



Code Number:

**A****Aakash****Medical | IIT-JEE | Foundations**

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**Time: 2 Hrs.****Mock Test for Class XII****Max. Marks: 60****MATHEMATICS**

$$1. \quad A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & -1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$A' = \begin{bmatrix} 3 & 0 & -2 \\ 2 & 4 & 1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 6 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 6 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 6 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 6 \end{bmatrix}$$

$$P' = \frac{1}{2} \begin{bmatrix} 6 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 6 \end{bmatrix} = P, \text{ is symmetric}$$

$$A - A' = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix}$$

$$\text{Let } Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix}$$

$$Q' = \frac{1}{2} \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix} = -Q, \text{ is skew-symmetric}$$

$$\text{Now } P + Q = \frac{1}{2} \begin{bmatrix} 6 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 6 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 6 & 4 & 2 \\ 0 & 8 & -2 \\ -4 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & -1 \\ -2 & 1 & 3 \end{bmatrix}$$

= A, is a square matrix. Hence verified.

2. i)  $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

ii)  $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(2\pi - \frac{5\pi}{6}\right) = \cos^{-1}\cos\frac{5\pi}{6} = \frac{5\pi}{6} \in [0, \pi]$

3. LHL =  $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x^2 - 1) = 2^2 - 1 = 3$

RHL =  $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (2x - 1) = 2(2) - 1 = 3$

LHL = RHL

Now  $\lim_{x \rightarrow 2} f(x) = 4$

Since LHL = RHL  $\neq f(2)$

$\therefore f(x)$  is not continuous at  $x = 2$

4. i)  $f(x)$  is decreasing in the interval  $(-\infty, 1)$  and decreasing in the interval  $(3, \infty)$

ii) Local maxima at  $x = 1$  and local minima at  $x = 3$

5.  $I = \int \frac{x - 1}{x^2 - 4x + 5} dx$

Let  $\frac{x - 1}{x^2 - 4x + 5} = \frac{x - 1}{(x + 1)(x - 5)} = \frac{A}{x + 1} + \frac{B}{x - 5} \dots\dots(1)$

$x - 1 = A(x - 5) + B(x + 1)$

Put  $x = 5$

$5 - 1 = A(0) + B(6)$

$\therefore 6B = 4$

$\therefore B = \frac{4}{6} = \frac{2}{3}$

Put  $x = -1$

$-1 - 1 = A(-1 - 5) + B(0)$

$-2 = -6A$

$A = \frac{-2}{-6} = \frac{1}{3}$

In (1)  $\frac{x - 1}{(x + 1)(x - 5)} = \frac{1/3}{x + 1} + \frac{2/3}{x - 5}$

$\therefore I = \frac{1}{3} \int \frac{dx}{x + 1} + \frac{2}{3} \int \frac{dx}{x - 5} = \frac{1}{3} \log|x + 1| + \frac{2}{3} \log|x - 5| + C$

6. i)  $\overline{AB} = (4 - 1)\hat{i} + (5 - 5)\hat{j} + (7 - 3)\hat{k} = 3\hat{i} + 0\hat{j} + 4\hat{k}$

ii)  $|\overline{AB}| = \sqrt{9 + 0 + 16} = \sqrt{25} = 5$

Unit vector in the direction of  $\overline{AB} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{3\hat{i} + 0\hat{j} + 4\hat{k}}{5} = \frac{3}{5}\hat{i} + \frac{0}{5}\hat{j} + \frac{4}{5}\hat{k}$

iii)  $-5\hat{j}$

7. i)  $f$  is many one and onto

ii)  $f(x_1) = f(x_2)$

$$4x_1 = 4x_2$$

$$x_1 = x_2$$

$\therefore f$  is one-one.

Now  $f(x) = 4x$

$$y = 4x$$

$$\therefore x = \frac{y}{4}$$

$$\therefore f\left(\frac{y}{4}\right) = 4 \times \frac{y}{4} = y$$

$\therefore f$  is onto

$\therefore f$  is bijective

$\therefore f$  is invertible

$$\therefore f^{-1} = \frac{y}{4}$$

$$\therefore f^{-1}(x) = \frac{x}{4}$$

8. Let  $E$ : the number 5 appears atleast once.

$F$ : Sum = 9

$$E = \{(1,5), (2,5), (3,5), (4,5), (5,5), (6,5), (5,1), (5,2), (5,3), (5,4), (5,6)\}$$

$$F = \{(5,4), (4,5), (3,6), (6,3)\}$$

$$P(E \cap F) = \frac{2}{36}$$

$$P(F) = \frac{4}{36}$$

$$\therefore P(E/F) = \frac{\frac{2}{36}}{\frac{4}{36}} = \frac{2}{4} = \frac{1}{2}$$

9. i)  $(1, 1), (2, 2)$  and  $(3, 3) \in R$

$\therefore R$  is reflexive

$$(1, 3) \in R, (3, 1) \in R \Rightarrow (1, 1) \in R$$

$\therefore R$  is transitive.

ii) Equivalence classes:  $\{1, 3\}, \{2\}, \{3, 1\}$

10. i)  $x - 1 = 2 \Rightarrow x = 2 + 1 = 3$

$$-2 - 2 = 2y \Rightarrow 2y = -4 \Rightarrow y = \frac{-4}{2} = -2$$

$$\text{ii) } AB = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix} [2 \quad -1 \quad -3] = \begin{bmatrix} 5 \times 2 & 5 \times -1 & 5 \times -3 \\ -2 \times 2 & -2 \times -1 & -2 \times -3 \\ 3 \times 2 & 3 \times -1 & 3 \times -3 \end{bmatrix} = \begin{bmatrix} 10 & -5 & -15 \\ -4 & 2 & 6 \\ 6 & -3 & -9 \end{bmatrix}$$

11. i) Let the side of the square be  $x$  cm

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

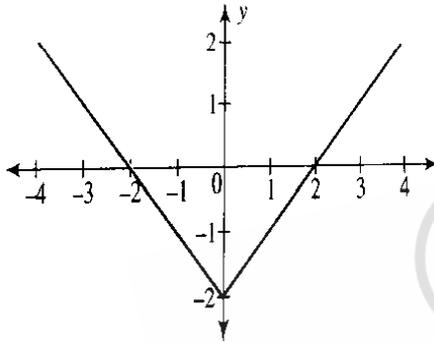
$$24 = 3 \times 6^2 \times \frac{dx}{dt}$$

$$\therefore \frac{dx}{dt} = \frac{24}{3 \times 6 \times 6} = \frac{2}{9} \text{ cm/s}$$

$$\text{Surface area, } A = 6x^2$$

$$\frac{dA}{dt} = 6 \cdot 2x \cdot \frac{dx}{dt} = 12 \times 6 \times \frac{2}{9} = 4 \times 2 \times 2 = 16 \text{ cm}^2/\text{s}$$

- ii) From the graph it is clear that local minimum value of the function is  $-2$



$$12. \quad \frac{x^2}{25} + \frac{y^2}{9} = 1 \Rightarrow \frac{y^2}{9} = 1 - \frac{x^2}{25} \Rightarrow \frac{y^2}{9} = \frac{25 - x^2}{25}$$

$$\Rightarrow y^2 = \frac{9}{25}(25 - x^2) \Rightarrow y = \frac{3}{5}\sqrt{25 - x^2}$$

$$\text{Area of the ellipse} = 4 \int_0^5 \frac{3}{5} \sqrt{5^2 - x^2} dx = 4 \times \frac{3}{5} \int_0^5 \sqrt{5^2 - x^2} dx$$

$$= 4 \times \frac{3}{5} \left[ \frac{x}{5} \sqrt{5^2 - x^2} + \frac{5^2}{2} \sin^{-1} \left( \frac{x}{5} \right) \right]_0^5$$

$$= 4 \times \frac{3}{5} \left[ \frac{5}{5} \sqrt{5^2 - 5^2} + \frac{5^2}{2} \sin^{-1} \left( \frac{5}{5} \right) - \left\{ \frac{5}{5} \sqrt{5^2 - 5^2} + \frac{5^2}{2} \sin^{-1} \left( \frac{5}{5} \right) \right\} \right]$$

$$= \frac{12}{5} \left[ \frac{25}{2} \times \frac{\pi}{2} - 0 \right] = \frac{12}{5} \times \frac{25\pi}{4} = 3 \times 5\pi = 15\pi \text{ square units}$$

13. i)  $y = e^x + 1$

ii)  $\frac{dy}{dx} = \frac{\sqrt{9 - y^2}}{x} \Rightarrow \frac{dy}{\sqrt{9 - y^2}} = \frac{dx}{x}$  is in variable separable

$$\int \frac{dy}{\sqrt{9 - y^2}} = \int \frac{dx}{x}$$

$$\int \frac{dy}{\sqrt{3^2 - y^2}} = \int \frac{dx}{x}$$

$$\sin^{-1}\left(\frac{y}{3}\right) = \log|x| + c$$

14. i)  $\vec{a} = 2\hat{i} + \hat{j}$ ,  $\vec{b} = 2\vec{a} = 4\hat{i} + 2\hat{j}$

$$\vec{a} \cdot \vec{b} = (2)(4) + (1)(2) = 10$$

$$|\vec{b}| = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{10}{2\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

ii)  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = (1 + 4)\hat{i} - (3 - 4)\hat{j} + (-3 - 1)\hat{k} = 5\hat{i} + \hat{j} - 4\hat{k}$

$$\therefore \text{Area of parallelogram} = |\vec{a} \times \vec{b}| = \sqrt{25 + 1 + 16} = \sqrt{42} \text{ square units}$$

15. Let  $A = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 - 1 & 1 - 1 & -1 - 0 \\ 2 & -1 & 2 \\ 3 & -5 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 2 \\ 3 & -5 & 2 \end{vmatrix}$

$$= 1[-2 + 10] - 0 - 1[-10 + 3] = 8 - 1(-7) = 8 + 7 = 15$$

$$B = \sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}$$

$$B = \sqrt{(-2 + 10)^2 + (6 - 4)^2 + (-10 + 3)^2} = \sqrt{64 + 4 + 49} = \sqrt{117}$$

$$\therefore \text{Shortest distance between the lines} = \frac{|A|}{|B|} = \frac{15}{\sqrt{117}} \text{ units}$$

16. i)  $P(A' \cap B') = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A)P(B)]$   
 $= 1 - [0.4 + 0.5 - 0.4 \times 0.5] = 1 - [0.9 - 0.2]$   
 $= 1 - 0.7 = 0.3$

ii)  $E_1$ : die shows 5

$E_2$ : die not shows 5

$$P(E_1) = \frac{1}{6}; P(E_2) = \frac{5}{6}$$

Let E: A man reports that it is 5

$$P(E/E_1) = \frac{4}{5}; P(E/E_2) = \frac{1}{5}$$

$$\text{Required probability} = P(E_1/E) = \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{1}{5} \times \frac{5}{6}} = \frac{1 \times 4}{1 \times 4 + 1 \times 5} = \frac{4}{9}$$

$$17. \quad |A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & -3 & 4 \end{vmatrix} = 1(4 + 3) - 2(8 - 1) + 1(-6 - 1)$$

$$= 1(7) - 2(7) + 1(-7) = 7 - 14 - 7 = -14 \neq 0$$

**Cofactors:**  $A_{11} = 7$  ;  $A_{12} = -7$  ;  $A_{13} = -7$

$$A_{21} = -11$$
 ;  $A_{22} = 3$  ;  $A_{23} = 5$

$$A_{31} = 1$$
 ;  $A_{32} = 1$  ;  $A_{33} = -3$

$$\text{Cofactor matrix of } A = \begin{bmatrix} 7 & -7 & -7 \\ -11 & 3 & 5 \\ 1 & 1 & -3 \end{bmatrix}$$

$$\text{Adjoint matrix of } A = \begin{bmatrix} 7 & -11 & 1 \\ -7 & 3 & 1 \\ -7 & 5 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj}A = \frac{1}{-14} \begin{bmatrix} 7 & -11 & 1 \\ -7 & 3 & 1 \\ -7 & 5 & -3 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{-14} \begin{bmatrix} 7 & -11 & 1 \\ -7 & 3 & 1 \\ -7 & 5 & -3 \end{bmatrix} \begin{bmatrix} 18 \\ 5 \\ 3 \end{bmatrix} = \frac{1}{-14} \begin{bmatrix} 126 - 55 + 3 \\ -126 + 15 + 3 \\ -126 + 25 - 9 \end{bmatrix} = \frac{1}{-14} \begin{bmatrix} 74 \\ -108 \\ -110 \end{bmatrix}$$

Hence,  $x = -\frac{74}{14}$  ;  $y = \frac{108}{14}$  and  $z = \frac{110}{14}$

18. i)  $y = \sqrt{\sin x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin x}} \times \cos x$$

ii)  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-a \sin t}{a(1 - \cos t)} = \frac{-2 \sin \frac{t}{2} \cos \frac{t}{2}}{2 \sin^2 \frac{t}{2}} = -\cot \frac{t}{2}$

iii)  $y = x^x$

$$\log y = x \log x$$

$$\frac{1}{y} \frac{dy}{dx} = x \times \frac{1}{x} + \log x \times 1$$

$$\frac{dy}{dx} = y(1 + \log x) = x^x(1 + \log x)$$

19. i)  $I = \int e^x \sin x \, dx$

Using integrating by parts, we have

$$= e^x (-\cos x) - \int e^x (-\cos x) \, dx = -e^x \cos x + \int e^x \cos x \, dx$$

$$= -e^x \cos x + e^x (\sin x) - \int e^x \sin x \, dx = -e^x \cos x + e^x (\sin x) - I$$

$$2I = e^x(\sin x - \cos x)$$

$$\therefore I = \frac{1}{2}e^x(\sin x - \cos x) + c$$

$$\text{ii) } I = \int_0^{\pi/2} \frac{\sin^3 x}{\cos^3 x + \sin^3 x} dx \quad \dots(i)$$

$$\text{We have } \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$\therefore I = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\cos^3\left(\frac{\pi}{2} - x\right) + \sin^3\left(\frac{\pi}{2} - x\right)} dx = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \quad \dots(2)$$

(1) + (2) we have,

$$2I = \int_0^{\pi/2} \frac{\sin^3 x}{\cos^3 x + \sin^3 x} dx + \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$2I = \int_0^{\pi/2} \left[ \frac{\sin^3 x}{\sin^3 x + \cos^3 x} + \frac{\cos^3 x}{\sin^3 x + \cos^3 x} \right] dx$$

$$2I = \int_0^{\pi/2} \left[ \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} \right] dx = \int_0^{\pi/2} 1 dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{4}$$

20.  $3x + 4y = 60$

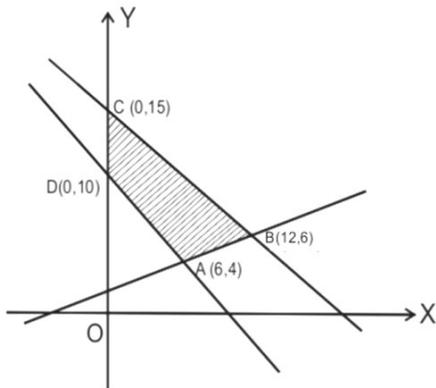
$x$	20	0
$y$	0	15

$x + y = 10$

$x$	10	0
$y$	0	10

$x - 3y = -6$

$x$	-6	0
$y$	0	2



The corner points are A(6, 4), B(12, 6), C(0, 15) and D(0, 10)

Points	$Z = 3x + 7y$
A(6,4)	$Z = 3(6) + 7(4) = 46$
B(12,6)	$Z = 3(12) + 7(6) = 72$
C(0,15)	$Z = 3(0) + 7(15) = 105$
D(0,10)	$Z = 3(0) + 7(10) = 70$

$\therefore Z_{\max} = 105$  at C(0, 15)

