

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Coefficient of x^{301} in $(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + \dots + x^{500}$ is equal to
- (1) $^{506}C_{306}$
 (2) $^{501}C_{300}$
 (3) $^{501}C_{301}$
 (4) $^{500}C_{300}$

Answer (3)

Sol. Coeff of $x^{301} = ^{500}C_{301} + ^{499}C_{300} + ^{498}C_{299} + \dots + ^{199}C_0$

$$= ^{500}C_{199} + ^{499}C_{199} + ^{498}C_{199} + \dots + ^{199}C_{199}$$

$$= ^{501}C_{200}$$

$$= ^{501}C_{301}$$

2. $\tan 15^\circ + \frac{1}{\tan 165^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then value of $\left(a + \frac{1}{a}\right)$ is

- (1) $4 - 2\sqrt{3}$ (2) $\frac{-4}{\sqrt{3}}$
 (3) 2 (4) $5 - \frac{3}{2}\sqrt{3}$

Answer (2)

Sol. $\tan 15^\circ + \cot 165^\circ + \cot 105^\circ + \tan 195^\circ$

$$= \tan 15^\circ - \cot 15^\circ - \tan 15^\circ + \tan 15^\circ$$

$$= \tan 15^\circ - \cot 15^\circ$$

$$= -2\sqrt{3}$$

$$\Rightarrow a = -\sqrt{3}$$

$$a + \frac{1}{a} = -\sqrt{3} - \frac{1}{\sqrt{3}} = \frac{-4}{\sqrt{3}}$$

3. If set $A = \{a, b, c\}$
 $R : A \rightarrow A$
 $R = \{(a, b), (b, c)\}$

How many elements should be added for making it symmetric and transitive.

- (1) 2 (2) 3
 (3) 4 (4) 7

Answer (4)

Sol. For symmetric

$$(a, b), (b, c) \in R$$

$$\Rightarrow (b, a), (c, b) \in R$$

For transitive.

$$(a, b), (b, c) \in R$$

$$\Rightarrow (a, c) \in R$$

Now,

$$(a, c) \in R$$

$$\Rightarrow (c, a) \in R \quad \text{\{For symmetric\}}$$

$$(a, b), (b, a) \in R$$

$$\Rightarrow (a, a) \in R$$

$$(b, c), (c, b) \in R$$

$$\Rightarrow (b, b) \in R$$

$$(c, b), (b, c) \in R$$

$$\Rightarrow (c, c) \in R$$

\therefore elements to be added

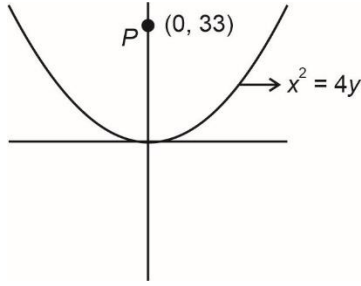
$$\{(b, a) (c, b) (b, b) (a, a) (a, c) (c, a) (c, c)\}$$

Total 7 elements

4. Let $P(h, k)$ be two points on $x^2 = 4y$ which is at shortest distance from $Q(0, 33)$ then difference of distances of $P(h, k)$ from directrix of $y^2 = 4(x + y)$ is
- (1) 2
 (2) 4
 (3) 6
 (4) 8

Answer (2)

Sol. For normal through (0, 33)



Normal at point $(2t, t^2)$

$$x = -ty + 2at + at^3$$

$$0 = -t \cdot 33 + 2t + t^3$$

$$\Rightarrow t = 0 \text{ OR } \pm\sqrt{31}$$

Points at which normal are drawn are

$$A(0, 0), B(2\sqrt{31}, 31), C(-2\sqrt{31}, 31)$$

Shortest distance

$$= PB = PC = \sqrt{124 + 4} = 8\sqrt{2} \text{ units}$$

Given parabola $(y - 2)^2 = 4(x + 1)$

Directrix is $x = -2$, that is line L

$$B_L - C_L = |(-2 + 2\sqrt{31}) - (2 + 2\sqrt{31})|$$

$$= 4$$

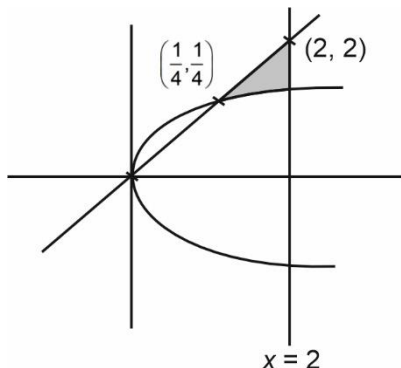
5. Area bounded by larger part in I quadrant by $x = 4y^2$, $x = 2$ and $y = x$ is A then $3A$ equals

$$(1) 6 + \frac{1}{32} - 2\sqrt{2} \quad (2) 2 + \frac{1}{96} - \frac{2\sqrt{2}}{3}$$

$$(3) \frac{2\sqrt{2}}{3} \quad (4) 96$$

Answer (1)

Sol.



Shaded area is the required area

$$A = \int_{1/4}^2 \left(x - \frac{\sqrt{x}}{2} \right) dx$$

$$= \frac{x^2}{2} - \frac{x^{3/2}}{3} \Big|_{1/4}^2$$

$$= \left(2 - \frac{2\sqrt{2}}{3} \right) - \left(\frac{1}{32} - \frac{1}{24} \right)$$

$$= 2 + \frac{1}{96} - \frac{2\sqrt{2}}{3}$$

$$\Rightarrow 3A = 6 + \frac{1}{32} - 2\sqrt{2} \text{ sq. units.}$$

6. A die with points $(2, 1, 0, -1, -2, 3)$ is thrown 5 times. The probability that the product of outcomes on all throws is positive is

$$(1) \frac{521}{2592}$$

$$(2) \frac{16}{81}$$

$$(3) \frac{41}{288}$$

$$(4) \frac{28}{81}$$

Answer (1)

Sol. Either all outcomes are positive or any two are negative.

$$\text{The required probability} = {}^5C_5 \left(\frac{1}{2}\right)^5 + {}^5C_2 \left(\frac{1}{3}\right)^2 \left(\frac{1}{2}\right)^3$$

$$+ {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{1}{2}\right)^1 = \frac{5}{162} + \frac{1}{32} + \frac{5}{36} = \frac{521}{2592}$$

7. Let $S = \{1, 2, 3, 4, 5\}$

if $f: S \rightarrow P(S)$, where $P(S)$ is power set of S . Then number of one-one functions f can be made is

$$(1) (32)^5$$

$$(2) \frac{32!}{27!}$$

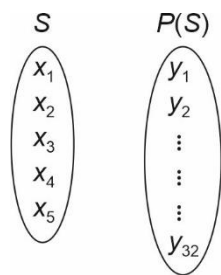
$$(3) {}^{32}C_{27}$$

$$(4) {}^{32}P_{27}$$

Answer (2)

Sol. $n(S) = 5$

$$n(P(S)) = 2^5 = 32$$



$$\therefore \text{No. of one-one function} = 32 \times 31 \times 30 \times 24 \times 28$$

$$= \frac{32!}{27!}$$

8. A line is cutting x axis and y axis at two points A and B, respectively, where $OA = a$, $OB = b$. A perpendicular is drawn from O (origin) to AB at an angle of $\frac{\pi}{6}$ from positive x-axis. If area of triangle

$$OAB = \frac{98\sqrt{3}}{3} \text{ sq. units, then } \sqrt{3} a + b \text{ is equal to}$$

- (1) 28 (2) 14
 (3) 12 (4) 7

Answer (1)

Sol. Let the perpendicular distance of line from origin is p .

$$\Rightarrow \text{Equation of AB: } \frac{x\sqrt{3}}{2} + \frac{y}{2} = p$$

$$\Rightarrow \frac{x}{\frac{2p}{\sqrt{3}}} + \frac{y}{2p} = 1$$

$$OA = \frac{2p}{\sqrt{3}}, OB = 2p$$

$$\frac{1}{2} \cdot \frac{2p}{\sqrt{3}} \cdot 2p = \frac{98}{\sqrt{3}}$$

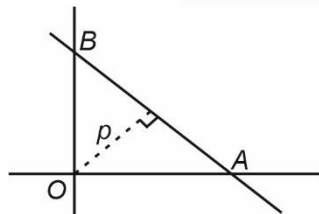
$$\Rightarrow p = 7$$

$$OA = a = \frac{14}{\sqrt{3}}$$

$$OB = b = 14$$

$$\sqrt{3}a + b$$

$$\Rightarrow 14 + 14 = 28$$



9. For solution of differential equation

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} y = -\frac{x^3 \tan^{-1} x^3}{\sqrt{1+x^6}}$$

given that $y(0) = 0$ then $y(1)$ is

- (1) $1 - e^{\frac{\pi}{4\sqrt{2}}}$
 (2) $1 - e^{\left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}\right)}$
 (3) $e^{\frac{1}{\sqrt{2}}} - e^{\frac{\pi}{4\sqrt{2}}}$
 (4) $e^{\frac{\pi}{4\sqrt{2}}}$

Answer (2)

Sol. IF = $\int \frac{-3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} dx$

Let $\tan^{-1}(x^3) = t$

$$IF = e^{-\int t \sin t} = e^{(t \cos t - \sin t)}$$

Solution of Differential equation

$$y \cdot e^{(t \cos t - \sin t)} = \int e^{(t \cos t - \sin t)} (-t \sin t) dt$$

$$y \cdot e^{(t \cos t - \sin t)} = e^{(t \cos t - \sin t)} + c$$

$$t = 0 \rightarrow y = 0$$

$$\therefore c = -1$$

When $x = 1$, $t = \frac{\pi}{4}$

$$y \cdot e^{\left(\frac{\pi}{4\sqrt{2}} - \frac{1}{\sqrt{2}}\right)} = e^{\left(\frac{\pi}{4\sqrt{2}} - \frac{1}{\sqrt{2}}\right)} - 1$$

$$y = 1 - e^{\left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}\right)}$$

10. $\frac{3(e-1)}{e} \int_1^2 x^2 e^{[x]+[x^3]} dx$ equals

- (1) $e^9 - e$ (2) $e^8 - 1$
 (3) $e^8 - e$ (4) $e^9 - 1$

Answer (3)

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

Sol. $I = \int_1^2 x^2 e^{[x]+[x^3]} dx = e \int_1^2 x^2 \cdot e^{[x^3]} dx$

Let $x^3 = t$

$I = e \int_1^8 \frac{dt}{3} e^{[t]} = \frac{e}{3} (e + e^2 + \dots + e^7)$

$= \frac{e^2}{3} \left(\frac{e^7 - 1}{e - 1} \right)$

So, $\frac{3(e-1)}{e} \cdot \frac{e^2}{3} \cdot \frac{e^7 - 1}{e - 1} = e^8 - e$

11. \hat{n} is a vector, $\vec{a} \neq 0, \vec{b} \neq 0$. If $\vec{n} \perp \vec{c}, \vec{a} = \alpha \vec{b} - \hat{n}$ and $\vec{b} \cdot \vec{c} = 12$ then the value of $|\vec{c} \times (\vec{a} \times \vec{b})|$ equals (where \hat{n} represents unit vector in the direction of \vec{n})

- (1) 144
- (2) $\sqrt{12}$
- (3) 12
- (4) 24

Answer (3)

Sol. $\vec{a} = \alpha \vec{b} - \hat{n}$

$\Rightarrow \vec{a} \times \vec{b} = -\hat{n} \times \vec{b}$

Now,

$|\vec{c} \times (\vec{a} \times \vec{b})|$

$= |\vec{c} \times (-\hat{n} \times \vec{b})|$

$= |\hat{n}(12) - \vec{b}(0)|$

$= 12$

- 12.
- 13.
- 14.
- 15.
- 16.
- 17.
- 18.
- 19.
- 20.

21. $\lim_{x \rightarrow 0} \frac{\int_0^x \frac{t^3}{1+t^6} dt}{x^4}$ equals

Answer (12)

Sol. $\lim_{x \rightarrow 0} \frac{48 \int_0^x \frac{t^3}{t^6+1} dt}{x^4}$

As $\frac{0}{0}$ form, applying L' hospital rule we get

$\lim_{x \rightarrow 0} 48 \frac{x^3}{(x^6+1) \cdot 4x^3} = 48 \cdot \frac{1}{4} = 12$

22. If $a_n = \frac{-2}{4n^2 - 16n + 15}$ and $a_1 + a_2 + \dots + a_{25} = \frac{m}{n}$

where m and n are coprime, then the value of $m + n$ is

Answer (191)

Sol. $a_n = \frac{-2}{4n^2 - 16n + 15} = \frac{-2}{(2n-3)(2n-5)}$

$= \frac{1}{2n-3} - \frac{1}{2n-5}$

$a_1 + a_2 + \dots + a_{25} = \left(\frac{1}{-1} - \frac{1}{-3} \right) + \dots + \left(\frac{1}{47} - \frac{1}{45} \right)$

$= \frac{1}{47} + \frac{1}{3} = \frac{50}{141}$

$\therefore m + n = 191$

23. If $z = 1 + i$ and $z_1 = \frac{i + \bar{z}(1-i)}{\bar{z}(1-z)} = z_1$, then find the value of $\frac{12}{\pi} \arg(z_1)$.

Answer (3)

Sol. $z_1 = \frac{i + \bar{z}(1-i)}{\bar{z}(1-z)} = \frac{i + (1-i)(1-i)}{(1-i)(-i)} = \frac{1}{1-i}$

$$\arg z_1 = \arg\left(\frac{1}{1-i}\right) = -\arg(1-i) = \frac{\pi}{4}$$

$$\frac{12}{\pi} \arg(z_1) = \frac{12}{\pi} \times \frac{\pi}{4} = 3$$

24. Mean & Variance of 7 observations are 8 & 16 respectively, if number 14 is omitted then a & b are new mean & variance. The value of $a + b$ is

Answer (19)

Sol. Let x_1, \dots, x_7 are observation

$$\text{New mean} = \frac{8 \times 7 - 14}{6} = 7$$

$$\therefore \frac{\sum_{i=1}^n x_i^2}{7} - 64 = 16 \Rightarrow \sum x_i^2 = 560$$

$$\sum x_{i(\text{new})}^2 = 560 - 14^2$$

$$\therefore b = \frac{364}{6} - 7^2 = \frac{70}{6} = \frac{35}{3}$$

$$\therefore a + b = 7 + \frac{35}{3} = \frac{56}{3} = 18.67$$

Rounding off gives 19

25. If coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to coefficient of x^{-15} in expansion of

$$\left(ax^{1/3} + \frac{1}{bx^3}\right)^{15} \text{ then } |ab - 5| \text{ is equal to}$$

Answer (04.00)

Sol. $a_n \left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15} \Rightarrow T_{r+1} = {}^{15}C_r a^{15-r} (x^3)^{15-r} b^{-r} x^{-\frac{r}{3}}$

$$45 - 3r - \frac{r}{3} = 15 \Rightarrow \frac{10r}{3} = 30$$

$$\boxed{r = 9}$$

$$a_n \left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15} \Rightarrow T_{r+1} = {}^{15}C_r a^{15-r} x^{\frac{15-r}{3}} b^{-r} x^{-3r}$$

$$\frac{15-r}{3} - 3r = -15$$

$$15 - r - 9r = -45$$

$$\Rightarrow r = 6$$

$$\text{So, } {}^{15}C_9 a^6 b^{-9} = {}^{15}C_6 a^9 b^{-6}$$

$$\Rightarrow a^{-3} b^{-3} = 1$$

$$\text{or } \boxed{ab = 1}$$

$$|ab - 5| = 4$$

26. Using 1, 2, 3, 5, 4-digit numbers are formed, where repetition is allowed. How many of them is divisible by 15?

Answer (21)

Sol. Units digit will be 5

$$\underline{a} \quad \underline{b} \quad \underline{c} \quad \underline{5}$$

$$a + b + c = (3\lambda + 1) \text{ type}$$

For (a, b, c) possibilities are

$$(2, 2, 3) (1, 1, 5) (1, 1, 2)$$

$$(3, 3, 1) (5, 5, 3) (2, 3, 5)$$

$$\text{For } (2, 2, 3) \Rightarrow \frac{3!}{2!} = 3$$

$$\text{For } (1, 1, 5) \Rightarrow \frac{3!}{2!} = 3$$

$$\text{For } (1, 1, 2) \Rightarrow \frac{3!}{2!} = 3$$

$$\text{For } (3, 3, 1) \Rightarrow \frac{3!}{2!} = 3$$

$$\text{For } (5, 5, 3) \Rightarrow \frac{3!}{2!} = 3$$

$$\text{For } (2, 3, 5) \Rightarrow 3! = 6$$

$$\text{Total} = 21$$

27. If $5f(x+y) = f(x) \cdot f(y)$ and $f(3) = 320$, then the value of $f(1)$ is

Answer (20)

Sol. $5f(x+y) = f(x) \cdot f(y)$... (i) $f(3) = 320$

Put $x = 1, y = 2$ in (i)

$$5f(3) = f(1) \cdot f(2)$$

$$\Rightarrow f(1) \cdot f(2) = 5 \times 320 = 1600 \quad \dots (ii)$$

Put $x = y = 1$ in (i)

$$5f(2) = (f(1))^2$$

$$\Rightarrow f(2) = \frac{(f(1))^2}{5} \quad \dots (iii)$$

Using (iii) in (ii),

$$f(1) \cdot \frac{(f(1))^2}{5} = 1600$$

$$(f(1))^3 = 8000$$

$$f(1) = 20$$

28. If for $\log_{\cos x}(\cot x) - 4\log_{(\sin x)} \cot x = 1$,

$$x = \sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right). \text{ Find } (\alpha + \beta), \text{ given } x \in \left(0, \frac{\pi}{2}\right)$$

Answer (04.00)

Sol. $\log_{\cos x} \cot x - 4\log_{\sin x} \cot x = 1$

$$1 - \log_{\cos x} \sin x - 4(\log_{\sin x} \cos x - 1) = 1$$

Let $\log_{\cos x} \sin x = t$

$$-t - 4\left(\frac{1}{t} - 1\right) = 0$$

$$\Rightarrow t + \frac{4}{t} = 4$$

$$\Rightarrow t = 2$$

$$\log_{\cos x} \sin x = 2$$

$$\Rightarrow \cos^2 x = \sin x$$

$$\Rightarrow 1 - \sin^2 x - \sin x = 0$$

$$\Rightarrow \sin^2 x + \sin x - 1 = 0$$

$$\text{So, } \sin x = \frac{-1 \pm \sqrt{5}}{2}$$

$$\alpha = -1, \beta = 5$$

$$\alpha + \beta = 4$$

29.

30.

