Date: 03/02/2024

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Answers & Solutions

Time : 3 hrs.

for

Max. Marks : 100

Indian National Astronomy Olympiad (INAO) 2024

(For Class XI & XII Students)

INSTRUCTIONS TO CANDIDATES

(1) There are total **5** questions. All questions are compulsory.

(2) Maximum marks are indicated in front of each sub-question.

(3) For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasonings / assumptions / approximations.

(4) Use of non-programmable scientific calculator is allowed.

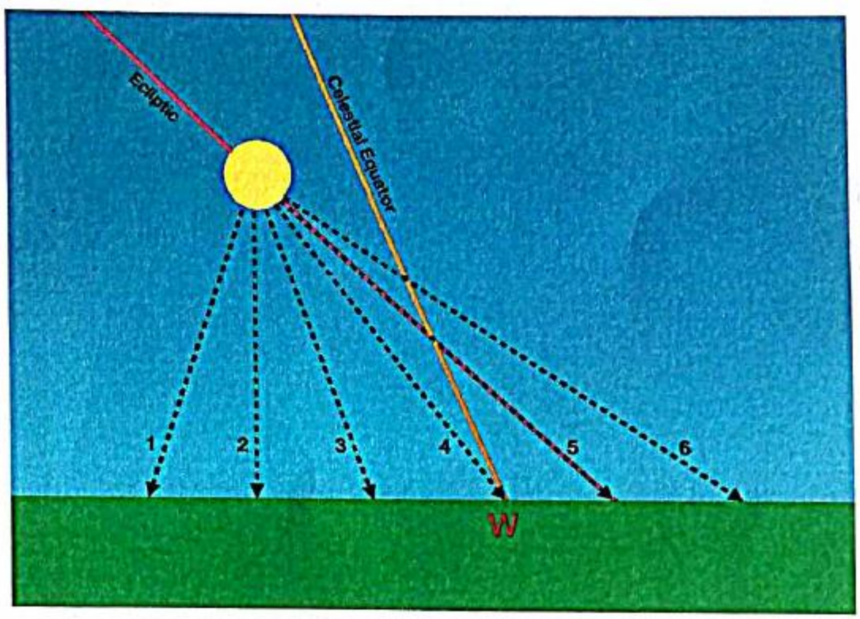
1. Answer each of the following sub-questions with a brief justification-

(a) **(2 marks)** Fatima observed a celestial object, with her unaided eyes, very near to the western horizon immediately after sunset on 17 July 2023. She continued to observe the same object at the same time everyday till 4 September 2023. She made the following observations-

- Object's separation from the Sun increases starting from 17 July.
- Object's separation from the Sun decreases after 11 August.
- She also made an observation that the object was never seen overhead at sunset within that year.

Name one object, which is not a comet, that fits these observations made by Fatima.

(b) **(2 marks)** In the image below, you see the Sun just above the western horizon on some day as seen from the city of Ujjain. The ecliptic, celestial equator and due west point (W) are marked in the image. Which of the paths marked (1 to 6) on the image closely represents the path that Sun will follow on its way to the horizon on that day.



(c) **(2 marks)** Three observers A, B and C are stationed on Moon, Venus and Mars respectively. Which of these observer(s) can see almost all the phases of Earth [new (no earth), crescent, half, gibbous, etc.].

(d) **(4 marks)** A certain star was observed to remain always above horizon over the course of one full day. During this period its maximum altitude was 50° and minimum altitude was 20°. What is/are the latitude/s for the place of observation?

Note: Altitude is the angular distance of a star from the horizon measured along a circular arc perpendicular to the horizon.

(e) **(2 marks)** In which month/s of the year, will there be a New Moon in the constellation of Leo?

A. August B. November C. February D. May

(f) **(4 marks)** Which of the following statement(s) CANNOT be inferred from Kepler's laws of motion?

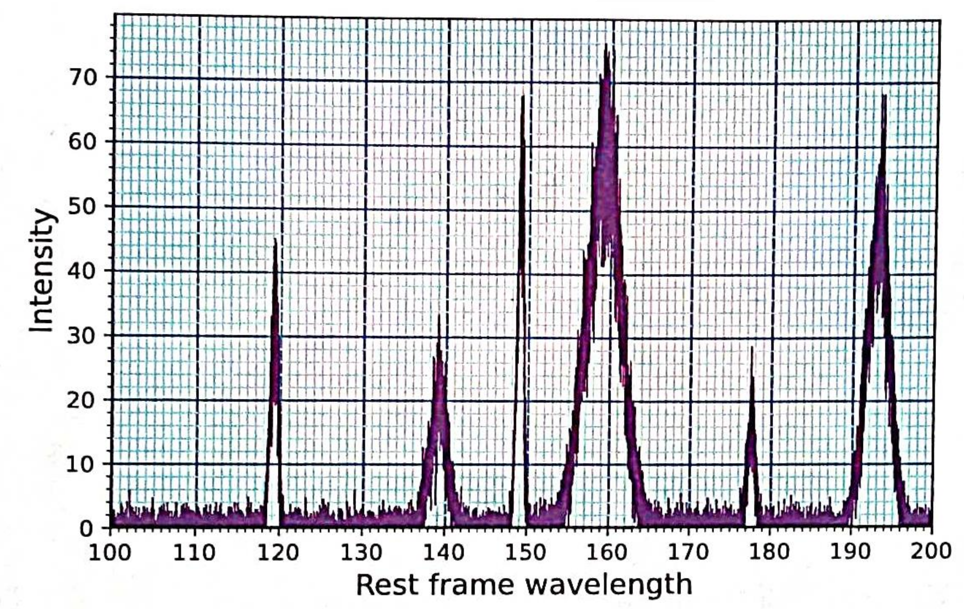
A. A planet moves in an elliptical orbit around the Sun.

B. The eccentricity of the orbits of all solar system planets is small.

C. A solar system planet has its highest tangential velocity when it is closest to the Sun.

D. All the planets move in elliptical orbits in roughly the same plane around the Sun.

(g) **(5 marks)** The presence of different elements in a distant astronomical object is inferred from the peaks in the emission spectrum of its soil, at certain wavelengths, characteristic to each element. One such spectrum for a moon (named *Soma*314 – *b* – 1) around a distant exoplanet observed by the hypothetical mission Chandrayaan-300, that landed on the surface of *Soma*314 – *b* – 1 in the future is shown below. The spectrometer is noisy *i.e.,* small random fluctuations get superimposed on top of the actual signal. The units of both the axes in the figure below are arbitrary.



A complete table of emission peaks (in same arbitrary length unit as in the above figure) of various hypothetical elements is given below. It is assumed that the strength of all the peaks indicated in the table is sufficiently high to be observed in the spectrum shown above if the corresponding element is present in the source.

Identify the elements present on the surface of *Soma*314 – *b* – 1.

Element	Wavelength-1	Wavelength-2	Wavelength-3
Fh	193.44	-	-
Dz	149.18	159.73	-
Ab	111.71	122.87	177.94
Hm	132.67	139.56	-
Cw	119.55	139.32	-
Xy	148.90	159.69	-

Table-1 : Emission lines

Sol. (a) As the object appears at west near Sun set it can be Mercury or Venus. As the motion is retrograde it must be Venus.

(b) The path followed by the Sun on its way to horizon is parallel to the celestial equator.

$\therefore$ Path (3) is the correct answer.

(c) Observer 'A' can see all the phases as from moon all the phases are visible.

From Venus new Earth is not visible from Mars full Earth is not visible completely.

(d) Altitude of place = $\dfrac{\theta_{max} + \theta_{min}}{2}$

$= \dfrac{50^\circ + 20^\circ}{2} = 35^\circ$

(e) New Moon occurs when the moon is between Earth and the Sun.

$\therefore$ Moon is in same zodiac as Sun + Leo, it will happen in August.

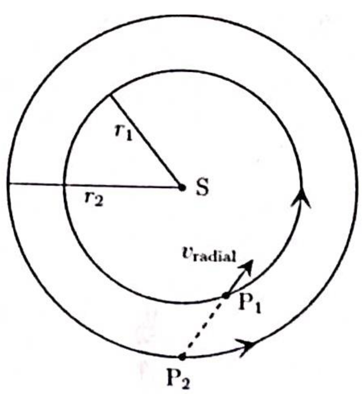
$\therefore$ Answer (A)

(f) Answer (D), theory based.

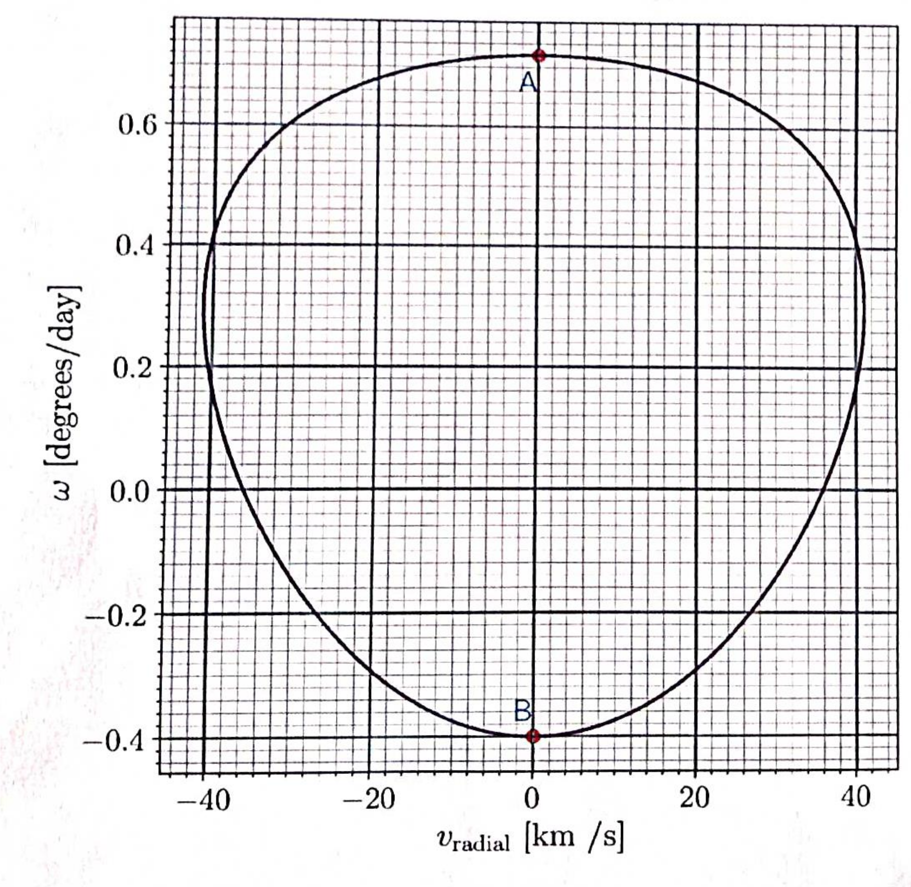
(g) Observing the emission spectrum checking for the non-zero intensity of particular wavelengths.

Element	
Fh	Present
Dz	Present
Ab	Absent
Hm	Absent
Cw	Present
Xy	Absent

2. Consider a system of two planets P_1 and P_2, as shown below, both revolving in the same direction in circular coplanar orbits of radii r_1 and r_2, respectively, around a star S of mass M much larger than the masses of the planets. Anilesh is stranded on the outer plane P_2. Both the star S and the planet P_1 are seen to move against the stellar background by Anilesh due to the orbital motions of P_1 and P_2. Taking the direction of motion of S as positive, he measures the angular velocity ω of P_1 against the stellar background. He also measures the velocity of P_1 along the line of sight (called the "radial velocity", v_{radial} in astronomy) as shown in the figure below.

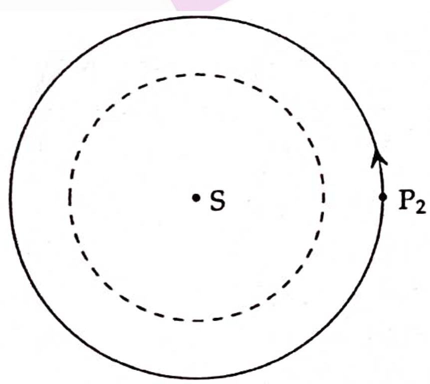


He obtained the following curve for the variation of the ω (in degrees/day, 1 day being 24 hours) versus v_{radial} (in km/s).



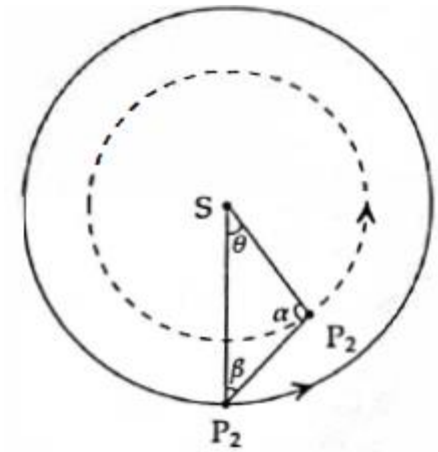
Plot of ω vs v_{radial}

(a) **(3 marks)** For the given position of P_2 as marked in the following figure; show the positions of P_1 corresponding to the points A and B on the graph above by marking appropriately on the orbit of P_1 (dashed circle in the figure in your answersheet).



(b) **(7 marks)** Find the ratio $\frac{r_1}{r_2}$.

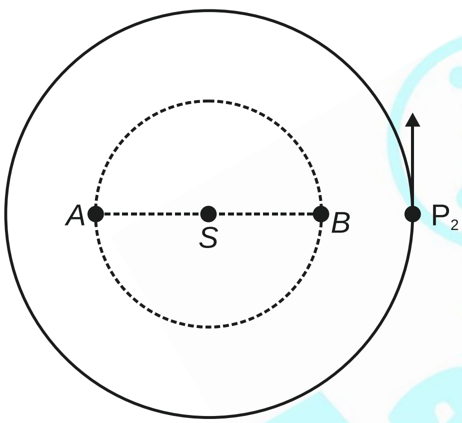
(c) **(8 marks)** Write the expression for $v(r)$ as a function of β (see figure below). Calculate the value of θ for which V_{radial} is maximum.



Hint: You may find it useful to use the sine rule here.

(d) **(8 marks)** Determine M, r_1 and r_2.

Sol. (a)



$\Rightarrow$ When P_1 is at A,

$\omega > 0$, $v_{radial} = 0$

And $\dfrac{v_1+v_2}{r_1+r_2} = (0.6) + \left(\dfrac{0.2}{8}\right) = 0.625$

$\Rightarrow$ When P_1 is at B

$\omega < 0$ also $v_{radial} = 0$

and $\omega = \ |-0.4| = \dfrac{v_1+v_2}{r_2-r_1}$

(b) $\dfrac{5}{8}(r_1+r_2) = \dfrac{c}{\sqrt{r_1}} + \dfrac{c}{\sqrt{r_2}}$

$\dfrac{2}{5}(r_2-r_1) = \dfrac{c}{\sqrt{r_1}} - \dfrac{c}{\sqrt{r_2}}$

$\dfrac{25}{16}\dfrac{(r_1+r_2)}{(r_2-r_1)} = \dfrac{\sqrt{r_1}+\sqrt{r_2}}{\sqrt{r_2}-\sqrt{r_1}}$

$$\frac{25}{16}\left(\frac{\frac{r_1}{r_2}+1}{1-\frac{r_1}{r_2}}\right)=\frac{\frac{\sqrt{r_1}}{\sqrt{r_2}}+1}{1-\frac{\sqrt{r_1}}{\sqrt{r_2}}}$$ $\left[\text{Let } \sqrt{\frac{r_1}{r_2}}=t\right]$

$$25\left(\frac{t^2+1}{1-t^2}\right)=16\left(\frac{t+1}{1-t}\right)$$

$$25(t^2+1)=16(t+1)^2$$

$25t^2 + 25 = 16\ (t^2 + 1 + 2t)$

$25t^2 + 25 = 16t^2 + 16 + 32t$

$9t^2 - 32t - 9 = 0$

$$t=\frac{32\pm\sqrt{32^2+324}}{18}$$

$$\sqrt{\frac{r_1}{r_2}}=0.84$$

$$\boxed{\frac{r_1}{r_2}=0.70}$$

(c) From sine rule

$$\frac{r_1}{\sin\beta}=\frac{r_2}{\sin\alpha}=\frac{x}{\sin\theta}$$

$$\because\quad \frac{r_1}{\sin\beta}=\frac{x}{\sin\theta}$$

$$\sin\theta=\frac{x}{r_1}\sin\beta$$

$$\cos\theta\left[\frac{d\theta}{dt}\right]=\frac{1}{r_1}\left[\frac{dx}{dt}\sin\beta+x\cos\beta\frac{d\beta}{dt}\right]$$

$$\because\quad \frac{dx}{dt}=v_r\quad and\quad \frac{d\theta}{dt}=\omega_2-\omega_1$$

$$\therefore\quad \cos\theta\left(\frac{d\theta}{dt}\right)=\frac{1}{r_1}\left[v_r\sin\beta+x\cos\beta\frac{d\beta}{dt}\right]$$

(d) M: mass of the star.

We have

$v_1 + v_2 = 0.625\ (r_1 + r_2)$

$$\therefore\quad \sqrt{\frac{GM}{r_1}}+\sqrt{\frac{GM}{r_2}}=\frac{5}{8}(r_1+r_2)\qquad \text{...(i)}$$

Also,

$v_1 - v_2 = 0.4\,(r_2 - r_1)$

$$\therefore \quad \sqrt{\frac{GM}{r_1}} - \sqrt{\frac{GM}{r_2}} = \frac{2}{5}(r_2 - r_1) \qquad \ldots\text{(ii)}$$

And

$$\frac{r_1}{r_2} = 0.7 \qquad \ldots\text{(iii)}$$

$$G = 6.67 \times 10^{-11}$$

from (i), (ii) & (iii) we can find the values of M, r_1 and r_2

3. Consider two containers A and B both with the same amount of liquid of volume V. Initially, A only has milk and B only has water. We transfer liquids back and forth between the two containers. One transfer is defined as completion of both the following steps:

- **Step 1:** Take some fixed volume L from container A, put it in B and mix it well.
- **Step 2:** Take the same volume L of the mixed fluid from container B and put it back in A and mix it well.

At the end of each transfer, both containers have liquids with exactly the same volume V. Let $C_M^A(n)$ denote the concentration of milk in container A at the end of the n-th transfer.

Here we define concentration as

$$C_M^A(n) = \frac{\text{Volume of milk in container } A}{\text{Total Volume in container } A} \text{ after } n \text{ complete transfers}$$

Similarly, you may define - C_W^A, C_M^B and C_W^B.

(a) **(4 marks)** Write down the expression for the concentration of milk in container A at the end of the first transfer. Express the concentration $C_M^A(1)$, after the *first* transfer, in terms of volumes L and V.

(b) **(8 marks)** Let us define ε as, $\varepsilon = \dfrac{L}{V}$. Write down the recursion relation for $C_M^A(n)$, in terms of the concentration at the end of the $(n-1)$th transfer *i.e* $C_M^A(n-1)$ and ε.

(c) If one does fairly large number of transfers, then the concentration of milk in either of the container will reach almost equilibrium value which is a function of ε. What will be the value of this equilibrium concentration?

(d) **(5 marks)** Define a new variable f_n such that,

$$f_n = C_M^A(n) - 1/2$$

and simplify the recursion relation between f_{n+1} and f_n to express $C_M^A(n)$ as a function of ε and n.

Sol. (a)

I – transfer

After Step-1

$C_M^A = 1$ $\quad$ $C_M^B = \dfrac{L}{V+L}, C_W^B = \dfrac{V}{V+L}$

$C_W^A = 0$

After Step-2

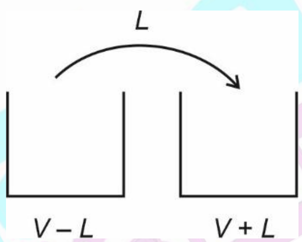
$$C_M^A(1) = \frac{V - L + \dfrac{L}{V+L} \times L}{V} = \frac{V - L + \dfrac{L^2}{V+L}}{V} = \frac{V^2}{V(V+L)} = \frac{V}{V+L}$$

$$C_W^A(1) = 1 - C_M^A = 1 - \frac{V}{V+L} = \frac{L}{V+L}$$

$C_M^B(1) = \dfrac{L}{V+L}$ (Same after in step-1), $C_W^B(1) = \dfrac{V}{L+V}$ (Same after step-1)

$$\boxed{\text{After I transfer : } C_M^A(1) = \frac{V}{V+L}; C_M^B(1) = \frac{L}{V+L}; C_W^A(1) = \frac{L}{V+L}; C_W^B(1) = \frac{V}{L+V}}$$

(b) After (n–1) transfer:



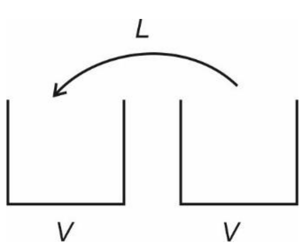
After step -1 of n^{th} transfer:

$$C_M^A = C_M^A(n-1), \ C_W^A = C_W^A(n-1)$$

$$C_M^B = \frac{L \cdot C_M^A(n-1) + V \cdot C_M^B(n-1)}{L+V} = \frac{\varepsilon C_M^A(n-1) + C_M^B(n-1)}{\varepsilon + 1}$$

Now after

Step -2 of n^{th} transfer:



$$C_M^B(n) = \frac{\varepsilon C_M^A(n-1) + C_M^B(n-1)}{\varepsilon + 1} \qquad \text{...(i)}$$

$$C_M^A(n) = \frac{C_M^A(n-1)\cdot(V-L) + C_M^B(n)(L)}{V}$$

$$= \frac{C_M^A(n-1)(V-L) + \left(\frac{\varepsilon C_M^A(n-1) + C_M^B(n-1)}{\varepsilon+1}\right)^L}{V} \qquad \text{from (i)}$$

$$= C_M^A(n-1)(1-\varepsilon) + \varepsilon\left(\frac{\varepsilon C_M^A(n-1) + C_M^B(n-1)}{\varepsilon+1}\right)$$

as, $V \cdot C_M^A(n-1) + VC_M^B(n-1) = V$

$$\Rightarrow C_M^B(n-1) = 1 - C_M^A(n-1)$$

$$\therefore C_M^A(n) = (1-\varepsilon)C_M^A(n-1) + \frac{\varepsilon}{\varepsilon+1}(\varepsilon C_M^A(n-1) + 1 - C_M^A(n-1))$$

$$= (1-\varepsilon)C_M^A(n-1) + \frac{\varepsilon}{\varepsilon+1}(C_M^A(n-1)(\varepsilon-1)+1)$$

$$= C_M^A(n-1)\left(1-\varepsilon + \frac{\varepsilon\cdot(\varepsilon-1)}{\varepsilon+1}\right) + \frac{\varepsilon}{\varepsilon+1}$$

$$= C_M^A(n-1)\left(\frac{1-\varepsilon^2+\varepsilon^2-\varepsilon}{\varepsilon+1}\right) + \frac{\varepsilon}{\varepsilon+1}$$

$$C_M^A(n) = C_M^A(n-1)\left(\frac{1-\varepsilon}{1+\varepsilon}\right) + \frac{\varepsilon}{\varepsilon+1}$$

(c) As $n \to \infty$

Let K = equilibrium concentration of milk in container A

$$\therefore C_M^A(n) = C_M^A(n-1) = K \ (let)$$

$$K = K\left(\frac{1-\varepsilon}{1+\varepsilon}\right) + \frac{\varepsilon}{1+\varepsilon}$$

$$\Rightarrow K\left(1 - \frac{1-\varepsilon}{1+\varepsilon}\right) = \frac{\varepsilon}{1+\varepsilon}$$

$$= K\left(\frac{2\varepsilon}{1+\varepsilon}\right) = \frac{\varepsilon}{1+\varepsilon}$$

$$\Rightarrow K = \frac{1}{2}$$

(d) $C_M^A(n+1) = f_{n+1} + \dfrac{1}{2}$

$$C_M^A(n)\left(\frac{1-\varepsilon}{1+\varepsilon}\right) + \frac{\varepsilon}{1+\varepsilon} = f_{n+1} = \frac{1}{2}$$

$$\Rightarrow \left(f_n + \frac{1}{2}\right)\left(\frac{1-\varepsilon}{1+\varepsilon}\right) + \frac{\varepsilon}{1+\varepsilon} = f_{n+1} + \frac{1}{2}$$

$$\Rightarrow f_{n+1} = \left(f_n + \frac{1}{2}\right)\left(\frac{1-\varepsilon}{1+\varepsilon}\right) + \frac{\varepsilon}{1+\varepsilon} - \frac{1}{2}$$

$$\Rightarrow f_{n+1} = f_n\left(\frac{1-\varepsilon}{1+\varepsilon}\right) \qquad \text{.... (after simplifying)}$$

Solving recursion relation

$$x^2 = x\left(\frac{1-\varepsilon}{1+\varepsilon}\right)$$

$$\Rightarrow x = \frac{1-\varepsilon}{1+\varepsilon} \qquad \text{(only one root)}$$

$$\therefore f_{(n+1)} = A\cdot\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{n+1} \qquad \text{(where } A \text{ is a constant)}$$

$n = 0$

$$f_1 = A\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{\prime}$$

$$C_M^A(1) - \frac{1}{2} = A\left(\frac{1-\varepsilon}{1+\varepsilon}\right) \qquad \left(C_M^A(1) = \frac{V}{V+L}\right)$$

$$= \frac{V}{V+L} = A\left(\frac{1-\varepsilon}{1+\varepsilon}\right) + \frac{1}{2}$$

$$= \frac{1}{1+\varepsilon} = A\left(\frac{1-\varepsilon}{1+\varepsilon}\right) + \frac{1}{2} \Rightarrow A = \frac{1}{2}$$

$$\therefore f_{(n+1)} = \frac{1}{2}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{n+1}$$

$$\therefore f_n = \frac{1}{2}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{n}$$

$$C_M^A(n) - \frac{1}{2} = \frac{1}{2}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{n}$$

$$\Rightarrow \boxed{C_M^A(n) = \left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{n}\cdot\frac{1}{2} + \frac{1}{2}}$$

4. The difference in magnitudes m_1 and m_2 of two celestial objects with flux (energy received from the object per unit area per unit time) F_1 and F_2, respectively, is given by Pogson's relation as

$$\Delta m = m_1 - m_2 = -2.5\log_{10}\left(\frac{F_1}{F_2}\right)$$

(a) **(3 marks)** Calculate the total energy per unit time received at the primary mirror of diameter 6 inches (1 inch = 2.54 cm) of a telescope from the star Sirius (magnitude m_{sirius} = –1.46). The flux of the star Vega, which has a magnitude m_{vega} = +0.03, is equal to 2.19×10^{-8} W/m^2.

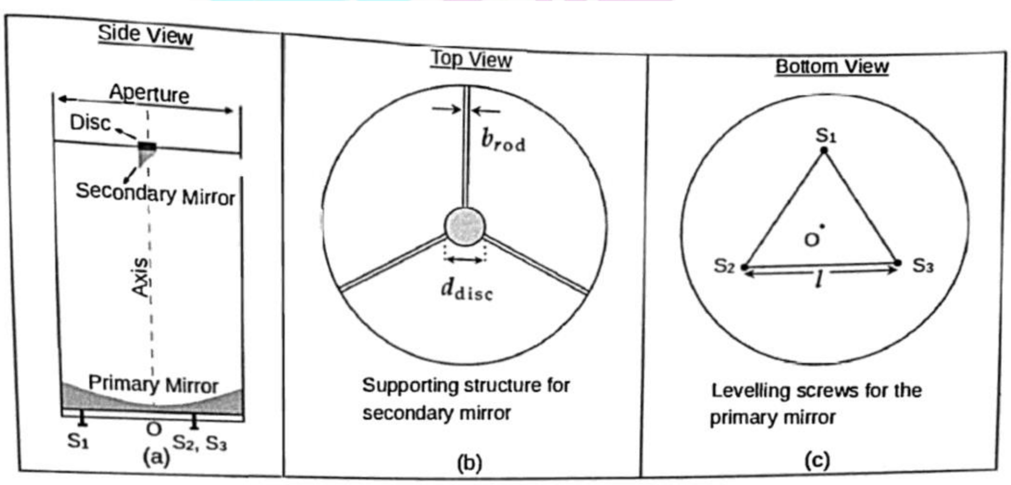
Consider a Newtonian reflecting telescope ((a) side, (b) top, (c) bottom views shown in the figure below) with a concave primary mirror of diameter 6 inches. A secondary mirror, mounted on a flat disc of diameter d_{disc} = 3.7 cm, is held in place on the axis of the telescope tube by three thin rods of thickness b_{rod} = 0.3 cm, as shown in the figure (b) below.

(b) **(3 marks)** Calculate the percentage reduction in flux at the primary mirror due to the supporting structure of the secondary mirror.

The optic axis of the primary mirror and the axis of the telescope tube are expected to be coincident. But sometimes, they may get misaligned due to mishandling or some other issues.

(c) **(3 marks)** Suppose that due to a misalignment, the angle between the two axes is θ = 5°. Calculate the percentage change in the flux received at the primary due to this fault. For this part ignore the effects of the secondary mirror and its supporting structure. Assume that the diameter of the primary mirror is very close to that of the telescope tube.

The alignment of the axis of the primary mirror is done by three levelling screws, S_1, S_2 and S_3 at the bottom of the telescope. As seen in the bottom view (figure (c)), these screws form an equilateral triangle of side l = 3 inches. The optic axis of the primary mirror is perpendicular to the plane of the paper in figure (c) and passes through the centroid O of this triangle.



(d) **(4 marks)** Suppose that the 5° tilt of the axis of the mirror described above has happened in the plane containing the axis of the tube and the line OS_1 (see figure (c)). Therefore, it should be possible to correct the tilt and realign the axes by moving the screw S_1 alone, without disturbing S_2 or S_3. If the pitch of the screw S_1 is 1.15 mm, how many turns of this screw are needed to bring the mirror back in alignment?

The maximum brightness of a source that can be observed with a telescope of a given aperture is limited by the tolerance of the human eye to bright light. The brightest object that can be observed safely with our telescope (primary mirror diameter of 6 inches) is of magnitude –10.50. To observe any source brighter than this limit the amount of light that reaches the primary mirror must be restricted by reducing the effective aperture of the telescope. This is accomplished by covering the top of the telescope with a lid that has a smaller hole (aperture) of appropriate size.

(e) **(4 marks)** Calculate the size of this smaller circular aperture on the lid so that a supernova of magnitude –12.525 can be safely observed. Once again, ignore the secondary mirror and its supporting structure.

Sol. (a) $m_1 = m_{\text{sirius}} = -1.46$

$m_2 = m_{\text{vega}} = +0.03$

Putting the values

$$\Rightarrow -1.46 - 0.03 = -2.5\log_{10}\left(\frac{F_1}{F_2}\right)$$

$$\Rightarrow \log_{10}\left(\frac{F_1}{F_2}\right) = \frac{1.49}{2.5} \simeq 0.6$$

$$\Rightarrow F_1 \simeq 2.19 \times 10^{-8}(10^{0.6})\ \text{W/m}^2$$

$\Rightarrow$ Energy received per unit time

$= F_1 \times$ Area

$$= 2.19 \times 10^{-7.4} \times \pi\left(\frac{3 \times 2.54}{100}\right)^2$$

$$\simeq 3.992 \times 10^{-9.4}\ \text{W}$$

$$\simeq \boxed{2.25 \times 10^{-9}\ \text{W}}$$

(b) Affected area $= \pi\left(\frac{d_{\text{disc}}}{2}\right)^2 + \left[R - \frac{d_{\text{disc}}}{2}\right] \times b_{\text{rod}} \times 3$

$$= \frac{\pi}{4}[3.7]^2 + [3 \times 2.54 - 1.85] \times 0.3 \times 3\ \text{cm}^2$$

$$\simeq 15.93\ \text{cm}^2$$

$$\Rightarrow \%\ \text{Reduction} = \frac{15.93}{\pi(3 \times 2.54)^2} \times 100$$

$$\simeq \boxed{8.73\%}$$

(c) When the axes do not coincide, the area would reduce.

Now the angle between the axes is 5°.

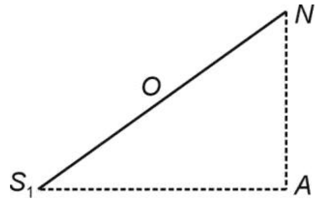
$\Rightarrow$ New radius $r' = r\cos(5°)$

$$\Rightarrow \%\ \text{change} = \frac{\pi r^2 - \pi(r')^2}{\pi r^2} \times 100$$

$= (1 - \cos^2(5°)) \times 100 \simeq 0.75\%$

(d) The movement of screw S_1 should be enough so that OS_1 again comes in the plane of the triangle.

For this, we take help of the following diagram:



From the above figure, change in height = $S_1M\sin(5°)$

Also, S_1M is height of equilateral triangle

$$\Rightarrow \quad \Delta H = \frac{3\sqrt{3}}{2} \times \sin(5°) \times 2.54 \text{ cm}$$

$$= 0.575 \text{ cm}$$

$$\Rightarrow \quad \text{Number of turns} = \frac{0.575 \text{ cm}}{0.115 \text{ cm}} = 5 \text{ turns.}$$

(e) Now, from the given equation, we put the appropriate values to get the new radius.

$$\Rightarrow \quad -12.525 + 10.5 = -2.5 \log \frac{\pi r_{new}^2}{\pi r^2}$$

$$\Rightarrow \quad r_{new} = \frac{r}{2.54}$$

$\Rightarrow$ r_{new} = 3 cm

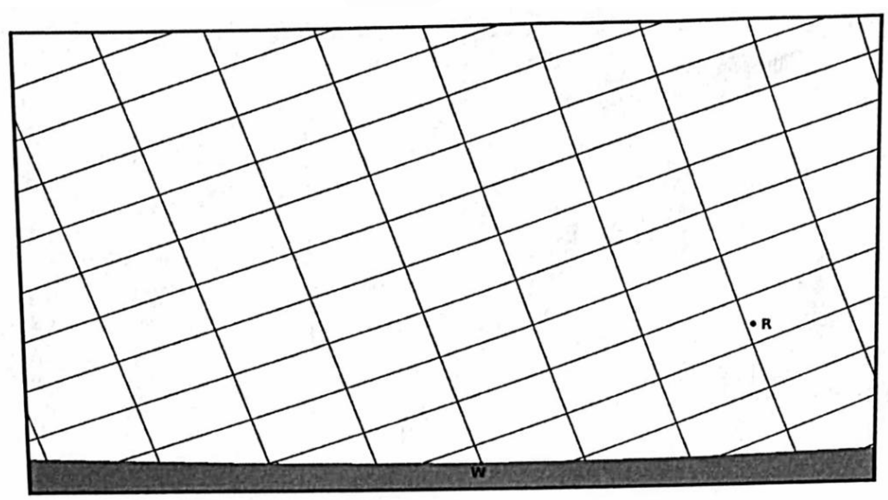
$\Rightarrow$ Size of the new aperture (diameter) would be 6 cm.

5. The image below shows a portion of the sky near the western horizon as observed from some place in the northern hemisphere. The horizon is shown as a green strip at the bottom of the image, and the letter 'W' marks the West cardinal point. The near-rectangular tilted grid on the sky is that of the RA and Dec coordinates. Each small rectangle corresponds to 10^m of RA and 5° of declination. A star, designated by the letter 'R', with its celestial coordinates (RA: 23^h 04^m & Dec: +15°12') is shown in the image. Same image is provided in your answersheet to mark/write your answers. Answer the following questions with the information given above.

(a) **(2 marks)** Identify and mark the Celestial Equator on the image provided in the answersheet.

(b) **(3 marks)** Determine the approximate geographic latitude of the location.

(c) **(4 marks)** The table below contains RA and Dec of four stars. Mark these stars on the image in your answersheet.



Sr. No.	Star name	Right Ascension	Declination
A	70 Peg	$23^h\ 29^m$	+12°45′
B	β Psc	$23^h\ 4^m$	+3°49′
C	ι Cet	$0^h\ 19^m$	–8°49′
D	71 Aqr	$23^h\ 16^m$	–9°5′

(d) **(2 marks)** Identify the star(s) from the table that will NOT be observable to an observer located at the South Pole. Give reason in support of your answers.

(e) **(3 marks)** At the specified date and time, the coordinates of the (centre of the) Sun are RA: $23^h\ 54^m$ and Dec: –0°36′. On the image in your answersheet, accurately draw a disc of appropriate size representing the Sun at these coordinates.

(f) **(2 marks)** At the given location, if the star B in the table above sets at 18:01 hrs local time, determine the approximate local time at which another star, identified by coordinates RA: $23^h\ 17^m$ and Dec: 3°49′, will set.

Notes: Celestial Coordinate System

The *Celestial Sphere* is a representation of the sky as a huge imaginary sphere, with its centre at the centre of the Earth, on which all the celestial objects can be seen. The *Celestial Equator* is the circle marking the intersection of the Earth's equatorial plane with the celestial sphere. Thus, for an observer standing on the Earth's equator, the Celestial Equator will be a circle passing through the cardinal points East and West, and the Zenith (overhead point). The North and South *Celestial Poles* are the intersection points of the rotational axis of the Earth with the *Celestial Sphere*.

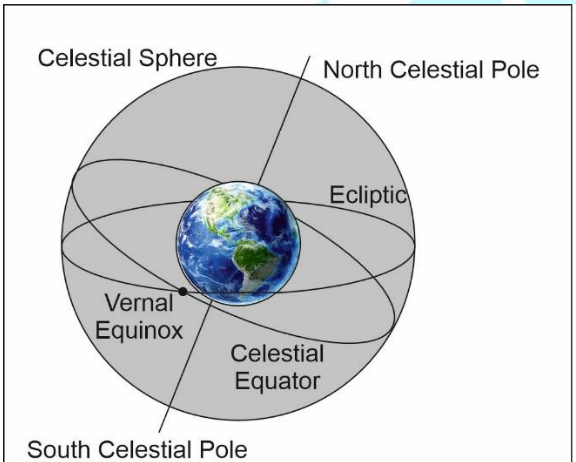


Diagram depicting the celestial sphere with Earth at its center.

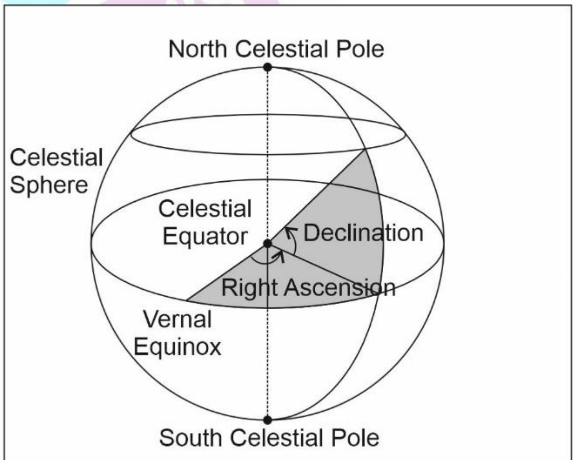


Diagram depicting the celestial sphere with Earth at its center,

With the equator and the poles thus defined, we can now define two celestial coordinates to describe the position of any celestial object, on the *Celestial Sphere*, namely *Right Ascension* (*RA*) and *Declination* (*Dec*). These celestial coordinates are analogous to the longitude and latitude on the Earth respectively.

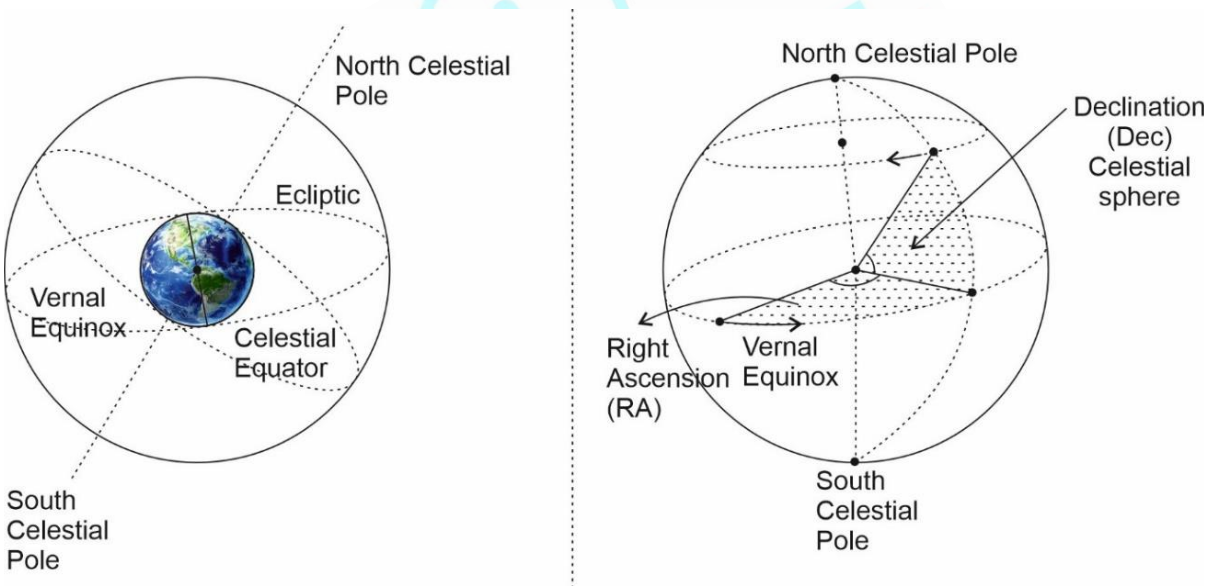
Right Ascension (*RA*) is the celestial analogue of the terrestrial longitude. The zero of RA passes through the Vernal Equinox, which marks the point where the Sun crosses the *Celestial Equator* into the northern part of the sky around 21 March every year. The corresponding terrestrial equivalent is the prime meridian, i.e., 0° longitude, which passes through Greenwich, UK.

As we trace celestial equator in our sky from west to east, the *RA* value keeps increasing. Although *RA* can be expressed in degrees (like longitude), it is customary to express it in hours, minutes, and seconds of time – from $00^h\ 00^m\ 00^s$ to $24^h\ 00^m\ 00^s$ – thus "fixing" the *RA* grid to the sky, as the sky appears to rotate due to the diurnal rotation of the Earth around its axis. The entire celestial sphere completes a full 360° rotation in approximately 24 hours, equating to about 15° per hour. Therefore, in one hour, the sky rotates by about one hour of RA. (Conversion: $1^h = 60^m = 3600^s$).

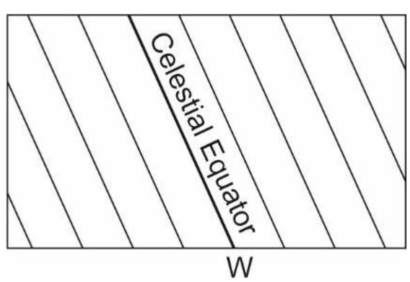
Declination (*Dec*) functions as the celestial counterpart of the terrestrial latitude, measured in degrees, arcminutes and arcseconds. Similar to latitude, positive and negative values indicate positions north and south of the *Celestial Equator*, respectively. The *Celestial Equator* has a declination of 0°, while the North and the South celestial poles have declinations of +90° and –90°, respectively. (Conversion: $1° = 60' = 3600''$).

Using these "equatorial coordinates", we can define the position of any object in the sky in much the same way as we use longitude and latitude to describe the location of any place on the Earth.

Sol. Star on celestial coordinate [RA and Dec] on the celestial sphere



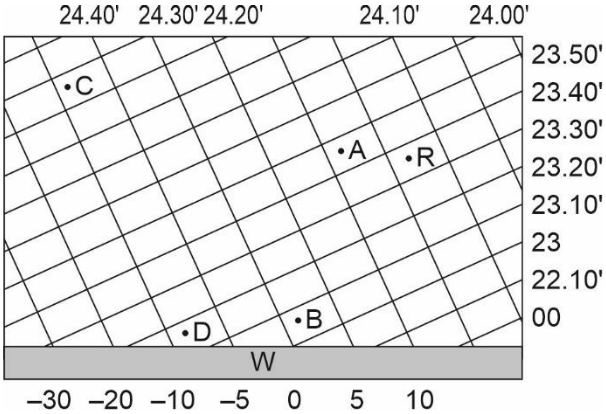
(a) The declination circle passing through west cardinal point (W) represent the celestial equator.



(b) The angle between Right Ascension (RA) and horizon gives approximate latitude of any place.

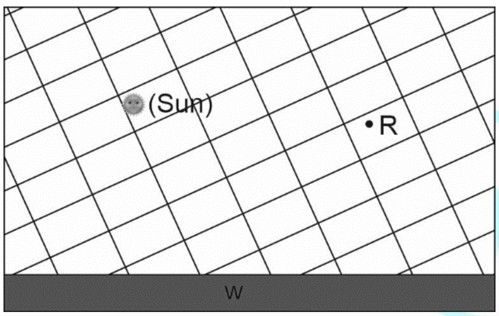
From give data, latitude of place is approximately equal to 25°·30′′.

(c)



(d) All the stars with positive declination (here A and B) are not visible from south pole as these star lie beyond equator and equator lie on the horizon as seen from south pole.

(e)



The angular size of sun cover 0.5° Dec and 2^m of RA. Its radius is on answer sheet will be 1.5 cm.

(f) The star with coordinate RA $23^h\ 17^m$ and Dec 3°49′ set at 18:14 hr

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