MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. 3, 8, 13, ……, 373 are in arithmetic series. The sum of numbers not divisible by three is

(1) 9310 (2) 8340

(3) 9525 (4) 7325

Answer (3)

Sol. $3 + 8 + 13 + 18 + \ldots.\ 373 = \frac{75}{2}[3+373] = 14100$

Now, $\underbrace{3+18+\ldots\ldots}_{25\text{ terms}} = \frac{25}{2}[6+24.15] = 4575$

$\therefore$ Required sum = 14100 – 4575

= 9525

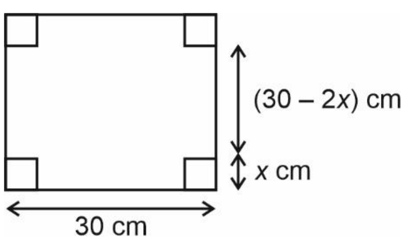
2. From a square of side 30 cm the squares of side x cm is cut off to make a cuboid of maximum volume. The surface area of cuboid with open top is

(1) 400 cm^2 (2) 464 cm^2

(3) 800 cm^2 (4) 900 cm^2

Answer (3)

Sol.



Volume of cuboid = $(30 - 2x)^2 \cdot x = V(x)$

$$\frac{dV}{dx} = (30-2x)^2 + 2x(30-2x)(-2) = 0$$

$\Rightarrow$ $(30 - 2x)(30 - 2x - 4x) = 0$

$\Rightarrow$ $x = 5, x = 15$ (not possible)

$\therefore$ Surface area = $(30 - 2x)(x) \times 4 + (30 - 2x)^2$

$= 20 \times 5 \times 4 + (20)^2$

= 800 cm^2

3. The negation of the statement $(p \vee q) \wedge \sim r$ is

(1) $(\sim p \wedge \sim q) \vee r$ (2) $(\sim p \wedge \sim q) \wedge r$

(3) $(\sim p \vee q) \vee r$ (4) $(p \vee \sim q) \wedge r$

Answer (1)

Sol. $\sim\left[(p \vee q) \wedge \sim r\right]$

$\therefore$ $\sim (p \vee q) \vee r$

$(\sim p \wedge \sim q) \vee r$

4. Slope of tangent to a curve at a variable point is $\frac{x^2+y^2}{2xy}$ and $y(2) = 0$, then $y(8) = 0$

(1) $\sqrt{3}$ (2) $2\sqrt{2}$

(3) $4\sqrt{3}$ (4) 6

Answer (3)

Sol. $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

$y = vx$ (let)

$y' = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(v+\frac{1}{v}\right) \Rightarrow x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v}-v\right) = \frac{1}{2}\left(\frac{1-v^2}{v}\right)$$

$$\therefore \int\frac{2v}{1-v^2}dv = \int\frac{dx}{x} \Rightarrow -\log\left|1-v^2\right| = \ln|x| + \ln c$$

$$\Rightarrow k = x\cdot\left(1-\frac{y^2}{x^2}\right) \Rightarrow k = \frac{x^2-y^2}{x}$$

$y(2) = 0$

$k = 2$

$$\Rightarrow 2 = \frac{x^2-y^2}{x}$$

$x = 8$

$$2 = \frac{64-y^2}{8} \Rightarrow y^2 = 64-16 = 48 = 4\sqrt{3}$$

5. Using the number 1, 2, 3 … 7, total numbers of 7 digit number which does not contain string 154 or 2367 is (Repetition is not allowed)

(1) 4897 (2) 4898

(3) 4896 (4) 4899

Answer (2)

Sol. Total numbers – when 154 comes as a n string – when 2367 comes as + 2 a string

7! – 5! – 4! + 2

5040 – 120 – 24 + 2

= 4898

6. If the order of matrix A is 3 × 3 and $|A| = 2$, then the value of $|3\text{adj}(|3A|A^2)|$ is

(1) $3^{10} \cdot 2^{21}$ (2) $2^{10} \cdot 3^{21}$

(3) $2^{12} \cdot 3^{15}$ (4) $3^{12} \cdot 2^{15}$

Answer (2)

Sol. $|3A| = 3^3 \cdot |A| = 2 \cdot 3^3$

$\text{adj}(|3A|A^2) = \text{adj}(2 \cdot 3^3 \cdot A^2) = (2 \cdot 3^3)^2 (\text{adj}A)^2$

$= 2^2 \cdot 3^6 (\text{adj}A)^2$

$|3\text{adj}(|3A|A^2)| = |2^2 \cdot 3^7 (\text{adj}A)^2|$

$= (2^2 \cdot 3^7)^3 \cdot |\text{adj}A|^2$

$= 2^6 \cdot 3^{21} \cdot (|A|^2)^2$

$= 2^6 \cdot 3^{21} \cdot 2^4 = 2^{10} \cdot 3^{21}$

7. Find the value of

$$96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\ldots\cos\left(\frac{16\pi}{33}\right)$$

(1) 0 (2) 1

(3) 2 (4) 3

Answer (4)

Sol. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\ldots\cos\left(\frac{16\pi}{33}\right)$

$$\frac{96.\sin\left(2^5\frac{\pi}{33}\right)}{2^5\sin\left(\frac{\pi}{33}\right)} = \frac{96}{32}.\frac{\sin\left(\frac{32\pi}{33}\right)}{\sin\left(\frac{\pi}{33}\right)} = 3$$

8. The coefficient of x^7 in $(1 - 2x + x^3)^{10}$ is

(1) 5140

(2) 2080

(3) 4080

(4) 6234

Answer (3)

Sol. $(1 - 2x + x^3)^{10}$

$$T_n = \frac{10!}{a!b!c!}(-2x)^b\left(x^3\right)^c = \frac{10!}{a!b!c!}(-2)^b \cdot x^{b+3c}$$

$b + 3c = 7$, $a + b + c = 10$

a	b	c
3	7	0
5	4	1
7	1	2

∴ Coefficient of

$$x^7 = \frac{10!}{3!7!0!}\times(-2)^7 + \frac{10!}{5!4!1!}\times(-2)^4 + \frac{10!}{7!1!2!}\times(-2)^1$$

= 120 × (–128) + 20160 + (–720)

= –15360 + 20160 – 720

= 4080

9. 9. Find the number of integral values of x which satisfy the inequality $x^2 - 10x + 19 < 6$.

(1) 5 (2) 11

(3) 7 (4) 8

Answer (3)

Sol. $x^2 - 10x + 13 < 0$

$\alpha < x < \beta$ where $\alpha, \beta = \frac{10 \pm \sqrt{48}}{2}$

i.e., $\alpha = 5 - 2\sqrt{3}$

and $\beta = 5 + 2\sqrt{3}$

$\Rightarrow$ $1.636 < x < 8.464$

$x = 2, 3, 4, 5, 6, 7, 8$

10. Shortest distance between lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-3}{1}$ is

(1) $\sqrt{29}$ (2) $2\sqrt{29}$

(3) $3\sqrt{29}$ (4) $4\sqrt{29}$

Answer (2)

Sol. $\vec{a} = \langle -1, -1, -1\rangle$ $\quad x_1 = 7\hat{i} - 6\hat{j} + \hat{k}$

$\vec{b} = \langle 3, 5, 7\rangle$ $\quad x_2 = \hat{i} - 2\hat{j} + \hat{k}$

$$d = \left|\frac{(\vec{a} - \vec{b})\cdot(\vec{x}_1 \times \vec{x}_2)}{|\vec{x}_1 \times \vec{x}_2|}\right|$$

$$\left|\frac{(4\hat{i} + 6\hat{j} + 8\hat{k})\cdot(4\hat{i} + 6\hat{j} + 8\hat{k})}{\sqrt{4^2 + 6^2 + 8^2}}\right|$$

$$d = \left|\frac{16 + 36 + 64}{\sqrt{16 + 36 + 64}}\right| = \sqrt{116}$$

11. If $a^2 + (ar)^2 + (ar^2)^2 = 33033$, $(a, r \in N)$, then the value of $a + ar + ar^2$ is

(1) 148

(2) 249

(3) 230

(4) 231

Answer (4)

Sol. $a^2(1 + r^2 + r^4) = 33033$, $a, r \in N$

$\Rightarrow a = 11$

$r = 4$

sum $= a + ar + ar^2$

$= 11 + 44 + 176$

$= 231$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If the coefficient of x^7 in expansion of $\left(ax - \frac{1}{bx^2}\right)^{13}$ is equal to coefficient of x^{-5} in expansion of $\left(ax + \frac{1}{bx^2}\right)^{13}$ then a^4b^4 is

Answer (22)

Sol. Coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$

$$T_{r+1} = {}^{13}C_r\,(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$$

$13 - 3r = 7$

$\Rightarrow r = 2$

Coeff = ${}^{13}C_2\,\frac{a^{11}}{b^2}$

Coeff of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$

$$T_{r+1} = {}^{13}C_r\,(ax)^{13-r}\left(\frac{1}{bx^2}\right)^r$$

$13 - 3r = -5$

$\Rightarrow r = 6$

Coeff = ${}^{13}C_6\,\frac{a^7}{b^6}$

Now,

$${}^{13}C_2\,\frac{a^{11}}{b^2} = {}^{13}C_6\,\frac{a^7}{b^6}$$

$$a^4b^4 = \frac{{}^{13}C_6}{{}^{13}C_2} = 22$$

22. Two dice are rolled and sum of numbers of two dice is N then probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are co-prime, then $11m - 3n$ is

Answer (85)

Sol. $\because$ $2^N < N!$ is true when $N \geq 24$

$\therefore$ When $N = 1$ (not possible)

$N = 2$, (1, 1)

$N = 3$ (1, 2) (2, 1)

$\therefore$ required probability $= \frac{36-3}{36} = \frac{33}{36}$

$= \frac{11}{12}$

$\therefore$ $m = 11$, $n = 12$

$\therefore$ $11m - 3n = 121 - 36$

$= 85$

23. If the number of ways in which a mixed double badminton can be played such that no couples played into a same game is 840. Then find the number of players

Answer (16)

Sol. Let total number of couples be n

then according to given condition

$${}^{n}C_2 \cdot {}^{n-2}C_2 \times 2 = 840$$

$\Rightarrow \quad n = 8$

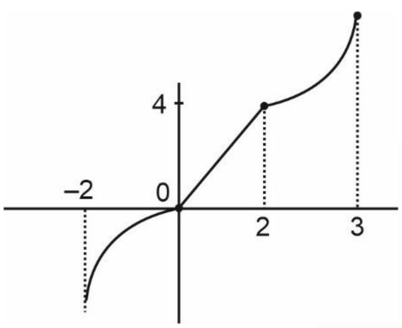
$\therefore \quad$ Total players = 8 × 2 = 16

24. Find number of points of non-differentiability for $f(x)$

$$f(x) = \begin{cases} x\,|x| & -2 < x \le 0 \\ |x-3| + |x+1| - 2|x-2| & 0 < x \le 2 \\ |x|\,(x^2 - x) & 2 < x \le 3 \end{cases}$$

Answer (2)

Sol.



Points of non-differentiability = 0, 2

25. Let f be a differentiable function

$$x^2 f(x) - x = 4\int_0^x t f(t)\,dt$$

If $f(1) = \frac{2}{3}$ then $18f(3)$ is

Answer (160)

Sol. $x^2 f(x) - x = 4\int_0^x t f(t)\,dt$

$2xf(x) + x^2 f'(x) - 1 = 4xf(x)$

$$x^2 \frac{dy}{dx} - 2xy = 1$$

$$\frac{dy}{dx} - \frac{2y}{x} = \frac{1}{x^2}$$

$$\text{I.F.} = e^{\int \frac{-2}{x}dx} = e^{-2\ln x} = \frac{1}{x^2}$$

$$\frac{y}{x^2} = \int \frac{1}{x^4}dx$$

$$\frac{y}{x^2} = \frac{-1}{3x^3} + c$$

Now, $y(1) = \frac{2}{3}$

$$\frac{2}{3} = -\frac{1}{3} + c$$

$\Rightarrow \quad c = 1$

$$\therefore \quad y = -\frac{1}{3x} + x^2$$

$$18f(3) = 18\left[-\frac{1}{9} + 9\right]$$

= –2 + 162

= 160

26. The mean of the data

0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
5	2	5	x	6

is 26, then variance of the data is

Answer (815)

Sol. $\bar{x} = \dfrac{25 + 30 + 125 + 35x + 270}{18 + x} = 26$

$\Rightarrow \quad \boxed{x = 2}$

$$\text{Variance} = \frac{5 \times 3^2 + 2 \times 13^2 + 5 \times 23^2 + 2 \times 33^2 + 6 \times 43^2}{20}$$

$$= \frac{45 + 338 + 2645 + 2178 + 11094}{20}$$

= 815

27.

28.

29.

30.

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