Code Number: **A**

Medical | IIT-JEE | Foundations

Corp. Office: Aakash Educational Services Limited, 3rd Floor, Incuspaze Campus- 2, Plot No. 13, Sector- 18, Udyog Vihar, Gurugram, Haryana - 122015

Time: 3 hrs.

Mock Test Paper for Class-XII

Max. Marks: 70

MATHEMATICS

Roll No.

GENERAL INSTRUCTIONS

Read the following instructions carefully and follow them:

1. The Question paper consists of parts **A, B, C, D** and **E**
2. **Part A - I** consists of **15 Multiple choice** questions,

   Part A – II consists of **5 fill up the blanks** questions
3. All the questions of **Part A – I and II** are to be answered **compulsorily**
4. **Part B** consists of **9 short answer type** questions carrying **2 marks** each, out of which **6 questions** to be answered
5. **Part C** consists of **9 short answer type** questions carrying **3 marks** each, out of which **6 questions** to be answered
6. **Part D** consists of **7 long answer type** questions carrying **5 marks** each, out of which **4 questions** to be answered.
7. **Part E** consists of linear programming questions use the graph sheet for question.

PART-A

I. Select the correct alternative from the choices given below: 15 x 1 = 15

1. Consider the following equivalence relation R on Z, the set of integers $R = \{(a,b) \mid 2 \text{ divides } a-b\}$. If $[x]$ represents the equivalence class of x, then $[0]$ is the set

   (a) $\{0, \pm 2, \pm 4, \pm 6, \pm 8, ...\}$ (b) $\{1, \pm 3, \pm 5, \pm 7, \pm 9, ...\}$

   (c) $\{0, \pm 1, \pm 2, \pm 3, \pm 4, ...\}$ (d) $\{........, -4, -1, 2, 5, 8, ...\}$

2. If $\sin^{-1} x = \theta$ (the principal value branch of $\sin^{-1} x$ where $0 \le x \le 1$, then the range in which θ lies

   (a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left[0, \frac{\pi}{2}\right]$ (c) $[0, \pi]$ (d) $-1 \le x \le 1$

3. The product of matrices A and B is equal to a diagonal matrix. If the order of the matrix B is 2×3, then order of the matrix A is

   (a) 3×3 (b) 2×2 (c) 3×2 (d) 2×3

4. Let A be a 2×2 square matrix such that $A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ then value of $|\text{adj}A|$ is

   (a) 25 (b) 5 (c) 0 (d) I

5. For some fixed $a > 0$ and $x > 0, y = a^x + x^a + a^a$ then find $\frac{dy}{dx} =$

   (a) $a^x \log_e a + a^x (a-1) + aa^{a-1}$ (b) $a^x \log_e a + x^{a-1}$

   (c) $a^x \log_e a + ax^{a-1}$ (d) $a^x \log_e a + x^a \log x + a^a \log a$

6. Choose the statement that is true from the options given below:

   (a) Every polynomial function is not continuous

   (b) Every rational function is continuous

   (c) Every differentiable function is continuous

   (d) Every continuous function is differentiable

7. Let C be the circumference and A be the area of the circle

   Statement 1: The rate of change of the area with respective to radius is equal to C

   Statement 2: The rate of change of the area with respective to diameter is C/2

   (a) Only statement 1 is true (b) Only statement 2 is true

   (c) Both statements are true (d) Both statements are false

8. Function $f(x) = a^x$ is increasing on R, if

   (a) $a > 0$ (b) $a < 0$ (c) $0 < a < 1$ (d) $a > 1$

9. The anti-derivative of $\frac{1}{x\sqrt{x^2-1}}, x > 1$ with respective to x

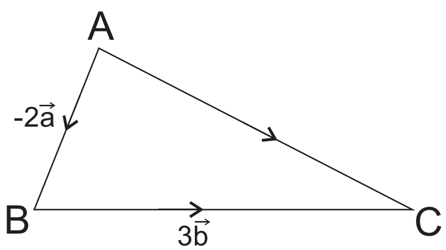
   (a) $\sin^{-1} x + C$ (b) $\cos^{-1} x + C$

   (c) $-\text{cosec}^{-1} x + C$ (d) $\cot^{-1} x + C$

10. Find the value of $\int_{-1}^{1} x^{99} dx =$

(a) 2 (b) 3 (c) 0 (d) 1

11. For the given figure, $\overline{AC}$ is



(a) $2\vec{a}-3\vec{b}$ (b) $3\vec{b}-2\vec{a}$ (c) $\vec{a}+\vec{b}$ (d) $2\vec{a}+\vec{b}$

12. The direction ratios of the vectors joining the points $P(2,3,0)$ & $Q(-1,-2,4)$ directed from P to Q are

(a) $(-3,-5,4)$ (b) $(-3,-5,-4)$ (c) $(-1,-2,-4)$ (d) $(1,1,1)$

13. The cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ then the vector equation is

(a) $\vec{r}=(-5\vec{i}+4\vec{j}-6\vec{k})+\lambda(3\vec{i}+7\vec{j}+2\vec{k})$ (b) $\vec{r}=(5\vec{i}+4\vec{j}-6\vec{k})+\lambda(3\vec{i}+7\vec{j}+2\vec{k})$

(c) $\vec{r}=(5\vec{i}-4\vec{j}+6\vec{k})+\lambda(3\vec{i}+7\vec{j}+2\vec{k})$ (d) $\vec{r}=(3\vec{i}+7\vec{j}+2\vec{k})+\lambda(5\vec{i}-4\vec{j}+6\vec{k})$

14. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black is

(a) $\frac{1}{26}$ (b) $\frac{1}{4}$ (c) $\frac{25}{102}$ (d) $\frac{25}{104}$

15. A and B are two events such that $P(A)=\frac{1}{2}, P(A\cup B)=\frac{3}{5}, P(B)=q$ then the value of q if A and B are mutually exclusive events

(a) $\frac{3}{10}$ (b) $\frac{1}{10}$ (c) $\frac{1}{5}$ (d) $\frac{7}{10}$

II. Fill in the blanks by choosing the appropriate word/words from those given below: 5 x 1 = 5

(-1, 6, 0, 1, 4, 3)

16. If $xy=81$, then $\frac{dy}{dx}$ at $x=9$ is ____________.

17. The absolute maximum value of the function f given by $f(x)=x^2, x\in[0,2]$ is ____________.

18. If m and n respectively are the order and degree of the differential equation $1+\left(\frac{dy}{dx}\right)^3=7\left(\frac{d^2y}{dx^2}\right)^2$ then $m-n=$ __________

19. The value of λ for which the vectors $2\vec{i}-3\vec{j}+4\vec{k}$ & $-4\vec{i}+\lambda\vec{j}-8\vec{k}$ are collinear is _______

20. If F be an event of a sample space S, then $P\left(\frac{S}{F}\right)=$ ___________

PART-B

III. Answer any SIX of the following questions in 3 – 5 sentences wherever applicable: 6 x 2 = 12

21. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

22. Find the area of the triangle whose, vertices are $(3,8),(-4,2)\ \&\ (5,1)$ using determinants

23. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right), -1<x<1$.

24. Find the interval in which of the function f given by $f(x)=x^2+2x-5$ is strictly increasing.

25. Find $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

26. Find the general solution of the differential equation $\frac{dy}{dx}=\frac{2x}{y^2}$

27. Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a}=3\vec{i}+\vec{j}+4\vec{k}$ and $\vec{b}=\vec{i}-\vec{j}+\vec{k}$

28. Find the angle between the pair of lines given by $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}\ \&\ \frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$

29. A die is thrown. If E is the event the number appearing is a multiple of '3' and F be the event the number appearing is 'even' then find whether E and F are independent?

PART-C

IV. Answer any SIX of the following questions: 6 x 3 = 18

30. Let L be the set of all lines in a plane and R be the relation in L defined as $R=\{(L_1,L_2): L_1 \text{ is perpendicular to } L_2\}$. Show that R is symmetric but neither reflexive nor transitive.

31. Find the simplest form of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x\neq 0$

32. Express the matrix $A=\begin{bmatrix}1 & 4\\ 6 & 7\end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.

33. Differentiate, $y=(\sin x)^x+\sin^{-1}x$ w.r.t.x.

34. Find the positive numbers whose sum is 15 and the sum of whose squares is minimum,

35. Integrate $\frac{x}{(x+1)(x+2)}$ with respect to x by partial fraction.

36. If $\vec{a},\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}|=3, |\vec{b}|=4\ \&\ |\vec{c}|=5$ and each vector is orthogonal to sum of the other two vectors then find $|\vec{a}+\vec{b}+\vec{c}|$

37. Derive the equation of the line in space passing through the point and parallel to the vector in the vector form.

38. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls, one of the two boxes I selected at random and a ball is drawn from the box which is found to be red. Find the probability that the red ball comes out from the box-II.

PART-D

V. Answer any FOUR of the following questions: 4 x 5 = 20

39. State weather the function $f: R \to R$ defined by $f(x) = 3 - 4x$ is one-one, onto or bijective. Justify your answer.

40. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ calculate AC, BC and $(A+B)C$, Also, verify that $(A+B)C = AC + BC$

41. Solve the system of linear equations by matrix method $4x + 3y + 2z = 60, 2x + 4y + 6z = 90$ & $6x + 2y + 3z = 70$

42. If $y = 3e^{2x} + 2e^{3x}$, then prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

43. Find the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and hence evaluate $\int \frac{dx}{\sqrt{25 - x^2}}$

44. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by the method of integration.

45. Find the general solution of the differential equation $\frac{dy}{dx} + 2y = \sin x$.

PART-E

VI. Answer the following questions 6 x 1 = 6

46. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^4 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4-x}}dx$

OR

Solve the following graphically, maximize $Z = 250x + 75y$ subjects to the constraints $x + y \le 60$, $25 + 5y \le 500, x \ge 0, y \ge 0$

47. If $A = \begin{bmatrix} 5 & 6 \\ 4 & 3 \end{bmatrix}$, satisfies the equation $A^2 - 8A - 9I = 0$ where I is 2×2 identify matrix and 0 is 2×2 zero matrix. Using this equation, find A^{-1}. **4 x 1 = 4**

OR

Find the value of k so that the function $f(x) = \begin{cases} kx + 1, & \text{if } x \le 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$ is a continuous at $x = 5$

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