

Date: 15/05/2026



# Aakash

Medical | IIT-JEE | Foundations

Corporate Office : AESL, 3rd Floor, Incuspaze Campus-2, Plot-13, Sector-18, Udyog Vihar,  
Gurugram, Haryana-122015

Question Paper Code

430/8/2

SET-2

Time: 3 Hrs.

## MATHEMATICS (Basic)

### CBSE Class-X (2026) Phase-2

### Answers & Solutions

Max. Marks: 80

#### GENERAL INSTRUCTIONS

Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **FIVE** Sections - **A, B, C, D** and **E**.
- (iii) In **Section-A**, Question numbers **1** to **18** are multiple choice questions (MCQs) and question numbers **19** and **20** are Assertion–Reason based questions of 1 mark each.
- (iv) In **Section-B**, Question numbers **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section-C**, Question numbers **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section-D**, Question numbers **32** to **35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section-E**, Question numbers **36** to **38** are case study based questions carrying **4** marks each. Internal choice is provided in **2** marks question in each case-study.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in **Section-B**, **2** questions in **Section-C**, **2** questions in **Section-D** and **3** questions **Section-E**.
- (ix) Draw neat diagrams wherever required. Take  $\pi = 22/7$  wherever required, if not stated.
- (x) Use of calculators is **NOT** allowed.

**SECTION-A**

Question numbers 1 to 20 are multiple type questions of 1 mark each.

1. If  $\alpha$  and  $\beta$  are the roots of quadratic equation  $x^2 - 7x + 11 = 0$ , then the value of  $(\alpha^2 + \beta^2)$  is [1]  
 (A) 27 (B) 29  
 (C) 31 (D) 33

**Answer (A)** [1]

**Sol.**  $\alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{1} = 7$

$\alpha\beta = \frac{c}{a} = \frac{11}{1} = 11$

$(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$   
 $= 7^2 - 2 \times 11$   
 $= 49 - 22$   
 $= 27$

2. Among the following pair of irrational number, whose product is a rational number? [1]  
 (A)  $\sqrt{8}, \sqrt{32}$  (B)  $\sqrt{3}, \sqrt{5}$   
 (C)  $\sqrt{8}, \sqrt{20}$  (D)  $\sqrt{2}, \sqrt{15}$

**Answer (A)** [1]

**Sol.**  $\sqrt{8} \times \sqrt{32} = \sqrt{2^3 \times 2^5} = \sqrt{2^8} = 2^4 = 16$   
 16 is a rational number.

3. The common difference of the A.P. 2, -2, -6, -10, ..... is [1]  
 (A) 4 (B) 10  
 (C) -4 (D) -16

**Answer (C)** [1]

**Sol.**  $d = -2 - 2 = -4$

4. The total surface area of a solid hemisphere of radius 7 cm is [1]  
 (A)  $447 \pi$  sq. cm (B)  $239 \pi$  sq. cm  
 (C)  $174 \pi$  sq. cm (D)  $147 \pi$  sq. cm

**Answer (D)** [1]

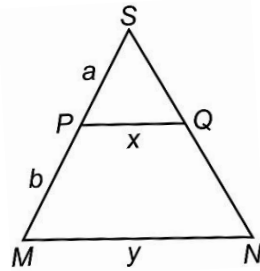
**Sol.**  $TSA = 3\pi r^2 = 3 \times \pi \times 7^2$   
 $= 147 \pi$  sq. cm

5. If  $x = 3$  is a zero of the polynomial  $p(x) = x^2 - 9k$ , then  $k$  is equal to [1]  
 (A) 1 (B) 10  
 (C) 0.1 (D) 100

**Answer (A)** [1]

**Sol.**  $p(x) = x^2 - 9k$   
 $p(3) = (3)^2 - 9k = 0$   
 $\Rightarrow 9 = 9k$   
 $\Rightarrow k = 1$

6. If in the given figure,  $PQ \parallel MN$ , then which of the following is true? [1]



(A)  $x = \frac{a+b}{ay}$

(B)  $\frac{x}{y} = \frac{a}{b}$

(C)  $x = \frac{ay}{a+b}$

(D)  $\frac{x}{y} = \frac{b}{a}$

**Answer (C)** [1]

**Sol.**  $\triangle SPQ \sim \triangle SMN$  (By AA similarity)

$$\frac{SP}{SM} = \frac{PQ}{MN}$$

$$\Rightarrow \frac{a}{a+b} = \frac{x}{y}$$

$$\Rightarrow x = \frac{ay}{a+b}$$

7. If L.C.M. of two numbers is 1890 and their H.C.F. is 30 and one of the numbers is 270, then the other number is [1]

(A) 210

(B) 220

(C) 230

(D) 240

**Answer (A)** [1]

**Sol.** Other number =  $\frac{1890 \times 30}{270}$   
= 210

8. If  $x = a \sin \theta$  and  $y = a \cos \theta$ , then the value of  $x^2 + y^2$  is [1]

(A) 1

(B)  $a^2$

(C)  $a$

(D) 0

**Answer (B)** [1]

**Sol.**  $x^2 + y^2 = (a \sin \theta)^2 + (a \cos \theta)^2$   
=  $a^2 \sin^2 \theta + a^2 \cos^2 \theta$   
=  $a^2 (\sin^2 \theta + \cos^2 \theta)$   
=  $a^2$

9. The pair of equations  $x = 2$  and  $y = 3$  represent the lines which are [1]

(A) intersecting at  $(-2, -3)$

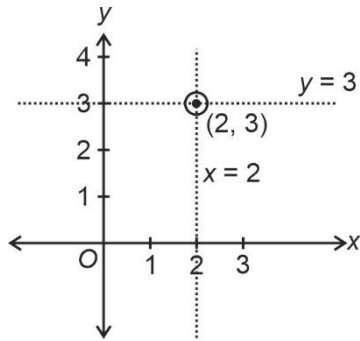
(B) intersecting at  $(2, 3)$

(C) parallel

(D) coincident

**Answer (B)** [1]

**Sol.** Interesting at (2, 3)



10. In an A.P., the ratio of the 7<sup>th</sup> term to the 3<sup>rd</sup> term is 5 : 3. If the sum of the first 10 terms is 255, then the 14<sup>th</sup> term is [1]

- (A) 42 (B) 47  
(C) 51 (D) 57

**Answer (C)** [1]

**Sol.**  $\frac{7^{\text{th}} \text{ term}}{3^{\text{rd}} \text{ term}} = \frac{a + 6d}{a + 2d} = \frac{5}{3}$

$a = 4d$

$S_{10} = 255 = \frac{10}{2} [2a + 9d]$

$\frac{255}{5} = 17d$

$d = 3$

$\therefore a = 12$

$\therefore a_{14} = a + 13d$   
 $= 12 + 13 \times 3$   
 $= 12 + 39$   
 $= 51$

11. The upper limit of median class in the following frequency distribution is : [1]

<b>Class</b>	0 – 10	10 – 10	20 – 30	30 – 40	40 – 50
<b>Frequency</b>	5	12	20	8	5

- (A) 10 (B) 20  
(C) 30 (D) 40

**Answer (C)** [1]

**Sol.**  $\frac{N}{2} = \frac{50}{2} = 25$

Class 20 – 30 has cumulative frequency just greater than 25 i.e. 37

$\therefore$  upper limit = 30

12. A chord of a circle of radius 10 cm, subtends a right angle at its centre. The length of the chord (in cm) is : [1]
- (A)  $5\sqrt{2}$  (B)  $10\sqrt{2}$   
(C)  $\frac{5}{\sqrt{2}}$  (D)  $10\sqrt{3}$

**Answer (B)** [1]

**Sol.** Length of chord = Length of hypotenuse

$$\therefore \text{length} = \sqrt{10^2 + 10^2} = 10\sqrt{2} \text{ cm}$$

13. If  $\sec\theta + \tan\theta = p$ , then  $\sin\theta$  is equal to [1]

- (A)  $\frac{p^2 - 1}{p^2 + 1}$  (B)  $\frac{p^2 + 1}{p^2 - 1}$   
(C)  $\frac{2p}{p^2 + 1}$  (D)  $\frac{p^2 - 1}{2p}$

**Answer (A)** [1]

**Sol.**  $\sec\theta + \tan\theta = p$

$$\therefore \sec\theta - \tan\theta = \frac{1}{p} \quad [\because \sec^2\theta - \tan^2\theta = 1]$$

$$\therefore \sec\theta = \frac{1}{2} \left( p + \frac{1}{p} \right) = \frac{p^2 + 1}{2p}$$

$$\therefore \cos\theta = \frac{2p}{p^2 + 1}$$

$$\Rightarrow \sin\theta = \sqrt{1 - \left( \frac{2p}{p^2 + 1} \right)^2}$$

$$= \frac{p^2 - 1}{p^2 + 1}$$

14. The fourth term of the A.P.  $\sqrt{5}, \sqrt{20}, \sqrt{45}, \dots$  is [1]

- (A) 20 (B)  $\sqrt{80}$   
(C)  $\sqrt{125}$  (D)  $\sqrt{85}$

**Answer (B)** [1]

**Sol.**  $\sqrt{5}, 2\sqrt{5}, 3\sqrt{5}, \dots$

$$\therefore 4^{\text{th}} \text{ term} = 4\sqrt{5} = \sqrt{80}$$

15. Two concentric circles have radii 5 cm and 13 cm. The length of a chord of the larger circle which is tangent to the smaller circle is [1]

- (A) 10 cm (B) 12 cm  
(C) 18 cm (D) 24 cm

**Answer (D)** [1]

**Sol.** Length of chord of larger circle

$$= 2 \times \sqrt{13^2 - 5^2}$$

$$= 24 \text{ cm}$$

16. The quadratic equation  $2x^2 - \sqrt{5}x + 1 = 0$  has [1]
- (A) two distinct real roots. (B) two equal real roots.  
(C) no real roots. (D) more than two real roots.

**Answer (C)** [1]

**Sol.**  $2x^2 - \sqrt{5}x + 1 = 0$

$$a = 2, b = -\sqrt{5}, c = 1$$

$$D = b^2 - 4ac$$

$$= -3$$

$\therefore$  As  $D < 0$ , no real roots.

17. A card is drawn at random from a well shuffled deck of 52 playing cards. The probability of it being a card of spade or an ace is [1]

- (A)  $\frac{7}{13}$  (B)  $\frac{5}{13}$   
(C)  $\frac{4}{13}$  (D)  $\frac{3}{13}$

**Answer (C)** [1]

**Sol.** There are 13 spade cards (including 1 ace) and 3 other ace cards.

$$P(\text{spade or ace}) = \frac{16}{52} = \frac{4}{13}$$

18. If  $EF$  is drawn parallel to the side  $BC$  of a  $\triangle ABC$  meeting  $AB$  at  $E$  and  $AC$  at  $F$  such that  $AB = 4 BE$  and  $CF = 3$  cm, then  $AF$  is equal to [1]

- (A) 6 cm (B) 9 cm  
(C) 12 cm (D) 15 cm

**Answer (B)**

**Sol.**  $\frac{AB}{BE} = \frac{4}{1} \Rightarrow \frac{AE}{BE} = \frac{3}{1}$

$$\therefore \frac{AF}{CF} = \frac{3}{1}$$

$$\therefore AF = 9 \text{ cm}$$

[1]

**(Assertion-Reason based questions)**

**Directions:** Question numbers 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)  
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A)  
(C) Assertion (A) is true, but Reason (R) is false.  
(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A):** Two congruent triangles are always similar.

**Reason (R):** If the area of two triangles are equal, then they are congruent.

[1]

**Answer (C)**

**Sol.** Assertion (A) is true, but Reason (R) is false.

[1]

20. **Assertion (A):** The mid-point of the line segment joining  $A(-4, 3)$  and  $B(4, 3)$  is  $(0, 3)$ .

**Reason (R):** The mid-point of the line segment joining two points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  can be determined by using section formula with ratio 1:1.

[1]

**Answer (A)**

**Sol.** Mid-point of  $AB \equiv \left( \frac{-4+4}{2}, \frac{3+3}{2} \right)$   
 $\equiv (0, 3)$

$\therefore$  Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)

[1]

### SECTION-B

**Question numbers 21 to 25 are very short answer type questions of 2 marks each.**

21. A group of students conducted a survey of 20 households in a neighbourhood to study family sizes. The following frequency distribution table shows the number of family members per household. Determine the mode of this data.

[2]

Family size	1-3	3-5	5-7	7-9	9-11
Number of families	7	8	2	2	1

**Sol.** Mode =  $l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

[½]

Now, modal class = 3-5

$\therefore l = 3, f_1 = 8, f_0 = 7$

$f_2 = 2, h = 2$

$\therefore$  Mode =  $3 + \left( \frac{8-7}{16-7-2} \right) \times 2$

[1]

$= 3 + \frac{2}{7}$

$= \frac{23}{7} \approx 3.286$

[½]

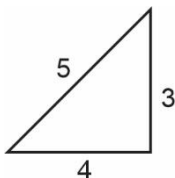
22. If  $\cos \theta = \frac{4}{5}$ , find the value of  $\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta}$ .

[2]

**Sol.**  $\therefore \cos \theta = \frac{4}{5}$ ,

[1]

$\therefore$  Using Pythagoras theorem



$$\text{Perp} = \sqrt{(\text{Hyp.})^2 - (\text{Base})^2}$$

$$= \sqrt{5^2 - 4^2}$$

$$= \sqrt{9}$$

$$= 3$$

$$\sec \theta = \frac{5}{4}$$

$$\tan \theta = \frac{3}{4}$$

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta}$$

$$= \frac{\frac{5}{4} - \frac{3}{4}}{\frac{5}{4} + \frac{3}{4}}$$

$$= \frac{1}{31}$$

[1]

23. (a) If  $A(a, b)$  is a point which is equidistant from points  $B(2, 4)$  and  $C(-2, -3)$ , then find the relation between  $a$  and  $b$ . [2]

OR

- (b) Find the ratio in which the line  $x + 3y = 12$  divides the line segment joining  $(1, 3)$  and  $(2, 5)$ . [2]

**Sol.** (a)  $A(a, b)$ ,  $B(2, 4)$  and  $C(-2, -3)$

We know,

$$AB = AC$$

[½]

$$\text{Or } AB^2 = AC^2$$

Using distance-formula we get

$$(a - 2)^2 + (b - 4)^2 = (a + 2)^2 + (b + 3)^2$$

[½]

$$\text{Or } (a - 2)^2 - (a + 2)^2 = (b + 3)^2 - (b - 4)^2$$

$$(a - 2 + a + 2)(a - 2 - a - 2) = (b + 3 + b - 4)(b + 3 - b + 4)$$

[½]

$$(2a)(-4) = (2b - 1)(7)$$

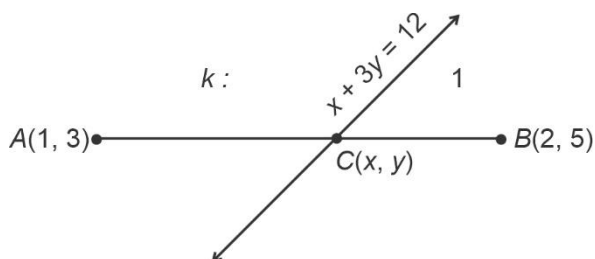
$$-8a = 14b - 7$$

$$\text{or } \boxed{8a + 14b = 7}$$

[½]

OR

- (b) Let the required ratio be  $k : 1$



C divides AB in the ratio  $k : 1$  [½]

$$x = \frac{2k+1}{k+1}, y = \frac{5k+3}{k+1} \quad [½]$$

C lies on AB

$$\therefore x + 3y = 12$$

$$\frac{2k+1}{k+1} + \frac{3(5k+3)}{k+1} = 12 \quad [½]$$

$$2k + 1 + 15k + 9 = 12k + 12$$

$$5k = 2$$

$$\boxed{k = \frac{2}{5}}$$

$\therefore$  The required ratio is 2 : 5. [½]

24. (a) Find the sum of first ten 2-digit numbers which are multiples of 7. [2]

**OR**

(b) The 3<sup>rd</sup> and the 8<sup>th</sup> terms of an A.P. are 13 and 38 respectively. Find its 11<sup>th</sup> term. [2]

**Sol.** (a) The first two-digit multiple of 7 are

14, 21, 28, 35, 42, 49, .... [½]

Let first term ( $a$ ) = 14

Common difference ( $d$ ) = (21 – 14)

$$= 7$$

Number of term ( $n$ ) = 10

$$S_n = \frac{n}{2} [2a + (n-1)d] \quad [½]$$

$$= \frac{10}{2} [2(14) + 9(7)] \quad [½]$$

$$= 5(28 + 63)$$

$$= 5 \times 91$$

$$= 455 \quad [½]$$

**OR**

(b) Let the first term of AP be  $a$  and common difference be  $d$

$$a_n = a + (n-1)d \quad [½]$$

3<sup>rd</sup> term:

$$a + 2d = 13 \quad \dots(i)$$

8<sup>th</sup> term:

$$a + 7d = 38 \quad \dots(ii)$$

subtract (i) from (ii), we get

$$5d = 25 \quad [½]$$

$$\Rightarrow d = 5$$

Put the value of  $d$  in equation (i)

$$a + 2(5) = 13$$

$$\Rightarrow a + 10 = 13$$

[½]

$$\Rightarrow a = 3$$

$$a_{11} = a + 10d$$

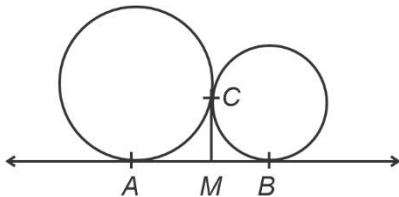
$$= 3 + 10(5)$$

$$= 3 + 50$$

$$= 53$$

[½]

25. The two touching circles have common tangents  $AB$  and  $CM$  as shown in the given figure. Show that  $M$  is the midpoint of line segment  $AB$ . [2]



**Sol.** As common tangent are  $AB$  and  $CM$ .

Now, tangents drawn from an external point are of equal length.

$$\therefore AM = CM$$

... (i)

[½]

$$\text{And } BM = CM$$

... (ii)

[½]

Using (i) and (ii)

$$AM = BM$$

[½]

$\Rightarrow M$  is the mid-point of line segment  $AB$

[½]

### SECTION-C

**Question numbers 26 to 31 are short answer type questions of 3 marks each.**

**6×3=18**

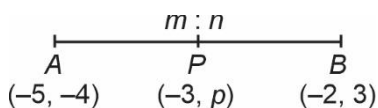
26. Find the ratio in which the point  $P(-3, p)$  divides the line segment joining  $A(-5, -4)$  and  $B(-2, 3)$ . Also, find the value of  $p$ . [3]

**Sol.** Let the point  $P(-3, p)$  divide the line segment joining  $A(-5, -4)$  and  $B(-2, 3)$  in ratio  $m : n$

Now, using section formula for the  $x$ -coordinates:

$$-3 = \frac{m(-2) + n(-5)}{m + n}$$

[0.5]



$$-3(m + n) = -2m - 5n$$

[0.5]

$$-3m - 3n = -2m - 5n$$

[0.5]

$$\boxed{m = 2n}$$

[0.5]

Hence, the ratio is  $2 : 1$

Now again using section formula for y-coordinates.

$$p = \frac{2(3) + 1(-4)}{2 + 1} \quad [0.5]$$

$$= \frac{6 - 4}{3}$$

$$\Rightarrow \boxed{p = \frac{2}{3}} \quad [0.5]$$

27. (a) In a certain fraction, if the numerator is multiplied by 4 and the denominator is decreased by 5, the result becomes  $\frac{8}{5}$ . But if the numerator is increased by 1 and denominator is increased by 2, the result becomes  $\frac{1}{3}$ . Find the original fraction. [3]

OR

- (b) A 2-digit number is such that its units digit is two less than the tens digit, also the sum of the number and the number formed by reversing the digits is 154. Find the number. [3]

**Sol.** (a) Let numerator of fraction be  $x$  and denominator be  $y$

According to question,

$$\frac{4x}{y - 5} = \frac{8}{5} \quad \dots(i) \quad [0.5]$$

Or  $5x = 2y - 10$

$$5x - 2y = -10 \quad \dots(ii) \quad [0.5]$$

Also,  $\frac{x + 1}{y + 2} = \frac{1}{3} \quad \dots(iii) \quad [0.5]$

Or  $3x + 3 = y + 2$

$$3x - y = -1 \quad \dots(iv) \quad [0.5]$$

Multiply (iv) by 2, we get

$$6x - 2y = -2 \quad \dots(v) \quad [0.5]$$

Subtract (ii) and (v)

$$-x = -8 \text{ or } \boxed{x = 8}$$

$$\therefore 5 \times 8 - 2y = -10$$

$$\boxed{y = 25}$$

$$\therefore \text{the fraction is } \frac{8}{25} \quad [0.5]$$

OR

(b) Let units digit =  $x$

Tens digit =  $y$

Number =  $10y + x$

$$x = y - 2 \quad \dots(i) \quad [0.5]$$

Also,

$$10y + x + 10x + y = 154$$

$$11(x + y) = 154$$

[0.5]

$$x + y = 14 \quad \dots(ii)$$

Put (i) in (ii)

$$y - 2 + y = 14$$

[0.5]

$$2y = 16$$

$$\boxed{y = 8}$$

[0.5]

$$\therefore \boxed{x = 6}$$

$\therefore$  the number is 86.

[0.5]

28. (a) Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468. [3]

OR

- (b) Prove that  $\sqrt{5}$  is an irrational number.

**Sol.** (a) The required number will be 17 less than the LCM of 520 and 468.

$$\text{Now, } 520 = 2^3 \times 5 \times 13$$

[0.5]

$$468 = 2^2 \times 3^2 \times 13$$

[0.5]

Now, LCM = Product of highest power of all factors.

$$\text{LCM} = 2^3 \times 3^2 \times 5 \times 13 = 4680$$

[1]

$$\therefore \text{Required Number} = 4680 - 17 = 4663$$

[1]

OR

- (b) Let us assume that  $\sqrt{5}$  is rational. Then, there exist co-prime positive integers  $a$  and  $b$  such that

$$\sqrt{5} = \frac{a}{b}$$

[0.5]

$$\Rightarrow b\sqrt{5} = a$$

[0.5]

Squaring both sides, we get

$$a^2 = 5b^2$$

[0.5]

Therefore,  $a^2$  is divisible by 5 and hence,  $a$  is also divisible by 5.

So, we can write  $a = 5r$ , for some integer  $r$ .

$$\text{Substituting for } a, \text{ we get } 25r^2 = 5b^2 \Rightarrow 5r^2 = b^2$$

[0.5]

This means,  $b^2$  is also divisible by 5 and so,  $b$  is also divisible by 5.

Therefore,  $a$  and  $b$  have at least one common prime factor, i.e. 5.

[0.5]

But, this contradicts the fact that  $a$  and  $b$  are co-prime.

Thus, our supposition is wrong.

Hence,  $\sqrt{5}$  is irrational.

[0.5]

29. The number of students absent in a school was recorded everyday for 140 days and the data was presented in the form of the following frequency table.

Number of absentees	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	Total
Number of days	1	5	11	14	16	13	14	60	6	140

Obtain the median and the mode of the data.

[3]

Sol. Here, total  $N = 140$

$$\therefore \frac{N}{2} = \frac{140}{2} = 70$$

Class $x_i$	Frequency $f$	Cumulative frequency $c.f.$
2-4	1	1
4-6	5	6
6-8	11	17
8-10	14	31
10-12	16	47
12-14	13	60
14-16	14	74
16-18	60	134
18-20	6	140
	<b><math>N = 140</math></b>	

Since,  $\frac{N}{2} = 70$  lies in the class 14-16, the median class is 14-16.

$$\therefore l = 14, h = 2, f = 14, cf = 60$$

[0.5]

Using formula,

$$\text{Median} = l + \left( \frac{\frac{N}{2} - c.f.}{f} \right) \times h$$

[0.5]

$$= 14 + \left( \frac{70 - 60}{14} \right) \times 2$$

$$= 14 + \frac{10}{14} \times 2$$

$$= 14 + 1.43$$

$$\text{Median} = 15.43$$

[0.5]

For mode,

$$L = 16$$

$$f_1 = 60$$

$$f_0 = 14$$

$$f_2 = 6$$

[0.5]

$$h = 2$$

$$\text{Mode} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

[0.5]

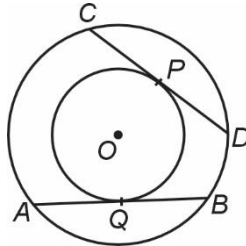
$$= 16 + \left( \frac{60 - 14}{2 \times 60 - 14 - 6} \right) \times 2$$

$$= 16 + \frac{46}{100} \times 2$$

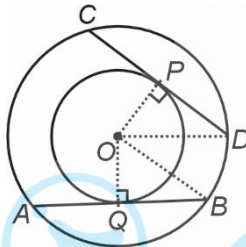
$$= 16.92$$

[0.5]

30. In the given figure, two concentric circles are drawn with centre  $O$ .  $AB$  and  $CD$  are two chords of the larger circle which are tangents to the smaller circle. Prove that  $AB = CD$ . [3]



**Sol.** Join  $OP$ ,  $OD$ ,  $OQ$  and  $OB$ .



In  $\triangle OPD$  and  $\triangle OQB$

$$\angle OPD = \angle OQB$$

[Radius and tangent are perpendicular]

$$OD = OB$$

[radii]

$$OP = OQ$$

[radii]

$$\therefore \triangle OPD \cong \triangle OQB$$

[RHS]

$$\Rightarrow PD = QB$$

... (i) [CPCT]

Since,  $OP \perp CD$  and  $OQ \perp AB$

... (ii)

$$\therefore CP = PD \text{ and } AQ = QB$$

From (i) and (ii)

We get

$$2PD = 2QB$$

$$\text{or } \boxed{CD = AB}$$

[0.5]

[0.5]

[0.5]

[0.5]

[0.5]

[0.5]

31. If  $\sin(A - B) = \frac{1}{2}$  and  $\cos(A + B) = \frac{1}{2}$ ,  $0 < A + B \leq 90^\circ$ ,  $A > B$ , then find the value of  $\sin A$ . [3]

**Sol.** Given :  $\sin(A - B) = \frac{1}{2}$

$$\sin(A - B) = \sin 30^\circ$$

$$\therefore A - B = 30^\circ$$

... (i)

[1]

$$\text{Also, } \cos(A + B) = \frac{1}{2}$$

$$\cos(A + B) = \cos 60^\circ$$

$$\therefore A + B = 60^\circ \quad \dots(ii) \quad [1]$$

On adding (i) and (ii)

$$A - B + A + B = 30^\circ + 60^\circ$$

$$\Rightarrow 2A = 90^\circ$$

$$\therefore A = 45^\circ$$

$$B = 60^\circ - 45^\circ = 15^\circ$$

$$\therefore \sin A = \sin 45^\circ$$

$$= \frac{1}{\sqrt{2}} \quad [1]$$

### SECTION-D

Question numbers 32 to 35 are long answer type questions of 5 marks each.

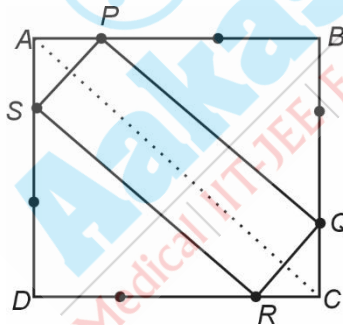
4×5=20

32. (a)  $ABCD$  is a quadrilateral and  $P, Q, R$  and  $S$  are the points of trisection of sides  $AB, BC, CD$  and  $DA$  respectively such that these are adjacent to  $A$  and  $C$ . Prove that  $PQRS$  is a parallelogram. [5]

OR

- (b) In an isosceles  $\triangle ABC$  with  $AB = AC$ , points  $P$  and  $Q$  are on the sides  $AB$  and  $AC$  respectively such that  $AP = AQ$ . Show that the points  $B, C, Q, P$  are concyclic. [5]

**Sol.** (a)  $\square PQRS$  is a parallelogram.



**Given :**  $P$  is on  $AB$  such that

$$AP = \frac{1}{3} AB$$

$$\therefore BP = \frac{2}{3} AB$$

Similarly,  $CQ = \frac{1}{3} BC$  and  $BQ = \frac{2}{3} BC$

$$CR = \frac{1}{3} CD \text{ and } DR = \frac{2}{3} CD$$

$$AS = \frac{1}{3} DA \text{ and } DS = \frac{2}{3} DA$$

**Construction:** Join  $AC$

**Proof :** In  $\triangle ABC$ ,

$$\frac{BP}{BA} = \frac{\frac{2}{3}AB}{AB} = \frac{2}{3}$$

$$\frac{BQ}{BC} = \frac{\frac{2}{3}BC}{BC} = \frac{2}{3}$$

$$\therefore \frac{BP}{BA} = \frac{BQ}{BC}$$

$\therefore PQ \parallel AC$  ... (i) [By corollary of BPT]

$$\therefore \boxed{PQ = \frac{2}{3}AC}$$
 ... (ii)

Again in  $\triangle ADC$ ,

$$\frac{DS}{DA} = \frac{\frac{2}{3}DA}{DA} = \frac{2}{3} \text{ and } \frac{DR}{DC} = \frac{\frac{2}{3}CD}{CD} = \frac{2}{3}$$

Here,  $\frac{DS}{DA} = \frac{DR}{DC}$

$\therefore SR \parallel AC$  ... (iii)

and  $\boxed{SR = \frac{2}{3}AC}$  ... (iv)

$\therefore$  From (i), (ii), (iii) and (iv)

$PQ \parallel SR$  [Both are parallel to AC]

$PQ = SR$  [Both are equal to  $\frac{2}{3}AC$ ]

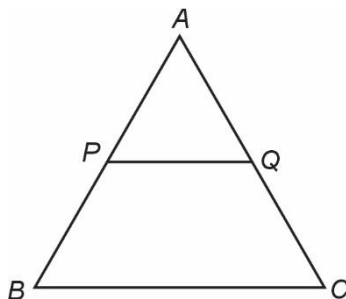
We know that, if one pair of opposite sides is both equal and parallel then the quadrilateral  $PQRS$  is a parallelogram. Proved.

**OR**

(b) **To prove :**  $B, C, Q, P \rightarrow$  are concyclic.

$\therefore$  Opposite angles are supplementary.

In  $\triangle ABC$



$AB = AC$

$P$  and  $Q \rightarrow$  are points on the sides  $AB$  and  $AC$  respectively, such that

$\therefore AP = AQ$

**To prove** : Point  $BC$ ,  $Q$  and  $P$  are concyclic

(or)

$$\angle PBC + \angle PQC = 180^\circ = \angle BPQ + \angle BCQ$$

**Proof** : In  $\triangle ABC$

$$AB = AC \text{ and } AP = AQ$$

$$AB - AP = AC - AQ$$

$$BP = CQ$$

Now,  $AP = AQ$  and  $PB = QC$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC} \quad [\text{B converse of BPT}]$$

$$PQ \parallel BC$$

$$\therefore \angle ABC = \angle APQ \quad [\text{Corresponding angles}]$$

$$\angle ABC + \angle BPQ = \angle APQ + \angle BPQ$$

$$\angle ABC + \angle BPQ = 180^\circ \quad [\text{Linear Pair angles}]$$

$$\angle ACB + \angle BPQ = 180^\circ \quad [\because AB = AC \therefore \angle ACB = \angle ABC]$$

Again,

$$PQ \parallel BC$$

$$\angle ACB = \angle AQP \quad [\text{Corresponding angles}]$$

$$\angle ACB + \angle CQP = \angle AQP + \angle CQP \quad [\text{Adding } \angle CQP \text{ to both sides}]$$

$$\angle ACB + \angle CQP = 180^\circ \quad [\text{Linear pair angles}]$$

$$\angle ABC + \angle CQP = 180^\circ \quad [\because \angle ABC = \angle ACB]$$

So, in quadrilateral  $BCQP$

$$\therefore \angle ACB + \angle BPQ = 180^\circ \text{ and } \angle ABC + \angle CQP = 180^\circ$$

Hence,  $BPQC$  is a cyclic quadrilateral.

$\therefore B, C, Q,$  and  $P$  are concyclic points.

**Proved**

33. (a) The sum of ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their ages respectively after five years. [5]

**OR**

- (b) Express the equation  $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}; (x \neq 3, -5)$  as a quadratic equation in standard form. Hence, solve the equation so formed. [5]

**Sol.** (a) Let the present age of father be  $x$  year present age of Son be  $y$  years.

$$x + y = 45 \quad \dots(i) \quad [1]$$

Five years ago,

$$(x - 5)(y - 5) = 124 \quad [1]$$

$$\Rightarrow xy - 5y - 5x + 25 = 124$$

$$\Rightarrow xy - 5(x + y) + 25 = 124 \quad [1]$$

Put the value of  $x + y$  from (i)

$$\Rightarrow xy - 5(45) + 25 = 124$$

$$\Rightarrow xy = 324$$

$$\therefore x + y = 45, xy = 324$$

$$x^2 - 45x + 324 = 0$$

$$\Rightarrow x^2 - 36x - 9x + 324 = 0$$

$$\Rightarrow x(x - 36) - 9(x - 36) = 0$$

$$\Rightarrow (x - 36)(x - 9) = 0$$

$$x = 36, y = 9$$

[1]

After five years

$$\text{Father's age} = 36 + 5 = 41 \text{ years}$$

[½]

$$\text{Son's age} = 9 + 5 = 14 \text{ years}$$

[½]

OR

$$(b) \frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$$

$$\Rightarrow \frac{(x+5) - (x-3)}{(x-3)(x+5)} = \frac{1}{6}$$

[½]

$$\Rightarrow \frac{x+5-x+3}{(x-3)(x+5)} = \frac{1}{6}$$

[½]

$$\Rightarrow \frac{8}{(x-3)(x+5)} = \frac{1}{6}$$

[½]

$$\Rightarrow 48 = (x-3)(x+5)$$

[½]

$$\Rightarrow x^2 + 2x - 15 = 48$$

[½]

$$\Rightarrow x^2 + 2x - 63 = 0$$

[½]

$$\Rightarrow x^2 + 9x - 7x - 63 = 0$$

[½]

$$\Rightarrow x(x+9) - 7(x+9) = 0$$

[½]

$$\Rightarrow (x+9)(x-7) = 0$$

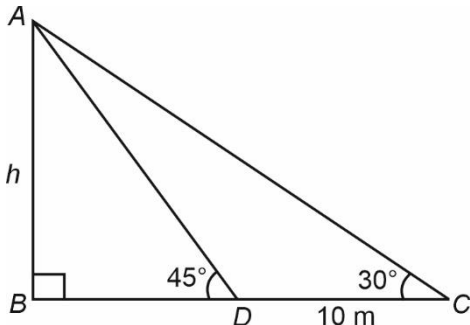
[½]

$$\Rightarrow x = -9 \text{ or } x = 7$$

[½]

34. A vertical tower casts a shadow on level ground. As the sun's angle of elevation decreases from  $45^\circ$  to  $30^\circ$ , the length of the shadow increases by 10 m. Calculate the height of the tower. [Use  $\sqrt{3} = 1.732$ ] [5]

Sol.



Let  $AB$  be height of Tower and  $BD$  be its shadow for  $45^\circ$  angle of elevation. [1/2]

$\therefore DC = 10$  m , Let  $AB = h$  m [1/2]

In  $\triangle ABD$

$$\frac{h}{BD} = \tan 45^\circ \quad [1/2]$$

$$h = BD$$

In  $\triangle ABC$

$$\frac{h}{BD + DC} = \tan 30^\circ \quad [1/2]$$

$$\frac{h}{h + 10} = \frac{1}{\sqrt{3}} \quad [1/2]$$

$$\sqrt{3}h = h + 10$$

$$(\sqrt{3} - 1)h = 10$$

$$h = \frac{10}{\sqrt{3} - 1} \text{ m} = \text{or } \frac{10(\sqrt{3} + 1)}{2} \quad [1/2]$$

$$\text{Or } h = 5(\sqrt{3} + 1) \quad [1/2]$$

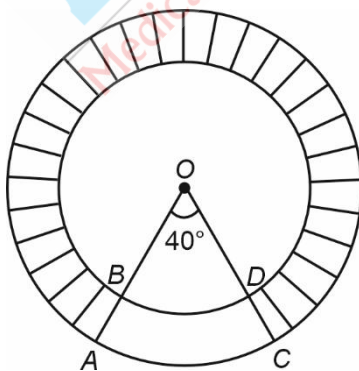
$$h = 5(1.732 + 1) \quad [1/2]$$

$$h = 5 \times 2.732$$

$$h = 13.66 \text{ m} \quad [1/2]$$

$\therefore$  the height of the tower is 13.66 m [1/2]

35. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm, where  $\angle AOC = 40^\circ$ . [5]



**Sol.** Area of a sector

$$= \pi r^2 \times \frac{\theta}{360^\circ} \quad [1/2]$$

Now, shaded area

$$= (\text{area of Major sector with } \theta = 360 - 40^\circ \text{ with radius} = 14 \text{ cm}) - (\text{area of major sector with } \theta = 360^\circ - 40^\circ \text{ and radius} = 7 \text{ cm}) \quad [1/2]$$

∴ Area of Sector with radius = 14 cm

$$= \frac{22}{7} \times 14 \times 14 \times \frac{320^\circ}{360^\circ} \quad [1/2]$$

$$= 22 \times 2 \times 14 \times \frac{8}{9}$$

$$= 547.55 \text{ cm}^2 \quad [1]$$

Area of major sector with radius = 7 cm

$$= \frac{22}{7} \times 7 \times 7 \times \frac{320^\circ}{360^\circ} \quad [1/2]$$

$$= 22 \times 7 \times \frac{8}{9}$$

$$= 136.88 \text{ cm}^2 \quad [1]$$

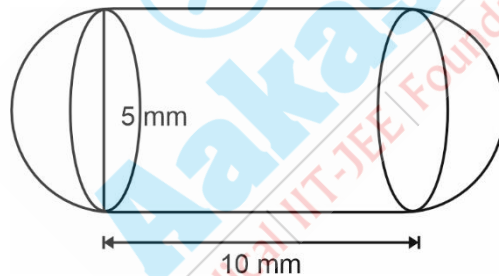
Required area = 547.55 – 136.88

$$= 410.67 \text{ cm}^2 \quad [1]$$

### SECTION-E

Question numbers 36 to 38 are case-based questions of 4 marks each.

36. A hospital stores medicine capsules shaped like cylinders with hemispherical ends. Each capsule has length of the cylindrical part of 10 mm and a diameter of 5 mm.



Based on the above, answer the following questions :

(i) Find the total surface area of one capsule. [1]

(ii) What is the capacity of the capsule? [1]

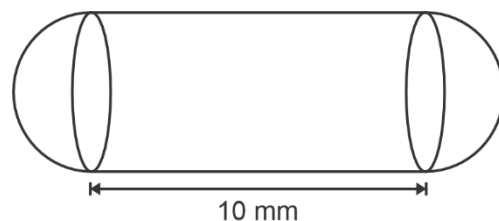
(iii) (a) If the capsule is filled with medicine, which costs ₹ 0.05/mm<sup>3</sup>. calculate the cost of medicine per capsule. [2]

OR

(b) How much extra medicine (in volume) can be stored in the capsule, if the capsule length is doubled? [2]

**Sol.** Here radius =  $r = \frac{5}{2}$  mm

Height of cylindrical part =  $h = 10$  mm



(i) T.S.A. of capsule = C.S.A. of cylindrical part + 2 × CSA of hemisphere

$$= 2\pi rh + 2 \times 2\pi r^2$$

$$= 2\pi rh + 4\pi r^2$$

$$= 2\pi r(h + 2r)$$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \left[ 10 + 2 \times \frac{5}{2} \right] \quad \left[ \frac{1}{2} \right]$$

$$= \frac{110}{7} \times 15$$

$$= \frac{1650}{7}$$

$$= 235.7 \text{ mm}^2 \quad \left[ \frac{1}{2} \right]$$

(ii) Volume (capacity) of capsule = Volume of cylindrical part + Volume of spherical part

$$= \pi r^2 h + \frac{4}{3} \pi r^3$$

$$= \pi r^2 \left[ h + \frac{4}{3} r \right]$$

$$= \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \left[ 10 + \frac{4}{3} \times \frac{5}{2} \right] \quad \left[ \frac{1}{2} \right]$$

$$= \frac{275}{14} \times \frac{40}{3}$$

$$= \frac{275}{7} \times \frac{20}{3} = 261.9 \text{ mm}^3 \quad \left[ \frac{1}{2} \right]$$

(iii) (a) ∴ The capacity of 1 capsule = 261.90 mm<sup>3</sup> [1]

∴ Cost of filled medicine = 261.90 × 0.05

= ₹ 13.09 [1]

**OR**

(iii) (b) If capsule length is doubled then new volume of capsule then the total length of capsule will be 30 mm

∴ Length of cylindrical part = 30 – 2.5 – 2.5 = 25 mm

$$= \pi r^2 H + \frac{4}{3} \pi r^3 \quad \left[ \frac{1}{2} \right]$$

$$= \pi r^2 \left[ H + \frac{4}{3} r \right]$$

$$= \pi r^2 \left[ H + \frac{4}{3} r \right] \quad \left[ \frac{1}{2} \right]$$

$$= \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \left[ 25 + \frac{4}{3} \times \frac{5}{2} \right]$$

$$= \frac{275}{14} \left[ 25 + \frac{10}{3} \right] \quad [1/2]$$

$$= \frac{275}{14} \times \frac{85}{3}$$

$$= 556.54 \text{ mm}^3 \quad [1/2]$$

$$\text{Extra volume} = 556.54 - 261.9 = 294.64 \text{ mm}^3$$

37. A school conducted a lucky draw during its annual function. The draw box contained 100 chits, each with a unique number from 1 to 100. The distribution of the chits is as follows :

- 30 chits had red markings.
- 20 chits were having numbers which are multiples of 5.
- 15 chits had both red markings and numbers which are multiples of 5.
- The rest were plain having no marking and no number.

One chit is drawn at random from the box.

Based on the above, answer the following questions :

- (i) What are the total number of outcomes ? [1]
- (ii) What is the probability that the chit drawn has a red marking only? [1]
- (iii) (a) What is the probability that the chit has a multiple of 5 but not red marking ? [2]

OR

- (iii) (b) What is the probability that the chit drawn is neither of red marking nor a multiple of 5? [2]

**Sol.** (i) Total outcomes = 100 [1]

(ii)  $P(\text{only red marking}) = \frac{30}{100} = \frac{3}{10}$  [1]

(iii) (a) There are 5 chits which here only multiple by 5. [1]

$\therefore$  Required probability =  $\frac{5}{100} = \frac{1}{20}$  [1]

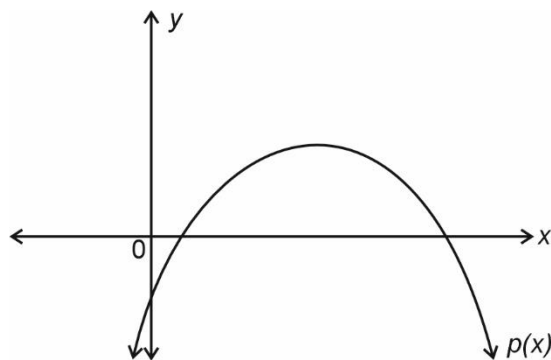
OR

(iii) (b) Chits with red marking or a multiple of 5 = 30 + 5 = 35 [1/2]

$\therefore P(\text{neither red marking nor multiple of 5}) = \frac{100 - 35}{100}$  [1/2]

$$= \frac{65}{100} = \frac{13}{20} \quad [1]$$

38. A parabolic fountain in a park is designed such that its shape follows the polynomial  $p(x) = -x^2 + 6x - 5$ , where  $p(x)$  is the height (in metres) above ground level at a horizontal distance  $x$  metre from the origin.



Based on the above, answer the following questions :

- (i) What are the zeroes of  $p(x)$ ? [1]  
 (ii) What is the height of the fountain at a horizontal distance of 2 metres from origin? [1]  
 (iii) (a) If  $q(x)$  is a reflection of  $p(x)$ , that is  $q(x) = -p(x)$ , write its value as polynomial and draw its figure. [2]

OR

- (iii) (b) At what horizontal distance from the origin does water hit the ground? [2]

**Sol.** (i)  $p(x) = -x^2 + 6x - 5$

$$= -x^2 + 5x + x - 5$$

$$= -x(x - 5) + (x - 5)$$

$$= -(x - 5)(x - 1)$$

$\therefore$  zeroes are 5 and 1

[1]

- (ii) Required height

$$= p(2)$$

$$= -2^2 + 6 \times 2 - 5$$

$$= -4 + 12 - 5$$

$$= 3 \text{ m}$$

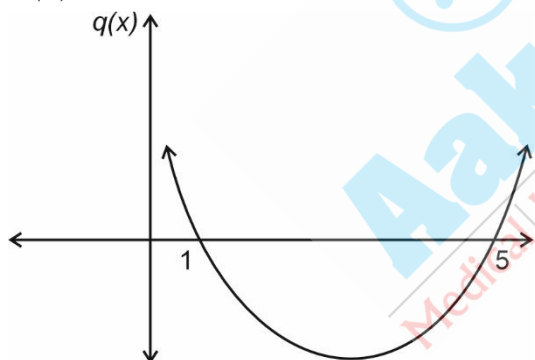
[1]

- (iii) (a)  $q(x) = -p(x)$

$$q(x) = -(-x^2 + 6x - 5)$$

[1]

$$q(x) = x^2 - 6x + 5$$



[1]

OR

- (iii) (b) At the point where water hits the ground, will be the point where graph cuts the  $x$ -axis and away from origin.

$$\therefore -x^2 + 6x - 5 = 0$$

$$\therefore x^2 - 6x + 5 = 0$$

[1]

$$x^2 - 5x - x + 5 = 0$$

$$x(x - 5) - (x - 5) = 0$$

$$(x - 5)(x - 1) = 0$$

$$x = 5 \text{ only is the point}$$

Required horizontal distance = 5 m

[1]

