04/04/2024

Morning

Aakash

Medical | IIT-JEE | Foundations

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6) **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let $f(x) = x^5 + 2^{e^x/4}$ for all $x \in$ R. Consider a function $g(x)$ such that $(gof)(x) = x$ for all $x \in$ R. Then the value of $8g'(2)$ is:

   (1) 16 (2) 8
   (3) 4 (4) 2

Answer (1)

Sol. $g(f(n)) = x$

Differentiate w.r.t x, we get

$g'(f(x))\cdot f'(x) = 1$

$g'(f(x)) = \dfrac{1}{f'(x)}$

We need to find $g'(2)$, so,

$f(0) = 2 \Rightarrow f(x) = x^5 + 2e^{x/4}$

$f'(x) = 5x^4 + \dfrac{1}{2}e^{x/4}$

$f'(x) = \dfrac{1}{2}$

$\therefore \quad g'(f(0)) = \dfrac{1}{f'(0)}$

$g'(2) = 2$

$\therefore \quad 8g'(2) = 16$

2. One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \dfrac{1}{x}$ is $\left(\dfrac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be $\dfrac{1}{24}\left(l\sqrt{5}+m\right) - n\log_e\left(1+\sqrt{5}\right)$, where $l, m, n \in N$. Then $l + m + n$ is equal to

   (1) 30 (2) 31
   (3) 32 (4) 29

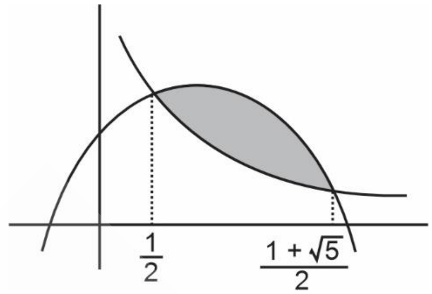
Answer (1)

Sol. Solving curves $y = 1 + 3x - 2x^2$ & $y = \dfrac{1}{x}$

$2x^3 - 3x^2 - x + 1 = 0$

$\Rightarrow \quad (2x - 1)(x^2 - x - 1) = 0$

$\Rightarrow \quad x = \dfrac{1}{2},\ x = \dfrac{1 \pm \sqrt{5}}{2}$



$$\text{Area} = \int_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}} \left(1 + 3x - 2x^2 - \frac{1}{x}\right)dx$$

$$= \left[x + \frac{3x^2}{2} - \frac{2x^3}{3} - \ln x\right]$$

$$= \frac{\sqrt{5}+1}{2} + \frac{3}{8}\left(\sqrt{5}+1\right)^2 - \frac{1}{12}\left(\sqrt{5}+1\right)^3 - \ln\left(\frac{\sqrt{5}+1}{2}\right) - \left(\frac{1}{2} + \frac{3}{8} - \frac{1}{12} - \ln\frac{1}{2}\right)$$

$$= \frac{1}{24}\left[12\left(\sqrt{5}+1\right) + 9\left(\sqrt{5}+1\right)^2 - 2\left(\sqrt{5}+1\right)^3 - 12 - 9 + 2\right] - \ln\left(\frac{\sqrt{5}+1}{2}\times 2\right)$$

$$= \frac{1}{24}\left[12\left(\sqrt{5}+1\right) + 9\left(6+2\sqrt{5}\right) - 2\left(5\sqrt{5}+1+3\sqrt{5}\left(\sqrt{5}+1\right)-19\right)\right] - \ln\left(\sqrt{5}+1\right)$$

$$= \frac{1}{24}\left[14\sqrt{5}+15\right] - \ln\left(\sqrt{5}+1\right)$$

$\therefore \quad l = 14,\ m = 15,\ n = 1$

$l + m + n = 30$

3. If the solution $y = y(x)$ of the differential equation $(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$ satisfies $y(-1) = -\frac{\pi}{4}$, then y(0) is equal to

(1) $-\frac{\pi}{12}$ (2) 0

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

Answer (4)

Sol. $(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$

$$\int dy = \int\left(\frac{2x^2 + 2x + 3}{x^4 + 2x^3 + 3x^2 + 2x + 2}\right)dx$$

$$\int dy = \int \frac{1}{x^2 + 1}dx + \int \frac{1}{x^2 + 2x + 2}dx$$

$y = \tan^{-1}(x) + \tan^{-1}(1 + x) + C$

$y(-1) = \tan^{-1}(-1) + \tan^{-1}(1 - 1) + C$

$y(-1) = -\frac{\pi}{4} + C = \left(\frac{-\pi}{4}\right)$ – {given}

$\Rightarrow \quad C = 0$

So, $y(x) = \tan^{-1}(x) + \tan^{-1}(1 + x)$

$y(0) = \tan^{-1}(0) + \tan^{-1}(1 + 0)$

$y(0) = \frac{\pi}{4}$

4. The vertices of a triangle are $A(-1, 3)$, $B(-2, 2)$ and $C(3, -1)$. A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is:

(1) $-x + y - (2 - \sqrt{2}) = 0$

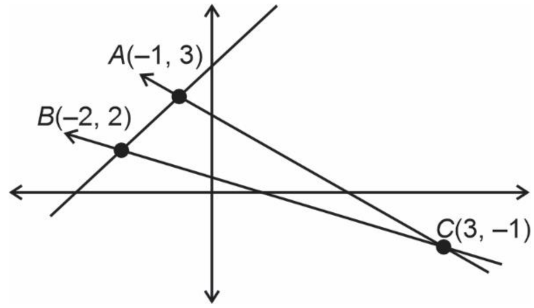
(2) $x + y - (2 - \sqrt{2}) = 0$

(3) $x + y + (2 - \sqrt{2}) = 0$

(4) $x - y - (2 + \sqrt{2}) = 0$

Answer (2)

Sol.



Equation of AC : $x + y = 2$

Equation of AB : $x - y + 4 = 0$

Equation of BC : $3x + 5y = 4$

The line nearest to origin is parallel to AC and inward. Let its equation is $x + y = C$.

$\therefore \quad \left|\frac{C - 2}{\sqrt{2}}\right| = 1$

$\therefore \quad C = 2 - \sqrt{2}$

$\therefore$ required equation line is :

$x + y - (2 - \sqrt{2}) = 0$

5. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is:

(1) $\frac{5}{16}$ (2) $\frac{7}{18}$

(3) $\frac{4}{17}$ (4) $\frac{5}{18}$

Answer (4)

Sol. Urn A contains 7 red, 5 black

Urn B contains 5 red, 7 black

Urn C contains 6 red, 6 black

By Baye's theorem,

$$P\left(\frac{\text{Ball drawn from } A}{\text{Ball drawn black}}\right)$$

$$= \frac{\frac{1}{3}\cdot\frac{5}{12}}{\frac{1}{3}\cdot\frac{5}{12} + \frac{1}{3}\cdot\frac{7}{12} + \frac{6}{12}\cdot\frac{1}{3}} = \frac{5}{18}$$

6. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B of a triangle ABC. Similarly there are 6 points $P_6, P_7, \ldots, P_{11}$ on the side BC and 7 points $P_{12}, P_{13}, \ldots, P_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points $P_1, P_2, \ldots, P_{18}$ as vertices, is:

(1) 776 (2) 771

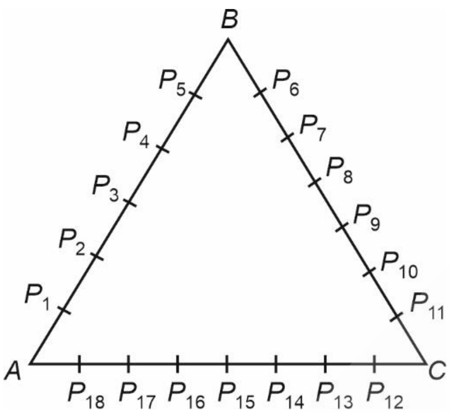
(3) 751 (4) 796

Answer (3)

Sol. Number of points on side AB = 5

Number of points on side BC = 6

Number of points on side AC = 7



Number of ways selecting three points from side

$AB = {}^5C_3$

Number of ways selecting three points from side

$BC = {}^6C_3$

Number of ways selecting three points from side

$AC = {}^7C_3$

Total number of triangle possible formed using the points $P_1P_2\ldots P_{18}$

$= {}^{18}C_3 - {}^5C_3 - {}^6C_3 - {}^7C_3$

$= 816 - 10 - 20 - 35$

$= 751$

7. If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right) + \log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to :

(1) 95 (2) 100

(3) 98 (4) 97

Answer (4)

Sol. $\sin^{-1}\left(\frac{3x-22}{2x-19}\right) + \log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$

$$-1 \le \frac{3x-22}{2x-19} \le 1$$

$$\frac{3x-22}{2x-19} + 1 \ge 0 \text{ and } \frac{3x-22}{2x-19} - 1 \le 0$$

$$\frac{3x-22+2x-19}{2x-19} \ge 0 \text{ and } \frac{3x-22-2x+19}{2x-19} \le 0$$

$$\Rightarrow \frac{5x-41}{2x-19} \ge 0 \text{ and } \frac{x-3}{2x-19} \le 0$$

$$x \in \left(-\infty, \frac{41}{5}\right] \cup \left(\frac{19}{2}, \infty\right) \text{ and } x \in \left[3, \frac{19}{2}\right)$$

$$\Rightarrow x \in \left[3, \frac{41}{5}\right] \quad \ldots(1)$$

$$\text{and, } \frac{3x^2-8x+5}{x^2-3x-10} > 0$$

$$\frac{(3x-5)(x-1)}{(x-5)(x-2)} > 0$$

$$\Rightarrow x \in (-\infty, -2) \cup \left[1, \frac{5}{3}\right] \cup (5, \infty) \ldots(2)$$

Taking intersection of individual domains

$$x \in \left(5, \frac{41}{5}\right]$$

$$\Rightarrow \alpha = 5 \text{ and } \beta = \frac{41}{5}$$

$\Rightarrow$ $3\alpha + 10\beta = 15 + 82$

$= 97$

$\therefore$ Option (4) is correct

8. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0$, $z \in C$. Then $4(\alpha^2 + \beta^2)$ is equal to :

(1) 8 (2) 6

(3) 4 (4) 2

Answer (3)

Sol. $(\bar{z})^2 + |z| = 0$ ...(1)

$z^2 + |\bar{z}| = 0$ ...(2)

From equation (1) and (2)

as $|z| = |\bar{z}|$

$\Rightarrow (\bar{z})^2 = z^2$

$\Rightarrow z = \bar{z}$ or $z = -\bar{z}$

$\Rightarrow$ Im(z) = 0 or Re(z) = 0

Case I : If Im(z) = 0

$\Rightarrow z = x$

Putting value of z in equation (1)

$x^2 + |x| = 0$

$\Rightarrow x = 0$ [Rejected]

Case II : If Re(z) = 0

$\Rightarrow z = iy$

Putting value of z in equation (1)

$-y^2 + |y| = 0$

$y = \pm 1$ as $y \neq 0$

Hence, $z = \pm i$ are the solution of the given equation

$\Rightarrow \alpha = i - i = 0$

and $\beta = i(-i) = 1$

$\Rightarrow 4(\alpha^2 + \beta^2) = 4\,(0 + 1)$

$= 4$

$\therefore$ Option (3) is correct

9. Let $f : R \to R$ be a function given by

$$f(x) = \begin{cases} \dfrac{1-\cos 2x}{x^2}, & x < 0 \\ \alpha, & x = 0 \\ \dfrac{\beta\sqrt{1-\cos x}}{x}, & x > 0 \end{cases}$$

Where $\alpha, \beta \in R$. If f is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to :

(1) 12 (2) 6

(3) 48 (4) 3

Answer (1)

Sol. $$f(x) = \begin{cases} \dfrac{1-\cos 2x}{x^2}, & x < 0 \\ \alpha, & x = 0 \\ \dfrac{\beta\sqrt{1-\cos x}}{x}, & x > 0 \end{cases}$$

$f(x)$ is continuous at $x = 0$

$$\Rightarrow f(0) = \lim_{x\to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$\lim_{x\to 0^-} f(x) = \alpha$$

$$\lim_{x\to 0^-} \left(\frac{1-\cos 2x}{x^2}\right) = \alpha$$

$$\Rightarrow \lim_{x\to 0^-} \frac{2\sin^2 h}{x^2} = \alpha$$

$$\Rightarrow \lim_{h\to 0} \frac{2\sin^2}{h^2} h = \alpha$$

$$\Rightarrow \boxed{\alpha = 2}$$

Also, $\lim_{x\to 0^+} f(x) = f(0)$

$$\Rightarrow \lim_{x\to 0^+} \frac{\beta\sqrt{1-\cos x}}{x} = 2$$

$$\Rightarrow \lim_{h\to 0} \frac{\beta\sqrt{\dfrac{1-\cos h}{h^2} \times h^2}}{h} = 2$$

$\Rightarrow \quad \dfrac{\beta}{\sqrt{2}} = 2$

$\Rightarrow \quad \boxed{\beta = 2\sqrt{2}}$

$\Rightarrow \quad \alpha^2 + \beta^2 = 4 + 8$

$= 12$

$\therefore$ Option (1) is correct

10. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}} + 5^{\frac{1}{3}}\right)^{15}$ is equal to :

(1) 6131 (2) 3133

(3) 931 (4) 633

Answer (2)

Sol. $T_{r+1} = {}^{15}C_r(2^{1/5})^{15-r}(5^{1/3})^r$

$= {}^{15}C_r\, 5^{r/3} 2^{\left(3 - \frac{r}{5}\right)}$

For rational terms,

$\dfrac{r}{3}$ and $\dfrac{r}{5}$ must be integer

3 and 5 divide $r \Rightarrow$ 15 divides $r \Rightarrow r = 0$ and $r = 15$

${}^{15}C_0 5^0 2^3 + {}^{15}C_{15} 5^5 2^{(0)}$

$= 8 + 3125$

$= 3133$

11. Let the sum of the maximum and the minimum values of the function $f(x) = \dfrac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$ be $\dfrac{m}{n}$, where $gcd(m, n) = 1$. Then $m + n$ is equal to

(1) 201 (2) 217

(3) 182 (4) 195

Answer (1)

Sol. $f(x) = \dfrac{2x^2 - 3x + 8}{2x^2 + 3x + 8} = y,\ 2x^2 + 3x + 8 > 0 \quad \forall\ x \in \mathbb{R}$

$\Rightarrow \quad x^2(2y - 2) + x(3y + 3) + 8y - 8 = 0$

Since $x \in \mathbb{R}$, the equation has real roots

$\Rightarrow \quad D \geq 0$

$\Rightarrow \quad (3y + 3)^2 - 4(2y - 2)(8y - 8) \geq 0$

$\Rightarrow \quad 9(y + 1)^2 - 64y(y - 1)^2 \geq 0$

$\Rightarrow \quad (3y + 3)^2 - (8y - 8)^2 \geq 0$

$\Rightarrow \quad (11y - 5)(-5y + 11) \geq 0$

$\Rightarrow \quad \left(y - \dfrac{5}{11}\right)\left(y - \dfrac{11}{5}\right) \leq 0$

$\Rightarrow \quad y \in \left[\dfrac{5}{11}, \dfrac{11}{5}\right]$

Sum of maximum and minimum value

$y_{max} + y_{min} = \dfrac{5}{11} + \dfrac{11}{5} = \dfrac{25 + 121}{55}$

$= \dfrac{146}{55} = \dfrac{m}{n} \Rightarrow m + n = 201$

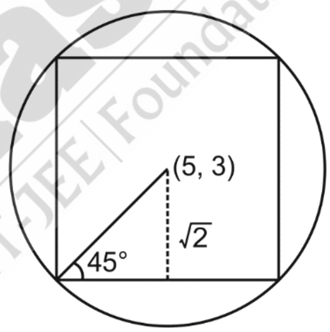
12. A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\Sigma(x_i^2 + y_i^2)$ is equal to :

(1) 160 (2) 148

(3) 152 (4) 156

Answer (3)

Sol.



One side of square is $y = x + k$

Distance of (5, 3) to the line $y = x + k$ is

$\dfrac{|3 - 5 - k|}{\sqrt{2}} = \sqrt{2}$

$= |-2 - k| = 2$

$\Rightarrow \quad k = 0$ or $k = -4$

So lines are $y = x$ and $y = x - 4$

Now, solving these lines with circle

$y = x$ and $x^2 + y^2 - 10x - 6y + 30 = 0$

$\Rightarrow\ 2x^2 - 16x + 30 = 0$

$\Rightarrow\ x = 3, y = 3$

$x = 5, y = 5$

$y = x - 4$ and $x^2 + y^2 - 10x - 6y + 30 = 0$

$\Rightarrow\ x = 5, y = 1$

$x = 7, y = 3$

$$\sum_{i=1}^{4} x_i^2 + y_i^2 = 9+9+25+25+25+1+49+9$$

$= 152$

13. Let a unit vector which makes an angle of 60° with $2\hat{i}+2\hat{j}-\hat{k}$ and an angle of 45° with $\hat{i}-\hat{k}$ be $\vec{C}$. Then $\vec{C}+\left(-\frac{1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$ is:

(1) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

(2) $-\frac{\sqrt{2}}{3}\hat{i}+\frac{\sqrt{2}}{3}\hat{j}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\hat{k}\right)$

(3) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{i}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{j}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{k}$

(4) $\frac{\sqrt{2}}{3}\hat{i}+\frac{3}{3\sqrt{2}}\hat{j}-\frac{1}{2}\hat{k}$

Answer (1)

Sol. Let $\vec{C} = a\hat{i}+b\hat{j}+c\hat{k}$

$$(a\hat{i}+b\hat{j}+c\hat{k})\cdot(2\hat{i}+2\hat{j}-\hat{k}) = 1\times 3\times\frac{1}{2}$$

$$2a+2b-c = \frac{3}{2} \qquad \ldots(1)$$

$$(a\hat{i}+b\hat{j}+c\hat{k}).(\hat{i}-\hat{k}) = 1\times\sqrt{2}\times\frac{1}{\sqrt{2}}$$

$a - c = 1 \qquad \ldots(2)$

$a^2 + b^2 + c^2 = 1 \qquad \ldots(3)$

Solving (1), (2) and (3)

$$a+2b = \frac{1}{2}$$

$$a^2+b^2+(a-1)^2 = 1$$

$2a^2 - 2a + b^2 = 0$

$$2a^2-2a+\left(\frac{2a-1}{4}\right)^2 = 0$$

$32a^2 - 32a + 4a^2 - 4a + 1 = 0$

$36a^2 - 36a + 1 = 0$

$$a = \frac{36\pm\sqrt{(36)^2-4(36)}}{2\times 36}$$

$$= \frac{1}{2}\pm\frac{\sqrt{2}}{3}$$

$$b = \frac{1-2a}{4} \Rightarrow b = \frac{1\pm\frac{2\sqrt{2}}{3}-1}{4}$$

$$= \mp\frac{1}{3\sqrt{2}}$$

$$C = -\frac{1}{2}\pm\frac{\sqrt{2}}{3}$$

$$C+\left(\frac{-1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$$

$$= \frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$$

14. Let $\alpha\in(0,\ \infty)$ and $A = \begin{bmatrix}1 & 2 & \alpha\\ 1 & 0 & 1\\ 0 & 1 & 2\end{bmatrix}$. If det(adj $(2A - A^T)$).adj$(A - 2A^T) = 2^8$, then $(\det(A))^2$ is equal to :

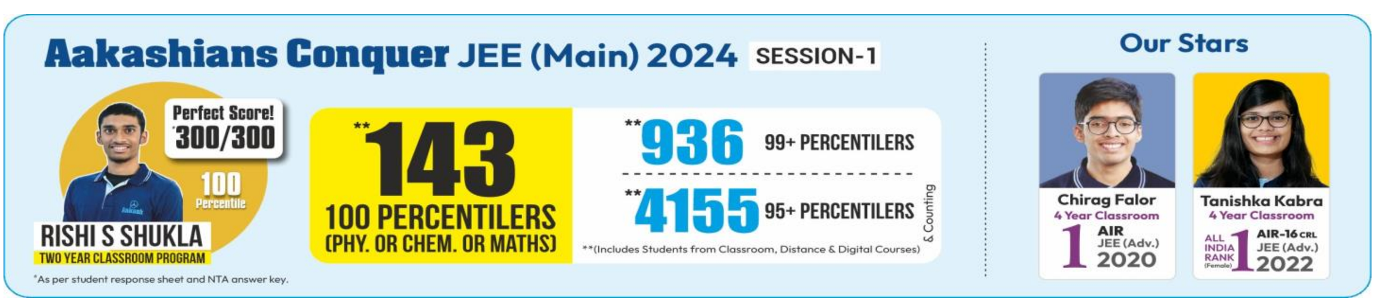
(1) 49

(2) 16

(3) 36

(4) 1

Answer (2)



Sol. $|\text{adj}(A-2A^T).\text{adj}(2A-A^T)| = 2^8$

$P = A - 2A^T$

$Q = 2A^T - A \Rightarrow Q^T = 2A^T - A = -P$

$|\text{adj}(P)\text{adj}(Q)| \Rightarrow |PQ| = -2^4$

$\Rightarrow |P|(-|P|) = -2^4 \Rightarrow |P| = 4$ and $|Q| = -4$

$|A - 2A^T| = 4$

$$A-2A^T = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ \alpha & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & \alpha \\ -3 & 0 & -1 \\ -2\alpha & -1 & -2 \end{bmatrix}$$

$\Rightarrow |A - 2A^T| = 1 + 3\alpha = 4 \Rightarrow \alpha = 1 \Rightarrow |A| = -4$

$\Rightarrow |A|^2 = 16$

15. Let $\alpha, \beta \in R$. Let the mean and the variance of 6 observations -3, 4, 7, -6, α, β be 2 and 23, respectively, The mean deviation about the mean of these 6 observations is:

(1) $\frac{14}{3}$

(2) $\frac{16}{3}$

(3) $\frac{11}{3}$

(4) $\frac{13}{3}$

Answer (4)

Sol. Mean = $\frac{-3+4+7+(-6)+\alpha+\beta}{6} = 2$

$\Rightarrow \alpha + \beta = 10$

Variance = $\frac{\sum x_i^2}{n} - \left(\frac{\bar{x}}{n}\right)^2 = 23$

$\Rightarrow \sum x_i^2 = 27 \times 6$

$\Rightarrow 9 + 16 + 49 + 36 + \alpha^2 + \beta^2 = 162$

$\Rightarrow \alpha^2 + \beta^2 = 52$

We get α and β as 4 and 6

So, mean deviation about mean

$= \frac{|-3-2|+|4-2|+|7-2|+|-6-2|+|4-2|+|6-2|}{6}$

$= \frac{5+2+5+8+2+4}{6}$

$= \frac{13}{3}$

16. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 - 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is:

(1) $x^2 + 10x + 16 = 0$

(2) $2x + 11x + 12 = 0$

(3) $4x^2 + 14x + 12 = 0$

(4) $x^2 + 8x + 12 = 0$

Answer (4)

Sol. $(x-2)(x-6) = 0$

$\Rightarrow x^2 - 8x + 12 = 0$

$\Rightarrow \frac{x^2}{12} - \frac{8x}{12} + 1 = 0$

$\therefore a = \frac{1}{12}, b = \frac{-2}{3}$

$\frac{1}{2a+b} = \frac{1}{\frac{1}{6}-\frac{2}{3}} \Rightarrow \frac{6}{-3} = -2$

$\frac{1}{6a+b} = \frac{1}{\frac{1}{2}-\frac{2}{3}} \Rightarrow \frac{6}{-1} = -6$

$\therefore (x+2)(x+6) = 0$

$\Rightarrow x^2 + 8x + 12 = 0$

17. Let the first three terms 2, p and q, which $q \neq 2$, of a G.P. be respectively the 7th, 8th and 13th terms of an A.P. If the 5th term of the G.P. is the nth terms of the A.P. then n is equal to

(1) 163

(2) 177

(3) 151

(4) 169

Answer (1)

Sol. Let $p = 2r$, $q = 2r^2$

$T_7 = 2$, $T_8 = 2r$, $T_{13} = 2r^2$

$d = 2r - 2 = 2(r-1)$

$2r^2 = T_7 + 6d = 2 + 6(2)(r-1) = 12r - 10$

$\Rightarrow r^2 - 6r + 5 = 0$

$\Rightarrow (r-1)(r-5) = 0$

$\therefore r = 1, 5$

$r = 1$ (rejected) as $q \neq 2$

$\therefore r = 5$

5[th] term of G.P = $2.r^4 = 2.5^4$

Let 1[st] term of A.P $ba = a$, $d = 8$

$2 = a + (6)(8) \Rightarrow a = -46$

n[th] term of A.P $= -46 + (n-1)8 = 8n - 54$

$2.5^4 = 8n - 54$

$\Rightarrow 1250 + 54 = 8n$

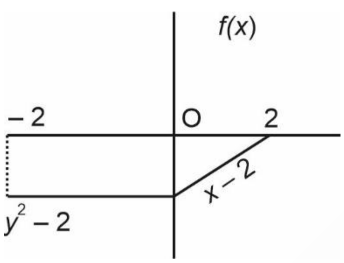
$\Rightarrow n = \dfrac{1304}{8} = 163$

18. Let $f(x) = \begin{cases} -2, & -2 \le x \le 0 \\ x-2, & 0 < x \le 2 \end{cases}$ and $h(x) = f(|x|) + |f(x)|$. Then $\int_{-2}^{2} h(x)dx$ is equal to:

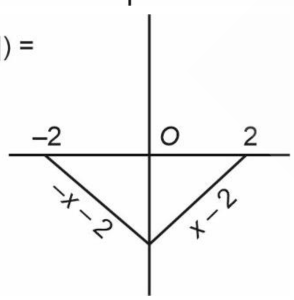
(1) 4
(2) 1
(3) 2
(4) 6

Answer (3)

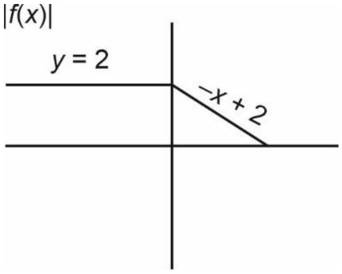
Sol. $f(x) = \begin{cases} -2 & -2 \le x \le 0 \\ x-2 & 0 < x \le 2 \end{cases}$ $h(x) = f|x| + |f(x)|$



$f(|x|) =$



$|f(x)|$



$$h(x) = \begin{cases} -x-2+2 = -x & -2 \le x \le 0 \\ 0 & 0 < x \le 2 \end{cases}$$

$$\therefore \int_{-2}^{2} h(x)dx = \int_{-2}^{0} -x dx + \int_{0}^{2} 0 dx$$

$$\left.\frac{x^2}{2}\right|_{-2}^{0} = \frac{4}{2} = 2$$

19. If the system of equations

$$x + \left(\sqrt{2}\sin\alpha\right)y + \left(\sqrt{2}\cos\alpha\right)z = 0$$

$$x + (\cos\alpha)y + (\sin\alpha)z = 0$$

$$x + (\sin\alpha)y - (\cos\alpha)z = 0$$

has a non-trivial solution, then $\alpha \in \left(0, \dfrac{\pi}{2}\right)$ is equal to:

(1) $\dfrac{7\pi}{24}$
(2) $\dfrac{3\pi}{4}$
(3) $\dfrac{5\pi}{24}$
(4) $\dfrac{11\pi}{24}$

Answer (3)

Sol. $x + \left(\sqrt{2}\sin\alpha\right)y + \left(\sqrt{2}\cos\alpha\right)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$x + (\sin\alpha)y - (\cos\alpha)z = 0$

$\because$ Non-trivial solution

$\Rightarrow D = 0$

$$\begin{vmatrix} 1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$1\left[-\cos^2\alpha - \sin^2\alpha\right] - 1\left[-\sqrt{2}\sin\alpha\cos\alpha - \sqrt{2}\sin\alpha\cos\alpha\right]$$

$$+1\left[\sqrt{2}\sin^2\alpha - \sqrt{2}\cos^2\alpha\right] = 0$$

$$-1+2\sqrt{2}\sin\alpha\cos\alpha+\sqrt{2}\left(\sin^2\alpha-\cos^2\alpha\right)=0$$

$$\sqrt{2}\sin2\alpha-\sqrt{2}\cos2\alpha=1$$

$$\frac{\sin2\alpha}{\sqrt{2}}-\frac{\cos2\alpha}{\sqrt{2}}=\frac{1}{2}$$

$$\sin\left(2\alpha-\frac{\pi}{4}\right)=\sin\frac{\pi}{6}$$

$$\Rightarrow\ 2\alpha-\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{6}\ \text{ for } n=0$$

$$\Rightarrow\ \alpha=\frac{5\pi}{24}$$

20. Let the point, on the line passing through the points $P(1, -2, 3)$ and $Q(5, -4, 7)$, farther from the origin and at a distance of 9 units from the point P, be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

(1) 160
(2) 155
(3) 150
(4) 165

Answer (2)

Sol. Line through PQ

$$\frac{x-1}{4}=\frac{y+2}{-2}=\frac{z-3}{4}$$

Any point on PQ. be $R(4\lambda + 1, -2\lambda -2, 4\lambda + 3)$

$PR = 9$ unit

$(PR)^2 = 81$

$(4\lambda + 1 - 1)^2 + (-2\lambda -2 + 2)^2 + (4\lambda + 3 - 3)^2 = 81$

$16\lambda^2 + 4\lambda^2 + 16\lambda^2 = 81$

$36\lambda^2 = 81$

$$\lambda=\pm\frac{9}{6}=\pm\frac{3}{2}$$

$\therefore$ R can be $(7, -5, 9)$ or $(-5, 1, -3)$

Distance from origin for both points be $\sqrt{49+25+81}$ and $\sqrt{25+1+9}=\sqrt{35}$

$\therefore$ Distance of $(7, -5, 9)$ is farthest from origin

$\therefore$ $(\alpha, \beta, \gamma) = (7, -5, 9)$

Now $7^2 + (-5)^2 + 9^2 = 155$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then $m + n$ is equal to______.

Answer (45)

Sol. $n(S) = 220$

$n(M) \in [125, 130]$, $n(P) \in [85, 95]$, $n(C) \in [75, 90]$

$n(M\cup P\cup C) = 220 - 10 = 210$

$n(M\cap P) = 40$, $n(P\cap C) = 30$, $n(M\cap C) = 50$

$$n(M\cup P\cup C)=\sum n(M)-\sum n(M\cap P)+n(M\cap P\cap C)$$

$\Rightarrow n(M\cap P\cap C) = 210 + (40 + 30 +50) - \Sigma n(M)$

$\because$ $(n(M\cap P\cap C))_{max} = n = \min(n(M\cap P), n(P\cap C), n(M\cap C)) = 30$

$$\because\ \left(\sum n(M)\right)_{max}=130+95+90=315$$

$\Rightarrow (n(M\cap P\cap C))_{min} = m = 330 - 315 = 15$

$\Rightarrow n + m = 45$

22. Let A be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is -3. If the points (x, y) satisfying $A^2 + xA + y I = O$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is l, then $e^4 + l^4$ is equal to______.

Answer (Bonus)

Sol. $|A| = 2$ $\sum \text{dia} = -3$

$\therefore$ character equation : $A^2 + 3A + 2I = 0$

$\Rightarrow x = 3 \quad y = 2$

$\because$ We are getting only one point (3, 2) but its given many points satisfy this equation.

Moreover hyperbola whose transverse axis is x-axis and passing through (3, 2) is not unique.

$\therefore$ multiple value of 'e' and $L(LR)$ is possible.

We'll not get a unique result.

23. Let A be a 3 × 3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$.

Then the maximum value of det(A) is______.

Answer (27)

Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Now

$$A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}3\\3\\3\end{bmatrix}$$

$$\begin{aligned} a_{11} + a_{12} + a_{13} &= 3 \\ a_{21} + a_{22} + a_{23} &= 3 \\ a_{31} + a_{32} + a_{33} &= 3 \end{aligned}$$

Now for maximum value of det(A) = $a_{ij}\begin{Bmatrix} 0 & i \neq j \\ 3 & i = j \end{Bmatrix}$

$\therefore |A| = 27$

24. If $\lim\limits_{x \to 1} \dfrac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}} = \dfrac{m\sqrt{5}}{n(2n)^{2/3}}$, where $gcd(m, n) = 1$, then $8m + 12n$ is equal to_____.

Answer (100)

Sol. $I = \lim\limits_{x \to 1} \dfrac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}}$

From: $\dfrac{0}{0}$, using L-H rule

$$I = \lim_{x \to 1} \frac{\frac{1}{3} \times 5(5x+1)^{-2/3} - \frac{1}{3}(x+5)^{-2/3}}{\frac{1}{2} \times 2(2x+3)^{-1/2} - \frac{1}{2}(x+4)^{-1/2}}$$

$$= \frac{\left(\frac{5}{3} - \frac{1}{3}\right)6^{-2/3}}{\frac{1}{2}5^{-1/2}} = \frac{8}{3} \times \frac{5^{1/2}}{6^{2/3}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$$

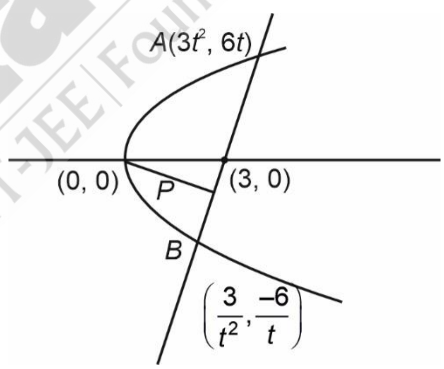
$\Rightarrow m = 8, n = 3$

$\Rightarrow 8m + 12n = 100$

25. Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p, then $10p^2$ is equal to ________.

Answer (72)

Sol.



$$AB = 15 \Rightarrow \left(3\left(t^2 - \frac{1}{t^2}\right)\right)^2 + \left(6\left(t + \frac{1}{t}\right)\right)^2 = 225$$

$$\Rightarrow \quad 9\left(t^2 - \frac{1}{t^2}\right) + 36\left(t + \frac{1}{t}\right)^2 = 225$$

$$\Rightarrow \left(t+\frac{1}{t}\right)^2\left[\left(t-\frac{1}{t}\right)^2+4\right]=25$$

$$\left(t+\frac{1}{t}\right)^2\left(t+\frac{1}{t}\right)^2=25 \Rightarrow \left(t+\frac{1}{t}\right)^4=25$$

$$\Rightarrow \ t+\frac{1}{t}=\pm\sqrt{5} \Rightarrow \left(t-\frac{1}{t}\right)=\pm 1$$

Equation of AB : $(y-6t)=\left(\dfrac{2t}{t^2-1}\right)(x-3t^2)$

$\Rightarrow$ Distance from $y - 6t = mx - 3mt^2$

$$\Rightarrow \ p=\frac{|3mt^2-6t|}{\sqrt{1+m^2}}=\frac{\left|\left(\frac{6t}{t^2-1}\right)\right|}{\sqrt{5}}=\frac{6}{\sqrt{5}}$$

$$\left[m=\frac{2t}{t^2-1}=\frac{2}{t-\frac{1}{t}}=\pm 2 \Rightarrow m^2=4\right]$$

$$\Rightarrow \ 10p^2=\frac{10\times 36}{5} \Rightarrow 72$$

26. If $\displaystyle\int_0^{\frac{\pi}{4}}\frac{\sin^2 x}{1+\sin x\cos x}dx=\frac{1}{a}\log_e\left(\frac{a}{3}\right)+\frac{\pi}{b\sqrt{3}}$, where a, $b \in N$, then $a + b$ is equal to ______.

Answer (8)

Sol. $I=\displaystyle\int_0^{\frac{\pi}{4}}\frac{\sin^2 x}{1+\sin x\cos x}dx=$

$$\int_0^{\frac{\pi}{4}}\frac{\sin^2 x}{\sin^2 x+\cos^2 x+\sin x\cos x}dx$$

$$I=\int_0^{\frac{\pi}{4}}\frac{\tan^2 x}{1+\tan x+\tan^2 x}dx$$

$$=\int_0^{\frac{\pi}{4}}\frac{\tan^2 x.\sec^2 x\,dx}{(1+\tan^2 x)(1+\tan x+\tan^2 x)}$$

Let $\tan x = t$

$$I=\int_0^1\frac{t^2}{(1+t^2)(1+t+t^2)}dt$$

$$=\int_0^1\left(\frac{x}{1+x^2}-\frac{x}{1+x+x^2}\right)dx$$

$$=\frac{1}{2}\int_0^1\frac{2x}{1+x^2}dx-\int_0^1\frac{\frac{1}{2}(2x+1)-\frac{1}{2}}{1+x+x^2}dx$$

$$=\frac{1}{2}\ln 2-\frac{1}{2}\ln 3+\frac{1}{2}\int_0^1\frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$=\frac{1}{2}\ln\frac{2}{3}+\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\left[\tan^{-1}\frac{2x+1}{\sqrt{3}}\right]_0^1$$

$$=\frac{1}{2}\ln\frac{2}{3}+\frac{1}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$$

$$=\frac{1}{2}\ln\frac{2}{3}+\frac{1}{\sqrt{3}}\cdot\frac{\pi}{6}$$

$\therefore$ $a = 2$, $b = 6$

$\therefore$ $a + b = 8$

27. Let $a=1+\dfrac{^2C_2}{3!}+\dfrac{^3C_2}{4!}+\dfrac{^4C_2}{5!}+\ldots,$

$$b=1+\frac{^1C_0+{}^1C_1}{1!}+\frac{^2C_0+{}^2C_1+{}^2C_2}{2!}+\frac{^3C_0+{}^3C_1+{}^3C_2+{}^3C_3}{3!}+\ldots$$

Then $\dfrac{2b}{a^2}$ is equal to _____.

Answer (8)

Sol. $a = 1 + \dfrac{{}^2C_2}{3!} + \dfrac{{}^3C_2}{4!} + \dfrac{{}^4C_2}{5!} + \ldots,$

$$b = 1 + \frac{{}^1C_0 + {}^1C_1}{1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{2!} + \ldots$$

$$b = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \ldots = e^2$$

Using $e^x = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$

$$a = 1 + \sum_{r=2}^{\infty} \frac{{}^rC_2}{(r+1)!} = 1 + \sum_{r=2}^{\infty} \frac{r(r-1)}{2(r+1)!}$$

$$= 1 + \frac{1}{2}\sum_{r=2}^{\infty} \frac{(r+1)r - 2r}{(r+1)!}$$

$$= 1 + \frac{1}{2}\sum_{r=2}^{\infty} \frac{1}{(r-1)!} - \frac{1}{2}\sum_{r=2}^{\infty} \frac{2r}{(r+1)!}$$

$$= 1 + \frac{1}{2}\left(\frac{1}{1!} + \frac{1}{2!} + \ldots\right) - \sum_{r=2}^{\infty} \frac{(r+1)-1}{(r+1)!}$$

$$= 1 + \frac{1}{2}(e-1) - \sum_{r=2}^{\infty} \frac{1}{r!} + \sum_{r=2}^{\infty} \frac{1}{(r+1)!}$$

$$= 1 + \frac{1}{2}(e-1) - \left(e - \frac{1}{1!} - \frac{1}{0!}\right) + \left(e - \frac{1}{1!} - \frac{1}{0!} - \frac{1}{2!}\right)$$

$$= 1 + \frac{e}{2} - \frac{1}{2} - e + 2 + e - 2 - \frac{1}{2} = \frac{e}{2}$$

$$\Rightarrow \quad \frac{2b}{a^2} = \frac{2e^2}{\frac{e^2}{4}} = 8$$

28. Let the solution $y = y(x)$ of the differential equation $\dfrac{dy}{dx} - y = 1 + 4\sin x$ satisfy $y(\pi) = 1$. Then $y\left(\dfrac{\pi}{2}\right) + 10$ is equal to _____.

Answer (7)

Sol. $\dfrac{dy}{dx} - y = 1 + 4\sin x$

Integrating factor = $e^{-\int dx} = e^{-x}$

Solution is $ye^{-x} = \int (1 + 4\sin x)e^{-x}dx$

$= -e^{-x} + 2.\, e^{-x}\,(-\sin x - \cos x) + C$

$y(\pi) = 1 \Rightarrow C = 0$

Hence $y(x) = -1 - 2(\sin x + \cos x)$

$y\left(\dfrac{\pi}{2}\right) + 10 = 7$

29. If the shortest distance between the lines

$$\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4} \text{ and } \frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$$

is $\dfrac{38}{3\sqrt{5}}k$, and $\int_0^k [x^2]dx = \alpha - \sqrt{\alpha}$, where $[x]$ denotes the greatest integer function, then $6\alpha^3$ is equal to______

Answer (48)

Sol. $L_1 : \dfrac{x+2}{2} = \dfrac{y+3}{3} = \dfrac{z-5}{4}$ $\quad \vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a}_1 = -2\hat{i} - 3\hat{j} + 5\hat{k}$

$L_2 = \dfrac{x-3}{1} = \dfrac{y-2}{-3} = \dfrac{z+4}{2}$ $\quad \vec{a}_2 = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{b}_2 = 1\hat{i} - 3\hat{j} + 2\hat{k}$

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$d = \left|\frac{(5\hat{i}+5\hat{j}-9\hat{k}).(18\hat{i}-9\hat{k})}{\sqrt{324+81}}\right|$$

$$|\vec{b}_1 \times \vec{b}_2| \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}$$

$$\Rightarrow \hat{i}(6+12)-\hat{j}(4-4)+\hat{k}(-6-3)$$

$$\Rightarrow (18\hat{i}-9\hat{k})$$

$$d = \left|\frac{90+81}{9\sqrt{5}}\right|$$

$$d = \frac{171}{9\sqrt{5}}$$

$$\frac{38}{3\sqrt{5}}k = \frac{171}{9\sqrt{5}}$$

$$\frac{38}{3\sqrt{5}}k = \frac{57}{3\sqrt{5}}$$

$$\boxed{k = \frac{57}{38} = \frac{3}{2}}$$

$$\int_0^{\frac{3}{2}} [x^2]\, dx$$

$$\int_0^1 0\, dx + \int_0^{\sqrt{2}} 1\, dx + \int_{\sqrt{2}}^{\frac{3}{2}} 2\, dx$$

$$0 + \left(\sqrt{2}-1\right) + 2\left(\frac{3}{2}-\sqrt{2}\right)$$

$$\sqrt{2}-1+3-2\sqrt{2}$$

$$\boxed{2-\sqrt{2}}$$

$$\boxed{\alpha = 2}$$

$$6\alpha^3 = 6(2)^3 = 48$$

30. Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{AB} = \hat{i}+2\hat{j}-7\hat{k}$, $\overrightarrow{BC} = a\hat{i}+b\hat{j}+c\hat{k}$ and $\overrightarrow{AC} = 6\hat{i}+d\hat{j}-2\hat{k}$, $d>0$. Then the square of the length of the largest side of the triangle ABC is _____.

Answer (54)

Sol. Area of triangle $ABC = 15\sqrt{2}$

$$\Rightarrow \frac{1}{2}\left|\overline{AB}\times\overline{AC}\right| = 15\sqrt{2} \qquad \ldots\text{(i)}$$

$$\overline{AB}\times\overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -7 \\ 6 & d & -2 \end{vmatrix}$$

$$= (7d-4)\hat{i} - 40\hat{j} + (d-12)\hat{k} \quad \ldots\text{(ii)}$$

From (i) and (ii) $5d^2 - 8d - 4 = 0$

$$\Rightarrow \quad d = \frac{-2}{5} \text{ (Rejected) or } d = 2$$

Also, $\overline{AB} + \overline{BC} = \overline{AC}$

$\Rightarrow \quad a + 1 = 6 \Rightarrow a = 5$

$b + 2 = d \Rightarrow b = 0$

and $c - 7 = -2 \Rightarrow c = 5$

$|\overline{AB}| = \sqrt{54}$, $|\overline{AC}| = \sqrt{44}$, $|\overline{BC}| = \sqrt{50}$

Largest side has length of $\sqrt{54}$ units

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. An electron is projected with uniform velocity along the axis inside a current carrying long solenoid. Then

(1) the electron will continue to move with uniform velocity along the axis of the solenoid.

(2) the electron will experience a force at 45° to the axis and execute a helical path.

(3) the electron will be accelerated along the axis.

(4) the electron path will be circular about the axis.

Answer (1)

Sol. $\because$ Electron is moving along the direction of magnetic field

So, $F_{net} = qV\beta \sin 0° = 0$

i.e. electron will move direction of its initial velocity without changing its motion and speed.

32. The co-ordinates of a particle moving in $x - y$ plane are given by :

$x = 2 + 4t$, $y = 3t + 8t^2$.

The motion of the particle is :

(1) non-uniformly accelerated.

(2) uniformly accelerated having motion along a straight line.

(3) uniform motion along a straight line.

(4) uniformly accelerated having motion along a parabolic path.

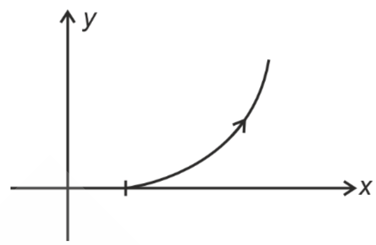
Answer (4)

Sol. $x = 2 + 4t$, $y = 3t + 8t^2$

$\Rightarrow V_x = 4$, $V_y = 3 + 16t$

$\Rightarrow a_x = 0$, $a_y = 16$

So, its motion should be uniformly accelerated and parabolic path.



33. If a rubber ball falls from a height h and rebounds upto the height of $h/2$. The percentage loss of total energy of the initial system as well as velocity ball before it strikes the ground, respectively, are

(1) 50%, $\sqrt{gh}$ (2) 50%, $\sqrt{\frac{gh}{2}}$

(3) 50%, $\sqrt{2gh}$ (4) 40%, $\sqrt{2gh}$

Answer (3)

Sol. Velocity of ball just before strike $= \sqrt{2gh}$

$$\% \text{ loss in energy } = \frac{mgh - \frac{mgh}{2}}{mgh} \times 100$$

$= 50\%$

34. The resistances of the platinum wire of a platinum resistance thermometer at the ice point and steam point are 8 Ω and 10 Ω respectively. After inserting in a hot bath of temperature 400°C, the resistance of platinum wire is

(1) 10 Ω (2) 16 Ω

(3) 8 Ω (4) 2 Ω

Answer (2)

Sol. At $t = 0°C \rightarrow R_0 = 8\ \Omega$

At $t = 100°C \rightarrow R = 10\ \Omega$

$10 = 8(1 + \alpha \times 100)$

$$\Rightarrow \quad \alpha = \frac{2}{800}$$

Again at $t = 400$

$$R = 8\left(1 + \frac{2}{800} \times 400\right)$$

$= 16\ \Omega$

35. On celcius scale the temperature of body increases by 40°C. The increase in temperature on Fahrenheit scale is

(1) 70°F
(2) 75°F
(3) 68°F
(4) 72°F

Answer (4)

Sol. Here $\frac{F-32}{9} = \frac{C}{5}$

$$\Rightarrow \quad \Delta C = \frac{5}{9}\Delta F$$

$$\Rightarrow \quad 40 = \frac{5}{9}\Delta F$$

$\Rightarrow \quad \Delta F = 72°F$

36. The electric field in an electromagnetic wave is given by $\vec{E} = \hat{i}\,40\cos\omega\left(t - \frac{z}{c}\right)$ NC^{-1}. The magnetic field induction of this wave is (in SI unit) :

(1) $\vec{B} = \hat{k}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(2) $\vec{B} = \hat{i}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(3) $\vec{B} = \hat{j}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(4) $\vec{B} = \hat{j}\,40\cos\omega\left(t - \frac{z}{c}\right)$

Answer (3)

Sol. $\vec{E} = \hat{i}\,40\cos\omega\left(t - \frac{z}{c}\right)\frac{\text{N}}{\text{C}}$

$$B_0 = \frac{40}{c}$$

and as $\vec{c} = \vec{E} \times \vec{B}$, B should be along y-axis.

$$\therefore \quad \vec{B} = \hat{j}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$$

37. Which of the following nuclear fragments corresponding to nuclear fission between neutron $\left({}^{1}_{0}n\right)$ and uranium isotope $\left({}^{235}_{92}\text{U}\right)$ is correct?

(1) ${}^{153}_{51}\text{Sb} + {}^{99}_{41}\text{Nb} + 3{}^{1}_{0}n$
(2) ${}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 4{}^{1}_{0}n$
(3) ${}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3{}^{1}_{0}n$
(4) ${}^{140}_{56}\text{Xe} + {}^{94}_{38}\text{Sr} + 3{}^{1}_{0}n$

Answer (3)

Sol. Nuclear fission of U^{235} takes place as

$${}^{1}_{0}n + {}^{235}_{92}\text{U} \longrightarrow {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3{}^{1}_{0}n$$

38. A body travels 102.5 m in n^{th} second and 115.0 m in $(n + 2)^{\text{th}}$ second. The acceleration is

(1) 6.25 m/s^2
(2) 12.5 m/s^2
(3) 5 m/s^2
(4) 9 m/s^2

Answer (1)

Sol. $S_n = u + \frac{a}{2}(2n-1)$

$$102.5 = u + \frac{a}{2}(2n-1) \quad \ldots\text{(i)}$$

$$115 = u + \frac{a}{2}[2(n+2)-1]$$

$$115 = u + \frac{a}{2}[2n+3] \quad \ldots\text{(ii)}$$

On solving $\rightarrow a = 6.25$ m/s^2

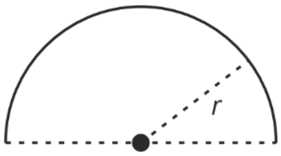
39. A metal wire uniform mass density having length L and mass M is bent to form a semicircular arc and a particle of mass m is placed at the centre of the arc. The gravitational force on the particle by the wire is

(1) $\frac{GMm\pi}{2L^2}$ (2) $\frac{GmM\pi^2}{L^2}$

(3) 0 (4) $\frac{2GmM\pi}{L^2}$

Answer (4)

Sol.



$r = \frac{L}{\pi}$

Field at centre due to arc, $I = \frac{2GM\pi}{L^2}$

$\therefore$ Net force on mass, $F = \frac{2GMm\pi}{L^2}$

40. To measure the internal resistance of a battery, potentiometer is used. For R = 10 Ω, the balance point is observed at ℓ = 500 cm and for R = 1 Ω the balance point is observed at ℓ = 400 cm. The internal resistance of the battery is approximately:

(1) 0.1 Ω (2) 0.2 Ω

(3) 0.4 Ω (4) 0.3 Ω

Answer (4)

Sol. $\frac{\ell_1}{\ell_2} = \left(\frac{ER_1}{R_1 + r}\right) \times \frac{R_2 + r}{ER_2}$

$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{R_1(R_2 + r)}{R_2(R_1 + r)}$

$\Rightarrow \frac{500}{400} = \frac{10(1 + r)}{1(10 + r)}$

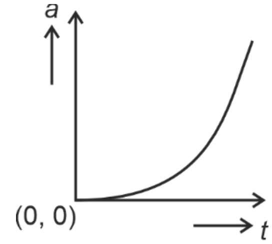
$\Rightarrow$ 10 + r = 8 + 8r

$\Rightarrow$ 2 = 7r

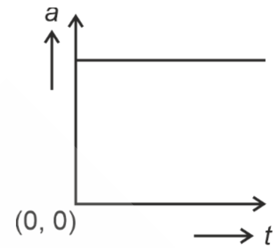
$\Rightarrow$ $r \approx 0.3\ \Omega$

41. A wooden block, initially at rest on the ground, is pushed by a force which increases linearly with time t. Which of the following curve best describes acceleration of the block with time

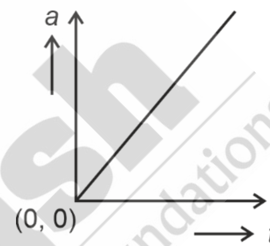
(1)



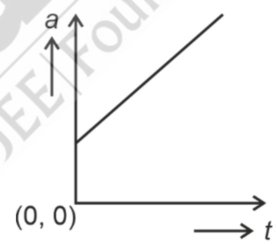
(2)



(3)



(4)



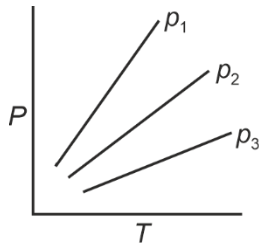
Answer (3)

Sol. Acceleration (a) $= \frac{f - F}{m}$

When applied force became equal to f_{max}, block will start moving.

As F increases linearly, so acceleration of also moving block will increase linearly.

42. *P–T* diagram of an ideal gas having three different densities ρ_1, ρ_2, ρ_3 (in three different cases) is shown in the figure. Which of the following is correct:



(1) $\rho_1 < \rho_2$ (2) $\rho_1 = \rho_2 = \rho_3$

(3) $\rho_2 < \rho_3$ (4) $\rho_1 > \rho_2$

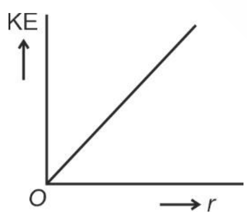
Answer (4)

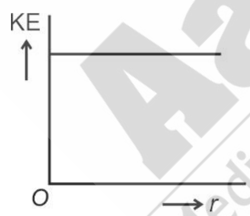
Sol. $PM = \rho RT$

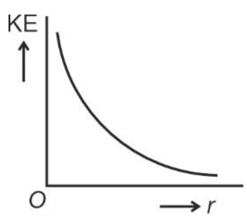
$$\Rightarrow \frac{P}{T} = \left(\frac{R}{m}\right)\rho = \text{slope}$$

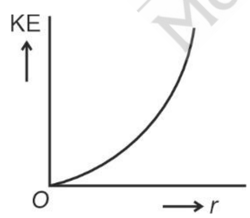
So from given curve, $\rho_1 > \rho_2 > \rho_3$

43. An infinitely long positively charged straight thread has a linear charge density λ Cm^{-1}. An electron revolves along a circular path having axis along the length of the wire. The graph that correctly represents the variation of the kinetic energy of electron as a function of radius of circular path from the wire is

(1)


(2)


(3)


(4)


Answer (2)

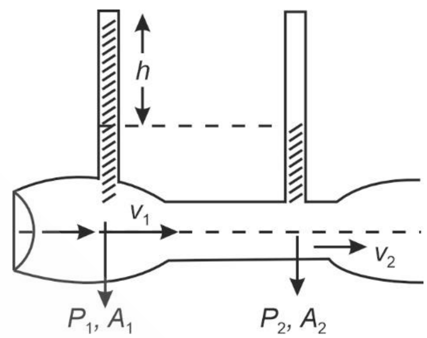
Sol. The electron revolves in a circle, so its kinetic energy remains same.

So option (2) best represent the given situation.

44. Given below are two statements

Statement I: When speed of liquid is zero everywhere, pressure difference at any two points depends on equation $P_1 - P_2 = \rho g(h_2 - h_1)$.

Statement-II: In ventury tube shown $2gh = v_1^2 - v_2^2$



In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **Statement I** and **Statement II** are correct.

(2) **Statement I** is correct but **Statement II** is incorrect.

(3) Both **Statement I** and **Statement II** are incorrect.

(4) **Statement I** is incorrect but **Statement II** is correct.

Answer (2)

Sol. If speed = 0

Then $P_1 + \rho g h_1 = P_2 + \rho g h_2$

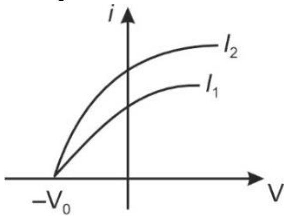
In given ventury tube,

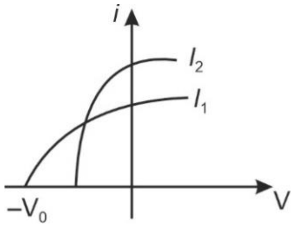
$$P_1 + \rho g h + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

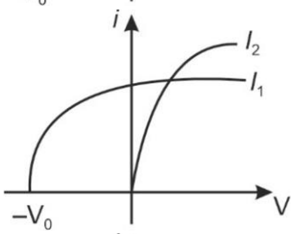
$$\Rightarrow \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = \left(P_2 - P_1\right) - \rho g h$$

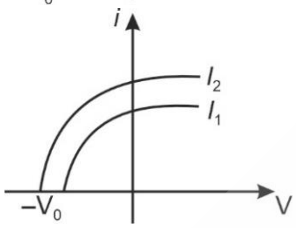
$$\Rightarrow v_1^2 - v_2^2 = \frac{2\left(P_2 - P_1\right)}{\rho} - 2gh$$

45. Which figure shows the correct variation of applied potential difference (V) with photoelectric current (I) at two different intensities of light ($I_1 < I_2$) of same wavelengths:

(1)


(2)


(3)


(4)


Answer (1)

Sol. Stopping potential is independent on intensity but photocurrent increases non-linearly on increasing intensity.

46. In an experiment to measure focal length (f) of convex lens, the least counts of the measuring scales for the position of object (u) and for the position of image (v) are Δu and Δv, respectively. The error in the measurement of the focal length of the convex lens will be:

(1) $f^2\left[\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}\right]$ (2) $2f\left[\frac{\Delta u}{u}+\frac{\Delta v}{v}\right]$

(3) $\frac{\Delta u}{u}+\frac{\Delta v}{v}$ (4) $f\left[\frac{\Delta u}{u}+\frac{\Delta v}{v}\right]$

Answer (1)

Sol. $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

$\Rightarrow \frac{df}{f^2}=\frac{dv}{v^2}-\frac{du}{u^2}$

For small change and maximum % error

$df \to \Delta \cdot f$

$\Delta f = f^2\left(\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}\right)$

47. The equation of stationary wave is:

$$y = 2a\sin\left(\frac{2\pi nt}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right).$$

Which of the following is NOT correct:

(1) The dimensions of x is $[L]$

(2) The dimensions of n/λ is $[T]$

(3) The dimensions of n is $[LT^{-1}]$

(4) The dimensions of nt is $[L]$

Answer (2)

Sol. $[x] = L$

$[nt] = L$

$[n] = LT^{-1}$

$\left[\frac{n}{-1}\right] = T^{-1}$

48. An effective power of a combination of 5 identical convex lenses which are kept in contact along the principal axis is 25 D. Focal length of each of the convex lens is

(1) 20 cm (2) 25 cm

(3) 50 cm (4) 500 cm

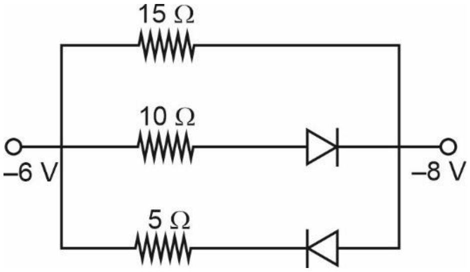
Answer (1)

Sol. $\frac{1}{F}=\frac{1}{f}+\frac{1}{f}........=\frac{5}{f}$

$\Rightarrow 25=\frac{5}{f}$

$\Rightarrow f=\frac{1}{5}m = 20 \text{ cm}$

49. The value of net resistance of the network as shown in the given figure is :

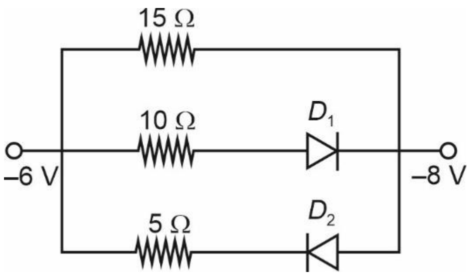


(1) 6 Ω (2) (15/4) Ω

(3) (30/11) Ω (4) (5/2) Ω

Answer (1)

Sol.



Here D_1 is forward wise while D_2 is reversed wise

So net resistance between end, $R = \frac{15 \times 10}{25} = 6\ \Omega$

50. In an ac circuit, the instantaneous current is zero, when the instantaneous voltage is maximum. In this case the source may be connected to :

A. pure inductor.

B. pure capacitor.

C. pure resistor.

D. combination of an inductor and capacitor.

Choose the **correct** answer from the options given below:

(1) A and B only (2) A, B and D only

(3) B, C and D only (4) A, B and C only

Answer (2)

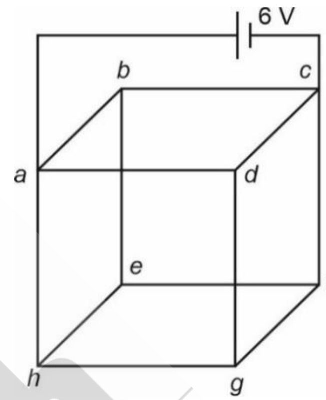
Sol. In this situation, phase difference between the current and the voltage is $\frac{\pi}{2}$.

This can be achieved by connecting L, C or combination of LC.

SECTION - B

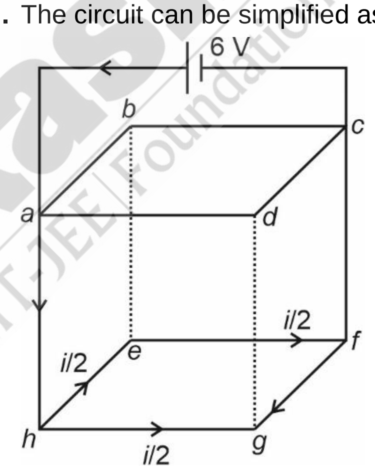
Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

51. Twelve wires each having resistance 2 Ω are joined to form *a* cube. A battery of 6 V emf is joined across point *a* and *c*. The voltage difference *e* and *f* is _______ V.



Answer (1)

Sol. The circuit can be simplified as



$R_{ac} = \frac{6 \times 2}{8} = \frac{3}{2}\ \Omega$

$i = 1$ Amp.

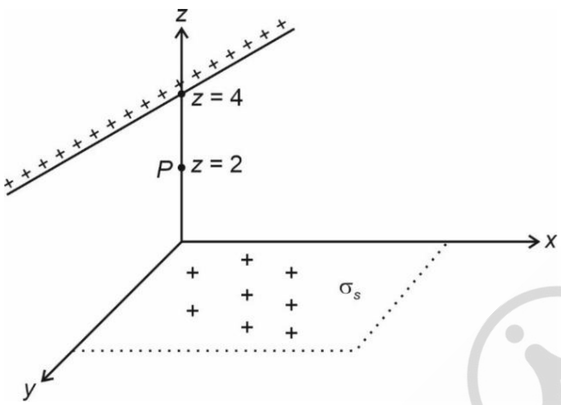
$V_{ef} = \left(\frac{i}{2}\right)2$

$= 1$ V

52. An infinite plane sheet of charge having uniform surface charge density $+\sigma_s$ C/m^2 is placed on $x-y$ plane. Another infinitely long line charge having uniform linear charge density $+\lambda_e$ C/m is placed at $z = 4$ m plane and parallel to y-axis. If the magnitude values $|\sigma_s| = 2\,|\lambda_e|$ then at point (0, 0, 2), the ratio of magnitudes of electric field values due to sheet charge to that of line charge is $\pi\sqrt{n}:1$. The value of n is _______.

Answer (16)

Sol.



Given $\sigma_s = 2\lambda_e$

At point P, $E_s = \dfrac{\sigma_s}{2\varepsilon_0}$

$$E_l = \frac{\lambda_e}{2\pi r\,\varepsilon_0}$$

$$\frac{E_s}{E_l} = 4\pi:1 = \pi\sqrt{n}:1$$

For value of $n = 16$

53. A soap bubble is blown to a diameter of 7 cm. 36960 erg of work is done in blowing it further. If surface tension of soap solution is 40 dyne/cm then the new radius is _______ cm Take $\left(\pi = \dfrac{22}{7}\right)$.

Answer (7)

Sol. $\Delta W = 8\pi\left(R_2^2 - R_1^2\right)T$

$$36960 = 8\times\frac{22}{7}\times 40\left(R_2^2 - \frac{49}{4}\right)$$

$R_2 = 7$ cm

54. An elastic spring under tension of 3 N has a length a. Its length is b under tension 2 N. For its length $(3a - 2b)$, the value of tension will be _______ N.

Answer (5)

Sol. Let natural length of spring = l_0

as give $\rightarrow$ $k(a - l_0) = 3$ …(i)

$\rightarrow$ $k(b - l_0) = 2$ …(ii)

$$\Rightarrow \frac{a - l_0}{b - l_0} = \frac{3}{2}$$

$\Rightarrow$ $2a - 2l_0 = 3b - 3l_0$

$\Rightarrow$ $l_0 = 3b - 2a$ and $k(a - b) = 1$

Again

$k(3a - 2b - l_0) = T$

$\Rightarrow$ $k(3a - 2b - 3b + 2a) = T$

$\Rightarrow$ $k(5a - 5b) = T$

$\Rightarrow$ $5k(a - b) = T$

$\Rightarrow$ $T = 5$

55. A hydrogen atom changes its state from $n = 3$ to $n = 2$. Due to recoil, the percentage change in the wave length of emitted light is approximately 1×10^{-n}. The value of n is __________.

[Given $Rhc = 13.6$ eV, $hc = 1242$ eV nm, $h = 6.6 \times 10^{-34}$ J s mass of the hydrogenatom $= 1.6 \times 10^{-27}$ kg]

Answer (7)

Sol. $\Delta E = 13.6 \text{ eV}\left(\frac{1}{4}-\frac{1}{9}\right)$

$$= \frac{68}{36} \text{ eV} = 1.89 \text{ eV}$$

Due to recoil of hydrogen atom, the energy of emitted photon will decrease by very small amount. So for approximate calculations,

$$\% \text{ charge} = \frac{\Delta E_{\text{atom}}}{\Delta E} \times 100$$

$$= \frac{\dfrac{\left(\frac{\Delta E}{C}\right)^2}{2m}}{\Delta E} \times 100$$

$$= \frac{\Delta E}{C^2 \times 2m} \times 100$$

$$= \frac{1.89 \times 1.6 \times 10^{-19} \times 100}{\left(3 \times 10^8\right)^2 \times 2 \times 1.6 \times 10^{-27}}$$

$= 1.05 \times 10^{-7}$ %

$\therefore \; n = 7$

56. A alternating current at any instant is given by $i = \left[6 + \sqrt{56}\sin\left(100\pi t + \frac{\pi}{3}\right)\right]$ A. The *rms* value of the current is _________ A.

Answer (8)

Sol. $i = \left[6 + \sqrt{56}\sin\left(100\pi t + \frac{\pi}{3}\right)\right]$ A.

$$i_{\text{rms}} = \sqrt{I_1^2 + \frac{I_2^2}{2}}$$

$$= \sqrt{36 + \frac{56}{2}}$$

$= 8$ A

57. Two wavelengths λ_1 and λ_2 are used in Young's double slit experiment. $\lambda_1 = 450$ nm and $\lambda_2 = 650$ nm. The minimum order of fringe produced by λ_2, which overlaps with the fringe produced by λ_1 is n. The value of n is _________.

Answer (9)

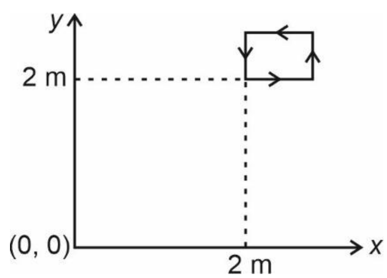
Sol. For overlap

$n_1\lambda_1 = n_2\lambda_2$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2}$$

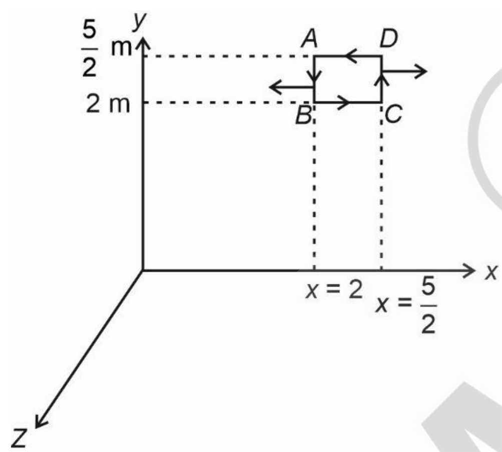
$$\frac{n_1}{n_2} = \frac{13}{9}$$

58. The magnetic field existing in a region is given by $\vec{B} = 0.2(1+2x)\hat{k}$T. A square loop of edge 50 cm carrying 0.5 A current is placed in x-y plane with its edges parallel to the x-y axes, as shown in figure. The magnitude of the net magnetic force experienced by the loop is _________ mN.



Answer (50)

Sol.



$\vec{F}_{BC} + \vec{F}_{DA} = 0$

$\vec{F}_{AB} = ilB = 0.5 \times 0.5(5) = 1.25 \text{ N} \times 0.2 = 0.25 \text{ N}$

$\vec{F}_{CD} = 0.5 \times 0.5(6) = 1.5 \times 0.2 = 0.3 \text{ N}$

F_{net} = 0.05 N

= 50 mN

59. Two forces $\vec{F_1}$ and $\vec{F_2}$ are acting on a body. One force has magnitude thrice that of the other force and the resultant of the two forces is equal to the force of larger magnitude. The angle between $\vec{F}_1$ and $\vec{F}_2$ is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of $|n|$ is _______.

Answer (6)

Sol. $F_1 = F$

$F_2 = 3F$

$F_{net} = 3F = F\sqrt{9+1+6\cos\theta}$

$\Rightarrow \quad 9 = 10 + 6\cos\theta$

$\Rightarrow \quad \cos\theta = -\frac{1}{6}$

$\therefore \quad |n| = 6$

60. A solid sphere and a hollow cylinder roll up without slipping on same inclined plane with same initial speed v. The sphere and the cylinder reaches upto maximum heights h_1 and h_2 respectively, above the initial level. The ratio $h_1 : h_2$ is $\frac{n}{10}$. The value of n is ________.

Answer (7)

Sol. If both having same mass and radius,

then for solid sphere, $\text{K.E.} = \frac{1}{2}(mv^2)\left(\frac{7}{5}\right) = mgh_1$

for hollow cylinder, K.E = $mv^2 = mgh_2$

$\frac{h_1}{h_2} = \frac{7}{10}$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Number of molecules/ions from the following in which the central atom is involved in sp^3 hybridization is ______.

NO_3^-, BCl_3, ClO_2^-, ClO_3

(1) 3 (2) 4

(3) 2 (4) 1

Answer (3)

Sol. ClO_2^- and ClO_3 are involved in sp^3 hybridisation.

62. The element which shows only one oxidation state other than its elemental form is

(1) Nickel (2) Titanium

(3) Cobalt (4) Scandium

Answer (4)

Sol. Scandium shows only one oxidation state other than its elemental form which is +3.

63. Given below are two statements :

Statement I : Acidity of α-hydrogens of aldehydes and ketones is responsible for Aldol reaction.

Statement II : Reaction between benzaldehyde and ethanal will NOT give Cross - Aldol product.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **Statement I** and **Statement II** are correct

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is correct but **Statement II** is incorrect

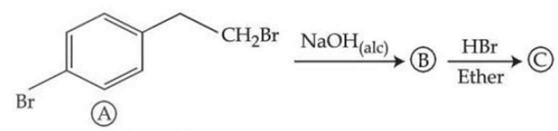
(4) **Statement I** is incorrect but **Statement II** is correct

Answer (3)

Sol. **Statement I** is correct as acidity of α-hydrogens is responsible for Aldol reaction.

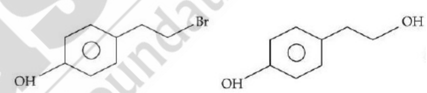
Statement II : Benzaldehyde and ethanal can give Cross-Aldol condensation reaction.

Hence, Statement II is incorrect.

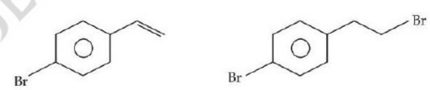
64. 

Identify (B) and (C) and how are (A) and (C) related?

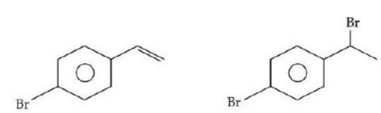
	(B)	(C)

(1) 

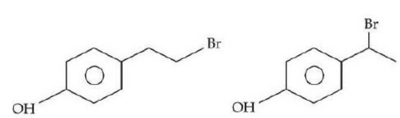
; functional group isomers

(2) 

; chain isomers

(3) 

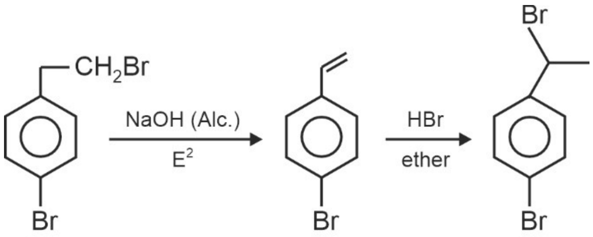
; position isomers

(4) 

; Derivative

Answer (3)

Sol.



(A) (B) (C)

(A) and (C) are position isomers.

$\therefore$ Option (3) is correct.

65. Which among the following is **incorrect** statement?

(1) Hydrogen ion (H^+) shows negative electromeric effect

(2) The electromeric effect is, temporary effect

(3) Electromeric effect dominates over inductive effect

(4) The organic compound shows electromeric effect in the presence of the reagent only

Answer (1)

Sol. Hydrogen ion does not show negative electromeric effect.

66. The correct order of first ionization enthalpy values of the following elements is

(A) O (B) N

(C) Be (D) F

(E) B

Choose the correct answer from the options given below.

(1) C < E < A < B < D

(2) A < B < D < C < E

(3) E < C < A < B < D

(4) B < D < C < E < A

Answer (3)

Sol. Correct ionization enthalpy order :

B < Be < O < N < F

or E < C < A < B < D

67. One of the commonly used electrode is calomel electrode. Under which of the following categories, calomel electrode comes?

(1) Metal - Insoluble Salt- Anion electrodes

(2) Oxidation - Reduction electrodes

(3) Gas - Ion electrodes

(4) Metal ion - Metal electrodes

Answer (1)

Sol. Calomel electrode is metal-insoluble salt – Anion electrode.

68. Which of the following nitrogen containing compound does not give Lassaigne's test?

(1) Hydrazine (2) Glycene

(3) Urea (4) Phenyl hydrazine

Answer (1)

Sol. Hydrazine (N_2H_4) doesn't contain any carbon atom and hence doesn't give Lassaigne test.

69. Number of elements from the following that CANNOT form compounds with valencies which match with their respective group valencies is ____.

B, C, N, S, O, F, P, Al, Si

(1) 6 (2) 5

(3) 7 (4) 3

Answer (4)

Sol. N, F and O will not satisfy the given condition.

70. What will be the decreasing order of basic strength of the following conjugate bases?

$^{-}OH, R\bar{O}, CH_3CO\bar{O}, C\bar{l}$

(1) $C\bar{l} > {}^{-}OH > R\bar{O} > CH_3CO\bar{O}$

(2) ${}^{-}OH > R\bar{O} > CH_3CO\bar{O} > C\bar{l}$

(3) $R\bar{O} > {}^{-}OH > CH_3CO\bar{O} > C\bar{l}$

(4) $C\bar{l} > R\bar{O} > {}^{-}OH > CH_3CO\bar{O}$

Answer (3)

Sol. Acidic strength order

$HCl > CH_3COOH > H_2O > ROH$

Basic strength order

$\overline{Cl} < CH_3CO\overline{O} < {}^{-}OH < R\overline{O}$

71. The Molarity (M) of an aqueous solution containing 5.85 g of NaCl in 500 mL water is :

(Given: Molar Mass Na : 23 and Cl : 35.5 $gmol^{-1}$)

(1) 0.2

(2) 20

(3) 4

(4) 2

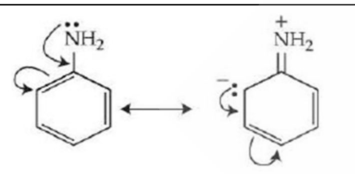
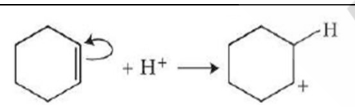
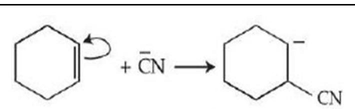
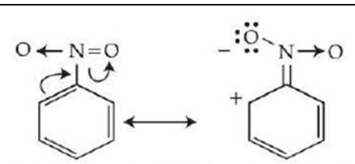
Answer (1)

Sol. Moles = 0.1

Volume = 0.5 L

Molarity $= \dfrac{0.1}{0.5} = 0.2$ M

72. Match **List I** with **List II** :

	List-I Mechanism steps		List-II Effect
(A)		(I)	–E effect
(B)		(II)	–R effect
(C)		(III)	+E effect
(D)		(IV)	+R effect

Choose the **correct** answer from the options given below.

(1) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

(2) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

(3) (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

(4) (A) - (III), (B) - (I), (C) - (II), (D) - (IV)

Answer (1)

Sol. A - +R(effect) (IV)

B - +E(effect) (III)

C - –E(effect) (I)

D - (–R effect) (II)

73. Identify the correct set of reagents or reaction conditions 'X' and 'Y' in the following set of transformation.

$$CH_3-CH_2-CH_2-Br \xrightarrow{\text{'X'}} \text{Product} \xrightarrow{\text{'Y'}} CH_3-\underset{\substack{|\\ Br}}{CH}-CH_3$$

(1) X = conc.alc. NaOH, 80°C, Y = $Br_2/CHCl_3$

(2) X = conc.alc. NaOH, 80°C, Y = HBr/acetic acid

(3) X = dil.aq. NaOH, 20°C, Y = $Br_2/CHCl_3$

(4) X = dil.aq. NaOH, 20°C, Y = HBr/acetic acid

Answer (2)

Sol. X : Alc. KOH

Y : HBr | Acetic acid

Product is $CH_3-CH=CH_2$

Final product is $CH_3-\underset{\substack{|\\ Br}}{CH}-CH_3$

74. What pressure (bar) of H_2 would be required to make emf of hydrogen electrode zero in pure water at 25°C?

(1) 10^{-14} (2) 0.5

(3) 10^{-7} (4) 1

Answer (1)

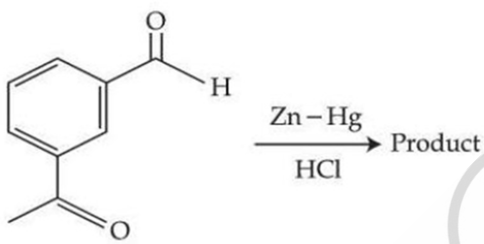
Sol. $[H^+] = 10^{-7}$ M

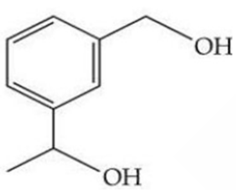
$H_2 \longrightarrow 2H^+ + 2e^-$

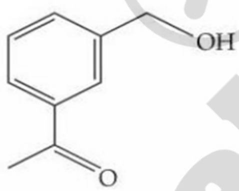
$P_{H_2} = [H^+]^2 = 10^{-14}$

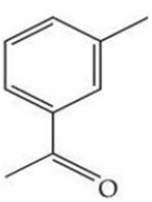
Option (1) is correct

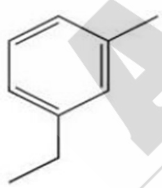
75. Identify the product in the following reaction :



(1) 

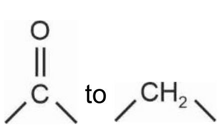
(2) 

(3) 

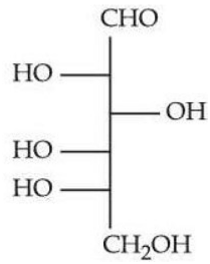
(4) 

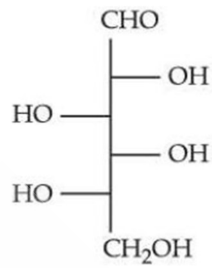
Answer (4)

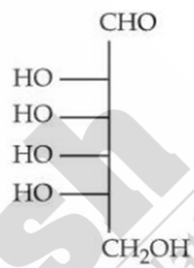
Sol. Clemmensen reduction will convert

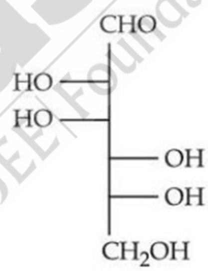


76. Which of the following is the correct structure of L-Glucose?

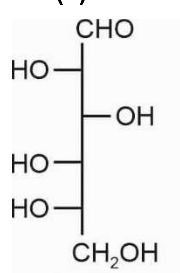
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

(L-Glucose)

77. Number of complexes from the following with even number of unpaired “d” electrons is _______.

$[V(H_2O)_6]^{3+}$, $[Cr(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{3+}$, $[Ni(H_2O)_6]^{3+}$, $[Cu(H_2O)_6]^{2+}$

[Given atomic numbers : V = 23, Cr = 24, Fe = 26, Ni = 28, Cu = 29]

(1) 1

(2) 5

(3) 2

(4) 4

Answer (3)

Sol. $[V(H_2O)_6]^{3+} \rightarrow$ 2 unpaired electrons

$[Cr(H_2O)_6]^{2+} \rightarrow$ 4 unpaired electrons

Above 2 complex have even number of unpaired electrons.

78. The correct sequence of ligands in the order of decreasing field strength is

(1) $NCS^- > EDTA^{4-} > CN^- > CO$

(2) $S^{2-} > {}^-OH > EDTA^{4-} > CO$

(3) $CO > H_2O > F^- > S^{2-}$

(4) ${}^-OH > F^- > NH_3 > CN^-$

Answer (3)

Sol. Field strength order : $CO > H_2O > F^- > S^{2-}$

79. In the precipitation of the iron group (III) in qualitative analysis, ammonium chloride is added before adding ammonium hydroxide to

(1) Prevent interference by phosphate ions

(2) Decrease concentration of ${}^-OH$ ions

(3) Increase concentration of Cl^- ions

(4) Increase concentration of NH_4^+ ions

Answer (2)

Sol. Ammonium chloride is added to increase NH_4^+ ions and hence decrease concentration of OH^- ions.

80. Which one of the following molecules has maximum dipole moment?

(1) NF_3

(2) NH_3

(3) PF_5

(4) CH_4

Answer (2)

Sol. NH_3 have more dipole moment than NF_3.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

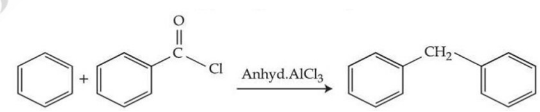
81. Number of molecules/species from the following having one unpaired electron is ______.

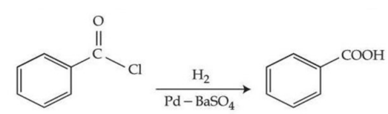
O_2, O_2^{-1}, NO, CN^{-1}, O_2^{2-}

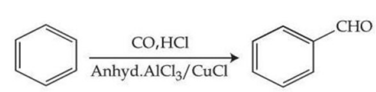
Answer (2)

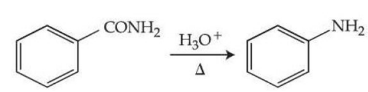
Sol. O_2^- and NO have 1 unpaired electron.

82. The number of the correct reaction(s) among the following is _______.

(A) 

(B) 

(C) 

(D) 

Answer (1)

Sol. Only reaction in option (C) is correct.

83. Only 2 mL of $KMnO_4$ solution of unknown molarity is required to reach the end point of a titration of 20 mL of oxalic acid (2 M) in acidic medium. The molarity of $KMnO_4$ solution should be _______ M.

Answer (8)

Sol. (M) × (2) × (5) = 2 × 20 × 2

M = 8

84. 2.5 g of a non-volatile, non-electrolyte is dissolved in 100 g of water at 25°C. The solution showed a boiling point elevation by 2°C. Assuming the solute concentration is negligible with respect to the solvent concentration, the vapour pressure of the resulting aqueous solution is ______ mm of Hg (nearest integer)

[Given : Molal boiling point elevation constant of water (K_b)] = 0.52 K. kg mol^{-1},

1 atm pressure = 760 mm of Hg, molar mass of water = 18 g mol^{-1}]

Answer (707)

Sol. $\Delta T_b = K_b(m)$

$$m = \frac{200}{52}$$

$$\frac{200}{52} = \frac{n_{solute}}{0.1}$$

$$n_{solute} = \frac{20}{52}$$

$$\frac{x}{760} = \frac{(20)(18)}{(52)(100)}$$

x = 52.61

$P_{solution}$ = 707.38

≈ 707 (Nearest integer)

85. Consider the following reaction

$$MnO_2 + KOH + O_2 \rightarrow A + H_2O$$

Product 'A' in neutral or acidic medium disproportionate to give products 'B' and 'C' along with water. The sum of spin-only magnetic moment values of B and C is _____BM. (nearest integer) (Given atomic number of Mn is 25)

Answer (4)

Sol. A is K_2MnO_4

B and C are $KMnO_4$ and MnO_2

$KMnO_4$ (μ = 0)

$MnO_2(Mn^{4+})$ (μ = 3.87)

Sum = 3.87 = 4 (Nearest integer)

86. The enthalpy of formation of ethane (C_2H_6) from ethylene by addition of hydrogen where the bond-energies of C — H, C — C, C = C, H — H are 414 kJ, 347 kJ, 615 kJ and 435 kJ respectively is – ______kJ

Answer (125)

Sol. $C_2H_4 + H_2 \longrightarrow C_2H_6$

ΔH = (615) + (435) – (347) – 2(414)

= 615 + 435 – 347 – 828

= –125 kJ

87. The number of different chain isomers for C_7H_{16} is________.

Answer (9)

Sol. (1) heptane

(2) 2-methylhexane

(3) 3-methylhexane

(4) 2,2-dimethylpentane

(5) 2,3-dimethylpentane

(6) 2,4-dimethylpentane

(7) 3,3-dimethylpentane

(8) 3-ethylpentane

(9) 2,2,3-trimethylbutane

88. Consider the following transformation involving first order elementary reaction in each step at constant temperature as shown below.

$$A + B \underset{\text{Step 3}}{\overset{\text{Step 1}}{\rightleftharpoons}} C \xrightarrow{\text{Step 2}} P$$

Some details of the above reactions are listed below:

Step	Rate constant (sec^{-1})	Activation energy ($kJ\ mol^{-1}$)
1	k_1	300
2	k_2	200
3	k_3	E_{a_3}

If the overall rate constant of the above transformation (k) is given as $k = \frac{k_1k_2}{k_3}$ and the overall activation energy (E_a) is 400 kJ mol^{-1}, then the value of E_{a_3} is ______ kJ mol^{-1} (nearest integer).

Answer (100)

Sol. $k = \frac{k_1k_2}{k_3}$

$E_{a_{eff}} = E_{a_1} + E_{a_2} - E_{a_3}$

$400 = 300 + 200 - E_{a_3}$

$400 = 500 - E_{a_3}$

$E_{a_3} = 100$ kJ mole^{-1}

89. X g of ethylamine is subjected to reaction with $NaNO_2/HCl$ followed by water; evolved dinitrogen gas which occupied 2.24 L volume at STP. X is ________ × 10^{-1} g.

Answer (45)

Sol. Moles of N_2 = 0.1

Mass of $C_2H_5NH_2$ = (0.1) × 45

= 4.5 gm

= 45 × 10^{-1}

= 45

90. The de-Broglie's wavelength of an electron in the 4th orbit is _______ (πa_0). (a_0 = Bohr's radius)

Answer (8)

Sol. $2\pi r = n\lambda$

$2\pi(16a_0) = 4\lambda$

$\lambda = 8\pi a_0$

❑ ❑ ❑