

MATHEMATICS

SECTION - A

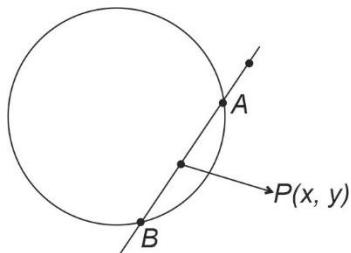
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let a circle $x^2 + y^2 = 16$ and line passing through $(1, 2)$ cuts the curve at A and B then the locus of the mid-point of AB is
 - (1) $x^2 + y^2 + x + y = 0$
 - (2) $x^2 + y^2 - x + 2y = 0$
 - (3) $x^2 + y^2 - x - 2y = 0$
 - (4) $x^2 + y^2 + x + 2y = 0$

Answer (3)

Sol.



Let $P(x_1, y_1)$ be the mid-point of AB

Then $T = S_1$

$$x_1^2 + y_1^2 - 16 = xx_1 + yy_1 - 16$$

$$\Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2 \quad \dots(i)$$

\therefore (i) passes through $(1, 2)$

$$\therefore x_1 + 2y_1 = x_1^2 + y_1^2$$

\therefore required locus

$$x^2 + y^2 - x - 2y = 0$$

2. Consider $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$, domain of $f(x)$ is $[\alpha, \beta] \cup [\gamma, \delta]$, then the value of $|3\alpha + 10\beta + 5\gamma + 21\delta|$ is
 - (1) 22
 - (2) 23
 - (3) 21
 - (4) 19

Answer (3)

Sol. $\frac{2x}{5x+3} \geq 1$ OR $\frac{2x}{5x+3} \leq -1$

$$\Rightarrow \frac{2x}{5x+3} - 1 \geq 0$$

$$\Rightarrow \frac{-3x-3}{5x+3} \geq 0$$

$$\Rightarrow \frac{x+1}{5x+3} \leq 0 \Rightarrow x \in \left[-1, \frac{-3}{5}\right)$$

$$\frac{2x}{5x+3} + 1 \leq 0$$

$$\Rightarrow \frac{7x+3}{5x+3} \leq 0 \Rightarrow x \in \left(-\frac{3}{5}, \frac{-3}{7}\right]$$

$$\therefore x \in \left[-1, \frac{-3}{5}\right] \cup \left(\frac{-3}{5}, \frac{-3}{7}\right]$$

$$\alpha = -1, \beta = \frac{-3}{5}, \gamma = \frac{-3}{5}, \delta = \frac{-3}{7}$$

$$|3\alpha + 10\beta + 5\gamma + 21\delta|$$

$$= |-3 - 6 - 3 - 9| = 21$$

3. 8 persons has to travel from A to B in 3 allotted cars. If a car can carry maximum 3 persons. Then find the number of ways they can travel.

- (1) 1880
- (2) 1800
- (3) 1680
- (4) 1600

Answer (3)

Sol. $C_1 \ C_2 \ C_3$
 $3 \ 3 \ 2 \rightarrow$

Total $\frac{8!}{3! 3! 2! 2!}$ groups

So, they can travel in

$$\frac{8!}{3! 3! 2! 2!} \times 3! \text{ ways}$$

$$= 1680$$

4. If $\frac{z+i}{4z+i}$ is purely real $\Delta z = x + iy$ ($x, y \in R$) then one of the possibility is

- (1) $x \neq 0, y \neq -1$
- (2) $x \neq 0, y = -1$
- (3) $x = -1, y = 1$
- (4) $x = 1, y \neq -1$

Answer (2)

Sol. $\operatorname{Im} \left(\frac{x+i(y+1)}{2x+i(y+1)} \cdot \frac{2x-i(y+1)}{2x-i(y+1)} \right)$

$$= \frac{2x(y+1) - x(y+1)}{4x^2 + (y+1)^2}$$

$$= \frac{x(y+1)}{4x^2 + (y+1)^2} = 0$$

$$\Rightarrow x = 0 \text{ or } y = -1$$

5. If $\int \left(\left(\frac{x}{e} \right)^{2x} + \left(\frac{e}{x} \right)^{2x} \right) \ln x \, dx = \alpha \left(\frac{x}{e} \right)^{2x} + \beta \left(\frac{e}{x} \right)^{2x} + c$

where c is constant of integration, then

(1) $\alpha + \beta = 0$ (2) $\alpha + \beta = 1$

(3) $\alpha\beta = \frac{1}{2}$ (4) $\alpha\beta = \frac{1}{4}$

Answer (1)

Sol. Let $\left(\frac{x}{e} \right)^{2x} = t$

$$2x(\ln x - 1) = \ln t$$

$$\left[2(\ln x - 1) + 2x \left(\frac{1}{x} \right) \right] dx = \frac{1}{t} dt$$

$$\ln x = \frac{1}{2t} dt$$

$$I = \int \left(t + \frac{1}{t} \right) \times \frac{1}{2t} dt = \frac{1}{2} \int \left(1 + \frac{1}{t^2} \right) dt$$

$$= \frac{1}{2} \left(t - \frac{1}{t} \right) + c$$

$$= \frac{1}{2} \left(\left(\frac{x}{e} \right)^{2x} - \left(\frac{e}{x} \right)^{2x} \right) + c$$

$$\Rightarrow \alpha = \frac{1}{2}, \beta = -\frac{1}{2}$$

6. If a dice is thrown n -times and probability of getting 7 times odd is equal to 9 times even. Then

$$P(2 \text{ even}) \text{ is } \frac{K}{2^{15}} \text{ then } K \text{ is}$$

(1) 58 (2) 60
 (3) 48 (4) 65

Answer (2)

Sol. $P(\text{getting odd 7 times}) = P(\text{getting even 9 times})$

$${}^n C_7 \left(\frac{1}{2} \right)^7 \left(\frac{1}{2} \right)^{n-7} = {}^n C_9 \left(\frac{1}{2} \right)^9 \left(\frac{1}{2} \right)^{n-9}$$

$${}^n C_7 = {}^n C_9$$

$$n = 9 + 7 = 16$$

$$P(2 \text{ times even}) = {}^{16} C_2 \left(\frac{1}{2} \right)^{14} \left(\frac{1}{2} \right)^2$$

$$= \frac{{}^{16} C_2}{2^{16}} = \frac{16!}{2! \times 14!} \times \frac{1}{2^{16}}$$

$$= \frac{15 \times 16}{2 \times 2^{16}}$$

$$= \frac{15 \times 4}{2^{15}} = \frac{K}{2^{15}}$$

$$\Rightarrow K = 60$$

7. If $\int_0^{t^2} (f(x) + x^2) dx = \frac{4}{3} t^3$, then $f(x)$ is

(1) $x^2 - 2\sqrt{x}$ (2) $x^2 + 2\sqrt{x}$
 (3) x^2 (4) $-x^2 + 2\sqrt{x}$

Answer (4)

Sol. $(f(t^2) + t^4) 2t = 4t^2$

$$f(t^2) + t^4 = 2t$$

$$f(x^2) = -x^4 + 2x$$

$$\text{Let } x^2 = u$$

$$f(u) = -u^2 + 2\sqrt{u}$$

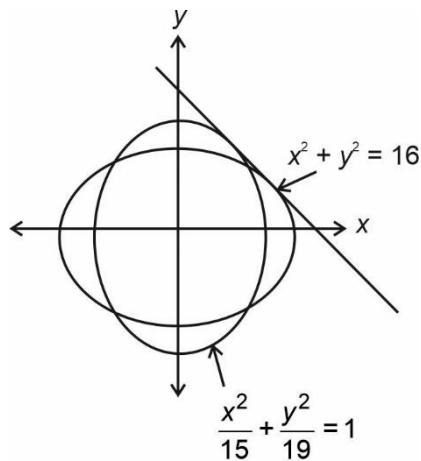
$$f(x) = -x^2 + 2\sqrt{x}$$

8. The equation of conic is $19x^2 + 15y^2 = 285$. A concentric circle with radius 4 units is given then angle of common tangent made by minor axis of ellipse is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$
 (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

Answer (1)

Sol.



$$T_n = 4 + \frac{(n-1)}{2} [14 + (n-2)3]$$

$$= 4 + \frac{(n-1)}{2} [8 + 3n]$$

$$T_n = 4 + \frac{1}{2} (3n^2 + 5n - 8)$$

$$\sum T_n = S_n = \frac{3}{2} \sum n^2 + \frac{5}{2} \sum n$$

$$= \frac{3}{2} \frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \frac{n(n+1)}{2}$$

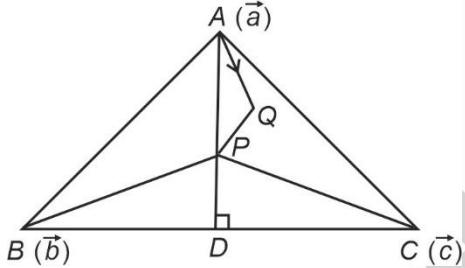
$$\frac{S_{29} - S_9}{60} = 223$$

23. In $\triangle ABC$, P is circumcentre, Q is orthocentre, then $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is

- (1) $2\overrightarrow{PQ}$ (2) \overrightarrow{PQ}
 (3) $3\overrightarrow{PQ}$ (4) $\frac{1}{2}\overrightarrow{PQ}$

Answer (2)

Sol.



Let P be origin then

$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{a} + \vec{b} + \vec{c}$$

$$\overrightarrow{PD} = \frac{\vec{b} + \vec{c}}{2}$$

$$\overrightarrow{PQ} = 2\overrightarrow{PD}$$

$$= \vec{b} + \vec{c}$$

$$\overrightarrow{PQ} = \vec{a} + (\vec{b} + \vec{c})$$

$$= \vec{a} + \vec{b} + \vec{c}$$

24. Let $S = \left\{ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10 \right\}$ and

$$\beta = \sum_{x \in S} \left(\frac{x}{3} \right). \text{ Then } \frac{1}{7}(\beta - 14)^2 \text{ is}$$

Answer (28)

Sol. Let $9^{\tan^2 x} = t$

$$\frac{9}{t} + t = 10$$

$$t^2 - 10t + 9 = 0$$

$$\Rightarrow t = 9 \text{ or } 1$$

$$9^{\tan^2 x} = 9$$

$$\Rightarrow \tan^2 x = 1$$

$$= \tan x = \pm 1$$

$$x = \pm \frac{\pi}{4}$$

or

$$\tan x = 0$$

$$\Rightarrow x = 0$$

$$\beta = \frac{0}{3} + \frac{\pi}{12} - \frac{\pi}{12} = 0$$

$$\therefore \frac{1}{7}(\beta - 14)^2 = \frac{14^2}{7} = 28$$

25. The coefficient of x and x^2 in $(1+x)^p (1-x)^q$ are 4 and -5, then $2p+3q$ is

Answer (63)

Sol. $(1+x)^p (1-x)^q = (1+px + \frac{p(p-1)}{2}x^2 + \dots)$

$$(1-qx + q \frac{(q-1)}{2}x^2 + \dots)$$

$$\therefore \text{Coefficient of } x = p - q = 4 \quad \dots(i)$$

$$\begin{aligned} \text{Coefficient of } x^2 &= \frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} \\ &= -5 \end{aligned}$$

$$\Rightarrow (p - q)^2 - (p + q) = -10$$

$$\therefore p + q = 26 \quad \dots(ii)$$

$$\text{By (i) and (ii) } p = 15, q = 11$$

$$\text{So, } 2p + 3q = 30 + 33 = 63$$

26. Let α be the remainder $(22)^{2022} + (2022)^{22}$ is divided by 3 and β be the remainder when the same is divided by 7 then $\alpha^2 + \beta^2$ is

Answer (05)

Sol. $(22)^{2022} + (2022)^{22}$

For α

$$(21+1)^{2022} + \underbrace{(2022)^{22}}_{\text{divisible by 3}}$$

$$= (3k_1 + 1)$$

For β

$$(21+1)^{2022} + (2023-1)^{22}$$

$$= (7\lambda + 1) + (7\mu + 1)$$

$$= 7k_2 + 2$$

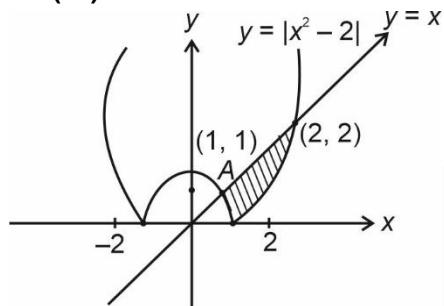
$$\text{So, } \alpha = 1, \beta = 2$$

$$\alpha^2 + \beta^2 = 5$$

27. If area bounded by region $\{x, y\} | |x^2 - 2| \leq y \leq x\}$ is A, then $6A + 16\sqrt{2}$ is

Answer (27)

Sol.



\therefore Required area

$$= \int_1^{\sqrt{2}} x - \{- (x^2 - 2)\} dx + \int_{\sqrt{2}}^2 \{x - (x^2 - 2)\} dx$$

$$= \int_1^{\sqrt{2}} (x^2 + x - 2) dx + \int_{\sqrt{2}}^2 (-x^2 + x + 2) dx$$

$$= \left(\frac{x^3}{3} + \frac{x^2}{2} - 2x \right)_1^{\sqrt{2}} + \left(\frac{-x^3}{3} + \frac{x^2}{2} + 2x \right)_{\sqrt{2}}^2$$

$$= \left(\frac{2\sqrt{2}}{3} + 1 - 2\sqrt{2} \right) - \left(\frac{1}{3} + \frac{1}{2} - 2 \right)$$

$$+ \left(\frac{-8}{3} + 2 + 4 \right) - \left(\frac{-2\sqrt{2}}{3} + 1 + 2\sqrt{2} \right)$$

$$\therefore 6A + 16\sqrt{2} = 27 - 16\sqrt{2} + 16\sqrt{2}$$

$$= 27$$

28.

29.

30.