomial $p(x) = lx + m$

Q3 (vii) Verify whether the following are zeroes of the polynomial, indicated against it. $p(x) = 3x^2 - 1,\ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Answer:

Given polynomial is $p(x) = 3x^2 - 1$

Now, at $x = -\frac{1}{\sqrt{3}}$ it's value is

$$p\left(-\frac{1}{\sqrt{3}}\right) = 3 \times \left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 1 - 1 = 0$$

And at $x = \frac{2}{\sqrt{3}}$

$$p\left(\frac{2}{\sqrt{3}}\right) = 3 \times \left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 4 - 1 = 3 \neq 0$$

Therefore, $x = -\frac{1}{\sqrt{3}}$ is a zeros of polynomial $p(x) = 3x^2 - 1$.

whereas $x = \frac{2}{\sqrt{3}}$ is not a zeros of polynomial $p(x) = 3x^2 - 1$

Q3 (viii) Verify whether the following are zeroes of the polynomial, indicated against it. $p(x) = 2x + 1,\ x = \frac{1}{2}$

Answer:

Given polynomial is $p(x) = 2x + 1$

Now, at $x = \frac{1}{2}$ it's value is

$$p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right) + 1 = 1 + 1 = 2 \neq 0$$

Therefore, $x = \frac{1}{2}$ is not a zeros of polynomial $p(x) = 2x + 1$

Q4 (i) Find the zero of the polynomial in each of the following cases: $p(x) = x + 5$

Answer:

Given polynomial is $p(x) = x + 5$

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

Now,

$$p(x) = 0$$

$$\Rightarrow x + 5 = 0$$

$\Rightarrow x = -5$

Therefore, **x = -5** is the zero of polynomial $p(x) = x + 5$

Q4 (ii) Find the zero of the polynomial in each of the following cases: $p(x) = x - 5$

Answer:

Given polynomial is $p(x) = x - 5$

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

Now,

$p(x) = 0$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

Therefore, **x = 5** is a zero of polynomial $p(x) = x - 5$

Q4 (iii) Find the zero of the polynomial in each of the following cases: $p(x) = 2x + 5$

Answer:

Given polynomial is $p(x) = 2x + 5$

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

Now,

$p(x) = 0$

$\Rightarrow 2x + 5 = 0$

$\Rightarrow x = -\frac{5}{2}$

Therefore, $x = -\frac{5}{2}$ is a zero of polynomial $p(x) = 2x + 5$

Q4 (iv) Find the zero of the polynomial in each of the following cases: $p(x) = 3x - 2$

Answer:

Given polynomial is $p(x) = 3x - 2$

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

Now,

$p(x) = 0$

$\Rightarrow 3x - 2 = 0$

$\Rightarrow x = \frac{2}{3}$

Therefore, $x = \frac{2}{3}$ is a zero of polynomial $p(x) = 3x - 2$

Q4 (v) Find the zero of the polynomial in each of the following cases: $p(x) = 3x$

Answer:

Given polynomial is $p(x) = 3x$

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

Now,

$p(x) = 0$

$\Rightarrow 3x = 0$

$\Rightarrow x = 0$

Therefore, $x = 0$ is a zero of polynomial $p(x) = 3x$

Q4 (vi) Find the zero of the polynomial in each of the following cases: $p(x) = ax,\ a \neq 0$

Answer:

Given polynomial is $p(x) = ax$

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

Now,

$p(x) = 0$

$\Rightarrow ax = 0$

$\Rightarrow x = 0$

Therefore, $x = 0$ is a zero of polynomial $p(x) = ax$

Q4 (vii) Find the zero of the polynomial in each of the following cases: $p(x) = cx + d, c \neq 0, c, d$ are real numbers

Answer:

Given polynomial is $p(x) = cx + d$

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

Now,

$p(x) = 0$

$\Rightarrow cx + d = 0$

$\Rightarrow x = -\frac{d}{c}$

Therefore, $x = -\frac{d}{c}$ is a zero of polynomial $p(x) = cx + d$

NCERT solutions for class 9 maths chapter 2 Polynomials Excercise: 2.3

Q1 (i) Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by $x + 1$

Answer:

When we divide $x^3 + 3x^2 + 3x + 1$ by $x + 1$.

By long division method, we will get

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
x+1 & x^3+3x^2+3x+1 \\
 & x^3+x^2 \\
 & -\quad - \\
\hline
 & 2x^2+3x+1 \\
 & 2x^2+2x \\
 & -\quad - \\
\hline
 & x+1 \\
 & x+1 \\
 & -\quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

Therefore, remainder is 0 .

Q1 (ii) Find the remainder when x^3+3x^2+3x+1 is divided by $x-\frac{1}{2}$

Answer:

When we divide x^3+3x^2+3x+1 by $x-\frac{1}{2}$.

By long division method, we will get

$$
\begin{array}{r|l}
 & x^2+\frac{7}{2}x+\frac{19}{4} \\
\hline
x-\frac{1}{2} & x^3+3x^2+3x+1 \\
 & x^3-\frac{x^2}{2} \\
 & -\quad + \\
\hline
 & \frac{7}{2}x^2+3x+1 \\
 & \frac{7}{2}x^2-\frac{7}{4}x \\
 & -\quad + \\
\hline
 & \frac{19}{4}x+1 \\
 & \frac{19}{4}x-\frac{19}{8} \\
 & -\quad + \\
\hline
 & \frac{27}{8} \\
\hline
\end{array}
$$

Therefore, the remainder is $\frac{27}{8}$

Q1 (iii) Find the remainder when x^3+3x^2+3x+1 is divided by x

Answer:

When we divide x^3+3x^2+3x+1 by x .

By long division method, we will get

$$
\begin{array}{r|l}
 & x^2 + 3x + 3 \\
\hline
x & x^3 + 3x^2 + 3x + 1 \\
 & x^3 \\
 & - \\
\hline
 & 3x^2 + 3x + 1 \\
 & 3x^2 \\
 & - \\
\hline
 & 3x + 1 \\
 & 3x \\
 & - \\
\hline
 & 1 \\
\hline
\end{array}
$$

Therefore, remainder is 1 .

Q1 (iv) Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by $x + \pi$

Answer:

When we divide $x^3 + 3x^2 + 3x + 1$ by $x + \pi$.

By long division method, we will get

$$
\begin{array}{r|l}
 & x^2+(3-\pi)x+(3-3\pi+\pi^2) \\
\hline
x+\pi & x^3+3x^2+3x+1 \\
 & x^3+\pi x^2 \\
 & -\quad - \\
\hline
 & (3-\pi)x^2+3x+1 \\
 & (3-\pi)x^2+(3-\pi)\pi x \\
 & -\qquad - \\
\hline
 & [3-3\pi+\pi^2]x+1 \\
 & [3-3\pi+\pi^2]x+(3-3\pi+\pi^2)\pi \\
 & -\qquad - \\
\hline
 & [1-3\pi+3\pi^2-\pi^3] \\
\hline
\end{array}
$$

Therefore, the remainder is $1 - 3\pi + 3\pi^2 - \pi^3$

Q1 (v) Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by $5 + 2x$

Answer:

When we divide $x^3 + 3x^2 + 3x + 1$ by $5 + 2x$.

By long division method, we will get

$$\begin{array}{r|l} & \frac{x^2}{2}+\frac{x}{4}+\frac{7}{8} \\ \hline 2x+5 & x^3+3x^2+3x+1 \\ & x^3+\frac{5}{2}x^2 \\ & -\quad - \\ \hline & \frac{x^2}{2}+3x+1 \\ & \frac{x^2}{2}+\frac{5x}{4} \\ & -\quad - \\ \hline & \frac{7x}{4}+1 \\ & \frac{7}{4}x+\frac{35}{8} \\ & -\quad - \\ \hline & -\frac{27}{8} \\ \hline \end{array}$$

Therefore, the remainder is $-\frac{27}{8}$

Q2 Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Answer:

When we divide $x^3 - ax^2 + 6x - a$ by $x - a$.

By long division method, we will get

$$
\begin{array}{r|l}
 & x^2+6 \\
\hline
x-a & x^3-ax^2+6x-a \\
 & x^3-ax^2 \\
 & -\quad + \\
\hline
 & \qquad 6x-a \\
 & \qquad 6x-6a \\
 & \qquad -\quad + \\
\hline
 & \qquad\quad 5a \\
\hline
\end{array}
$$

Therefore, remainder is $5a$

Q3 Check whether $7+3x$ is a factor of $3x^3+7x$.

Answer:

When we divide $3x^3+7x$ by $7+3x$.

We can also write $3x^3+7x$ as $3x^3+0x^2+7x$

By long division method, we will get

$$
\begin{array}{r l}
 & x^2 - \frac{7}{3}x + \frac{70}{9} \\
3x+7 \,) & 3x^3 + 0x^2 + 7x \\
 & 3x^3 + 7x^2 \\
 & - \quad - \\
\hline
 & -7x^2 + 7x \\
 & -7x^2 - \frac{49x}{3} \\
 & + \quad + \\
\hline
 & \frac{70x}{3} \\
 & \frac{70x}{3} + \frac{490}{9} \\
 & - \quad - \\
\hline
 & -\frac{490}{9} \\
\hline
\end{array}
$$

Since, remainder is not equal to 0

Therefore, $7 + 3x$ is not a factor of $3x^3 + 7x$

NCERT solutions for class 9 maths chapter 2 Polynomials Excercise: 2.4

Q1 (i) Determine which of the following polynomials has $(x+1)$ a factor : $x^3 + x^2 + x + 1$

Answer:

Zero of polynomial $(x+1)$ is **-1.**

If $(x+1)$ is a factor of polynomial $p(x) = x^3 + x^2 + x + 1$

Then, $p(-1)$ must be equal to zero

Now,

$\Rightarrow p(-1)=(-1)^{3}+(-1)^{2}-1+1$

$\Rightarrow p(-1)=-1+1-1+1=0$

Therefore, $(x+1)$ is a factor of polynomial $p(x)=x^{3}+x^{2}+x+1$

Q1 (ii) Determine which of the following polynomials has $(x+1)$ a factor : $x^{4}+x^{3}+x^{2}+x+1$

Answer:

Zero of polynomial $(x+1)$ is **-1.**

If $(x+1)$ is a factor of polynomial $p(x)=x^{4}+x^{3}+x^{2}+x+1$

Then, $p(-1)$ must be equal to zero

Now,

$\Rightarrow p(-1)=(-1)^{4}+(-1)^{3}+(-1)^{2}-1+1$

$\Rightarrow p(-1)=1-1+1-1+1=1 \neq 0$

Therefore, $(x+1)$ is not a factor of polynomial $p(x) = x^4 + x^3 + x^2 + x + 1$

Q1 (iii) Determine which of the following polynomials has $(x+1)$ a factor : $x^4 + 3x^3 + 3x^2 + x + 1$

Answer:

Zero of polynomial $(x+1)$ is **-1.**

If $(x+1)$ is a factor of polynomial $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Then, $p(-1)$ must be equal to zero

Now,

$\Rightarrow p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 - 1 + 1$

$\Rightarrow p(-1) = 1 - 3 + 3 - 1 + 1 = 1 \neq 0$

Therefore, $(x+1)$ is not a factor of polynomial $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Q1 (iv) Determine which of the following polynomials has $(x+1)$ a factor : $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Answer:

Zero of polynomial $(x+1)$ is **-1.**

If $(x+1)$ is a factor of polynomial $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Then, $p(-1)$ must be equal to zero

Now,

$\Rightarrow p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$\Rightarrow p(-1) = -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = 2\sqrt{2} \neq 0$

Therefore, $(x+1)$ is not a factor of polynomial $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Q2 (i) Use the Factor Theorem to determine whether g(x) is a factor of p(x) in the following case: $p(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$

Answer:

Zero of polynomial $g(x) = x + 1$ is -1

If $g(x) = x + 1$ is factor of polynomial $p(x) = 2x^3 + x^2 - 2x - 1$

Then, $p(-1)$ must be equal to zero

Now,

$\Rightarrow p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$\Rightarrow p(-1) = -2 + 1 + 2 - 1 = 0$

Therefore, $g(x) = x + 1$ is factor of polynomial $p(x) = 2x^3 + x^2 - 2x - 1$

Q2 (ii) Use the Factor Theorem to determine whether g(x) is a factor of p(x) in the following case: $p(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

Answer:

Zero of polynomial $g(x) = x + 2$ is -2

If $g(x) = x + 2$ is factor of polynomial $p(x) = x^3 + 3x^2 + 3x + 1$

Then, $p(-2)$ must be equal to zero

Now,

$\Rightarrow p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$\Rightarrow p(-2) = -8 + 12 - 6 + 1 = -1 \neq 0$

Therefore, $g(x) = x + 2$ is not a factor of polynomial $p(x) = x^3 + 3x^2 + 3x + 1$

Q2 (iii) Use the Factor Theorem to determine whether g(x) is a factor of p(x) in the following case: $p(x) = x^3 - 4x^2 + x + 6,\ g(x) = x - 3$

Answer:

Zero of polynomial $g(x) = x - 3$ is 3

If $g(x) = x - 3$ is factor of polynomial $p(x) = x^3 - 4x^2 + x + 6$

Then, $p(3)$ must be equal to zero

Now,

$\Rightarrow p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$\Rightarrow p(3) = 27 - 36 + 3 + 6 = 0$

Therefore, $g(x) = x - 3$ is a factor of polynomial $p(x) = x^3 - 4x^2 + x + 6$

Q3 (i) Find the value of *k* , if $x - 1$ is a factor of p(x) in the following case: $p(x) = x^2 + x + k$

Answer:

Zero of polynomial $x - 1$ is 1

If $x - 1$ is factor of polynomial $p(x) = x^2 + x + k$

Then, $p(1)$ must be equal to zero

Now,

$\Rightarrow p(1) = (1)^2 + 1 + k$

$\Rightarrow p(1) = 0$

$\Rightarrow 2+k=0$

$\Rightarrow k=-2$

Therefore, value of k is -2

Q3 (ii) Find the value of k , if $x-1$ is a factor of p(x) in the following case: $p(x)=2x^2+kx+\sqrt{2}$

Answer:

Zero of polynomial $x-1$ is 1

If $x-1$ is factor of polynomial $p(x)=2x^2+kx+\sqrt{2}$

Then, $p(1)$ must be equal to zero

Now,

$\Rightarrow p(1)=2(1)^2+k(1)+\sqrt{2}$

$\Rightarrow p(1)=0$

$\Rightarrow 2+k+\sqrt{2}=0$

$\Rightarrow k=-(2+\sqrt{2})$

Therefore, value of k is $-(2+\sqrt{2})$

Q3 (iii) Find the value of k , if $x-1$ is a factor of p(x) in the following case: $p(x)=kx^2-\sqrt{2}x+1$

Answer:

Zero of polynomial $x-1$ is 1

If $x-1$ is factor of polynomial $p(x)=kx^2-\sqrt{2}x+1$

Then, $p(1)$ must be equal to zero

Now,

$\Rightarrow p(1)=k(1)^2-\sqrt{2}(1)+1$

$\Rightarrow p(1)=0$

$\Rightarrow k-\sqrt{2}+1=0$

$\Rightarrow k=-1+\sqrt{2}$

Therefore, value of k is $-1+\sqrt{2}$

Q3 (iv) the value of k , if $x-1$ is a factor of p(x) in the following case: $p(x)=kx^2-3x+k$

Answer:

Zero of polynomial $x-1$ is 1

If $x-1$ is factor of polynomial $p(x)=kx^2-3x+k$

Then, $p(1)$ must be equal to zero

Now,

$\Rightarrow p(1)=k(1)^2-3(1)+k$

$\Rightarrow p(1)=0$

$\Rightarrow k-3+k=0$

$\Rightarrow k=\frac{3}{2}$

Therefore, value of k is $\frac{3}{2}$

Q4 (i) Factorise : $12x^2-7x+1$

Answer:

Given polynomial is $12x^2-7x+1$

We need to factorise the middle term into two terms such that their product is equal to $12\times 1=12$and their sum is equal to -7

We can solve it as

$\Rightarrow 12x^2 - 7x + 1$

$\Rightarrow 12x^2 - 3x - 4x + 1 \, (\because -3 \times -4 = 12 \;\; and \;\; -3 + (-4) = -7)$

$\Rightarrow 3x(4x - 1) - 1(4x - 1)$

$\Rightarrow (3x - 1)(4x - 1)$

Q4 (ii) Factorise : $2x^2 + 7x + 3$

Answer:

Given polynomial is $2x^2 + 7x + 3$

We need to factorise the middle term into two terms such that their product is equal to $2 \times 3 = 6$ and their sum is equal to 7

We can solve it as

$\Rightarrow 12x^2 - 7x + 1$

$\Rightarrow 2x^2 + 6x + x + 3 \, (\because 6 \times 1 = 6 \;\; and \;\; 6 + 1 = 7)$

$\Rightarrow 2x(x + 3) + 1(x + 3)$

$\Rightarrow (2x + 1)(x + 3)$

Q4 (iii) Factorise : $6x^2 + 5x - 6$

Answer:

Given polynomial is $6x^2 + 5x - 6$

We need to factorise the middle term into two terms such that their product is equal to $6 \times -6 = -36$ and their sum is equal to 5

We can solve it as

$\Rightarrow 6x^2 + 5x - 6$

$\Rightarrow 6x^2 + 9x - 4x - 6 \, (\because 9 \times -4 = -36 \;\; and \;\; 9 + (-4) = 5)$

$\Rightarrow 3x(2x + 3) - 2(2x + 3)$

$\Rightarrow (2x + 3)(3x - 2)$

Q4 (iv) Factorise : $3x^2 - x - 4$

Answer:

Given polynomial is $3x^2 - x - 4$

We need to factorise the middle term into two terms such that their product is equal to $3 \times -4 = -12$ and their sum is equal to -1

We can solve it as

$\Rightarrow 3x^2 - x - 4$

$\Rightarrow 3x^2 - 4x + 3x - 4 \; (\because 3 \times -4 = -12 \;\; and \;\; 3 + (-4) = -1)$

$\Rightarrow x(3x - 4) + 1(3x - 4)$

$\Rightarrow (x + 1)(3x - 4)$

Q5 (i) Factorise : $x^3 - 2x^2 - x + 2$

Answer:

Given polynomial is $x^3 - 2x^2 - x + 2$

Now, by hit and trial method we observed that $(x + 1)$ is one of the factors of the given polynomial.

By long division method, we will get

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x+1 & x^3 - 2x^2 - x + 2 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -3x^2 - x + 2 \\
 & -3x^2 - 3x \\
 & + \quad + \\
\hline
 & 2x + 2 \\
 & 2x + 2 \\
 & - \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

We know that Dividend = (Divisor × Quotient) + Remainder

$x^3 - 2x^2 - x + 2 = (x+1)(x^2 - 3x + 2) + 0$

$= (x+1)(x^2 - 2x - x + 2)$

$= (x+1)(x-2)(x-1)$

Therefore, on factorization of $x^3 - 2x^2 - x + 2$ we will get $(x+1)(x-2)(x-1)$

Q5 (ii) Factorise : $x^3 - 3x^2 - 9x - 5$

Answer:

Given polynomial is $x^3 - 3x^2 - 9x - 5$

Now, by hit and trial method we observed that $(x+1)$ is one of the factors of the given polynomial.

By long division method, we will get

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
\hline
x+1 & x^3 - 3x^2 - 9x - 5 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -4x^2 - 9x - 5 \\
 & -4x^2 - 4x \\
 & + \quad + \\
\hline
 & -5x - 5 \\
 & -5x - 5 \\
 & + \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

We know that Dividend = (Divisor × Quotient) + Remainder

$x^3 - 3x^2 - 9x - 5 = (x+1)(x^2 - 4x - 5)$

$= (x+1)(x^2 - 5x + x - 5)$

$= (x+1)(x-5)(x+1)$

Therefore, on factorization of $x^3 - 3x^2 - 9x - 5$ we will get $(x+1)(x-5)(x+1)$

Q5 (iii) Factorise : $x^3 + 13x^2 + 32x + 20$

Answer:

Given polynomial is $x^3 + 13x^2 + 32x + 20$

Now, by hit and trial method we observed that $(x+1)$ is one of the factore of given polynomial.

By long division method, we will get

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & 12x^2 + 32x \\
 & 12x^2 + 12x \\
 & - \quad - \\
\hline
 & 20x + 20 \\
 & 20x + 20 \\
 & - \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

We know that Dividend = (Divisor $\times$ Quotient) + Remainder

$$x^3 + 13x^2 + 32x + 20 = (x+1)(x^2 + 12x + 20)$$

$= (x+1)(x^2+10x+2x+20)$

$= (x+1)(x+10)(x+2)$

Therefore, on factorization of $x^3+13x^2+32x+20$ we will get $(x+1)(x+10)(x+2)$

Q5 (iv) Factorise : $2y^3+y^2-2y-1$

Answer:

Given polynomial is $2y^3+y^2-2y-1$

Now, by hit and trial method we observed that $(y-1)$ is one of the factors of the given polynomial.

By long division method, we will get

$$
\begin{array}{r|l}
 & 2y^2+3y+1 \\
\hline
y-1 & 2y^3+y^2-2y-1 \\
 & 2y^3-2y^2 \\
 & -\quad + \\
\hline
 & 3y^2-2y-1 \\
 & 3y^2-3y \\
 & -\quad + \\
\hline
 & y-1 \\
 & y-1 \\
\hline
 & 0 \\
\hline
\end{array}
$$

We know that Dividend = (Divisor × Quotient) + Remainder

$$2y^3 + y^2 - 2y - 1 = (y-1)(2y^2 + 3y + 1)$$

$$= (y-1)(2y^2 + 2y + y + 1)$$

$$= (y-1)(2y+1)(y+1)$$

Therefore, on factorization of $2y^3 + y^2 - 2y - 1$ we will get $(y-1)(2y+1)(y+1)$

NCERT solutions for class 9 maths chapter 2 Polynomials Excercise: 2.5

Q1 (i) Use suitable identities to find the following product: $(x+4)(x+10)$

Answer:

We will use identity

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

Put $a = 4 \;\; and \;\; b = 10$

$$(x+4)(x+10) = x^2 + (10+4)x + 10 \times 4$$

$$= x^2 + 14x + 40$$

Therefore, $(x+4)(x+10)$ is equal to $x^2 + 14x + 40$

Q1 (ii) Use suitable identities to find the following product: $(x+8)(x-10)$

Answer:

We will use identity

$$(x+a)(x+b)=x^2+(a+b)x+ab$$

Put $a=8 \;\; and \;\; b=-10$

$$(x+8)(x-10)=x^2+(-10+8)x+8\times(-10)$$

$$=x^2-2x-80$$

Therefore, $(x+8)(x-10)$ is equal to $x^2-2x-80$

Q1 (iii) Use suitable identities to find the following product: $(3x+4)(3x-5)$

Answer:

We can write $(3x+4)(3x-5)$ as

$$(3x+4)(3x-5)=9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)$$

We will use identity

$$(x+a)(x+b)=x^2+(a+b)x+ab$$

Put $a = \frac{4}{3} \;\; and \;\; b = -\frac{5}{3}$

$= 9x^2 - 3x - 20$

Therefore, $(3x + 4)(3x - 5)$ is equal to $9x^2 - 3x - 20$

Q1 (iv) Use suitable identities to find the following product: $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2})$

Answer:

We will use identity

$(x + a)(x - a) = x^2 - a^2$

Put $x = y^2 \;\; and \;\; a = \frac{3}{2}$

$(y^2 + \frac{3}{2})(y^2 - \frac{3}{2}) = (y^2)^2 - \left(\frac{3}{2}\right)^2$

$= y^4 - \frac{9}{4}$

Therefore, $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2})$ is equal to $y^4 - \frac{9}{4}$

Q1 (v) Use suitable identities to find the following product: $(3 - 2x)(3 + 2x)$

Answer:

We can write $(3-2x)(3+2x)$ as

$$(3-2x)(3+2x) = -4\left(x-\frac{3}{2}\right)\left(x+\frac{3}{2}\right)$$

We will use identity

$$(x+a)(x-a) = x^2 - a^2$$

Put $a = \frac{3}{2}$

$$-4(x+\frac{3}{2})(x-\frac{3}{2}) = -4\left((x)^2 - \left(\frac{3}{2}\right)^2\right)$$

$$= 9 - 4x^2$$

Therefore, $(3-2x)(3+2x)$ is equal to $9-4x^2$

Q2 (i) Evaluate the following product without multiplying directly: 103×107

Answer:

We can rewrite 103×107 as

$$\Rightarrow 103 \times 107 = (100+3) \times (100+7)$$

We will use identity

$(x+a)(x+b)=x^2+(a+b)x+ab$

Put $x=100, a=3 \quad and \quad b=7$

$(100+3)\times(100+7)=(100)^2+(3+7)100+3\times 7$

$=10000+1000+21=11021$

Therefore, value of 103×107 is 11021

Q2 (ii) Evaluate the following product without multiplying directly: 95×96

Answer:

We can rewrite 95×96 as

$\Rightarrow 95\times 96=(100-5)\times(100-4)$

We will use identity

$(x+a)(x+b)=x^2+(a+b)x+ab$

Put $x=100, a=-5 \quad and \quad b=-4$

$(100-5)\times(100-4)=(100)^2+(-5-4)100+(-5)\times(-4)$

$= 10000 - 900 + 20 = 9120$

Therefore, value of 95×96 is 9120

Q2 (iii) Evaluate the following product without multiplying directly: 104×96

Answer:

We can rewrite 104×96 as

$\Rightarrow 104 \times 96 = (100 + 4) \times (100 - 4)$

We will use identity

$(x + a)(x - a) = x^2 - a^2$

Put $x = 100 \quad and \quad a = 4$

$(100 + 4) \times (100 - 4) = (100)^2 - (4)^2$

$= 10000 - 16 = 9984$

Therefore, value of 104×96 is 9984

Q3 (i) Factorise the following using appropriate identities: $9x^2 + 6xy + y^2$

Answer:

We can rewrite $9x^2 + 6xy + y^2$ as

$\Rightarrow 9x^2 + 6xy + y^2 = (3x)^2 + 2 \times 3x \times y + (y)^2$

Using identity $\Rightarrow (a + b)^2 = (a)^2 + 2 \times a \times b + (b)^2$

Here, $a = 3x \quad and \quad b = y$

Therefore,

$9x^2 + 6xy + y^2 = (3x + y)^2 = (3x + y)(3x + y)$

Q3 (ii) Factorise the following using appropriate identities: $4y^2 - 4y + 1$

Answer:

We can rewrite $4y^2 - 4y + 1$ as

$\Rightarrow 4y^2 - 4y + 1 = (2y)^2 - 2 \times 2y \times 1 + (1)^2$

Using identity $\Rightarrow (a - b)^2 = (a)^2 - 2 \times a \times b + (b)^2$

Here, $a = 2y \quad and \quad b = 1$

Therefore,

$4y^2 - 4y + 1 = (2y - 1)^2 = (2y - 1)(2y - 1)$

Q3 (iii) Factorise the following using appropriate identities: $x^2 - \frac{y^2}{100}$

Answer:

We can rewrite $x^2 - \frac{y^2}{100}$ as

$$\Rightarrow x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2$$

Using identity $\Rightarrow a^2 - b^2 = (a-b)(a+b)$

Here, $a = x \quad and \quad b = \frac{y}{10}$

Therefore,

$$x^2 - \frac{y^2}{100} = \left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$$

Q4 (i) Expand each of the following, using suitable identities: $(x+2y+4z)^2$

Answer:

Given is $(x+2y+4z)^2$

We will Use identity

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Here, $a = x, b = 2y \quad and \quad c = 4z$

Therefore,

$$(x+2y+4z)^2=(x)^2+(2y)^2+(4z)^2+2.x.2y+2.2y.4z+2.4z.x$$

$$=x^2+4y^2+16z^2+4xy+16yz+8zx$$

Q4 (ii) Expand each of the following, using suitable identities: $(2x-y+z)^2$

Answer:

Given is $(2x-y+z)^2$

We will Use identity

$$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$$

Here, $a=2x, b=-y \quad and \quad c=z$

Therefore,

$$(2x-y+z)^2=(2x)^2+(-y)^2+(z)^2+2.2x.(-y)+2.(-y).z+2.z.2x$$

$$=4x^2+y^2+z^2-4xy-2yz+4zx$$

Q4 (iii) Expand each of the following, using suitable identities: $(-2x+3y+2z)^2$

Answer:

Given is $(-2x+3y+2z)^2$

We will Use identity

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

Here, $a=-2x, b=3y \ \ and \ \ c=2z$

Therefore,

$$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+2.(-2x).3y+2.3y.2z+2.z.(-2x)$$

$$= 4x^2+9y^2+4z^2-12xy+12yz-8zx$$

Q4 (iv) Expand each of the following, using suitable identities: $(3a-7b-c)^2$

Answer:

Given is $(3a-7b-c)^2$

We will Use identity

$$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

Here, $x=3a, y=-7b \ \ and \ \ z=-c$

Therefore,

$(3a-7b-c)^2 = (3a)^2+(-7b)^2+(-c)^2+2.3a.(-7b)+2.(-7b).(-c)+2.(-c).3a$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

Q4 (v) Expand each of the following, using suitable identities: $(-2x + 5y - 3z)^2$

Answer:

Given is $(-2x + 5y - 3z)^2$

We will Use identity

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = -2x, b = 5y \ \ and \ \ c = -3z$

Therefore,

$(-2x + 5y - 3z)^2$
$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2.(-2x).5y + 2.5y.(-3z) + 2.(-3z).(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

Q4 (vi) Expand each of the following, using suitable identities: $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Answer:

Given is $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$

We will Use identity

$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x=\frac{a}{4}, y=-\frac{b}{2} \quad and \quad z=1$

Therefore,

$\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$

$=\frac{a^2}{16}+\frac{b^2}{4}+1-\frac{ab}{4}-b+\frac{a}{2}$

Q5 (i) Factorise: $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Answer:

We can rewrite $4x^2+9y^2+16z^2+12xy-24yz-16xz$ as

$\Rightarrow 4x^2+9y^2+16z^2+12xy-24yz-16xz$
$=(2x)^2+(3y)^2+(-4z)^2+2.2x.3y+2.3y.(-4z)+2.(-4z).2x$

We will Use identity

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Here, $a = 2x, b = 3y \quad and \quad c = -4z$

Therefore,

$$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz = (2x + 3y - 4z)^2$$

$$= (2x + 3y - 4z)(2x + 3y - 4z)$$

Q5 (ii) Factorise: $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Answer:

We can rewrite $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$ as

$$\Rightarrow 2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$$
$$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2.(-\sqrt{2}).y + 2.y.2\sqrt{2}z + 2.(-\sqrt{2}x).2\sqrt{2}z$$

We will Use identity

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Here, $a = -\sqrt{2}x, b = y \quad and \quad c = 2\sqrt{2}z$

Therefore,

$$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz = (-\sqrt{2}x + y + 2\sqrt{2}z)^2$$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

Q6 (i) Write the following cubes in expanded form: $(2x+1)^3$

Answer:

Given is $(2x+1)^3$

We will use identity

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Here, $a = 2x \quad and \quad b = 1$

Therefore,

$(2x+1)^3 = (2x)^3 + (1)^3 + 3.(2x)^2.1 + 3.2x.(1)^2$

$= 8x^3 + 1 + 12x^2 + 6x$

Q6 (ii) Write the following cube in expanded form: $(2a-3b)^3$

Answer:

Given is $(2a-3b)^3$

We will use identity

$$(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$$

Here, $x = 2a \quad and \quad y = 3b$

Therefore,

$$(2a-3b)^3 = (2a)^3 - (3b)^3 - 3.(2a)^2.3b + 3.2a.(3b)^2$$

$$= 8a^3 - 9b^3 - 36a^2b + 54ab^2$$

Q6 (iii) Write the following cube in expanded form: $\left[\frac{3}{2}x + 1\right]^3$

Answer:

Given is $\left[\frac{3}{2}x + 1\right]^3$

We will use identity

$$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

Here, $a = \frac{3x}{2} \quad and \quad b = 1$

Therefore,

$$= \frac{27x^3}{8} + 1 + \frac{27x^2}{4} + \frac{9x}{2}$$

Q6 (iv) Write the following cube in expanded form: $\left[x - \frac{2}{3}y\right]^3$

Answer:

Given is $\left[x - \frac{2}{3}y\right]^3$

We will use identity

$$(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$$

Here, $a = x \ and \ \ b = \frac{2y}{3}$

Therefore,

$$= x^3 - \frac{8y^3}{27} - 2x^2y + \frac{4xy^2}{3}$$

Q7 (i) Evaluate the following using suitable identities: $(99)^3$

Answer:

We can rewrite $(99)^3$ as

$\Rightarrow (99)^3 = (100-1)^3$

We will use identity

$(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

Here, $a = 100 \; and \;\; b = 1$

Therefore,

$(100-1)^1 = (100)^3 - (1)^3 - 3.(100)^2.1 + 3.100.1^2$

$= 1000000 - 1 - 30000 + 300 = 970299$

Q7 (ii) Evaluate the following using suitable identities: $(102)^3$

Answer:

We can rewrite $(102)^3$ as

$\Rightarrow (102)^3 = (100+2)^3$

We will use identity

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Here, $a = 100 \; and \;\; b = 2$

Therefore,

$$(100+2)^1 = (100)^3 + (2)^3 + 3.(100)^2.2 + 3.100.2^2$$

$$= 1000000 + 8 + 60000 + 1200 = 1061208$$

Q7 (iii) Evaluate the following using suitable identities: $(998)^3$

Answer:

We can rewrite $(998)^3$ as

$$\Rightarrow (998)^3 = (1000-2)^3$$

We will use identity

$$(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$$

Here, $a = 1000 \; and \;\; b = 2$

Therefore,

$$(1000-2)^1 = (1000)^3 - (2)^3 - 3.(0100)^2.2 + 3.1000.2^2$$

$$= 1000000000 - 8 - 6000000 + 12000 = 994011992$$

Q8 (i) Factorise the following: $8a^3 + b^3 + 12a^2b + 6ab^2$

Answer:

We can rewrite $8a^3 + b^3 + 12a^2b + 6ab^2$ as

$$\Rightarrow 8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + (b)^3 + 3.(2a)^2.b + 3.2a.(b)^2$$

We will use identity

$$(x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$$

Here, $x = 2a \quad and \quad y = b$

Therefore,

$$8a^3 + b^3 + 12a^2b + 6ab^2 = (2a+b)^3$$

$$= (2a+b)(2a+b)(2a+b)$$

Q8 (ii) Factorise the following: $8a^3 - b^3 - 12a^2b + 6ab^2$

Answer:

We can rewrite $8a^3 - b^3 - 12a^2b + 6ab^2$ as

$$\Rightarrow 8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - (b)^3 - 3.(2a)^2.b + 3.2a.(b)^2$$

We will use identity

$$(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$$

Here, $x = 2a \quad and \quad y = b$

Therefore,

$$8a^3 - b^3 - 12a^2b + 6ab^2 = (2a-b)^3$$

$$= (2a-b)(2a-b)(2a-b)$$

Q8 (iii) Factorise the following: $27 - 125a^3 - 135a + 225a^2$

Answer:

We can rewrite $27 - 125a^3 - 135a + 225a^2$ as

$$\Rightarrow 27 - 125a^3 - 135a + 225a^2 = (3)^3 - (25a)^3 - 3.(3)^2.5a + 3.3.(5a)^2$$

We will use identity

$$(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$$

Here, $x = 3 \quad and \quad y = 5a$

Therefore,

$$27 - 125a^3 - 135a + 225a^2 = (3-5a)^3$$

$$= (3-5a)(3-5a)(3-5a)$$

Q8 (iv) Factorise the following: $64a^3 - 27b^3 - 144a^2b + 108ab^2$

Answer:

We can rewrite $64a^3 - 27b^3 - 144a^2b + 108ab^2$ as

$$\Rightarrow 64a^3 - 27b^3 - 144a^2b + 108ab^2 = (4a)^3 - (3b)^3 - 3.(4a)^2.3b + 3.4a.(3b)^2$$

We will use identity

$$(x - y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$$

Here, $x = 4a \quad and \quad y = 3b$

Therefore,

$$64a^3 - 27b^3 - 144a^2b + 108ab^2 = (4a - 3b)^2$$

$$= (4a - 3b)(4a - 3b)(4a - 3b)$$

Q8 (v) Factorise the following: $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Answer:

We can rewrite $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ as

$$\Rightarrow 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \left(\frac{1}{6}\right)^3 - 3.(3p)^2.\frac{1}{6} + 3.3p.\left(\frac{1}{6}\right)^2$$

We will use identity

$$(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$$

Here, $x = 3p \quad and \quad y = \frac{1}{6}$

Therefore,

$$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = \left(3p - \frac{1}{6}\right)^3$$

$$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$$

Q9 (i) Verify: $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

Answer:

We know that

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

Now,

$$\Rightarrow x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

$$\Rightarrow x^3 + y^3 = (x+y)\left((x+y)^2 - 3xy\right)$$

$$\Rightarrow x^3 + y^3 = (x+y)\left(x^2 + y^2 + 2xy - 3xy\right) (\because (a+b)^2 = a^2 + b^2 + 2ab)$$

$$\Rightarrow x^3 + y^3 = (x+y)\left(x^2 + y^2 - xy\right)$$

Hence proved.

Q9 (ii) Verify: $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Answer:

We know that

$$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$$

Now,

$$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$$

$$\Rightarrow x^3 - y^3 = (x-y)\left((x-y)^2 + 3xy\right)$$

$$\Rightarrow x^3 - y^3 = (x-y)\left(x^2 + y^2 - 2xy + 3xy\right) (\because (a-b)^2 = a^2 + b^2 - 2ab)$$

$$\Rightarrow x^3 - y^3 = (x-y)\left(x^2 + y^2 + xy\right)$$

Hence proved.

Q10 (i) Factorise the following: $27y^3 + 125z^3$

Answer:

We know that

$$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

Now, we can write $27y^3 + 125z^3$ as

$$\Rightarrow 27y^3 + 125z^3 = (3y)^3 + (5z)^3$$

Here, $a = 3y \ \ and \ \ b = 5z$

Therefore,

$$27y^3 + 125z^3 = (3y + 5z)\left((3y)^2 + (5z)^2 - 3y.5z\right)$$

$$27y^3 + 125z^3 = (3y + 5z)\left(9y^2 + 25z^2 - 15yz\right)$$

Q10 (ii) Factorise the following: $64m^3 - 343n^3$

Answer:

We know that

$$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$$

Now, we can write $64m^3 - 343n^3$ as

$\Rightarrow 64m^3 - 343n^3 = (4m)^3 - (7n)^3$

Here, $a = 4m \quad and \quad b = 7n$

Therefore,

$64m^3 - 343n^3 = (4m - 7n)\left((4m)^2 + (7n)^2 + 4m.7n\right)$

$64m^3 - 343n^3 = (4m - 7n)\left(16m^2 + 49n^2 + 28mn\right)$

Q11 Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Answer:

Given is $27x^3 + y^3 + z^3 - 9xyz$

Now, we know that

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Now, we can write $27x^3 + y^3 + z^3 - 9xyz$ as

$\Rightarrow 27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + (y)^3 + (z)^3 - 3.3x.y.z$

Here, $a = 3x, b = y \quad and \quad c = z$

Therefore,

$$27x^3 + y^3 + z^3 - 9xyz = (3x + y + z)\left((3x)^2 + (y)^2 + (z)^2 - 3x.y - y.z - z.3x\right)$$

$$= (3x + y + z)\left(9x^2 + y^2 + z^2 - 3xy - yz - 3zx\right)$$

Q12 Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$

Answer:

We know that

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

Now, multiply and divide the R.H.S. by 2

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$$

$$= \frac{1}{2}(x + y + z)(x^2 + y^2 - 2xy + x^2 + z^2 - 2zx + y^2 + z^2 - 2yz)$$

$$= \frac{1}{2}(x + y + z)\left((x - y)^2 + (y - z)^2 + (z - x)^2\right)\left(\because a^2 + b^2 - 2ab = (a - b)^2\right)$$

Hence proved.

Q13 If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Answer:

We know that

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

Now, It is given that $x+y+z=0$

Therefore,

$$x^3+y^3+z^3-3xyz=0(x^2+y^2+z^2-xy-yz-zx)$$

$$x^3+y^3+z^3-3xyz=0$$

$$x^3+y^3+z^3=3xyz$$

Hence proved.

Q14 (i) Without actually calculating the cubes, find the value of each of the following: $(-12)^3+(7)^3+(5)^3$

Answer:

Given is $(-12)^3+(7)^3+(5)^3$

We know that

If $x+y+z=0$ then , $x^3+y^3+z^3=3xyz$

Here, $x=-12, y=7 \quad and \quad z=5$

$\Rightarrow x + y + z = -12 + 7 + 5 = 0$

Therefore,

$(-12)^3 + (7)^3 + (5)^3 = 3 \times (-12) \times 7 \times 5 = -1260$

Therefore, value of $(-12)^3 + (7)^3 + (5)^3$ is -1260

Q14 (ii) Without actually calculating the cubes, find the value of the following: $(28)^3 + (-15)^3 + (-13)^3$

Answer:

Given is $(28)^3 + (-15)^3 + (-13)^3$

We know that

If $x + y + z = 0$ then , $x^3 + y^3 + z^3 = 3xyz$

Here, $x = 28, y = -15 \quad and \quad z = -13$

$\Rightarrow x + y + z = 28 - 15 - 13 = 0$

Therefore,

$(28)^3 + (-15)^3 + (-13)^3 = 3 \times (28) \times (-15) \times (-13) = 16380$

Therefore, value of $(28)^3 + (-15)^3 + (-13)^3$ is 16380

Q15 (i) Give possible expressions for the length and breadth of the following rectangle, in which its area is given:

$25a^2 - 35a + 12$

Answer:

We know that

Area of rectangle is = $length \times breadth$

It is given that area = $25a^2 - 35a + 12$

Now, by splitting middle term method

$\Rightarrow 25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12$

$= 5a(5a - 4) - 3(5a - 4)$

$= (5a - 3)(5a - 4)$

Therefore, two answers are possible

case (i) :- Length = $(5a - 4)$ and Breadth = $(5a - 3)$

case (ii) :- Length = $(5a - 3)$ and Breadth = $(5a - 4)$

Q15 (ii) Give possible expressions for the length and breadth of the following rectangle, in which its area is given:

$35y^2 + 13y - 12$

Answer:

We know that

Area of rectangle is = $length \times breadth$

It is given that area = $35y^2 + 13y - 12$

Now, by splitting the middle term method

$\Rightarrow 35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (7y - 3)(5y + 4)$

Therefore, two answers are possible

case (i) :- Length = $(5y + 4)$ and Breadth = $(7y - 3)$

case (ii) :- Length = $(7y - 3)$ and Breadth = $(5y + 4)$

Q16 (i) What are the possible expressions for the dimensions of the cuboid whose volumes is given below?

Volume : $3x^2 - 12x$

Answer:

We know that

Volume of cuboid is = $length \times breadth \times height$

It is given that volume = $3x^2 - 12x$

Now,

$\Rightarrow 3x^2 - 12x = 3 \times x \times (x - 4)$

Therefore,one of the possible answer is possible

Length = 3 and Breadth = x and Height = $(x - 4)$

Q16 (ii) What are the possible expressions for the dimensions of the cuboid whose volumes is given below?

Volume : $12ky^2 + 8ky - 20k$

Answer:

We know that

Volume of cuboid is = $length \times breadth \times height$

It is given that volume = $12ky^2 + 8ky - 20k$

Now,

$\Rightarrow 12ky^2 + 8ky - 20k = k(12y^2 + 8y - 20)$

$= k(12y^2 + 20y - 12y - 20)$

$= k\left(4y(3y+5) - 4(3y+5)\right)$

$= k(3y+5)(4y-4)$

$= 4k(3y+5)(y-1)$

Therefore,one of the possible answer is possible

Length = $4k$ and Breadth = $(3y+5)$ and Height = $(y-1)$