



Code Number:

A**Aakash****Medical | IIT-JEE | Foundations**

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Time: 3 hrs.

Mock Test Paper for Class-XII

Max. Marks: 75

MATHEMATICS**Paper - II(A)****Answers & Solutions**

1. Answer

$$Z = 7 + 24i$$

$$\frac{1}{Z} = \frac{1}{7 + 24i} = \frac{1}{7 + 24i} \times \frac{7 - 24i}{7 - 24i} = \frac{7 - 24i}{625} = \frac{7}{625} - \frac{24i}{625}$$

2. Answer

$$Z_1 = -i = \cos\pi + i\sin\pi \quad \text{Arg } Z_1 = \pi, \quad Z_2 = i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

$$\text{Arg } Z_2 = \frac{\pi}{2}$$

$$\text{Arg} = \left(\frac{Z_1}{Z_2} \right) = \text{Arg } Z_1 - \text{Arg } Z_2 = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

3. Answer

$$(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6$$

$$= (-\omega - \omega)^6 + (-\omega^2 - \omega^2)^6$$

$$= (-2\omega)^6 + (-2\omega^2)^6$$

$$= 2^6(\omega^6 + \omega^{12}) = 2^6(1 + 1) = 2^7 = 128$$

4. Answer

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\frac{\beta^2 + \alpha^2}{\alpha^2\beta^2} + \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{\left(\frac{-b}{a}\right)^2 - 2\frac{c}{a}}{\left(\frac{c}{a}\right)^2} = \frac{b^2 - 2ac}{c^2}$$

5. Answer

$$\alpha\beta\gamma = 9$$

$$\frac{a}{4} = 9$$

$$a = 36$$

6. Answer

$$n = 6$$

$$\frac{(n-1)!}{2} = \frac{5!}{2} = 60$$

7. Answer

$$r + 1 = 3r - 5$$

$$2r = 6$$

$$r = 3$$

$$r + 1 + 3r - 5 = 12$$

$$4r = 16$$

$$r = 4$$

8. Answer

$$\left(Z \left(1 + \frac{3}{2}x \right) \right)^{\frac{3}{2}}$$

$$\left| \frac{3}{2}x \right| < 1$$

$$|x| < \frac{2}{3}$$

$$(x) \in \left(-\frac{2}{3}, \frac{2}{3} \right)$$

9. Answer

$$\bar{X} = \frac{500}{10} = 50$$

Mean deviation from Mean

$$\frac{\sum |x_i - \bar{X}|}{n} = \frac{12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6}{10} = \frac{84}{10} = 8.4$$

10. Answer

$$n = 10, P = 0.1, q = 0.9$$

$$PP(X = 1) = {}^{10}C_1 (0.1)^1 \times (0.9)^9 = (0.9)^9$$

11. Answer

$$\text{Let, } y = \frac{x}{x^2 - 5x + 9} \Rightarrow x^2y - 5xy + 9y = x$$

$$\Rightarrow x^2y - 5xy + 9y - x = yx^2 + (-5y - 1)x + 9y = 0$$

It is in the form of $ax^2 + bx + c = 0$, where $a = y$, $b = -5y - 1$, $c = 9y$

Since, 'x' is real $\Rightarrow \Delta \geq 0 \Rightarrow b^2 - 4ac \geq 0$

$$\Rightarrow [-(5y + 1)]^2 - 4y(9y) \geq 0 \Rightarrow 25y^2 + 1 + 10y - 36y^2 \geq 0 \Rightarrow -11y^2 + 10y + 1 \geq 0$$

$$\Rightarrow 11y^2 - 10y - 1 \leq 0 \quad \Rightarrow 11y^2 - 11y + y - 1 \leq 0$$

$$\Rightarrow (y-1)(11y+1) \leq 0 \Rightarrow \frac{-1}{11} \leq y \leq 1$$

i.e. 'y' lies in between $\frac{-1}{11}$ and 1

Hence, the given expression lies between $\frac{-1}{11}$ and 1

12. Answer

$$\text{Let } A = (-2 + 7i) = (-2 - 7), B = \left(\frac{-3}{2} + \frac{1}{2}i\right) = \left(\frac{-3}{2}, \frac{1}{2}\right),$$

$$C = (4 - 3i) = (4, -3), D = \frac{7}{2}(1+i) = \left(\frac{7}{2}, \frac{7}{2}\right), \overline{AB} = \sqrt{\left(\frac{-3}{2} + 2\right)^2 + \left(\frac{1}{2} - 7\right)^2} = \sqrt{\frac{1}{4} + \frac{169}{4}} = \sqrt{\frac{170}{4}}$$

[∵ The distance between the two point (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$]

$$\overline{BC} = \sqrt{\left(4 + \frac{3}{2}\right)^2 + \left(-3 - \frac{1}{2}\right)^2} = \sqrt{\frac{121}{4} + \frac{49}{4}} = \sqrt{\frac{170}{4}}; \quad \overline{CD} = \sqrt{\left(\frac{7}{2} - 4\right)^2 + \left(\frac{7}{2} - 3\right)^2} = \sqrt{\frac{1}{4} + \frac{169}{4}} = \sqrt{\frac{170}{4}}$$

$$\overline{DA} = \sqrt{\left(-2 - \frac{7}{2}\right)^2 + \left(7 - \frac{7}{2}\right)^2} = \sqrt{\frac{121}{4} + \frac{49}{4}} = \sqrt{\frac{170}{4}};$$

$$\overline{AC} = \sqrt{(4+2)^2 + (-3-7)^2} = \sqrt{36+100} = \sqrt{136};$$

$$\overline{BD} = \sqrt{\left(\frac{7}{2} + \frac{3}{2}\right)^2 + \left(\frac{7}{2} - \frac{1}{2}\right)^2} = \sqrt{\frac{100}{4} + \frac{36}{4}} = \sqrt{\frac{136}{4}};$$

$$\therefore \overline{AB} = \overline{BC} = \overline{CD} = \overline{DA} \text{ and } \overline{AC} \neq \overline{BD}$$

Small, all the four sides are equal in length and diagonals are not equal in length \Rightarrow the given points form a rhombus.

13. The dictionary order of the letters of the word MASTER is A, E, M, R, S, T

No of words that begin with A = 5 !

No of words that begin with E = 5 !

No of words that begin with MAE = 3 !

No of words that begin with MAR = 3 !

No of words that begin with MASE = 2 !

No of words that begin with MASR = 2 !

No of words that begin with MASTER = 0! = 1

Rank of word MASTER = 5!+5!+3!+3!+2!+2!+1 = 120+120+6+6+5 = 257

$$14. \text{L.H.S} = \frac{{}^{4n}C_{2n}}{2n C_n} = \frac{\frac{(4n)!}{((4n-2n)!2n!)}}{\frac{(2n)!}{((2n-n)!n!)}} = \frac{\frac{(4n)!}{(2n!2n!)}}{\frac{(2n)!}{n!n!}} = \frac{4n!}{(2n!)^2} \times \frac{4n!}{(2n!)^2} \left[\because n C_r = \frac{n!}{(n-r)!r!} \right]$$

$$= \frac{4n(4n-1)(4n-2)\dots 5.4.3.2.1}{(2n(2n-1)(2n-2)\dots 5.4.3.2.1)^2} \times \frac{(n!)^2}{(2n!)}$$

$$\begin{aligned}
&= \frac{(4n-1)(4n-3)\dots 5 \cdot 3 \cdot 1 [4n(4n-2)\dots 4 \cdot 2]}{(2n-1)(2n-3)\dots 5 \cdot 3 \cdot 1)^2 [2n(2n-2)\dots 4 \cdot 2]^2} \times \frac{(n!)^2}{(2n!)} \\
&= \frac{(4n-1)(4n-3)\dots 5 \cdot 3 \cdot 1 [2^{2n}(2n)(2n-2)\dots 2 \cdot 1]}{(2n-1)(2n-3)\dots 5 \cdot 3 \cdot 1)^2 2^{2n} [n(n-1)\dots (2 \cdot 1)]^2} \times \frac{(n!)^2}{(2n!)} \\
&= \frac{[(4n-1)(4n-3)\dots 5 \cdot 3 \cdot 1] \cdot (2n!)}{(2n-1)(2n-3)\dots 5 \cdot 3 \cdot 1)^2 (n!)^2} \times \frac{(n!)^2}{(2n!)} \\
&= \frac{[(4n-1)(4n-3)\dots 5 \cdot 3 \cdot 1]}{(2n-1)(2n-3)\dots 5 \cdot 3 \cdot 1)^2} = \frac{1 \cdot 3 \cdot 5 \dots (4n-3)(4n-1)}{[1 \cdot 3 \cdot 5 \dots (2n-3)(2n-1)]^2} = \text{R.H.S.}
\end{aligned}$$

15. Answer

Let, $x-1=t \Rightarrow x=1+t$

Now,
$$\frac{3x^3 - 8x^2 + 10}{(x-1)^4} = \frac{3(1+t)^3 + 8(1+t)^2 + 10}{t^4} = \frac{3(1+t^3 + 3t^2 + 3t) - 8(1+t^2 + 2t) + 10}{t^4}$$

$$= \frac{3 + 3t^3 + 9t^2 - 8 - 8t^2 - 16t + 10}{t^4} = \frac{3t^3 + t^2 - 7t + 5}{t^4} = \frac{3}{t} + \frac{1}{t^2} - \frac{7}{t^3} + \frac{5}{t^4}$$

$\therefore \frac{3x^3 - 8x^2 + 10}{(x-1)^4} = \frac{3}{x-1} + \frac{1}{(x-1)^2} - \frac{7}{(x-1)^3} + \frac{5}{(x-1)^4}$

16. Given that A, B, C are three horses.

Let P(A), P(B), P(C) be the probabilities of winning the horses A, B, C respectively.

Given $P(A) = 2P(B)$ ---(1) and $P(B) = 2P(C)$ ---(2)

$P(A) + P(B) + P(C) = 1$

$2P(B) + P(B) + P(C) = 1$

$3P(B) + P(C) = 1$

$6P(C) + P(C) = 1$

$P(C) = 1/7$

$P(B) = 2/7$

$P(A) = 4/7$

17. Given that, $P(A \cap \bar{B} \cap \bar{C}) = \frac{1}{4}$ (1) ; $P(\bar{A} \cap \bar{B} \cap \bar{C}) = \frac{1}{8}$ (2) & $P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{4}$ (3)

Now,
$$\frac{(2)}{(3)} \Rightarrow \frac{P(\bar{A} \cap \bar{B} \cap \bar{C})}{P(\bar{A} \cap B \cap \bar{C})} = \frac{1/8}{1/4} = \frac{1}{2} \Rightarrow \frac{P(B)}{P(\bar{B})} = \frac{1}{2} \Rightarrow 2P(B) = P(\bar{B})$$

[\because If A, B, C are independent events, then $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$]

$$\Rightarrow 2P(B) = 1 - P(B) \Rightarrow 3[P(B)] = 1 \Rightarrow P(B) = \frac{1}{3}$$

Again,
$$\frac{(1)}{(3)} = \frac{P(A \cap \bar{B} \cap \bar{C})}{P(\bar{A} \cap \bar{B} \cap \bar{C})} = \frac{1/4}{1/4} = 1 \Rightarrow \frac{P(A)}{P(\bar{A})} = 1 \Rightarrow P(A) = P(\bar{A})$$

$$\Rightarrow P(A) = 1 - P(\bar{A}) = 2P(A) = 1 \Rightarrow P(A) = \frac{1}{2}$$

$$\text{From eq. (3); } P(\bar{A})P(\bar{B})P(\bar{C}) = \frac{1}{4} \Rightarrow [1-P(A)][1-P(B)][1-P(C)] = \frac{1}{4}$$

$$\Rightarrow \frac{1}{2} \left(\frac{2}{3} \right) [1-P(C)] = \frac{1}{4} \Rightarrow 1-P(C) = \frac{3}{4} \Rightarrow P(C) = 1 - \frac{3}{4} \Rightarrow P(C) = \frac{1}{4}$$

$$\therefore P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$$

18. Given that $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$

$$\text{Now, } (\cos \alpha + i \sin \alpha) + (\cos \beta + i \sin \beta) + (\cos \gamma + i \sin \gamma) = (\cos \alpha + \cos \beta + \cos \gamma) + i(\sin \alpha + \sin \beta + \sin \gamma) = 0 + i0$$

$$\Rightarrow (\cos \alpha + i \sin \alpha) + (\cos \beta + i \sin \beta) + (\cos \gamma + i \sin \gamma) = 0 \dots \dots (1)$$

Let, $x = \text{cis } \alpha, y = \text{cis } \beta, z = \text{cis } \gamma$ then $x + y + z = 0$ [by eq. (1)]

$$\text{By squaring on both side } (x + y + z)^2 = 0 \Rightarrow x^2 + y^2 + z^2 = 2(xy + yz + zx) = -2xyz \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)$$

$$= -2xyz [\cos \alpha - i \sin \alpha + \cos \beta - i \sin \beta + \cos \gamma]$$

$$= -2xyz [\cos \alpha + \cos \beta + \cos \gamma] - i [\sin \alpha + \sin \beta + \sin \gamma] = -2xyz(0 - i0) \quad (\because \text{from given})$$

$$\therefore x^2 + y^2 + z^2 = 0$$

$$\Rightarrow (\cos \alpha + i \sin \alpha)^2 + (\cos \beta + i \sin \beta)^2 + (\cos \gamma + i \sin \gamma)^2 = 0$$

19. Answer

$$\text{Given equation is } 6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0 \dots (1)$$

$$\text{Now, by comparing the given equation } a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n = 0$$

$$\text{We get } a_0 = 6, a_1 = 25, a_2 = 31, a_3 = -31, a_4 = 25, a_5 = -6.$$

The degree of the given equation is $n = 6$, which is even

$$\text{Also } a_k = a_n - k \forall k = 0, 1, 2, 3, 4, 5, 6.$$

Hence the given equation is a reciprocal equation of class-II of even degree.

$$\text{Hence } 1, -1 \text{ are the roots of } 6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$$

\therefore By Horner's synthetic division, we get

-1	6	-25	31	0	-31	25	-6
	0	-6	31	-62	62	-31	6
1	6	-31	62	-62	-31	-6	0
	0	6	-25	37	-25	6	
	6	-25	37	-25	6	0	

$$\text{The reduced equation is } 6x^4 - 25x^3 + 37x^2 - 25x + 6 = 0 \dots (2)$$

Clearly, eq. (2) is an even degree reciprocal equation of class-I by dividing eq. (2) with x^2 .

$$\text{We get } 6x^2 - 25x + 37 - 25 \frac{1}{x} + \frac{6}{x^2} = 0 \Rightarrow x^2 + \frac{1}{x^2} = a^2 - 2.$$

$$\text{From eq. (3). We get } 6(a^2 - 2) - 25a + 37 = 0 \Rightarrow 6a^2 - 12 - 25a + 37 = 0 \Rightarrow 6a^2 - 25a + 25 = 0$$

$$\Rightarrow 6a^2 - 15a - 10a + 25 = 0 \Rightarrow 3a(2a - 5) - 5(2a - 5) = 0$$

$$\Rightarrow (2a - 5)(3a - 5) = 0 \Rightarrow a = \frac{5}{2} \text{ (or) } a = \frac{5}{3}.$$

Case - I : if $a = \frac{5}{2}$ then $x + \frac{1}{x} = a \Rightarrow x + \frac{1}{x} = \frac{5}{2} \Rightarrow \frac{x^2 + 1}{x} = \frac{5}{2}$

$$\Rightarrow 2x^2 + 2 = 5x \Rightarrow 2x^2 - 5x + 2 = 0 \Rightarrow 2x^2 - 4x - x + 2 = 0$$

$$\Rightarrow 2x(x - 2) - 1(x - 2) = 0 \Rightarrow (x - 2)(2x - 1) = 0 \Rightarrow x = 2 \text{ (or) } x = \frac{1}{2}.$$

Case- II : if $a = \frac{5}{3}$, Then $x + \frac{1}{x} = a \Rightarrow x + \frac{1}{x} = \frac{5}{3} \Rightarrow \frac{x^2 + 1}{x} = \frac{5}{3}$

$$\Rightarrow 3x^2 + 3 = 5x \Rightarrow 3x^2 - 5x + 3 = 0$$

$$\Rightarrow x = \frac{5 \pm \sqrt{25 - 4(3)(3)}}{2(3)} \Rightarrow x = \frac{5 \pm \sqrt{25 - 36}}{6} \Rightarrow x = \frac{5 \pm i\sqrt{11}}{6} \left[\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$

$\therefore -1, 1, 2, \frac{1}{2}, \frac{5 \pm i\sqrt{11}}{6}$ are the roots of given equation.

20. Answer

Given equation is $(a + x)^n$.

2nd term $T_2 = T_{1+1} = {}^n C_1 \cdot (a)^{n-1} \cdot (x)^1 = 240 \dots (1)$

(\because General term in $(a + x)^n$ is $T_{r+1} = {}^n C_r \cdot (a)^{n-r} \cdot (x)^r$)

3rd Term $T_3 = T_{2+1} = {}^n C_2 \cdot (a)^{n-2} \cdot (x)^2 = 720 \dots (2)$

4th Term $T_4 = T_{3+1} = {}^n C_3 \cdot (a)^{n-3} \cdot (x)^3 = 1080 \dots (3)$

Now $\frac{\text{eq}(2)}{\text{eq}(1)} \Rightarrow \frac{{}^n C_2 \cdot (a)^{n-2} \cdot (x)^2}{{}^n C_1 \cdot (a)^{n-1} \cdot (x)^1} = \frac{720}{240} = 3 \Rightarrow \frac{n-1}{2} \cdot a^{-1} \cdot x = 3$

$$\Rightarrow \left(\frac{n-2}{3} \right) \left(\frac{x}{a} \right) = 3 \Rightarrow \frac{x}{a} = \frac{9}{2(n-2)} \dots (5)$$

From equation (4) and (5), we get $\frac{6}{n-1} = \frac{9}{2(n-2)} \Rightarrow \frac{2}{n-1} = \frac{3}{2(n-2)}$

$$\Rightarrow 4n - 8 = 3n - 3 \Rightarrow n = 5$$

From (4) $\Rightarrow \frac{x}{a} = \frac{6}{5-1} \Rightarrow \frac{x}{a} = \frac{6}{4} = \frac{3}{2} \Rightarrow x = \frac{3a}{2} \dots (6)$

By substituting the values of x, n in equation (1), we get ${}^5 C_1 (a)^{5-1} \left[\frac{3a}{2} \right] = 240$

$$\Rightarrow a^5 = 32, \Rightarrow a^5 = 2^5 \Rightarrow a = 2$$

By substituting $a = 2$ in equation (6), we get $x = \frac{6}{2} = 3$

$\therefore a = 2, x = 3, n = 5.$

21. Answer

$$\text{Given that, } x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \dots = \frac{1.3}{1.2} \left(\frac{1}{3}\right)^2 + \frac{1.3.5}{1.2.3} \left(\frac{1}{3}\right)^3 + \dots$$

$$\text{By adding } 1 + \frac{1}{3} \text{ on both sides, } 1 + \frac{1}{3} + x = 1 + \frac{1}{1!} \left(\frac{1}{3}\right) + \frac{1.3}{2!} \left(\frac{1}{3}\right)^2 + \dots$$

$$\text{Now R.H.S is of the form } 1 + \frac{p}{1!} \left(\frac{x}{q}\right) + \frac{p(p+q)}{2!} \left(\frac{x}{q}\right)^2 + \dots$$

$$\text{Where } p = 1, p+q = 3 \Rightarrow q = 2, \text{ and } \frac{x}{q} = \frac{1}{3} \Rightarrow x = \frac{2}{3}$$

$$\left[\because 1 + \frac{p}{1!} \left(\frac{x}{q}\right) + \frac{p(p+q)}{2!} \left(\frac{x}{q}\right)^2 + \dots = (1-x)^{-p/q} \right]$$

$$\Rightarrow 1 + \frac{1}{3} + x = \left(1 - \frac{2}{3}\right)^{-1/2} \Rightarrow \frac{4}{3} + x = \left(\frac{1}{3}\right)^{-1/2} = \sqrt{3} \Rightarrow 3x + 4 = 3\sqrt{3}$$

$$\text{By squaring on both sides, } (3x+4)^2 = (3\sqrt{3})^2$$

$$9x^2 + 24x + 16 = 27 \Rightarrow 9x^2 + 24x = 11$$

22. Answer

We taken the assumed mean $A = 65$. Here $C = 10$, then $d_i = \frac{x_i - 65}{10}$.

Now, we construct the table with given data.

Class Intervals (C.I)	Frequency (f_i)	Mid point of C.I (x_i)	$d_i = \frac{x_i - 65}{10}$	d_i^2	$f_i d_i$	$f_i d_i^2$
30-40	3	35	-3	9	-9	27
40-50	7	45	-2	4	-14	28
50-60	12	55	-1	1	-12	12
60-70	15	65	0	0	0	0
70-80	8	75	1	1	8	8
80-90	3	85	2	4	6	12
90-100	2	95	3	9	6	18
	$\sum f_i = N = 50$				$\sum f_i d_i = -15$	$\sum f_i d_i^2 = 105$

Here, $N = 50$, $\sum f_i d_i = -15$, $\sum f_i d_i^2 = 105$

$$\bar{X} = A + \left(\frac{\sum f_i d_i}{N}\right) \times h$$

$$= 65 - 3 = 62$$

$$\text{Variance } \sigma^2 = C^2 \left[\frac{\sum f_i d_i}{N} - \left(\frac{\sum f_i d_i}{N} \right)^2 \right]$$

$$= 10^2 \left[\frac{105}{50} - \left(\frac{-15}{50} \right)^2 \right] = 100 \left[\frac{105}{50} - \frac{225}{2500} \right] = 100 \left[\frac{(105)(50) - 225}{2500} \right] = \frac{100(5025)}{2500} = 201$$

Standard deviation (σ_x) = $\sqrt{201} = 14.18$ (approximately)

23. Answer

Given that,

Box	White	Black	Red	Total No. Of Balls
I	1	2	3	6
II	2	1	1	4
III	4	5	3	12

Let B_1, B_2, B_3 be a events of selecting the boxes I, II, III respectively.

$$\therefore P(B_1) = \frac{1}{3}, P(B_2) = \frac{1}{3}, P(B_3) = \frac{1}{3}.$$

Let , 'R' be an event of drawing a red ball from a box.

$$\therefore P(R/B_1) = \frac{3}{6} = \frac{1}{2}, P(R/B_2) = \frac{1}{4}, P(R/B_3) = \frac{3}{12} = \frac{1}{4}$$

\therefore The probability that the red from box- II is

[\therefore by Baye's theorem]

$$P(B_2/R) = \frac{P(B_2)P(R/B_2)}{P(B_1)P(R/B_1) + P(B_2)P(R/B_2) + P(B_3)P(R/B_3)}$$

$$= \frac{\frac{1}{3} \cdot \frac{1}{4}}{\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{4}} = \frac{\frac{1}{3} \cdot \frac{1}{4}}{\frac{1}{3} \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{4} \right)} = \frac{2+1+1}{4} = \frac{1}{4}$$

24. Answer

Given,

$X = x_i$	0	1	2	3	4	5	6	7
$P(X = x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

i) We have $\sum_{i=0}^7 P(X = x_i) = 1$

[\therefore Sum of the probabilities is 1]

$$\Rightarrow 0 + k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1 \Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow 10k^2 - k - 1 = 0$$

$$\Rightarrow 10k(k+1) - 1(k+1) = 0 \Rightarrow (k+1)(10k-1) = 0 \Rightarrow k = \frac{1}{10}$$

ii) Mean (μ) = $\sum_{i=0}^7 P(X = x_i)$

$$\Rightarrow 0(0) + 1k + 2(2k) + 3(2k) + 4(3k) + 5(k^2) + 6(2k^2) + 7(7k^2 + k)$$

$$\Rightarrow 0 + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k = 66k^2 + 30k$$

$$= 66\left(\frac{1}{100}\right) + 30\left(\frac{1}{10}\right) = \frac{66}{100} + \frac{300}{100} = \frac{366}{100} = 3.66$$

$$\text{iii) } P(0 < X < 5) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= k + 2k + 2k + 3k = 8k = 8\left(\frac{1}{10}\right) = \frac{4}{5}$$

