Code Number: **A**

Aakash

Medical | IIT-JEE | Foundations

Corp. Office: Aakash Educational Services Limited, 3rd Floor, Incuspaze Campus- 2, Plot No. 13, Sector- 18, Udyog Vihar, Gurugram, Haryana - 122015

Time: 3 hrs. **Mock Test Paper for Class-XII** **Max. Marks: 75**

MATHEMATICS

Paper - II(B)

Answers & Solutions

1 Given centre (h, k) = (1, 4)

Radius (r) = 5

Equation of circle $(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 4)^2 = 5^2$

$x^2 + 1 - 2x + y^2 \;\; 16 - 8y = 25$

$x^2 + y^2 - 2x - 8y + 17 - 25 = 0$

$x^2 + y^2 - 2x - 8y - 8 = 0$

2 Given points A(1, 3) B(2, k)

Given circle $x^2 + y^2 = 35$

Given A(1, 3) B(2, k) are conjugate points

$S_{12} = 0$

$x_1x_2 + y_1y_2 - 35 = 0$

(1) (2) + 3(k) –35 = 0

2 + 3k – 35 = 0

3k = 33

k = 11

3 Given circles

$S = 2x^2 + 2y^2 + 3x + 6y - 5 = 0$; $S' = 3x^2 + 3y^2 - 7x + 8y - 11 = 0$

$S = x^2 + y^2 + 3/2x + 3y - 5/2 = 0$ $\quad = x^2 + y^2 - \frac{7}{3}x + \frac{8}{3}y - \frac{11}{3}$

Radical axis is $S - S^1 = 0$

$$x^2 + y^2 + \frac{3}{2}x + 3y - \frac{5}{2} - \left(x^2 + y^2 - \frac{7}{3}x + \frac{8}{3}y - \frac{11}{3}\right) = 0$$

$$x^2+y^2+\frac{3}{2}x+3y-\frac{5}{2}-x^2-y^2+\frac{7}{3}x-\frac{8}{3}y+\frac{11}{3}=0$$

$$\left(\frac{3}{2}+\frac{7}{3}\right)x+\left(3-\frac{8}{3}\right)y-\frac{5}{2}+\frac{11}{3}=0$$

$$\left(\frac{9+14}{6}\right)x+\left(\frac{9-8}{3}\right)y+\left(\frac{-15+22}{6}\right)=0$$

$$\frac{23x}{6}+\frac{y}{3}+\frac{7}{6}=0$$

$$23x+2y+7=0$$

4 Let P (x_1, y_1) be any point on the parabola

$y^2 = 8x$

$4a = 8 \Rightarrow a = 2$

Focal distance = 10

$$|x_1+a|=10$$

$$x_1+2=10 \Rightarrow x_1=8$$

$$\therefore y_1^2=8x_1 \Rightarrow y_1^2=8(8) \Rightarrow y_1=\pm 8$$

Required points are (8, 8) & (8, –8)

5 Given $e=\frac{5}{4}$

W.K.T $\frac{1}{e^2}+\frac{1}{e_1^2}=1$

$$\frac{1}{\left(\frac{25}{16}\right)}+\frac{1}{e_1^2}=1$$

$$\frac{16}{25}+\frac{1}{e_1^2}=1$$

$$\frac{1}{e_1^2}=1-\frac{16}{25}$$

$$\frac{1}{e_1^2}=\frac{25-16}{25}$$

$$\frac{1}{e_1^2}=\frac{9}{25} \Rightarrow \frac{1}{e_1}=\frac{3}{5}$$

Conjugate eccentricity = $\frac{5}{3}$

6 $\int e^x \sin(e^x)dx$

Put $e^x = t$

Diff. w.r.t to ‘x’

$$e^x = \frac{dt}{dx} \Rightarrow e^x.dx = dt$$

$$\int \sin(t)dt = -\cos t + c = -\cos(e^x) + c$$

7 $\int e^x(\sin x + \cos x)dx$

$f(x) = \sin x; f'(x) = \cos x$

$$\boxed{\therefore \int e^x(f(x) + f'(x))dx = e^x f(x) + c}$$

$= e^x.\sin x + c$

8 $\int_2^3 \frac{2x}{1+x^2}dx$

$$\boxed{\therefore \int \frac{f'(x)}{f(x)}dx = \log|f(x)| + c}$$

$= \left[\log|1+x^2|\right]_2^3$

$= |\log 10 - \log 5|$

$= \log\left(\frac{10}{5}\right)$

= log 2

9 $\int_0^{\pi/2} \sin^7 x\, dx$

$\therefore$ n is odd $\because$ n = 7

$$\boxed{\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}......\frac{2}{3}}$$

$= \frac{7-1}{7}, \frac{7-3}{7-2}, \frac{7-5}{7-4}$

$= \frac{6}{7}.\frac{4}{5}.\frac{2}{3} = \frac{16}{35}$

10 Given $\frac{dy}{dx} = \frac{2y}{x}$

$\frac{dy}{y} = 2.\frac{dx}{x}$

$\int \frac{1}{y}dy = 2.\int \frac{1}{x}dx$

$\log y = 2\log x + \log c$

$\log y = \log x^2 + \log c$

$y = cx^2$

11 Given line x + y + 2 = 0

Given circle $x^2 + y^2 - 4x + 6y - 12 = 0$

$2g = -4;\ 2f = 6,\ c = -12$

$g = -2;\ f = 3$

$r^2 = g^2 + f^2 - c$

$= 4 + 9 + 12$

$= 25$

$$\text{pole} = \left(-g + \frac{lr^2}{lg + mf - n}, -f + \frac{mr^2}{lg + mf - n}\right)$$

$$= \left(2 + \frac{1(25)}{1(-2) + 1(3) - 2}, -3 + \frac{1(25)}{1(-2) + 1(3) - 2}\right)$$

$$= \left(2 + \frac{25}{-1}, -3 + \frac{25}{-1}\right)$$

$$= (-23, -28)$$

12 Given circles

$S = x^2 + y^2 + 2x + 2y + 1 = 0$

$S^1 = x^2 + y^2 + 4x + 3y + 2 = 0$

Equation of common chord $S - S^1 = 0$

$$x^2 + y^2 + 2x + 2y + 1 - x^2 - y^2 - 4x - 3y - 2 = 0$$

$$-2x - y - 1 = 0$$

$$2x + y + 1 = 0$$

Centre of the circle S = 0

$C_1 = (-1, -1)$

Radius $(r_1) = \sqrt{1 + x - x}$

$$\boxed{r_1 = 1}$$

d = $\perp^{er}$ distance from C_1 to the line 2x + y + 1 = 0

$$d = \frac{|2(-1) + 1(-1) + 1|}{\sqrt{2^2 + 1^2}} = \frac{|-2 - 1 + 1|}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

Length of common chord $AB = 2\sqrt{r^2 - d^2}$

$$= 2\sqrt{1^2 - \left(\frac{2}{\sqrt{5}}\right)^2} = 2\sqrt{1 - \frac{4}{5}} = 2\sqrt{\frac{5-4}{5}}$$

$$\boxed{AB = 2\sqrt{\frac{1}{5}}}$$

13 Given ellipse $9x^2 + 16y^2 = 144$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$a^2 = 16,\ b^2 = 9,\ a > b$

(i) center (h, k) = (0, 0)

(ii) Eccentricity (e) = $\sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{\frac{16-9}{16}} = \frac{\sqrt{7}}{4}$

(iii) Vertices = $(h \pm a, k) = (\pm 4, 0)$

(iv) foci = $(h \pm ae, k) = \left(0 \pm 4\frac{\sqrt{7}}{4}, 0\right) = (\pm\sqrt{7}, 0)$

(v) Length of major axis = 2a = 2(a) = 8

(vi) length of minor axis = 2(b) = 2(3) = 6

(vii) length of latus rectum $= \frac{2b^2}{a} = \frac{2(4)}{4} = \frac{9}{2}$

(viii) equation of the directrices are $x = h \pm \frac{a}{e}$

$$x = 0 \pm \frac{4(4)}{\sqrt{7}}$$

$$\sqrt{7}x = \pm 16$$

15 Given hyperbola $3x^2 - 4y^2 = 12$

$$\frac{3x^2}{12} - \frac{4y^2}{12} = 1$$

$$\frac{x^2}{4} - \frac{y^2}{3} = 1 \qquad a^2 = 4;\ b^2 = 3$$

Given line y = x – 7 $\Rightarrow$ slope m = 1

(i) parallel slope = 1 (ii) perpendicular slope = –1

Equation of tangent is	equation tangent
$y = mx \pm \sqrt{a^2m^2 - b^2}$	$y = mx \pm \sqrt{a^2m^2 - b^2}$
$y = (1)x \pm \sqrt{4(1) - 3}$	$= (-1)x \pm \sqrt{4(1) - 3}$
$= x \pm \sqrt{1}$	$y = -x \pm 1$
$y = x \pm 1$	$x + y \pm 1 = 0$
$x - y \pm 1$	

16 Let $I = \int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx$ ---(1) $\qquad \because \int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$I = \int_0^{\pi/2} \frac{\sin^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx$$

$$= \int_0^{\pi/2} \frac{\cos^5(x)}{\cos^5 x + \sin^5 x} dx \text{ ----(2)}$$

(1) + (2)

$$I + I = \int_0^{\pi/2} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^{5-} x} dx$$

$$2I = \int_0^{\pi/2} dx$$

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$\boxed{I = \frac{\pi}{4}}$$

17 Given $\frac{dy}{dx} + y \tan x = \cos^2 x$

It is a linear equation in 'y'

$P = \text{Tan } x$; $Q = \cos^2 x$

$I.F = e^{\int P dx} = e^{\int \text{Tan}\, x} = e^{\log|\sec x|} = \sec x$

General solution is y (I.F) = $\int Q.(IF)\, dx + C$

$$y \sec x = \int \cos^2 x . \sec x dx + c$$

$$= \int \cos^2 \frac{1}{\cos x} dx + c$$

$$= \int \cos x dx + c = \sin x + C$$

18 Given points A(2, 0), B(0, 1), C(4,5), D(0, C)

Required circle equation

$x^2 + y^2 + 2gx + 2fy + c = 0$ ----(1)

equation (1) passing through at A(2,0)

4 + 0 + 4g + 0 + c = 0

4g + c + 4 = 0 -----(2)

Equation (1) passing through at B(0, 1)

0 + 1 + 0 + 2f + c = 0

2f + c + 1 = 0 -----(3)

Equation (1) passing through at c(4, 5)

16 + 25 + 8g + 10f + c = 0

8g + 10f + c + 41 = 0 ----- (4)

Solving (2) & (3)

4g + 0 + c + 4 = 0

0 + 2f + c + 1 = 0

– – – –

4g – 2f + 3 = 0 ---(5)

Solving (3) & (4)

0 + 2f + c + 1 = 0

8g + 10f + c + 41 = 0

– – – –

–8g – 8f – 40 = 0

g + f + 5 = 0 ------ (6)

solving (5) & (6)

g	f	1	
–2	3	4	–2
1	5	1	1

$$\frac{g}{-10-3}=-\frac{f}{3-20}=\frac{1}{4+2}$$

$$\frac{g}{-13}=\frac{f}{-17}=\frac{1}{6}$$

$$\boxed{g=-\frac{13}{6}};\boxed{f=-\frac{17}{6}}$$

‘g’ value sub in (2)

$$4\left(-\frac{13}{6}\right)+c+4=0$$

$$-\frac{26}{3}+4+c=0$$

$$c=-4+\frac{26}{3}$$

$$\boxed{c=-\frac{12+26}{3}=\frac{14}{3}}$$

G, f, c, values sub in (1)

$$x^2+y^2+2\left(-\frac{13}{6}\right)x+2\left(-\frac{17}{6}\right)y+\frac{14}{3}=0$$

$3\left(x^2+y^2\right)-13x-17y+14=0$ ---- (7)

(0, c) passing through in above equation

$3(0 + c^2) – 13(0) – 17c + 14 = 0$

$3c^2 – 17c + 14 = 0$

$3c^2 – 14c – 3c + 14 = 0$

c(3c – 14) -1 (3c – 14) = 0

(3c – 1) (c – 1) = 0

3c – 14 = 0 ; c – 1 = 0

$\boxed{c=\frac{14}{3}};c=1$

19 Given circles

S = $x^2 + y^2 – 4x – 6y – 12 = 0$; $S'=x^2+y^2+6x+18y+26=0$

Compare with $x^2+y^2+2gx+2fy+c=0$

2g = –4; 2f = –6; c = –12

g = –2 ; f = –3

centre (c_1) = 2,3)

radius (r_1) = $\sqrt{(-2)^2+(-3)^2+12}=\sqrt{4+9+12}=\sqrt{25}$ =5

$2g' = 6; 2f'=18; c' = 26$

$g'=3; f' = 9$

center (c_2) = (–3, –9)

radius (r_2) = $\sqrt{9+81-26}$

$= \sqrt{90-26}$

$= \sqrt{64}$

$\boxed{r_2 = 8}$

Distance between centers (C_1C_2)

$C_1C_2 = \sqrt{(2+3)^2+(3+9)^2}$

$= \sqrt{25+144}$

$= \sqrt{169}$

$\boxed{c_1c_2 = 13}$ $\quad r_1 + r_2 = 8 + 5 = 13$

$\boxed{c_1c_2 = r_1 + r_2}$

Given two circles touch each other externals. The point of contact divides c_1c_2 in the ratio $r_1 : r_2$ internally

$$p = \left(\frac{5(-3)+8(2)}{5+8}, \frac{5(-9)+8(3)}{5+8}\right) = \left(\frac{-15+16}{13}, \frac{-45+24}{13}\right) = \left(\frac{1}{13}, \frac{-21}{13}\right)$$

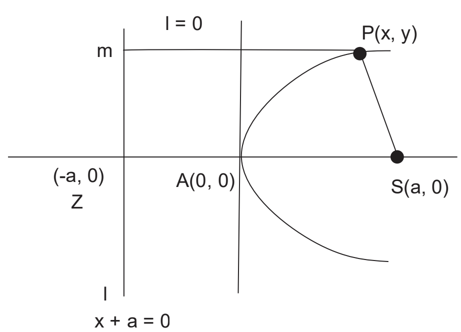
Equation common tangent S – S' = 0

$x^2 + y^2 - 4x - 6y - 12 - x^2 - y^2 - 6x - 18y - 26 = 0$

$-10x - 24y - 38 = 0$

$5x + 12y + 19 = 0$

20
- Let 'S' be the focus and 'l' be the directrix of parabola.
- Let Z be the projection of 'S' on l.
- A be the mid point of ZS



AS = AZ = a

A(0, 0), S(a, 0) Z(–a, 0)

Equation of directrix (l) = x + a = 0

Let P(x_1, y) be any point on the parabola

$\frac{SP}{PM} = 1$

SP = PM

SP = perpendicular distance from 'p' to directrix.

$$\sqrt{(x-a)^2+y^2} = \frac{|x+a|}{\sqrt{1^2+0^2}}$$

S.O.B.S

$$(x-a)^2+y^2=(x+a)^2$$

$$x^2+a^2-2ax+y^2=x^2+a^2+2ax$$

$$\boxed{y^2=4ax}$$

$\therefore y^2=4ax$ is the standard from of parabola.

21 $$\int \frac{x+1}{x^2+3x+12}dx$$

$$px+q=A\frac{d}{dx}(ax^2+bx+c)+B$$

$$x+1=A.\frac{d}{dx}(x^2+3x+12)+B$$

$$x+1=A(2x+3)+B$$

Comparing coeff. Of x & constant

2A = 1 3A + B = 1

$$\boxed{A=\frac{1}{2}} \qquad 3\left(\frac{1}{2}\right)+B=1$$

$$B=1-\frac{3}{2}$$

$$\boxed{B=-\frac{1}{2}}$$

$$x+1=\frac{1}{2}(2x+3)-\frac{1}{2}$$

$$\int\frac{x+1}{x^2+3x+12}dx=\int\frac{\frac{1}{2}(2x+3)-\frac{1}{2}}{x^2+3x+12}dx$$

$$=\frac{1}{2}\int\frac{2x+3}{x^2+3x+12}dx-\frac{1}{2}\int\frac{1}{x^2+3x+12}dx \quad \because \int\frac{f'(x)}{f(x)}=\log|f(x)|+c$$

$$=\frac{1}{2}\log|x^2+3x+12|-\frac{1}{2}\int\frac{dx}{x^2+2x\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+12}dx$$

$$=\frac{1}{2}\log|x^2+3x+12|-\frac{1}{2}\int\frac{1}{\left(x+\frac{3}{2}\right)^2-\left(\frac{9}{4}\right)+12}dx$$

$$= \frac{1}{2}\log|x^2+3x+12| - \frac{1}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\frac{39}{4}}dx$$

$$= \frac{1}{2}\log|x^2+3x+12| - \frac{1}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\left(\frac{\sqrt{39}}{2}\right)^2}dx \quad \left(\because \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\mathrm{Tan}^{-1}\left(\frac{x}{a}\right)\right)$$

$$= \frac{1}{2}\log|x^2+3x+12| - \frac{1}{2}.\frac{1}{\left(\frac{\sqrt{39}}{2}\right)}.\mathrm{Tan}^{-1}\left(\frac{x+\frac{3}{2}}{\frac{\sqrt{39}}{2}}\right)$$

$$= \frac{1}{2}\log|x^2+3x+12| - \frac{1}{\sqrt{39}}.\mathrm{Tan}^{-1}\left(\frac{2x+3}{\sqrt{39}}\right)+C$$

22 $I_n = \int \sin^n x dx$

$$\int \underbrace{\sin^{n-1} x}_{U}.\underbrace{\sin x}_{V} dx$$

$$\boxed{\therefore \int Uvdx = U\int vdx - \int\left[\frac{d}{dx}(U).\int Vdx\right]dx}$$

$$= \sin^{n-1}x.\int \sin xdx - \int\left[\frac{d}{dx}(\sin^{n-1}x).\int \sin xdx\right]dx$$

$$= \sin^{n-1}x.(-\cos x) - \int\left[(n-1)\sin^{n-2}x(\cos x)(-\cos x)\right]dx$$

$$= -\cos x\sin^{n-1}x + \int\left[(n-1)\sin^{n-2}x.\cos^2 x\right]dx$$

$$= -\cos x\sin^{n-1}x + \int\left[(n-1)\sin^{n-2}x.(1-\sin^2 x)\right]dx$$

$$= -\cos x\sin^{n-1}x + (n-1)\int \sin^{n-2}xdx - (n-1)\int \sin^n xdx$$

$$I_n = -\cos x\sin^{n-1}x + (n-1)I_{n-2} - (n-1)I_n$$

$$I_n + (n-1)I_n = -\cos x\sin^{n-1}x + (n-1)I_{n-2}$$

$$I_n(1+n-1) = -\cos x\sin^{n-1}x + (n-1)I_{n-2}$$

$$I_n = \frac{-\cos x\sin^{n-1}x}{n} + \left(\frac{n-1}{n}\right)I_{n-2}$$

Now $I_4 = \int \sin^4 xdx = \dfrac{-\cos x\sin^3 x}{4} + \dfrac{3}{4}I_2$

$$= \frac{-\cos x\sin^3 x}{4} + \frac{3}{4}\left(-\frac{\sin x\cos x}{2} + \frac{1}{2}I_0\right)$$

$$= \frac{-\sin^3 x\cos x}{4} - \frac{3}{8}\sin x\cos x + \frac{3}{8}(x) + c$$

23 $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx = \frac{1}{20}\log 3$

Let sin x - cosx = t

Diff. w.r.to 't'

$(\cos x + \sin x)dx = dt$

sin x – cos x = t

Also $(\sin x - \cos x)^2 = t^2$

$\sin^2 x + \cos^2 x - 2\sin x \cos x = t^2$

$1 - \sin 2x = t^2$

$1 - t^2 = \sin 2x$

$U.L. x = \frac{\pi}{4} \Rightarrow t = \sin\frac{\pi}{4} - \cos\frac{\pi}{4} = 0$

$L.L.\ x = 0 \Rightarrow t = \sin 0 - \cos 0 = -1$

LHS = $\int_{-1}^{0} \frac{dt}{9 + 16(1 - t^2)}$

$= \int_{-1}^{0} \frac{dt}{25 - 16t^2} = \int_{-1}^{0} \frac{dt}{5^2 - (4t)^2} \left(\because \int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x} + c\right| \right)$

$= \frac{1}{2(5)}\left[\frac{\log\left|\frac{5+4t}{5-4t}\right|}{4}\right]_{-1}^{0} = \frac{1}{40}\left[\log\left(\frac{5}{5}\right) - \log\left(\frac{5-4}{5+4}\right)\right]^{-1}$

$= \frac{1}{40}\left[\log(1) - \log\left(\frac{1}{9}\right)\right] = \frac{1}{40}\left[0 - (\log 1 - \log 9)\right]$

$= \frac{1}{40}[\log 9] = \frac{1}{40}\left[\log 3^2\right] = \frac{1}{40} 2\log 3 = \frac{1}{20}\log 3$

24 $\left(x^2 + y^2\right)dx = 2xydy$

$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$

Which is a homogeneous equation

$v + x.\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2x(vx)} \quad \left(\begin{matrix}\text{put } y = vx \\ \frac{dy}{dx} = v + x\frac{dv}{dx}\end{matrix}\right)$

$x.\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$

$x.\frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{2v}$

$$\int \frac{2v}{1-v^2}\,dv = \int \frac{dx}{x}$$

$$-\log\left(1-v^2\right) = \log x + \log c$$

$$\log x + \log\left(1-v^2\right) = \log c$$

$$\log\left[x\left(1-v^2\right)\right] = \log c$$

$$(\because -\log c = \log c)$$

$$x\left(1-v^2\right) = c \qquad \left(\because \text{where } v = \frac{y}{x}\right)$$

$$x\left(1-\frac{y^2}{x^2}\right) = c$$

$$x^2 - y^2 = cx$$