

## MATHEMATICS

### SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. If  $f(x) + f(\pi - x) = \pi^2$  then  $\int_0^\pi f(x) \sin x dx$

(1)  $\pi^2$

(2)  $\frac{\pi^2}{4}$

(3)  $2\pi^2$

(4)  $\frac{\pi^2}{8}$

**Answer (1)**

**Sol.**  $I = \int_0^\pi f(x) \sin x dx$

$$I = \int_0^\pi f(\pi - x) \sin x dx$$

$$2I = \int_0^\pi (\sin x)(f(x) + f(\pi - x)) dx$$

$$2I = \pi^2 \int_0^\pi \sin x dx$$

$$2I = 2\pi^2 \int_0^{\frac{\pi}{2}} \sin x dx$$

$$I = \pi^2 (-\cos x) \Big|_0^{\frac{\pi}{2}}$$

$$= \pi^2$$

2. The system of the equations

$$x + y + z = 6$$

$$x + 2y + \alpha z = 5$$
 has

$$x + 2y + 6z = \beta$$

- (1) Infinitely many solution for  $\alpha = 6, \beta = 3$
- (2) Infinitely many solution for  $\alpha = 6, \beta = 5$
- (3) Unique solution for  $\alpha = 6, \beta = 5$
- (4) No solution for  $\alpha = 6, \beta = 5$

**Answer (2)**

**Sol.** Let  $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 2 & 6 \end{vmatrix} = 6 - \alpha$

$\therefore$  for  $\alpha \neq 6$  system has unique solution

Now, when  $\alpha = 6$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 2 & 6 \\ \beta & 2 & 6 \end{vmatrix} = 0 - (30 - 6\beta) + (10 - 2\beta)$$

$$= 4(\beta - 5)$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 2 \\ 1 & 5 & 6 \\ 1 & \beta & 6 \end{vmatrix} = -4(\beta - 5)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 5 \\ 1 & 2 & \beta \end{vmatrix} = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & -1 \\ 0 & 1 & \beta - 6 \end{vmatrix} = \beta - 5$$

Clearly at  $\beta = 5, \Delta_i = 0$  for  $i = 1, 2, 3$

$\therefore$  at  $\alpha = 6, \beta = 5$  system has infinite solutions.

3. Let statement 1 :  $(2002)^{2023} - (1919)^{2002}$  is divisible by 8.

Statement 2 :  $13 \cdot 13^n - 12n - 13$  is divisible by 144  
 $\forall n \in \mathbb{N}$ , then

- (1) Statement-1 and statement-2 both are true
- (2) Only statement-1 is true
- (3) Only statement-2 is true
- (4) Neither statement-1 nor statement-2 are true

**Answer (3)**

**Sol.**  $\because (2002)^{2023} = 8m$

$\therefore (2002)^{2023}$  is divisible by 8

and  $(1919)^{2002}$  is not divisible by 8

$\therefore (2002)^{2023} - (1919)^{2002}$  is not divisible by 8.

Now

$$13 \cdot (13)^n - 12n - 13$$

$$= 13(1 + 12)^n - 12n - 13$$

$$= 13[1 + 12n + n_{C_2} 12^2 + \dots] - 12n - 13$$

$$= 144n + 144n_{C_2} + \dots$$

$$= 144[n + n_{C_2} + \dots]$$

$$= 144K$$

$\therefore$  Statement-2 is correct

4. If the coefficient of  $x^7$  in  $\left(\alpha x^2 + \frac{1}{2\beta x}\right)^{11}$  and  $x^{-7}$  in  $\left(x + \frac{1}{3\beta x^2}\right)^{11}$  are equal then

$$(1) \alpha^6 \beta = \frac{2^5}{3^6}$$

$$(2) \alpha^6 \beta = \frac{2^6}{3^5}$$

$$(3) \alpha \beta^6 = \frac{2^5}{3^6}$$

$$(4) \alpha \beta^6 = \frac{2^6}{3^5}$$

**Answer (1)**

**Sol.** Coefficient of  $x^7$  in  $\left(\alpha x^2 + \frac{1}{2\beta x}\right)^{11}$

$$T_{r+1} = {}^{11}C_r \left(\alpha x^2\right)^{11-r} \left(\frac{1}{2\beta x}\right)^r$$

$$\text{Now, } 22 - 2r - r = 7$$

$$r = 5$$

$$\text{Coeff.} = {}^{11}C_5 \frac{\alpha^6}{(2\beta)^5}$$

Coeff. of  $x^{-7}$  in  $\left(x + \frac{1}{3\beta x^2}\right)^{11}$  will be, if  $r = 6$  is  $\frac{{}^{11}C_6}{3^6 \beta^6}$

$${}^{11}C_5 \frac{\alpha^6}{2^5 \beta^5} = \frac{{}^{11}C_5}{3^6 \beta^6}$$

$$\alpha^6 \beta = \frac{2^5}{3^6}$$

5. If  $(21)^{18} + 20 \cdot (21)^{17} + (20)^2 \cdot (21)^{16} + \dots + (20)^{18} = k(21^{19} - 20^{19})$  then  $k =$

$$(1) \frac{21}{20} \quad (2) 1$$

$$(3) \frac{21}{20} \quad (4) \frac{20}{21}$$

**Answer (2)**

**Sol.**  $a = (21)^{18}, r = \frac{20}{21}, n = 19$

$$S = (21)^{18} \frac{\left(1 - \left(\frac{20}{21}\right)^{19}\right)}{1 - \frac{20}{21}}$$

$$\Rightarrow \frac{(21)^{19}}{(21)^{19}} \left((21)^{19} - (20)^{19}\right)$$

$$= (21)^{19} - (20)^{19}$$

6. If  $1^2 - 2^2 + 3^2 - 4^2 + \dots - (2022)^2 + (2023)^2 = m^2 n$ , where  $m, n \in N$  and  $m > 19$  then  $n^2 - m$  is
- 615
  - 562
  - 812
  - 264

**Answer (1)**

$$\begin{aligned} & 1^2 - 2^2 + 3^2 - 4^2 + \dots - (2021)^2 + (2022)^2 + (2023)^2 \\ &= \underbrace{-3 - 7 - 11 - \dots}_{1011 \text{ terms}} + (2023)^2 \end{aligned}$$

$$= -\frac{1011}{2}(6 + (1010)4) + (2023)^2$$

$$= -1011(3 + 2020) + (2023)^2$$

$$= (2023)(-1011) + (2023)^2$$

$$= (2023)(2023 - 1011)$$

$$= 2023(1012)$$

$$= (17)^2 \cdot 7(2^2 \cdot 253) = (34)^2 (1771)$$

$$= m = 34, n = 1771$$

$$\therefore n - m^2$$

$$= 1771 - 1156$$

$$= 615$$

7. If  $a \neq b$  and are purely real,  $z \in \text{complex number}$ ,  $\operatorname{Re}(az^2 + bz) = a$  and  $\operatorname{Re}(bz^2 + az) = b$  then number of value of  $z$  possible is

$$(1) 0$$

$$(2) 1$$

$$(3) 2$$

$$(4) 3$$

**Answer (1)**

$$\text{Sol. } a(x^2 - y^2) + bx = a \quad \dots(i)$$

$$b(x^2 - y^2) + ax = b \quad \dots(ii)$$

$$(i) - (ii)$$

$$(a - b)(x^2 - y^2) + (b - a)x = a - b \quad (a \neq b)$$

$$\Rightarrow x^2 - y^2 - x = 1$$

$$(i) + (ii)$$

$$(a + b)(x^2 - y^2) + x(a + b) = a + b \quad (a \neq -b)$$

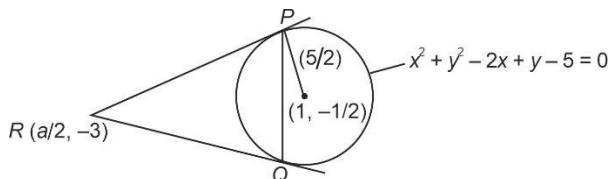
$$\Rightarrow x^2 - y^2 + x = 1$$

$$\Rightarrow x = 0$$

$$\Rightarrow y^2 = -1 \quad (\text{not possible as } y \in R)$$

$\therefore$  No complex number possible.

8. Two tangents are drawn from point  $R\left(\frac{a}{2}, -3\right)$  intersect the circle  $x^2 + y^2 - 2x + y = 5$  at P and Q then the area of  $\Delta PQR$  is
- (1)  $\frac{1710}{290}$       (2)  $\frac{1715}{296}$   
 (3)  $\frac{296}{1715}$       (4)  $\frac{290}{1710}$

**Answer (2)**
**Sol.**


$$\text{Area} = \frac{RL^3}{R^2 + L^2} r$$

$$L = \frac{7}{2}$$

$$\text{Area} = \frac{\frac{5}{2} \times \left(\frac{7}{2}\right)^3}{\left(\frac{5}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$$

$$= \frac{\frac{1715}{16}}{\frac{25+49}{4}} = \frac{1715 \times 4}{16 \times 74} = \frac{1715}{296}$$

9. Equation of plane passing through intersection of  $P_1 : \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$  and  $P_2 : \vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$  and  $(0, 2, -2)$  is P. Then square of distance of  $(12, 12, 18)$  from P is

- (1) 310      (2) 1240  
 (3) 155      (4) 620

**Answer (4)**
**Sol.**  $P_1 + \lambda P_2 = 0$ 

$$(x + y + z - 6) + \lambda (2x + 3y + 4z + 5) = 0$$

$$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + 5\lambda - 6 = 0$$

 Passing through  $(0, 2, -2)$ 

$$\Rightarrow \lambda = 2$$

 ∴ Plane :  $5x + 7y + 9z = -4$ 

$$\text{Distance} = \sqrt{\frac{5(12) + 7(12) + 9(18) + 4}{5^2 + 7^2 + 9^2}}$$

$$= 620$$

10. If V is volume of parallelopiped whose three coterminous edges are  $\vec{a}, \vec{b}, \vec{c}$ , then volume of a parallelopiped whose coterminous edges are  $\vec{a}, \vec{b} + \vec{c}, \vec{a} + 2\vec{b} + 3\vec{c}$  is

- (1)  $6V$       (2)  $V$   
 (3)  $2V$       (4)  $3V$

**Answer (2)**
**Sol.**  $[\vec{a} \ \vec{b} + \vec{c} \ \vec{a} + 2\vec{b} + 3\vec{c}]$ 

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} [\vec{a} \ \vec{b} \ \vec{c}]$$

$$= [\vec{a} \ \vec{b} \ \vec{c}]$$

$$= V$$

11.  $S_1 : (p \Rightarrow q) \vee (\neg p \wedge q)$  is a tautology  
 $S_2 : (q \Rightarrow p) \Rightarrow (\neg p \wedge q)$  is a contradiction
- (1) Both  $S_1$  and  $S_2$  are true  
 (2) Neither  $S_1$  Nor  $S_2$  are true  
 (3) Only  $S_1$  are true  
 (4) Only  $S_2$  are true

**Answer (2)**

$$\begin{aligned} \text{Sol. } & (p \Rightarrow q) \vee (\neg p \wedge q) \\ & \equiv (\neg p \vee q) \vee \neg p \wedge q \\ & \equiv \neg p \vee q \vee \neg pq \\ & \equiv \neg p \vee q \end{aligned}$$

Which is not a tautology

$$\begin{aligned} & (q \Rightarrow p) \Rightarrow (\neg p \wedge q) \\ & \equiv (\neg q \vee p) \Rightarrow (\neg p \wedge q) \\ & \equiv \neg(\neg q \vee p) \vee (\neg p \wedge q) \\ & \equiv (q \wedge (\neg p)) \vee (\neg p \wedge q) \\ & \equiv \neg p \wedge q \end{aligned}$$

Which is not a contradiction.

12.  $(1 + \ln x) \frac{dx}{dy} + x \ln x = e^y$

 Solution of this differential equation satisfies  $(1, 90)$  and  $(\alpha, 92)$  then  $\alpha^\alpha$  is

- (1)  $\frac{e^{90}}{90}$       (2)  $\frac{e^{92} - e^{90}}{45}$   
 (3)  $e^{\left(\frac{e^{92} - e^{90}}{90}\right)}$       (4)  $e^{92} - e^{90}$

**Answer (3)**

**Sol.**  $(1 + \ln x)dx + x \ln x dy = e^y dy$

$$d(y \cdot x \ln x) = d(e^y)$$

$$\Rightarrow xy \ln x = e^y + C$$

Through  $(1, 90) \Rightarrow C = -e^{90}$

$$xy \ln x = e^y - e^{90}$$

$$\therefore \alpha \cdot 92 \ln \alpha = e^{92} - e^{90}$$

$$\ln \alpha^\alpha = \frac{e^{92} - e^{90}}{92}$$

$$\alpha^\alpha = e^{\left(\frac{e^{92} - e^{90}}{92}\right)}$$

13.

14.

15.

16.

17.

18.

19.

20.

### SECTION - B

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The rank of the word "PUBLIC" is

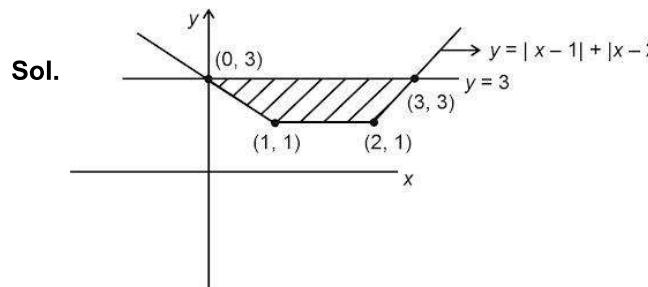
**Answer (198)**

$$\begin{array}{ccccccc}
 5 & 6 & 1 & 4 & 3 & 2 \\
 P & U & B & L & I & C \\
 \therefore & 4 & 4 & 0 & 2 & 1 & 0 \\
 4! & 4! & 3! & 2! & 1! & 0!
 \end{array}$$

$$\begin{aligned}
 \therefore \text{Rank} &= (1 \times 1! + 2 \times 2! + 4 \times 4! + 4 \times 4!) + 1 \\
 &= (1 + 4 + 96 + 96) + 1 \\
 &= 198
 \end{aligned}$$

22. The area enclosed by  $y = |x-1| + |x-2|$  and  $y = 3$  is

**Answer (04)**



$$\begin{aligned}
 &= \frac{1}{2}[1+3] \times 2 \\
 &= 4
 \end{aligned}$$

23. If the number of all 4 letter words with 2 vowels and 2 consonants from the word UNIVERSE is  $n$ , then  $n - 500$  is

**Answer (4)**

**Sol.** Vowels  $\rightarrow$  I, U, E, E

Consonants  $\rightarrow$  N, V, R, S

(I) 2 Vowels same      (II) 2 Vowels different

$$4C_2 \times \frac{4!}{2!} = 72 \quad 3C_2 \times 4C_2 \times 4! = 432$$

$$72 + 432 = 504$$

24. Three dice are thrown. Then the probability that no outcomes is similar is  $\frac{p}{q}$  then  $q-p$  is (where  $p$  and  $q$  are co-prime)

**Answer (04)**

$$\text{Sol. } P(E) = \frac{6 \times 5 \times 4}{6 \times 6 \times 6}$$

$$= \frac{20}{36} = \frac{5}{9} = \frac{p}{q}$$

$$q-p = 9-5 = 4$$

25.  $P^2 = I - P$

$$P^\alpha + P^\beta = \gamma I - 2qp$$

$$P^\alpha - P^\beta = \delta I - 13P$$

Then find the value of  $\alpha + \beta + \gamma - \delta$

**Answer (24)**

**Sol.**  $P^3 = P - P^2$   
 $= P - (I - P) = 2P - I$   
 $P^4 = 2P^2 - P$   
 $= 2(I - P) - P = 2I - 3P$   
 Similarly  
 $P^6 = 5I - 8P$   
 and  $P^8 = 13I - 21P$   
 $P^8 + P^6 = 18I - 29P$

$$P^8 - P^6 = 8I - 13P$$
 $\alpha = 8, \beta = 6, \gamma = 18, \delta = 8$ 
 $\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$ 

26.  
27.  
28.  
29.  
30.

□ □ □

