Aakash
Medical | IIT-JEE | Foundations

Corp. Office: Aakash Educational Services Limited, 3rd Floor, Incuspaze Campus- 2, Plot No. 13, Sector- 18, Udyog Vihar, Gurugram, Haryana - 122015

Time: 3 hrs. **Mock Test Paper for Class-XII** **Max. Marks: 70**

MATHEMATICS

Roll No. | | | | | | | | | | | | |

GENERAL INSTRUCTIONS

Read the following instructions carefully and follow them:

1. The Question paper consists of parts **I, II, II, and IV**
2. **Part - I** consists of **15 Multiple choice** questions,
3. **Part - II** consists of **9 short answer type** questions carrying **2 marks** each, out of which **6 questions** to be answered
4. **Part - III** consists of **9 short answer type** questions carrying **3 marks** each, out of which **6 questions** to be answered
5. **Part – IV** consists of **5 long answer type** questions carrying **5 marks** each, answer all the questions.
6. Use Blue or Black ink to write and underline and pencil to draw diagrams.
7. Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

PART - I

Answer all the questions **20 x 1 = 20**

1. The area between y^2 = 4x and its latus rectum is :

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{3}$ (d) $\frac{4}{3}$

2. The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is :

(a) $\frac{3\pi a^2}{8}$ (b) $\frac{\pi a^3}{16}$ (c) $\frac{3\pi a^4}{8}$ (d) $\frac{3\pi a^4}{16}$

3. If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$, then $PF_1 + PF_2$ is :

(a) 10 (b) 8 (c) 12 (d) 6

4. If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$ then the value of $|z_1 + z_2 + z_3|$ is :

(a) 3 (b) 1 (c) 4 (d) 2

5. The number of rows in the truth table of $(p \vee q) \wedge (p \vee r)$ is :

(a) 3 (b) 1 (c) 4 (d) 8

6. If a vector $\overline{a}$ lies in the plane of $\overline{\beta}$ and $\overline{\gamma}$, then :

(a) $[\overline{a}, \overline{\beta}\, \overline{y}] = 0$ (b) $[\overline{a}, \overline{\beta}\, \overline{y}] = 1$ (c) $[\overline{a}, \overline{\beta}\, \overline{y}] = 2$ (d) $[\overline{a}, \overline{\beta}\, \overline{y}] = -1$

7. The differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary constants is :

(a) $\frac{dy}{dx} + y = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$ (c) $\frac{dy}{dx} - y = 0$ (d) $\frac{d^2y}{dx^2} - y = 0$

8. The horizontal asymptote of $f(x) = \frac{1}{x}$ is :

(a) $x = c$ (b) $y = 0$ (c) $y = c$ (d) $x = 0$

9. If $f(x) = \frac{x}{x+1}$, then its differential is given by :

(a) $\frac{1}{x+1}dx$ (b) $\frac{-1}{(x+1)^2}dx$ (c) $\frac{-1}{x+1}dx$ (d) $\frac{1}{(x+1)^2}dx$

10. If $(1+i)(1+2i)(1+3i) \ldots (1+ni) = x + iy$ Then $2.5.10\ldots(1+n^2)$ is :

(a) $x^2 + y^2$ (b) 1 (c) $1 + n^2$ (d) i

11. The Number given by the Rolle's theorem for the function $x^3 - 3x^2, x \epsilon [0,3]$ is :

(a) $\frac{3}{2}$ (b) 1 (c) 2 (d) $\sqrt{2}$

12. The Type of conic section for $x^2 - 3 = 5x + 3y$ is :

(a) hyperbola (b) ellipse (c) circle (d) parabola

13. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $\left(A^T\right)^{-1} =$

(a) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$

14. The angle between the line $\bar{r} = \left(\bar{i} + 2\bar{j} - 3\bar{k}\right) + t\left(2\bar{i} + \bar{j} - 2\bar{k}\right)$ and the plane $\bar{r}\left(\bar{i} + \bar{j}\right) + 4 = 0$ is :

(a) 45^0 (b) 0^0 (c) 90^0 (d) 30^0

15. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal

(a) $\frac{2}{\sqrt{5}}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{5}}$

16. If α, β and γ are zeros of $x^3 + px^2 + qx + r$ then $\sum \frac{1}{\alpha}$ is :

(a) $\frac{q}{r}$ (b) $-\frac{q}{r}$ (c) $-\frac{q}{p}$ (d) $-\frac{p}{r}$

17. The value of Var(3) is :

(a) 0 (b) 3 (c) Var(3) (d) 9

18. The random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is :

(a) 3 (b) 6 (c) 2 (d) 4

19. If A, B and C are invertible matrices of some order, then which one of the following is not true?

(a) $\det A^{-1} = (\det A)^{-1}$ (b) $\text{adj } A = |A|A^{-1}$

(c) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$ (d) $\text{adj }(AB) = (\text{adj } A)(\text{adj } B)$

20. A zero of $x^3 + 64$ is :

(a) 4i (b) 0 (c) -4 (d) 4

PART - II

Answer any seven questions. Question No. 30 is Compulsory. **7 × 2 = 14**

21. Simplify : $\sum_{n=1}^{12} i^n$

22. If α and β are the roots of the quadratic equation $x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2

23. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 3$ and $dx = 0.02$.

24. Find the differential equation for the family of all straight lines passing through the origin.

25. For the random variable X with the given probability mass function.

$$f(x) = \begin{cases} 2(x-1) & 1 < x < 2 \\ 0 & \text{oterwise} \end{cases}.$$

Find the Mean.

26. Find the general equation of a circle with centre (–3, –4) and radius 3 units

27. Find the rank of the matrix $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

28. Evaluate: $\int_0^{\frac{\pi}{2}} \sin^{10} x \, dx$

29. Evaluate: $\lim_{x \to 1} \frac{x^2 - 3X + 2}{x^2 - 4X + 3}$

30. Show that the vectors $2\vec{i} - \vec{j} + 3\vec{k}, \vec{i} - \vec{j}$ *and* $3\vec{i} - \vec{j} + 6\vec{k}$ are coplanar.

PART-III

Answer any seven questions. Question No. 40 is Compulsory. **7 × 3 = 21**

31. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right) = \sec^{-1} x, |x| > 1$.

32. Find the equation of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at (1, –3).

33. Prove that $\left[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}\right] = 0$

34. If $u(x, y) = \frac{x^2 + y^2}{\sqrt{x + y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

35. Find two positive numbers whose sum is 12 and their product is maximum.

36. Evaluate: $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1 + \sqrt{\tan x}} \, dx$

37. Simplify $\left[\frac{1+i}{1-i}\right]^3 - \left[\frac{1+i}{1-i}\right]^3$ into rectangular form

38. solve: $(1 + x^2)\frac{dy}{dx} = 1 + y^2$

39. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

40. If A= $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$,then find $|\text{adj}(\text{adj} A)|$.

PART - IV

Answer all the questions. **7 × 5 = 35**

41. (a) Find the angle between the curves $y = x^2$ and $y = (x - 3)^2$

(OR)

(b) solve: $\tan^{-1}\left[\frac{x-1}{x-2}\right] + \tan^{-1}\left[\frac{x+1}{x+2}\right] = \frac{\pi}{4}$

42. (a) A six sided die is marked ‘1’ on one face, ‘3’ on two of its faces, and ‘5’ on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find:

(i) the probability mass function

(ii) the cumulative distribution function

(iii) $P(4 \le X < 10)$

(OR)

(b) if $z = x + iy$ is a complex number such that $\text{Im}\left[\frac{2z+1}{iz+1}\right] = 0$, show that the locus of z is

$2x^2 + 2y^2 + x - 2y = 0$

43. (a) A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

(OR)

(b) Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

44. (a) Assume that water issuing from the end of horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

(OR)

(b) Solve the Linear differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$

45. (a) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

(OR)

(b) Find the vector and Cartesian equations of the plane containing $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$ and the parallel to the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1}$

46. (a) Find the area of the region bounded by the ellipse.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(OR)

(b) Find the vertex, focus and equation of the directrix of the parabola $y^2 - 4y - 8x + 12 = 0$.

47. (a) Show that $p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge ((\sim q) \vee p)$

(OR)

(b) Solve the system of linear equations by Cramer’s Rule.

$$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0,$$

$$\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0,$$

$$\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$$

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