Date: 11/03/2024

Question Paper Code

430/1/3

SET-3

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Time: 3 Hrs.

MATHEMATICS (Basic)

Max. Marks: 80

CBSE Class-X (2024)

Answers & Solutions

GENERAL INSTRUCTIONS

Read the following instructions carefully and follow them :

(i) This question paper contains **38** questions. **All** questions are compulsory.

(ii) Question paper is divided into **FIVE** sections – **Section A, B, C, D** and **E.**

(iii) In **section A,** question number **1** to **18** are Multiple Choice Questions (MCQs) and question number **19** and **20** are Assertion-Reason based questions of **1** mark each.

(iv) In **section B,** question number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.

(v) In **section C,** question number **26** to **31** are Short Answer (SA) type questions carrying **3** marks each.

(vi) In **section D,** question number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.

(vii) In **section E,** question number **36** to **38** are **case-based integrated units** of assessment questions carrying **4** marks each. Internal choice is provided in **2** marks question in each case-study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section **B**, 2 questions in Section **C**, 2 questions in Section **D** and 3 questions in Section **E**.

(ix) Draw neat figures wherever required. Take π **= 22/7** wherever required if not stated.

(x) Use of calculators is **NOT allowed.**

SECTION-A

Q. No. 1 to 20 are Multiple Choice Questions of 1 mark each.

1. If $\sin\theta = \frac{1}{3}$, then $\sec\theta$ is equal to : **[1]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{3}{2\sqrt{2}}$

(c) 3 (d) $\frac{1}{\sqrt{3}}$

Answer (b)

2. If the roots of quadratic equation $4x^2 - 5x + k = 0$ are real and equal, then value of k is : **[1]**

(a) $\frac{5}{4}$ (b) $\frac{25}{16}$

(c) $-\frac{5}{4}$ (d) $-\frac{25}{16}$

Answer (b)

3. If a certain variable x divides a statistical data arranged in order into two equal parts, then the value of x is called the : **[1]**

(a) mean (b) median

(c) mode (d) range

of the data.

Answer (b)

4. The curved surface area of a right circular cone of radius 7 cm is 550 sq cm. The slant height of the cone is : **[1]**

(a) 24 cm (b) 25 cm

(c) 22 cm (d) 20 cm

Answer (b)

5. The distance between the points (2, –3) and (–2, 3) is : **[1]**

(a) $2\sqrt{13}$ units (b) 5 units

(c) $13\sqrt{2}$ units (d) 10 units

Answer (a)

6. The mid-point of the line segment joining the points (–1, 3) and $\left(8, \frac{3}{2}\right)$ is : **[1]**

(a) $\left(\frac{7}{2}, -\frac{3}{4}\right)$ (b) $\left(\frac{7}{2}, \frac{9}{2}\right)$

(c) $\left(\frac{9}{2}, -\frac{3}{4}\right)$ (d) $\left(\frac{7}{2}, \frac{9}{4}\right)$

Answer (d)

Sol. $\left(\frac{7}{2}, \frac{9}{4}\right)$ is the mid-point of line segment

7. The mean and median of a statistical data are 21 and 23 respectively. The mode of the data is : **[1]**

(a) 27 (b) 22

(c) 17 (d) 23

Answer (a)

Sol. 27 is the mode of the data

8. The value of k for which the pair of linear equations $5x + 2y - 7 = 0$ and $2x + ky + 1 = 0$ don't have a solution, is **[1]**

(a) 5 (b) $\frac{4}{5}$

(c) $\frac{5}{4}$ (d) $\frac{5}{2}$

Answer (b)

Sol. $k = \frac{4}{5}$

9. If HCF (96, 404) = 4, then LCM (96, 404) is **[1]**

(a) 9600 (b) 96 × 404

(c) 404 (d) 9696

Answer (d)

Sol. LCM (96, 404) = 9696

10. The height and radius of a right circular cone are 24 cm and 7 cm respectively. The slant height of the cone is **[1]**

(a) 24 cm (b) 31 cm

(c) 26 cm (d) 25 cm

Answer (d)

Sol. Slant height of the cone = 25 cm

11. For what value of θ, $\sin^2\theta + \sin\theta + \cos^2\theta$ is equal to 2 ? **[1]**

(a) 45° (b) 0°

(c) 90° (d) 30°

Answer (c)

12. In an A.P., if $a = 8$ and $a_{10} = -19$, then value of d is **[1]**

(a) 3 (b) $-\frac{11}{9}$

(c) $-\frac{27}{10}$ (d) –3

Answer (d)

13. Which of the following cannot be the probability of an event? **[1]**

(a) 52% (b) $\frac{1}{3}\%$

(c) 0.99 (d) $\frac{1}{0.99}$

Answer (d)

14. The diameter of a circle is of length 6 cm. If one end of the diameter is (–4, 0), the other end on x-axis is at: **[1]**

(a) (0, 2) (b) (6, 0)

(c) (2, 0) (d) (4, 0)

Answer (c)

15. Two dice are rolled together. The probability of getting at least one 6, is: **[1]**

(a) $\frac{1}{3}$ (b) $\frac{11}{36}$

(c) $\frac{1}{6}$ (d) $\frac{10}{36}$

Answer (b)

16. A card is drawn from a well shuffled deck of 52 playing cards. The probability that drawn card is a red queen, is: **[1]**

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$

(c) $\frac{1}{52}$ (d) $\frac{1}{26}$

Answer (d)

17. If one of the zeroes of the quadratic polynomial $(\alpha - 1)x^2 + \alpha x + 1$ is –3, then the value of α is: **[1]**

(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{3}$ (d) $\frac{3}{4}$

Answer (c)

18. For what value of k, the product of zeroes of the polynomial $kx^2 - 4x - 7$ is 2 ? **[1]**

(a) $-\frac{1}{14}$ (b) $-\frac{7}{2}$

(c) $\frac{7}{2}$ (d) $-\frac{2}{7}$

Answer (b)

Directions :

In Q. No. 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Select the correct option from the following options :

(a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.

(b) Both, Assertion (A) and Reason (R) are true. Reason (R) does not explain Assertion (A).

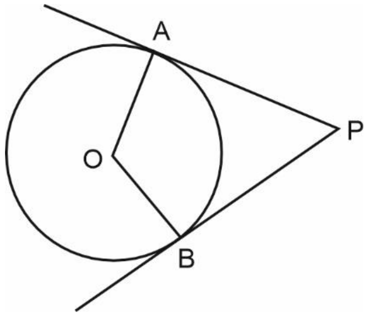
(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A):** Zeroes of a polynomial $p(x) = x^2 - 2x - 3$ are –1 and 3.

Reason (R): The graph of polynomial $p(x) = x^2 - 2x - 3$ intersects x-axis at (–1, 0) and (3, 0). **[1]**

Answer (a)

20. 

Assertion (A): If PA and PB are tangents drawn to a circle with centre O from an external point P, then the quadrilateral OAPB is a cyclic quadrilateral.

Reason (R): In a cyclic quadrilateral, opposite angles are equal. **[1]**

Answer (c)

SECTION-B

Q. No. 21 to 25 are Very Short Answer Questions of 2 marks each.

21. A bag contains 4 red, 5 white and some yellow balls. If probability of drawing a red ball at random is $\frac{1}{5}$, then find the probability of drawing a yellow ball at random. **[2]**

Sol. Let number of yellow balls be x.

Total number of outcomes = 4 + 5 + x

= 9 + x **[½]**

Probability of drawing a red ball = $\frac{1}{5} = \frac{4}{9+x}$

$\Rightarrow \quad \frac{4}{9+x} = \frac{1}{5}$

$\Rightarrow \quad x = 11$ **[½]**

Probability of drawing a yellow ball = $\frac{11}{20}$ **[1]**

22. (A) Prove that $6-4\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number. **[2]**

OR

(B) Show that 11 × 19 × 23 + 3 × 11 is not a prime number. **[2]**

Sol. (A) Let us assume, to the contrary, that $6-4\sqrt{5}$ is rational.

We can find coprime integers a and b $(b \neq 0)$ such that $6-4\sqrt{5} = \frac{a}{b}$ **[½]**

Rearranging, we get $\frac{6b-a}{4b} = \sqrt{5}$ **[½]**

Since a and b are integers, so $\frac{6b-a}{4b}$ is rational and so is $\sqrt{5}$ is rational. **[½]**

But this contradicts the fact that $\sqrt{5}$ is irrational.

So, we conclude $6-4\sqrt{5}$ is an irrational number. **[½]**

OR

(B) Given $11 \times 19 \times 23 + 3 \times 11$

$= 11 \times (19 \times 23 + 3)$ **[½]**

$= 11 \times 440$ **[½]**

This number is multiple of two integers.

Hence it has more than two factors.

Hence, it is a composite number. **[1]**

23. (A) Solve the following pair of linear equations for x and y algebraically:

$x + 2y = 9$ and $y - 2x = 2$ **[2]**

OR

(B) Check whether the point (–4, 3) lies on both the lines represented by the linear equations $x + y + 1 = 0$ and $x - y = 1$. **[2]**

Sol. (A) Given equations are

$x + 2y = 9$ … (i)

and $y - 2x = 2$

$\Rightarrow y = 2 + 2x$ … (ii)

Put $y = 2 + 2x$ in equation (i), we get

$x + 2(2 + 2x) = 9$ **[½]**

$\Rightarrow x + 4 + 4x = 9$

$\Rightarrow 5x = 5$

$\therefore x = 1$ **[½]**

$\Rightarrow y = 2 + 2(1)$ **[½]**

$y = 4$ **[½]**

OR

(B) Given, $x + y + 1 = 0$ …(i)

and $x - y = 1$ … (ii)

Given point is (–4, 3)

Now, put (–4, 3) in equation (i) and (ii) respectively.

Taking LHS in equation (i),

$x + y + 1$

$\Rightarrow (-4) + 3 + 1$ **[½]**

$\Rightarrow 0 = \text{R.H.S}$

$\therefore$ (–4, 3) lies on the line $x + y + 1 = 0$ **[½]**

Put (–4, 3) in LHS of equation (ii) i.e. $x - y = 1$

$\Rightarrow -4 - 3$

$\Rightarrow -7 \neq 1$ **[½]**

$\Rightarrow \text{L.H.S} \neq \text{R.H.S}$

$\therefore$ (–4, 3) does not lie on the line $x - y = 1$ **[½]**

24. If $\sin A = \frac{1}{2}$ and $\cos B = \frac{1}{\sqrt{2}}$, then find the value of sin A sin B + cos A cos B. **[2]**

Sol. $\sin A = \frac{1}{2} \Rightarrow A = 30°$

$\cos B = \frac{1}{\sqrt{2}} \Rightarrow B = 45°$ **[½]**

Now, sin B = sin 45°

$= \frac{1}{\sqrt{2}}$ **[½]**

cos A = cos 30°

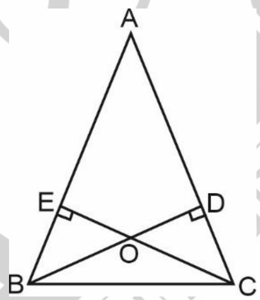
$= \frac{\sqrt{3}}{2}$ **[½]**

sin A sin B + cos A cos B = $\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

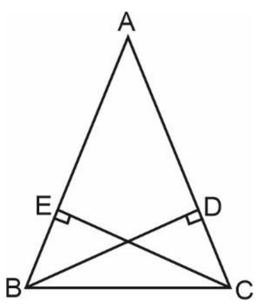
$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{2}\left(1+\sqrt{3}\right)}{4}$ **[½]**

25. In the given figure, in ΔABC, BD and CE are perpendicular to AC and AB respectively. Prove that : AE × BD = AD × CE **[2]**



Sol.



In ΔBDA and ΔCEA, we have

$\angle ADB = \angle AEC$ [Each 90°] **[½]**

$\angle BAD = \angle EAC$ [common] **[½]**

So, $\Delta BDA \sim \Delta CEA$ [By AA criteria] **[½]**

$\frac{BD}{AD} = \frac{CE}{AE}$

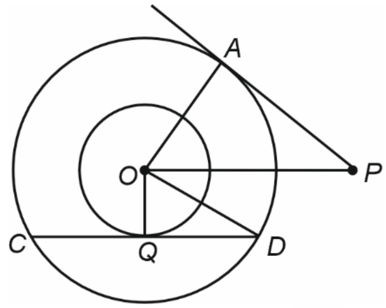
or AE × BD = AD × CE **[½]**

Hence, proved.

SECTION-C

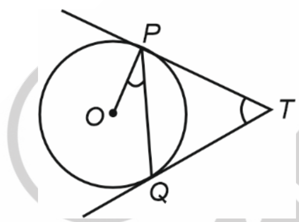
Q. No. 26 to 31 are Short Answer Questions of 3 marks each.

26. (A) In two concentric circles, the radii are OA = r cm and OQ = 6 cm, as shown in the figure. Chord CD of larger circle is a tangent to smaller circle at Q. PA is tangent to larger circle. If PA = 16 cm and OP = 20 cm, find the length CD. **[3]**



OR

(B) In given figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$ **[3]**



Sol. (A) $\angle OAP = 90^\circ$ [A tangent to a circle is perpendicular to the radius at the point of contact] **[½]**

$OA^2 + AP^2 = OP^2$

$\Rightarrow OA^2 + 16^2 = 20^2$

$\Rightarrow OA^2 = 400 - 256$

$\Rightarrow OA^2 = 144$

$\Rightarrow OA = 12$ cm **[½]**

$OA = OD$ ($\because$ Radii of larger circle)

In ΔOQD,

$OQ^2 + QD^2 = OD^2 \quad (OQ \perp CD)$ **[½]**

$\Rightarrow (6)^2 + QD^2 = (12)^2$

$\Rightarrow QD^2 = 144 - 36$

$\Rightarrow QD^2 = 108$ **[½]**

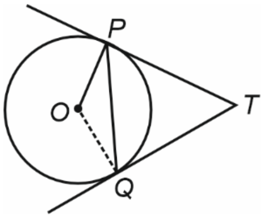
$\Rightarrow QD = 6\sqrt{3}$ cm **[½]**

$\therefore CD = 2QD = 2\left(6\sqrt{3}\right)$ cm [Perpendicular from the centre bisects the chord]

$= 12\sqrt{3}$ cm **[½]**

OR

(B) Join OQ



Let $\angle OPQ = x°$

$\angle OPQ = \angle OQP = x°$ [$\because$ Radii of circle]

$\angle OPT = \angle OQT = 90°$ **[½]**

[$\because$ A tangent to a circle is perpendicular to the radius at the point of contact]

$\angle QPT = \angle PQT = 90° - x°$ **[½]**

In ΔPQT,

$\angle PQT + \angle QPT + \angle PTQ = 180°$ **[½]**

$\Rightarrow 90° - x° + 90° - x° + \angle PTQ = 180°$

$\Rightarrow \angle PTQ = 180° - 180° + 2x°$ **[½]**

$\Rightarrow \angle PTQ = 2x°$ **[1]**

$\therefore \angle PTQ = 2\angle OPQ$

Hence proved

27. (A) A solid is in the form of a cylinder with hemi-spherical ends of same radii. The total height of the solid is 20 cm and the diameter of the cylinder is 14 cm. Find the surface area of the solid. **[3]**

OR

(B) A juice glass is cylindrical in shape with hemi-spherical raised up portion at the bottom. The inner diameter of glass is 10 cm and its height is 14 cm. Find the capacity of the glass. (use $\pi = 3.14$) **[3]**

Sol. (A) Height of cylinder = Total height – (2 × radius of hemi-spheres)

= 20 – 14 = 6 cm **[1]**

Surface area of solid = Curved surface area of cylinder + 2(Curved surface area of hemi-spheres) **[½]**

$= 2\pi rh + 4\pi r^2$ **[½]**

$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 7(6 + 14)$

$= 880 \text{ cm}^2$ **[1]**

OR

(B) Inner radius of glass = 5 cm

and height = 14 cm

Apparent capacity of glass = $\pi r^2 h$

$= 3.14 \times 5^2 \times 14 \text{ cm}^3$

$= 1099 \text{ cm}^3$ **[1]**

But actual capacity of glass is less by the volume of the hemi-sphere at the base of the glass.

It is less by $\frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 5^3\ \text{cm}^3$

$= 261.66\ \text{cm}^3$ **[1]**

So, actual capacity of glass = Apparent capacity of glass – Volume of hemi-sphere

$= (1099 - 261.66)\ \text{cm}^3$

$= 837.33\ \text{cm}^3$ **[1]**

28. Two alarm clocks ring their alarms at regular intervals of 20 minutes and 25 minutes respectively. If they first beep together at 12 noon, at what time will they deep again together next time? **[3]**

Sol. Given that, two alarm clocks ring their alarm at an interval of 20 minutes and 25 minutes respectively. **[½]**

$\therefore$ LCM (20, 25) is 100 minutes. **[1]**

i.e., 1 hour and 40 minutes. **[½]**

$\because$ First beep together at 12 noon.

$\therefore$ For the next time, it will beep at 1:40 PM. **[1]**

29. Prove that : $\sin^6\theta + \cos^6\theta + 3\sin^2\theta \cos^2\theta = 1$ **[3]**

Sol. LHS = $\sin^6\theta + \cos^6\theta + 3\sin^2\theta \cos^2\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta \cos^2\theta$ **[½]**

$= [a^3 + b^3 = (a + b)^3 - 3ab\ (a + b)]$ **[1]**

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta\ (\sin^2\theta + \cos^2\theta) + 3\sin^2\theta \cos^2\theta$ **[1]**

$= 1[\sin^2\theta + \cos^2\theta = 1]$ **[½]**

= RHS

Hence Proved.

30. The greater of two supplementary angles exceeds the smaller by 18°. Find measures of these two angles. **[3]**

Sol. Let the two supplementary angles be x and y (y > x).

Now; x + y = 180° (Sum of supplementary angles is 180°) …(1) **[½]**

According to the question;

y – x = 18° …(2) **[½]**

Adding (1) and (2):

2y = 198° **[1]**

y = 99°

Putting y = 99° in (2)

99° – x = 18° **[½]**

x = 81° **[½]**

So the angles are 99° and 81°.

31. If A(2, –1), B(a, 4), C(–2, b) and D(–3, –2) are vertices of a parallelogram ABCD taken in order, then find the values of a and b. Also, find the length of the sides of the parallelogram. **[3]**

Sol. In a parallelogram diagonals bisect each other that means the mid-points of diagonals AC and BD will coincide

Midpoint of AC = $\left(\frac{2-2}{2}, \frac{-1+b}{2}\right)$ **[½]**

$= \left(0, \frac{b-1}{2}\right)$

Midpoint of BD = $\left(\frac{a-3}{2}, \frac{4-2}{2}\right)$ **[½]**

$= \left(\frac{a-3}{2}, 1\right)$

Now, $\left(0, \frac{b-1}{2}\right) = \left(\frac{a-3}{2}, 1\right)$

$\Rightarrow 0 = \frac{a-3}{2}$

$\Rightarrow$ a = 3 **[½]**

Also, $\frac{b-1}{2} = 1$

$b = 3$ **[½]**

Now, AB = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(2-3)^2 + (-1-4)^2}$

$= \sqrt{1+25}$

$= \sqrt{26}$ units **[½]**

BC = $\sqrt{(3+2)^2 + (4-3)^2}$

$= \sqrt{25+1}$

$= \sqrt{26}$ units **[½]**

Now, AB = CD and BC = AD

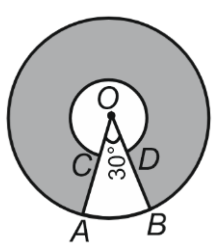
Hence in parallelogram ABCD

AB = BC = CD = AD = $\sqrt{26}$ units

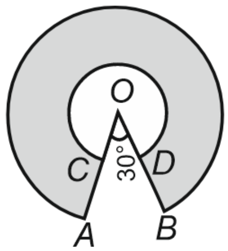
SECTION-D

Q. No. 32 to 35 are Long Answer Questions of 5 marks each.

32. In the given figure, two concentric circles with centre O have radii 14 cm and 7 cm. If $\angle$AOB = 30°, find the area of the shaded region. **[5]**



Sol. Angles for major sectors at O = 360° – 30° = 330° **[1]**



Area of shaded portion = Area of major sector of radius 14 cm – Area of major sector of radius 7 cm **[1]**

$= \left(\frac{330^\circ}{360^\circ} \times \pi \times 14^2 - \frac{330^\circ}{360^\circ} \times \pi \times 7^2\right) \text{cm}^2$ **[1]**

$= \frac{11}{12} \times \frac{22}{7}\left(14^2 - 7^2\right) \text{cm}^2$ **[1]**

$= \frac{11}{12} \times \frac{22}{7} \times 21 \times 7 \text{ cm}^2$

$= 423.5 \text{ cm}^2$ **[1]**

33. (A) The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower and the length of original shadow. (use $\sqrt{3} = 1.73$) **[5]**

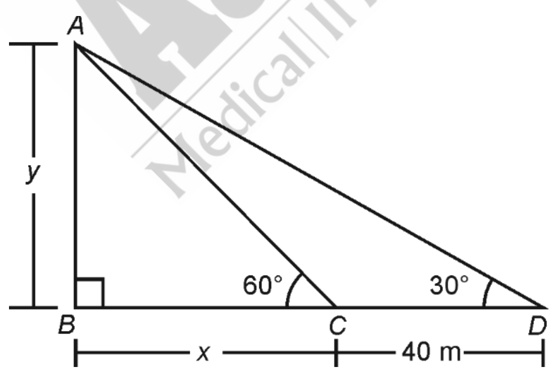
OR

(B) The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building and the distance between the two building. (use $\sqrt{3} = 1.73$) **[5]**

Sol. (A) Let $AB = y$ metres be the height of tower. **[½]**

Consider, $BC = x$ metres be the length of shadow (where Sun's altitude is 60°) **[½]**

So, $BD = (x + 40)$ m be the length of shadow (when Sun's altitude is 30°)



In ΔABC,

$\tan 60^\circ = \frac{y}{x}$

$\sqrt{3} = \frac{y}{x}$

$\Rightarrow \quad y = \sqrt{3}x \qquad \ldots(i)$ **[1]**

In ΔABD,

$$\tan 30° = \frac{y}{x+40}$$ **[½]**

$$\frac{1}{\sqrt{3}} = \frac{y}{x+40}$$

$\Rightarrow \quad \sqrt{3}y = x+40 \qquad \ldots(ii)$ **[½]**

From equation (i) and (ii), we get

$\sqrt{3}(\sqrt{3}x) = x+40$ **[½]**

$3x = x + 40$

$\therefore \quad \boxed{x = 20 \text{ metres}}$ **[½]**

Now, from equation (i), we get

$y = \sqrt{3}(20)$ **[½]**

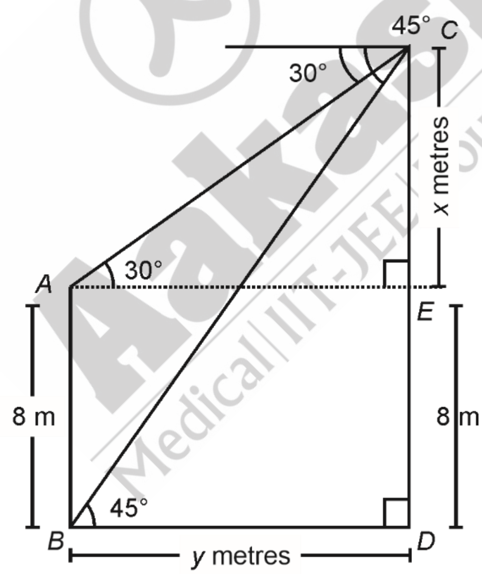
$= 20 \times 1.73$

$\therefore \; y = 34.6$ metres **[½]**

OR

(B) Consider AB be the building and CD be the multi-storeyed building. **[½]**

Let $CE = x$ m, $AB = 8$ m and $CD = (x + 8)$ m, y be the horizontal distance between building and multi-storeyed building. **[½]**



In ΔCDB,

$$\tan 45° = \frac{x+8}{y}$$ **[½]**

$$1 = \frac{x+8}{y}$$

$\Rightarrow \quad y = x + 8 \qquad \ldots(i)$ **[½]**

In ΔCEA,

$\tan 30° = \dfrac{x}{y} \quad [\because AE||BD \Rightarrow AE = BD = y]$ **[½]**

$$\frac{1}{\sqrt{3}} = \frac{x}{y}$$

$\Rightarrow \quad y = \sqrt{3}x$ …(ii) **[½]**

From equations (i) and (ii), we get

$\sqrt{3}x = x + 8$ **[½]**

$$\Rightarrow \quad x = \frac{8}{\sqrt{3}-1} = \frac{8}{1.73-1}$$

$x = 10.92$ m **[½]**

Now from equation (i)

$y = \sqrt{3}(10.92)$ **[½]**

$y = 1.73(10.92)$

$= 18.89$ m **[½]**

34. In an A.P. of 50 terms, the sum of first 10 terms is 250 and the sum of its last 15 terms is 2625. Find the A.P. so formed. **[5]**

Sol. Let a, d be the first term and common difference of the given A.P respectively.

Now, $S_{10} = 250$

Using $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = \frac{10}{2}(2a + (10-1)d)$ **[½]**

$S_{10} = 5(2a + 9d)$

$250 = 10a + 45d$

$50 = 2a + 9d$ …(1) **[½]**

Now, sum of last 15 terms = Sum of 50 terms – Sum of first 35 terms. **[½]**

$= \frac{50}{2}[2a + (50-1)d] - \frac{35}{2}[2a + (35-1)d]$

$= 25[2a + 49d] - 35[a + 17d]$ **[½]**

$= 50a + 1225d - 35a - 595d$

$2625 = 15a + 630d$ **[½]**

$175 = a + 42d$

$350 = 2a + 84d$ …(2) **[½]**

Subtracting (1) and (2), we get

$2a + 84d = 350$

$2a + 9d = 50$ **[½]**

(–) (–) (–)

$75d = 300$

$d = 4$ **[½]**

Putting $d = 4$ in (1)

$50 = 2a + 36$

$a = 7$ **[½]**

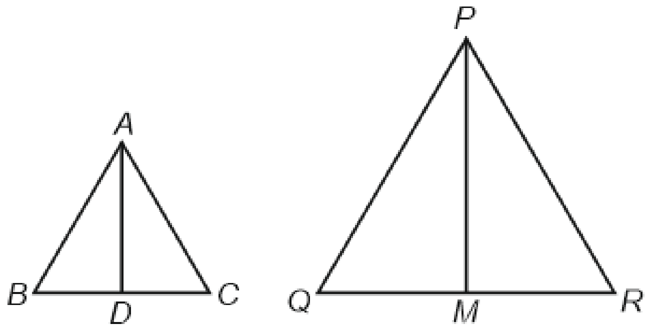
So the required A.P. is : 7, 7 + 4, 7 + 2(4), 7 + 3(4), …..

7, 11, 15, 19, …. **[½]**

35. (A) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that other two sides are divided in the same ratio. **[5]**

OR

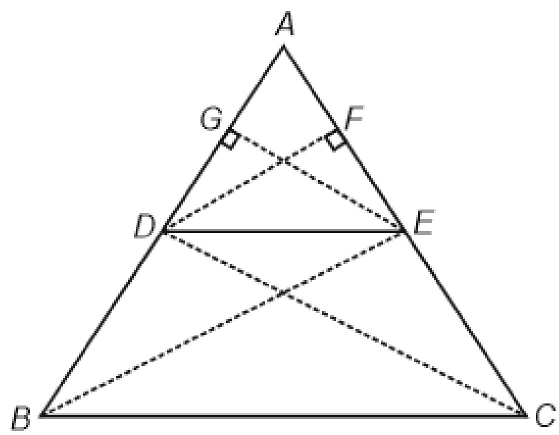
(B) Sides AB and AC and median AD of a ΔABC are respectively proportional to sides PQ and PR and median PM of ΔPQR. Show that $\Delta ABC \sim \Delta PQR$. **[5]**



Sol. (A) Given : ΔABC, in which DE is drawn parallel to BC.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join CD and BE. Draw $DF \perp AE$ and $EG \perp AD$ **[½]**



Proof :

$\text{ar}(\Delta ADE) = \frac{1}{2} \times AD \times EG$ ...(i) **[½]**

$\text{ar}(\Delta BDE) = \frac{1}{2} \times BD \times EG$ ...(ii) **[½]**

Dividing (i) by (ii), we get

$$\frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EG}{\frac{1}{2} \times BD \times EG} = \frac{AD}{BD} \quad \text{...(iii)}$$ **[½]**

Similarly,

$\text{ar}(\Delta ADE) = \frac{1}{2} \times DF \times AE$ **[½]**

$\text{ar}(\Delta CDE) = \frac{1}{2} \times CE \times DF$ **[½]**

$\frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CDE)} = \frac{\frac{1}{2} \times DF \times AE}{\frac{1}{2} \times DF \times CE} = \frac{AE}{CE}$ …(iv) **[½]**

Now, ar(ΔBDE) = ar(ΔCDE) [$\because$ Triangles on the same base and between the same parallel lines are equal in area] **[½]**

$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CDE)}$ **[½]**

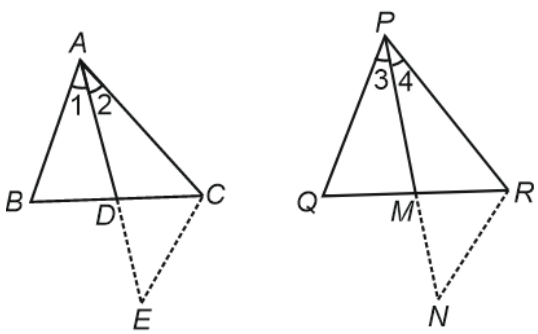
$\therefore$ From (iii) and (iv), we get

$\frac{AD}{DB} = \frac{AE}{EC}$ **[½]**

Hence proved.

OR

(B)



Given, two triangles ΔABC and ΔPQR in which AD and PM are medians such that

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ **[½]**

To prove that $\Delta ABC \sim \Delta PQR$

Construction : Produce AD to E so that $AD = DE$. Join CE.

Similarly produce PM to N such that $PM = MN$, also join RN. **[½]**

Proof :

In ΔABD and ΔCDE, we have

$AD = DE$ [By construction]

$BD = DC$ [$\because$ AD is the median]

And, $\angle ADB = \angle CDE$ [Vertically opposite angles] **[½]**

$\therefore$ $\Delta ABD \cong \Delta ECD$ [By SAS criterion of congruence] **[½]**

$\Rightarrow$ $AB = CE$ [by CPCT] …(i)

Also, in ΔPQM and ΔMNR, we have

$PM = MN$ [By construction]

$QM = MR$ [$\therefore$ PM is the median]

And, $\angle PMQ = \angle NMR$ [Vertically opposite angles]

$\therefore \quad \Delta PQM \cong \Delta NRM$ [By SAS criterion of congruence] **[½]**

$\Rightarrow \quad PQ = RN$ [CPCT] ...(ii) **[½]**

Now, $\dfrac{AB}{PQ} = \dfrac{AC}{PR} = \dfrac{AD}{PM}$

$\Rightarrow \quad \dfrac{CE}{RN} = \dfrac{AC}{PR} = \dfrac{AD}{PM}$ [From (i) and (ii)]

$\Rightarrow \quad \dfrac{CE}{RN} = \dfrac{AC}{PR} = \dfrac{2AD}{2PM}$

$\Rightarrow \quad \dfrac{CE}{RN} = \dfrac{AC}{PR} = \dfrac{AE}{PN}$ [$\therefore$ $2AD = AE$ and $2PM = PN$]

$\therefore \quad \Delta ACE \sim \Delta PRN$ [By SSS similarity criterion] **[½]**

Therefore, $\angle 2 = \angle 4$

Similarly, $\angle 1 = \angle 3$

$\therefore \quad \angle 1 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow \quad \angle A = \angle P$ …(iii) **[½]**

Now, in ΔABC and ΔPQR, we have

$\dfrac{AB}{PQ} = \dfrac{AC}{PR}$ (Given) **[½]**

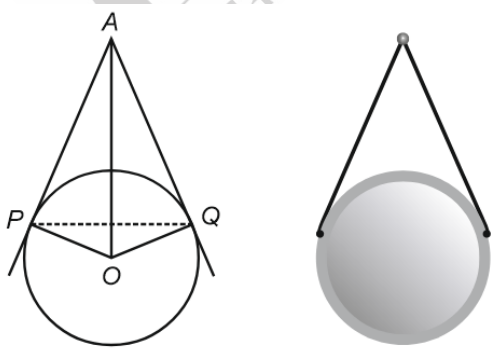
$\angle A = \angle P$ [From (iii)]

$\therefore \quad \Delta ABC \sim \Delta PQR$ [By SAS similarity criterion] **[½]**

SECTION-E

Q. No. 36 to 38 are Case-Based Questions of 4 marks each.

36. The picture given below shows a circular mirror hanging on the wall with a cord. The diagram represents the mirror as a circle with centre O. AP and AQ are tangents to the circle at P and Q respectively such that AP = 30 cm and $\angle PAQ$ = 60º.



Based on the above information, answer the following questions :

(i) Find the length PQ. **[1]**

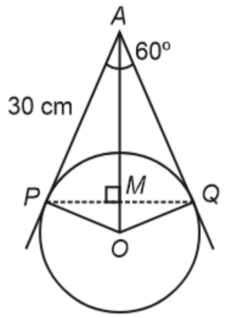
(ii) Find m$\angle POQ$. **[1]**

(iii) (a) Find the length OA. **[2]**

OR

(b) Find the radius of the mirror. **[2]**

Sol. (i)



$\angle PAQ = 60^\circ$

$\angle PAM = 30^\circ$

In ΔPAM

$$\sin 30^\circ = \frac{PM}{AP}$$ **[½]**

$\Rightarrow \quad \frac{1}{2} = \frac{PM}{30}$

$\Rightarrow \quad 2PM = 30$ cm

$\Rightarrow \quad PQ = 2PM = 30$ cm **[½]**

(ii) $\angle POQ + \angle OQA + \angle QAP + \angle APO = 360^\circ$ **[½]**

$\Rightarrow \quad \angle POQ + 90^\circ + 60^\circ + 90^\circ = 360^\circ$

m$\angle POQ = 120^\circ$ **[½]**

(iii) (a) In ΔOAP **[½]**

$$\cos 30^\circ = \frac{AP}{OA}$$ **[½]**

$$\frac{\sqrt{3}}{2} = \frac{30}{OA}$$ **[½]**

$OA = 20\sqrt{3}$ cm **[½]**

(b) Radius of mirror = OP

In ΔOAP

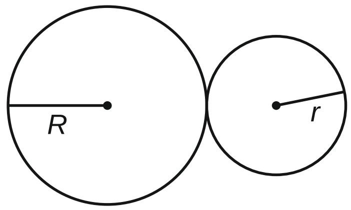
$$\tan 30^\circ = \frac{OP}{AP}$$ **[1]**

$$\frac{1}{\sqrt{3}} = \frac{OP}{30}$$ **[½]**

$OP = 10\sqrt{3}$ cm **[½]**

37. To keep the lawn green and cool, Sadhna uses water sprinklers which rotate in circular shape and cover a particular area.

The diagram below shows the circular areas covered by two sprinklers :



$(R > r)$

Two circles touch externally. The sum of their areas is 130π sq. m and the distance between their centres is 14 m.

Based on above information, answer the following questions.

(i) Obtain a quadratic equation involving R and r from above. **[1]**

(ii) Write a quadratic equation involving only r. **[1]**

(iii) (a) Find the radius r and the corresponding area irrigated. **[2]**

OR

(b) Find the radius R and the corresponding area irrigated. **[2]**

Sol. (i) Sum of areas of both circles = 130π

$\Rightarrow \pi R^2 + \pi r^2 = 130\pi$ **[½]**

$\Rightarrow R^2 + r^2 = 130$ **[½]**

(ii) $R = 14 - r$ $(R + r = 14)$

Now, $\pi R^2 + \pi r^2 = 130\pi$

$\Rightarrow (14 - r)^2 + r^2 = 130$ **[½]**

$\Rightarrow 2r^2 + 196 - 28r - 130 = 0$

$\Rightarrow 2r^2 - 28r + 66 = 0$

$\Rightarrow r^2 - 14r + 33 = 0$ **[½]**

(iii) (a) $r^2 - 14r + 33 = 0$ **[½]**

$r^2 - 11r - 3r + 33 = 0$

$r(r - 11) - 3(r - 11) = 0$ **[½]**

$r = 3$ m **[½]**

$r = 11$ m (not possible as R becomes 3 m which is not possible)

Area irrigated = $\pi(3)^2$

$= 9\pi$ m^2 **[½]**

OR

(b) $r^2 - 14r + 33 = 0$ **[½]**

$r^2 - 11r - 3r + 33 = 0$

$r(r - 11) - 3(r - 11) = 0$ **[½]**

$r = 3$ m

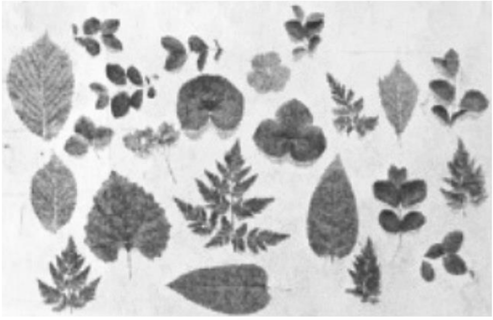
$r = 11$ m (not possible as R becomes 3 m which is not possible)

$R = 14 - 3$

$= 11$ m **[½]**

Area irrigated = $\pi(11)^2 = 121\pi$ m^2 **[½]**

38. Gurpreet is very fond of doing research on plants. She collected some leaves from different plants and measured their lengths in mm.



The data obtained is represented in the following table :

Length (in mm) :	70–80	80–90	90–100	100–110	110–120	120–130	130–140
Number of leaves :	3	5	9	12	5	4	2

Based on the above information, answer the following questions.

(i) Write the median class of the data. **[1]**

(ii) How many leaves are of length equal to or more than 10 cm? **[1]**

(iii) (a) Find median of the data. **[2]**

OR

(b) Write the modal class and find the mode of the data. **[2]**

Sol.

Length	Number of leaves	*cf*
70 – 80	3	3
80 – 90	5	8
90 – 100	9	17
100 – 110	12	29
110 – 120	5	34
120 – 130	4	38
130 – 140	2	40

(i) Median class of the data = 100 – 110 **[1]**

(ii) Number of leaves with length equal to or more than 10 cm (100 mm)

= 12 + 5 + 4 + 2 **[½]**

= 23 **[½]**

(iii) (a) Median of the data = $l + \left\{\dfrac{\frac{N}{2} - cf}{f}\right\} \times h$ **[½]**

$l = 100$

$N = 40$

$cf = 17$

$f = 12$

$h = 10$ **[½]**

$100 + \left\{\frac{20-17}{12}\right\} \times 10 = 102.5$ mm **[1]**

OR

(b) Modal class = 100 – 110 **[½]**

Mode = $l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$ **[½]**

$= 100 + \left(\frac{12-9}{24-9-5}\right) \times 10$ **[½]**

= 103 mm **[½]**

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