Code Number: **A**

Medical | IIT-JEE | Foundations

Corp. Office: Aakash Educational Services Limited, 3rd Floor, Incuspaze Campus- 2, Plot No. 13, Sector- 18, Udyog Vihar, Gurugram, Haryana - 122015

Time: 3 hrs. **Mock Test Paper for Class-XII** **Max. Marks: 75**

MATHEMATICS

Paper - II(A)

Answers & Solutions

1 Multiplicative Inverse of $7+24i = \frac{1}{7+24i}$

$$= \frac{1}{7+24i} \times \frac{7-24i}{7-24i} = \frac{7-24i}{49+576} = \frac{1}{625}(7-24i)$$

2 $z = -\sqrt{7} + i\sqrt{21}$

Polar form of Z =argument of z = $\pi - \alpha$ where $\alpha = \text{Tan}^{-1}\left|\frac{y}{x}\right|$

$$= \pi - \frac{\pi}{3} \qquad = \text{Tan}^{-1}\left(\frac{\sqrt{21}}{\sqrt{7}}\right)$$

$$= \frac{2\pi}{3} \qquad = \text{Tan}^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$

$$|z| = \sqrt{7+21} = \sqrt{28} = 2\sqrt{7}$$

$$\therefore |z| = |z|(\cos\theta + i\sin\theta)$$

$$= 2\sqrt{7}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

3 $x = \text{cis}\theta = \cos\theta + i\sin\theta$

$$x^6 = (\cos\theta + i\sin\theta)^6 = \cos 6\theta + i\sin 6\theta$$

$$\frac{1}{x^6} = \frac{1}{\cos 6\theta + i\sin 6\theta} \times \frac{\cos 6\theta - i\sin 6\theta}{\cos 6\theta - i\sin 6\theta} = \frac{\cos 6\theta - i\sin 6\theta}{\cos^2 6\theta + \sin^2 6\theta}$$

$$= \frac{\cos 6\theta - i\sin 6\theta}{1} = \cos 6\theta - i\sin 6\theta$$

$$x^6 + \frac{1}{x^6} = \cos 6\theta + i\sin 6\theta + \cos 6\theta - i\sin 6\theta = 2\cos 6\theta$$

4 $(x-\alpha)(x-\beta)=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-(7+2\sqrt{5}+7-2\sqrt{5})x+(7+2\sqrt{5})(7-2\sqrt{5})=0$

$x^2-14x+29=0$

5 Let $\alpha=\alpha,\beta=2,\gamma-1$, are roots of $2x^3+x^2-7x-6=0$

$\alpha.\beta.\gamma$ = product of roots $=-\frac{d}{a}=\frac{-(-6)}{2}=3$

$\alpha.2.(-1)=3$

$\boxed{\alpha=-\frac{3}{2}}$

6 ${}^nP_r=5040$ ${}^nC_r=210$

$\frac{n!}{(n-r)!}=5040$ ---(1) $\frac{n!}{(n-r)!r!}=210$ -----(2)

Divide, $\frac{(1)}{(2)}$ $\frac{\frac{n!}{(n-r)!}}{\frac{n!}{(n-r)!(r!)}}=\frac{5040}{210}$

$\Rightarrow r!=\frac{168}{7}=24=4!$

$\boxed{r=4}$

Using $\frac{n!}{(n-r)!}=5040$

$\Rightarrow \frac{n!}{(n-4)!}=5040$

$\Rightarrow \frac{n.(n-1).(n-2).(n-3).(n-r)!}{(n-r)!}=5040$

$7\times 8+9\times 10=5040$

$\therefore \boxed{n=10}$

7 N = 12

No. of diagonals of an 'n' sided polygon

$={}^n C_2-n$

$={}^{12}C_2-12=\frac{12!}{10!2!}-12$

$=\frac{12\times 11}{2}-12$

= 66 – 12

= 54

8 $(2x+3y+z)^7$

The number of terms in above expansion

$= {}^{n+r-1}C_{r-1} = {}^{7+3-1}C_{3-1} = {}^{9}C_2$

$= \frac{9\times 8}{2} = 36$ terms

9 c.v. $= \frac{\sigma}{\mu}\times 100$ = co-eff. Of variation

$60 = \frac{21}{\mu_1}\times 100$

$\mu_1 = \frac{21}{60}\times 100$

= 7 × 5

=35

$70 = \frac{16}{\mu_2}\times 100$

$\mu_2 = \frac{16\times 100}{70} = \frac{160}{7} = (22)\frac{6}{7}$ = 22.85

10 $P(X=1) = P(X=2)$

$\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$

$\frac{\lambda}{1} = \frac{\lambda^2}{2}$

$2\lambda = \lambda^2$

$\lambda(\lambda-2) = 0$

$\lambda = 0,2$

λ = mean must be a positive number

Calculating

$P(x=5)$ for $\lambda = 2$

$P(x=5) = \frac{e^{-2}2^5}{5!}$

$= \frac{32}{120}\times e^{-2}$

$= \frac{4}{15}\times e^{-2}$

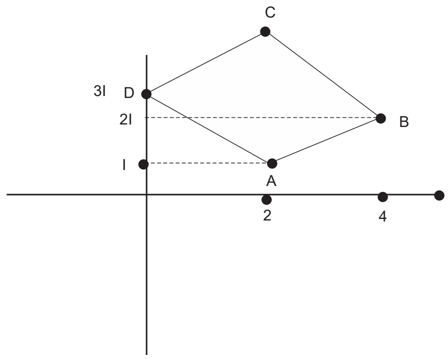
11 Let A = 2+I, B = 4+2i, C = 2 + 5i, D = 3!,

A = (2, 1)

B = (4, 2)

C = (2, 5)

D = (0, 3)



$AB = \sqrt{(4-2)^2 + (2-1)^2 1} = \sqrt{5}$

$BC = \sqrt{(4-2)^2 + (2-5)^2} = \sqrt{2^2 - (-3)^2} = \sqrt{13}$

$CD = \sqrt{2^2 + 2^2} = \sqrt{8}$

$AD = \sqrt{2^2 + 2^2} = \sqrt{8}$

$AC = \sqrt{0^2 + 4^2} = 4$

The vertices do not form a square $\because AB \neq BC \neq CD$

12 Let y_0 a value of given function. Then

$\exists x \in R$ such that, $y = \dfrac{x^2 + 34x - 71}{x^2 + 2x - 7}$

If y = 1, then clearly $y \notin (5,9)$

Let $y \notin 1$. Then the equation,

$y(x^2 + 2x - 7) = x^2 + 34x - 71$ is a quadratic equation and x is a real root of it.

$\therefore$ $(y - 1)x^2 + (2y - 34)x - (7y - 71) = 0$ is a quadratic equation having a real root x. Since all the coefficient of this quadratic equation are real, the other root of equation is also real. i.e,.

$\Delta = (2y - 34)^2 + 4(y-1)(7y-71) \geq 0$

$\Rightarrow 4y^2 + 1156 - 136y + 4(7y^2 - 71y - 7y + 71) \geq 0$

$\Rightarrow 32y^2 - 447y + 1440 \geq 0$

$\Rightarrow y^2 - 14y + 45 \geq 0$

$\Rightarrow (y-5)(y-9) \geq 0$

R lies between 5 & 9

13 Req. Sum = Sum of n digits × (n – 1)! (1111)

= 25 × (5 – 1)! × 1111 $\begin{pmatrix} n = 5 \\ 1111 \rightarrow 4\text{digits} \end{pmatrix}$

= 25 × 24 × 1111

= 666600

14 Consider,

$$\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{\dfrac{(4n)!}{(4n-2n)!(2n)!}}{\dfrac{2n!}{(2n-n)!n!}}$$

$$= \frac{(4n)!(2n)!(2n)!}{(2n)!n!n!} = \frac{(4n)!n!n!}{(2n)!(2n)!(2n)!}$$

$$= \frac{(n!)^2[4n(4n-1)(4n-2).........3.2.1]}{(2n)![(2n)(2n-1)(2n-2).......3.2.1]^2}$$

$$= \frac{(n!)^2[4n(4n-2).....4.2][(4n-1)(4n-3).....3.1]}{(2n)![(2n)(2n-2).........4.2]^2[(2n-1)(2n-3)........3.1]^2}$$

$$= \frac{(n!)^2[2^{2n}(2n)(2n-1).....2.1][(4n-1)(4n-3).....3.1]}{(2n)![2^n(n)(n-1)(n-2).........2.1]^2[(2n-1)(2n-3)........3.1]^2}$$

$$= \frac{(n!)^2[2^{2n}(2n)!][(4n-1)(4n-3)......3.1]}{(2n)![2^n n!]^2[(2n-1)(2n-3)......3.1]^2}$$

$$= \frac{(n!)^2.2^{2n}.(2n)![1.3.5........(4n-1)]}{(2n)!.2^2.(n!)^2[1.3.5.....(2n-1)]^2}$$

$$= \frac{[1.3.5.......(4n-1)]}{\{1.3.5.........(2n-1)\}^2}$$

$$\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1.3.5......(4n-1)}{\{1.3.5.......(2n-1)\}^2}$$

15 $$\frac{x+4}{(x^2-4)(x+1)} = \frac{x+4}{(x-2)(x+2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x+1}$$

$$\frac{x+4}{(x-2)(x+2)(x+1)} = \frac{A(x+2)(x+1)+B(x-2)(x+1)+c(x-2)(x+2)}{(x-2)(x+2)(x+1)}$$

$$x+4 = A(x+2)(x+1)+B(x-2)(x+1)+c(x-2)(x+2)$$

Put x = 2

$6 = 12A \Rightarrow \boxed{A = \frac{1}{2}}$

Put x = –1

$3 = C(-1-2)(-1+2)$

$3 = -3C \Rightarrow \boxed{c = -1}$

Put x = –2

$-2+4 = B(-2-2)(-2+1)$

$2 = -4B(-1)$

$2 = 4B$

$5=\frac{2}{4}=\frac{1}{2}$

$B=\frac{1}{2}$

$\boxed{B=\frac{-5}{6}}$

$$\therefore \frac{x+4}{(x^2-4)(x+1)}=\frac{\left(\frac{1}{2}\right)}{x-2}+\frac{\left(\frac{1}{2}\right)}{x+2}+\frac{-1}{x+1}$$

$$=\frac{1}{2(x-2)}+\frac{1}{2(x+2)}-\frac{1}{x+1}$$

16 $P(r)=\frac{2}{3}$

$P(b)=\frac{5}{9}$

$P(r\cup b)=\frac{5}{9}$

$P(r\cap b)=?$

$P(r\cap b)=p(r)+p(b)-p(r\cup b)$

$=\frac{2}{3}+\frac{5}{9}-\frac{5}{9}$

$=\frac{2}{3}$

Using inclusion exclusion principle.

17 $P(A\cap B)=P(A).P(B)$ ($\because$ A & B are independent events)

$P(A)=0.6, P(B)=0.7$

(i) $P(A\cap B)=P(A).P(B)=0.6\times 0.7=0.42$

(ii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

= 0.6 + 0.7 – 0.42 = 1.3 – 0.42

= 0.88

$=\frac{0.7}{P}=\frac{0.7\times 0.6}{0.7}$

(iii) $P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{P(B).P(A)}{P(A)}$

$=\frac{0.7\times 0.6}{0.6}=0.7=\frac{7}{10}$

(iv) $P(\overline{A}\cap\overline{B})=P(\overline{A\cup B})$

$=1-P(A\cup B)$

= 1 – 0.88 (from (ii))

= 0.12

$P(A^C\cap B^C)$

$=P(A^C).P(B^C)=[1-PA][1-P(B)]=(1-0.6)(1-0.7)$

$=(0.4)(0.3)$

= 0.12

18 $x^2 - 2x + 4 = 0$

$$x = \frac{2 \pm \sqrt{-12}}{2} = 1 \pm i\sqrt{3}$$

$$x = 1 \pm i\sqrt{3} = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2(\cos 60^\circ + \sin 60^\circ) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$\alpha^n = 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right) \text{-----}(1)$$

Similarly

$$\beta^n = 2^n\left(\cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right)$$ ($\because \alpha \,\&\, \beta$ are conjugate to each other)

$$(1)+(2)\alpha^n + \beta^n = 2^n\left[\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3} + \cos\frac{n\pi}{3} - \sin\frac{n\pi}{3}\right] = 2.2^n\left[\cos\frac{n\pi}{3}\right] = 2^{n+1}\cos\frac{n\pi}{3}$$

19 $4x^3 - 24x^2 + 23x + 18 = 0$

Standard form = $ax^3 + bx^2 + cx + d = 0$

$a = 4$, $b = -24$, $c = 23$, $d = 18$

Let the roots are $\alpha - \beta, \alpha, \alpha + \beta$ (λ = common difference)

Sum of the roots $= -\frac{b}{a} = -\frac{(-24)}{4} = 6$

$$\alpha - \beta + \alpha + \alpha + \beta = 3\alpha = 6 \boxed{\alpha = 2}$$

Product of roots = $(\alpha - \beta)\alpha(\alpha + \beta) = \alpha(\alpha^2 - \beta^2) = -\frac{18}{4}$

$$2(2^2 - \beta^2) = -\frac{9}{2}$$

$$4 - \beta^2 = -\frac{9}{4}$$

$$\beta^2 = \frac{25}{4}$$

$$\beta = \pm\frac{5}{2} \quad \left\langle \begin{matrix} +\frac{5}{2} \\ -\frac{5}{2} \end{matrix} \right.$$

Now $\beta = \frac{5}{2}$ $\qquad \alpha + \beta = 2 + \frac{5}{2} = \frac{9}{2}$

$$\alpha - \beta = 2 - \frac{5}{2} = \frac{4-5}{2}$$

$$\alpha - \beta = \frac{-1}{2}$$

$$\alpha = 2$$

$$\alpha + \beta = 2 + \frac{5}{2} = \frac{9}{2}$$

$\therefore$ The roots are $\left(\frac{-1}{2}, 2, \frac{9}{2}\right)$

20 The coefficient of r^{th}, $(r + 1)^{th}$, $(r + 2)^{th}$ terms in $(1 + x)^n$ are ${}^nC_{r-1}$, ${}^nC_r$, ${}^nC_{r+1}$

Given that ${}^nC_{r-1}$, ${}^nC_r$, ${}^nC_{r+1}$ are in A.P.

$$\Rightarrow 2.{}^nC_r = {}^nC_{r-1} + {}^nC_{r+1} \Rightarrow 2 = \frac{{}^nC_{r-1}}{{}^nC_r} + \frac{{}^nC_{r+1}}{{}^nC_r}$$

$$\Rightarrow 2 = \frac{r}{n-r+1} + \frac{n-r}{r+1}\left(\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \text{and} \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1}\right)$$

$$\Rightarrow 2 = \frac{r(r+1)+(n-r)(n-r+1)}{(n-r+1)(r+1)}$$

$$\Rightarrow 2(n-r+1)(r+1)+(n-r)(n-r+1)$$

$$\Rightarrow 2nr + 2n - 2r^2 - 2r + 2r + 2 = r^2 + r + n^2 - nr + n - nr + r^2 - r$$

$$\Rightarrow n^2 - 4nr - n + 4r^2 - 2 = 0 \Rightarrow n^2 - n(4r+1) + 4r^2 - 2 = 0$$

21 The series is.

$$n = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + ...\infty \text{----(1)}$$

It's a binomial expansion of form

$$(1+y)^n = 1 + ny + \frac{n(n-1)y^2}{2!} + \frac{n(n-1)(n-2)y^3}{3!} + ...\infty \text{----(2)}$$

Compare (2) with the given terms of equation (1)

$$ny = \frac{1}{5} \;\&\; \frac{n(n-1)y^2}{2!} = \frac{1.3}{5.10} = \frac{3}{50}$$

$$\boxed{y = \frac{1}{5n}}$$

$$\frac{n(n-1)}{2!}\left(\frac{1}{5n}\right)^2 = \frac{3}{50}$$

$$\frac{n(n-1)}{2}\frac{1}{25n^2} = \frac{3}{50}$$

$$\frac{n-1}{50n} = \frac{3}{50}$$

$$n - 1 = 3n$$

$$2n = -1$$

$$n = \frac{-1}{2}$$

Finding ‘g’

$$y = \frac{1}{5n} = -\frac{2}{5}$$

Now, $(1+x^2) = (1+y)^{n\times 2} = \left(1-\frac{2}{5}\right)^{-\frac{1}{2}\times 2} = \left(\frac{3}{5}\right)^{-\frac{1}{2}\times 2}$

$$x^2 + 2x + 1 = \frac{5}{3}$$

$$3(x^2 + 2x + 1) = 3 \times \frac{5}{3}$$

$$3x^2 + 6x + 3 = 5$$

$3x^2 + 6x = 2$ (A misprint in question)

22 The mean deviation about the mean is computed using the table shown below.

Marks Obtained (C.I)	No. of Students (f_i)	Mid Point (x_i)	$d_i = \frac{x_i - A}{h}$	$f_i d_i$	$\lvert x_1 - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
0-10	5	5	–2	–10	22	110
10-20	8	15	–1	–8	12	96
20-30	15	25	0	0	2	30
30-40	16	35	1	16	8	128
40-0	6	45	2	12	18	108
	N = 50			$\sum f_i d_i = 10$		$\sum f_i \lvert x_i - \bar{x}\rvert = 472$

Here, h = 10, N = 50; Let, Assumed mean (A) =25

Mean, $\bar{x} = A + \left[\frac{\left(\sum f_i d_i\right)}{N}\right] \times h = 25 + \left(\frac{10}{50}\right)10 = 27$

$\therefore$ Mean deviation from the mean = $= \frac{1}{N}\sum f_i \lvert x_i - \bar{x}\rvert = \frac{1}{50}(472) = 9.44$

23 Let B_1, B_2 and B_3 be events of choosing

Box 1, 2 and 3

$$\rightarrow P(B_1) = \frac{2}{6} = \frac{1}{3}$$

$$\rightarrow P(B_2) = \frac{2}{6} = \frac{1}{3}$$

$$\rightarrow P(B_3) = \frac{2}{6} = \frac{1}{3}$$

Let R be the event of drawing a red ball.

Total balls in B_1 = 2W + 1B + 2R = 5

$\therefore P(R/B_1) = \frac{2}{5}$

Total balls in B_2 = 3W + 2B + 4R = 9

$\therefore P(R/B_2) = \frac{4}{9}$

$\Rightarrow$ Total balls in B_3 = 4W + 3B + 2R = 9

$\therefore P(R/B_3) = \frac{2}{9}$

$P(R) = P(R/B_1).P(B_1) + P(R/B_2).P(B_2) + P(R/B_3).P(B_3)$

$P(R) = \frac{2}{5}.\frac{1}{3} + \frac{4}{9}.\frac{1}{3} + \frac{2}{9}.\frac{1}{3} = \frac{2}{15} + \frac{4}{27} + \frac{2}{27} = \frac{2}{15} + \frac{6}{27} = \frac{18+30}{135} = \frac{48}{135} = \frac{16}{45}$

Using Baye's theorem

$$P(B_1/R) = \frac{P(R/B_1)P(B_1)}{P(R)} = \frac{P(B_2)P(R/R_2)}{P(B_1).P(R/B_1) + P(B_2)P(R)(B_2) + P(B_3)P(R/B_3)}$$

$$= \frac{\frac{1}{3}\times\frac{4}{9}}{\frac{1}{3}\times\frac{2}{5} + \frac{1}{3}\times\frac{4}{9} + \frac{1}{3}\times\frac{2}{9}} = \frac{\frac{4}{27}}{\frac{18+20+10}{3\times3\times9}} = \frac{20}{48} = \frac{5}{12}$$

24 (i) Sum of all the probabilities = 1

$k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k - 1 = 0$

$$k = \frac{-9 \pm \sqrt{81+40}}{2.10} = \frac{-9 \pm 11}{20} = -\frac{20}{20} \bigg| \frac{2}{10\times2} = -1, \frac{1}{10}$$

$\therefore k = \frac{1}{10}$

(ii) The mean (μ)

$= 1K + 2(2k) + 3(2k) + 4(3k) + 5(k^2) + 6(2k^2) + 7(7k^2 + K)$

$= k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$

$= 30k + 66k^2$

$= 30 \times \frac{1}{10} + \frac{66}{100} = \frac{300}{100} + \frac{66}{100} = \frac{366}{100} = 3.66$

(iii) $P(0 < x < 5) =$

$= P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$k = \frac{1}{10}$

$= 0.1 + 0.2 + 0.2 + 0.3 = 0.8$

$P(0 < x < 5) = 0.8$