05/04/2024

Morning

Aakash

Medical | IIT-JEE | Foundations

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B.**

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6) **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

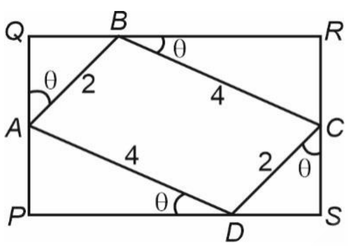
1. Let a rectangle *ABCD* of sides 2 and 4 be inscribed in another rectangle *PQRS* such that the vertices of the rectangle *ABCD* lie on the sides of the rectangles *PQRS*. Let *a* and *b* be the sides of the rectangle *PQRS* when its area is maximum. Then $(a + b)^2$ is equal to

(1) 80 (2) 64

(3) 60 (4) 72

Answer (4)

Sol.



$PD = 4\cos\theta \Rightarrow PS = 4\cos\theta + 2\sin\theta$

$DS = 2\sin\theta$

$AP = 4\sin\theta$

$QA = 2\cos\theta \Rightarrow PQ = 2\cos\theta + 4\sin\theta$

$\Rightarrow$ Area of *PQRS* $= 4(2\cos\theta + \sin\theta)(\cos\theta + 2\sin\theta)$

$= 4[2\cos^2\theta + 2\sin^2\theta + 5\sin\theta\cos\theta]$

$= 8 + 10\sin 2\theta$

Area is maximum when $\sin 2\theta = 1 \Rightarrow \theta = 45^\circ$

$\Rightarrow$ Maximum area = 8 + 10 = 18

$$\therefore\ PS = 4\times\frac{1}{\sqrt{2}} + 2\times\frac{1}{\sqrt{2}} = \frac{6}{\sqrt{2}}$$

$$PQ = 2\times\frac{1}{\sqrt{2}} + 4\times\frac{1}{\sqrt{2}} = \frac{6}{\sqrt{2}}$$

$$\therefore\ (a+b)^2 = \left(\frac{6}{\sqrt{2}} + \frac{6}{\sqrt{2}}\right)^2 = \left(\frac{12}{\sqrt{2}}\right)^2 = \left(6\sqrt{2}\right)^2 = 72$$

2. Let two straight lines drawn from the origin *O* intersect the line $3x + 4y = 12$ at the points *P* and *Q* such that ΔOPQ is an isosceles triangle and $\angle POQ = 90^\circ$. If $l = OP^2 + PQ^2 + QO^2$, then the greatest integer less than or equal to *l* is

(1) 44 (2) 42

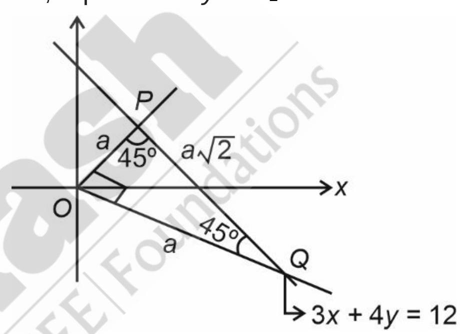
(3) 46 (4) 48

Answer (3)

Sol. $OP = OQ$ (ΔPQR is isosceles triangle)

Let slope of line $OP \to m_1$

So, equation $\to y = m_1x$



$$\tan 45^\circ = \left|\frac{m_1 - m_2}{m_1m_2}\right|$$

$$1 = \left|\frac{m_1 + \frac{3}{4}}{1 - \frac{3}{4}m_1}\right|$$

$$\Rightarrow\ 1 - \frac{3}{4}m_1 = m_1 + \frac{3}{4}$$

$$\frac{1}{4} = \frac{7}{4}m_1 \Rightarrow m_1 = \frac{1}{7}$$

Equation $OP \to y = \frac{1}{7}x$

Point of intersection of OP & line $3x + 4y = 12$

is $P\left(\frac{84}{25}, \frac{12}{25}\right)$

$$\Rightarrow \quad OP^2 = a^2 = \left(\frac{84}{25}\right)^2 + \left(\frac{12}{25}\right)^2 = \frac{288}{25}$$

$\therefore \quad l = OP^2 + PQ^2 + QO^2$

$= a^2 + a^2 + 2a^2$

$= 4a^2$

$= 4 \times \frac{288}{25}$

$l = 46.08$

$[l] = 46$

3. The integral $\int_0^{\pi/4} \frac{136\sin x}{3\sin x + 5\cos x} dx$ is equal to

(1) $3\pi - 25\log_e 2 + 10\log_e 5$

(2) $3\pi - 50\log_e 2 + 20\log_e 5$

(3) $3\pi - 10\log_e(2\sqrt{2}) + 10\log_e 5$

(4) $3\pi - 30\log_e 2 + 20\log_e 5$

Answer (2)

Sol. $\int_0^{\pi/4} \frac{136\sin x}{3\sin x + 5\cos x} dx$

$\sin x = A(3\sin x + 5\cos x) + B(3\cos x - 5\sin x)$

$$\left.\begin{matrix} 3A - 5B = 1 \\ 5A + 3B = 0 \end{matrix}\right\rangle \boxed{A = \frac{3}{34}} \quad \boxed{B = \frac{-5}{34}}$$

$$\int_0^{\pi/4} \frac{136\left[\frac{3}{34}(3\sin x + 5\cos x) - \frac{5}{34}(3\cos x - 5\sin x)\right]}{3\sin x + 5\cos x} dx$$

$$\int_0^{\pi/4} 12dx - 20\int_0^{\pi/4} \frac{3\cos x - 5\sin x}{3\sin x + 5\cos x} dx$$

$$12 \times \frac{\pi}{4} - 20\left[\ln\left|\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{2}}\right| - \ln 5\right]$$

$$3\pi - 20\ln 2^{5/2} + 20\ln 5$$

$\Rightarrow \quad 3\pi - 50\ln 2 + 20\ln 5$

4. If $A(1, -1, 2)$, $B(5, 7, -6)$, $C(3, 4, -10)$ and $D(-1, -4, -2)$ are the vertices of quadrilateral $ABCD$, then its area is

(1) $48\sqrt{7}$

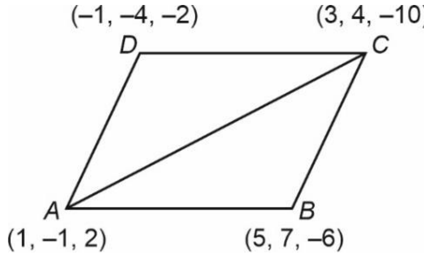
(2) $24\sqrt{7}$

(3) $24\sqrt{29}$

(4) $12\sqrt{29}$

Answer (4)

Sol.



Area of quadrilateral $ABCD$ = area of ΔABC + area of ΔADC

In ΔABC

$\overrightarrow{AB} = 4, 8, -8$

$\overrightarrow{BC} = -2, -3, -4$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 8 & -8 \\ -2 & -3 & -4 \end{vmatrix}$$

$= -56\hat{i} + 32\hat{j} + 4\hat{k}$

Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$

$= \frac{1}{2}\sqrt{56^2 + 32^2 + 4^2}$

$= \frac{1}{2}\sqrt{4176} = \frac{12\sqrt{29}}{2} = 6\sqrt{29}$

In ΔADC = $\overrightarrow{AD} = -2, -3, -4$

$\overrightarrow{DC} = 4, 8, -8$

$$\overrightarrow{AD} \times \overrightarrow{DC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -3 & -4 \\ 4 & 8 & -8 \end{vmatrix}$$

$= 56\hat{i} - 32\hat{j} - 4k$

Area of $\frac{1}{2}\left|\overrightarrow{AD} \times \overrightarrow{DC}\right|$

$= \frac{1}{2}\sqrt{4176}$

$= 6\sqrt{29}$

Area of $ABCD = 6\sqrt{29} + 6\sqrt{29}$

$= 12\sqrt{29}$

5. Consider the following two statements:

Statement I : For any two non-zero complex numbers z_1, z_2, $(|z_1|+|z_2|)\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le 2(|z_1|+|z_2|)$,

and **Statement II** : If x, y and z are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$, then

$\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = 1$

Between the two statements

(1) Both **Statement I** and **Statement II** are correct

(2) **Statement I** is incorrect but **Statement II** is correct

(3) Both **Statement I** and **Statement II** are incorrect

(4) **Statement I** is correct but **Statement II** is incorrect

Answer (4)

Sol. Statement II

$$\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|} = \lambda$$

$\Rightarrow$ $a^2 = \lambda^2|(y-z)|^2$

$b^2 = \lambda^2|(z-x)|^2$

$c^2 = \lambda^2|(x-y)|^2$

$$\frac{a^2\left(\overline{y-z}\right)}{(y-z)\left(\overline{y-z}\right)} = \frac{a^2\left(\bar{y}-\bar{z}\right)}{|y-z|^2} = \frac{a^2\left(\bar{y}-\bar{z}\right)}{\frac{a^2}{\lambda^2}} = \lambda^2\left(\bar{y}-\bar{z}\right)$$

$$\Rightarrow \sum\left(\frac{a^2}{y-z}\right) = \lambda^2\left(\bar{y}-\bar{z}+\bar{z}-\bar{x}+\bar{x}-\bar{y}\right) = 0 \ne 1$$

Statement I

$$(|z_1|+|z_2|)\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le 2(|z_1|+|z_2|)$$

$\Rightarrow z_1 = |z_1|e^{i\theta_1}$

$z_2 = |z_2|e^{i\theta_2}$

$\Rightarrow \frac{z_1}{|z_1|} = e^{i\theta_1}$

$\Rightarrow \frac{z_2}{|z_2|} = e^{i\theta_2}$

$\Rightarrow |e^{i\theta_1} + e^{i\theta_2}|$

$= \left|\sqrt{2+2\cos(\theta_1-\theta_2)}\right|$

$= \left|2\cos\left(\frac{\theta_1-\theta_2}{2}\right)\right| \le 2$

6. The coefficients a, b, c in the quadratic equation $ax^2+bx+c=0$ are chosen from the set {1,2,3,4,5,6,7,8}. The probability of this equation having repeated roots is

(1) $\frac{3}{256}$

(2) $\frac{1}{64}$

(3) $\frac{3}{128}$

(4) $\frac{1}{128}$

Answer (2)

Sol. Given quadratic equation is

$ax^2 + bx + c = 0$ where $a, b, c \in \{1,2,3,\ldots,8\}$

For repeated roots,

$b^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac$

$\Rightarrow$ ac must be a perfect square

(a, c) $\in$ {(1, 1), (1, 4), (2, 2), (2, 8), (3, 3), (4, 1), (4, 4), (5, 5), (6, 6), (7, 7), (8, 2), (8, 8)}

Corresponding b must lie in set {1, 2, 3, …8}

$(a, b, c) \in \{(1, 2, 1), (1, 2, 4), (2, 4, 2), (2, 8, 8), (3, 6, 3), (4, 4, 1), (4, 8, 4), (8, 8, 2)\}$

$\therefore$ probability $= \dfrac{8}{8^3}$

$= \dfrac{1}{64}$

7. If $\dfrac{1}{\sqrt{1}+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+...+\dfrac{1}{\sqrt{99}+\sqrt{100}}=m$ and $\dfrac{1}{1\cdot2}+\dfrac{1}{2\cdot3}+....+\dfrac{1}{99\cdot100}=n$, then the point (m, n) lies on the line

(1) $11(x-1)-100(y-2)=0$

(2) $11(x-2)-100(y-1)=0$

(3) $11(x-1)-100y=0$

(4) $11x-100y=0$

Answer (4)

Sol. $\dfrac{1}{\sqrt{1}+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+...+\dfrac{1}{\sqrt{99}+\sqrt{100}}=m$

and $\dfrac{1}{1\cdot2}+\dfrac{1}{2\cdot3}+...+\dfrac{1}{99\cdot100}=n$

$\dfrac{1}{\sqrt{1}+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+...+\dfrac{1}{\sqrt{99}+\sqrt{100}}$

$=\dfrac{1}{\sqrt{1}+\sqrt{2}}\times\dfrac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}}+\dfrac{1}{\sqrt{3}+\sqrt{2}}\times\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$+...+\dfrac{1}{\sqrt{99}+\sqrt{100}}\times\dfrac{\sqrt{100}-\sqrt{99}}{\sqrt{100}-\sqrt{99}}$

$=\sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+...+\sqrt{100}-\sqrt{99}$

$=\sqrt{100}-\sqrt{1}$

$= 10 - 1$

$\Rightarrow \boxed{m=9}$

and $\dfrac{1}{1\cdot2}+\dfrac{1}{2\cdot3}+...+\dfrac{1}{99\cdot100}=n$

$\dfrac{2-1}{1\times2}+\dfrac{3-2}{2\times3}+...+\dfrac{100-99}{100\times99}=n$

$\Rightarrow 1-\dfrac{1}{2}+\dfrac{1}{2}-\dfrac{1}{3}+...+\dfrac{1}{99}-\dfrac{1}{100}=n$

$\Rightarrow n=1-\dfrac{1}{100}$

$\Rightarrow \boxed{n=\dfrac{99}{100}}$

$(m,n)=\left(9,\dfrac{99}{100}\right)$

Satisfies the line $11x - 100y = 0$

8. The value of $\displaystyle\int_{-\pi}^{\pi}\frac{2y(1+\sin y)}{1+\cos^2 y}dy$ is

(1) π^2

(2) $\dfrac{\pi}{2}$

(3) $\dfrac{\pi^2}{2}$

(4) $2\pi^2$

Answer (1)

Sol. $I=\displaystyle\int_{-\pi}^{\pi}\frac{2y(1+\sin y)}{1+\cos^2 y}dy$

$=\displaystyle\int_0^{\pi}\left(\frac{2y(1+\sin y)}{1+\cos^2 y}+\frac{-2y(1-\sin y)}{1+\cos^2 y}\right)dy$

$=\displaystyle\int_0^{\pi}\left(\frac{2y+2y\sin y-2y+2y\sin y}{1+\cos^2 y}\right)dy$

$I=4\displaystyle\int_0^{\pi}\left(\frac{y\sin y}{1+\cos^2 y}\right)dy$ ...(1)

$I=4\displaystyle\int_0^{\pi}\left(\frac{(\pi-y)\sin(\pi-y)}{1+\cos^2(\pi-y)}\right)dy$

$I=4\left[\displaystyle\int_0^{\pi}\left(\frac{\pi\sin y}{1+\cos^2 y}\right)dy-\int_0^{\pi}\frac{y\sin y}{1+\cos^2 y}dy\right]$ ...(2)

Adding equation (1) and (2)

$$2I = 4\int_0^{\pi}\left(\frac{\pi \sin y}{1+\cos^2 y}\right)dy$$

$$I = 2\pi\int_0^{\pi}\frac{\sin y}{1+\cos^2 y}dy$$

$$= 2\pi \times \frac{\pi}{2}$$

$= \pi^2$

9. For the function, $f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x)$, where $x \in \left[0, \frac{\pi}{2}\right]$, consider the following two statements:

(I) f is increasing in $\left(0, \frac{\pi}{2}\right)$

(II) f' is decreasing in $\left(0, \frac{\pi}{2}\right)$

Between the above two statements

(1) Neither (I) nor (II) is true

(2) Only (I) is true

(3) Both (I) and (II) are true

(4) Only (II) is true

Answer (3)

Sol. $f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x)$, where $x \in \left[0, \frac{\pi}{2}\right]$

$$f'(x) = \cos x + 3 - \frac{2}{\pi}(2x+1)$$

$$= \cos x - \frac{4x}{\pi} - \frac{2}{\pi} + 3$$

as $x \in \left[0, \frac{\pi}{2}\right]$

$$\frac{4x}{\pi} \in [0, 2]$$

$$\Rightarrow \quad 3 - \frac{2}{\pi} - \frac{4x}{\pi} > 0$$

and also cos$x$ > 0 when $x \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \quad f'(x) > 0$

$\Rightarrow \quad f(x)$ is increasing

Now, $f''(x) = -\sin x - \frac{4}{\pi} < 0 \; \forall \; x \in \left[0, \frac{\pi}{2}\right]$

Hence, $f'(x)$ is decreasing

$\therefore$ Both statements (I) and (II) are true

10. If the system of equations

$11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to

(1) 51 (2) 47

(3) 49 (4) 45

Answer (2)

Sol. $11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

$$\Delta = 0 \Rightarrow \begin{vmatrix} 11 & 1 & \lambda \\ 2 & 3 & 5 \\ 8 & -19 & -39 \end{vmatrix} = 0$$

$11(-39.3 + 19.5) - 1(-39.2 - 40) + \lambda(-38 - 24) = 0$

$= 11(-117 + 95) - 1(-118) - 62\lambda = 0$

$= -242 + 118 = 62\lambda$

$\Rightarrow \quad \lambda = -2$

$\Delta_2 = 0$

$$\Rightarrow \begin{vmatrix} 11 & 1 & -5 \\ 2 & 3 & 3 \\ 8 & -19 & \mu \end{vmatrix} = 0$$

$11(3\mu + 57) - 1(2\mu - 24) - 5(-38-24) = 0$

$33\mu + 627 - 2\mu + 24 + 310 = 0$

$\mu = -31$

$\Rightarrow \lambda^4 - 31$

$= 16 + 31$

$= 47$

11. Let A and B be two square matrices of order 3 such that $|A| = 3$ and $|B| = 2$. Then $|A^T A(\text{adj}(2A))^{-1}(\text{adj}(AB))^{-1}AA^T|$ is equal to

(1) 108 (2) 64

(3) 32 (4) 81

Answer (2)

Sol. $|A| = 3$

$|B| = 2$

$|A^T|\,|A||(\text{adj}(2A))^{-1}||\text{adj}(4B)||(\text{adj}(AB))^{-1})\,|A||A^T|$

$$3\cdot 3\,\frac{1}{64\cdot 9}(64)^2\cdot 4\cdot\frac{1}{9\cdot 4}3\cdot 3$$

$= 64$

12. Suppose $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solution of $4\cos\theta - 3\sin\theta = 1$. Then $\cos\theta$ is equal to

(1) $\frac{6+\sqrt{6}}{(3\sqrt{6}+2)}$ (2) $\frac{6-\sqrt{6}}{(3\sqrt{6}-2)}$

(3) $\frac{4}{(3\sqrt{6}+2)}$ (4) $\frac{4}{(3\sqrt{6}-2)}$

Answer (4)

Sol. $4\cos\theta - 3\sin\theta = 1$

$4\cos\theta - 1 = 3\sin\theta$

$16\cos^2\theta + 1 - 8\cos\theta = 9(1 - \cos^2\theta)$

$\Rightarrow 25\cos^2\theta - 8\cos\theta - 8 = 0$

$$\Rightarrow \cos\theta = \frac{8 \pm \sqrt{64 + 4\times 25\times 8}}{2.25}$$

$$= \frac{8 \pm 4\sqrt{4+50}}{2.25}$$

$$= \frac{4 \pm 2\sqrt{54}}{25}$$

As $\theta \in \left[0, \frac{\pi}{4}\right]$

$$\Rightarrow \cos\theta = \frac{4+6\sqrt{6}}{25} = \frac{4}{3\sqrt{6}-2}$$

13. Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12)$. Then the total number of one-one maps $f : A \to B$, such that $f(1) + f(3) = 14$, is:

(1) 180

(2) 120

(3) 480

(4) 240

Answer (4)

Sol. $f(1) + f(3) = 14$

Case I $f(1) = 2, f(3) = 12$

$f(1) = 12, f(3) = 2$

Total one-one function

$= 2 \times 5 \times 4 \times 3$

$= 120$

Case II $f(1) = 4, f(3) = 10$

$f(1) = 10, f(3) = 4$

Total one-one function

$= 2 \times 5 \times 4 \times 3$

$= 120$

Total cases = 120 + 120 = 240

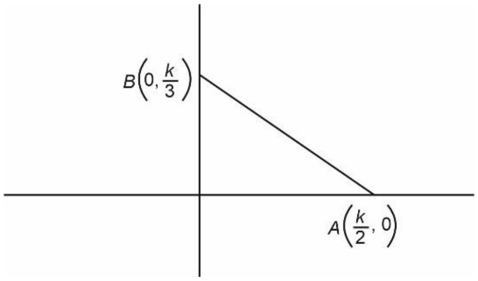
14. Let the line $2x + 3y - k = 0$, $k > 0$, intersect the x-axis and y-axis at the points A and B, respectively. If the equation of the circle having the line segment AB as a diameter is $x^2 + y^2 - 3x - 2y = 0$ and the length of the latus rectum of the ellipse $x^2 + 9y^2 = k^2$ is $\frac{m}{n}$, where m and n are coprime, then $2m+n$ is equal to

(1) 10 (2) 13

(3) 11 (4) 12

Answer (3)

Sol.



Equation of circle with AB as diameter

$$\left(x-\frac{k}{2}\right)x+y\left(y-\frac{k}{3}\right)=0$$

$$\Rightarrow x^2+y^2-\frac{kx}{2}-\frac{ky}{3}=0$$

Comparing, $k = 6$

Latus rectum of ellipse

$x^2 + 9y^2 = k^2 = 6^2$

$$\Rightarrow \frac{x^2}{6^2}+\frac{y^2}{2^2}=1$$

$$\text{L.R} = \frac{2b^2}{a}=\frac{2\times 4}{6}=\frac{4}{3}$$

$m = 4$

$n = 3$

$2m + n = 8 + 3 = 11$

15. If the function

$$f(x)=\frac{\sin 3x+\alpha\sin x-\beta\cos 3x}{x^3}, x\in \mathbf{R},$$

is continuous at $x = 0$, then $f(0)$ is equal to

(1) –4 (2) 4

(3) 2 (4) –2

Answer (1)

Sol. $\lim\limits_{x\to 0} f(x)=f(0)$ (continuous at $x = 0$)

$$\lim_{x\to 0}\frac{\sin 3x+\alpha\sin x-\beta\cos 3x}{x^3}$$

For limit to exist $\beta = 0$

$$\Rightarrow \lim_{x\to 0}\frac{\sin 3x+\alpha\sin x}{x^3}$$

$$\Rightarrow \lim_{x\to 0}\frac{(3+\alpha)\sin x-4\sin^3 x}{x^3}$$

For limit to exist $\alpha + 3 = 0 \Rightarrow \alpha = -3$

$$\Rightarrow \lim_{x\to 0}\frac{-4\sin^3 x}{x^3}=-4=f(0)$$

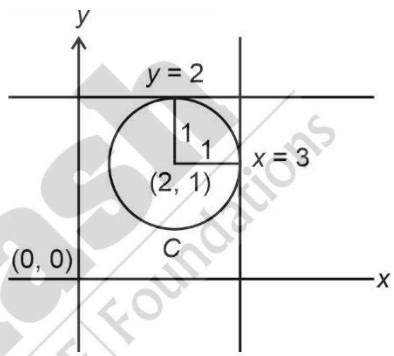
16. Let a circle C of radius 1 and closer to the origin be such that the lines passing through the point (3, 2) and parallel to the coordinate axes touch it. Then the shortest distance of the circle C from the point (5, 5) is:

(1) 4 (2) $4\sqrt{2}$

(3) $2\sqrt{2}$ (4) 5

Answer (1)

Sol.



Shortest distance of circle C form (5, 5)

$= \sqrt{9+16}-1$

$= 5 - 1 = 4$

17. Let d be the distance of the point of intersection of the lines $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$ and $\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$ from the point (7,8,9). Then $d^2 + 6$ is equal to

(1) 78 (2) 69

(3) 75 (4) 72

Answer (3)

Sol. $P_1 : (3k - 6, 2k, k - 1)$

$P_2\ (4\alpha + 7, 3\alpha + 9, 2\alpha + 4)$

$P_1 \equiv P_2$

$3k - 6 = 4\alpha + 7 \Rightarrow 3k - 4\alpha = 13$

$2k = 3\alpha + 9 \Rightarrow 2k - 3\alpha = 9$

$\therefore k = 3, \alpha = -1$

$\therefore P_1 : (3, 6, 2)$

Distance of (3, 6, 2) and (7, 8, 9)

$= \sqrt{16+4+49} = \sqrt{69} = d$

$d^2 + 6 = 69 + 6 = 75$

18. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + 2y = \sin(2x), y(0) = \frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to

(1) $e^{-\frac{\pi}{8}}$ (2) $e^{\frac{\pi}{4}}$

(3) $e^{-\frac{\pi}{4}}$ (4) $e^{\frac{\pi}{8}}$

Answer (3)

Sol. $\frac{dy}{dx} + 2y = \sin 2x$

$\text{IF} = e^{\int 2dx} = e^{2x}$

$y.e^{2x} = \int e^{2x} \sin 2x\, dx + c$

$= \frac{e^{2x}}{8}(2\sin 2x - 2\cos 2x) + c$

$y(0) = \frac{3}{4}$

$\frac{3}{4} = \frac{1}{8}(-2) + c \Rightarrow c = 1$

Put $x = \frac{\pi}{8}$

$y = \frac{1}{8} \times 2\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) + e^{-\pi/4}$

$y = e^{-\pi/4}$

19. Let $f(x) = x^5 + 2x^3 + 3x + 1$, $x \in \mathbf{R}$, and $g(x)$ be a function such that $g(f(x)) = x$ for all $x \in \mathbf{R}$. Then $\frac{g(7)}{g'(7)}$ is equal to

(1) 14 (2) 1

(3) 7 (4) 42

Answer (1)

Sol. $f(x) = x^5 + 2x^3 + 3x + 1$

$g(f(x)) = x.$ $\Rightarrow g'(f(x))f'(x) = 1$

Now $\frac{g(7)}{g'(7)}$

$g(7) \Rightarrow f(x) = 7$

$x^5 + 2x^3 + 3x + 1 = 7$

$\Rightarrow x(x^4 + 2x^2 + 3) = 0$

$\Rightarrow x = 1$

$\therefore g(7) \Rightarrow g(f(1)) = 1$

& $g'(f(x)) = \frac{1}{f'(x)}$

$g'(7)$

$\Rightarrow f(x) = 7 \Rightarrow x = 1$

$\therefore g'(7) = \frac{1}{f'(1)}$

$= \frac{1}{5x^4 + 6x^2 + 3}$

$= \frac{1}{14}$

$\therefore \frac{g(7)}{g'(7)} = \frac{1}{\frac{1}{14}} = 14$

20. If the line $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4-z$ makes a right angle with the line $\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$, then $4\lambda + 9\mu$ is equal to

(1) 13 (2) 6

(3) 4 (4) 5

Answer (2)

Sol. $L_1: \dfrac{x-2}{(-3)} = \dfrac{y-\frac{2}{3}}{\left(\frac{4\lambda+1}{3}\right)} = \dfrac{z-4}{(-1)}$

$L_2: \dfrac{x+3}{3\mu} = \dfrac{y-\frac{1}{2}}{-3} = \dfrac{z-5}{-7}$

$\because L_1 \perp L_2$

$\Rightarrow (-3)(3\mu) + \left(\dfrac{4\lambda+1}{3}\right)(-3) + (-1)(-7) = 0$

$-9\mu - 4\lambda - 1 + 7 = 0$

$\Rightarrow 4\lambda + 9\mu = 6$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If S = {$a \in$ **R:** $|2a - 1| = 3[a] + 2\{a\}$}, where $[t]$ denotes the greatest integer less than or equal to t and $\{t\}$ represents the fractional part of t, then $72\sum_{a\in S} a$ is equal to ______

Answer (18)

Sol. $S: \{a \in R : |2a - 1| = 3[a] + 2\{a\}\}$

$|2a - 1| = 3[a] + 2(a - [a])$

$|2a - 1| = [a] + 2a$

Case I: If $0 < a < \frac{1}{2}$

$1 - 2a = 0 + 2a$

$\Rightarrow a = \frac{1}{4}$

Case II: If $\frac{1}{2} < a < 1$

$2a - 1 = 0 + 2a$

No solution

Case III: If $1 \le a < 2$

$2a - 1 = 1 + 2a$

$\Rightarrow$ No solution

$\therefore$ only solution is $a = \frac{1}{4}$

$72\sum_{a\in S} a = 72 \times \frac{1}{4} = 18$

22. If the constant term in the expansion of $(1 + 2x - 3x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is p, then $108p$ is equal to _____

Answer (54)

Sol. General term of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$T_{r+1} = {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r = {}^9C_r\,(-1)^r\,3^{9-2r}\,2^{r-9}\,x^{18-35}$

Constant term in expansion of $(1 + 2x - 3x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$= T_7 - 3T_8 = {}^9C_6\,3^{-3}.\,2^{-3} + 3\,{}^9C_7 \cdot 3^{-5}\,.\,2^{-2}$

$= \dfrac{3\times4\times7}{3^3.2^3} + \dfrac{3\times9\times4}{3^5\times2^2} =$

$p = \dfrac{42+12}{108} = \dfrac{54}{108}$

$108p = 54$

23. The number of distinct real roots of the equation $|x|\,|x+2| - 5|x+1| - 1 = 0$ is _____

Answer (3)

Sol. $|x| \quad |x+2| - 5\,|x+1| - 1 = 0$

(I) if $x < -2$,

$x^2 + 2x + 5x + 5 - 1 = 0$

$x^2 + 7x + 4 = 0 \Rightarrow$ one root satisfying $x < -2$

(II) if $-2 \le x < -1$

$-x^2 - 2x + 5x + 5 - 1 = 0$

$x^2 - 3x - 4 = 0 \Rightarrow$ not root satisfying $-2 \le x < -1$

(III) if $-1 \le x < 0$

$-x^2 - 2x - 5x - 5 - 1 = 0$

$x^2 + 7x + 6 = 0$

x = – 1 is only root satisfying $-1 \le x < 0$

(IV) if $x \ge 0$

$x^2 + 2x - 5x - 5 - 1 = 0$

$x^2 - 3x - 6 = 0$

one root satisfying $x \ge 0$

$\Rightarrow$ The number of distinct real roots are three.

24. The number of ways of getting a sum 16 on throwing a dice four times is ______

Answer (125)

Sol. Number of ways = coefficient of x^{16} in $(x + x^2 + ... + x^6)^4$

= coefficient of x^{16} in $(1 - x^6)^4 (1 - x)^{-4}$

= coefficient of x^{16} in $(1 - 4x^6 + 6x^{12} ...) (1 - x)^{-4}$

$= {}^{15}C_3 - 4 \cdot {}^{9}C_3 + 6 = 125$

25. The area of the region enclosed by the parabolas $y = x^2 - 5x$ and $y = 7x - x^2$ is _____

Answer (72*)

Sol. $y = x^2 - 5x,\ y = 7x - x^2 \Rightarrow \quad x^2 - 5x = 7x - x^2$

$\Rightarrow \quad x = 0,\ x = 6$

$$\text{Area} = \int_0^6 \left[(7x - x^2) - (x^2 - 5x)\right] dx$$

$$= \int_0^6 (12x - 2x^2) dx = 6x^2 - \frac{2x^3}{3}\Bigg|_0^6$$

= 216 – 144 = 72 sq. unit

But answer is 198 as per NTA.

26. From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2, then $96\sigma^2$ is equal to _______

Answer (56)

Sol.

x	0	1	2	3
$P(x)$	$\frac{{}^7C_5}{{}^{10}C_5} = \frac{1}{12}$	$\frac{{}^7C_4 \cdot {}^3C_1}{{}^{10}C_5} = \frac{5}{12}$	$\frac{{}^7C_3 \cdot {}^3C_2}{{}^{10}C_5} = \frac{5}{12}$	$\frac{{}^7C_2 \cdot {}^3C_3}{{}^{10}C_5} = \frac{1}{12}$
$xP(x)$	0	$\frac{5}{12}$	$\frac{10}{12}$	$\frac{3}{12}$

$$\mu = \sum xP(x) = 0 + \frac{5}{12} + \frac{10}{12} + \frac{3}{12} = \frac{3}{2}$$

$$\sigma^2 = \sum (x - \mu)^2 P(x) = \sum \left(x - \frac{3}{2}\right)^2 P(x)$$

$$= \frac{9}{4} \times \frac{1}{12} + \frac{1}{4} \times \frac{5}{12} + \frac{1}{4} \times \frac{5}{12} + \frac{9}{4} \times \frac{1}{12} = \frac{7}{12}$$

$\Rightarrow \quad \sigma^2 . 96 = 8 \times 7 = 56$

27. Let f be a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim\limits_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $2f(2) + 3f(3)$ is equal to ______

Answer (24)

Sol. $\lim\limits_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{(t - x)} = 1 \qquad \left(\frac{0}{0}\text{form}\right)$

$$\lim_{t \to x} \frac{2tf(x) - x^2 f'(t)}{1} = 1$$

$\Rightarrow \quad 2xf(x) - x^2 f'(x) = 1$

$$\frac{dy}{dx} - \frac{2xy}{x^2} = \frac{-1}{x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} - \left(\frac{2}{x}\right) y = \frac{-1}{x^2}$$

$$\Rightarrow \quad I.F. = e^{\int \frac{-2}{x} dx} = e^{-2\ln x} = x^{-2} = \frac{1}{x^2}$$

$$\Rightarrow \quad y\left(\frac{1}{x^2}\right) = \int \left(\frac{-1}{x^2}\right)\left(\frac{1}{x^2}\right) dx + C$$

$$\frac{y}{x^2} = \frac{1}{3x^3} + C \quad \text{at } x = 1,\ y = 1$$

$$\Rightarrow \quad 1 = \frac{1}{3} + C \Rightarrow C = \frac{2}{3}$$

$$\Rightarrow \quad y = \frac{1}{3x} + \frac{2}{3}x^2 = f(x)$$

$\Rightarrow \quad 2f(2) + 3f(3) = 24$

28. Suppose AB is a focal chord of the parabola $y^2 = 12x$ of length l and slope $m < \sqrt{3}$. If the distance of the chord AB from the origin is d, then ld^2 is equal to

Answer (108)

Sol. Equation of focal chord

$y - 0 = \tan\theta.\ (x - 3)$

Distance from origin

$$d = \left|\frac{-3\tan\theta}{\sqrt{1+\tan^2\theta}}\right|$$

$l = 4 \times 3\ \text{cosec}^2\theta$

$$l.\ d^2 = \frac{9\tan^2\theta}{1+\tan^2\theta} \times 12\,\text{cosec}^2\theta$$

$$= \frac{108\,\text{cosec}^2\theta}{1+\cot^2\theta} = 108$$

29. Let $\vec{a} = \hat{i} - 3\hat{j} + 7\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$. If $\vec{a} \cdot \vec{c}$ = 130, then $\vec{b} \cdot \vec{c}$ is equal to ____

Answer (30)

Sol. $(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$

$$\Rightarrow \quad \vec{b} \times \vec{c} + 2(\vec{a} \times \vec{c}) = 0$$

$$(\vec{b} + 2\vec{a}) \times \vec{c} = 0$$

$$\vec{c} = \lambda(\vec{b} + 2\vec{a})$$

$$\vec{c}.\ \vec{a} = 130 \Rightarrow \lambda = 1$$

$$\vec{c} = 4\hat{i} - 7\hat{j} + 15\hat{k}$$

$$\vec{b}.\ \vec{c} = 30$$

30. Let a_1, a_2, a_3, …… be in arithmetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153$, $A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to ________

Answer (910)

Sol. Let $a_n = a + (n - 1)d\ \forall\ n \in N$

$$A_k = (a_1^2 - a_2^2) + (a_3^2 - a_4^2) + \ldots a_{2k-1}^2 - a_{2k}^2$$

$= (-d)\ (a_1 + a_2 + \ldots + a_{2k})$

$A_k = (-dk)\ (2a + (2k - 1)d)$

$\Rightarrow \quad A_3 = (-3d)\ (2a + 5d) = -153$

$\Rightarrow \quad d(2a + 5d) = 51 \qquad \ldots(i)$

$A_5 = (-5d)\ (2a + 9d) = -435$

$\Rightarrow \quad d(2a + 9d) = 87$

$\Rightarrow \quad 4d^2 = 36 \Rightarrow d = \pm 3$ ($d = 3$ positive terms)

$\Rightarrow \quad 3(2a + 27) = 87$

$\Rightarrow \quad 2a = 29 - 27$

$\Rightarrow \quad a = 1$

$a_{17} - A_7 = (a + 16d) - (-7d)\ (2 + 13d)$

$= 49 + 7 \times 3(2 + 39)$

$= 49 + 21 \times 41 = 910$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. A simple pendulum doing small oscillations at a place R height above earth surface has time period of $T_1 = 4$ s. T_2 would be it's time period if it is brought to a point which is at a height $2R$ from earth surface. Choose the correct relation [R = radius of earth]

(1) $T_1 = T_2$ (2) $3T_1 = 2T_2$

(3) $2T_1 = 3T_2$ (4) $2T_1 = T_2$

Answer (2)

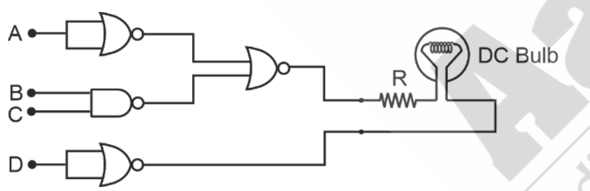
Sol. $T = 2\pi\sqrt{\frac{l}{g}}$ & $g = \frac{GM}{r^2}$

$\therefore \quad T \propto r$

$\Rightarrow \frac{T_1}{T_2} = \frac{(R+R)}{(R+2R)} = \frac{2}{3}$

$\Rightarrow \quad 3T_1 = 2T_2$

32. Following gates section is connected in a complete suitable circuit.



For which of the following combination, bulb will glow (ON)

(1) A = 1, B = 1, C = 1, D = 0

(2) A = 0, B = 1, C = 1, D = 1

(3) A = 0, B = 0, C = 0, D = 1

(4) A = 1, B = 0, C = 0, D = 0

Answer (4)

Sol. For bulb to glow, there should be low-high combination across the bulb.

If $A = 1$, $B = 0$, $C = 0$ & $D = 0$

$\overline{1+1} = 0$ & $\bar{0} = 1$

So, bulb will glow.

33. In hydrogen like system the ratio of coulombian force and gravitational force between an electron and a proton is in the order of

(1) 10^{19} (2) 10^{39}

(3) 10^{36} (4) 10^{29}

Answer (2)

Sol. $\frac{Fe}{F_G} = \frac{Ke^2}{Gm_e m_p}$

$= \frac{(9\times10^9)(1.6\times10^{-19})^2}{(6.67\times10^{-11})(9.1\times10^{-31})(1.67\times10^{-27})}$

$\simeq 1.5 \times 10^{39}$

$\Rightarrow$ order is 10^{39}

34. If G be the gravitational constant and u be the energy density then which of the following quantity have the dimensions as that of the $\sqrt{uG}$:

(1) Force per unit mass

(2) Pressure gradient per unit mass

(3) Energy per unit mass

(4) Gravitational potential

Answer (1)

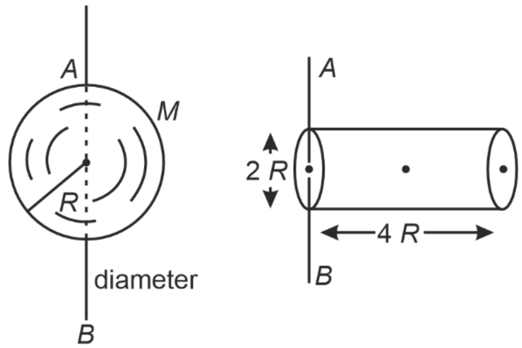
Sol. $[\sqrt{uG}] = \sqrt{\frac{[ML^2T^{-2}]}{[L^3]}[M^{-1}L^3T^{-2}]}$

$= \sqrt{L^2T^{-4}}$

$= [LT^{-2}]$

$\Rightarrow \quad [LT^{-2}] = \left[\frac{\text{Force}}{\text{Mass}}\right]$

35. Ratio of radius of gyration of a hollow sphere to that of a solid cylinder of equal mass, for moment of Inertia about their diameter axis AB as shown in figure is $\sqrt{\frac{8}{x}}$. The value of x is



(1) 67 (2) 51

(3) 17 (4) 34

Answer (1)

Sol. For hollow sphere

$$I_1 = \frac{2}{3}mR^2 = \left(\sqrt{\frac{2}{3}}R\right)^2 m$$

$$\therefore \quad K_1 = \sqrt{\frac{2}{3}}R$$

For solid cylinder

$$I_2 = \frac{1}{4}mR^2 + \frac{m}{3}(4R)^2$$

$$= \left(\sqrt{\frac{67}{12}}R\right)^2 m$$

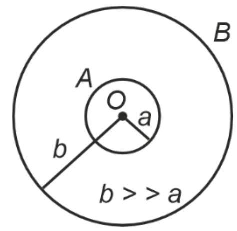
$$\therefore \quad K_2 = \sqrt{\frac{67}{12}}R$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \sqrt{\frac{2}{3}\times\frac{12}{67}}$$

$$= \sqrt{\frac{8}{67}}$$

$$\Rightarrow \quad x = 67$$

36. Two conducting circular loops A and B are placed in the same plane with their centres coinciding as shown in figure. The mutual inductance between them is



(1) $\frac{\mu_0}{2\pi}\cdot\frac{a^2}{b}$ (2) $\frac{\mu_0 \pi b^2}{2a}$

(3) $\frac{\mu_0 \pi a^2}{2b}$ (4) $\frac{\mu_0}{2\pi}\cdot\frac{b^2}{a}$

Answer (3)

Sol. $\frac{\phi_{A/B}}{I_B} = M$

$$\Rightarrow \quad M = \frac{\mu_0 I_B \times \pi a^2}{2b \times I_B}$$

$$= \frac{\mu_0 \pi a^2}{2b}$$

37. In a co-axial straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero:

(1) Inside the outer conductor

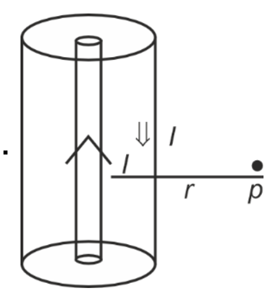
(2) In between the two conductors

(3) Inside the inner conductor

(4) Outside the cable

Answer (4)

Sol.



$B_p = 0$

$\because\ B_p = \frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2\pi r}$

$\therefore$ Magnetic field is zero outside the conductor.

38. Light emerges out of a convex lens when a source of light kept at its focus. The shape of wavefront of the light is:

(1) Both spherical and cylindrical

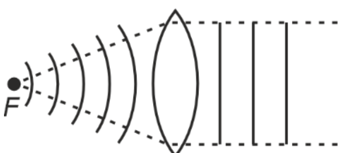
(2) Cylindrical

(3) Spherical

(4) Plane

Answer (4)

Sol.



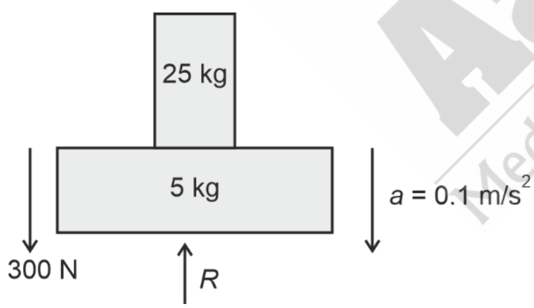
39. A wooden block of mass 5 kg rests on a soft horizontal floor. When an iron cylinder of mass 25 kg is placed on the top of the block, the floor yields and the block and they cylinder together go down with an acceleration of 0.1 ms^{-2}. The action force of the system on the floor is equal to :

(1) 294 N (2) 291 N

(3) 297 N (4) 196 N

Answer (3)

Sol.



$\because\ 300 - R = 30\ (0.1)$

$\Rightarrow\ \boxed{R = 297\ \text{N}}$

40. The angle between vector $\vec{Q}$ and the resultant of $\left(2\vec{Q} + 2\vec{P}\right)$ and $\left(2\vec{Q} - 2\vec{P}\right)$ is :

(1) 0° (2) $\tan^{-1}\left(\frac{P}{Q}\right)$

(3) $\tan^{-1}\left(\frac{2\vec{Q} - 2\vec{P}}{2\vec{Q} + 2\vec{P}}\right)$ (4) $\tan^{-1}\left(\frac{2Q}{P}\right)$

Answer (1)

Sol. $\vec{R} = 4\vec{Q}$

$\therefore$ angle between $\vec{Q}$ and $4\vec{Q}$ will be zero.

41. Match **List-I and List-II** :

	List-I		List-II
(A)	Kinetic energy of planet	(I)	$\frac{-GMm}{a}$
(B)	Gravitation Potential energy of sun-planet system	(II)	$\frac{GMm}{2a}$
(C)	Total mechanical energy of planet	(III)	$\frac{Gm}{r}$
(D)	Escape energy at the surface of planet for unit mass object	(IV)	$\frac{-GMm}{2a}$

(Where a = radius of planet orbit, r = radius of planet, M = mass of sum, m = mass of planet)

Choose the **correct** answer from the options given below:

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(2) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(4) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Answer (1)

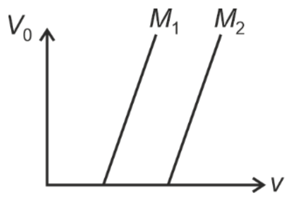
Sol. K.E $= \dfrac{GMm}{2a}$ (II)

$U_G = \dfrac{-GMm}{a}$ (I)

M.E $= \dfrac{-GMm}{2a}$ (IV)

and Escape Energy $= \dfrac{Gm}{r}$ (III)

42. Given below are two statements:



Statement I : Figure shows the variation of stopping potential with frequency (v) for the two photosensitive materials M_1 and M_2. The slope gives value of $\dfrac{h}{e}$, where h is Planck's constant, e is the charge of electron.

Statement II : M_2 will emit photoelectrons of greater kinetic energy for the incident radiation having same frequency.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **Statement I** and **Statement II** are correct

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is correct and **Statement II** is incorrect

(4) **Statement I** is incorrect but **Statement II** is correct

Answer (3)

Sol. $V_0 = \dfrac{h}{e}f - \dfrac{h}{e}f_0$

$\Rightarrow$ Slope $= \dfrac{h}{e}$ (S-I is correct)

$\because$ $(f_0)_1 < (f_0)_2$

$\therefore$ (S-II is incorrect)

$\therefore$ Option (3) is correct.

43. Time periods of oscillation of the same simple pendulum measured using four different measuring clocks were recorded as 4.62 s, 4.632 s, 4.6 s and 4.64 s. The arithmetic mean of these readings in correct significant figure is

(1) 4.6 s (2) 5 s

(3) 4.623 s (4) 4.62 s

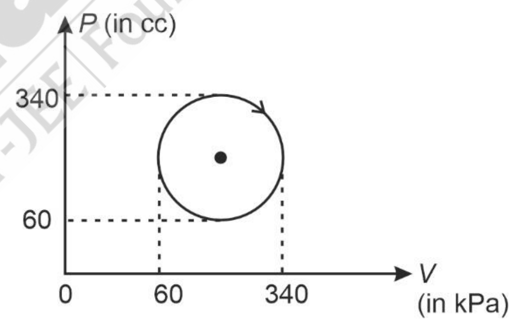
Answer (1)

Sol. $\langle T \rangle = \dfrac{4.62 + 4.632 + 4.6 + 4.64}{4}$

$= \dfrac{18.492}{4}$

$= 4.6$ s

44. The heat absorbed by a system in going through the given cyclic process is



(1) 431.2 J (2) 61.6 J

(3) 19.6 J (4) 616 J

Answer (2)

Sol. $\Delta Q = \Delta U + w$

$= \pi(140)^2 \times 10^3 \times 10^{-6}$

$= 61.6$ J

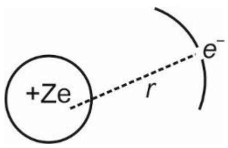
45. An electron rotates in a circle around a nucleus having positive charge Ze. Correct relation between total energy (E) of electron to its potential energy (U) is

(1) $E = 2U$ (2) $E = U$

(3) $2E = U$ (4) $2E = 3U$

Answer (3)

Sol.

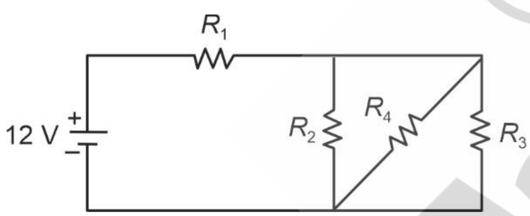


$$E = -\frac{kZe^2}{2r}$$

and $U = -\dfrac{kZe^2}{r}$

$$\Rightarrow \frac{E}{U} = \frac{1}{2}$$

46. In the given figure $R_1 = 10\ \Omega$, $R_2 = 8\ \Omega$, $R_3 = 4\ \Omega$ and $R_4 = 8\ \Omega$. Battery is ideal with emf 12 V. Equivalent resistant of the circuit and current supplied by battery are respectively



(1) 12 Ω and 1 A (2) 10.5 Ω and 1 A

(3) 10.5 Ω and 1.14 A (4) 12 Ω and 11.4 A

Answer (1)

Sol. $R_{eq} = 12\ \Omega$

and, $I = \dfrac{E}{R_{eq}}$

$= \dfrac{12}{12}$

$= 1$ A

47. If the collision frequency of hydrogen molecules in a closed chamber at 27° C is Z, then the collision frequency of the same system at 127° C is

(1) $\dfrac{3}{4}Z$ (2) $\dfrac{2}{\sqrt{3}}Z$

(3) $\dfrac{\sqrt{3}}{2}Z$ (4) $\dfrac{4}{3}Z$

Answer (2)

Sol. $\nu = \sqrt{2}\pi d^2 \dfrac{N}{V}\sqrt{\dfrac{3RT}{m}}$

$\nu \propto \sqrt{T}$

$\therefore \quad \nu = \dfrac{2}{\sqrt{3}}Z$

48. An alternating voltage of amplitude 40 V and frequency 4 kHz is applied directly across the capacitor of 12 μF. The maximum displacement current between the plates of the capacitor is nearly

(1) 12 A (2) 10 A

(3) 8 A (4) 13 A

Answer (1)

Sol. $I_d = C\dfrac{dv}{dt}$

$(I_d)_{max} = CV\omega$

$= (12 \times 10^{-6})(40)(2\pi \times 4 \times 10^3)$

≈ 12 A

49. Given below are two statements :

Statement I : When a capillary tube is dipped into a liquid, the liquid neither rises nor falls in the capillary. The contact angle may be 0°.

Statement II : The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well.

In the light of the above statement, choose the **correct** answer from the options given below.

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is true and **Statement II** is false

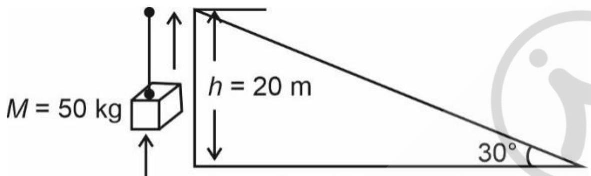
Answer (2)

Sol. $h = \dfrac{2T\cos\theta}{r\rho g}$

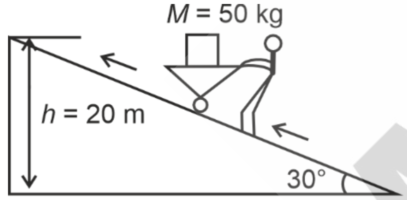
If $h = 0$, then $\theta \neq 0°$

Also contact angle is the property of the materials in contact.

50. A body of mass 50 kg is lifted to a height of 20 m from the ground in the two different ways as shown in the figures. The ratio of work done against the gravity in both the respective case, will be :



Case-1 → Pulled straight up



Case-2 → Along the ramp

(1) 1 : 1

(2) 1 : 2

(3) $\sqrt{3} : 2$

(4) 2 : 1

Answer (1)

Sol. Work done in both cases is equal to $-mg\Delta h$

$\therefore$ Ratio = 1 : 1

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

51. The electric field between the two parallel plates of a capacitor of 1.5 μF capacitance drops to one third of its initial value in 6.6 μs when the plates are connected by a thin wire.

The resistance of this wire is _______ Ω. (Given, log3 = 1.1)

Answer (4)

Sol. If E_0 changes to $\dfrac{E_0}{3}$

$\therefore$ q_0 changes to $\dfrac{q_0}{3}$

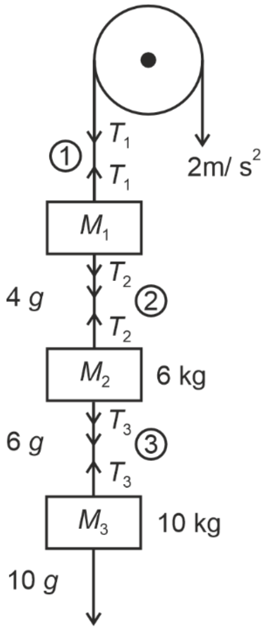
Also $\dfrac{q_0}{3} = q_0\, e^{-\frac{t}{RC}}$

$\Rightarrow \ln\left(\dfrac{1}{3}\right) = \dfrac{-t}{RC}$

$\Rightarrow R = \dfrac{t}{C\ln 3}$

$= 4\ \Omega$

52. Three blocks M_1, M_2, M_3 having masses 4 kg, 6 kg and 10 kg respectively are hanging from a smooth pully using rope 1, 2 and 3 as shown in figure. The tension in the rope 1, T_1 when they are moving upward with acceleration of 2ms^{-2} is _______ N (if g = 10 m/s^2).



Answer (240)

Sol. $T_1 - (4 + 6 + 10)\, g = (4 + 6 + 10)\,(2)$

$\Rightarrow T_1 = 20(10 + 2)$

$= 240$ N

53. If three helium nuclei combine to form a carbon nucleus then the energy released in this reaction is ________ × 10^{-2} MeV. (Given 1 u = 931 MeV/c^2, atomic mass of helium = 4.002603 u)

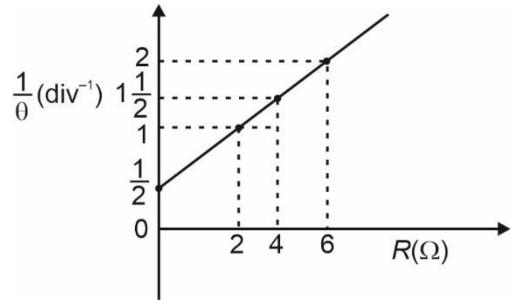
Answer (727)

Sol. $3\,{}^4_2\text{He} \rightarrow {}^{12}_{6}\text{C}$

Q value = 3(4.002603) – 12

= 727 × 10^{-2} MeV

54. In the experiment to determine the galvanometer resistance by half-deflection method, the plot of $\frac{1}{\theta}$ vs the resistance (R) of the resistance box is shown in the figure. The figure of merit of the galvanometer is ________ × 10^{-1} A/division. [The source has emf 2 V]



Answer (5)

Sol. $\frac{1}{3}\text{A} \longrightarrow \frac{1}{2}\text{div}$

$\frac{1}{2}\text{A} \longrightarrow \frac{2}{3}\text{div}$

$$\text{Figure of merit} = \frac{\Delta i}{\Delta\theta} = \frac{\frac{1}{2}-\frac{1}{3}}{\frac{2}{3}-\frac{1}{2}}$$

$= 0.5$

$= 5 \times 10^{-1}$ A/div

55. A body moves on a frictionless plane starting from rest. If S_n is distance moved between $t = n - 1$ and $t = n$ and S_{n-1} is distance moved between $t = n - 2$ and $t = n - 1$, then the ratio $\frac{S_{n-1}}{S_n}$ is $\left(1-\frac{2}{x}\right)$ for $n = 10$. The value of x is __________.

Answer (19)

Sol. $\because$ acceleration is constant

$$\frac{S_{n-1}}{S_n} = \frac{[2(n-1)-1]}{[2n-1]}$$

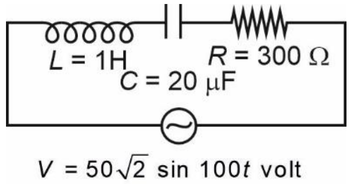
$$= 1 - \frac{2}{2n-1}$$

$\therefore\ x = 2n - 1$

At $n = 10$

$\boxed{x = 19}$

56. An ac source is connected in given series LCR circuit. The rms potential difference across the capacitor of 20 μF is _______ V.



Answer (50)

Sol. $(V_{rms})_c = \frac{V_{rms}}{Z} X_c$

$= \frac{50}{\sqrt{(500-100)^2 + 300^2}} \times 500$

= 50 V

57. The density and breaking stress of a wire are 6×10^4 kg/m^3 and 1.2×10^8 N/m^2 respectively. The wire is suspended from a rigid support on a planet where acceleration due to gravity is $\frac{1^{rd}}{3}$ of the value on the surface of earth. The maximum length of the wire with breaking is ____________m (take, g = 10 m/s^2).

Answer (600)

Sol. Breaking stress $= \frac{T}{A} = \frac{mg}{3}$

$\Rightarrow$ $1.2 \times 10^8 = \frac{\rho A l g}{3A}$

$l = \frac{1.2 \times 10^8 \times 3}{6 \times 10^4 \times 10}$

= 600 m

58. In Young's double slit experiment, carried out with light of wavelength 5000 Å, the distance between the slits is 0.3 mm and the screen is at 200 cm from the slits. The central maximum is at x = 0 cm. The value of x for third maxima is _______ mm.

Answer (10)

Sol. $x = \frac{3\lambda D}{d}$

$= \frac{3 \times 5000 \times 10^{-10} \times 200 \times 10^{-2}}{0.3 \times 10^{-3}}$

= 10 mm

59. Three capacitors of capacitances 25 μF, 30 μF and 45 μF are connected in parallel to a supply of 100 V. Energy stored in the above combination is E. When these capacitors are connected in series to the same supply, the stored energy is $\frac{9}{x}E$. The value of x is ____________.

Answer (86)

Sol. $E = \frac{1}{2}(25+30+45)(100)^2$ …(i)

Also, $\frac{9}{x}E = \frac{1}{2}\frac{1}{\left(\frac{1}{25}+\frac{1}{30}+\frac{1}{45}\right)}(100)^2$ …(ii)

From (i) and (ii)

$\boxed{x = 86}$

60. A 2 A current carrying straight metal wire of resistance 1 Ω resistivity 2×10^{-6} Ωm, area of cross-section 10 mm^2 and mass 500 g is suspended horizontally in mid air by applying a uniform magnetic field $\vec{B}$. The magnitude of B is________ $\times 10^{-1}$ T (given, g = 10 m/s^2).

Answer (5)

Sol. $iLB = mg$ and $L = \frac{AR}{\rho}$

$\therefore$ $B = \frac{mg\rho}{iAR}$

$= \frac{0.5 \times 10 \times 2 \times 10^{-6}}{2 \times 10 \times 10^{-6} \times 1}$

= 0.5 T

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. The statement(s) that are **correct** about the species O^{2-}, F^{-}, Na^{+} and Mg^{2+}.

(A) All are isoelectronic

(B) All have the same nuclear charge

(C) O^{2-} has the largest ionic radii

(D) Mg^{2+} has the smallest ionic radii

Choose the **most appropriate** answer from the options given below.

(1) (A), (B), (C) and (D)

(2) (B), (C) and (D) only

(3) (A), (C) and (D) only

(4) (C) and (D) only

Answer (3)

Sol. O^{2-}, F^{-}, Na^{+} and Mg^{2+} all has $10e^{-}$ and hence, they are isoelectronic.

The number of protons in their nucleus are different, hence different nuclear charge.

Size of $Mg^{2+} < Na^{+} < F^{-} < O^{2-}$

62. Which of the following gives a positive test with ninhydrin?

(1) Starch (2) Cellulose

(3) Egg albumin (4) Polyvinyl chloride

Answer (3)

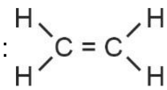
Sol. Ninhydrin test is given by amino acids having free $-NH_2$ group. Egg albumin is a protein which on hydrolysis gives number of α-amino acids.

63. Number of σ and π bonds present in ethylene molecule is respectively

(1) 5 and 2 (2) 3 and 1

(3) 4 and 1 (4) 5 and 1

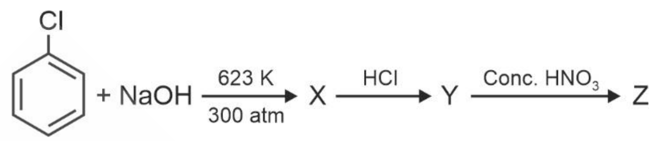
Answer (4)

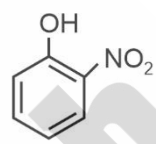
Sol. Ethylene : C_2H_4 : 

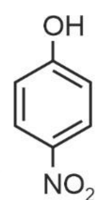
Number of σ bonds = 5

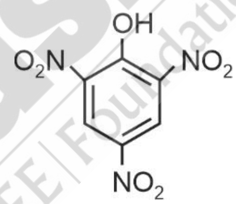
Number of π bonds = 1

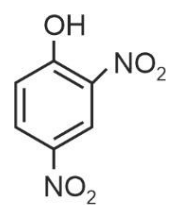
64. Identify compound (Z) in the following reaction sequence.



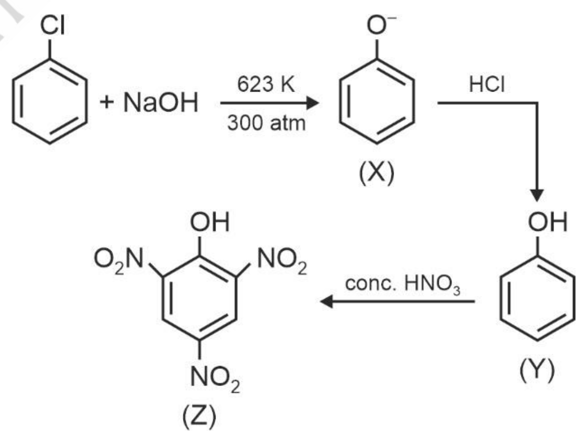
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

65. The metal that shows highest and maximum number of oxidation state is

(1) Ti (2) Co
(3) Mn (4) Fe

Answer (3)

Sol. Mn shows highest oxidation state of +7.

Ti, Co and Fe shows highest oxidation state of +4, +4 and +6 respectively.

66. The **incorrect** postulates of the Dalton's atomic theory are

(A) Atoms of different elements differ in mass.

(B) Matter consists of divisible atoms.

(C) Compounds are formed when atoms of different element combine in a fixed ratio.

(D) All the atoms of given element have different properties including mass.

(E) Chemical reactions involve reorganisation of atoms.

Choose the **correct** answer from the options given below.

(1) (C), (D), (E) only
(2) (B), (D) only
(3) (A), (B), (D) only
(4) (B), (D), (E) only

Answer (2)

Sol. Matter consists of non-divisible atoms.

All the atoms of given element have same properties including mass.

Hence, statements (B) and (D) are incorrect.

67. The following reaction occurs in the Blast furnance where iron ore is reduced to iron metal

$$Fe_2O_{3(s)} + 3CO_{(g)} \rightleftharpoons Fe_{(l)} + 3CO_{2(g)}$$

Using the Le-chatelier's principle, predict which one of the following will not disturb the equilibrium.

(1) Addition of Fe_2O_3 (2) Addition of CO_2
(3) Removal of CO_2 (4) Removal of CO

Answer (1)

Sol. For the reaction :

$$Fe_2O_{3(s)} + 3CO_{(g)} \rightleftharpoons Fe_{(l)} + 3CO_{2(g)}$$

Addition or removal of $Fe_2O_{3(s)}$ and/or $Fe_{(l)}$ will not affect the equilibrium quotient and the equilibrium.

68. Which one of the following complexes will exhibit the least paramagnetic behaviour?

[Atomic number, Cr = 24, Mn = 25, Fe = 26, Co = 27]

(1) $[Mn(H_2O)_6]^{2+}$ (2) $[Co(H_2O)_6]^{2+}$
(3) $[Cr(H_2O)_6]^{2+}$ (4) $[Fe(H_2O)_6]^{2+}$

Answer (2)

Sol. $[Mn(H_2O)_6]^{2+}$: Mn^{2+} : | ↑ | ↑ | ↑ | ↑ | ↑ |

$[Co(H_2O)_6]^{2+}$: Co^{2+} : | ↑↓ | ↑↓ | ↑ | ↑ | ↑ |

$[Cr(H_2O)_6]^{2+}$: Cr^{2+} : | ↑ | ↑ | ↑ | ↑ | |

$[Fe(H_2O)_6]^{2+}$: Fe^{2+} : | ↑↓ | ↑ | ↑ | ↑ | ↑ |

So, the complex with minimum number of unpaired $e^{\ominus}$ is option (2) $[Co(H_2O)_6]^{2+}$.

69. Molar ionic conductivities of divalent cation and anion are 57 S cm^2 mol^{-1} and 73 S cm^2 mol^{-1} respectively. The molar conductivity of solution of an electrolyte with the above cation and anion will be:

(1) 260 S cm^2 mol^{-1} (2) 130 S cm^2 mol^{-1}
(3) 65 S cm^2 mol^{-1} (4) 187 S cm^2 mol^{-1}

Answer (2)

Sol. The compound with divalent cation (A^{2+}) and anion (B^{2-}) will be AB.

Molar conductivity of its solution will be

57 + 73 = 130 S cm^2 mol^{-1}.

70. Given below are two statements:

Statement I : Nitration of benzene involves the following step

$$H-\overset{\overset{H}{|}}{\underset{\bullet\bullet}{O}}{}^{\oplus}-NO_2 \rightleftharpoons H_2O+\overset{\oplus}{N}O_2$$

Statement II : Use of Lewis base promotes the electrophilic substitution of benzene.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is correct but **Statement II** is incorrect

(2) Both **Statement I** and **Statement II** are incorrect

(3) Both **Statement I** and **Statement II** are correct

(4) **Statement I** is incorrect but **Statement II** is correct

Answer (1)

Sol. Due to Lewis base formation electrophile stops/greatly slowed. Hence statement 2 is false while statement 1 is true.

71. The reaction at cathode in the cells commonly used in clocks involves.

(1) reduction of Mn from +4 to +3

(2) oxidation of Mn from +2 to +7

(3) reduction of Mn from +7 to +2

(4) oxidation of Mn from +3 to +4

Answer (1)

Sol. Reduction of Mn^{4+} to Mn^{3+} is observed at cathode in the cells commonly used in clocks.

72. An organic compound has 42.1% carbon, 6.4% hydrogen and remainder is oxygen. If its molecular weight is 342, then its molecular formula is :

(1) $C_{14}H_{20}O_{10}$ (2) $C_{12}H_{20}O_{12}$

(3) $C_{11}H_{18}O_{12}$ (4) $C_{12}H_{22}O_{11}$

Answer (4)

Sol. Let, the molecular formula is $C_xH_yO_z$.

$$\%C=\frac{12x}{342}\times100=42.1 \Rightarrow x=12$$

$$\%H=\frac{y}{342}\times100=6.4 \Rightarrow y=22$$

$$\%O=\frac{16z}{342}\times100=51.5 \Rightarrow z=11$$

So, molecule is $C_{12}H_{22}O_{11}$

73. The correct order of ligands arranged in increasing field strength.

(1) $Br^- < F^- < H_2O < NH_3$

(2) $F^- < Br^- < I^- < NH_3$

(3) $H_2O < {}^-OH < CN^- < NH_3$

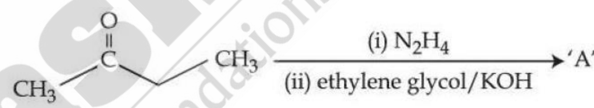
(4) $Cl^- < {}^-OH < Br^- < CN^-$

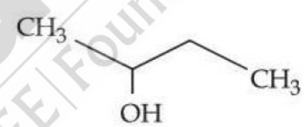
Answer (1)

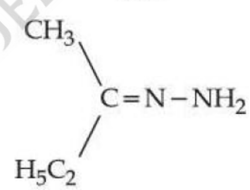
Sol. The correct order of ligands field strength is :

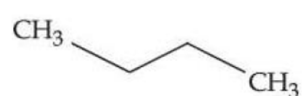
$Br^- < F^- < H_2O < NH_3$

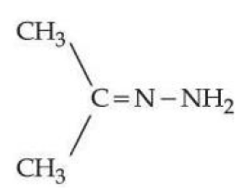
74. Identify 'A' in the following reaction :



(1) 

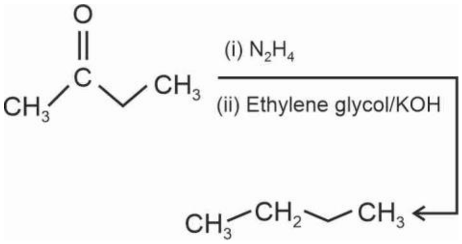
(2) 

(3) 

(4) 

Answer (3)

Sol. This is Wolff-Kishner reduction :



75. For the compounds :

(A) $H_3C - CH_2 - O - CH_2 - CH_2 - CH_3$

(B) $H_3C - CH_2 - CH_2 - CH_2 - CH_3$

(C) $CH_3 - CH_2 - \underset{\underset{O}{\|}}{C} - CH_2 - CH_3$

(D) $H_3C - \underset{\underset{OH}{|}}{CH} - CH_2 - CH_2 - CH_3$

The increasing order of boiling point is :

Choose the **correct** answer from the options given below :

(1) (B) < (A) < (D) < (C) (2) (D) < (C) < (A) < (B)

(3) (B) < (A) < (C) < (D) (4) (A) < (B) < (C) < (D)

Answer (3)

Sol. Boiling point order will be :

(B) < (A) < (C) < (D) due to H bonding in (D) and dipole dipole interactions in (A) and (C).

76. Given below are two statements :

Statement I : Bromination of phenol in solvent with low polarity such as $CHCl_3$ or CS_2 requires Lewis acid catalyst.

Statement II : The Lewis acid catalyst polarises the bromine to generate Br^+.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are false

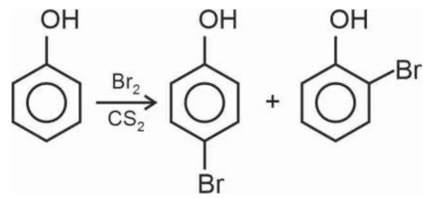
(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Answer (4)

Sol.



Presence of Lewis acid promotes the reaction due to ease in formation of electrophile.

77. Given below are two statements :

Statement I : In group 13, the stability of +1 oxidation state increases down the group.

Statement II : The atomic size of gallium is greater than that of aluminium.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **Statement I** and **Statement II** are incorrect

(2) **Statement I** is correct but **Statement II** is incorrect

(3) Both **Statement I** and **Statement II** are correct

(4) **Statement I** is incorrect but **Statement II** is correct

Answer (2)

Sol. Stability of +1 oxidation state increases down the group due to inert pair effect. Radius of Ga is smaller than Al due to *d*-Block contraction.

78. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Cis form of alkene is found to be more polar than the trans form.

Reason (R) : Dipole moment of trans isomer of 2-butene is zero.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

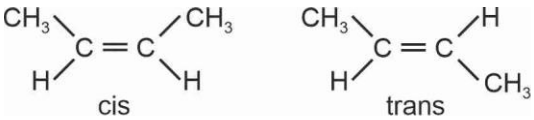
(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

(4) **(A)** is false but **(R)** is true

Answer (1)

Sol.



Dipole moment of cis > trans and $\mu_{trans} = 0$

79. The number of neutrons present in the more abundant isotope of boron is 'x'. Amorphous boron upon heating with air forms a product, in which the oxidation state of boron is 'y'. The value of x + y is _______.

(1) 3 (2) 6

(3) 4 (4) 9

Answer (4)

Sol. The most abundant isotope of Boron is $_5B^{11}$.

No. of neutrons in it = x = 6

$$2B(s) + N_2(g) \xrightarrow{\Delta} 2BN(s)$$

Oxidation state of boron in BN = y = +3

So, x + y = 6 + 3

= 9

80. Given below are two statements : One is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A) : Enthalpy of neutralisation of strong monobasic acid with strong monoacidic base is always –57 kJ mol^{-1}

Reason (R) : Enthalpy of neutralisation is the amount of heat liberated when one mole of H^+ ions furnished by acid combine with one mole of ^-OH ions furnished by base to form one mole of water.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Answer (4)

Sol. Enthalpy of neutralisation of strong acids and bases is –57 kJ/mol. which is fixed for reaction of 1 mole of H^+ with 1 mole of OH^- to form 1 mole of water.

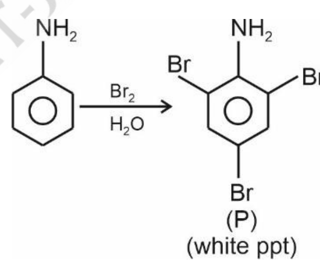
SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

81. 9.3 g of pure aniline is treated with bromine water at room temperature to give a white precipitate of the product 'P'. The mass of product 'P' obtaind is 26.4 g. The percentage yield is _______%.

Answer (80)

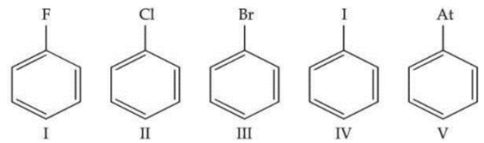
Sol.



moles of aniline taken = $\frac{9.3}{93} = 0.1$

% yield = $\frac{26.4}{0.1 \times 330} \times 100 = 80\%$

82. The number of halobenzenes from the following that can be prepared by Sandmeyer's reaction is _______.



Answer (2)

Sol. Only II and III can be prepared from Sandmeyer's reaction.

83. The heat of combustion of solid benzoic acid at constant volume is –321.30 kJ at 27°C. The heat of combustion at constant pressure is (–321.30 – xR) kJ, the value of x is ______.

Answer (150)

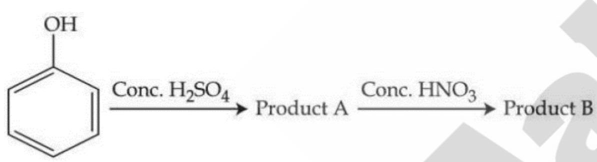
Sol. $C_6H_5COOH(s) + \frac{15}{2}O_2(g) \longrightarrow 7CO_2(g) + 3H_2O(l)$

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta H = -321.30 - \frac{1}{2}R \times 300$

So, x = 150

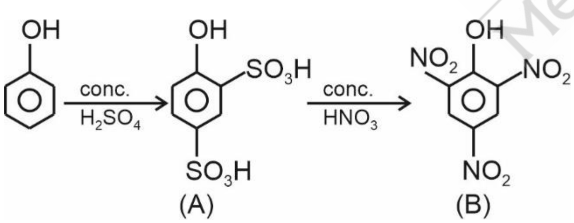
84. Consider the given chemical reaction sequence :



Total sum of oxygen atoms in Product A and Product B are ______.

Answer (14)

Sol.



Total no. of oxygen in A and B = 7 + 7 = 14

85. The spin-only magnetic moment value of the ion among Ti^{2+}, V^{2+}, Co^{3+} and Cr^{2+}, that acts as strong oxidising agent in aqueous solution is ______BM (Near integer).

(Given atomic numbers : Ti : 22, V : 23, Cr : 24, Co : 27)

Answer (5)

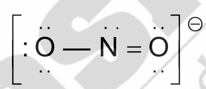
Sol. The ion which acts as strong oxidising agent in aqueous solution is Cr^{2+} : $[Ar]4s^{\circ}3d^4$

$\mu = \sqrt{4(4+2)} = 4.89 \Rightarrow 5$

86. In the lewis dot structure for NO_2^-, total number of valence electrons around nitrogen is _____.

Answer (8)

Sol. Lewis dot structure of NO_2^- is:



Total number of valence e⊖ around N = 8

87. The value of Rydberg constant (R_H) is 2.18 × 10^{-18}J. The velocity of electron having mass 9.1 × 10^{-31} kg in Bohr's first orbit of hydrogen atom = _______ × 10^5 ms^{-1} (nearest integer).

Answer (22)

Sol. K.E. = $R_H.\frac{z^2}{n^2} = \frac{1}{2}mv^2$

$v^2 = \frac{2 \times 2.18 \times 10^{-18}}{9.1 \times 10^{-31}} \times \frac{1}{1} = 0.479 \times 10^{13}$

v = 21.88 × 10^5 m/s

88. During Kinetic study of reaction $2A + B \rightarrow C + D$, the following results were obtained :

	A[M]	B [M]	Initial rate of formation of D
I	0.1	0.1	6.0×10^{-3}
II	0.3	0.2	7.2×10^{-2}
III	0.3	0.4	2.88×10^{-1}
IV	0.4	0.1	2.40×10^{-2}

Based on above data, overall order of the reaction is_____.

Answer (3)

Sol. Rate = $k[A]^x [B]^y$

$$\frac{6 \times 10^{-3}}{2.4 \times 10^{-2}} = \left(\frac{0.1}{0.4}\right)^x \Rightarrow x = 1$$

$$\frac{7.2 \times 10^{-2}}{2.88 \times 10^{-1}} = \left(\frac{0.2}{0.4}\right)^y \Rightarrow y = 2$$

Hence order of reaction is 1 + 2 = 3

89. An artificial cell is made by encapsulating 0.2 M glucose solution within a semipermeable membrane. The osmotic pressure developed when the artificial cell is placed with a 0.05 M solution of NaCl at 300 K is _______$\times 10^{-1}$ bar. (nearest integer).

[Given : R = 0.083 L bar $mol^{-1}K^{-1}$]

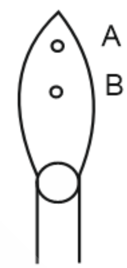
Assume complete dissociation of NaCl

Answer (25)

Sol. $\pi = (i_1C_1 - i_2C_2)\,RT = (1 \times 0.2 - 2 \times 0.05)\,0.083 \times 300$

$= 2.5$ bar $= 25 \times 10^{-1}$ bar

90. In a borax bead test under hot condition, a metal salt (one from the given) is heated at point B of the flame resulted in green colour salt bead. The spin-only magnetic moment value of the salt is ____BM (Nearest integer)



[Given atomic number of Cu = 29, Ni = 28, Mn = 25, Fe = 26]

Answer (6)

Sol. Green coloured salt bead represents the metal ion taken is Fe^{3+} so, Fe^{3+} : [Ar] $4s^0 3d^5$

So, $\mu = \sqrt{5 \times 7} = 5.9 = 6$

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