Date: 15/03/2024

Question Paper Code

T24 511

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Time: 2½ Hrs.

MATHEMATICS

Max. Marks: 80

ICSE Board Class X Exam (2024)

Answers & Solutions

GENERAL INSTRUCTIONS

Read the following instructions very carefully and follow them:

(i) You will not be allowed to write during first 15 minutes.

(ii) Attempt all questions from Section A and any four questions from Section B.

(iii) Omission of essential working will result in loss of marks.

(iv) The intended marks for questions or parts of questions are given in brackets [ ].

(v) Mathematical tables and graph papers are provided.

SECTION-A (40 Marks)

*(Attempt **all** questions from this Section.)*

1. Choose the correct answers to the questions from the given options. **[15]**

(i) For an Intra-state sale, the CGST paid by a dealer to the Central government is ₹120. If the marked price of the article is ₹2000, the rate of GST is

(a) 6% (b) 10%

(c) 12% (d) 16.67%

Answer (c) **[1]**

Sol. Rate of G.S.T. = $\frac{2\times 120}{2000}\times 100$

= 12%

(ii) What must be subtracted from the polynomial $x^3 + x^2 - 2x + 1$, so that the result is exactly divisible by $(x - 3)$?

(a) –31 (b) –30

(c) 30 (d) 31

Answer (d) **[1]**

Sol. $3^3 + 3^2 - 2(3) + 1 = 31$

(iii) The roots of the quadratic equation $px^2 - qx + r = 0$ are real and equal if:

(a) $p^2 = 4qr$ (b) $q^2 = 4pr$

(c) $-q^2 = 4pr$ (d) $p^2 > 4qr$

Answer (b) **[1]**

Sol. $(-q)^2 - 4pr = 0$

$\Rightarrow \quad q^2 = 4pr$

(iv) If matrix $A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ and $A^2 = \begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$, then the value of x is

(a) 2 (b) 4

(c) 8 (d) 10

Answer (c) **[1]**

Sol. $A^2 = \begin{bmatrix} 4 & 8 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$

$x = 8$

(v) The median of the following observations arranged in ascending order is 64.

Find the value of x:

27, 31, 46, 52, x, $x + 4$, 71, 79, 85, 90

(a) 60 (b) 61

(c) 62 (d) 66

Answer (c) **[1]**

Sol. $\frac{x+(x+4)}{2} = 64 \Rightarrow x = 62$

(vi) Points $A(x, y)$, $B(3, -2)$ and $C(4, -5)$ are collinear. The value of y in terms of x is

(a) $3x - 11$ (b) $11 - 3x$

(c) $3x - 7$ (d) $7 - 3x$

Answer (d) **[1]**

Sol. $\frac{3-x}{-2-y} = \frac{4-3}{-5+2} \Rightarrow y = 7 - 3x$

(vii) The given table shows the distance covered and the time taken by a train moving at a uniform speed along a straight track.

Distance (in m)	60	90	y
Time (in sec)	2	x	5

The values of x and y are:

(a) $x = 4, y = 150$ (b) $x = 3, y = 100$

(c) $x = 4, y = 100$ (d) $x = 3, y = 150$

Answer (d) **[1]**

Sol. Distance and time are directly proportional

$x = 3$ seconds

$y = 150$ meters

(viii) The 7th term of the given Arithmetic Progression (A.P.)

$\frac{1}{a}, \left(\frac{1}{a}+1\right), \left(\frac{1}{a}+2\right)...$ is

(a) $\left(\frac{1}{a}+6\right)$ (b) $\left(\frac{1}{a}+7\right)$

(c) $\left(\frac{1}{a}+8\right)$ (d) $\left(\frac{1}{a}+7^7\right)$

Answer (a) **[1]**

Sol. $a_7 = a + (7-1)d$

$= \frac{1}{a} + 6(1)$

$= \frac{1}{a} + 6$

(ix) The sum invested to purchase **15** shares of a company of nominal value **₹75** available at a discount of **20%** is

(a) ₹60 (b) ₹90

(c) ₹1350 (d) ₹900

Answer (d) **[1]**

Sol. Money invested = Number of shares × Market value of 1 share

= 15 × (75 – 20% of 75)

= 15 × 60

= ₹900

(x) The circumcentre of a triangle is the point which is

(a) at equal distance from the three sides of the triangle.

(b) at equal distance from the three vertices of the triangle.

(c) the point of intersection of the three medians.

(d) the point of intersection of the three altitudes of the triangle.

Answer (b) **[1]**

Sol. Circumcentre of a triangle is the point which is at equal distance from the three vertices of the triangle.

(xi) Statement 1: $\sin^2\theta + \cos^2\theta = 1$

Statement 2: $\text{cosec}^2\theta + \cot^2\theta = 1$

Which of the following is valid?

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

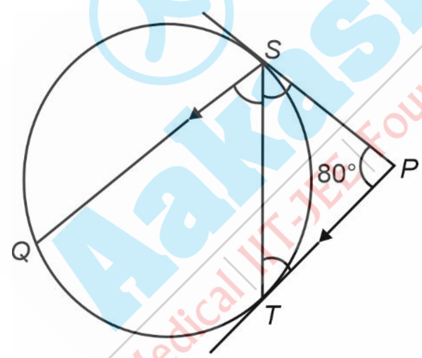
Answer (a) **[1]**

Sol. We know that, $\sin^2\theta + \cos^2\theta = 1$ and

$\text{cosec}^2\theta - \cot^2\theta = 1$

$\therefore$ Only Statement 1 is valid.

(xii) In the given diagram, *PS* and *PT* are the tangents to the circle. *SQ* || *PT* and $\angle SPT = 80°$. The value of $\angle QST$ is

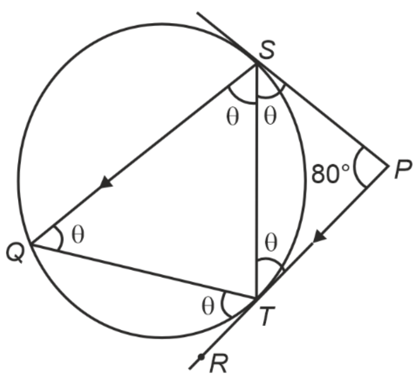


(a) 140° (b) 90°

(c) 80° (d) 50°

Answer (d) **[1]**

Sol.



Let $\angle QST = \theta$

$\Rightarrow$ $\angle STP = \angle QST = \theta$ [Alternate interior angles]

$\Rightarrow \quad \angle SQT = \angle STP = \theta$ [Alternate segment theorem]

$\Rightarrow \quad \angle PST = \angle SQT = \theta$ [Alternate segment theorem]

Now,

In ΔPST,

$\theta + \theta + 80° = 180°$

$\Rightarrow \quad \theta = 50°$

(xiii) **Assertion (A):** A die is thrown once and the probability of getting an even number is $\mathbf{\frac{2}{3}}$.

Reason (R): The sample space for even numbers on a die is **{2, 4, 6}**

(a) A is true, R is false
(b) A is false, R is true
(c) Both A and R are true
(d) Both A and R are false

Answer (b) **[1]**

Sol. A is false, R is true

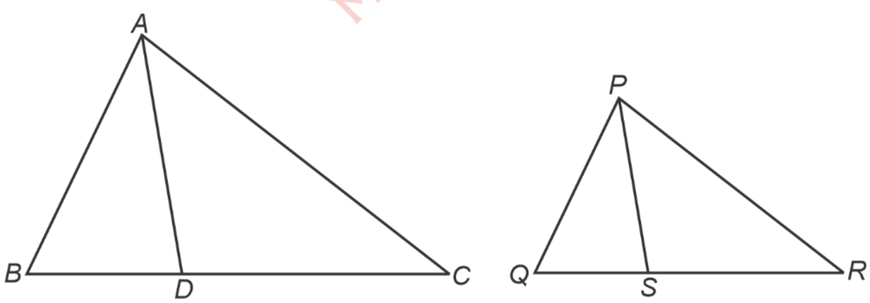
(xiv) A rectangular sheet of paper of size 11 cm × 7 cm is first rotated about the side **11 cm** and then about the side **7 cm** to form a cylinder, as shown in the diagram. The ratio of their curved surface areas is

(a) 1 : 1
(b) 7 : 11
(c) 11 : 7
(d) $\frac{11\pi}{7} : \frac{7\pi}{11}$

Answer (a) **[1]**

Sol. 1 : 1

(xv) In the given diagram, $\mathbf{\Delta ABC \sim \Delta PQR}$. If AD and PS are bisectors of $\angle \mathbf{BAC}$ and $\angle \mathbf{QPR}$ respectively then



(a) $\Delta ABC \sim \Delta PQS$
(b) $\Delta ABD \sim \Delta PQS$
(c) $\Delta ABD \sim \Delta PSR$
(d) $\Delta ABC \sim \Delta PSR$

Answer (b) **[1]**

Sol. $\Delta ABD \sim \Delta PQS$

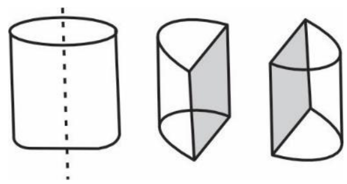
2. (i) $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$ **[4]**

Find the values of x and y, if $AB = C$.

(ii) A solid metallic cylinder is cut into two identical halves along its height (as shown in the diagram). The diameter of the cylinder is 7 cm and the height is 10 cm. Find : **[4]**

(a) The total surface area (both the halves)

(b) The total cost of painting the two halves at the rate of ₹30 per cm^2. $\left(\text{Use } \pi = \dfrac{22}{7}\right)$



(iii) 15, 30, 60, 120… are in G.P. (Geometric Progression). **[4]**

(a) Find the n^{th} term of this G.P. in terms of n.

(b) How many terms of the above G.P. will give the sum 945?

Sol. (i) $AB = C$

$$\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$$ **[½]**

$$\Rightarrow \begin{bmatrix} 4x & 0 \\ 4+y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$$ **[1]**

$\Rightarrow 4x = 4$ **[1]**

$x = 1$ **[½]**

$4 + y = x = 1$ **[½]**

$\Rightarrow y = -3$ **[½]**

(ii) (a) Total surface area of (both halves)

$= 2 \times$ Total surface area of one half

$= 2\pi r(r + h) + 2 \times 2r \times h$ **[1]**

$= 2 \times \dfrac{22}{7} \times \dfrac{7}{2}\left[\dfrac{7}{2} + 10\right] + 2 \times 7 \times 10$ **[1]**

$= 437$ cm^2 **[1]**

(b) Total cost of painting

$= 30 \times 437$ **[½]**

$=$ ₹13,110 **[½]**

(iii) (a) $a = 15$

$r = \dfrac{30}{15} = 2$ **[½]**

$a_n = ar^{n-1}$ **[1]**

$a_n = 15(2)^{n-1}$ **[½]**

(b) $S_n = \dfrac{a(r^n - 1)}{r - 1}$ **[½]**

Now, $945 = \dfrac{15(2^n - 1)}{2 - 1}$ **[½]**

$\Rightarrow$ $945 = 15(2^n - 1)$ **[½]**

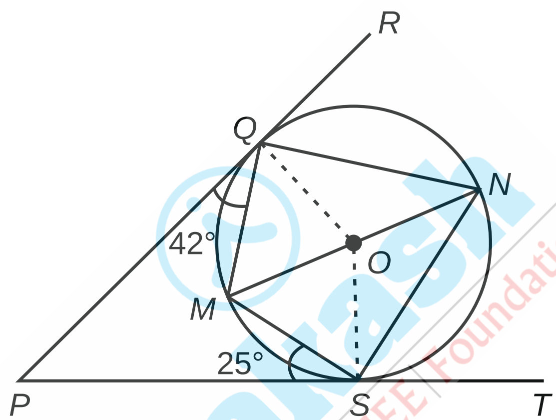
$\Rightarrow$ $2^n = 64$

$\Rightarrow$ $n = 6$ **[½]**

3. (i) Factorize : $\sin^3\theta + \cos^3\theta$ **[4]**

Hence, prove the following identity : $\dfrac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta = 1$

(ii) In the given diagram, *O* is the centre of the circle. *PR* and *PT* are two tangents drawn from the external point *P* and touching the circle at *Q* and *S* respectively. *MN* is a diameter of the circle. Given $\angle PQM = 42°$ and $\angle PSM = 25°$. **[4]**



Find :

(a) $\angle OQM$

(b) $\angle QNS$

(c) $\angle QOS$

(d) $\angle QMS$

(iii) Use graph sheet for this question. Take 2 cm = 1 unit along the axes. **[5]**

(a) Plot *A*(0, 3), *B*(2, 1) and *C*(4, –1).

(b) Reflect point *B* and *C* in *y*-axis and name their images as *B′* and *C′* respectively. Plot and write coordinates of the points *B′* and *C′*.

(c) Reflect point *A* in the line *BB′* and name its images as *A′*.

(d) Plot and write coordinates of point *A′*.

(e) Join the points *ABA′B′* and give the geometrical name of the closed figure so formed.

Sol. (i) $\sin^3\theta + \cos^3\theta = (\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)$ [Using $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$] **[1]**

$= (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$ …(i) [$\because$ $\sin^2\theta + \cos^2\theta = 1$] **[1]**

Now, LHS

$$\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$$

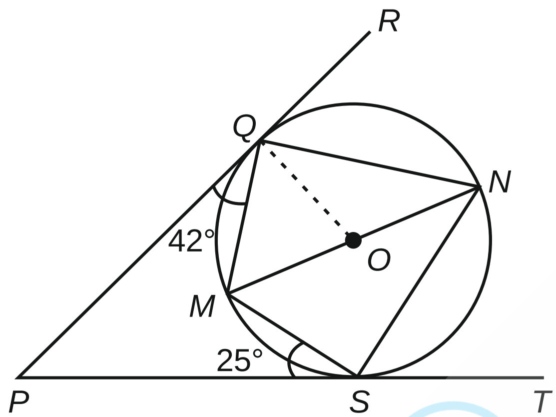
$$=\frac{(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)}{(\sin\theta+\cos\theta)}+\sin\theta\cos\theta \quad \text{[Using (i)]}$$ **[1]**

= 1 – sinθ cosθ + sinθ cosθ

= 1 = RHS **[1]**

Hence proved

(ii) (a)



Join OQ,

$\angle OQP = 90°$ [radius ⊥ tangent]

$\Rightarrow$ $\angle PQM + \angle OQM = 90°$ **[½]**

$\Rightarrow$ $42° + \angle OQM = 90°$

$\Rightarrow$ $\angle OQM = 48°$ **[½]**

(b) $\angle SNM = \angle PSM$ [Alternate segment theorem]

$\Rightarrow$ $\angle SNM = 25°$

Also, $\angle QNM = \angle PQM$ [Alternate segment theorem]

$\Rightarrow$ $\angle QNM = 42°$ **[½]**

Now, $\angle QNS = \angle SNM + \angle QNM$

$= 25° + 42° = 67°$ **[½]**

(c) $\angle QOS = 2\angle QNS$ **[½]**

[Angle made by an arc at the centre is double the angle made by it on remaining part of circle]

$\therefore$ $\angle QOS = 2 \times 67°$ [$\because$ $\angle QNS = 67°$]

$= 134°$ **[½]**

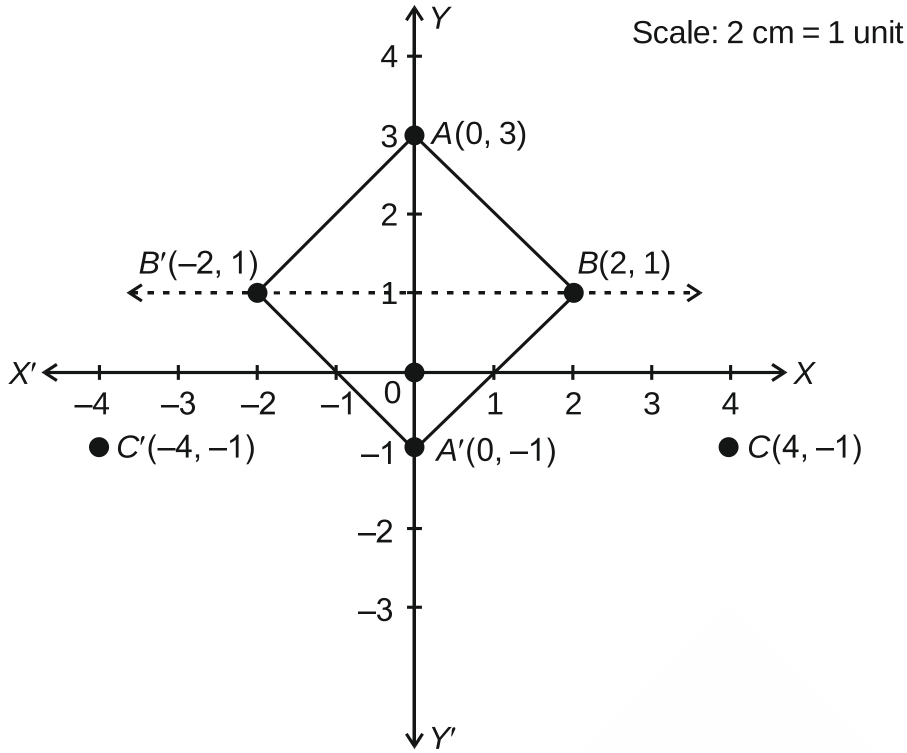
(d) $\angle QMS + \angle QNS = 180°$

[$\because$ Opposite angles of cyclic quadrilateral are supplementary] **[½]**

$\Rightarrow$ $\angle QMS + 67° = 180°$ [$\because$ $\angle QNS = 67°$]

$\Rightarrow$ $\angle QMS = 113°$ **[½]**

(iii)



(a) Points $A(0, 3)$, $B(2, 1)$ and $C(4, –1)$ are plotted. **[1]**

(b) Coordinates of B' are (–2, 1) and coordinates of C' are (–4, –1). **[1]**

(c) Reflection of point $A(0, 3)$ about BB' is shown. **[1]**

(d) A' (0, –1) **[1]**

(e) On joining $AB\,A'\,B'$, the figure formed is square. **[1]**

SECTION-B (40 Marks)

*(Attempt **any four** questions from this Section.)*

4. (i) Suresh has a recurring deposit account in a bank. He deposits ₹2000 per month and the bank pays interest at the rate of 8% per annum. If he gets ₹1040 as interest at the time of maturity, find in years total time for which the account was held. **[3]**

(ii) The following table gives the duration of movies in minutes. **[3]**

Duration (in minutes)	100 - 110	110 - 120	120 - 130	130- 140	140 -150	150 - 160
No. of movies	5	10	17	8	6	4

Using step - deviation method, find the mean duration of the movies.

(iii) If $\dfrac{(a+b)^3}{(a-b)^3} = \dfrac{64}{27}$ **[4]**

(a) Find $\dfrac{a+b}{a-b}$

(b) Hence using properties of proportion, find $a : b$.

Sol. (i) Money deposited = ₹2000 per month

r = 8% p.a

SI = 1040

n = ?

$$\text{SI} = \frac{P \times n(n+1) \times r}{2 \times 12 \times 100}$$ **[½]**

$$1040 = \frac{2000 \times n(n+1) \times 8}{2 \times 12 \times 100}$$ **[½]**

$n(n + 1) = 156$ **[1]**

$n^2 + n - 156 = 0$

$n^2 + 13n - 12n - 156 = 0$

$n(n + 13) - 12(n + 13) = 0$ **[½]**

$n = 12$, $n = -13$ (not possible)

n = 12 months **[½]**

(ii)

Duration (in minutes)	No. of movies (f_i)	Mid point (x_i)	$v_i = \frac{x_i - 125}{10}$	$f_i v_i$
100 - 110	5	105	–2	–10
110 - 120	10	115	–1	–10
120 - 130	17	125 = A	0	0
130 - 140	8	135	1	8
140 - 150	6	145	2	12
150 - 160	4	155	3	12
	Σf_i = 50			**$\Sigma f_i v_i$ = 12**

[2]

Mean, $\bar{x} = A + h \times \frac{\Sigma f_i v_i}{\Sigma f_i}$

$$= 125 + 10 \times \frac{12}{50}$$ **[½]**

$$= 125 + \frac{12}{5}$$

= 127.4 **[½]**

(iii) (a) $\frac{(a+b)^3}{(a-b)^3} = \frac{64}{27}$

$$\Rightarrow \frac{(a+b)^3}{(a-b)^3} = \frac{4^3}{3^3}$$ **[½]**

$$\Rightarrow \frac{a+b}{a-b} = \frac{4}{3}$$ [By taking cube root on both sides] **[½]**

(b) $\frac{a+b}{a-b}=\frac{4}{3}$

Using componendo and dividendo rule :

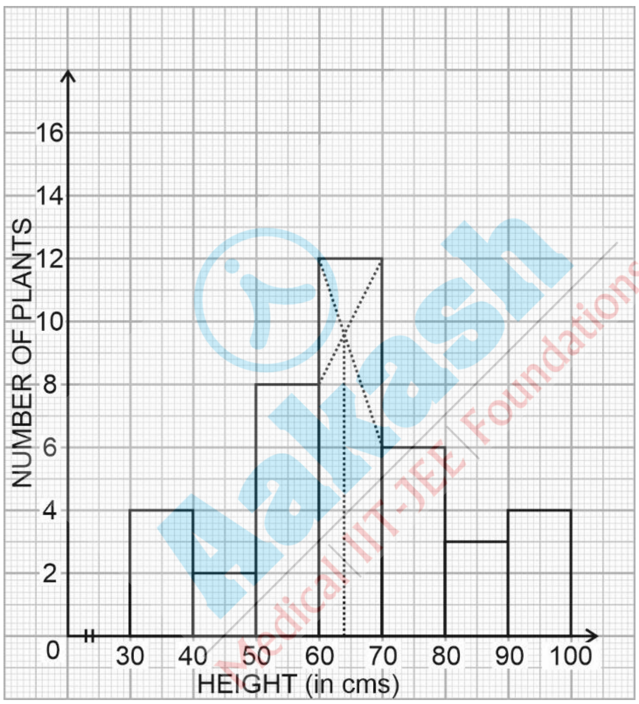
$$\frac{a+b+(a-b)}{a+b-(a-b)}=\frac{4+3}{4-3}$$ **[1]**

$$\frac{a+b+a-b}{a+b-a+b}=\frac{7}{1}$$ **[1]**

$$\frac{2a}{2b}=\frac{7}{1}$$

$a : b = 7 : 1$ **[1]**

5. (i) The given graph with a histogram represents the number of plants of different heights grown in a school campus. Study the graph carefully and answer the following questions: **[5]**



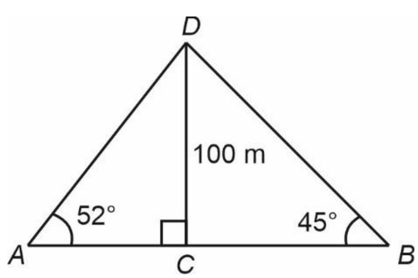
(a) Make a frequency table with respect to the class boundaries and their corresponding frequencies.

(b) State the modal class.

(c) Identify and note down the mode of the distributions.

(d) Find the number of plants whose height range is between 80 cm to 90 cm.

(ii) The angle of elevation of the top of a 100 m high tree from two points A and B on the opposite side of the tree are 52° and 45° respectively. Find the distance AB, to the nearest metre. **[5]**



Sol. (i) (a) Required frequency table :

Height (in cm)	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
No. of plants	4	2	8	12	6	3	4

[1]

(b) Since the modal class is the class with the highest frequency,

$\therefore$ 60 - 70 is the modal class. **[1]**

(c) Mode of the distribution is given by

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$ **[½]**

where $l = 60$

$f_1 = 12$

$f_0 = 8$

$f_2 = 6$

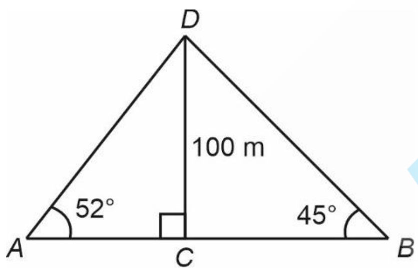
$h = 10$ **[½]**

$$\therefore \quad \text{Mode} = 60 + \left(\frac{12-8}{24-8-6}\right) \times 10$$ **[½]**

$= 64$ **[½]**

(d) Number of plants whose height range is between 80 cm to 90 cm is 3. **[1]**

(ii)



In ΔACD,

$$\tan 52° = \frac{100}{AC}$$ **[½]**

$$\Rightarrow \quad AC = \frac{100}{\tan 52°}$$ **[½]**

$$\Rightarrow \quad AC = \frac{100}{1.28}$$ **[½]**

$AC = 78.125$ m ...(i) **[1]**

In ΔBCD,

$$\tan 45° = \frac{100}{BC}$$ **[½]**

$$\Rightarrow \quad BC = \frac{100}{1}$$ **[½]**

$\Rightarrow$ $BC = 100$ m ...(ii)

$AC + BC = 78.125 + 100$ [From (i) and (ii)] **[½]**

$AB = 178.125$ m **[½]**

$\therefore$ Distance AB = 178 metres approximately. **[½]**

6. (i) Solve the following quadratic equation for x and give your answer correct to three significant figures: **[3]**

$2x^2 - 10x + 5 = 0$

(Use mathematical tables if necessary)

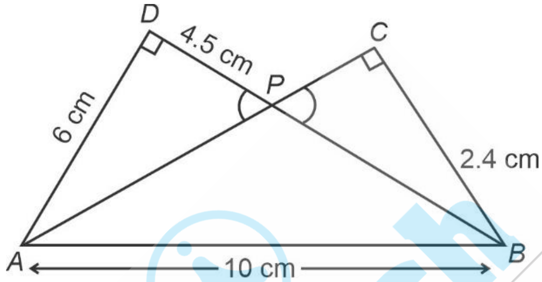
(ii) The $\boldsymbol{n^{th}}$ term of an Arithmetic Progression **(A.P.)** is given by the relation $\boldsymbol{T_n = 6(7 - n)}$. **[3]**

Find:

(a) its first term and common difference

(b) sum of its first **25** terms

(iii) In the given diagram ΔADB and ΔACB are two right angled triangles with $\angle ADB = \angle BCA = 90°$. If $AB = 10$ cm, $AD = 6$ cm, $BC = 2.4$ cm and $DP = 4.5$ cm **[4]**



(a) Prove that $\Delta APD \sim \Delta BPC$

(b) Find the length of BD and PB

(c) Hence, find the length of PA

(d) Find area ΔAPD : area ΔBPC

Sol. (i) $2x^2 - 10x + 5 = 0$ [Given]

$\because$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **[½]**

where $a = 2$, $b = -10$, $c = 5$ **[½]**

$\therefore$ $x = \dfrac{-(-10) \pm \sqrt{(-10)^2 - (4 \cdot 2 \cdot 5)}}{2 \cdot 2}$ **[½]**

$= \dfrac{10 \pm \sqrt{100 - 40}}{4}$

$= \dfrac{10 \pm \sqrt{60}}{4}$

$= \dfrac{10 \pm 2\sqrt{15}}{4}$ **[½]**

$= \dfrac{10 \pm 2(3.873)}{4}$ **[½]**

$= \dfrac{10 \pm 7.746}{4}$

$\therefore$ $x = 4.436$ or $x = 0.563$ **[½]**

(ii) $T_n = 6(7 - n)$ [Given]

(a) $T_1 = 6(7 - 1)$

$= 6 \times 6$

$= 36$ **[½]**

Also, common difference $= T_2 - T_1$ **[½]**

$= 6(7 - 2) - 6(7 - 1)$

$= 30 - 36$

$= -6$ **[½]**

(b) $T_n = 6(7 - n)$

$\because$ $T_1 = 36$ and common difference 'd' $= -6$

$\therefore$ $S_{25} = \frac{25}{2}[2 \times 36 + (25 - 1)(-6)]$ **[½]**

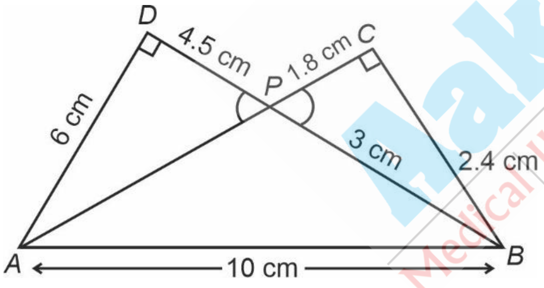
$= \frac{25}{2}[72 + 24 \times (-6)]$

$= \frac{25}{2}[72 - 144]$ **[½]**

$= \frac{-25}{2} \times 72$

$= -900$ **[½]**

(iii) (a)



$\because$ $\angle ADP = \angle BCP = 90°$ [Given]

Also, $\angle APD = \angle BPC$ [Vertically opposite angles] **[½]**

$\therefore$ By AA similarity,

$\Delta APD \sim \Delta BPC$ **[½]**

(b) $\because$ $\Delta APD \sim \Delta BPC$

$\Rightarrow$ $\frac{AP}{BP} = \frac{PD}{PC} = \frac{AD}{BC}$

$\Rightarrow$ $\frac{AP}{BP} = \frac{4.5}{PC} = \frac{6}{2.4}$

$\therefore$ $\Rightarrow PC = 1.8$ cm

In ΔBCP,

$BP^2 = BC^2 + PC^2 = (2.4)^2 + (1.8)^2$

$BP = 3$ cm **[½]**

$\therefore \quad BD = BP + PD$

$= 3$ cm $+ 4.5$ cm

$BD = 7.5$ cm **[½]**

(c) In ΔPDA,

$AP^2 = AD^2 + PD^2$ **[½]**

$= 6^2 + (4.5)^2$

$AP = 7.5$ cm **[½]**

(d) $$\frac{\text{ar. } \Delta APD}{\text{ar. } \Delta BPC} = \frac{\frac{1}{2} \times 6 \times 4.5}{\frac{1}{2} \times 2.4 \times 1.8}$$ **[½]**

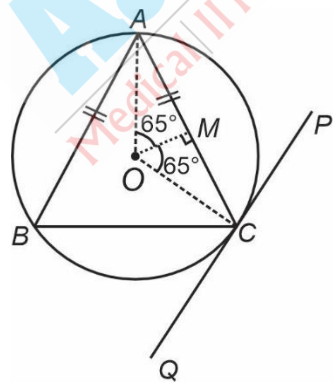
$$= \frac{27}{4.32}$$

$$= \frac{25}{4}$$ **[½]**

* This question has ambiguity. Since the triangles are similar therefore $AB = 10$ cm is not possible but it is not required to find other sides.

So, the question can be considered as bonus.

7. (i) In the given diagram, an isosceles ΔABC is inscribed in a circle with centre O. PQ is a tangent to the circle at C. OM is perpendicular to chord AC and $\angle COM = 65°$. **[3]**



Find :

(a) $\angle ABC$

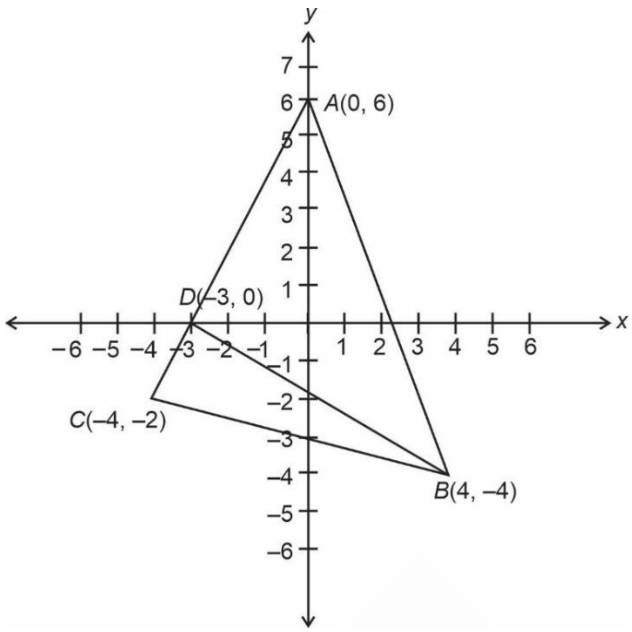
(b) $\angle BAC$

(c) $\angle BCQ$

(ii) Solve the following inequation, write down the solution set and represent it on the real number line. **[3]**

$$-3 + x \le \frac{7x}{2} + 2 < 8 + 2x,\ x \in I$$

(iii) In the given diagram, ABC is a triangle, where $B(4, -4)$ and $C(-4, -2)$. D is a point on AC. **[4]**



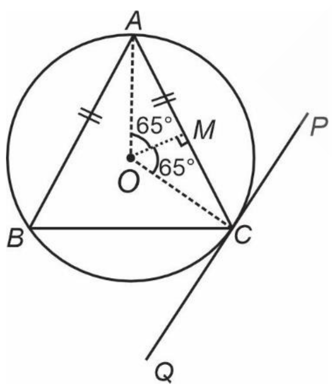
(a) Write down the coordinates of A and D.

(b) Find the coordinates of the centroid of ΔABC.

(c) If D divides AC in the ratio $\boldsymbol{k : 1}$, find the value of $\boldsymbol{k}$.

(d) Find the equation of the line BD.

Sol. (i)



(a) $\angle OAM = \angle OCM = 90° - 65° = 25°$ [$\because OM \perp AC$]

$\Rightarrow$ $\angle ACP = 90° - 25° = 65°$ [$\because OC \perp PQ$] **[½]**

$\therefore$ $\angle ABC = \angle ACP = 65°$ [Alternate Segment Theorem]

$\Rightarrow$ $\angle ACB = 65°$ [$\because AB = AC$] **[½]**

(b) $\angle BAC + \angle ABC + \angle ACB = 180°$ [Angle sum property of triangle]

$\Rightarrow$ $65° + 65° + \angle BAC = 180°$ **[½]**

$\Rightarrow$ $\angle BAC = 180° - 130° = 50°$ **[½]**

(c) $\angle BCQ = \angle BAC = 50°$ [Alternate segment Theorem] **[1]**

(ii) $-3+x \le \dfrac{7x}{2}+2<8+2x,\ x \in I$

$\Rightarrow \quad -3+x \le \dfrac{7x}{2}+2$ and $\dfrac{7x}{2}+2<8+2x$ [½]

$\Rightarrow \quad -6+2x \le 7x+4$ and $7x+4<16+4x$ [½]

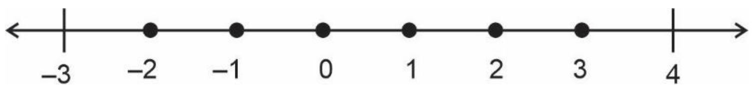
$\Rightarrow \quad -10 \le 5x$ and $3x<12$

$\Rightarrow \quad x \ge -2$ and $x<4$ [½]

$\Rightarrow \quad x \in [-2, 4)$ and $x \in I$ [½]

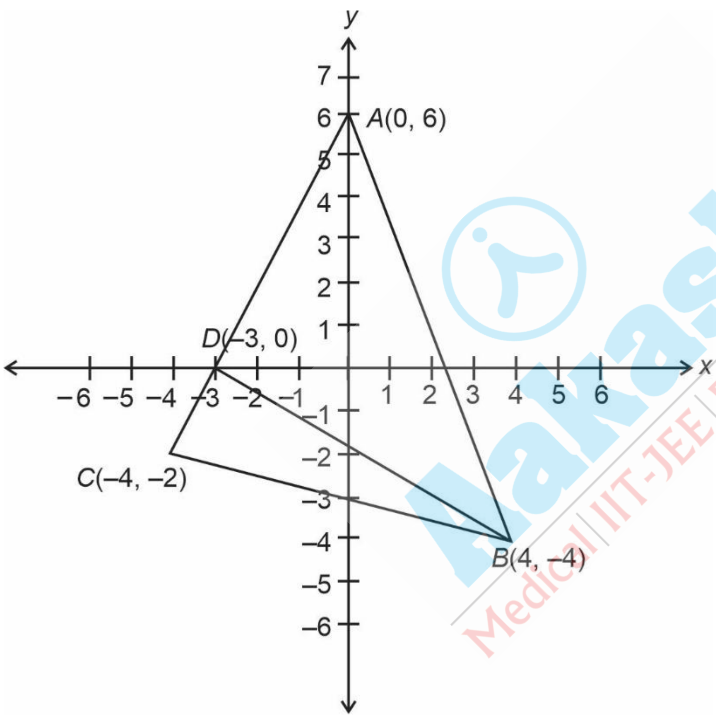
$\Rightarrow \quad x = \{-2, -1, 0, 1, 2, 3\}$

$\therefore$ Solution set = $\{-2, -1, 0, 1, 2, 3\}$ [½]



[½]

(iii)



(a) Coordinates of A are (0, 6) [½]

Coordinates of D are (–3, 0) [½]

(b) Coordinates of centroid of ΔABC are $\left(\dfrac{0-4+4}{3}, \dfrac{6-2-4}{3}\right)$ = (0, 0) [½]

(c) Coordinates of D are (–3, 0) = $\left(\dfrac{-4k+0}{k+1}, \dfrac{-2k+6}{k+1}\right)$ [½]

$\Rightarrow \quad (-3, 0) = \left(\dfrac{-4k}{k+1}, \dfrac{-2k+6}{k+1}\right)$ [½]

$\Rightarrow \quad 0(k+1) = -2k+6$

$\Rightarrow \quad k = 3$ [½]

(d) Coordinates of B and D are (4, –4) and (–3, 0) respectively.

$\therefore$ Equation of line BD is given by,

$$(y-0)=\left(\frac{0-(-4)}{-3-4}\right)(x-(-3)) \qquad \left[\because \text{ slope of } BD=\frac{0-(-4)}{-3-4}\right]$$ **[½]**

$\Rightarrow$ $y=\frac{-4}{7}(x+3)$

$\Rightarrow$ $7y = -4x - 12$

$\Rightarrow$ $4x + 7y + 12 = 0$ **[½]**

8. (i) The polynomial $\mathbf{3x^3 + 8x^2 - 15x + k}$ has $\mathbf{(x - 1)}$ as a factor. Find the value of $\boldsymbol{k}$. Hence factorize the resulting polynomial completely. **[3]**

(ii) The following letters **A, D, M, N, O, S, U, Y** of the English alphabet are written on separate cards and put in a box. The cards are well shuffled and one card is drawn at random. What is the probability that the card drawn is a letter of the word, **[3]**

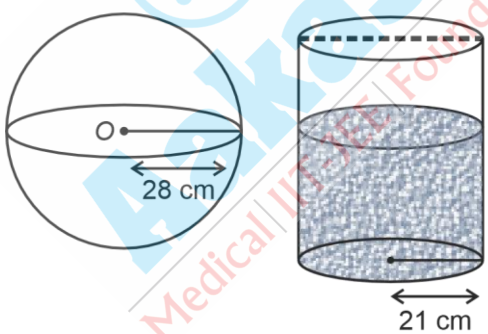
(a) MONDAY?

(b) which does not appear in MONDAY?

(c) which appears both in SUNDAY and MONDAY?

(iii) Oil is stored in a spherical vessel occupying **3/4** of its full capacity. Radius of this spherical vessel is **28 cm**. This oil is then poured into a cylindrical vessel with a radius of **21 cm**. Find the height of the oil in the cylindrical vessel **(correct to the nearest cm)**. **[4]**

Take $\pi=\frac{22}{7}$



Sol. (i) If $(x - 1)$ is factor of $3x^3 + 8x^2 - 15x + k$, then $3(1)^3 + 8(1)^2 - 15(1) + k = 0$ **[½]**

$3 + 8 - 15 + k = 0$

$k = 15 - 11$

$\boxed{k=4}$ **[½]**

$f(x) = 3x^3 + 8x^2 - 15x + 4$

$$
\begin{array}{r|l}
x-1 & 3x^3 + 8x^2 - 15x + 4 \;\big(\; 3x^2 + 11x - 4 \\
 & \underline{\overset{-}{3x^3} \overset{+}{-} 3x^2} \\
 & \quad 11x^2 - 15x \\
 & \quad \underline{\overset{-}{11x^2} \overset{+}{-} 11x} \\
 & \qquad -4x + 4 \\
 & \qquad \underline{\overset{+}{-} 4x \overset{-}{+} 4} \\
 & \qquad\quad 0
\end{array}
$$ **[1]**

$\because$ $(x-1)$ is a factor of $3x^3 + 8x^2 - 15x + 4$

$\therefore$ $3x^3 + 8x^2 - 15x + 4 = (x-1)(3x^2 + 11x - 4)$

$= (x-1)(3x^2 + 12x - x - 4)$ **[½]**

$= (x-1)[3x(x+4) - 1(x+4)]$

$= (x-1)(3x-1)(x+4)$ **[½]**

(ii) (a) Letters given are A, D, M, N, O, S, U, Y which are 8 in count.

To form MONDAY, 6 letters are desired letters.

$\therefore$ Probability $= \dfrac{\text{Favourable number of cases}}{\text{Total number of cases}}$

$= \dfrac{6}{8}$

$= \dfrac{3}{4}$ **[1]**

(b) Probability (Not appear in MONDAY)

$= \dfrac{2}{8}$ [As S, U are not there is MONDAY]

$= \dfrac{1}{4}$ **[1]**

(c) Probability (appear in SUNDAY and MONDAY)

$= \dfrac{4}{8}$ [As 4 letters N, D, A, Y are common]

$= \dfrac{1}{2}$ **[1]**

(iii) Volume of oil in spherical vessel = Volume of oil in cylindrical vessel

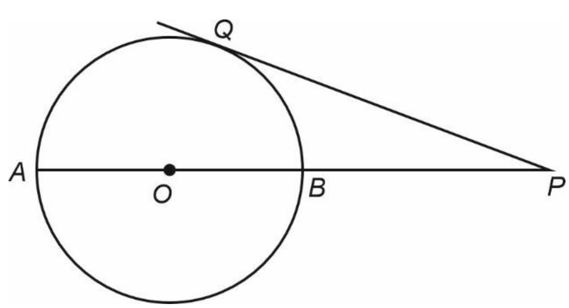
$\dfrac{3}{4} \times \dfrac{4}{3}\pi r^3 = \pi R^2 h$; where r, R, h are the radius of sphere, radius of cylinder and height upto which oil is filled in the cylinder. **[2]**

$\pi(28)^3 = \pi(21)^2 h$ **[1]**

$h = \dfrac{28^3}{21^2} = 49.777$ cm **[½]**

≈ 50 cm (approximately) **[½]**

9. (i) The figure shows a circle of radius 9 cm with O as the centre. The diameter AB produced meets the tangent PQ at P. If $PA = 24$ cm, find the length of tangent PQ. **[3]**



(ii) Mr. Gupta invested ₹33000 in buying ₹100 shares of a company at 10% premium. The dividend declared by the company is 12%. Find: **[3]**

(a) the number of shares purchased by him.

(b) his annual dividend.

(iii) A life insurance agent found the following data for distribution of ages of 100 policy holders: **[4]**

Age in years	Policy Holders (frequency)	Cumulative frequency
20 - 25	2	2
25 - 30	4	6
30 - 35	12	18
35 - 40	20	38
40 - 45	28	66
45 - 50	22	88
50 - 55	8	96
55 - 60	4	100

On a graph sheet draw an ogive using the given data. Take 2 cm = 5 years along one axis and 2 cm = 10 policy holders along the other axis. Use your graph to find:

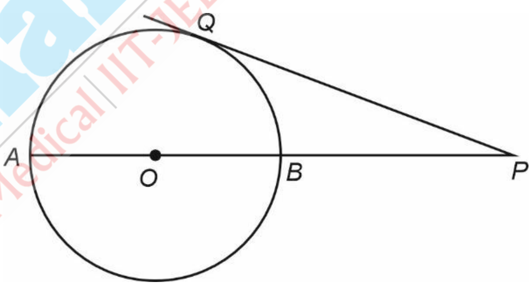
(a) The median age.

(b) Number of policy holders whose age is above 52 years.

Sol. (i) $PA = 24$ cm

$PB = 24 - 2(9)$

$= 6$ cm **[½]**



Now,

$PQ^2 = PA \times PB$ **[1]**

$PQ^2 = 24 \times 6$ **[½]**

$PQ^2 = 144$ **[½]**

$PQ = 12$ cm **[½]**

(ii) (a) Nominal value of each share = ₹100

Dividend = 12% **[½]**

(Number of shares) × 110 = 33000 **[½]**

Number of shares $= \dfrac{33000}{110}$

$= 300$ **[½]**

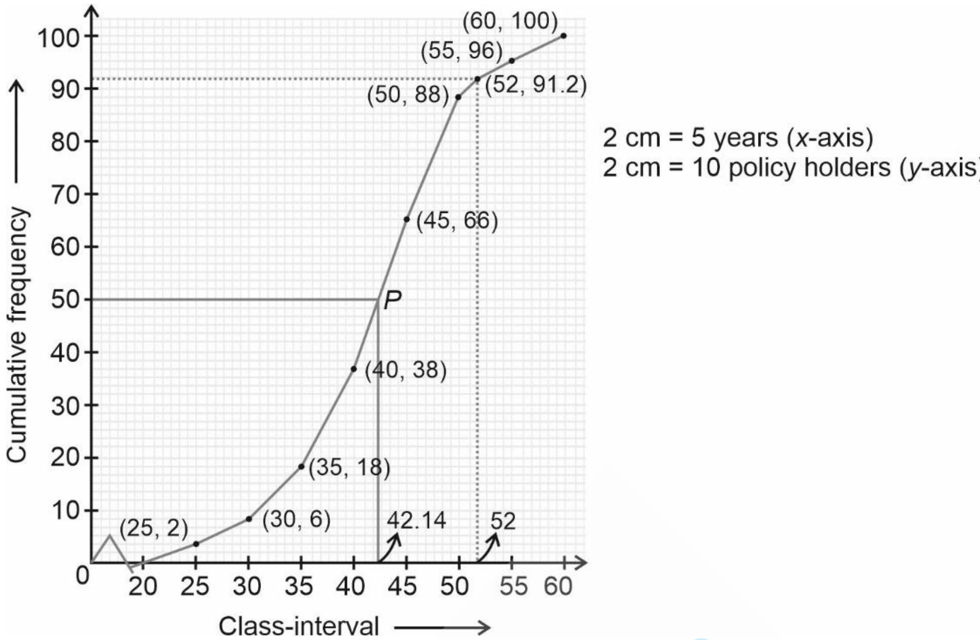
(b) Total amount of dividend = Dividend on one share × Number of shares **[½]**

= (12% of ₹100) × 300 **[½]**

= ₹3600 **[½]**

(iii) (a) From the given table, we have N = 100 (even) **[2]**

The position of median is given by $\frac{N}{2} = 50$



Median age = 42.14 (approximately)

(b) Number of policy holders whose age is above 52 years = 100 – 92 **[2]**

= 8 people

10. (i) Rohan bought the following eatables for his friends : **[3]**

Soham Sweet Mart : Bill				
S. No.	**Item**	**Price**	**Quantity**	**Rate of GST**
1	Laddu	₹500 per kg	2 kg	5%
2	Pastries	₹100 per piece	12 pieces	18%

Calculate :

(a) Total GST paid.

(b) Total bill amount including GST.

(ii) (a) If the lines $kx - y + 4 = 0$ and $2y = 6x + 7$ are perpendicular to each other, find the value of k. **[3]**

(b) Find the equation of a line parallel to $2y = 6x + 7$ and passing through (–1, 1)

(iii) Use ruler and compass to answer this question. Construct $\angle ABC = 90°$, where AB = 6 cm, BC = 8 cm. **[4]**

(a) Construct the locus of points equidistant from B and C.

(b) Construct the locus of points equidistant from A and B.

(c) Mark the point which satisfies both the conditions (a) and (b) as O. Construct the locus of point keeping a fixed distance OA from the fixed point O.

(d) Construct the locus of points which are equidistant from BA and BC.

Sol. (i)

Soham Sweet Mart : Bill							
S. No.	**Item**	**Price**	**Quantity**	**Rate of GST**	**Item Price**	**GST Paid**	**Price with GST**
1	Laddu	₹500 per kg	2 kg	5%	2 × 500 = ₹1000	5% of 1000 $= \frac{5}{100}\times 1000$ = ₹50	₹(1000 + 50) = ₹1050
2	Pastries	₹100 per piece	12 pieces	18%	12 × 100 = ₹1200	18% of 1200 $= \frac{18}{100}\times 1200$ = ₹216	₹(1200 + 216) = ₹1416
	Total				= ₹2200	= ₹266	= ₹2466

[2]

(a) ∴ Total GST paid = ₹266 **[½]**

(b) Total bill amount including GST = ₹2466 **[½]**

(ii) (a) Here, $kx - y + 4 = 0$

$$\text{Slope} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } y} \quad \textbf{[½]}$$

$$= \frac{-k}{-1} = k$$

∴ $2y = 6x + 7$

⇒ $6x - 2y + 7 = 0$

$$\text{Slope} = \frac{-6}{-2} = 3 \quad \textbf{[½]}$$

If lines are perpendicular, then

$k \times 3 = -1$

$$\Rightarrow \quad k = \frac{-1}{3} \quad \textbf{[½]}$$

(b) $2y = 6x + 7$

$$\Rightarrow \quad y = 3x + \frac{7}{2}$$

⇒ $y = mx + c$

∴ $m = 3$ **[½]**

When lines are parallel,

$m_1 = m_2 = 3$

Now,

Equation of line

$y - y_1 = m(x - x_1)$ **[½]**

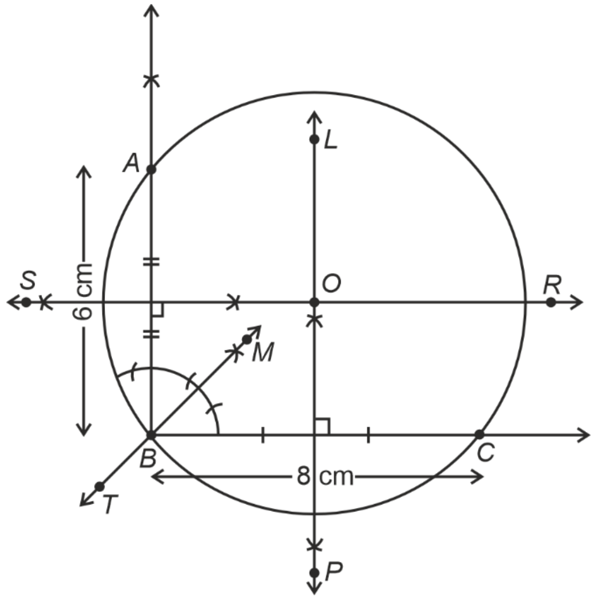
⇒ $y - 1 = 3\,[x - (-1)]$

⇒ $y - 1 = 3\,(x + 1)$

⇒ $y - 1 = 3x + 3$

⇒ $y = 3x + 4$ **[½]**

(iii)



$\angle ABC = 90°$, $AB = 6$ cm and $BC = 8$ cm is drawn.

(a) Perpendicular bisector of BC is locus of points equidistant from B and C represented by LP in figure. **[1]**

(b) Line SR represents locus of points equidistant from AB which is perpendicular bisector of AB. **[1]**

(c) Point 'O' which is intersection of SR and LP satisfies both (a) and (b). Circle with radius OA represents locus of points with fixed distance OA. **[1]**

(d) TM as angle bisector of $\angle ABC$ is drawn which is locus of points which are equidistant from BA and BC. **[1]**

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