04/04/2024
Evening

Medical | IIT-JEE | Foundations

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6) **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in [–1, 1] such that $\cos^{-1}x - \sin^{-1}y = \alpha$, $\frac{-\pi}{2} \le \alpha \le \pi$.

   Then, the minimum value of $x^2 + y^2 + 2xy \sin\alpha$ is

   (1) –1 (2) $\frac{1}{2}$

   (3) $\frac{-1}{2}$ (4) 0

Answer (4)

Sol. $\cos^{-1}x - \frac{\pi}{2} + \cos^{-1}y = \alpha$

$$\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2} + \alpha$$

$$\because \quad \alpha \in \left[\frac{-\pi}{2}, \pi\right]$$

then $\frac{\pi}{2} + \alpha \in \left(0, \frac{3\pi}{2}\right)$

$$\cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \frac{\pi}{2} + \alpha$$

$$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\sin\alpha$$

$$xy + \sin\alpha = \sqrt{1-x^2}\sqrt{1-y^2}$$

$$x^2y^2 + \sin^2\alpha + 2xy\sin\alpha = 1 - x^2 - y^2 + x^2y^2$$

$$\underbrace{x^2 + y^2 + 2xy\sin\alpha}_{E} = \cos^2\alpha$$

Now, minimum value of E is 0.

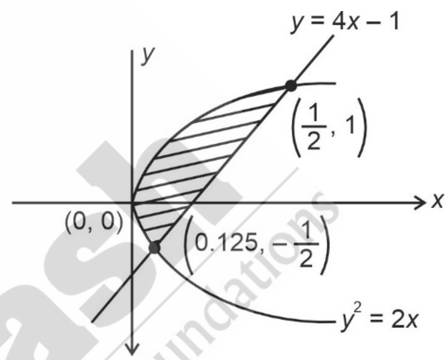
2. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x, \text{ and } y \ge 4x - 1\}$ is

   (1) $\frac{9}{32}$ (2) $\frac{11}{12}$

   (3) $\frac{11}{32}$ (4) $\frac{8}{9}$

Answer (1)

Sol. Area $= \int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{4} - \frac{y^2}{2}\right)dy$



$$= \left[\frac{y^2}{8} + \frac{y}{4} - \frac{y^3}{6}\right]_{-\frac{1}{2}}^{1}$$

$$= \left(\frac{1}{8} + \frac{1}{4} - \frac{1}{6}\right) - \left(\frac{1}{32} - \frac{1}{8} + \frac{1}{48}\right)$$

$$= \frac{5}{24} + \frac{7}{96}$$

$$= \frac{27}{96}$$

$$= \frac{9}{32}$$

3. Let $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ and $B = I + \text{adj}(A) + (\text{adj } A)^2 + \cdots + (\text{adj } A)^{10}$.
Then, the sum of all the elements of the matrix B is:
(1) 22 (2) –88
(3) –124 (4) –110

Answer (2)

Sol. $\text{adj}(A)=\begin{bmatrix}1 & -2\\ 0 & 1\end{bmatrix}$

$(\text{adj}A)^2=\begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix}$

$(\text{adj}A)^3=\begin{bmatrix}1 & -6\\ 0 & 1\end{bmatrix}$

$(\text{adj}A)^4=\begin{bmatrix}1 & -8\\ 0 & 1\end{bmatrix}$

$(\text{adj}A)^r=\begin{bmatrix}1 & (-2r)\\ 0 & 1\end{bmatrix}$

$$B=\sum_{r=0}^{10}(\text{adj}A)^r=\begin{bmatrix}\sum_{r=0}^{10}1 & \sum_{r=0}^{10}(-2r)\\ \sum_{r=0}^{10}(0) & \sum_{r=0}^{10}(1)\end{bmatrix}$$

$$B=\begin{bmatrix}11 & -110\\ 0 & 11\end{bmatrix}$$

Sum of elements = –110 + 11 + 11 = –88

4. For $\lambda > 0$, let θ be the angle between the vectors $\vec{a}=\hat{i}+\lambda\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$. If the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to
(1) 20 (2) 25
(3) 40 (4) 50

Answer (2)

Sol. Given $\vec{a}=\hat{i}+\lambda\hat{j}-3\hat{k}$

$\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$

$\vec{a}+\vec{b}=4\hat{i}+(\lambda-1)\hat{j}-\hat{k}$

$\vec{a}-\vec{b}=-2\hat{i}+(\lambda+1)\hat{j}-5\hat{k}$

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=0$

$-8+\lambda^2-1+5=0$

$\lambda^2 = 4$

$\lambda = \pm 2 \because \lambda > 0$ (Given)

$\therefore \boxed{\lambda=2}$

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta=\frac{3-2-6}{\sqrt{14}\times\sqrt{14}}=\frac{-5}{14}$$

$$(14\cos\theta)^2=\left(14\times\frac{-5}{14}\right)^2=25$$

5. Let $f(x)=\int_0^x(t+\sin(1-e^t))dt, x\in\mathbb{R}$. Then, $\lim_{x\to0}\frac{f(x)}{x^3}$ is equal to
(1) $-\frac{1}{6}$ (2) $\frac{2}{3}$
(3) $-\frac{2}{3}$ (4) $\frac{1}{6}$

Answer (1)

Sol. Given $f(x)=\int_0^x(t+\sin(1-e^t))dt$

Now, $\lim_{x\to0}\frac{f(x)}{x^3}\left(\frac{0}{0}\text{ form}\right)$

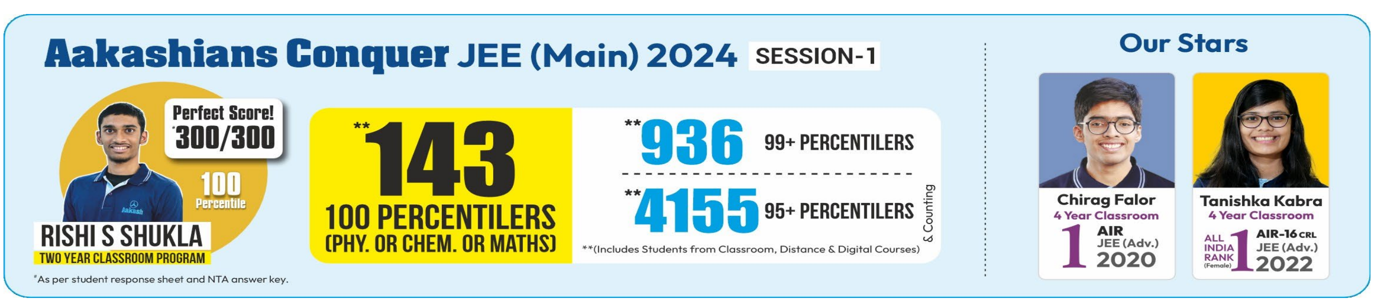
$$=\lim_{x\to0}\frac{\int_0^x(t+\sin(1-e^t))dt}{x^3}$$

$$=\lim_{x\to0}\frac{x+\sin(1-e^x)}{3x^2}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to0}\frac{1+\cos(1-e^x)(-e^x)}{6x}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to0}\frac{-\sin(1-e^x)(e^x)^2+\cos(1-e^x)(-e^x)}{6}$$

$$=-\frac{1}{6}$$



6. Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at (4, 1). Then, which of the following point lies on the line passing through the points P and Q?

(1) (3, –3)
(2) $\left(\frac{3}{2}, -16\right)$
(3) (2, –9)
(4) $\left(\frac{1}{2}, -20\right)$

Answer (4)

Sol. $y^2 = 12x$

Chord PQ having mid-point $(x_1, y_1) = (4, 1)$ equation of chord PQ

$T = S_1$

$$yy_1 - 12\frac{(x + x_1)}{2} = y_1^2 - 12x_1$$

$y - 6(x + 4) = 1 - 12 \times 4$

$y - 6x - 24 = -47$

$y - 6x + 23 = 0$

From option (4) $x = \frac{1}{2}$ & $y = -20$

$$-20 - 6 \times \frac{1}{2} + 23 = 0$$

7. Let $f(x) = 3\sqrt{x - 2} + \sqrt{4 - x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f, then $\alpha^2 + 2\beta^2$ is equal to

(1) 42
(2) 38
(3) 44
(4) 24

Answer (1)

Sol. $f(x) = 3\sqrt{x - 2} + \sqrt{4 - x}$

Let $x = 2\sin^2\theta + 4\cos^2\theta$

$$= 3\sqrt{2\sin^2\theta + 4\cos^2\theta - 2} + \sqrt{4 - 2\sin^2\theta - 4\cos^2\theta}$$

$$= 3\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}$$

$$= 3\sqrt{2}\,|\cos\theta| + \sqrt{2}\,|\sin\theta|$$

$$\Rightarrow 3\sqrt{2}\cos\theta + \sqrt{2}\sin\theta \le \sqrt{18 + 2}$$

$$\Rightarrow 3\sqrt{2}\cos\theta + \sqrt{2}\sin\theta \le \sqrt{20}$$

Minimum value exists when $\theta = \frac{\pi}{2}$

So, minimum value = $\sqrt{2}$

$\Rightarrow \alpha = \sqrt{2}$ and $\beta = \sqrt{20}$

$\Rightarrow \alpha^2 + 2\beta^2 = 2 + 40$

= 42

8. Let $\vec{a} - \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}, x \in \mathbb{R}.$.

If $\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$ such that $\vec{a} \cdot \vec{d} = 1$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to

(1) 6
(2) 9
(3) 3
(4) 11

Answer (4)

Sol. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k},$

$\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}, x \in R$

also, $\vec{b} + \vec{c} = (x + 2)\hat{i} + 6\hat{j} - 2\hat{k}$

$\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$

$$|\vec{b} + \vec{c}| = \sqrt{(x + 2)^2 + 6^2 + 2^2}$$

$$= \sqrt{40 + (x + 2)^2}$$

$$\vec{d} = \frac{x + 2}{\sqrt{40 + (x + 2)^2}}\hat{i} + \frac{6}{\sqrt{40 + (x + 2)^2}}\hat{j} - \frac{2}{\sqrt{40 + (x + 2)^2}}\hat{k}$$

$\vec{a} \cdot \vec{d} = 1$

$$\frac{x + 2 + 6 - 2}{\sqrt{40 + (x + 2)^2}} = 1$$

$$x + 6 = \sqrt{40 + (x + 2)^2}$$

$$(x+6)^2 = 40+(x+2)^2$$

$$x^2+36+12x = 40+x^2+4+4x$$

$8x = 8$

$\Rightarrow x = 1$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = [\vec{a}\ \vec{b}\ \vec{c}]$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & -5 \\ 1 & 2 & 3 \end{bmatrix}$$

= 1 (12+10) – 1 (6+5) + 1 (4 – 4)

= 22 – 11 = 11

9. If the value of the integral $\int_{-1}^{1}\frac{\cos ax}{1+3^x}dx$ is $\frac{2}{\pi}$. Then, a value of α is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Answer (2)

Sol. Given, $\int_{-1}^{1}\frac{\cos\alpha x}{1+3^x}dx = \frac{2}{\pi}$

$$I = \int_{-1}^{1}\frac{\cos\alpha x}{1+3^x}dx$$

$$\Rightarrow I = \int_{0}^{1}\left(\frac{\cos\alpha x}{1+3^x}+\frac{\cos\alpha x}{1+3^{-x}}\right)dx$$

$$= \int_{0}^{1}\cos\alpha x\, dx$$

$$= \left(\frac{\sin\alpha x}{\alpha}\right)_0^1$$

$$= \frac{\sin\alpha}{\alpha}$$

$$\Rightarrow \frac{\sin\alpha}{\alpha} = \frac{2}{\pi}$$

$$\Rightarrow \boxed{\alpha = \frac{\pi}{2}}$$

10. If the coefficient of x^4, x^5 and x^6 in the expansion of $(1 + x)^n$ are in the arithmetic progression, then the maximum value of n is:

(1) 28 (2) 7

(3) 21 (4) 14

Answer (4)

Sol. $(1+x)^n = {}^nC_0 + {}^nC_1x^1 + {}^nC_2x^2 + \ldots {}^nC_nx^n$

${}^nC_4, {}^nC_5$ & ${}^nC_6$ are in A.P.

$${}^nC_5 - {}^nC_4 = {}^nC_6 - {}^nC_5$$

$$\Rightarrow \frac{n!}{5!(n-5)!}-\frac{n!}{4!(n-4)!} = \frac{n!}{6!(n-6)!}-\frac{n!}{5!(n-5)!}$$

$\Rightarrow 30(n-9)(n-6) = 5(n-4)(n-11)$

$\Rightarrow 30n^2 - 450n + 1620 = 5n^2$

$$\Rightarrow \frac{1}{n-5}\left[\frac{n-4-5}{5(n-4)}\right] = \frac{1}{5}\left[\frac{n-5-6}{6(n-5)}\right]$$

$$\Rightarrow \frac{n-9}{5(n-4)} = \frac{1}{5}\left[\frac{n-11}{6}\right]$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$n_{max} = 14$

11. Let a relation R on N × N be defined as:

(x_1,y_1) R (x_2,y_2) if and only if $x_1 \le x_2$ or $y_1 \le y_2$.

Consider the two statements:

(I) R is reflexive but not symmetric.

(II) R is transitive

Then which one of the following is true?

(1) Both (I) and (II) are correct.

(2) Neither (I) nor (II) is correct.

(3) Only (I) is correct.

(4) Only (II) is correct.

Answer (3)

Sol. $(x_1, y_1) R (x_2, y_2)$

If $x_1 \le x_2$ or $y_1 \le y_2$

For reflexive;

$(x_1, y_1) R (x_1, y_1)$

$\Rightarrow x_1 \le x_1$ or $y_1 \le y_1$

So, R is reflexive

For symmetric

When $(x_1, y_1)\ R\ (x_2, y_2)$

$\Rightarrow x_1 \le x_2$ or $y_1 \le y_2$

For $(x_2, y_2)\ R\ (x_1, y_1)$

$\Rightarrow x_2 \le x_1$ or $y_2 \le y_1$

Not true for (1, 2) and (3, 4)

For transitive

Take pairs as (3, 9), (4, 6), (2, 7)

(3, 9) R (4, 6)

as $4 \ge 3$

(4, 6) R (2, 7)

As $7 \ge 6$

But (3, 9) R (2, 7)

As neither $2 \ge 3$ nor $7 \ge 9$

So not transitive

12. The value of $\dfrac{1\times 2^2 + 2\times 3^2 + \ldots + 100\times(101)^2}{1^2\times 2 + 2^2\times 3 + \ldots 100^2\times 101}$ is

(1) $\dfrac{32}{31}$ (2) $\dfrac{31}{30}$

(3) $\dfrac{306}{305}$ (4) $\dfrac{305}{301}$

Answer (4)

Sol. $\dfrac{1\times 2^2 + 2\times 3^2 + \ldots + 100\times(101)^2}{1^2\times 2 + 2^2\times 3 + \ldots + 100^2\times 101}$

$$\Rightarrow \frac{\sum_{n=1}^{100} n(n+1)^2}{\sum_{n=1}^{100} n^2(n+1)}$$

$$\Rightarrow \frac{\sum_{n=1}^{100} n^3 + 2n^2 + n}{\sum_{n=1}^{100} n^3 + n^2}$$

$$= \frac{\left(\frac{100(101)}{2}\right)^2 + \frac{2\cdot 100(101)(201)}{6} + \frac{100(101)}{2}}{\left(\frac{100(101)}{2}\right)^2 + \frac{100(101)(201)}{6}}$$

$$= \frac{300(101) + 4(201) + 6}{300(101) + 2(201)} = \frac{5185}{5117} = \frac{305}{301}$$

13. If the mean of the following probability distribution of a random variable X:

X	0	2	4	6	8
$P(X)$	a	$2a$	$a + b$	$2b$	$3b$

is $\dfrac{46}{9}$, then the variance of the distribution is

(1) $\dfrac{173}{27}$ (2) $\dfrac{566}{81}$

(3) $\dfrac{581}{81}$ (4) $\dfrac{151}{27}$

Answer (2)

Sol.

X	0	2	4	6	8
$P(X)$	a	$2a$	$a + b$	$2b$	$3b$

Mean = $\sum x_i P(x_i)$

$$\frac{46}{9} = 4a + 4a + 4b + 12b + 24b$$

$$\frac{46}{9} = 8a + 40b$$

$$\frac{23}{9} = 4a + 20b$$

$36a + 180b = 23$ …(1)

Sum of probability is 1

$\Rightarrow$ $4a + 6b = 1$ …(2)

Solving (1) and (2)

$$a = \frac{1}{12}, b = \frac{1}{9}$$

$$\sigma^2 = \sum x_i^2 P(x_i) - \left(\sum x_i P(x_i)\right)^2$$

$$= 4 \times 2a + 16(a+b) + 36(2b) + 64(3b) - \left(\frac{46}{9}\right)^2$$

$$= 8(a + 2(a+b) + 9b + 24b) - \left(\frac{46}{9}\right)^2$$

$$= 8(3a + 35b) - \left(\frac{46}{9}\right)^2$$

$$= 8\left(\frac{3}{12} + \frac{35}{9}\right) - \left(\frac{46}{9}\right)^2$$

$$= 8\left(\frac{149}{36}\right) - \left(\frac{46}{9}\right)^2 = \frac{566}{81}$$

14. Let three real numbers a, b, c be in arithmetic progression and $a + 1$, b, $c + 3$ be in geometric progression. If $a > 10$ and the arithmetic mean of a, b and c is 8, then the cube of the geometric mean of a, b and c is

(1) 120 (2) 316

(3) 312 (4) 128

Answer (1)

Sol. $2b = a + c$ ...(1)

$b^2 = (a + 1)(c + 3)$ ...(2)

$\frac{a+b+c}{3} = 8$ ...(3)

$\Rightarrow \frac{3b}{3} = 8$

$b = 8$

$\Rightarrow ac + 3a + c + 3 = 64$

$3a + c + ac = 61$ ...(4)

$a + c = 16$

$c = 16 - a$

from equation (4)

$3a + 16 - a + a(16 - a) = 61$

$\Rightarrow (a - 15)(a - 3) = 0$

$a = 15$ $(a > 10)$

$\Rightarrow a = 15, b = 8, c = 1$

$$\left((a \cdot b \cdot c)^{\frac{1}{3}}\right)^3 = 15 \times 8 \times 1 = 120$$

15. The area (in sq. units) of the region

$S = \{z \in \mathbb{C} : |z - 1| \le 2; (z + \bar{z}) + i(z - \bar{z}) \le 2, \text{Im}(z) \ge 0\}$

is

(1) $\frac{17\pi}{8}$ (2) $\frac{3\pi}{2}$

(3) $\frac{7\pi}{3}$ (4) $\frac{7\pi}{4}$

Answer (2)

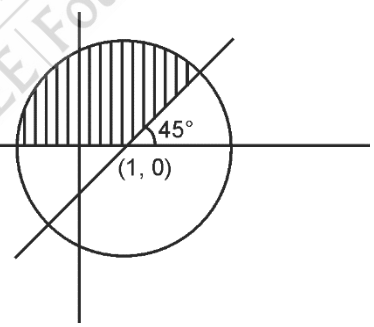
Sol. $|z - 1| \le 2$ $\Rightarrow (x - 1)^2 + y^2 = 4$

$z + \bar{z} + i(z - \bar{z}) \le 2$

$\Rightarrow x - y \le 1$

$\text{Im}(z) \ge 0$

$\Rightarrow y \ge 0$



$$\text{Required area} = \left(\frac{\frac{3\pi}{4}}{2\pi}\right)(\pi)(2)^2$$

$$= \frac{3}{2}\pi$$

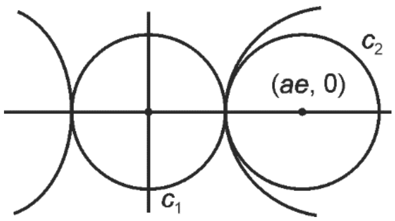
16. Consider a hyperbola H having centre at the origin and foci on the x-axis. Let C_1 be the circle touching the hyperbola H and having the centre at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the centre at the one of its foci. If areas (in sq. units) of C_1 and C_2 are 36π and 4π, respectively, then the length (in units) of latus rectum of H is

(1) $\frac{11}{3}$ (2) $\frac{14}{3}$

(3) $\frac{10}{3}$ (4) $\frac{28}{3}$

Answer (4)

Sol.



$C_1 : x^2 + y^2 = a^2 \quad \Rightarrow$ area $= \pi a^2 = 36\pi \quad \Rightarrow a = 6$

$C_2 : x^2 + (y - ae)^2 = (ae - a)^2$

$\therefore \quad \pi(ae - a)^2 = 4\pi$

$\Rightarrow \quad 36(e - 1)^2 = 4$

$\Rightarrow \quad e - 1 = \frac{1}{3} \quad \Rightarrow \quad e = \frac{4}{3}$

$\Rightarrow \quad b^2 = 28$

$\therefore \quad LR = \frac{2b^2}{a} = \frac{2 \times 28}{6} = \frac{28}{3}$ units

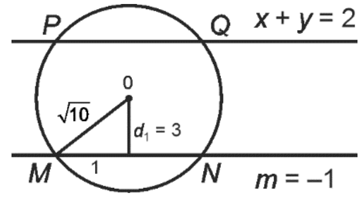
17. Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line $x + y = 2$ intersects the circle C at the points P and Q. Let MN be a chord of C of length 2 unit and slope -1. Then, a distance (in units) between the chord PQ and the chord MN is

(1) $\sqrt{2} - 1$ (2) $2 - \sqrt{3}$

(3) $3 - \sqrt{2}$ (4) $\sqrt{2} + 1$

Answer (3)

Sol. Let the line by $x + y = \lambda$



$\therefore \quad \left|\frac{\lambda}{\sqrt{2}}\right| = 3$

$\therefore \quad \lambda = \pm 3\sqrt{2}$

$\therefore$ distance between lines

$x + y = 2$ and $x + y = 3\sqrt{2}$ is

$$\frac{3\sqrt{2} - 2}{\sqrt{2}} = 3 - \sqrt{2}$$

18. Let $y = y(x)$ be the solution of the differential equation $(x^2 + 4)^2 dy + (2x^3 y + 8xy - 2)dx = 0$. If $y(0) = 0$, then $y(2)$ is equal to

(1) $\frac{\pi}{8}$ (2) 2π

(3) $\frac{\pi}{32}$ (4) $\frac{\pi}{16}$

Answer (3)

Sol. $\frac{dy}{dx} + \frac{y(2x^3 + 8x)}{(x^2 + 4)^2} = \frac{2}{(x^2 + 4)^2}$

$$\text{IF} = e^{\int \frac{2x^3 + 8x}{(x^2+4)^2} dx}$$

Let $(x^2 + 4)^2 = t \quad \Rightarrow \quad 2(x^2 + 4)(2x)dx = dt$

$$= e^{\int \frac{dt}{2t}} = e^{\log \sqrt{t}} = \sqrt{t} = (x^2 + 4)$$

$\therefore \quad y(x^2 + 4) = \int \frac{2}{x^2 + 4} + c$

$\Rightarrow \quad y(x^2 + 4) = \tan^{-1}\left(\frac{x}{2}\right) + c$

$y(0) = 0$

$\Rightarrow \quad 0 = 0 + c \qquad \Rightarrow \quad c = 0$

put $x = 2$

$y(8) = \frac{\pi}{4} \qquad \Rightarrow \quad y = \frac{\pi}{32}$

19. Let P be the point of intersection of the lines $\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1}$ and $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$. Then, the shortest distance of P from the line $4x = 2y = z$ is

(1) $\frac{\sqrt{14}}{7}$ (2) $\frac{6\sqrt{14}}{7}$

(3) $\frac{5\sqrt{14}}{7}$ (4) $\frac{3\sqrt{14}}{7}$

Answer (4)

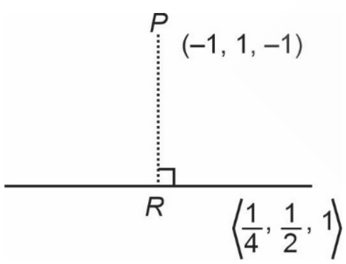
Sol. $L_1 : \frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1}$

$L_2 : \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$

Point of intersection of L_1 and L_2 is (–1, 1, –1)

Distance of point P from L_3 :$4x = 2y = z$

$L_3 : \frac{x}{\frac{1}{4}} = \frac{y}{\frac{1}{2}} = \frac{z}{1}$



Any point on L_3 be

$\left(\frac{\lambda}{4}, \frac{\lambda}{2}, \lambda\right)$

$PR : \left\langle \frac{\lambda}{4}+1, \frac{\lambda}{2}-1, \lambda+1 \right\rangle$

$\because \; PR \perp \left\langle \frac{1}{4}, \frac{1}{2}, 1 \right\rangle$

$\Rightarrow \quad \left(\frac{\lambda}{4}+1\right)\frac{1}{4} + \frac{1}{2}\left(\frac{\lambda}{2}-1\right) + \lambda + 1 = 0$

$\frac{\lambda}{16} + \frac{1}{4} + \frac{\lambda}{4} - \frac{1}{2} + \lambda + 1 = 0$

$\Rightarrow \quad \lambda = \frac{-4}{7}$

$\therefore \quad R\left(\frac{-1}{7}, \frac{-2}{7}, \frac{-4}{7}\right)$

Now RP : $\sqrt{\left(\frac{-1}{7}+1\right)^2 + \left(\frac{-2}{7}-1\right)^2 + \left(\frac{-4}{7}+1\right)^2}$

$= \sqrt{\frac{36}{49} + \frac{81}{49} + \frac{9}{49}} = \frac{\sqrt{126}}{7} = \frac{3\sqrt{14}}{7}$

20. If the function $f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, & x \neq 0 \\ a\log_e 2 \, \log_e 3 & , x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a^2 is equal to

(1) 746 (2) 1152

(3) 968 (4) 1250

Answer (2)

Sol. $f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, & x \neq 0 \\ a\log_e 2 \, \log_e 3 & , x = 0 \end{cases}$

$\because \; f(x)$ is continuous at $x = 0$

$\Rightarrow \quad \lim_{x\to 0} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}$

$\lim_{x\to 0} \frac{(9^x - 1)(8^x - 1)\left(\sqrt{2} + \sqrt{1+\cos x}\right)}{\frac{(1-\cos x)}{x^2} \times x^2}$

$= (\ln 9 \cdot \ln 8)\ \left(2\sqrt{2}\right) \times 2$

$= 4\sqrt{2} \times 2 \times 3\ln 2 \cdot \ln 3$

$24\sqrt{2} \cdot \ln 2 \cdot \ln 3$

$\Rightarrow \quad a = 24\sqrt{2}$

$a^2 = 1152$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $f : R \to R$ be a thrice differentiable function such that $f(0) = 0$, $f(1) = 1$, $f(2) = -1$, $f(3) = 2$ and $f(4) = -2$. Then, the minimum number of zeroes of $(3f'f'' + ff''')(x)$ is ________

Answer (5)

Sol. $\because$ $f : R \to R$ and $f(0) = 0$, $f(1) = 1$, $f(2) = -1$, $f(3) = 2$ and $f(4) = -2$ then

$f(x)$ has atleast 4 real roots.

Then $f'(x)$ has atleast 3 real roots and $f''(x)$ has atleast 2 real roots.

Now we know that

$$\frac{d}{dx}(f^3 \cdot f'') = 3f^2 \cdot f' \cdot f'' + f^3 \cdot f'''$$

$$= f^2(3f' \cdot f'' + f \cdot f''')$$

Here $f^3 \cdot f''$ has atleast 6 roots.

Then its differentiation has atleast 5 distinct roots.

22. Consider the function $f : R \to R$ defined by $f(x) = \frac{2x}{\sqrt{1-9x^2}}$. If the composition of f, $\underbrace{(fofofo \cdots of)}_{10 \text{ times}}(x) = \frac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the value of $\sqrt{3\alpha+1}$ is equal to ________.

Answer (1024)

Sol. $f(x) = \frac{2x}{\sqrt{1+9x^2}}$

$$(fof)(x) = \frac{2f(x)}{\sqrt{1+9(f(x))^2}} = \frac{\frac{4x}{\sqrt{1+9x^2}}}{\sqrt{1+9 \times \frac{4x^2}{1+9x^2}}} = \frac{4x}{\sqrt{1+45x^2}}$$

$$(fofof)(x) = \frac{4 \times \frac{2x}{\sqrt{1+9x^2}}}{\sqrt{1+45 \times \frac{4x^2}{1+9x^2}}} = \frac{8x}{\sqrt{1+21 \times 9x^2}}$$

$$(fofofof)(x) = \frac{16x}{\sqrt{1+85 \times 9x^2}}$$

$\Rightarrow$ α is 10th term of 1, 5, 21, 85, …

α is 10th term of

$$\frac{(2^1)^2-1}{3}, \frac{(2^2)^2-1}{3}, \frac{(2^3)^2-1}{3}, \frac{(2^4)^2-1}{3}, \ldots$$

$$\Rightarrow \alpha = \frac{(2^{10})^2-1}{3}$$

$$\Rightarrow \sqrt{3\alpha+1} = 2^{10} = 1024$$

23. Let $S = \{\sin^2 2\theta : (\sin^4\theta + \cos^4\theta)x^2 + (\sin 2\theta)x + (\sin^6\theta + \cos^6\theta) = 0$ has real roots$\}$. If α and β be the smallest and largest elements of the set S, respectively, then $3((\alpha - 2)^2 + (\beta - 1)^2)$ equal _____.

Answer (4)

Sol. For real roots

$D \geq 0$

$$\sin^2 2\theta \geq 4(\sin^4\theta + \cos^4\theta)(\sin^6\theta + \cos^6\theta)$$

Put $\sin^2 2\theta = t$

$$\Rightarrow t \geq 4\left(1-\frac{t}{2}\right)\left(1-\frac{3t}{4}\right)$$

$2t \geq (2 - t)(4 - 3t)$

$3t^2 - 12t + 8 \leq 0$

$$t^2 - 4t + \frac{8}{3} \leq 0$$

$$(t-2)^2 + \frac{8}{3} - 4 \leq 0$$

$$(t-2)^2 \leq \frac{4}{3}$$

$$-\frac{2}{\sqrt{3}} \leq t-2 \leq \frac{2}{\sqrt{3}}$$

$$2-\frac{2}{\sqrt{3}} \leq t \leq 2+\frac{2}{\sqrt{3}}$$

$\because$ $t \in [0, 1]$

$$\Rightarrow 2-\frac{2}{\sqrt{3}} \leq t \leq 1$$

$$\alpha = 2-\frac{2}{\sqrt{3}}, \ \beta = 1$$

$$\Rightarrow 3[(\alpha-2)^2 + (\beta-1)^2] = 4$$

24. If $\int \text{cosec}^5 x dx = \alpha \cot x \,\text{cosec} x \left(\text{cosec}^2 x + \frac{3}{2}\right)$ $+ \beta \log_e \left|\tan \frac{x}{2}\right| + c$ where α, $\beta \in R$ and C is the constant of integration, then the value of $8(\alpha + \beta)$ equals ________.

Answer (1)

Sol. $I = \int (\text{cosec}\, x)^5 dx = \int (\text{cosec}\, x)^3 (\text{cosec}\, x)^2 dx$

$= (\text{cosec}\, x)^3 \int \text{cosec}^2 x dx -$

$\int \left(\frac{d}{dx}(\text{cosec}\, x)^3 \int \text{cosec}^2 x dx\right) dx$

$= -\cot x (\text{cosec}\, x)^3 - \int 3\,\text{cosec}^2 x \cdot (-\text{cosec}\, x \cot x)(-\cot x) dx$

$= -\cot x (\text{cosec}\, x)^3 - \int 3\,\text{cosec}^3 x \cot^2 x dx$

$= -\cot x (\text{cosec}\, x)^3 - 3\int (\text{cosec}\, x)^3 (\text{cosec}^2 x - 1) dx$

$I = -\cot x (\text{cosec}\, x)^3 - 3I + 3\int (\text{cosec}\, x)^3 dx$

$4I = -\cot x (\text{cosec}\, x)^3 + 3\int (\text{cosec}\, x)^3 dx$

$= -\cot x (\text{cosec}\, x)^3 + 3I_1$

$I_1 = \int \text{cosec}\, x \cdot \text{cosec}^2 x dx = \text{cosec}\, x(-\cot x) -$

$\int (-\text{cosec}\, x \cot x)(-\cot x) dx$

$I_1 = -\text{cosec}\, x \cot x - \int \text{cosec}\, x (\text{cosec}^2 x - 1) dx$

$I_1 = -\text{cosec}\, x \cot x - I + \int \text{cosec}\, x dx$

$2I_1 = -\text{cosec}\, x \cot x + \ln\left|\tan \frac{x}{2}\right|$

$\Rightarrow \quad 4I = -\cot x (\text{cosec}\, x)^3 - \frac{3}{2}\text{cosec}\, x \cot x$

$+ \frac{3}{2} \ln\left|\tan \frac{x}{2}\right| + C$

$I = -\frac{1}{4}\text{cosec}\, x \cot x \left(\text{cosec}^2 x + \frac{3}{2}\right) + \frac{3}{8}\ln\left|\tan \frac{x}{2}\right| + C$

$\Rightarrow \quad \alpha = -\frac{1}{4}, \ \beta = \frac{3}{8} \ \Rightarrow \ 8(\alpha + \beta) = 1$

25. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is ______

Answer (5626)

Sol.

Group A	Group B	Ways	
$4m$	$4w$	${}^4C_4 \cdot {}^4C_4$	= 1
$3m + 1w$	$1m + 3w$	${}^4C_1 \cdot {}^5C_1 \cdot {}^5C_1 {}^4C_3$	= 400
$2m + 2w$	$2m + 2w$	${}^4C_2 \cdot {}^5C_2 {}^5C_2 {}^4C_2$	= 3600
$1m + 3w$	$3m + w$	${}^4C_1 {}^5C_3 {}^5C_3 {}^4C_1$	= 1600
$4w$	$4m$	${}^5C_4 {}^5C_4$	= 25

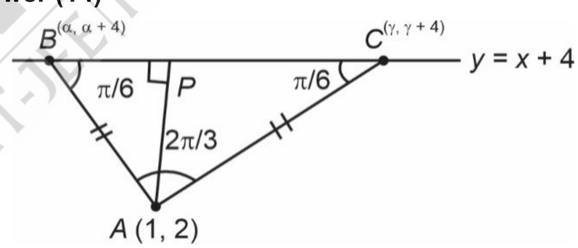
Total ways = 1 + 400 + 3600 + 1600 + 25

= 5626

26. Consider a triangle ABC having the vertices $A(1, 2)$, $B(\alpha, \beta)$ and $C(\gamma, \delta)$ and angles $\angle ABC = \frac{\pi}{6}$ and $\angle BAC = \frac{2\pi}{3}$. If the points B and C lie on the line $y = x + 4$, then $\alpha^2 + \gamma^2$ is equal to ______.

Answer (14)

Sol.



$P = \frac{|2 - 1 - 4|}{\sqrt{1^2 + 1^2}} = \frac{3}{\sqrt{2}}$

$\sin\left(\frac{\pi}{6}\right) = \frac{3/\sqrt{2}}{AB} = \frac{1}{2} \quad \Rightarrow \quad AB = \frac{6}{\sqrt{2}}$

$\Rightarrow \quad (\alpha - 1)^2 + (\alpha + 4 - 2)^2 = 18$

$\Rightarrow\ 2\alpha^2 + 2\alpha - 13 = 0 \rightarrow \alpha$ and γ satisfy same equation

$2x^2 + 2x - 13 = 0 \begin{smallmatrix}\nearrow \alpha \\ \searrow \gamma\end{smallmatrix}$

$\Rightarrow\ \alpha^2 + \gamma^2 = (\alpha + \gamma)^2 - 2\alpha\gamma$

$= (-1)^2 - 2\left(\frac{-13}{2}\right) = 1 + 13 = 14$

27. In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that team loses. If the probability $P(|x - y| \le 2)$ is p, then $3^9 p$ equals ________.

Answer (8288)

Sol. $x + y = 10$

$A = x - y$

$P(|A| < 2)$ is P

$\Rightarrow\ |A| = 2, 1, 0 \Rightarrow A = 0, 1, -1, 2, -2$

$\Rightarrow\ x = \frac{10 + A}{2} \Rightarrow A \in$ even as $x \in$ integer

$\Rightarrow A = 0, -2, 2$

$\Rightarrow P(|A| \le 2) = P(A = 0) + P(A = -2) + P(A = 2)$

(1) $A = 0 \Rightarrow x = 5 = y$

$P(A = 0) = {}^{10}C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^5$

(2) $A = -2$

$\Rightarrow x = 4$ and $y = 6$

$P(A = -2) = {}^{10}C_4 \cdot \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^6$ and

Similarly, $P(A = 2) = {}^{10}C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^4$

$\Rightarrow P(|A| \le 2)3^9 = 3\left({}^{10}C_5 \cdot 2^5 + {}^{10}C_4 \cdot 2^6 + {}^{10}C_6 \cdot 2^4\right)$

$= 8288$

28. Let $y = y(x)$ be the solution of the differential equation $(x + y + 2)^2\, dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$, $\gamma, \delta \in \mathbb{Z}$, then $\gamma + \delta$ equals ________

Answer (31)

Sol. $\frac{dy}{dx} = (x + y + z)^2$

Put $x + y + z = t$

$\Rightarrow\ 1 + \frac{dy}{dx} = \frac{dt}{dx}$

Given DE $\Rightarrow \frac{dt}{dx} - 1 = t^2$

$\Rightarrow \frac{dt}{1 + t^2} = dx \ \Rightarrow \tan^{-1} t = x + c$

$\Rightarrow x + y + z = \tan(x + c)$

$\Rightarrow y(x) = \tan(x + c) - x - 2$

$\because\ y(0) = -2 \ \Rightarrow -2 = \tan c - 0 - 2$

$\Rightarrow\ c = 0$

$\Rightarrow\ y(x) = \tan x - x - 2$

$\frac{dy}{dx} = \sec^2 x - 1 \ge 0$

$\Rightarrow\ y(x)$ is increasing if $x \in \left(0, \frac{\pi}{3}\right)$

$\Rightarrow\ \alpha = y\left(\frac{\pi}{3}\right),\ \beta = y(0)$

$\Rightarrow\ \alpha = -\frac{\pi}{3} - 2 + \sqrt{3}$ and $\beta = -2$

Now, $(3\alpha + \pi)^2 + \beta^2 = (6 + 3\sqrt{3})^2 + (-2)^2$

$= 67 - 36\sqrt{3} = y + \delta\sqrt{3}$.

$\Rightarrow\ \gamma + \delta = 31$

29. Let A be a 2 × 2 symmetric matrix such that $A\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}3\\7\end{bmatrix}$ and the determinant of A be 1. If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2 × 2, then $\alpha + \beta$ equals _____

Answer (5)

Sol. Let $A=\begin{bmatrix}a & b\\ b & c\end{bmatrix}$

$|A| = 1 \Rightarrow ac - b^2 = 0 \quad \ldots\text{(i)}$

Given $\begin{bmatrix}a & b\\ b & c\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}3\\7\end{bmatrix}$

$\Rightarrow \; a + b = 3 \quad \ldots\text{(ii)}$

and $b + c = 7 \quad \ldots\text{(iii)}$

from (i), (ii) and (iii) $a = 1$, $b = 2$, $c = 5$

$\Rightarrow \; A=\begin{bmatrix}1 & 2\\ 2 & 5\end{bmatrix} \Rightarrow A^{-1}=\begin{bmatrix}5 & -2\\ -2 & 1\end{bmatrix}$

Given $A^{-1} = \alpha A + \beta I$

$\Rightarrow \begin{bmatrix}5 & -2\\ -2 & 1\end{bmatrix}=\alpha\begin{bmatrix}1 & 2\\ 2 & 5\end{bmatrix}+\beta\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

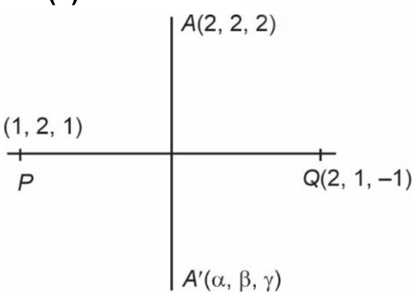
$\Rightarrow \; \alpha = -1$ and $\beta = 6$

$\alpha + \beta = 5$

30. Consider a line L passing through the points $P(1, 2, 1)$ and $Q(2, 1, -1)$. If the mirror image of the point $A(2, 2, 2)$ in the line L is (α, β, γ), then $\alpha + \beta + 6\gamma$ is equal to _______.

Answer (6)

Sol.



$A(2, 2, 2)$

$PQ: \dfrac{x-1}{1}=\dfrac{y-2}{-1}=\dfrac{z-1}{-2}$

General point,

$(k + 1, -k + 2, -2k + 1)$

$\overrightarrow{OA}=(k+1-2)\hat{i}+(-k+2-2)\hat{j}+(-2k+1-2)\hat{k}$

$\overrightarrow{OA}=(k-1)\hat{i}-k\hat{j}+(-2k-1)\hat{k}$

$\overrightarrow{PQ}=1\hat{i}-\hat{j}-2\hat{k}$

$\overrightarrow{OA}\cdot\overrightarrow{PQ}=0$

$(k-1)+k+2(2k+1)=0$

$k-1+k+4k+2=0$

$6k+1=0$

$\boxed{k=\dfrac{-1}{6}}$

$0\left(\dfrac{-1}{6}+1, \dfrac{+1}{6}+2, -2\left(\dfrac{-1}{6}\right)+1\right)$

$0\left(\dfrac{5}{6}, \dfrac{13}{6}, \dfrac{-8}{6}\right)$

$0\left(\dfrac{5}{6}, \dfrac{13}{6}, \dfrac{8}{6}\right)=\left(\dfrac{\alpha+2}{2}, \dfrac{\beta+2}{2}, \dfrac{\gamma+2}{2}\right)$

$\alpha+2=\dfrac{10}{6}$

$\alpha=\dfrac{10}{6}-2$

$\alpha=\dfrac{-2}{6}$

$\boxed{\alpha=-\dfrac{1}{3}}$

$\beta+2=\dfrac{26}{6}$

$\beta=\dfrac{26-12}{6}$

$\beta=\dfrac{14}{6}$

$\beta=\dfrac{7}{3}$

$\gamma+2=\dfrac{16}{6}$

$\gamma=\dfrac{16-12}{6}$

$\gamma=\dfrac{4}{6}$

$\Rightarrow \; \alpha + \beta + 6\gamma$

$\Rightarrow \; \dfrac{-1}{3}+\dfrac{7}{3}+6\times\dfrac{4}{6}$

$\Rightarrow \; 2+4=6$

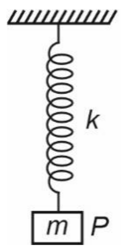
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. In simple harmonic motion, the total mechanical energy of given system is E. If mass of oscillating particle P is doubled then the new energy of the system for same amplitude is



(1) $\frac{E}{\sqrt{2}}$ (2) $E\sqrt{2}$

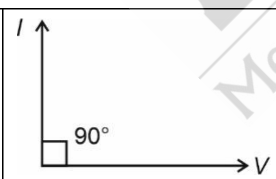
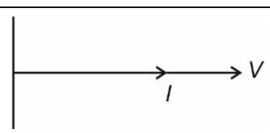
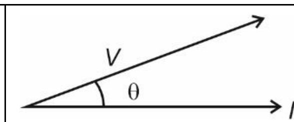
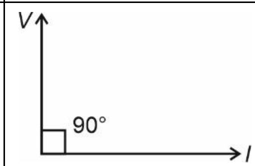
(3) $2E$ (4) E

Answer (4)

Sol. $\because \quad E = \frac{1}{2}kA^2$

Here energy depends on A and k and not on mass.

32. Match List I with List II.

	LIST I		LIST II
A.	Purely capacitive circuit	I.	
B.	Purely inductive circuit	II.	
C.	LCR series at resonance	III.	
D.	LCR series circuit	IV.	

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III (2) A-I, B-IV, C-III, D-II

(3) A-IV, B-I, C-III, D-II (4) A-I, B-IV, C-II, D-III

Answer (4)

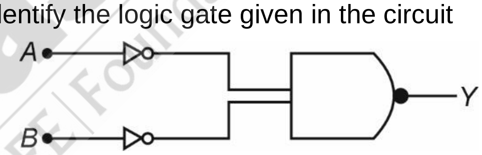
Sol. For pure capacitive circuit, I lead by 90° to V

For pure inductive circuit, V lead by 90° to I

At series LCR resonance, I and V are in same phase.

For LCR series circuit, V and I may suffer some phase difference.

33. Identify the logic gate given in the circuit



(1) NOR gate (2) OR-gate

(3) AND gate (4) NAND-gate

Answer (2)

Sol.

A	B	y
0	0	0
1	0	1
0	1	1
1	1	1

So, $Y = A + B$

34. Correct formula for height of a satellite from earths surface is

(1) $\left(\frac{T^2R^2g}{4\pi}\right)^{1/2}-R$

(2) $\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3}-R$

(3) $\left(\frac{T^2R^2}{4\pi^2g}\right)^{1/3}-R$

(4) $\left(\frac{T^2R^2g}{4\pi^2}\right)^{-1/3}+R$

Answer (2)

Sol. $T=2\pi\sqrt{\frac{r^3}{GM}}$

$$T^2=\frac{4\pi^2}{GM}(R+h)^3$$

$$h=\left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}-R$$

$$=\left(\frac{GM\cdot R^2}{R^2}\cdot\frac{T^2}{4\pi^2}\right)^{\frac{1}{3}}-R$$

$$=\left(\frac{T^2R^2g}{4\pi^2}\right)^{\frac{1}{3}}-R$$

35. A 2 kg brick begins to slide over a surface which is inclined at an angle of 45° with respect to horizontal axis. The co-efficient of static friction between their surfaces is

(1) 0.5　　(2) $\frac{1}{\sqrt{3}}$

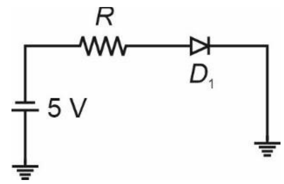
(3) 1.7　　(4) 1

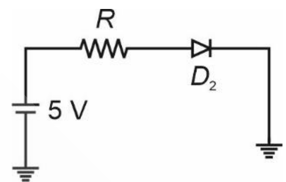
Answer (4)

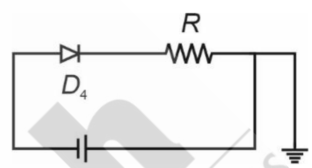
Sol. $mg\sin\theta=\mu.mg\cos\theta$

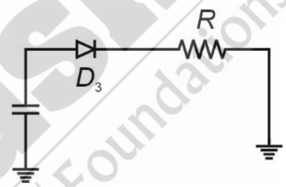
$\Rightarrow\ \tan\theta=\mu=1$

36. Which of the diode circuit shows correct biasing used for the measurement of dynamic resistance of $p-n$ junction diode:

(1)


(2)


(3)


(4)


Answer (2)

Sol. In the given diagram, only the diode given in option (2) is forward biase, so this circuit can be used to measure dynamic resistance of p-n junction diode.

37. An electric bulb rated 50 W – 200 V is connected across a 100 V supply. The power dissipation of the bulb is

(1) 12.5 W　　(2) 100 W

(3) 50 W　　(4) 25 W

Answer (1)

Sol. $R = \frac{V^2}{P} = \frac{200 \times 200}{50} = 800\ \Omega$

So power at given voltage,

$$P = \frac{100 \times 100}{800} = 12.5\text{ W}$$

38. The translational degrees of freedom (f_t) and rotational degrees of freedom (f_r) of CH_4 molecule are

(1) $f_t = 2$ and $f_r = 3$

(2) $f_t = 3$ and $f_r = 2$

(3) $f_t = 3$ and $f_r = 3$

(4) $f_t = 2$ and $f_r = 2$

Answer (3)

Sol. For non-linear polyatomic molecules, both translational and rotational degree of freedom have same value and is equal to 3.

39. The magnetic moment of a bar magnet is 0.5 Am^2. It is suspended in a uniform magnetic field of 8×10^{-2} T. The work done in rotating it from its most stable to most unstable position is

(1) 8×10^{-2} J (2) 4×10^{-2} J

(3) Zero (4) 16×10^{-2} J

Answer (1)

Sol. $W = \int_0^{180} d\tau \cdot d\theta$

$= mB(\cos 0 - \cos 180)$

$= 0.5 \times 8 \times 10^{-2}(2)$

$= 8 \times 10^{-2}$ J

40. Given below are two statements :

Statement I : The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well.

Statement II : The rise of a liquid in a capillary tube does not depend on the inner radius of the tube.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (3)

Sol. The rise in capillary is given as $h = \frac{2T\cos\theta}{2\rho r}$

i.e., $h \propto \frac{1}{r}$

So, statement-II is incorrect.

41. Applying the principle of homogeneity of dimensions, determine which one is correct, where T is time period, G is gravitational constant, M is mass, r is radius of orbit.

(1) $T^2 = \frac{4\pi^2 r}{GM^2}$ (2) $T^2 = 4\pi^2 r^3$

(3) $T^2 = \frac{4\pi^2 r^3}{GM}$ (4) $T^2 = \frac{4\pi^2 r^2}{GM}$

Answer (3)

Sol. $[G] = M^{-1}L^3T^{-2}$

$[M] = M$

$[r] = L$

$[T^2] = T^2$

$$\Rightarrow \frac{4\pi^2 r^3}{Gm} = \frac{L^3}{M^{-1}L^3T^{-2}M} = T^2$$

42. A charge q is placed at the centre of one of the surface of a cube. The flux linked with the cube is:

(1) $\frac{q}{8 \epsilon_0}$ (2) $\frac{q}{2 \epsilon_0}$

(3) Zero (4) $\frac{q}{4 \epsilon_0}$

Answer (2)

Sol. Charge inside the cube, $q_{in} = \frac{q}{2}$

So flux through surface, $\phi = \frac{q}{2 \epsilon_0}$

43. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Number of photons increases with increase in frequency of light.

Reason R : Maximum kinetic energy of emitted electrons increases with the frequency of incident radiation.

In the light of above statements, choose the **most appropriate** answer from the options given below.

(1) **A** is not correct but **R** is correct.

(2) **A** is correct but **R** is not correct.

(3) Both **A** and **R** are correct and **R** is **NOT** the correct explanation of **A**.

(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

Answer (1)

Sol. Number of photons independent on frequency of light. But K.E. of emitted electrons increases on increasing frequency of incident light.

44. The width of one of the two slits in a Young's double slit experiment is 4 times that of the other slit. The ratio of the maximum of the minimum intensity in the interference pattern is:

(1) 4 : 1 (2) 1 : 1

(3) 9 : 1 (4) 16 : 1

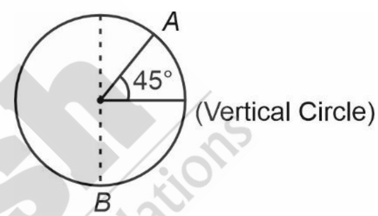
Answer (3)

Sol. $I_{max} = \left(\sqrt{4I_0} + \sqrt{I_0}\right)^2 = \left(3\sqrt{I_0}\right)^2 = 9I_0$

$I_{min} = \left(\sqrt{4I_0} - \sqrt{I_0}\right)^2 = I_0$

$\frac{I_{max}}{I_{min}} = 9:1$

45. A body of m kg slides from rest along the curve of vertical circle from point A to B in friction less path. The velocity of the body at B is:



(given, R = 14 m, g = 10 m/s^2 and $\sqrt{2} = 1.14$)

(1) 16.7 m/s (2) 10.6 m/s

(3) 19.8 m/s (4) 21.9 m/s

Answer (4)

Sol. From energy conservation $\rightarrow$

$mg(R + R\sin 45) = \frac{1}{2}mv^2$

$\Rightarrow \quad 10\left(1 + \frac{1}{\sqrt{2}}\right) \times 14 = \frac{1}{2}v^2$

$\Rightarrow \quad 10\left(1 + \frac{\sqrt{2}}{2}\right) \times 28 = v^2$

$\Rightarrow \quad 10(1 + 0.7) \times 28 = v^2$

$\Rightarrow \quad v = 21.81$

46. According to Bohr's theory, the moment of momentum of an electron revolving in 4th orbit of hydrogen atom is

(1) $2\frac{h}{\pi}$

(2) $8\frac{h}{\pi}$

(3) $\frac{h}{2\pi}$

(4) $\frac{h}{\pi}$

Answer (1)

Sol. From Bohr's quantization,

$$\vec{L} = n\frac{h}{2\pi}$$

$$= 4\frac{h}{2\pi}$$

$$= 2\frac{h}{\pi}$$

47. Arrange the following in the ascending order of wavelength

A. Gamma rays (λ_1)

B. x-rays (λ_2)

C. Infrared waves (λ_3)

D. Microwaves (λ_4)

Choose the **most appropriate** answer from the options given below

(1) $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$

(2) $\lambda_2 < \lambda_1 < \lambda_4 < \lambda_3$

(3) $\lambda_4 < \lambda_3 < \lambda_2 < \lambda_1$

(4) $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$

Answer (4)

Sol. Wavelengths are as

Gamma rays < 1nm

x-ray < (1 – 10)nm

Infrared < ($700 - 10^5$)nm

Microwave < ($10^5 - 10^8$)nm

48. A 90 kg body placed at 2*R* distance from surface of earth experiences gravitational pull of

(*R* = Radius of earth, *g* = 10 ms^{-2})

(1) 100 N

(2) 300 N

(3) 120 N

(4) 225 N

Answer (1)

Sol. $F = 90 \times \frac{Gm}{9R^2}$

$$= \frac{900}{9}$$

= 100 N

49. A sample of gas at temperature *T* is adiabatically expanded to double its volume. Adiabatic constant for the gas is γ = 3/2. The work done by the gas in the process is

(μ = 1 mole)

(1) $RT\left[1-2\sqrt{2}\right]$

(2) $RT\left[2\sqrt{2}-1\right]$

(3) $RT\left[\sqrt{2}-2\right]$

(4) $RT\left[2-\sqrt{2}\right]$

Answer (4)

Sol. $w = \dfrac{-nR}{\gamma - 1}(\Delta T)$

$= \dfrac{-R}{1/2}\left(\dfrac{T}{\sqrt{2}} - T\right)$

$= 2R\left(\dfrac{\sqrt{2}T - T}{\sqrt{2}}\right)$

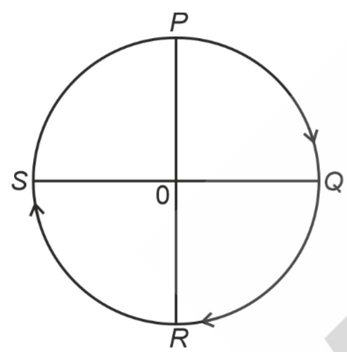
$= RT\left(2 - \sqrt{2}\right)$

$\therefore \quad TV\gamma^{-1} = \text{cons.}$

$TV\gamma^{-1} = T_f(2V)^{\gamma - 1}$

$T_f = \dfrac{T}{\sqrt{2}}$

50. A cyclist starts from the point P of a circular ground of radius 2 km and travels along its circumference to the point S. The displacement of a cyclist is



(1) 4 km

(2) $\sqrt{8}$ km

(3) 6 km

(4) 8 km

Answer (2)

Sol. Displacement = $\sqrt{2}R$

$= \sqrt{8}$ km

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

51. The disintegration energy Q for the nuclear fission of ${}^{235}U \rightarrow {}^{140}Ce + {}^{94}Zr + n$ is ________ MeV.

Given atomic masses of

${}^{235}U : 235.0439u$; ${}^{140}Ce : 139.9054u$,

${}^{94}Zr : 93.9063u$; $n : 1.0086u$,

Value of c^2 = 931 *MeV/u*.

Answer (208)

Sol. Q. value

$= \{(235.0439) - [39.9054 + 93.9063 + 1.0086]\} \times 931 \text{ MeV}$

= 208 MeV

52. A light ray is incident on a glass slab of thickness $4\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence is equal to the critical angle for the glass slab with air. The lateral displacement of ray after passing through glass slab is ______ cm.

(Given sin15° = 0.25)

Answer (2)

Sol. $\mu = \sqrt{2}$

$\sin\theta_C = \dfrac{1}{\sqrt{2}}$

$Q_C = 45°$

$i = Q_C = 45°$

(ϕ) lateral displacement = $= \dfrac{t\sin(i-r)}{\cos r}$

$\sin 45^\circ = \sqrt{2}\ \sin r$

$\Rightarrow\ r = 30^\circ$

$\therefore\ d = \dfrac{4\sqrt{3}\sin(45^\circ - 30^\circ)}{\cos 30^\circ}$

$= \dfrac{4\sqrt{3}\times\frac{1}{4}}{\frac{\sqrt{3}}{2}} = 2$

53. Two wires A and B are made up of the same material and have the same mass. Wire A has radius of 2.0 mm and wire B has radius of 4.0 mm. The resistance of wire B is 2Ω. The resistance of wire A is _______ Ω.

Answer (32)

Sol. $R = \rho\dfrac{l}{A} = \rho\dfrac{V}{A^2}$

and $\pi r_1^2 l_1 = \pi r_2^2 l_2$

$A_1 l_1 = A_2 l_2$

So $\dfrac{R_1}{R_2} = \left(\dfrac{A_2}{A_1}\right)^2$

$\Rightarrow\ \dfrac{R}{2} = \left(\dfrac{r_2}{r_1}\right)^4$

$\Rightarrow\ R = 32$

54. In a system two particles of masses m_1= 3 kg and m_2 = 2 kg are placed at certain distance from each other. The particle of mass m_1 is moved towards the center of mass of the system through a distance 2 cm. In order to keep the center of mass of the system at the original position, the particle of mass m_2 should move towards the center of mass by the distance _______ cm.

Answer (3)

Sol. $m_1\Delta x_1 = m_2\Delta x_2$

$\Rightarrow\ 3 \times 2\text{ cm} = 2\Delta x_2$

$\Rightarrow\ \Delta x_2 = 3\text{ cm}$

55. A bus moving along a straight highway with speed of 72 km/h is brought to halt within 4 s after applying the brakes. The distance travelled by the bus during this time (Assume the retardation is uniform) is _______ m.

Answer (40)

Sol. $u = 72\times\dfrac{5}{18} = 20\text{ m/s}$

$v = 0$

$t = 4$

$\Rightarrow\ 0 = 20 + 4a$

$\Rightarrow\ a = -5\text{ m/s}^2$

$\therefore\ S = 20\times 4 - \dfrac{1}{2}\times 5\times 16$

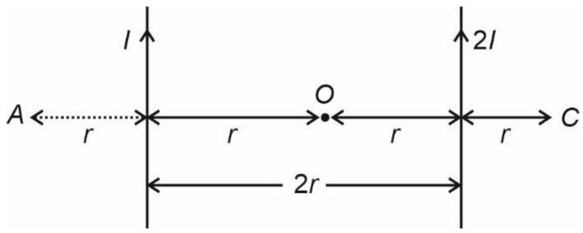
$= 40\text{ m}$

56. A rod of length 60 cm rotates with a uniform angular velocity 20 rad s^{-1} about its perpendicular bisector, in a uniform magnetic filed 0.5 T. The direction of magnetic field is parallel to the axis of rotation. The potential difference between the two ends of the rod is _________ V.

Answer (0)

Sol. Both end having same potential, so potential difference between end will be zero.

57. Two parallel long current carrying wire separated by a distance $2r$ are shown in the figure. The ratio of magnetic field at A to the magnetic field produced at C is $\frac{x}{7}$. The value of x is ___________ .



Answer (5)

Sol. At point A

$$B_A = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 (2I)}{2\pi(3r)}$$

At point C

$$B_C = \frac{\mu_0 I}{2\pi(3r)} + \frac{\mu_0 (2I)}{r}$$

$$\Rightarrow \frac{B_A}{B_C} = \frac{5}{7}$$

58. Mercury is filled in a tube of radius 2 cm up to a height of 30 cm. The force exerted by mercury on the bottom of the tube is ________ N.

(Given, atmospheric pressure = 10^5 Nm^{-2}, density of mercury = 1.36×10^4 kg m^{-3}, g = 10 m s^{-2}, $\pi = \frac{22}{7}$)

Answer (177)

Sol. $F = (p_0 + \rho g h)\,A$

$$= \left(10^5 + 1.36 \times 10^4 \times 10 \times \frac{3}{10}\right)\frac{22}{7}\left(\frac{2}{100}\right)^2$$

= 177 N

59. The displacement of a particle executing SHM is given by $x = 10\sin\left(\omega t + \frac{\pi}{3}\right)$ m. The time period of motion is 3.14 s. The velocity of the particle at $t = 0$ is _________ m/s.

Answer (10)

Sol. At $t = 0$, $x = 10\sin\frac{\pi}{3}$

$$= 10 \times \frac{\sqrt{3}}{2}$$

$$= 5\sqrt{3}$$

$$\omega = \frac{2\pi}{T}$$

= 2 rad/s

$$\therefore \quad v = \omega\sqrt{A^2 - x^2}$$

$$= 2\sqrt{100 - 75}$$

= 10 m/s

60. A parallel plate capacitor of capacitance 12.5 pF is charged by a battery connected between its plates to potential difference of 12.0 V. The battery is now disconnected and a dielectric slab (ϵ_r = 6) is inserted between the plates. The change in its potential energy after inserting the dielectric slab is _________ 10^{-12} J.

Answer (750)

Sol. $E_1 = \frac{1}{2}\left(\frac{25}{2}\right) \times 10^{-12} \times 144$

= 900×10^{-12} J

$$E_2 = \frac{1}{2}\left(6 \times \frac{25}{2} \times 10^{-12}\right)\left(\frac{12}{6}\right)^2 = 150 \times 10^{-12} \text{ J}$$

$\Delta E = 750 \times 10^{-12}$ J

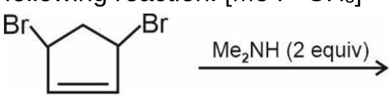
CHEMISTRY

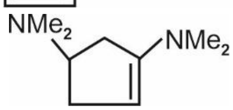
SECTION - A

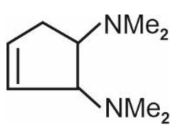
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

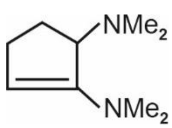
Choose the correct answer :

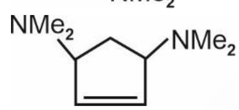
61. Find out the major product formed from the following reaction. [Me : $-CH_3$]



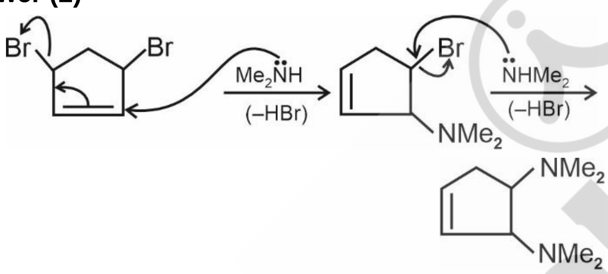
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

62. Given below are two statements :

Statement I : The correct order of first ionization enthalpy values of Li, Na, F and Cl is Na < Li < Cl < F.

Statement II : The correct order of negative electron gain enthalpy values of Li, Na, F and Cl is Na < Li < F < Cl

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Answer (3)

Sol. First ionization enthalpy of F > Cl as first IE decreases down the group.

Similarly, first IE of Li > Na.

So, statement-I is correct.

Electron gain enthalpy of given elements are negative. Considering the magnitude the given order is correct.

Thus, statement-II is correct

63. For a strong electrolyte, a plot of molar conductivity against $(\text{concentration})^{1/2}$ is a straight line, with a negative slope, the correct unit for the slope is

(1) $S\ cm^2\ mol^{-1}\ L^{1/2}$

(2) $S\ cm^2\ mol^{-3/2}\ L^{1/2}$

(3) $S\ cm^2\ mol^{-3/2}\ L^{-1/2}$

(4) $S\ cm^2\ mol^{-3/2}\ L$

Answer (2)

Sol. Units of molar conductivity = $S\ cm^2\ mol^{-1}$

Units of $\sqrt{\text{concentration}}$ = $mol^{1/2}\ L^{-1/2}$

$$\text{Units for slope} = \frac{\text{units of molar conductivity}}{\text{units of }\sqrt{\text{concentration}}}$$

$$= S\ cm^2\ mol^{-3/2}\ L^{1/2}$$

64. When MnO_2 and H_2SO_4 is added to a salt (A), the greenish yellow gas liberated as salt (A) is :

(1) CaI_2

(2) NaBr

(3) KNO_3

(4) NH_4Cl

Answer (4)

Sol. Cl^- in NH_4Cl is oxidized to Cl_2 gas which is greenish yellow.

65. Match **List I** with **List II**

	LIST I		LIST II
A.	α-Glucose and α-Galactose	I.	Functional isomers
B.	α-Glucose and β-Glucose	II.	Homologous
C.	α-Glucose and α-Fructose	III.	Anomers
D.	α-Glucose and α-Ribose	IV.	Epimers

Choose the **correct** answer from the options give below :

(1) A-III, B-IV, C-I, D-II

(2) A-IV, B-III, C-I, D-II

(3) A-IV, B-III, C-II, D-I

(4) A-III, B-IV, C-II, D-I

Answer (2)

Sol. α-Glucose and α-Galactose differ in configuration of one asymmetric carbon. Thus they are epimers.

α and β-glucose differ at anomeric carbon. Thus they are anomers.

α-Glucose and α-Fructose have different functional groups. Thus they are functional isomers.

66. The adsorbent used in adsorption chromatography is/are-

A. Silica gel

B. Alumina

C. Quick lime

D. Magnesia

Choose the **most appropriate** answer from the options given below :

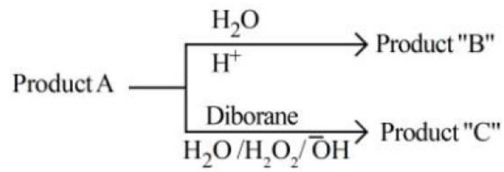
(1) A only (2) B only

(3) A and B only (4) C and D only

Answer (3)

Sol. Commonly used adsorbents in adsorption chromatography are silica gel and alumina.

67. $CH_3-CH_2-CH_2-Br+NaOH\xrightarrow{C_2H_5OH}$ Product 'A'



Consider the above reactions, identify product B and product C.

(1) B = 1-Propanol C = 2-Propanol

(2) B = 2-Propanol C = 1-Propanol

(3) B = C = 2-Propanol

(4) B = C = 1-Propanol

Answer (2)

Sol. Product 'A' is $CH_3-CH=CH_2$

$$CH_3-CH=CH_2\xrightarrow[H^+]{H_2O}CH_3-\underset{}{\overset{OH}{CH}}-CH_3\ (B)$$

2-Propanol

$$CH_3-CH=CH_2\xrightarrow[H_2O/H_2O_2/OH^-]{Diborane}CH_3-CH_2-CH_2-OH\ (C)$$

1-Propanol

68. A first row transition metal in its +2 oxidation state has a spin-only magnetic moment value of 3.86 BM. The atomic number of the metal is

(1) 23 (2) 26

(3) 22 (4) 25

Answer (1)

Sol. $\mu=3.86=\sqrt{n(n+2)}$

$\Rightarrow$ n = 3

Electronic configuration of ion could be $[Ar]3d^3$.

So, E.C. of atom could be $[Ar]\ 3d^34s^2$

$\Rightarrow$ Atomic number = 23

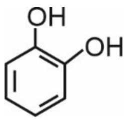
69. Common name of Benzene-1, 2-diol is

(1) Resorcinol (2) Catechol

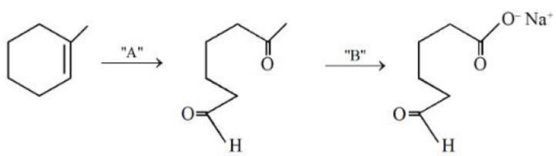
(3) o-cresol (4) Quinol

Answer (2)

Sol. Benzene-1, 2-diol is also called catechol



70.



In the above chemical reaction sequence "A" and "B" respectively are

(1) H_2O, H^+ and $NaOH_{(alc)}/I_2$

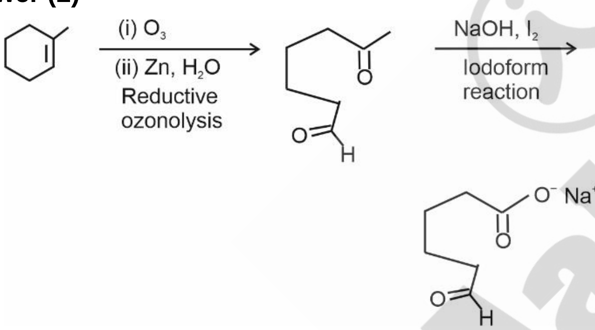
(2) O_3, Zn/H_2O and $NaOH_{(alc)}/I_2$

(3) O_3, Zn/H_2O and $KMnO_4$

(4) H_2O, H^+ and $KMnO_4$

Answer (2)

Sol.



71. The correct order of the first ionization enthalpy is

(1) B > Al > Ga

(2) Tl > Ga > Al

(3) Ga > Al > B

(4) Al > Ga > Tl

Answer (2)

Sol. $I.E_1$ of Tl = 589 kJ/mol

$I.E_1$ of Ga = 579 kJ/mol

$I.E_1$ of Al = 577 kJ/mol

72. Fuel cell, using hydrogen and oxygen as fuels,

A. has been used in spaceship

B. has as efficiency of 40% to produce electricity

C. uses aluminum as catalysts

D. is eco-friendly

E. is actually a type of Galvanic cell only

Choose the **correct** answer from the options given below:

(1) A, B, C only

(2) A, B, D, E only

(3) A, B, D only

(4) A, D, E only

Answer (4)

Sol. Fuel cells produce electricity with an efficiency of about 70%.

Fuel cells are pollution free, thus, eco-friendly.

Fuel cells are type of Galvanic cells only.

73. The equilibrium constant for the reaction

$$SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$$

is $K_c = 4.9 \times 10^{-2}$. The value of K_c for the reaction given below is

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g) \text{ is}$$

(1) 41.6 (2) 4.9

(3) 416 (4) 49

Answer (3)

Sol. The reaction

$$2SO_2 + O_2 \rightleftharpoons 2SO_3$$

can be formed from the given reaction by reverting it and multiplying coefficients by 2.

$$\text{Thus, } K_c' = K_c^{-2} = \frac{1}{K_c^2} = \left(\frac{1}{4.9 \times 10^{-2}}\right)^2$$

$$= 416$$

74. If an iron (III) complex with the formula $[Fe(NH_3)_x(CN)_y]^-$ has no electron in its e_g orbital, then the value of x + y is

(1) 3 (2) 5

(3) 4 (4) 6

Answer (4)

Sol. Balancing charges,

$3 - y = -1$

$\Rightarrow$ $y = 4$

$x = 2$

$x + y = 6$

75. Choose the **Incorrect** Statement about Dalton's Atomic Theory

(1) All the atoms of a given element have identical properties including identical mass

(2) Matter consists of indivisible atoms

(3) Chemical reactions involve reorganization of atoms

(4) Compounds are formed when atoms of different elements combine in any ratio

Answer (4)

Sol. According to Dalton's theory, compounds are formed when atoms of different elements combine in fixed ratio.

76. The number of species from the following that have pyramidal geometry around the central atom is______

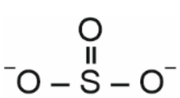
$S_2O_3^{2-}$, SO_4^{2-}, SO_3^{2-}, $S_2O_7^{2-}$

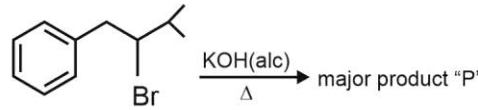
(1) 2 (2) 3

(3) 1 (4) 4

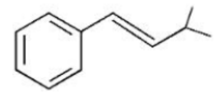
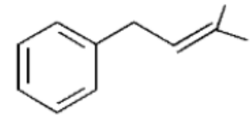
Answer (3)

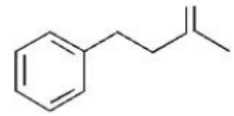
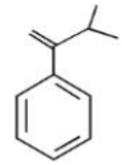
Sol. SO_3^{2-} is the only species with pyramidal geometry.



77. 

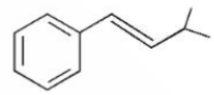
Product P is

(1)  (2) 

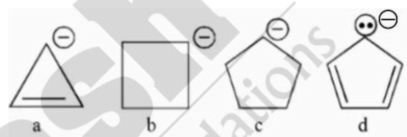
(3)  (4) 

Answer (1)

Sol. More stable double bond will be formed.

In , double bond is in resonance with the ring, thus, more stable.

78. Correct order of stability of carbanion is



(1) d > a > c > b (2) d > c > b > a

(3) c > b > d > a (4) a > b > c > d

Answer (2)

Sol. (d) is aromatic. So it is most stable.

(a) is anti-aromatic. So it is least stable.

79. the number of unpaired d-electrons in $[Co(H_2O)_6]^{3+}$ is________.

(1) 2 (2) 4

(3) 1 (4) 0

Answer (4)

Sol. $[Co(H_2O)_6]^{3+}$ is an inner orbital complex.

So, electronic configuration is $t_{2g}^6 e_g^0$.

All electrons are paired.

80. The correct statement/s about Hydrogen bonding is/are

A. Hydrogen bonding exists when H is covalently bonded to the highly electro negative atom

B. Intermolecular H bonding is present in o-nitro phenol

C. Intramolecular H bonding is present in HF

D. The magnitude of H bonding depends on the physical state of the compound

E H-bonding has powerful effect on the structure and properties of compounds

Choose the **correct** answer from the options given below.

(1) A, D, E only (2) A, B, D only

(3) A only (4) A, B, C only

Answer (1)

Sol. In o-nitrophenol intra molecular hydrogen bonding is present.

In HF intermolecular hydrogen bonding is present.

Other statements are correct except B and C.

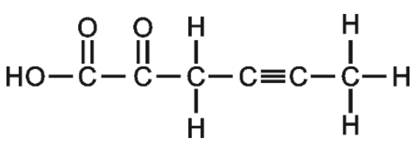
SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

81. The total number of 'sigma' and 'Pi' bonds in 2-oxohex-4-ynoic acid is ______.

Answer (18)

Sol.



Total number of σ & π bonds = 18

82. The maximum number of orbitals which can be identified with n = 4 and m_l = 0 is _____.

Answer (4)

Sol. Possible combination of first three quantum numbers are

n = 4, l = 3, m = 0

n = 4, l = 2, m = 0

n = 4, l = 1, m = 0

n = 4, l = 0, m = 0

83. Three moles of an ideal gas are compressed isothermally from 60 L to 20 L using constant pressure of 5 atm. Heat exchange Q for the compression is _____ Lit/atm.

Answer (200)

Sol. For isothermal process

$Q = -W$

$\Rightarrow Q = -5 \times 40$

$|Q| = +200$ Lit atm

84. A first row transition metal with highest enthalpy of atomisation, upon reaction with oxygen at high temperature forms oxides of formula M_2O_n (where n = 3, 4, 5). The 'spin-only' magnetic moment value of the amphoteric oxide from the above oxides is___ BM (near integer)

(Given atomic number : Sc : 21, Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26, Co : 27, Ni : 28, Cu : 29, Zn : 30)

Answer (0)

Sol. Vanadium has highest enthalpy of atomization among first row transition elements.

V_2O_5 is amphoteric

In V^{5+} there are no unpaired electrons.

Thus, $\mu = 0$

85. 2.7 kg of each of water and acetic acid are mixed. The freezing point of the solution will be –x°C. Consider the acetic acid does not dimerise in water, nor dissociates in water. x = ________ (nearest integer)

[Given : Molar mass of water = 18 g mol^{-1}, acetic acid = 60 g mol^{-1}

$K_{f\,H_2O}$: 1.86 K kg mol^{-1}

$K_{f\ \text{acetic acid}}$: 3.90 K kg mol^{-1}

freezing point : H_2O = 273 K, acetic acid = 290 K]

Answer (31)

Sol. Molality of acetic acid = $\frac{2700}{60}\times\frac{1}{2.7}$ mol / kg

$= 16.667$

$\Delta T_f = K_f \times 16.667$

$= 1.86 \times 16.667$

$= 31$ K

86. Number of compounds / species from the following with non-zero dipole moment is _______.

$BeCl_2$, BCl_3, NF_3, XeF_4, CCl_4, H_2O, H_2S, HBr, CO_2, H_2, HCl

Answer (5)

Sol. NF_3, H_2O, H_2S, HBr, HCl have non zero dipole moments.

87. From 6.55 g of aniline, the maximum amount of acetanilide that can be prepared will be _______ × 10^{-1} g.

Answer (95)

Sol. Moles of aniline = $\frac{6.55}{93}$

= 0.0704

= moles of acetanilide

mass of acetanilide = 0.0704 × 135

= 9.504 g

= 95.04 × 10^{-1} g

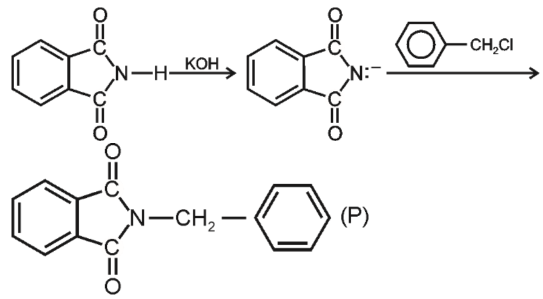
88. Phthalimide is made to undergo following sequence of reactions.

$$\text{Phthalimide} \xrightarrow[\text{(ii) Benzylchloride}]{\text{(i) KOH}} \text{'P'}$$

Total number of π bonds present in product 'P' is/are ______.

Answer (8)

Sol.



Total number of π bonds in P = 8

89. Consider the following reaction, the rate expression of which is given below

A + B → C

rate = $k[A]^{1/2}[B]^{1/2}$

The reaction is initiated by taking 1 M concentration of A and B each. If the rate constant (k) is 4.6×10^{-2} s^{-1}, then the time taken for A to become 0.1 M is _____sec. (nearest integer)

Answer (50)

Sol. A + B → C

$$\frac{-d[A]}{dt} = k[A]^{1/2}[B]^{1/2}$$

Since, [A] = [B]

$$\Rightarrow \frac{-d[A]}{dt} = k[A]$$

$$\Rightarrow kt = \ln\frac{[A]_0}{[A]}$$

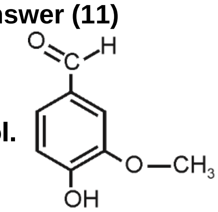
$$\Rightarrow t = \frac{1}{4.6\times10^{-2}}\times\ln\left(\frac{1}{0.1}\right)$$

$$= \frac{2.303}{4.6}\times100 \approx 50$$

90. Vanillin compound obtained from vanilla beans, has total sum of oxygen atoms and π electrons is ________.

Answer (11)

Sol.



Vanillin, shown above, has 8π electrons and 3 oxygen atoms.

Total = 8 + 3 = 11

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