Date: 11/03/2024

Question Paper Code

30/1/3

SET-3

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Time: 3 Hrs. Max. Marks: 80

MATHEMATICS (Standard)

CBSE Class-X (2024)

Answers & Solutions

GENERAL INSTRUCTIONS

Read the following instructions carefully and follow them:

(i) This question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This question paper is divided into **five** Sections - **A, B, C, D** and **E**.

(iii) In **Section A**, Question numbers **1** to **18** are multiple choice questions (MCQs) and question numbers **19** and **20** are Assertion–Reason based questions of 1 mark each.

(iv) In **Section B**, Question numbers **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.

(v) In **Section C**, Question numbers **26** to **31** are short answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D**, Question numbers **32** to **35** are long answer (LA) type questions carrying **5** marks each.

(vii) In **Section E**, Question numbers **36** to **38** are **case-study based integrated** questions carrying **4** marks each. Internal choice is provided in **2** marks question in each case-study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section **B**, 2 questions in Section **C**, 2 questions in Section **D** and 3 questions of **2** marks in Section **E**.

(ix) Draw neat diagrams wherever required. Take π = 22/7 wherever required, if not stated.

(x) Use of calculators is **NOT allowed.**

SECTION-A

This section consists of 20 questions of 1 mark each.

1. If two positive integers p and q can be expressed as $p = 18a^2b^4$ and $q = 20a^3b^2$, where a and b are prime numbers, then LCM (p, q) is **[1]**

(a) $2a^2b^2$ (b) $180a^2b^2$

(c) $12a^2b^2$ (d) $180a^3b^4$

Answer (d) **[1]**

Sol. $180a^3b^4$

2. In an A.P., if the first term (a) = –16 and the common difference (d) = –2, then the sum of first 10 terms is **[1]**

(a) –200 (b) –70

(c) –250 (d) 250

Answer (c) **[1]**

Sol. –250

3. For some data $x_1, x_2, ... x_n$ with respective frequencies $f_1, f_2, ... f_n$, the value of $\sum_1^n f_i(x_i - \bar{x})$ is equal to **[1]**

(a) $n\bar{x}$ (b) 1

(c) $\sum f_i$ (d) 0

Answer (d) **[1]**

Sol. 0

4. The volume of the largest right circular cone that can be carved out from a solid cube of edge 2 cm is **[1]**

(a) $\frac{4\pi}{3}$ cu cm (b) $\frac{5\pi}{3}$ cu cm

(c) $\frac{8\pi}{3}$ cu cm (d) $\frac{2\pi}{3}$ cu cm

Answer (d) **[1]**

Sol. $\frac{2\pi}{3}$ cu cm

5. A solid sphere is cut into two hemispheres. The ratio of the surface areas of sphere to that of two hemispheres taken together, is **[1]**

(a) 1 : 1 (b) 1 : 4

(c) 2 : 3 (d) 3 : 2

Answer (c) **[1]**

Sol. 2 : 3

6. The centre of a circle is at (2, 0). If one end of a diameter is at (6, 0), then the other end is at **[1]**

(a) (0, 0) (b) (4, 0)

(c) (–2, 0) (d) (–6,0)

Answer (c) **[1]**

Sol. (–2, 0)

7. One card is drawn at random from a well shuffled deck of 52 playing cards. The probability that it is a red ace card, is **[1]**

(a) $\frac{1}{13}$ (b) $\frac{1}{26}$

(c) $\frac{1}{52}$ (d) $\frac{1}{2}$

Answer (b) **[1]**

Sol. $\frac{1}{26}$

8. The middle most observation of every data arranged in order is called **[1]**

(a) mode (b) median

(c) mean (d) deviation

Answer (b) **[1]**

Sol. Median

9. For $\theta = 30°$, the value of $(2 \sin \theta \cos \theta)$ is **[1]**

(a) 1 (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3}{2}$

Answer (b) **[1]**

Sol. $2 \sin 30° \cos 30° = \frac{\sqrt{3}}{2}$

10. If the roots of equation $ax^2 + bx + c = 0$, $a \neq 0$ are real and equal, then which of the following relation is true? **[1]**

(a) $a = \frac{b^2}{c}$ (b) $b^2 = ac$

(c) $ac = \frac{b^2}{4}$ (d) $c = \frac{b^2}{a}$

Answer (c) **[1]**

Sol. $b^2 = 4ac$

11. From the data 1, 4, 7, 9, 16, 21, 25, if all the even numbers are removed, then the probability of getting at random a prime number from the remaining is **[1]**

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$

Answer (b) **[1]**

Sol. $\frac{1}{5}$

12. *AD* is a median of ΔABC with vertices $A(5, -6)$, $B(6, 4)$ and $C(0, 0)$. Length *AD* is equal to : **[1]**

(a) $\sqrt{68}$ units (b) $2\sqrt{15}$ units

(c) $\sqrt{101}$ units (d) 10 units

Answer (a) **[1]**

Sol. $\sqrt{68}$ units

13. Two dice are rolled together. The probability of getting sum of numbers on the two dice as 2, 3 or 5, is: **[1]**

(a) $\frac{7}{36}$ (b) $\frac{11}{36}$

(c) $\frac{5}{36}$ (d) $\frac{4}{9}$

Answer (a) **[1]**

Sol. $\frac{7}{36}$

14. If the distance between the points $(3, -5)$ and $(x, -5)$ is 15 units, then the values of x are: **[1]**

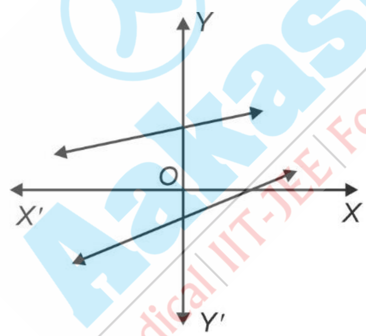
(a) 12, –18 (b) –12, 18

(c) 18, 5 (d) –9, –12

Answer (b) **[1]**

Sol. –12, 18

15. In the given figure, graphs of two linear equations are shown. The pair of these linear equations is: **[1]**



(a) consistent with unique solution.

(b) consistent with infinitely many solutions.

(c) inconsistent.

(d) inconsistent but can be made consistent by extending these lines.

Answer (a) **[1]**

Sol. consistent with unique solution.

16. If α, β are the zeroes of the polynomial $6x^2 - 5x - 4$, then $\frac{1}{\alpha}+\frac{1}{\beta}$ is equal to **[1]**

(a) $\frac{5}{4}$ (b) $-\frac{5}{4}$

(c) $\frac{4}{5}$ (d) $\frac{5}{24}$

Answer (b) **[1]**

Sol. $-\frac{5}{4}$

17. If sec θ – tan θ = m, then the value of sec θ + tan θ is **[1]**

(a) $1-\frac{1}{m}$ (b) $m^2 - 1$

(c) $\frac{1}{m}$ (d) $-m$

Answer (c) **[1]**

Sol. $\frac{1}{m}$

18. The zeroes of a polynomial $x^2 + px + q$ are twice the zeroes of the polynomial $4x^2 - 5x - 6$. The value of p is **[1]**

(a) $-\frac{5}{2}$ (b) $\frac{5}{2}$

(c) –5 (d) 10

Answer (a) **[1]**

Sol. $-\frac{5}{2}$

Directions :

In Q. No. 19 and 20 a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

(a) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A)

(b) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A)

(c) Assertion (A) is true but Reason (R) is false

(d) Assertion (A) is false but Reason (R) is true

19. **Assertion (A):** The tangents drawn at the end points of a diameter of a circle, are parallel.

Reason (R): Diameter of a circle is the longest chord. **[1]**

Answer (b) **[1]**

20. **Assertion (A):** If the graph of a polynomial touches x-axis at only one point, then the polynomial cannot be a quadratic polynomial.

Reason (R): A polynomial of degree $n(n > 1)$ can have at most n zeroes. **[1]**

Answer (d) **[1]**

SECTION-B

This section consists of 5 questions of 2 marks each.

21. In a pack of 52 playing cards one card is lost. From the remaining cards, a card is drawn at random. Find the probability that the drawn card is queen of heart, if the lost card is a black card. **[2]**

Sol. One card lost is a black card

Total number of outcomes = 51 **[½]**

Number of favourable outcome = 1 **[½]**

Required probability = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{51}$ **[1]**

22. (A) Evaluate : $2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ$ **[2]**

OR

(B) If A = 60° and B = 30°, verify that : **[2]**

sin(A + B) = sin A cos B + cos A sin B

Sol. (A) $2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ$

$= 2\sqrt{2}\times\frac{1}{\sqrt{2}}\times\frac{1}{2}+2\sqrt{3}\times\frac{\sqrt{3}}{2}$ **[1]**

= 1 + 3 **[½]**

= 4 **[½]**

OR

(B) L.H.S. = sin(A + B) = sin(60° + 30°)

= sin 90° = 1 **[½]**

= 1

R.H. S = sin$A$ cos$B$ + cos$A$ sin$B$

= sin60° cos30° + cos60° sin30°

$= \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}+\frac{1}{2}\times\frac{1}{2}$ **[½]**

$= \frac{3}{4}+\frac{1}{4}$ **[½]**

$= \frac{4}{4}$

= 1 = L.H. S. **[½]**

∴ LHS = RHS

Hence verified

23. (A) Prove that $5-2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number. **[2]**

OR

(B) Show that the number 5 × 11 × 17 + 3 × 11 is a composite number. **[2]**

Sol. (A) Let us assume that

$5-2\sqrt{3}$ is rational. **[½]**

It means we can find integers a and b ($b \neq 0$) such that $5-2\sqrt{3}=\frac{a}{b}$ **[½]**

∴ $5-\frac{a}{b}=2\sqrt{3}$

Rearranging this, we get

$\sqrt{3}=\frac{5b-a}{2b}$

Since a and b are integers, we get $\frac{5b-a}{2b}$ as rational **[½]**

And so $\sqrt{3}$ is rational

But it contradicts the fact that $\sqrt{3}$ is irrational

So our assumption is wrong

Hence $5-2\sqrt{3}$ is irrational **[½]**

OR

(B) $5 \times 11 \times 17 + 3 \times 11$

$= 11\ (5 \times 17 + 3)$ **[½]**

$= 11 \times (85 + 3)$

$= 11 \times 88$

$= 2^3 \times 11^2$ **[½]**

As we can see $5 \times 11 \times 17 + 3 \times 11$ can be factorised as $2^3 \times 11^2$. It means it has factors other than 1 and itself **[½]**

$\therefore$ $5 \times 11 \times 17 + 3 \times 11$ is a composite number **[½]**

24. Solve the following system of linear equations algebraically :

$2x + 5y = -4;\ 4x - 3y = 5$ **[2]**

Sol. $2x + 5y = -4$ …(i)

$4x - 3y = 5$ …(ii)

Multiplying equation (i) by 2, we get

$4x + 10y = -8$ …(iii) **[½]**

Now subtracting (ii) from (iii), we get **[½]**

$$\begin{array}{rcl} 4x + 10y & = & -8 \\ 4x - 3y & = & 5 \\ - \quad + & & - \\ \hline 13y & = & -13 \\ y & = & -1 \end{array}$$

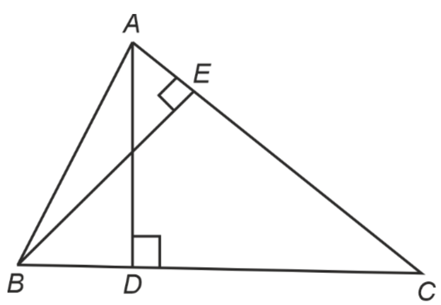
Putting the values of y in equation (i), we get

$2x + 5(-1) = -4$ **[½]**

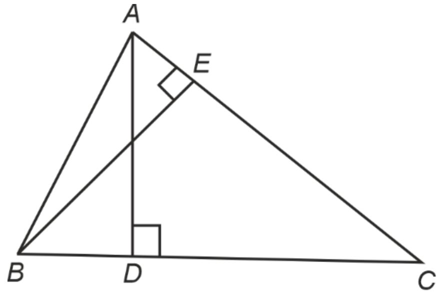
$2x = 1$

$x = \dfrac{1}{2}$ **[½]**

25. In ΔABC, altitudes AD and BE are drawn. If $AD = 7$ cm, $BE = 9$ cm and $EC = 12$ cm then, find the length of CD. **[2]**



Sol. Given AD = 7 cm, BE = 9 cm and EC = 12 cm



In ΔADC and ΔBEC

$\angle ADC = \angle BEC = 90°$ **[½]**

$\angle ACD = \angle BCE$ (Common)

$\Rightarrow \Delta ADC \sim \Delta BEC$ [By AA similarity criterion] **[½]**

$\Rightarrow \dfrac{AD}{BE} = \dfrac{CD}{EC}$ **[½]**

$\Rightarrow \dfrac{7}{9} = \dfrac{CD}{12}$

$\Rightarrow CD = \dfrac{28}{3}$ cm **[½]**

SECTION-C

This section consists of 6 questions of 3 marks each.

26. The sum of the digits of a 2-digit number is 14. The number obtained by interchanging its digits exceeds the given number by 18. Find the number. **[3]**

Sol. Let the number be 10x + y

Sum of digits of two digit number = 14

$\therefore$ $x + y = 14$ …(i) **[½]**

Number obtained by interchanging its digits exceeds the given number by 18

$\therefore$ $10y + x = 10x + y + 18$ **[½]**

$\Rightarrow$ $9y = 9x + 18$

$\Rightarrow$ $y = x + 2$ **[½]**

On putting the value of y in equation (i), we get

$\Rightarrow$ $x + (x + 2) = 14$ **[½]**

$\Rightarrow$ $2x + 2 = 14$

$\Rightarrow$ $2x = 12$

$\Rightarrow$ $x = 6$ **[½]**

On putting the value of x in equation (i), we get

$6 + y = 14$

$\Rightarrow$ $y = 8$

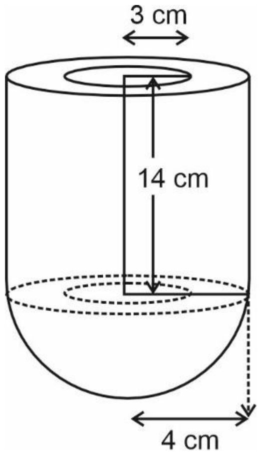
$\therefore$ Required number $= 10x + y$

$= 10(6) + 8$ **[½]**

$= 68$

27. The inner and outer radii of a hollow cylinder surmounted on a hollow hemisphere of same radii are 3 cm and 4 cm respectively. If height of the cylinder is 14 cm, then find its total surface area (inner and outer). **[3]**

Sol.



Total surface area of solid = Inner and outer curved surface areas of cylinder and hemisphere

$= 2\pi rh + 2\pi Rh + 2\pi r^2 + 2\pi R^2$

Where,

$r = 3$ cm

$R = 4$ cm

$h = 14$ cm

$= 2\pi h(r + R) + 2\pi(r^2 + R^2)$ **[½]**

$= 2\times\frac{22}{7}\times14(3+4)+2\times\frac{22}{7}(9+16)$ **[½]**

$= 616+\frac{1100}{7}$ **[½]**

$= \frac{616\times7+1100}{7}$ **[½]**

$= \frac{5412}{7}$ cm^2 **[½]**

$= 773.14$ cm^2 (approx.) **[½]**

28. In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject. **[3]**

Sol. The number of rooms will be minimum if every room has maximum number of participants.

$\therefore$ Number of teachers in each room must be HCF of 48, 80 and 144. **[½]**

$\therefore$ $48 = 2^4 \times 3$

$80 = 2^4 \times 5$ **[½]**

$144 = 2^4 \times 3^2$

$\therefore$ HCF (48, 80, 144) $= 2^4 = 16$ **[½]**

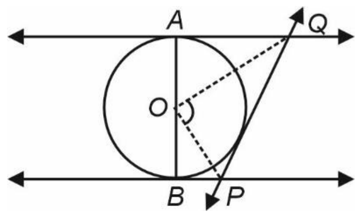
$\therefore$ In each room, 16 teachers can be seated.

$\therefore$ Minimum number of rooms required $= \dfrac{48+80+144}{16}$ **[½]**

$= 17$ **[½]**

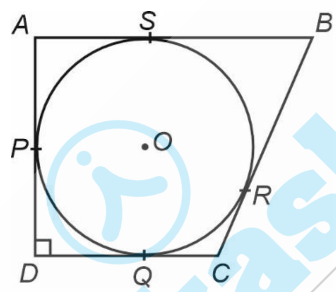
$\therefore$ Minimum number of rooms = 17 **[½]**

29. (A) In the given figure, AB is a diameter of the circle with centre O. AQ, BP and PQ are tangents to the circle. Prove that $\angle POQ = 90°$. **[3]**

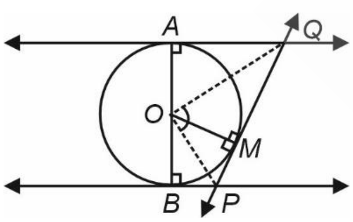


OR

(B) A circle with centre O and radius 8 cm is inscribed in a quadrilateral $ABCD$ in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC, BC = 30 cm and BS = 24 cm, then find the length DC. **[3]**



Sol. (A) Given : AQ, BP and PQ are tangents. AB is diameter of circle with centre O.



To Prove : $\angle POQ = 90°$ **[½]**

Proof : Let tangent PQ touches circle at M.

Join OM

In ΔOAQ and ΔOMQ,

$AQ = QM$ [Lengths of tangents from an external point are equal]

$OQ = OQ$

$OA = OM$ [radius]

$\therefore$ $\Delta OAQ \cong \Delta OMQ$. [By SSS congruence criterion] **[1]**

$\therefore$ $\angle AOQ = \angle MOQ$ [By CPCT] ...(i) **[½]**

Similarly,

$\angle BOP = \angle MOP$ ...(ii)

Now, As AB is diameter which is a straight line.

$\therefore$ $\angle AOQ + \angle MOQ + \angle BOP + \angle MOP = 180°$ [Linear Pair axiom]

$\Rightarrow \quad \angle MOQ + \angle MOQ + \angle MOP + \angle MOP = 180°$ [using (i) and (ii)] **[½]**

$\Rightarrow \quad 2\angle MOP + 2\angle MOQ = 180°$

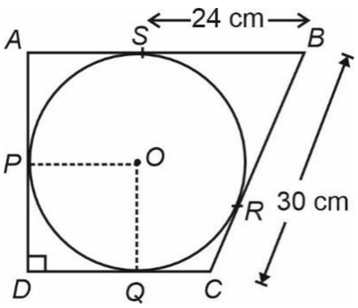
$\Rightarrow \quad \angle MOP + \angle MOQ = 90°$

$\Rightarrow \quad \angle POQ = 90°$ **[½]**

Hence, Proved.

OR

(B)



Given that BS = 24 cm, BC = 30 cm

and radius = 8 cm

Join OP and OQ.

Now,

$BR = BS$ [Length of tangents from an external point are equal]

$\Rightarrow \quad BR$ = 24 cm …(i) [$\because$ BS = 24 cm] **[½]**

Also, BC = 30 cm

$\Rightarrow \quad BR + CR$ = 30 cm

$\Rightarrow \quad 24 + CR$ = 30 cm [using (i)]

$\Rightarrow \quad CR$ = 6 cm … (ii) **[1]**

$CQ = CR$ [As lengths of tangents from an external point are equal]

$\therefore \quad CQ$ = 6 cm …(iii) [using (ii)] **[½]**

In $\square POQD$,

$\angle OPD + \angle PDQ + \angle DQO + \angle POQ = 360°$

$\Rightarrow \quad 90° + 90° + 90° + \angle POQ = 360°$ [By angle sum property]

$\Rightarrow \quad \angle POQ = 90°$

$\therefore \quad \square POQD$ is a square as two adjacent sides are equal also. [$OP = OQ$] **[½]**

$\therefore \quad DQ = OQ$ = radius = 8 cm …(iv)

From (iii) and (iv)

$DC = DQ + CQ$

= 8 + 6

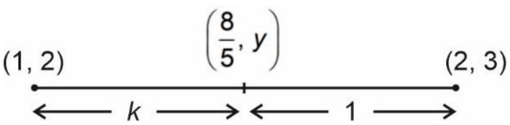
= 14 cm **[½]**

30. (A) Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points (1, 2) and (2, 3). Also, find the value of y. **[3]**

OR

(B) *ABCD* is a rectangle formed by the points *A* (–1, –1), *B* (–1, 6), *C* (3, 6) and *D* (3, –1). *P*, *Q*, *R* and *S* are mid-points of sides *AB*, *BC*, *CD* and *DA* respectively. Show that diagonals of the quadrilateral *PQRS* bisect each other. **[3]**

Sol. (A) Let the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points (1, 2) and (2, 3) in the ratio k : 1,



[½]

$$\Rightarrow \quad \frac{8}{5} = \frac{2k+1}{k+1}$$ **[½]**

$$\Rightarrow \quad 8k + 8 = 10k + 5$$

$$2k = 3$$

$$k = \frac{3}{2}$$ **[½]**

Required ratio = 3 : 2

and $y = \dfrac{k(3)+1(2)}{k+1}$ **[½]**

$$y = \frac{\frac{3}{2}\times 3+2}{\frac{3}{2}+1}$$

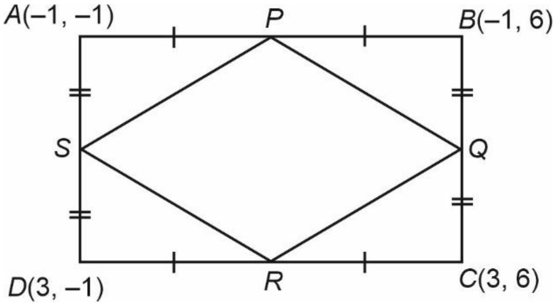
$$y = \frac{\frac{9}{2}+2}{\frac{5}{2}}$$ **[½]**

$$y = \frac{13}{2}\times\frac{2}{5}$$

$$\boxed{y = \frac{13}{5}}$$ **[½]**

OR

(B)



$$P = \left(\frac{-1-1}{2}, \frac{-1+6}{2}\right)$$

$$= \left(-1, \frac{5}{2}\right)$$ **[½]**

$$Q = \left(\frac{3-1}{2}, \frac{6+6}{2}\right)$$

$$= (1, 6)$$ **[½]**

$$R = \left(\frac{3+3}{2}, \frac{6-1}{2}\right)$$

$$= \left(3, \frac{5}{2}\right)$$ **[½]**

$$S = \left(\frac{-1+3}{2}, \frac{-1-1}{2}\right)$$

$$= (1, -1)$$ **[½]**

Mid point of diagonal $PR = \left(\dfrac{-1+3}{2}, \dfrac{\frac{5}{2}+\frac{5}{2}}{2}\right)$

$$= \left(1, \frac{5}{2}\right)$$

Mid point of diagonal $QS = \left(\dfrac{1+1}{2}, \dfrac{6-1}{2}\right)$

$$= \left(1, \frac{5}{2}\right)$$ **[½]**

Here, midpoint of *PR* is same as mid point of *QS*

$\therefore$ *PR* and *QS* bisect each other.

So, diagonals of quadrilateral *PQRS* bisect each other. **[½]**

31. Prove that : $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\text{cosec}\,\theta$ **[3]**

Sol. LHS $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$ [½]

$$=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$ [½]

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$ [½]

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$ [½]

$$=\frac{1}{\sin\theta-\cos\theta}\left[\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right]$$ [½]

$$=\frac{1}{\sin\theta-\cos\theta}\left[\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta}\right]$$

$$=\frac{1}{\sin\theta-\cos\theta}\left[\frac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{\cos\theta\sin\theta}\right]\qquad\left[\because \sin^2\theta+\cos^2\theta=1\right]$$

$$=\frac{1}{\cos\theta\sin\theta}+\frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

= 1 + sec θ cosec θ = RHS

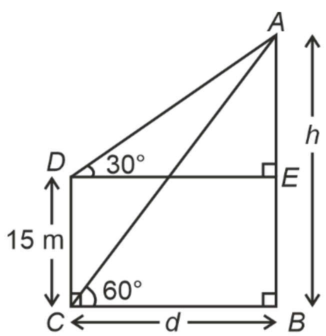
Hence, Proved. [½]

SECTION-D

This section consists of 4 questions of 5 marks each.

32. From the top of a 15 m high building, the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower and the distance between tower and the building. **[5]**

Sol. [½]



Let AB and CD represent the tower of height h m and building of height 15 m respectively and let the distance between the building and the tower be d m, then

In ΔABC, **[½]**

$\tan 60° = \frac{h}{d}$

$\Rightarrow \quad \sqrt{3}d = h$ ...(i) **[½]**

And in ΔADE,

$\tan 30° = \frac{h-15}{d}$ **[½]**

$\Rightarrow \quad \frac{d}{\sqrt{3}} = h - 15$ ...(ii)

$\Rightarrow \quad \frac{h}{\sqrt{3}\times\sqrt{3}} = h - 15$ [From (i) and (ii)] **[½]**

$\Rightarrow \quad h = 3h - 45$

$\Rightarrow \quad h = \frac{45}{2}$ **[½]**

$= 22.5$ m **[½]**

Substituting it in (i), we get

$d = \frac{45}{2\sqrt{3}} = \frac{45\sqrt{3}}{6}$ **[½]**

$= \frac{15\sqrt{3}}{2}$ m

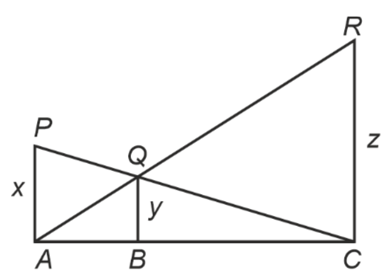
$= 12.99$ m (approx.) **[½]**

$\therefore$ Height of tower is 22.5 m and distance between tower and the building is 12.99 m approximately. **[½]**

33. (A) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. **[5]**

OR

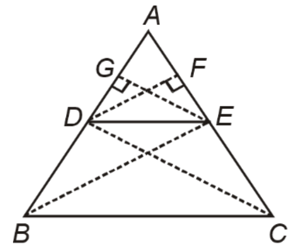
(B) In the given figure PA, QB and RC are each perpendicular to AC. If $AP = x$, $BQ = y$ and $CR = z$, then prove that $\frac{1}{x}+\frac{1}{z}=\frac{1}{y}$ **[5]**



Sol. (A) **Basic Proportionality Theorem (Thales' Theorem)**

Statement : In a triangle, a line drawn parallel to one side of a triangle intersecting the other two sides in distinct points, divides the other two sides in the same ratio.

Proof of the Theorem



Given : ΔABC, in which DE is drawn parallel to BC.

To Prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ [½]

Construction : Join CD and BE. Draw $DF \perp AE$ and $EG \perp AD$. [½]

Proof : $\text{ar}(\Delta ADE) = \dfrac{1}{2} \times AD \times EG$ ...(i) [½]

$\text{ar}(\Delta BDE) = \dfrac{1}{2} \times BD \times EG$ ...(ii) [½]

Dividing (i) by (ii), we get

$$\frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EG}{\frac{1}{2} \times BD \times EG} = \frac{AD}{BD} \quad \text{...(iii)}$$ [1]

Similarly,

$\text{ar}(\Delta ADE) = \dfrac{1}{2} \times DF \times AE$

and $\text{ar}(\Delta CDE) = \dfrac{1}{2} \times CE \times DF$

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CDE)} = \frac{\frac{1}{2} \times DF \times AE}{\frac{1}{2} \times DF \times CE} = \frac{AE}{CE} \quad \text{...(iv)}$$ [1]

Now, $\text{ar}(\Delta BDE) = \text{ar}(\Delta CDE)$ [∵ Triangles on the same base and between the same parallel lines are equal in area]

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CDE)}$$ [½]

∴ From (iii) and (iv), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$ [½]

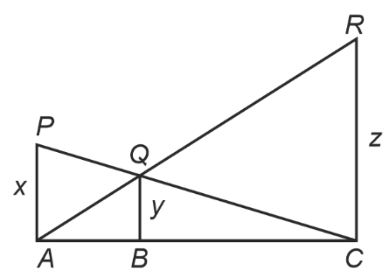
Hence proved.

OR

(B) **Given :** $AP \perp AC$,

$BQ \perp AC$ and $CR \perp AC$

$AP = x$, $BQ = y$, $CR = Z$



To Prove : $\dfrac{1}{x} + \dfrac{1}{z} = \dfrac{1}{y}$

Proof : In ΔPAC and ΔQBC,

$\angle PAC = \angle QBC = 90°$

$\angle PCA = \angle QCB$ (Common)

$\therefore \Delta PAC \sim \Delta QBC$ (By AA similarity criterion) **[1]**

$\therefore \dfrac{BQ}{AP} = \dfrac{BC}{AC}$ [Corresponding sides are in same ratio] **[½]**

$\dfrac{y}{x} = \dfrac{BC}{AC}$ ...(i) **[½]**

Similarly,

$\Delta ACR \sim \Delta ABQ$ [By AA similarity criterion] **[½]**

$\dfrac{BQ}{CR} = \dfrac{AB}{AC}$

$\Rightarrow \dfrac{y}{Z} = \dfrac{AB}{AC}$ ...(ii) **[½]**

Adding (i) and (ii), we get

$\dfrac{y}{x} + \dfrac{y}{z} = \dfrac{BC}{AC} + \dfrac{AB}{AC}$ **[1]**

$\Rightarrow y\left(\dfrac{1}{x} + \dfrac{1}{z}\right) = \dfrac{BC + AB}{AC}$

$\Rightarrow y\left(\dfrac{1}{x} + \dfrac{1}{z}\right) = \dfrac{AC}{AC} = 1$ **[½]**

$\Rightarrow \dfrac{1}{x} + \dfrac{1}{z} = \dfrac{1}{y}$ **[½]**

Hence, proved

34. (A) The sum of first and eighth terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms. **[5]**

OR

(B) In an A.P. of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of A.P. Also, find the sum of all the terms of the A.P. **[5]**

Sol. (A) Let the first term of A.P. be a, common difference be d and eighth term of A.P. be b,

$b = a + 7d$ ...(i)

Given,

$a + b = 32$ ...(ii)

$a \cdot b = 60$ ...(iii)

From (i) and (ii), we get

$a + a + 7d = 32$ **[½]**

$2a + 7d = 32$ ...(iv)

From (i) and (iii), we get

$a(a + 7d) = 60$ **[½]**

$\Rightarrow a(32 - a) = 60$ [From (iv)]

$\Rightarrow$ $32a - a^2 = 60$ **[½]**

$\Rightarrow$ $a^2 - 32a + 60 = 0$

$a^2 - 30a - 2a + 60 = 0$ **[½]**

$a = 2$ and $a = 30$ **[½]**

For $a = 2$ [Using (iv)]	For $a = 30$ [Using (iv)]
$d = 4$	$d = -4$ **[½]**

For $a = 2$, Sum of first 20 terms $= \frac{20}{2}[2(2)+19(4)]$

$= 10(4 + 76)$ **[½]**

$= 800$ **[½]**

For $a = 30$, sum of first 20 terms

$= \frac{20}{2}[2(30)+19(-4)]$ **[½]**

$= 10(60 - 76)$

$= -160$ **[½]**

OR

(B) Let the first term and common difference of A.P. be a and d respectively.

According to question,

$\frac{9}{2}[2a+(9-1)d] = 153$ **[½]**

$\Rightarrow$ $2a + 8d = 34$

$\Rightarrow$ $a + 4d = 17$ ...(i) **[½]**

Sum of last 6 terms = Sum of 40 terms – Sum of first 34 terms

$687 = \frac{40}{2}[2a+39d] - \frac{34}{2}[2a+33d]$ **[½]**

$\Rightarrow$ $687 = 20(2a + 39d) - 17(2a + 33d)$ **[½]**

$\Rightarrow$ $687 = 6a + 219d$ ...(ii) **[½]**

From (i) and (ii), we get

$687 = 6(17 - 4d) + 219d$ **[½]**

$687 = 102 - 24d + 219d$

$687 = 102 + 195d$

$d = 3$ **[½]**

$a = 17 - 4(3)$

$a = 5$ **[½]**

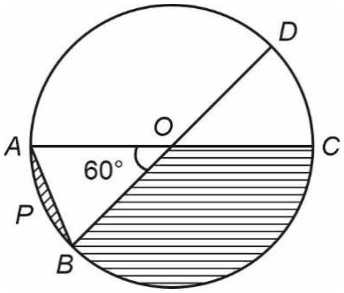
Sum of 40 terms $= \frac{40}{2}[2(5)+39(3)]$

$= 20(10 + 117)$ **[½]**

$= 20\ (127)$

$= 2540$ **[½]**

35. In the given figure, diameters AC and BD of the circle intersect at O. If $\angle AOB = 60°$ and $OA = 10$ cm, then : **[5]**



(i) find the length of the chord AB.

(ii) find the area of shaded region. (Take $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Sol. Since diameters AC and BD intersect at O.

$\therefore$ O is the centre of the circle.

$\therefore$ $OA = OB$ [radii of circle] **[½]**

$\therefore$ $\angle OAB = \angle OBA = 60°$

$\therefore$ ΔOAB is an equilateral triangle. **[½]**

(i) Length of the chord $AB = 10$ cm **[1]**

(ii) Area of $\Delta OAB = \frac{\sqrt{3}}{4}(\text{side})^2$ **[½]**

$= \frac{\sqrt{3}}{4}(10)^2$

$= 25\sqrt{3}\ \text{cm}^2$ **[½]**

Area of shaded region = Area of semicircle ABC – Area of equilateral ΔOAB **[½]**

$= \frac{\pi(10)^2}{2} - 25\sqrt{3}$ **[½]**

$= 50\pi - 25\sqrt{3}$

$= 50(3.14) - 25(1.73)$ **[½]**

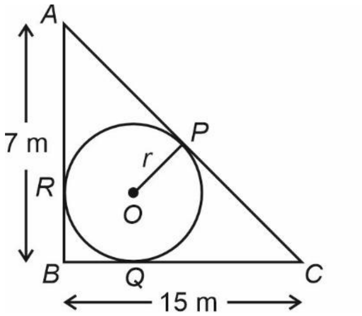
$= (157 - 43.25)$

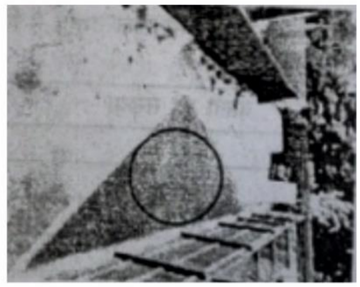
$= 113.75\ \text{cm}^2$ **[½]**

SECTION-E

This section consists of 3 Case-Study Based Questions of 4 marks each.

36. A backyard is in the shape of a triangle *ABC* with right angle at *B*. *AB* = 7 m and *BC* = 15 m. A circular pit was dug inside it such that it touches the walls *AC*, *BC* and *AB* at *P*, *Q* and *R* respectively such that *AP* = *x* m.





Based on the above information, answer the following questions:

(i) Find the length of *AR* in terms of *x*. **[1]**

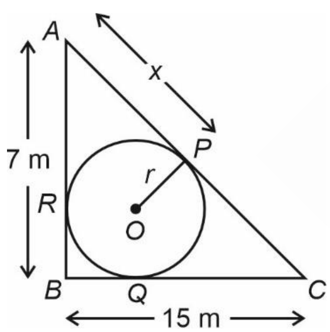
(ii) Write the type of quadrilateral *BQOR*. **[1]**

(iii) (a) Find the length *PC* in terms of *x* and hence find the value of *x*. **[2]**

OR

(b) Find *x* and hence find the radius *r* of circle. **[2]**

Sol. (i) $AR = AP = x$ m [$\because$ Length of tangents from an external point to a circle are equal] **[1]**



(ii) $OR \perp AB$ and $OQ \perp BC$ [$\because$ Radius $\perp$ Tangent]

$\Rightarrow \angle ORB = \angle OQB = \angle RBQ = 90°$

$\Rightarrow \angle QOR = 90°$ [By angle sum property] **[½]**

Also,

$OR = OQ$

$\Rightarrow$ *BQOR* is a square **[½]**

(iii) (a) $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{7^2 + 15^2}$

$= \sqrt{274}$ units **[½]**

$PC = AC - AP$

$\therefore\ PC = \left(\sqrt{274} - x\right)$ units **[½]**

Now,

$AR = AP = x$

$\Rightarrow BR = BQ = 7 - x$

$\Rightarrow CQ = PC = 15 - (7 - x)$

$= x + 8$ **[½]**

$\Rightarrow AC = x + x + 8$

$= 2x + 8$

$\Rightarrow \sqrt{274} = 2x + 8$

$\Rightarrow x = \left(\frac{\sqrt{274} - 8}{2}\right)$ m **[½]**

OR

(iii) (b) $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{7^2 + 15^2}$ **[½]**

$= \sqrt{274}$ m

Now,

$AR = AP = x$

$\Rightarrow BR = BQ = 7 - x$

$\Rightarrow CQ = CP = 15 - (7 - x)$

$= x + 8$ **[½]**

$\Rightarrow AC = x + x + 8$

$= 2x + 8$

$\Rightarrow \sqrt{274} = 2x + 8$

$\Rightarrow x = \left(\frac{\sqrt{274} - 8}{2}\right)$ m **[½]**

And $r = 7 - x = 7 - \left(\frac{\sqrt{274} - 8}{2}\right)$ [As *BQOR* is square. So, $r = BR = 7 - x$]

$= \left(\frac{22 - \sqrt{274}}{2}\right)$ m **[½]**

Case Study-Based Questions: **[3×4=12]**

37. A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information. **[1]**

(ii) Write the corresponding quadratic equation in standard form. **[1]**

(iii) (a) Find the value of x, the length of side of a tile by factorisation. **[2]**

OR

(b) Solve the quadratic equation for x, using quadratic formula. **[2]**

Sol. (i) According to question, Area of rectangular floor = Number of tiles × area of each tile

Total area of rectangular floor = $200x^2$

$200x^2 = 128(x + 1)^2$ **[1]**

(ii) $200x^2 = 128(x^2 + 1 + 2x)$

$72x^2 - 256x - 128 = 0$ **[½]**

$9x^2 - 32x - 16 = 0$ **[½]**

(iii) (a) $9x^2 - 32x - 16 = 0$

$9x^2 - 36x + 4x - 16 = 0$ **[½]**

$9x(x - 4) + 4(x - 4) = 0$ **[½]**

$x = 4,\ x = \frac{-4}{9}$ (Not possible) **[½]**

Length of side of tile = 4 units **[½]**

OR

(b) By using quadratic formula : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{32 \pm \sqrt{1024 - 4(9)(-16)}}{2(9)}$ **[½]**

$x = \frac{32 \pm \sqrt{1024 + 576}}{18}$ **[½]**

$x = \frac{32 \pm 40}{18}$ **[½]**

$x = 4$ and $x = \frac{-4}{9}$ **[½]**

38. BINGO is game of chance. The host has 75 balls numbered 1 through 75. Each player has a BINGO card with some numbers written on it.

The participant cancels the number on the card when called out a number written on the ball selected at random. Whosoever cancels all the numbers on his/her card, says BINGO and wins the game.

The table given below, shows the data of one such game where 48 balls were used before Tara said 'BINGO'.

Numbers announced	Number of times
0-15	8
15-30	9
30-45	10
45-60	12
60-75	9

Based on the above information, answer the following :

(i) Write the median class. **[1]**

(ii) When first ball was picked up, what was the probability of calling out an even number? **[1]**

(iii) (a) Find median of the given data. **[2]**

OR

(b) Find mode of the given data. **[2]**

Sol. (i) We draw cumulative frequency table.

Numbers announced	Number of times	Cumulative frequency
0-15	8	8
15-30	9	17
30-45	10	27
45-60	12	39
60-75	9	48

Here n = sum of all frequencies

= 48

$\therefore \quad \frac{n}{2} = \frac{48}{2} = 24$ **[½]**

We find that class whose cumulative frequency is just greater than 24.

$\therefore$ Median class = 30 – 45 as this class has frequency = 27 **[½]**

(ii) Probability (Even number)

$$= \frac{\text{Count of even numbers}}{\text{Total count of numbers}}$$

$$= \frac{37}{75}$$ [$\because$ Even numbers are 2, 4, …., 74] **[1]**

(iii) (a)

Class	Frequency	Cumulative frequency
0-15	8	8
15-30	9	17
30-45	10	27
45-60	12	39
60-75	9	48

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$ **[½]**

Here, median class = 30 – 45

$\therefore$ $l = 30$, $n = 48$, $cf = 17$, $f = 10$, $h = 15$

$$\therefore \quad \text{Median} = 30 + \left(\frac{\frac{48}{2} - 17}{10}\right) \times 15$$ **[1]**

$$= 30 + \frac{7}{10} \times 15$$

$$= 40.5$$ **[½]**

OR

(b)

Class	Frequency
0-15	8
15-30	9
30-45	10
45-60	12
60-75	9

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$ **[½]**

As class 45-60 has highest frequency

Modal class = 45 – 60 **[½]**

$l = 45$, $f_1 = 12$, $f_0 = 10$, $f_2 = 9$, $h = 15$

$$\therefore \quad \text{Mode} = 45 + \left(\frac{12-10}{2\times 12-10-9}\right)\times 15$$ **[½]**

$$= 45 + \frac{2}{5}\times 15$$

$$= 51$$ **[½]**

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