Code Number: **A**

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Time: 3 hrs. **Mock Test Paper for Class-XII** **Max. Marks: 70**

MATHEMATICS

Answers & Solutions

1. Answer (a)
2. Answer (b)
3. Answer (c)
4. Answer (a)
5. Answer (c)
6. Answer (c)
7. Answer (c)
8. Answer (d)
9. Answer (c)
10. Answer (c)
11. Answer (b)
12. Answer (a)
13. Answer (c)
14. Answer (c)
15. Answer (b)
16. -1
17. 4
18. 0
19. 6
20. 1
21. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]=\tan^{-1}\left[2\cos\left(2,\frac{\pi}{6}\right)\right]$ $\left(\because \sin\frac{\pi}{6}=\frac{1}{2}\right)$

    $=\tan^{-1}\left(2\cos\frac{\pi}{3}\right)=\tan^{-1}\left(2.\frac{1}{2}\right)=\tan^{-1}1=\frac{\pi}{4}$
22. Let $A=(3,8)=(x_1,y_1),B=(-4,2)=(x_2,y_2)$ and $C=(5,1)=(x_3,y_3)$

Area of triangle ABC is given by $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

$= \frac{1}{2}\left[3(2-1) - 8(-4-5) + 1(-4-10)\right]$

$= \frac{1}{2}\left[3(1) - 8(-9) + 1(-14)\right]$

$= \frac{1}{2}[3 + 72 - 14] = \frac{1}{2}(61)$

Area of triangle ABC $= \frac{61}{2}$ Sq.units

23. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}x$

w.r.t x

$\frac{dy}{dx} = \frac{2}{1+x^2}$

24. Let, $f(x) = x^2 + 2x - 5 \Rightarrow f'(x) = 2x + 2$

Now, $f'(x) > 0 \Rightarrow 2x + 2 > 0 \Rightarrow x > -1$

Thus, $f(x)$ is increasing in $(-1, \infty)$

25. $I = \int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

Put, $\sin^{-1}x = t \Rightarrow \frac{1}{\sqrt{1-x^2}}dx = dt$, Also, $x = \sin t$

$I = \int t\sin t\, dt$

$= t(-\cos t) - \int(-\cos t).1\, dt$

$= -t\cos t + \sin t + c$

$I = -\sqrt{1-x^2}\sin^{-1}x + x + c$

26. Given, $\frac{dy}{dx} = \frac{2x}{y^2}$

Now, $\frac{dy}{dx} = \frac{2x}{y^2} \Rightarrow y^2dy = 2xdx$ (by separating the variables)

On integration we get,

$\int y^2 dy = \int 2x\, dx + c \Rightarrow \frac{y^3}{3} = x^2 + c$

27. Given $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$,

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$

$\vec{a} \times \vec{b} = i(1+4) - j(3-4) + \hat{k}(-3-1)$

$\vec{a} \times \vec{b} = 5\hat{i} + \hat{j} - 4\hat{k}$

Area of the parallelogram $= \left|\vec{a} \times \vec{b}\right| = \sqrt{(5)^2 + (1)^2 + (-4)^2} = \sqrt{25+1+16} = \sqrt{42}$ squnits

28. Let $\overrightarrow{b_1} = 3\hat{\imath} + 5\,\hat{\jmath} + 4\hat{k} \Rightarrow \left|\overrightarrow{b_1}\right| = \sqrt{3^2 + 5^2 + 4^2} = \sqrt{50} = 5\sqrt{2}$

$\overrightarrow{b_2} = \hat{\imath} + \hat{\jmath} + 2\hat{k} \Rightarrow \left|\overrightarrow{b_2}\right| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$

Now $\overrightarrow{b_1}.\overrightarrow{b_2} = (3,5,4),(1,1,2) = 3 + 5 + 8 = 16$

Let θ be the angle between $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ then

$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|.\left|\overrightarrow{b_2}\right|} = \frac{16}{5\sqrt{2}.\sqrt{6}} = \frac{16}{5\sqrt{12}} = \frac{16}{5.2\sqrt{3}} = \frac{8}{5\sqrt{3}}$

$\Rightarrow \theta = \cos^{-1}\frac{8}{5\sqrt{3}}$

29. The sample space is $S = \{1,2,3,4,5,6\}, n(S) = 6$

Now, $E = \{3,6\}, F = \{2,4,6\}$ and $E \cap F = \{6\}$

$\Rightarrow P(E) = \frac{2}{6} = \frac{1}{3}, P(F) = \frac{3}{6} = \frac{1}{2}$, and $P(E \cap F) = \frac{1}{6}$

Clearly, $P(E \cap F) = P(E),P(F) \Rightarrow$ the events E and F are independent.

30. <u>Reflexive:</u>

Since no line is perpendicular to itself $\Rightarrow (L_1, L_1) \notin R \forall L_1 \in L$

$\therefore$ R is not reflexive.

<u>Symmetric:</u>

$\forall L_1, L_2 \in L$, If $(L_1, L_2) \in R \Rightarrow L_1$ is perpendicular to line L_2

$\Rightarrow L_2$ is perpendicular to line L_1

$\Rightarrow (L_2, L_1) \in R$

$\therefore$ R is symmetric

<u>Transitive:</u>

$\forall L_1, L_2, L_3 \in L, (L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow L_1$ is perpendicular to line L_2 and the line L_2 is perpendicular to line L_3

$\Rightarrow L_1$ is not perpendicular to line L_3

$\Rightarrow (L_1, L_3) \notin R$

$\therefore$ R is not transitive

Hence R is symmetric but neither reflexive nor transitive.

31. $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$ Put, $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$ $\left(\because 1+\tan^2\theta = \sec^2\theta\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$ $\left(\text{writing } \sec\theta = \frac{1}{\cos\theta} \text{ and } \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$ $\left(\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right)$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

32. Consider, $A + A' = \begin{bmatrix} 1 & 4 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 10 & 14 \end{bmatrix}$

$P = \frac{1}{2}(A + A') = \begin{bmatrix} 1 & 5 \\ 5 & 7 \end{bmatrix}$

$P' = \begin{bmatrix} 1 & 5 \\ 5 & 7 \end{bmatrix} = P$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Consider, $A - A' = \begin{bmatrix} 1 & 4 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -Q$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Consider, $P + Q = \begin{bmatrix} 1 & 5 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 6 & 7 \end{bmatrix} = A$

Thus, the given matrix A is a sum of symmetric (P) and skew-symmetric (Q) matrices.

33. $y=(\sin x)^{x}+\sin^{-1}x$

Let $u=(\sin x)^{x}$ and $v=\sin^{-1}x$

$\therefore\ y=u+v$

D. w. r. t. x,

$\Rightarrow \quad \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$ ------------------ (1)

Now, $u=(sinx)^{x}$

Taking log

$\log u=\log(sinx)^{x}$

$logu=xlog(sinx)$

D w.r.t. x, we get

$\Rightarrow \quad \frac{1}{u}\frac{du}{dx}=x\frac{d}{dx}\log(sinx)+\log(sinx)\ \frac{d}{dx}(x)$

$=\frac{x}{sinx}cosx+\log(sinx)$

$\frac{du}{dx}=u[xcotx+logsinx]$

$\frac{du}{dx}=(sinx)^{x}[xcotx+logsinx]$ ----------------- (2)

Again, $v=sin^{-1}x$

D. w.r.t. x, we get

$\frac{dv}{dx}=\frac{1}{\sqrt{1-x^2}}$ ---------------- (3)

From eqns. (1), (2) and (3) we can conclude that

$\frac{dy}{dx}=(sinx)^{x}[xcotx+logsinx]+\frac{1}{\sqrt{1-x^2}}$

34. Let one of the number be x. Then the other number will be $15-x$.

$f''(x)=4$

$f''\left(\frac{15}{2}\right)=4$

Consider, $f(x)=x^2+(15-x)^2$

$f'(x)=2x+2(15-x)(-1)=4x-30$

For maxima or minima,

$f'(x)=0 \Rightarrow 4x-30=0 \Rightarrow x=\frac{15}{2}$

So $x=\frac{15}{2}$ is point of minima as $f''(x)$ is +ve

Hence, $f(x)$ will be minimum, when the numbers are $\frac{15}{2}$ *and* $\frac{15}{2}$

35. Let, $I=\int \frac{x}{(x+1)(x+2)}dx$

Let, $\frac{x}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$;

$x=A(x+2)+B(x+1)$

Put, $x=-1,\quad -1=A(-1+2)\Rightarrow A=-1$

Put. $x=-2,\quad -2=B(-2+1)\Rightarrow B=2$

$\int \frac{x}{(x+1)(x+2)}dx=\int\left(\frac{-1}{x+1}+\frac{2}{x+2}\right)dx$

$=-\log(x+1)+2\log(x+2)+c$

36. By data, $\vec{a}.(\vec{b}+\vec{c})=0, \vec{b}.(\vec{c}+\vec{a})=0$ and $\vec{c}.(\vec{a}+\vec{b})=0$. Now,

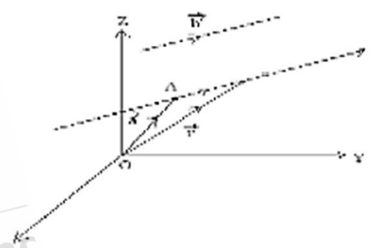
$|\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})$

$=\vec{a}.\vec{a}+\vec{b}.\vec{b}+\vec{c}.\vec{c}+\vec{a}.(\vec{b}+\vec{c})+\vec{b}.(\vec{c}+\vec{a})+\vec{c}.(\vec{a}+\vec{b})$

$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+0+0+0=9+16+25=0$

Hence, $|\vec{a}+\vec{b}+\vec{c}|=\sqrt{50}=5\sqrt{2}$

37. Consider a line l passing through a point A with position vector $\vec{a}$ and parallel to the vector $\vec{b}$. Let P be any point on l whose position vector is $\vec{r}$.



The line $\overrightarrow{AP}$ is parallel to the vector $\vec{b}$ if and only if

$\overrightarrow{AP}=\lambda\vec{b}$

$\overrightarrow{OP}-\overrightarrow{OA}=\lambda\vec{b}$

$\vec{r}-\vec{a}=\lambda\vec{b}$ --------(1)

$\therefore\ \vec{r}=\vec{a}+\lambda\vec{b}$

This is the equation of the line in vector form.

38. **Ans:** Let, E_1 be the event that Bag I is selected $\Rightarrow P(E_1)=\frac{1}{2}$

E_2 be the event that Bag II is selected $\Rightarrow P(E_2)=\frac{1}{2}$

Let A: Drawn ball is red

$P\left(\frac{A}{E_1}\right)=P$ (red ball from the bag I) $=\frac{3}{9}=\frac{1}{3}$. $P\left(\frac{A}{E_2}\right)=P(\text{red ball from the bag II})=\frac{5}{10}=\frac{1}{2}$

By Baye's theorem

$B\left(\frac{E_2}{A}\right)=\frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)}=\frac{\left(\frac{1}{2}\cdot\frac{1}{2}\right)}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{1}{2}}=\frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{3}+\frac{1}{2}\right)}=\frac{3}{5}$

39. **For One-One:**

Let x_1 and $x_2\in R(domain)$,then

$f(x_1)=f(x_2)$

$\Rightarrow 3-4x_1=3-4x_2$

$\Rightarrow 4x_1=4x_2$

$\Rightarrow x_1=x_2$

Thus, f is a one – one function

For Onto:

Let, $y \in R(Codomain)\ and\ f(x) = y$

$\Rightarrow 3 - 4x = y$

$\Rightarrow 4x = 3 - y$

$\Rightarrow x = \frac{3-y}{4} \in \mathrm{R}$ (domain)

That is, for all $y \in R$, there exist, $x = \frac{3-y}{4}\ such\ that,\ f(x) = y.$

Thus, f is onto function

Hence f is a bijective function.

40. Consider,

$$AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}_{3\times3} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 0-12+21 \\ -12+0+24 \\ 14+16+0 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}_{3\times1}$$

$$BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}_{3\times3} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 0-2+3 \\ 2+0+6 \\ 2-4+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}_{3\times1}$$

$$\text{Now, } AC + BC = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}_{3\times1} \text{ ------(1)}$$

$$\text{Consider, } A + B = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix}$$

$$(A+B)C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0-14+24 \\ -10+0+30 \\ 16+12+0 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}_{3\times1} \text{ ------(2)}$$

Clearly from (1) & (2), $(A+B)C = AC + BC$

41. **Ans:** The matrix form of the system is $AX = B$

where, $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}\ and\ B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

The cofactors of the elements of A are,

$A_{11} = +(12-12) = 0$	$A_{21} = -(9-4) = -5$	$A_{31} = +(18-8) = 10$
$A_{12} = -(6-36) = 30$	$A_{22} = +(12-12) = 0$	$A_{32} = -(24-4) = -20$
$A_{13} = +(4-24) = -20$	$A_{23} = -(8-18) = 10$	$A_{33} = +(16-6) = 10$

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$
$$= 4(0) + 3(30) - 2(20) = 50$$

Now, $X = A^{-1}B$

$$X = \frac{1}{|A|}(adjA).B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0-450+700 \\ 1800+0-1400 \\ -1200+900+700 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$$\Rightarrow x = 5, y = 8, z = 8$$

42. **Ans:** Given $y = 3e^{2x} + 2e^{3x}$

D.w.r.t. x,

$$\frac{dy}{dx} = 6e^{2x} + 6e^{3x}$$

Again D.w.r.t. x,

$$\frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x}$$

Consider,

$$LHS = \frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y$$

$$= 12e^{2x} + 18e^{3x} - 5(6e^{2x} + 6e^{3x}) + 6(3e^{2x} + 2e^{3x})$$

$$= 12e^{2x} + 18e^{3x} - 30e^{2x} - 30e^{3x} + 18e^{2x} + 12e^{3x}$$

$$= 0 = RHS$$

$$\therefore \frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

43. **Ans:** $I = \int \frac{dx}{\sqrt{a^2-x^2}}$ Put $x = a\sin\theta \Rightarrow dx = a\cos\theta\, d\theta$.

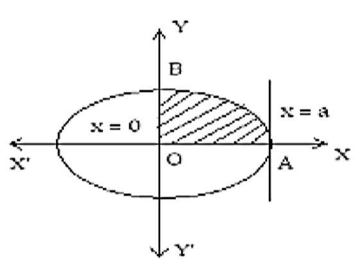
$= \int \frac{a\cos\theta\, d\theta}{\sqrt{a^2-a^2\sin^2\theta}}$ Also, $\theta = \sin^{-1}\frac{x}{a}$

$$= \int \frac{a\cos\theta d\theta}{a\sqrt{1-\sin^2\theta}}$$

$$= \int \frac{\cos\theta d\theta}{\cos\theta} = \int 1.d\theta$$

$$= \theta + C = \sin^{-1}\frac{x}{a} + C$$

Consider $I = \int \frac{1}{\sqrt{25-x^2}}dx = \int \frac{1}{\sqrt{5^2-x^2}}dx = \sin^{-1}\left(\frac{x}{5}\right) + C$

44. 

Ans: We have, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{y^2}{b^2}=1-\frac{x^2}{a^2}$

$\Rightarrow y=\frac{b}{a}\sqrt{a^2-x^2}$

From the figure we have,

Area of ellipse = 4 $(area\ OABO)$

= $4(area\ bounded\ by\ the$

curve, x – axis and the ordinates x = 0 and x = a)

$\Rightarrow$ Area of ellipse = $4\int_0^a y\,dx$

$=4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}$

$=\frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$=\frac{4b}{a}\left[\frac{a}{2}.0+\frac{a^2}{2}\sin^{-1}1\right] = \frac{4b}{a}\frac{a^2}{2}\frac{\pi}{2}=\pi ab$

45. **Ans:** We have, $\frac{dy}{dx}+2y=sinx$. This is a linear differential equation, with $P=2\ and\ Q=sinx$

Now, I.F. = $e^{\int Pdx}=e^{\int 2dx}\Rightarrow I.F=e^{2x}$

The general solution is given by

$y.e^{2x}=\int e^{2x}.sinxdx+c \Rightarrow y.e^{2x}=I+c$ ---------------- (1)

Consider,

$I=\int e^{2x}.sinx\,dx=e^{2x}(-cosx)-\int(-cosx).2e^{2x}dx$

$\Rightarrow I=-e^{2x}cosx+2e^{2x}sinx-\int sinx.4e^{2x}dx$

$\Rightarrow I=-e^{2x}cosx+2e^{2x}sinx-4I$

$\Rightarrow 5I=e^{2x}(2sinx-cosx)$

$\Rightarrow I=\frac{1}{5}e^{2x}(2sinx-cosx)$

Thus (1) becomes,

$y.e^{2x}=\frac{1}{5}e^{2x}(2sinx-cosx)+c$

$\Rightarrow y=\frac{1}{5}(2sinx-cosx)+ce^{-2x}$

46. **Ans:** Consider, $I=\int_0^a f(x)dx$

Put, $x=a-t \Rightarrow dx=-dt$

When, $x=0 \Rightarrow a-t=0 \Rightarrow t=a$,

$x=a \Rightarrow a-t=a \Rightarrow t=0$

$\therefore I=\int_0^a f(x)dx=\int_0^a f(a-t).(-dt)$

$=-\int_a^0 f(a-t)dt$

$I=\int_0^a f(a-x)dx$

Hence $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

Let, $I=\int_0^4 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}dx$........(i)

Replace $x\ by\ 4-x$

$I=\int_0^4 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}dx$.........(ii)

Adding (i) and (ii)we get,

$2I=\int_0^4 \frac{\sqrt{x}+\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx=\int_0^4 dx$

$\Rightarrow 2I=x]_0^4 \Rightarrow 2I=4 \Rightarrow I=\frac{4}{2}=2$

OR

Ans: Given $x + y \leq 60$ ---(1)

$x + y = 60$

$\Rightarrow \frac{x}{60} + \frac{y}{60} = 1$ (Divide both sides by 60)

$\therefore$The points on the line (1) are $(60,0)$ &$(0,60)$

Put $x = y = 0$ in (1) $\Rightarrow 0 \leq 60$ which is true

Therefore Solution of (1) is towards origin

And $25x + 5y \leq 500$

$5x + y \leq 100$ ---(2)

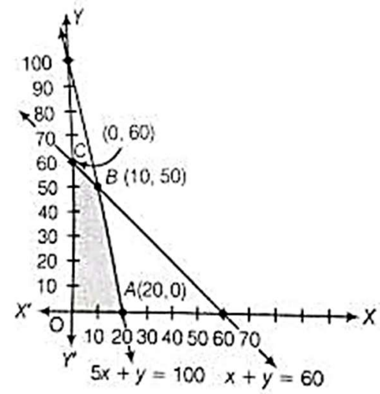
$25x + 5y = 500$

$\Rightarrow \frac{x}{20} + \frac{y}{100} = 1$ (Divide both sides by 500)

$\therefore$The points on the line (2) are $(20,0)$ &$(0,100)$

Put $x = y = 0$ in (2) $\Rightarrow 0 \leq 500$ which is true

Therefore Solution of (2) is towards origin



Here feasible region is bounded and corner points are (0,0), (0,60), (20,0) and (10,50).

Corner points	$Z = -3x + 4y$
(0,0)	0
(0,60)	4500
(20,0)	5000
(10,50)	6250

$\therefore Z_{max} = 6250\ at(10,50)$.

47. **Ans:** $A^2 = \begin{bmatrix} 5 & 6 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 25+24 & 30+18 \\ 20+12 & 24+9 \end{bmatrix} = \begin{bmatrix} 49 & 48 \\ 32 & 33 \end{bmatrix}$

Consider $A^2 - 8A - 9I = \begin{bmatrix} 49 & 48 \\ 32 & 33 \end{bmatrix} - \begin{bmatrix} 40 & 48 \\ 32 & 24 \end{bmatrix} - \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

$A^2 - 8A - 9I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 - 8A - 9I = O$

To find A^{-1} *consider*,

$A^2 - 8A - 9I = O$

$\Rightarrow -9I = -A^2 + 8A$

$\Rightarrow -9I.A^{-1} = (-A^2 + 8A)A^{-1}$ (Post multiply by A^{-1})

$\Rightarrow -9I.A^{-1} = -AAA^{-1} + 8AA^{-1}$

$\Rightarrow -9I.A^{-1} = -AI + 8I$ $\quad (\because AA^{-1} = I)$

$\Rightarrow -9A^{-1} = -A + 8I$ $\quad (\because AI = A)$

$\Rightarrow -9A^{-1} = \begin{bmatrix} -5 & -6 \\ -4 & -3 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$\Rightarrow A^{-1} = -\frac{1}{9}\begin{bmatrix} 3 & -6 \\ -4 & 5 \end{bmatrix}$

OR

Ans: LHL = $\lim_{x\to 5^-} f(x) = \lim_{x\to 5} kx + 1 = k(5) + 1 = 5k + 1$

RHL = $\lim_{x\to 5^+} f(x) = \lim_{x\to 5} 3x - 5 = 3(5) - 5 = 10$

But, $f(x)$ is continuous at $x = 5$

$\therefore LHL = RHL$

$5k + 1 = 10$

$5k = 9$

$\therefore k = \frac{9}{5}$