

## MATHEMATICS

### SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. Find the sum of series

$$2 \times 2^2 - 2 \times 3^2 + 2 \times 4^2 \dots \quad (20 \text{ terms})$$

- (1) 462                          (2) -462  
 (3) 460                           (4) -460

**Answer (4)**

$$\begin{aligned} \text{Sol. } S &= 2[2^2 - 3^2 + 4^2 \dots + 20^2 - 21^2] \\ &= 2[(2^2 + 4^2 + 6^2 + \dots + 20^2) - (3^2 + 5^2 + \dots + 21^2)] \\ &= 2[2(2^2 + 4^2 + 6^2 + \dots + 20^2) - (2^2 + 3^2 + 4^2 + \dots + 21^2)] \\ &= 2[2^3(1^2 + 2^2 + 3^2 + \dots + 10^2) - (2^2 + 3^2 + 4^2 + \dots + 21^2)] \\ &= 2\left[\frac{8 \times 10 \times 11 \times 21}{6} - \frac{21 \times 22 \times 43}{6} + 1\right] \\ &= 2[3080 - 3311 + 1] \\ &= 2[-230] \\ &= -460 \end{aligned}$$

2. The number of seven digit numbers made using 1, 2, 3, 4 whose sum of digits is 12 is

- (1) 413                            (2) 311  
 (3) 308                           (4) 393

**Answer (1)**

$$\text{Sol. } \underline{\underline{1}} \quad \underline{\underline{1}} \quad \underline{\underline{1}} \quad \underline{\underline{1}} \quad \underline{\underline{1}} \quad \underline{\underline{4}} \quad \underline{\underline{3}} = \frac{7!}{5!} = 42$$

$$\underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{3} \quad \underline{3} \quad \underline{2} = \frac{7!}{4! 2!} = 105$$

$$\underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{3} \quad \underline{1} \quad \underline{1} \quad \underline{1} = \frac{7!}{3! 3!} = 140$$

$$\underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{1} \quad \underline{1} = \frac{7!}{5! 2!} = 21$$

$$\underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{4} \quad \underline{2} \quad \underline{2} = \frac{7!}{4! 2!} = 105$$

$$\text{Total} = \underline{\underline{413}}$$

3. If  $\frac{dy}{dx} = y + 7$  &  $y(0) = 0$ , then the value of  $y(1) =$   
 (1)  $7(e - 1)$                           (2)  $2(e - 1)$   
 (3)  $7e$                                       (4) None of these

**Answer (1)**

$$\begin{aligned} \text{Sol. } \frac{dy}{y+7} &= dx \\ \Rightarrow \log|y+7| &= x + c \\ y(0) = 0 & \\ \Rightarrow c &= \log 7 \\ \log|y+7| &= x + \log 7 \\ \text{Now put } x = 1 & \\ \log|y+7| &= 1 + \log 7 \\ |y+7| &= 7e \\ \therefore y &= 7(e - 1) \end{aligned}$$

4. If  $g(x) = \sqrt{x+1}$  and  $f(g(x)) = 3 - \sqrt{x+1}$  then value of  $f(0)$  is  
 (1) 2                                      (2) 3  
 (3) 4                                      (4) -4

**Answer (2)**

- Sol.** For  $f(0)$ , put  $x = -1$  in  $f(g(x))$

$$\begin{aligned} \therefore f(g(-1)) &= 3 + \sqrt{-1+1} \\ \Rightarrow f(0) &= 3 \end{aligned}$$

5. If  $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$ , then  $f(3) =$   
 (1) 2                                      (2) -3  
 (3) -4                                      (4) None of these

**Answer (2)**

$$\text{Sol. } 3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10 \quad \dots(i)$$

$$x \rightarrow \frac{1}{x}$$

$$3f\left(\frac{1}{x}\right) + 2f(x) = x - 10 \quad \dots(ii)$$

3(i) – 2(ii)

$$9f(x) - 4f(x) = \frac{3}{x} - 30 - 2x + 20$$

$$\Rightarrow 5f(x) = \frac{3}{x} - 2x - 10$$

$$x = 3$$

$$5f(3) = 1 - 6 - 10 = -15$$

$$f(3) = -3$$

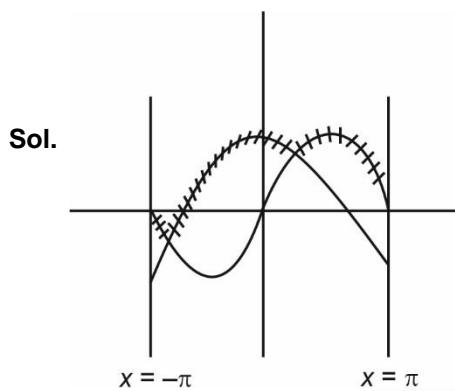
6. Find area bounded by the curves  $y = \max \{\sin x, \cos x\}$  and  $x$ -axis between  $x = -\pi$  and  $x = \pi$

$$(1) 2 + \sqrt{2}$$

$$(2) \sqrt{2}$$

$$(3) 1 + \sqrt{2}$$

$$(4) 2\sqrt{2}$$

**Answer (4)**

$$\int_{-\pi}^{-3\pi/4} \sin x \, dx + \int_{-3\pi/4}^{\pi/4} \cos x \, dx + \int_{\pi/4}^{\pi} \sin x \, dx$$

$$= -\cos x \Big|_{-\pi}^{-3\pi/4} + \sin x \Big|_{-3\pi/4}^{\pi/4} + -\cos x \Big|_{\pi/4}^{\pi}$$

$$= \left( \frac{1}{\sqrt{2}} - 1 \right) + \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) + \left( 1 + \frac{1}{\sqrt{2}} \right)$$

$$= 2\sqrt{2}$$

7. The negation of  $((A \cap (B \cup C)) \Rightarrow (B \cap C)) \Rightarrow A$  is equivalent to

$$(1) \sim(A \cup B)$$

$$(2) \sim A$$

$$(3) A$$

$$(4) \text{None of these}$$

**Answer (2)****Sol.** Let  $P = (((A \cap (B \cup C)) \Rightarrow (B \cap C)) \Rightarrow A)$ 

$$P = \sim(((A \cap (B \cup C)) \Rightarrow (B \cap C)) \cup A)$$

$$= \sim((\sim(A \cap (B \cup C)) \cup (B \cap C)) \cup A)$$

$$= A$$

$$P \equiv A$$

$$\sim P \equiv \sim A$$

8.  $f(x) = x - 2\sin x \cos x + \frac{1}{3}\sin 3x$ ,  $x \in [0, \pi]$  then maximum value of  $f(x)$  is

$$(1) \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3} \quad (2) 0$$

$$(3) \frac{5\pi}{6} + \frac{\sqrt{3}}{2} - \frac{1}{3} \quad (4) \frac{5\pi}{6}$$

**Answer (1)**

**Sol.**  $f(x) = x - \sin 2x + \frac{1}{3}\sin 3x$

$$f'(x) = 1 - 2\cos 2x + \cos 3x = 0$$

$$x = \frac{5\pi}{6}, \frac{\pi}{6}$$

$$\therefore f'(x) = 4\sin 2x - 3\sin 3x$$

$$f'(x) \text{ at } \frac{5\pi}{6} \text{ is } < 0$$

$$\Rightarrow \left(\frac{5\pi}{6}\right) \text{ is point of maxima}$$

$$f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3}$$

9. If  $\vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$$\vec{b} = 3\hat{i} + 3\hat{j} + 7\hat{k}$$

$$\vec{c} = 7\hat{i} + 8\hat{j} + 9\hat{k}$$

If  $\vec{a} \times \vec{b} = \vec{c} + \vec{d}$  then  $|\vec{d}|$  is

$$(1) \sqrt{174} \quad (2) 8\sqrt{2}$$

$$(3) \sqrt{168} \quad (4) 5\sqrt{5}$$

**Answer (1)**

**Sol.**  $\vec{a} \times \vec{b} = 6\hat{i} + \hat{j} - 3\hat{k}$

$$\vec{d} = -\hat{i} - 7\hat{j} - 12\hat{k}$$

$$|\vec{d}| = \sqrt{174}$$



14. Plane  $P_3$  is passing through  $(1, 1, 1)$  and point of intersection of  $P_1$  and  $P_2$  where  $P_1 : 2x - y + z = 5$  and  $P_2 : x + 3y + 3z + 2 = 0$ , then distance of  $(1, 1, 10)$  from  $P_3$  is

(1)  $\frac{53}{85}$

(2)  $\sqrt{85}$

(3)  $\frac{54}{\sqrt{85}}$

(4) 53

**Answer (3)**

**Sol.**  $P_1 + \lambda P_2 = 0$

$$(2x - y + z - 5) + \lambda(x + 3y + 3z + 2) = 0$$

Passing through  $(1, 1, 1)$

$$(2 - 1 + 1 - 5) + \lambda(1 + 3 + 3 + 2) = 0$$

$$-3 + \lambda(9) = 0$$

$$\lambda = \frac{1}{3}$$

$$P_3 = 3(2x - y + z - 5) + (x + 3y + 3z + 2) = 0$$

$$7x + 6z = 13$$

Distance of  $(1, 1, 10)$  is

$$\left| \frac{7 \times 1 + 6 \times 10 - 13}{\sqrt{7^2 + 6^2}} \right| = \left| \frac{7 + 60 - 13}{\sqrt{49 + 36}} \right|$$

$$= \frac{54}{\sqrt{85}} \text{ unit}$$

15.

16.

17.

18.

19.

20.

## SECTION - B

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Let the number of matrices of order  $3 \times 3$  are possible using the digits  $\{0, 1, 2, 3, \dots, 10\}$  is  $m^n$ , then  $(m + n)$  is, (where  $m$  is a prime number)

**Answer (20)**

**Sol.** So, we have 9 places and 11 numbers

$$\therefore \text{Number of matrices} = 11^9$$

$$\therefore m + n = 20.$$

22. Remainder when  $2^{2022}$  is divided by 15 is equal to

**Answer (4)**

$$\begin{aligned} \text{Sol. } & (2^8)^{252} \cdot 2^6 \\ & = (255 + 1)^{252} \cdot 64 \\ & = \left( {}^{252}C_0 (255)^{252} + {}^{252}C_1 (255)^{251} + \dots \right. \\ & \quad \left. + {}^{252}C_{252} (255)^0 \right) \cdot 64 \\ & = 64 (15k + 1) \end{aligned}$$

$$\text{Remainder} = 4$$

23. Let 10 APs are there whose first terms are  $(1, 2, 3, \dots, 10)$  respectively & common differences are  $(1, 3, 5, \dots)$  respectively and  $S_i$  denotes sum of 10 terms of  $i^{\text{th}}$  A.P then  $\sum_{i=1}^{10} S_i$  is

**Answer (5050)**

$$\begin{aligned} \text{Sol. } S_i &= \frac{10}{2} [2i + (10-1)(2i-1)] \\ &= 5[2i + 18i - 9] \\ &= 5[20i - 9] \\ &\therefore \sum_{i=1}^{10} S_i = 5 \left[ 20 \times \frac{10 \times 11}{2} - 9 \times 10 \right] \\ &= 5[1100 - 90] \\ &= 5 \times 1010 = 5050 \end{aligned}$$

24. For the data

$x_i$	1	3	5	7	9
$f_i$	4	24	28	$\alpha$	$\beta$

If mean of data is 5 and mean deviation about mean

is  $M$  and variance is  $\sigma^2$  then  $\frac{3\alpha}{M + (\sigma^2)}$  is

**Answer (08)**

$$\text{Sol. } \bar{x} = \frac{4 + 72 + 140 + 7\alpha + 72}{(64 + \alpha)} = 5$$

$$\therefore \boxed{\alpha = 16}$$

$$\text{Now } M = \frac{4 \times 4 + 24 \times 2 + 28 \times 0 + 16 \times 2 + 8 \times 4}{80}$$

$$= \frac{8}{5}$$

$$\sigma^2 = \frac{4 \times 4^2 + 24 \times 2^2 + 28 \times 0^2 + 16 \times 2^2 + 8 \times 4^2}{80}$$

$$= \frac{22}{5}$$

$$\therefore \frac{3\alpha}{M + \sigma^2} = \frac{3 \times 16}{\frac{8}{5} + \frac{22}{5}} = 08$$

25.

26.

27.

28.

29.

30.

