Code Number: **A**

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Time: 3 hrs. **Mock Test Paper for Class-XII** **Max. Marks: 70**

Mathematics

Answers & Solutions

1. (a) $\frac{8}{3}$
2. (d) $\frac{3\pi a^2}{16}$
3. (a) 10
4. (d) 2
5. (d) 8
6. (a) $[\alpha,\beta,\gamma]=0$
7. (d) $\frac{d^2y}{dx^2}-y=0$
8. (b) $y=0$
9. (d) $\frac{1}{(x+1)^2}dx$
10. (a) x^2+y^2
11. (c) 2
12. (d) Parabola
13. (c) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$
14. (a) 45^0
15. (d) $\frac{1}{\sqrt{5}}$
16. (b) $-\frac{q}{r}$
17. (a) 0
18. (c) 2
19. (d) $\text{Adj}(AB)=(\text{adj}A)(\text{adj}B)$
20. (c) -4

21. $\sum_{n=1}^{12} i^2 = (i^1 + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) + (i^9 + i^{10} + i^{11} + i^{12})$

$= (i - 1 - i + 1) + (i^{4+1} + i^{4+2} + i^{4+3} + (i^4)^2) + (i^{8+1} + i^{8+2} + i^{8+3} + (i^4)^3)$

$= 0 + (i + i^2 + i^3 + i^4) + (i^1 + i^2 + i^3 + i^4)$ [\ $i^2 = -1$, $i^3 = -i$, $i^4 = 1$]

$= 0 + (i - 1 - i + 1) + (i - 1 - i + 1)$

$= 0 + 0 + 0 = 0$

22. Since α and β are the roots of the quadratic equation, we have $\alpha + \beta = \frac{7}{2}$ and $\alpha\beta = \frac{13}{2}$ Thus, to construct a new quadratic equation,

Sum of the roots = $a^2 + b^2 = (a + b)^2 - 2ab = \frac{-3}{4}$

Product of the roots = $a^2.b^2 = (a\,b)^2 = \frac{169}{4}$

Thus a required quadratic equation is $x^2 + \frac{3}{4}x + \frac{169}{4} = 0$

$4x^2 + 3x + 169 = 0$ is a quadratic equation with roots α^2 and β^2

23. $x = 3$ and $dx = 0.02$

When x = 3 and dx = 0.02

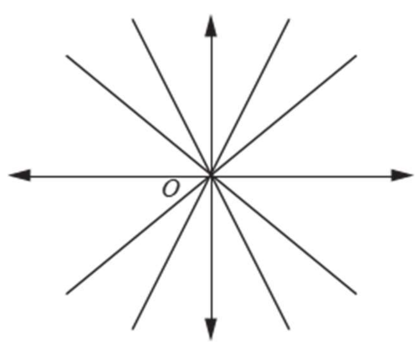
$df = (2x + 3)dx = [2(3) + 3]\,(0.02)$

$df = (6 + 3)\,(0.02) = 9(0.02) = 0.18$

24. The family of straight lines passing through the origin is $y = mx$, where m is an arbitrary constant. … (1)
Differentiating both sides with respect to x,

We get $\frac{dy}{dx} = m$………(2)

From (1) and (2) , we get $y = x\frac{dy}{dx}$. This is the required differential equation.



Observe that the given equation $y = mx$ contains only one arbitrary constant and thus we get the differential equation of order one.

25. $f(x) = \begin{cases} 2(x-1), & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

Mean – E(X) $= \int_1^2 f(x)\,dx = \int_1^2 2(x-1)\,dx$

$= \left[\frac{2x^2}{3} - x^2\right]_1^2$

$= \frac{16}{3} - 4 - \frac{2}{3} + 1 = \frac{5}{3}$

26. Here, $c\,(h, k) = c\,(-3, -4)$ and $r = 3$

Equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$

$\Rightarrow (x - (-3))^2 + (y - (-4))^2 = 3^2$

$\Rightarrow (x + 3)^2 + (y + 4)^2 = 9$

$\Rightarrow x^2 + y^2 + 6x + 8y + 16 = 0$

27. Let $A = \begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

A is a matrix of order 3 × 2

∴ P(A) = 2

We find that there is a second order minor, $\begin{vmatrix} -1 & 3 \\ 4 & -7 \end{vmatrix} = 7 - 12 = -5 \neq 0$

∴ P(A) = 2

28. $I_n = \int_0^{\frac{\pi}{2}} \sin^n x = \frac{n-1}{n} I_{n-2}, n \geq 2$

Let $I_{10} = \int_0^{\frac{\pi}{2}} \sin^{10} x\,dx = \frac{9}{10} I_8 = \frac{9}{10} \times \frac{7}{8} \times I_6$

$= \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times I_4 = \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times \frac{3}{4} I_2$

$= \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times I_4 = \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times \frac{3}{4} I_2$

$= \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}$

$= \frac{315}{1280} \times \frac{\pi}{2} = \frac{63\pi}{256(2)} = \frac{63\pi}{512}$

29. $\lim_{x \to 1} \left(\frac{x^2 + 3x + 2}{x^2 - 4x + 3} \right) \left(\frac{0}{0} \text{form} \right)$

Appling l′Hôpital Rule, we get,

$\lim_{x \to 1} \left(\frac{2x - 3}{2x - 4} \right) \quad \left[\because \frac{x^2 - 3x + 2}{x^2 - 4x + 3} = \frac{(x-1)(x-2)}{(x-1)(x-3)} \right] \quad = \frac{2(1) - 3}{2(1) - 4} = \frac{-1}{-2} = \frac{1}{2}$

30. Let $\vec{a} = 2\vec{i} - \vec{j} + 3\vec{k}; \vec{b} = \vec{i} - \vec{j}, \vec{c} = 3\vec{i} - \vec{j} + 6\vec{k}$

We know that if $\vec{a}, \vec{b}, \vec{c}$ are coplanar

$\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$

$$\left[\vec{a}, \vec{b}, \vec{c}\right] = \begin{vmatrix} + & - & + \\ 2 & -1 & 3 \\ 1 & -1 & 0 \\ 3 & -1 & 6 \end{vmatrix}$$

$= 2(-6-0) + 1(6) + 3(-1+3)$

$= -12 + 6 + 6 = -12 + 12 = 0$

Therefore, the three given vectors are coplanar.

31. Let $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \alpha$

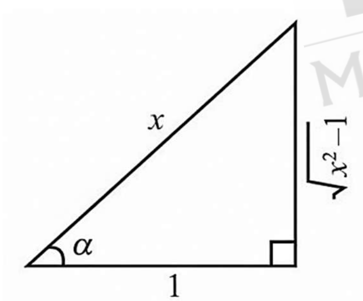
Then, $\cot\alpha = \frac{1}{\sqrt{x^2-1}}$ and α is acute.

We construct a right triangle with the given data. From the triangle, $\sec\alpha = \frac{x}{1} = x$.

Thus, $\alpha = \sec^{-1}x$.

Hence, $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1}x$.

Hence, $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1}x, |x| > 1$.



32. Equation of parabola is $x^2 + 6x + 4y + 5 = 0$.

$x^2 + 6x + 9 - 9 + 4^y + 5 = 0$.

$(x+3)^2 = -4(y-1)\ldots(1)$

Let $X = x+3, Y = y-1$

Equation (1) takes the standard form

$X^2 = -4Y$

Equation of tangent is $XX_1 = -2(Y + Y_1)$

At $(1,-3)\, X_1 = 1+3 = 4; Y_1 = -3-1 = -4$

Therefore, the equation of tangent at $(1,-3)$ is

$(x+3)4 = -2(y-1-4)$

$2x+6 = -7+5$

$2x+y+1 = 0$

Slope of tangent at $(1,-3)$ is -2, so slope of normal at $(1,-3)$ is $\frac{1}{2}$

Therefore, the equation of normal at $(1,-3)$ is given by

$y+3 = \frac{1}{2}(x-1)$

$2y+6 = x-1$

$x-2y-7 = 0.$

33. $LHS = \left[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}\right]$

$[\because \text{cross product is distributive}]$

$= \left(\vec{a}-\vec{b}\right).\left[\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right]$

$= \left(\vec{a}-\vec{b}\right).\left[\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right]$

$= \left(\vec{a}-\vec{b}\right).\left[\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{0}+\vec{c}\times\vec{a}\right]$

$\left[\because \vec{c}\times\vec{c} = \vec{0}\right]$

$\left[\vec{a}\vec{b}\vec{c}\right]-0+0-0+0-\left[\vec{b}\vec{c}\vec{a}\right]$

$\left[\because \left[\vec{a}\vec{b}\vec{a}\right] = \left[\vec{b}\vec{b}\vec{c}\right] = 0\right]$

34. Given $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$

$u(\lambda x+\lambda y) = \frac{\lambda^2 x^2+\lambda^2\lambda y^2}{\sqrt{\lambda x+\lambda y}}$

$= \frac{\lambda^2\left(x^2+y^2\right)}{\sqrt{\lambda}\left(\sqrt{x+y}\right)} = \lambda^{2-\frac{1}{2}u(x,y)} = \lambda^{\frac{3}{2}u(x,y)}$

$\therefore u(x,y)$ is a homogeneous function of degree $\frac{3}{2}$.

$\therefore$ By Euler's theorem,

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = n.u \Rightarrow x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \frac{3}{2}u$

Hence, proved.

35. Let x, y be the two numbers then the sum $x+y=12$ gives $\Rightarrow y=12-x$

Product of the numbers $P=xy=x(12-x)$

$P=12x-x^2$

$P'=12-2x$

$P''=-2$

Substituting $P'=0$, we get

$12-2x=0$

$2x=12$ gives

$x=6$

Since $P''=-2<0$, Product P is maximum at $x=6$. Then $y=12-x$,

gives $y=12-6=6$. The required two numbers are 6, 6.

36. $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}$

$$I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}dx=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\frac{\sqrt{\sin x}}{\cos x}}dx$$

$$=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \quad \ldots 1$$

By the property, $\int_a^b f(x)dx=\int_a^b f(a+b-x)ax$

We get

$$I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{\cos\left(\frac{\pi}{8}+\frac{3\pi}{8}+x\right)}}{\sqrt{\cos\left(\frac{\pi}{8}+\frac{3\pi}{8}-x\right)}+\sqrt{\sin\left(\frac{\pi}{8}+\frac{3\pi}{8}-x\right)}}$$

$$=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$$

$$I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \quad \ldots(2)$$

$(1)+(2)\rightarrow$

$$2I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx+\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$$=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx+\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}dx=[x]_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$$

$$2I=\frac{3\pi}{8}-\frac{\pi}{8}=\frac{2\pi}{8}=\frac{\pi}{4}\Rightarrow 2I=\frac{\pi}{4}\therefore I=\frac{\pi}{8}$$

37. We consider

$$\frac{1+i}{1-i}=\frac{(1+i)(1+i)}{(1-i)(1+i)}=\frac{1+2i-1}{1+1}=\frac{2i}{2}=i \text{ and } \frac{1-i}{1+i}=\left(\frac{1+i}{1-i}\right)^{-1}=\frac{1}{i}=-i$$

Therefore,

$$\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3=i^3-(-i)^3=-i-i=-2i$$

38. Given that

$$\left(1+x^2\right)\frac{dy}{dx}=1+y^2. \qquad \ldots(1)$$

The given equation is written in the variables separable form

$$\frac{dy}{1+y^2}=\frac{dx}{1+x^2}. \qquad \ldots(2)$$

Integrating both sides of (2), we get

$$\tan^{-1}y=\tan^{-1}x+C. \qquad \ldots(3)$$

But $\tan^{-1}y-\tan^{-1}x=\tan^{-1}\left(\frac{y-x}{1+xy}\right)\ldots(4)$

Using (4) in (3) leads to $\left(\frac{y-x}{1+xy}\right)=C$,

which implies $\frac{y-x}{1+xy}=\tan C=a(say)$,

Thus, $y-x=a(1+xy)$ gives the required solution.

39. Sample space $s=\{HHH,HHT,HTH,THH,HTT,THT,TTH,TTT\}$

Let x denote the number of heads occurred.

$p(x=0)$

$p(\text{no head})=p(TTT)=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

$p(x=1)=p(1\text{ head})=p(THT,HTT,TTH)$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$ $\because p(H) = \frac{1}{2}$ and $p(T) = \frac{1}{2}$

$p(x = 2) = p(2 \text{ heads}) = p(HHT, HTH, THH)$ $= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$p(x = 3) = p(3 \text{ heads}) = p(HHH) = \frac{1}{8}$

$\therefore$ X take the value $0, 1, 2, 3$.

Probability mass function is

x	0	1	2	3
$f(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$\therefore f(x) = \begin{cases} \frac{1}{8} & \text{for } x = 0, 3 \\ \frac{3}{8} & \text{for } x = 1, 2 \end{cases}$$

40. Given $\text{adj } A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$

We know that $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

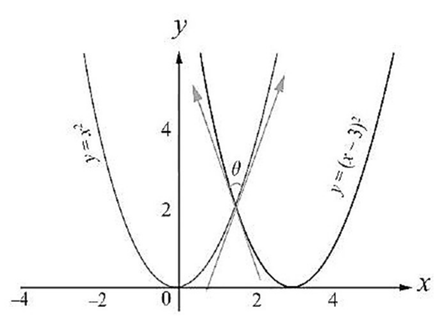
Here $n = 3$, $|\text{adj}(\text{adj})| = |A|^{(3-1)^2} = |A|^4$

$$|A| = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = 2(9-2) + 1(-15+3) + 3(-10+9) = 2(7) + 1(-12) + 3(-1) = 14 - 12 - 3 = 14 - 15 = -1$$

$|\text{adj}(\text{adj } A)| = -1$

41. (a) Let us now find the point of intersection of the two given curves. Equating $x^2 = (x-3)^2$ we get, $x = \frac{3}{2}$. Therefore, the point of intersection is $\left(\frac{3}{2}, \frac{9}{4}\right)$. Let θ be the angle between the curves.

The slopes of the curves are as follows:



For the curve $y = x^2$, $\frac{dy}{dx} = 2x$.

Let $m_1 = \frac{dy}{dx}$ at $\left(\frac{3}{2}, \frac{9}{4}\right) = 3.$

For the curve $y = (x-3)^2$, $\frac{dy}{dx} = 2(x-3).$

Let $m_2 = \frac{dy}{dx}$ at $\left(\frac{3}{2}, \frac{9}{4}\right) = -3.$

Using (3), we get $\tan\theta = \left|\frac{3-(-3)}{1-9}\right| = \frac{3}{4}$

Hence, $\theta = \tan^{-1}\left(\frac{3}{4}\right).$

(OR)

(b) Now, $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x-1}{x-2}\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$

Thus, $\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x-1}{x-2}\left(\frac{x+1}{x+2}\right)} = 1,$

which on simplification gives $2x^2 - 4 = -3.$

Thus, $x^2 = \frac{1}{2}$ gives $x = \pm\frac{1}{\sqrt{2}}.$

42. (a) The numbers on the dice are $1,3,3,5,5,5$ since X denotes the total score in two throws, it takes the values $2,4,6,8,10.$

Sample space s

I/II	1	3	3	5	5	5
1	2	4	4	6	6	6
3	4	6	6	8	8	8
3	4	6	6	8	8	8
5	6	8	8	10	10	10
5	6	8	8	10	10	10
5	6	8	8	10	10	10

From the sample space *s* we have

Values of the random variable	2	4	6	8	10	Total
No. of elements in inverse images	1	4	10	12	9	36

P(x=2)= $\frac{1}{36}$

$\Rightarrow$ P(x=4) = $\frac{4}{36}$

P(x=6) = $\frac{10}{36}$

$\Rightarrow$ P(x=8) = $\frac{12}{36}$

P(x=10)= $\frac{9}{36}$

(i) Probability mass function is

x	2	4	6	8	10	Total
f(*x*)	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$	1

(ii) Cumulative distribution function.

F(x) = p (x $\le$ x)

$\Rightarrow$ F(2) = $\frac{1}{36}$

F(4) = $\frac{1}{36}+\frac{4}{36}+\frac{5}{36}$

$\Rightarrow$ F(6) = $\frac{1}{36}+\frac{4}{36}+\frac{6}{36}=\frac{15}{36}$

F(8) = $\frac{15}{36}+\frac{12}{36}+\frac{27}{36}$

$\Rightarrow$ F(10) = $\frac{27}{36}+\frac{9}{36}=\frac{36}{36}=1$

$\because$ The cumulative distribution function

$$F(x)=\begin{cases} 0 \text{ for} & x<2 \\ \frac{1}{36} \text{ for} & 2\le x4 \\ \frac{5}{36} \text{ for} & 4\le x<6 \\ \frac{15}{36} \text{ for} & 6\le x<8 \\ \frac{27}{36} \text{ for} & 8\le x<10 \\ 1 \text{ for} & 10\le x<\infty \end{cases}$$

$p(4\le x<10)=p(x=4)+p(x=6)+p(x=8)$

$=\frac{4}{36}+\frac{10}{36}+\frac{12}{36}=\frac{26}{36}=\frac{13}{18}$

(Or)

(b) Given $z=x+iy$

$$\text{Im}\left(\frac{2z+1}{iz+1}\right)=0$$

$$\Rightarrow \text{Im}\left(\frac{2(x+iy)+1}{i(x+iy)+1}\right)=0$$

$$\Rightarrow \text{Im}\left(\frac{2(x+1)+2iy}{ix+i^2\ y+1}\right)=0$$

$$\Rightarrow \text{Im}\left(\frac{(2x+1)+2iy}{ix-y+1}\right)=0$$

$$\Rightarrow \text{Im}\left(\frac{(2x+1)+2iy}{(1-y)+ix}\right)=0$$

Multiply and divide by the conjugate of the denominator

We get $\text{Im}\left(\frac{(2x+1)+2iy}{(1-y)+ix}\times\frac{(1-y)-ix}{(1-y)-ix}\right)=0$

$$\Rightarrow \left(\frac{(2x+1)+2iy\times(1-y)-ix}{(1-y)^2+x^2}\right)=0$$

Choosing the imaginary part we get,

$$\left(\frac{(2x+1)(-x)+2y(1-y)}{(1-y)^2+x^2}\right)=0$$

$$\Rightarrow (2x+1)(-x)+2y(1-y)=0$$

$$\Rightarrow -2x^2-x+2y-2y^2=0$$

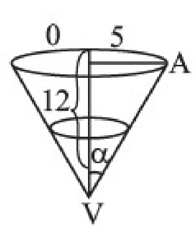
$$\Rightarrow 2x^2+2y^2+x-2y=0$$

Hance, locus of z is $2x^2+2y^2+x-2y=0$

43. (a)

Given $h = 12$ m

$r = 5$ m



Let V be the volume of the cone and α be the semi-vertical angle of the cone:

$$\therefore \tan\alpha=\frac{OA}{VO}=\frac{5}{12}=\frac{r}{h}$$

$$\Rightarrow 12r = 5h$$

$\Rightarrow \quad r = \frac{5h}{12}$

We know $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{5h}{12}\right)^2 .h$ [Form (1)]

$V = \frac{1}{3}\pi\left(\frac{25h^3}{144}\right) = \frac{25\pi h^2}{3\times 144}$

Differentiating with respect to 't' we get,

$\frac{dv}{dt} = \frac{25\mu}{3\times 144}3h^2\frac{dh}{dt} = \frac{25\pi}{144}h^2\frac{dh}{dt}$

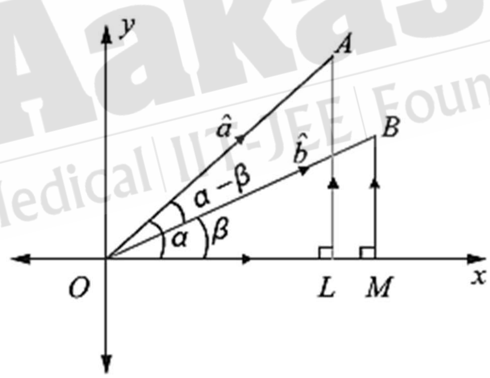
$\Rightarrow \quad 10 = \frac{25\pi}{144}(8)^2\frac{dh}{dt}$

$\left[\therefore \text{Given} h = 8, \frac{dv}{dt} = 10m^3 / \text{min}\right]$

$\Rightarrow \quad \frac{10\times 144}{25\pi\times 64} = \frac{dh}{dt} \quad \Rightarrow \quad \frac{dh}{dt} = \frac{9}{10\pi} m / \text{min}$

(OR)

(b) let $\hat{a} = \overline{OA}$ and $\hat{b} = \overline{OB}$ be the unit vectors making angles α *and* β respectively, with positive x-axis, where A and B are as shown in the Figure. Then, we get $\hat{a} = cos\alpha\hat{\imath} + sin\alpha\hat{\jmath}$ and $\hat{b} = cos\beta\hat{\imath} + sin\beta\hat{\jmath}$, The angle between $\hat{a}$ and $\hat{b}$ is $\alpha - \beta$ and, the vectors $\hat{a}, \hat{b}, \hat{k}$ *bak* form a right-handed system.



Hence, we get

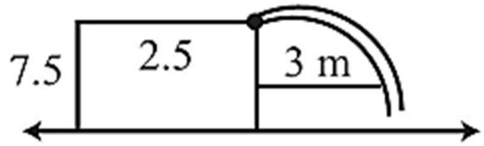
$\hat{b}\times\hat{a} = |\hat{b}||\hat{a}|\sin(\alpha - \beta)\hat{k} = \sin(\alpha - \beta)\hat{k}$

On the other hand, …… (1)

$\hat{b}\times\hat{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} = (\sin\alpha\cos\beta - \cos\alpha\sin\beta)\hat{k}$ …(2)

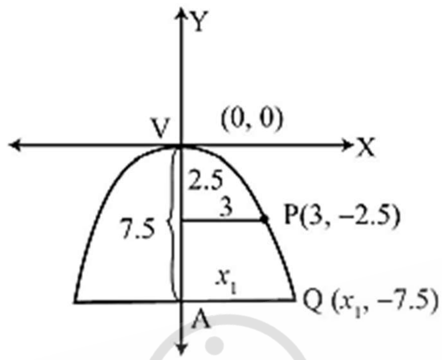
Hence, equations (1) and (2), leads to $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

44. **(a)** As per the given information, we can take the parabola as open downward.



$\therefore$ Its equation is $x^2 = -4ay$

Let P be a point on the flow paths, 2.5 m below the line of the pipe and 3 m beyond the vertical line through the end of the pipe.



$\therefore$ P is $(3, -2.5)$

$\therefore$ (1) becomes $3^2 = -4a(-2.5)$

$\Rightarrow \frac{9}{2.5} = 4a$

$\therefore$ (1) becomes, $x^2 = -\frac{9}{2.5}y$

Let x_1 be the distance between the bottom of the vertical line on the ground from the pipe end and the point on which *this water touches the ground. But the height of the pipe from the ground is 7.5 m.*

$\therefore (x_1, -7.5)$ *lies on*

$\therefore$ *(2) becomes,* $x_1^2 = -\frac{9}{2.5}(-7.5)$

$\Rightarrow x_1^2 = 9(3)$

$\Rightarrow x_1 = \sqrt{9 \times 3} = 3\sqrt{3}\,\text{m}$

The water strikes the ground $3\sqrt{3}$ *m beyond the vertical line.*

(OR)

(b) *This is a linear differential equation.*

$\therefore P = \frac{1}{x}; \; Q = \sin x$

$\int p\,dx = \int \frac{1}{x}dx = \log x$

$I.F. = e^{\int p\,dx} = e^{\log x} = x$

∴ *The solution is*

$ye^{\int p\,dx} = \int Qe^{\int p\,dx}\,dx + c$

$u = x;\ dv = \sin x$

$du = dx;\ v = -\cos x$

$\therefore \int u\,dv = uv - \int v\,du$

$\Rightarrow yx = x\sin\int x\,dx + c \Rightarrow xy = -x\cos x + \cos\int x\,dx \Rightarrow xy = -x\cos x + \sin x + c \Rightarrow xy + x\cos x = \sin x + c$

$\Rightarrow x(y + \cos x) = \sin x + c$

45. **(a)** Let A be the number of bacteria at any time t.

Given $\frac{dA}{A} \propto A$

$\Rightarrow \frac{dA}{A} = kA$

$\Rightarrow \frac{dA}{A} = k\,dt$

$\Rightarrow \int \frac{dA}{A} = k\int dt \Rightarrow \log A = kt + \log c \Rightarrow \log\left(\frac{A}{c}\right) = kt$

$\Rightarrow \frac{A}{c} = e^{kt} \Rightarrow A = ce^{kt}$

Initially when $t = 0$, *assume that* $A = A_0$.

(1) becomes

$A_0 = ce^0 \Rightarrow c = A_0$

$\therefore\ A = A_0e^{kt} \ldots(2)$

Given when $t = 5$, $A = 3A_0$

$3A_0 = A_0e^{5k} \Rightarrow 3 = e^{5k}$

When $t = 10$, *(2) becomes,*

$A = A_0e^{10k} = A_0(e^{5k})^2 = A_0(3)^2 \Rightarrow A = 9A_0$

Hence after 10 hours the number of bacteria is 9 times the original number of bacteria.

(OR)

(b) The required plane contains the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$

i.e. It passes through the point (2,2,1) and is parallel to the vector $3\vec{i} + 2\vec{j} + \vec{k}$.

Also given that the required plane is parallel to the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1}$,

i.e., it is parallel to the vector $3\vec{i} + 2\vec{j} + \vec{k}$.

Therefore $\vec{a} = 2\vec{i} + 2\vec{j} + \vec{k}; \vec{u} = 2\vec{i} + 3\vec{j} + 3\vec{k} \ \vec{v} = 3\vec{i} + 2\vec{j} + \vec{k}$

Vector Equation: The required vector equation is $\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$

$$\vec{r} = \left(2\vec{i} + 2\vec{j} + \vec{k}\right) + s\left(2\vec{i} + 3\vec{j} + 3\vec{k}\right) + t\left(3\vec{i} + 2\vec{j} + \vec{k}\right)$$

Cartesian form: *The Cartesian equation of the required plane is*

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

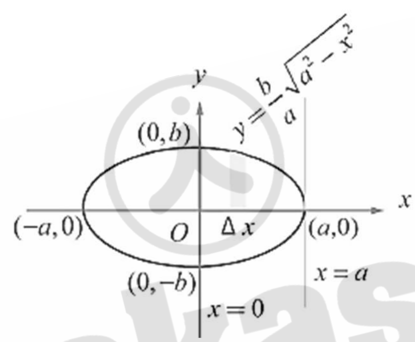
$\Rightarrow (x-2)(3-6) - (y-2)(2-9) + (z-1)(4-0) = 0$

$\Rightarrow -3x + 6 + 7y - 14 - 5z + 5 = 0$

$\Rightarrow -3x + 7y - 5z - 3 = 0$

$\Rightarrow 3x - 7y + 5z + 3 = 0$

46. (A) The ellipse is symmetric about both major and minor axes. It is sketched as in Figure.



So, viewing in the positive direction of y-axis, the required are A is four times the area of the region bounded by the portion of the ellipse in the first quadrant $\left(y = \frac{b}{a}\sqrt{a^2 - x^2}, 0 < x < a\right)$

x-axis, x=0 and x=a.

Hence, by taking vertical strips, we get

$$A = 4\int_0^a y\,dx = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

$$= \frac{4b}{a}\left[\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a = \frac{4b}{a} \times \frac{\pi a^2}{4} = \pi ab$$

(OR)

(b) $y^2 - 4y - 8x + 12 = 0$

$y^2 - 4y = 8x - 12$

Adding 4 both sides, we get,

$y^2 - 4y + 4 = 8x - 12 + 4 = 8x - 8$

$(y-2)^2 = 8(x-1)$

This is a right open parabola and latus rectum is 4a=8 ⟹a=2.

(a) Vertex is (1,2) ⟹ h=1, k=2

(b) Focus is (h+a, 0+k)

⟹ (1+2,0+2) ⟹ (3,2)

(c) Equation of directrix is x= h-a

⟹x = 1-2 ⟹ -1 (or) x+1=0

47. (a)

p	q	$p \leftrightarrow q$	~p	~q	(~p) ∨q	(~q) ∨p	((~p) ∨q) ∧((~q) ∨p)
T	T	T	F	F	T	T	T
T	F	F	F	T	F	T	F
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T
		(1)					(2)

From (1) and (2) we get $p \leftrightarrow q \equiv \left((\sim p) \vee q \wedge (\sim q) \vee p\right)$

(OR)

(b) put $\frac{1}{x} = X, \frac{1}{y}$ = Y, $\frac{1}{z}$ = Z

We get 3X-4Y-2Z= 1, X+2Y+Z=2,

2X-5Y-4Z=-1 $\therefore \Delta = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix} = 3\begin{vmatrix} 2 & 1 \\ -5 & -4 \end{vmatrix} + 4\begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & -5 \end{vmatrix}$

$= 3(-8+5) + 4(-4-2) - 2(-5-4) = 3(-3) + 4(-6) - 2(-9)$

$= -9 - 24 + 18 = -15$

$\Delta_x = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix} = 1\begin{vmatrix} 2 & 1 \\ -5 & -4 \end{vmatrix} + 4\begin{vmatrix} 2 & 1 \\ -1 & -4 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ -1 & -5 \end{vmatrix} = 1(-8+5) + 4(-8+1) - 2(-10+2)$

$= 1(-3) + 4(-7) - 2(-8) = -3 - 28 + 16 = -15$

$\Delta_y = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix}$

$= 3\begin{vmatrix} 2 & 1 \\ -1 & -4 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

$= 3(-8+1) - 1(-4-2) - 2(-1-4) = 3(-7) - 1(-6) - 2(-5) = -21 + 6 + 10 = -5$

$$\Delta_z = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix} = 3\begin{vmatrix} 2 & 2 \\ -5 & -1 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 2 & -5 \end{vmatrix}$$

$$= 3(-2+10) + 4(-1-4) + 1(-5-4) = 3(8) + 4(-5) + 1(-9) = 24 - 20 - 9 = -5$$

$$\therefore X = \frac{\Delta_x}{\Delta} = \frac{-15}{-15} = 1 \Rightarrow \frac{1}{x} = 1$$

$$\Rightarrow x = 1$$

$$Y = \frac{\Delta_y}{\Delta} = \frac{-5}{-15} = \frac{1}{3}$$

$$\Rightarrow \frac{1}{y} = \frac{1}{3} \Rightarrow y = 3$$

$$Z = \frac{\Delta_z}{\Delta} = \frac{-5}{-15} = \frac{1}{3} \Rightarrow \frac{1}{z} = \frac{1}{3} \Rightarrow z = 3$$

$$\therefore x = 1, y = 3, z = 3$$