MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

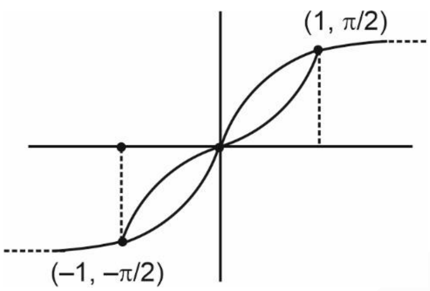
1. If $\sin^{-1}x = 2\tan^{-1}x$, then number of integral values of x is equal to

(1) 0 (2) 1

(3) 2 (4) More than 2

Answer (4)

Sol.



$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$\Rightarrow$ clearly 1, 0, –1 are the real integral solution

2. If $x^2 - \sqrt{2}x + 2 = 0$ has roots α and β then $\alpha^{14} + \beta^{14}$ is

(1) –256 (2) –128

(3) $-128\sqrt{2}$ (4) $-256\sqrt{2}$

Answer (2)

Sol. $\alpha, \beta = \dfrac{\sqrt{2} \pm i\sqrt{6}}{2}$

$$= \frac{\sqrt{2}}{2}\left(1+\sqrt{3}i\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{3} \pm i\sin\frac{\pi}{3}\right)$$

$$\alpha^{14} + \beta^{14} = 2^7\left[\left(\cos\frac{14\pi}{3} + i\sin\frac{14\pi}{3}\right) + \left(\cos\frac{14\pi}{3} - i\sin\frac{14\pi}{3}\right)\right]$$

$$= 2^7 \cdot 2\cos\frac{2\pi}{3}$$

$$= -128$$

3. The range of $\dfrac{4+(\sin x)^4}{1+x^2}$ is

(1) [0, 1] (2) (0, 4]

(3) (0, 3] (4) None of these

Answer (2)

Sol. $f(x) = \dfrac{4+(\sin x)^4}{1+x^2}$

$\because$ $f(-x) = f(x)$

$\Rightarrow$ $f(x)$ is even

Now,

$$f'(x) = \frac{\left(1+x^2\right)4\sin^3 x\cos x - 4 + (\sin x)^4 2x}{\left(1+x^2\right)^2}$$

$\Rightarrow$ $f(x)$ is decreasing for $x \in (0, \infty)$

$f(x)$ is increasing for $x \in (-\infty, 0)$

$\therefore$ $f(x)_{max}$ will be for $x = 0$

$f(x) \in (0, 4]$

4. The coefficient of x^4 in $\left(2x^3 - \dfrac{1}{3x^8}\right)^5$ is

(1) $-\dfrac{80}{3}$ (2) $\dfrac{80}{3}$

(3) $\dfrac{40}{3}$ (4) $-\dfrac{40}{3}$

Answer (1)

Sol. $T_{r+1} = {}^5C_r(2x^3)^{5-r}\left(-\dfrac{1}{3x^8}\right)^r$

$$= {}^5C_r\, 2^{5-r}\left(-\frac{1}{3}\right)^r \cdot x^{15-11r}$$

For 4 : $15 - 11r = 4$

$11r = 1$

$\boxed{r = 1}$

$\therefore$ Coefficient of $x^4 = {}^5C_1 2^4\left(-\dfrac{1}{3}\right)$

$$= -\frac{80}{3}$$

5. The number of six-digit number formed by using the digits {1, 2, 3, 4, 5, 6} which are divisible by 6 (Repetition is not allowed)

(1) 120 (2) 360

(3) 240 (4) 720

Answer (2)

Sol. 1 + 2 + 3 + 4 + 5 + 6 = 21 is divisible by 3, so all numbers are divisible by 3.

To be divisible by 2

Total numbers = 3.5!

= 360

6. $\int_0^{\frac{\pi}{4}} \frac{\tan^{50} x}{\tan^{51} x + \tan^{49} x} dx =$

(1) $\frac{1}{4}$ (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) $\frac{1}{2}$

Answer (1)

Sol. $\int_0^{\frac{\pi}{4}} \frac{dx}{\tan x + \cot x}$

$\Rightarrow \int_0^{\frac{\pi}{4}} \frac{2\sin x \cdot \cos x}{2} dx$

$\Rightarrow \frac{1}{2}\int_0^{\frac{\pi}{4}} \sin 2x dx$

$= -\frac{1}{2}\frac{\cos 2x}{2}\Big|_0^{\frac{\pi}{4}}$

$= -\frac{1}{4}(0-1) = \frac{1}{4}$

7. For matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ \alpha & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ and $|A|$ = 2 then the value of $|\alpha \,\text{adj}(\alpha \text{ adj}(\alpha A))|$ is

(1) 2^{25} (2) 2^{24}

(3) 2^{20} (4) 2^{16}

Answer (1)

Sol. $\because \quad |A| = 2$

$\Rightarrow 1(3-2) - 2(\alpha - 6) + 1(\alpha - 9) = 2$

$\Rightarrow \boxed{\alpha = 2}$

Now, $|\alpha \text{ adj}(\alpha \text{ adj}(\alpha A))| = |2 \text{ adj}(2 \text{ adj}(2A))|$

$= 2^3 |\text{adj } (2 \text{ adj}(2A))|$

$= 2^3 |2 \text{ adj}(2A)|^2$

$= 2^3 \cdot 2^6 |2A|^4$

$= 2^9 \cdot 2^{12} \cdot 2^4$

$= 2^{25}$

8. In a given data set mean of 40 observations is 50 and standard deviation is 12. Two readings which were 20 and 25, were mistakenly taken as 40 and 45. Find correct variance of data set

(1) 169 (2) 150

(3) 178 (4) 180

Answer (3)

Sol. $\sum x_{i(\text{Correct})} = 40{\cdot}50{-}40$

= 1960

$\bar{x}_{i(\text{Correct})} = 49$

$\frac{\sum x^2_{i(\text{Wrong})}}{40} - (50)^2 = 144$

$\sum x^2_{i(\text{Wrong})} = 2644{\cdot}40$

$\sum x^2_{i(\text{Correct})} = 40{\cdot}2644 - (2600)$

Correct variance $= \frac{40{\cdot}2644 - 2600}{40} - (49)^2$

= 2644 – 65 – 2401

= 2644 – 2466

= 178

9. If for a complex number z, $\bar{z} = i\left(z^2 + \text{Re}(z)\right)$ then $|z|^2$ is sum of values of all

(1) 1 (2) 2

(3) 3 (4) 4

Answer (4)

Sol. Let $z = x + iy$

$x - iy = i(x^2 - y^2 + 2ixy + x)$

$x - iy = -2xy + i(x^2 - y^2 + x)$

$x = -2xy$ or $x^2 - y^2 + x + y = 0$

$x = 0$ for $x = 0 \quad y = 0, 1$

or

$y = \frac{-1}{2}$ for $y = \frac{-1}{2}$

$$x^2 + x - \frac{1}{4} - \frac{1}{2} = 0$$

$$x^2 + x - \frac{3}{4} = 0$$

$$4x^2 + 4x - 3 = 0$$

$$4x^2 + 6x - 2x - 3 = 0$$

$$(2x - 1)(2x + 3) = 0$$

$z = 0 + 0i \rightarrow |z| = 0$

$z = 0 + i \rightarrow |z| = 1$

$z = \frac{1}{2} - \frac{1}{2}i \rightarrow |z| = \sqrt{\frac{1}{2}}$

$z = \frac{-3}{2} - \frac{1}{2}i \rightarrow |z| = \sqrt{\frac{10}{4}}$

Sum of $|z|^2 = 0 + 1 + \frac{1}{2} + \frac{10}{4}$

$$= \frac{16}{4} = 4$$

10. The area bounded by the curve

$x^2 \le y \le |x^2 - 4|$ and $y \ge 1$ is

(1) $4\sqrt{2} + 1$

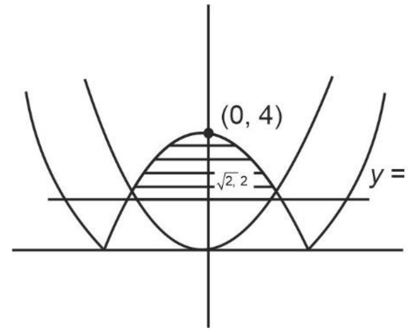
(2) $\frac{4}{3}(4\sqrt{2} - 1)$

(3) $\frac{4}{3}(4\sqrt{2} + 1)$

(4) $\frac{2}{3}(4\sqrt{2})$

Answer (2)

Sol.



Required area $= 2\left[\int_1^2 \sqrt{y}\,dy + \int_2^4 \sqrt{4-y}\,dy\right]$

$$= 2\left[\left.\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right|_1^2 - \left.\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right|_2^4\right]$$

$$= \frac{4}{3}\left[\left(2\sqrt{2} - 1\right) - \left(0 - 2\sqrt{2}\right)\right]$$

$$= \frac{4}{3}\left(4\sqrt{2} - 1\right)$$

11. The geometric mean of 5th and 7th term is 2 and the product of 3rd and 6th term of the GP is $\frac{1}{3}$. If a_n is the nth term then $(a_3 + a_4).(a_5 + a_6)$ is

(1) $\frac{1 + (12)^{1/3}}{(12)^{1/3}}$

(2) $\frac{1 + (12)^{1/3}}{3}$

(3) $\left(\frac{1 + (12)^{1/3}}{3 \times (12)^{1/3}}\right)^2$

(4) $\frac{(1 + (12)^{1/3})^2}{3 \times (12)^{1/3}}$

Answer (4)

Sol. $(a_5 a_7)^{1/2} = 2$

$ar^4 ar^6 = 4$

$a^2 r^{10} = 4$ …(1)

$a_3 a_6 = \frac{1}{3}$

$ar^2 ar^5 = \frac{1}{3}$

$a^2 r^7 = \frac{1}{3}$ …(2)

$\frac{(1)}{(2)} = r^3 = 12$

$\Rightarrow r = (12)^{1/3}$

$a^2 (12)^{7/3} = \frac{1}{3}$

$$a^2 = \left(\frac{1}{3(12)^{7/3}}\right)$$

$\because$ $(a_3 + a_4) . (a_5 + a_6)$

$= (ar^2 + ar^3)\ (ar^4 + ar^5)$

$$= r^6a^2(1+r)^2 = \underbrace{\frac{1}{3(12)^{7/3}}\times 144\left(1+(12)^{1/3}\right)^2}$$

$$= \frac{(1+(12)^{1/3})^2}{3\times(12)^{1/3}}$$

12. The statement $((\sim p) \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is equivalent to

(1) Tautology (2) Fallacy

(3) $(p \vee q)$ (4) $\sim (p \wedge q)$

Answer (4)

Sol. $(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

$= \sim p \wedge (q \vee \sim q) \vee (p \wedge \sim q)$

$= \sim p \vee (p \wedge \sim q)$

$= (\sim p \vee p) \wedge (\sim p \vee \sim q)$

$= T \wedge \sim(p \wedge q)$

$= \sim(p \wedge q)$

13. Given $\frac{x+3}{-3} = \frac{y-2}{2} = \frac{z-5}{5}$ which of the following lines in options is coplanar with the given line?

(1) $\frac{x+1}{-1} = \frac{y-1}{1} = \frac{z-5}{5}$

(2) $\frac{x+}{1} = \frac{y+1}{-1} = \frac{z-5}{5}$

(3) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

(4) $\frac{x-1}{-1} = \frac{y+2}{-2} = \frac{z-5}{4}$

Answer (1)

Sol. For non-parallel lines to be coplanar

$(\vec{r}_1 - \vec{r}_2)\cdot(\vec{a}_1 \times \vec{a}_2) = 0$

For option A $\vec{r}_1 - \vec{r}_2 = -2\hat{i} + \hat{j}$

$\vec{a}_1 = -3\hat{j} + 2\hat{j} + 5\hat{k}$

$\vec{a}_2 = -\hat{i} + \hat{j} + 5\hat{k}$

$$\begin{vmatrix} -2 & 1 & 0 \\ -3 & 2 & 5 \\ -1 & 1 & 5 \end{vmatrix} = 0$$

$\therefore$ Option (A) is correct

Similarly, we can also check other option which comes out to be non-coplanar

14. For $\vec{a}, \vec{b}, \vec{c}, |\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$ then $\left|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})\right|^2$ is

(1) 280 (2) 980

(3) 480 (4) 1764

Answer (2)

Sol. $\left|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})\right|^2$

$= \left|-3(\vec{a}\times\vec{b}) + 4(\vec{b}\times\vec{a})\right|^2$

$= \left|7(\vec{b}\times\vec{a})\right|^2 = 49\left(|\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2\right)$

$= 49\ (36 - 16)$

$= 980$

15. A line is passing through $A(4, 5, 8)$ and $B(1, -7, 5)$ from point $C(1, 2, 5)$ a perpendicular is drawn on AB. If foot of perpendicular is N then distance of N from plane $2x - 2y + 2z - 3 = 0$ is

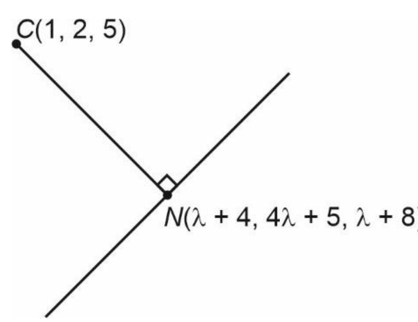
(1) $\frac{9}{2\sqrt{3}}$ (2) $\frac{15}{2\sqrt{3}}$

(3) $\frac{8}{3\sqrt{3}}$ (4) $\frac{7}{4\sqrt{3}}$

Answer (2)

Sol. $L: \frac{x-4}{3} = \frac{y-5}{12} = \frac{z-8}{3}$

$L: \frac{x-4}{1} = \frac{y-5}{4} = \frac{z-8}{1}$



Now, $NC \perp AB$

$< \lambda + 3, 4\lambda + 3, \lambda + 3 > \cdot < 1, 4, 1> = 0$

$\lambda + 3 + 16\lambda + 12 + \lambda + 3 = 0$

$18\lambda + 18 = 0$

$\lambda = -1$

$\therefore$ $N(3, 1, 7)$

$$\text{Distance} = \left|\frac{6-2+14-3}{\sqrt{2^2+2^2+2^2}}\right|$$

$$= \frac{15}{2\sqrt{3}} \text{ unit}$$

16. If centroid of triangle formed by the lines $2x + y = 10$, $x + 3y = 7$ and $3x - y = 5$ is (α, β). The quadratic equation whose roots are $\alpha + 2\beta$ and $2\alpha + \beta$ is

(1) $225x^2 + 3645x - 14690 = 0$

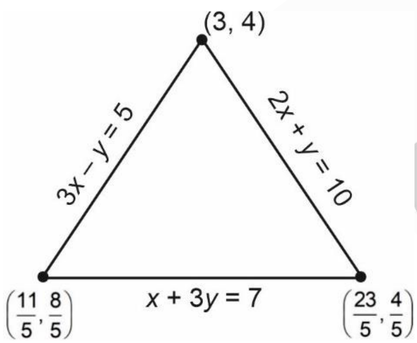
(2) $225x^2 - 3645x + 14690 = 0$

(3) $225x^2 - 3645x - 14690 = 0$

(4) $225x^2 + 3645x + 14690 = 0$

Answer (2)

Sol.



$$\therefore \text{ Centroid} = (\alpha, \beta) = \left(\frac{3+\frac{23}{5}+\frac{11}{5}}{3}, \frac{4+\frac{4}{5}+\frac{8}{5}}{3}\right)$$

$$\Rightarrow (\alpha, \beta) = \left(\frac{49}{15}, \frac{32}{15}\right)$$

$$\alpha + 2\beta = \frac{113}{15}$$

$$2\alpha + \beta = \frac{130}{15}$$

$$\therefore \text{ required equation : } x^2 - \left(\frac{243}{15}\right)x + \frac{14690}{225} = 0$$

$\Rightarrow$ $225x^2 - 3645x + 14690 = 0$

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The remainder when 7^{103} is divided by 17 is

Answer (12)

Sol. $7 \equiv 7 \pmod{17}$

$7^2 \equiv -2 \pmod{17}$

$7^6 \equiv -8 \pmod{17}$

$7^8 \equiv -1 \pmod{17}$

$7^{96} \equiv 1 \pmod{17}$

$7^{103} \equiv 12 \pmod{17}$

$\therefore$ Remainder = 12

22. The value of

$\left[\sqrt{1}\right] + \left[\sqrt{2}\right] + \left[\sqrt{3}\right] + \ldots \left[\sqrt{120}\right]$ is equal to, where

$[\cdot]$ denotes greatest integer function

Answer (825)

Sol. $E = 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 4 + 4 + 4 + 4 + 4 + \ldots$

$E = 3 \times 1 + 5 \times 2 + 7 \times 3 + \ldots + 19 \times 9 + 10 \times 21$

$$= \sum_{r=1}^{10} (2r+1)r = 2\left[\frac{10 \times 11 \times 21}{6}\right] + \frac{10 \times 11}{2}$$

$= 770 + 55 = 825$

23. Rank of Monday in English dictionary if all alphabets are arranged in order?

Answer (327)

Sol.

3	5	4	2	1	6
M	O	N	D	A	Y
2	3	2	1	0	0
5!	4!	3!	2!	1!	0!

$\therefore$ Rank = $(2 \times 5! + 3 \times 4! + 2 \times 3! + 1 \times 2!) + 1$

$= 240 + 72 + 12 + 2 + 1 = 327$

24.

25.

26.

27.

28.

29.

30.

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