06/04/2024
Evening

Aakash
Medical | IIT-JEE | Foundations

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6) **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to

(1) 150 (2) 160

(3) 180 (4) 125

Answer (1)

Sol. Let the work done by each computer = k.

$\therefore$ Total work = $17mk$

Work done on day 1 = mk

Work done on day 2 = $(m-4)k$

Work done on day 3 = $(m-8)k$

$\vdots$

Work done on day 25 = $(m-96)k$

$\therefore \quad k(m+(m-4)+(m-8)+\cdots+(m-96)) = 17mk$

$25m-(4+8+\cdots+96) = 17m$

$$\frac{24}{2}[4+96] = 8m$$

$\Rightarrow \quad m = 150$

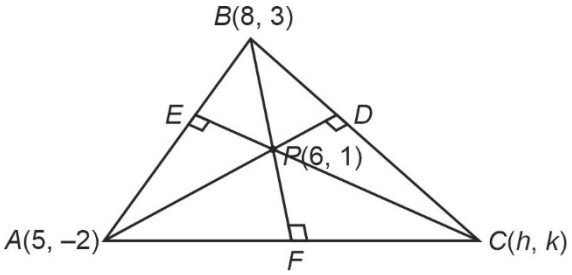
2. If $P(6, 1)$ be the orthocentre of the triangle whose vertices are $A(5, -2)$, $B(8, 3)$ and $C(h, k)$, then the point C lies on the circle

(1) $x^2 + y^2 - 74 = 0$ (2) $x^2 + y^2 - 65 = 0$

(3) $x^2 + y^2 - 61 = 0$ (4) $x^2 + y^2 - 52 = 0$

Answer (2)

Sol.



Slope of $BF = (m_{BF}) = \dfrac{3-1}{8-6} = 1$

$$m_{AC} = \frac{k+2}{h-5} = -1 \ (AC \perp BF)$$

$\Rightarrow \quad k + 2 = 5 - h$

$\Rightarrow \quad k = 3 - h$ ...(i)

$$m_{AD} = \frac{1+2}{6-5} = 3$$

$$m_{BC} = \frac{k-3}{h-8} = -\frac{1}{3} \ (AD \perp BC)$$

$\Rightarrow \quad 3k - 9 = 8 - h$ ...(ii)

From (i) & (ii)

$h = -4$, $k = 7$

Now, $h^2 + k^2 = 16 + 49 = 65$

$h^2 + k^2 - 65 = 0$

Locus is $x^2 + y^2 - 65 = 0$

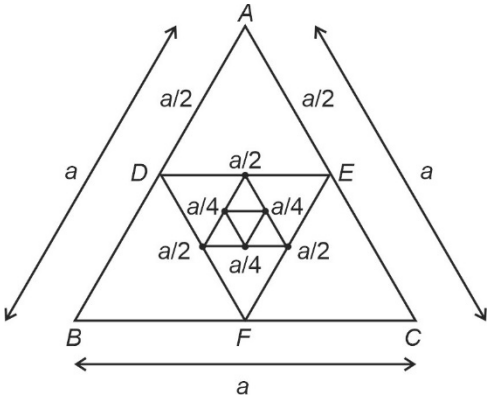
3. Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is be the sum of areas of all the triangles formed in this process, then

(1) $P^2 = 72\sqrt{3}Q$ (2) $P^2 = 36\sqrt{3}Q$

(3) $P = 36\sqrt{3}Q^2$ (4) $P^2 = 6\sqrt{3}Q$

Answer (2)

Sol.



ΔABC is an equilateral triangle having side = a unit

Now, perimeter = sum of all sides = $3a$

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

Now, In ΔDEF, $DE = \frac{a}{2} = EF = DF$

$$\text{Perimeter} = 3 \times \frac{a}{2} = \frac{3a}{2}$$

$$\text{Area} = \frac{\sqrt{3}}{4} \times \left(\frac{a}{2}\right)^2 = \frac{\sqrt{3}a^2}{16}$$

Now, $P = 3a + \frac{3a}{2} + \frac{3a}{4} + \cdots$

$$Q = \frac{\sqrt{3}}{4}a^2 + \frac{\sqrt{3}}{16}a^2 + \frac{\sqrt{3}}{64}a^2 + \cdots$$

$$P = \frac{3a}{1-\frac{1}{2}} = 3a \times 2 \Rightarrow \boxed{P = 6a} \qquad \ldots(i)$$

$$Q = \frac{\frac{\sqrt{3}}{4}a^2}{1-\frac{1}{4}} = \frac{4}{3} \times \frac{\sqrt{3}}{4}a^2 \quad \boxed{Q = \frac{\sqrt{3}}{3}a^2} \qquad \ldots(ii)$$

From equation (i) & (ii)

$\boxed{P = 6a}$

$$Q = \frac{\sqrt{3}}{3}a^2$$

$$Q = \frac{\sqrt{3}}{3} \times \frac{P^2}{36}$$

$\boxed{P^2 = 36\sqrt{3}Q}$

4. Let $f(x) = \frac{1}{7 - \sin 5x}$ be a function defined on **R**. Then the range of the function $f(x)$ is equal to

(1) $\left[\frac{1}{8}, \frac{1}{6}\right]$ (2) $\left[\frac{1}{8}, \frac{1}{5}\right]$

(3) $\left[\frac{1}{7}, \frac{1}{6}\right]$ (4) $\left[\frac{1}{7}, \frac{1}{5}\right]$

Answer (1)

Sol. $f(x) = \frac{1}{7 - \sin 5x}$

$-1 \le \sin 5x \le 1$

$-1 \le -\sin 5x \le 1$

$-1 + 7 \le 7 - \sin 5x \le 1 + 7$

$6 \le 7 - \sin 5x \le 8$

$\frac{1}{8} \le \frac{1}{7 - \sin 5x} \le \frac{1}{6}$

$\frac{1}{8} \le f(x) \le \frac{1}{6}$

Range $= \left[\frac{1}{8}, \frac{1}{6}\right]$

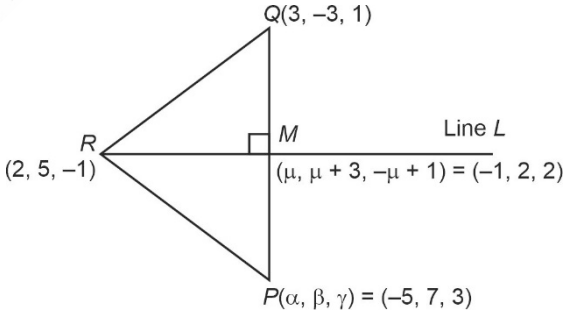
5. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(3, -3, 1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14K$, then K is equal to

(1) 81 (2) 36

(3) 72 (4) 18

Answer (1)

Sol.



Line $L \Rightarrow \frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1} = \mu$

Point $M(\mu, \mu+3, -\mu+1)$

Direction vector of $QM = (\mu-3, \mu+6, -\mu)$

Direction vector of line $L = (1, 1, -1)$

Both direction vector are perpendicular so

$1 \times (\mu-3) + 1 \times (\mu+6) - 1 \times (-\mu) = 0$

$\mu = -1$

Point $M(-1, 2, 2)$ midpoint of PQ

So point $P(\alpha, \beta, \gamma) = (-5, 7, 3)$

Area of $\Delta PQR = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{QR}\right|$

$\overrightarrow{PQ} = (8, -10, -2)$

$\overrightarrow{QR} = (-1, 8, -2)$

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -10 & -2 \\ -1 & 8 & -2 \end{vmatrix} = 36\hat{i} + 18\hat{j} + 54\hat{k}$$

Area of $\Delta PQR = \lambda$

$$\lambda = \frac{1}{2}\sqrt{(36)^2 + (18)^2 + (54)^2}$$

$$\lambda^2 = \frac{1}{4}(4536)$$

$\lambda^2 = 1134$

$\lambda^2 = 14K$ (given)

$14K = 1134$

$$K = \frac{1134}{14} = 81$$

$$\boxed{K = 81}$$

6. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is

(1) $\frac{4}{25}$ (2) $\frac{18}{25}$

(3) $\frac{12}{25}$ (4) $\frac{6}{25}$

Answer (3)

Sol. We have 3 letters and 5 addresses, where 3 letters are posted to exactly 2 addresses. First, we will select 2 addresses in ${}^5C_2$ ways.

Now, 3 letters can be posted to exactly 2 addresses in 6 ways.

$\therefore$ Probability $= \frac{{}^5C_2 \times 6}{5^3}$

$$= \frac{60}{125} = \frac{12}{25}$$

7. If $\int \frac{1}{a^2\sin^2 x + b^2\cos^2 x}dx = \frac{1}{12}\tan^{-1}(3\tan x) +$ constant, then then maximum value of $a\sin x + b\cos x$, is

(1) $\sqrt{42}$ (2) $\sqrt{40}$

(3) $\sqrt{41}$ (4) $\sqrt{39}$

Answer (2)

Sol. $\int \frac{1}{a^2\sin^2 x + b^2\cos^2 x}dx = \frac{1}{12}\tan^{-1}(3\tan x) + c$

$$I = \int \frac{\sec^2 x}{b^2 + a^2\tan^2 x}dx$$

$\tan x = t$

$\Rightarrow \sec^2 \times dx = dt$

$$I = \int \frac{dt}{b^2 + a^2t^2}$$

$$= \frac{1}{ba}\tan^{-1}\left(\frac{at}{b}\right) + c$$

$$I = \frac{1}{ab}\tan^{-1}\left(\frac{a}{b}\tan x\right) + c$$

$\Rightarrow ab = 12$ and $\frac{a}{b} = 3$

$\Rightarrow a^2 = 36$ and $b^2 = 4$

$\Rightarrow$ Maximum value of $a\sin x + b\cos x$ is $\sqrt{a^2 + b^2}$

$$= \sqrt{36 + 4}$$

$$= \sqrt{40}$$

8. If z_1, z_2 are two distinct complex number such that $\left|\frac{z_1-2z_2}{\frac{1}{2}-z_1\bar{z}_2}\right|=2$, then

(1) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1

(2) Either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$

(3) Either z_1 lies on circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1

(4) Both z_1 and z_2 lie on the same circle

Answer (2)

Sol. $\left|\frac{z_1-2z_2}{\frac{1}{2}-z_1\bar{z}_2}\right|=2$

$|z_1-2z_2|=|1-2z_1\bar{z}_2|$

$\Rightarrow (z_1-2z_2)(\bar{z}_1-2\bar{z}_2)=(1-2z_1\bar{z}_2)(1-2\bar{z}_1z_2)$

$\Rightarrow |z_1|^2+4|z_2|^2-2\bar{z}_1z_2-2\bar{z}_2z_1$

$=1+4|z_1|^2|z_2|^2-2z_1\bar{z}_2-2\bar{z}_1z_2$

$\Rightarrow |z_1|^2+4|z_2|^2-4|z_1|^2|z_2|^2-1=0$

$\Rightarrow \left(|z_1|^2-1\right)\left(1-4|z_2|^2\right)=0$

$\Rightarrow |z_1|=1$ and $|z_2|=\frac{1}{2}$

9. Let $\vec{a}=2\hat{i}+\hat{j}-\hat{k}, \vec{b}=\left(\left(\vec{a}\times\left(\hat{i}+\hat{j}\right)\right)\times\hat{i}\right)\times\hat{i}$. Then the square of the projection of $\vec{a}$ on $\vec{b}$ is

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$

(3) $\frac{1}{5}$ (4) 2

Answer (4)

Sol. $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$

$\vec{b}=\left((\vec{a}\times(\hat{i}+\hat{j}))\times\hat{i}\right)\times\hat{i}$

$\vec{a}\times(\hat{i}+\hat{j})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&1&-1\\1&1&0\end{vmatrix}$

$=\hat{i}(1)-\hat{j}(1)+\hat{k}(2-1)$

$=\hat{i}-\hat{j}+\hat{k}$

$\left(\vec{a}\times(\hat{i}+\hat{j})\right)\times\hat{i}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&1\\1&0&0\end{vmatrix}$

$=\hat{i}(0)-\hat{j}(-1)+\hat{k}(1)$

$=\hat{j}+\hat{k}$

$\left((\vec{a}\times(\hat{i}+\hat{j})\times\hat{i}\right)\times\hat{i}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&1\\1&0&0\end{vmatrix}$

$=\hat{i}(0)-\hat{j}(-1)+\hat{k}(-1)$

$\vec{b}=\hat{j}-\hat{k}$

Projection of $\vec{a}$ on $\vec{b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$=\frac{2}{\sqrt{2}}=\sqrt{2}$

Square of projection = 2

10. If A is a square matrix of order 3 such that det(A) = 3 and det (adj(–4 adj(– 3 adj(3 adj$(2A)^{-1}$))))) = $2^m\,3^n$, then $m + 2n$ is equal to

(1) 2 (2) 6

(3) 3 (4) 4

Answer (4)

Sol. $|A| = 3$

$\left|adj\left(-4adj\left(-3adj\left(3adj(2A)^{-1}\right)\right)\right)\right|$

$=\left|-4adj\left(-3adj\left(3adj\left((2A)^{-1}\right)\right)\right)\right|^2$

$= 4^6 \left|-3adj\left(3adj\left((2A)^{-1}\right)\right)\right|^4$

$= 4^6 \cdot 3^{12} \left|3adj\left((2A)^{-1}\right)\right|^8$

$= 4^6 \cdot 3^{12} \cdot 3^{24} \left|(2A)^{-1}\right|^{16}$

$= 4^6 \cdot 3^{36} 2^{-48} \left|A^{-1}\right|^{16}$

$= \frac{2^{-36}3^{36}}{3^{16}} = 2^{-36}3^{20}$

$m = -36$

$n = 20$

$m + 2n = 4$

11. Suppose the solution of the differential equation $\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta\gamma - 4\alpha)}$ represents a circle passing through origin. Then the radius of this circle is

(1) $\sqrt{17}$ (2) 2

(3) $\frac{1}{2}$ (4) $\frac{\sqrt{17}}{2}$

Answer (4)

Sol. $\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta\gamma - 4\alpha)}$

$\beta x dy - 2\alpha y dy - (\beta\gamma - 4\alpha)dy$

$= 2xdx + \alpha x dx - \beta y dx + 2dx$

$\beta\int(xdy + ydy) - \alpha y^2 - (\beta\gamma - 4x)y = x^2 + \frac{\alpha x^2}{2} + 2x$

$\beta xy - \alpha y^2 - (\beta\gamma - 4\alpha)y = x^2 + \frac{\alpha x^2}{2} + 2x$

$\left(1 + \frac{\alpha}{2}\right)x^2 + \alpha y^2 - \beta xy + 2x + (\beta\gamma - 4\alpha)y = 0$

$\because$ this represents circle passing through origin

$\Rightarrow \beta = 0$ and $1 + \frac{\alpha}{2} = \alpha$

$\Rightarrow \boxed{\alpha = 2}$

$\therefore C : 2x^2 + 2y^2 + 2x - 8y = 0$

$x^2 + y^2 + x - 4y = 0$

Radius = $\sqrt{\frac{1}{4} + 4 - 0}$

$= \frac{\sqrt{17}}{2}$

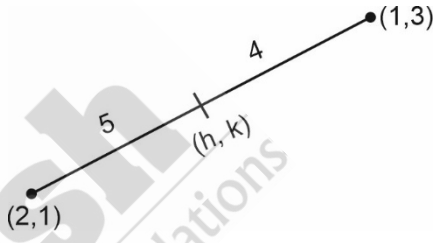
12. If the locus of the point, whose distances from the point (2, 1) and (1, 3) are in the ratio 5 : 4, is $ax^2 + by^2 + cxy + dx + ey + 170 = 0$, then the value of $a^2 + 2b + 3c + 4d + e$ is equal to

(1) –27 (2) 437

(3) 5 (4) 37

Answer (4)

Sol.



$\therefore \frac{(h-2)^2 + (k-1)^2}{(h-1)^2 + (k-3)^2} = \frac{25}{16}$

$\frac{h^2 + k^2 - 4h - 2k + 5}{h^2 + k^2 - 2h - 6k + 10} = \frac{25}{16}$

Replacing $h \to x$ and $k \to y$.

$16x^2 + 16y^2 - 64x - 32y + 80$

$= 25x^2 + 25y^2 - 50x - 150y + 250$

$9x^2 + 9y^2 + 14x - 118y + 170 = 0$

$\Rightarrow a = 9, b = 9, c = 0, d = 14, e = -118$

$a^2 + 2b + 3c + 4d + e$

81 + 18 + 56 – 118

$\Rightarrow$ 37

13. Let $0 \le r \le n$. If ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 55 : 35 : 21$, then $2n + 5r$ is equal to

(1) 50 (2) 60

(3) 55 (4) 62

Answer (1)

Sol. $\frac{n+1}{r+1} \times {}^{n}C_{r} : {}^{n}C_{r} : \frac{r}{n} {}^{n}C_{r} = 55 : 35 : 21$

$\Rightarrow \frac{n+1}{r+1} = \frac{55}{35}$ and $\frac{n}{r} = \frac{35}{21}$

$\Rightarrow 7n - 11r = 4$ ...(1)

$3n - 5r = 0$ ...(2)

Solving (1) and (2)

$r = 6$ and $n = 10$

$\Rightarrow 2n + 5r = 20 + 30 = 50$

14. $$\lim_{n\to\infty} \frac{(1^2-1)(n-1)+(2^2-2)(n-2)+\ldots+\left((n-1)^2-(n-1)\right)\cdot 1}{\left(1^3+2^3+\ldots+n^3\right)-\left(1^2+2^2+\ldots+n^2\right)}$$

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$

(3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Answer (2)

Sol. $$\lim_{n\to\infty} \frac{(1^2-1)(n-1)+(2^2-2)(n-2)+\ldots+\left((n-1)^2-(n-1)\right)\times 1}{\left(1^3+2^3+\ldots+n^3\right)-\left(1^2+2^2+\ldots+n^2\right)}$$

Numerator = $\sum_{r=1}^{n-1}\left((r-1)^2-(r-1)\right)(n-r)$

$= \sum_{r=1}^{n-1}(r-1)-(r-2)(n-r)$

$= \sum_{r=1}^{n-1} -r^3+(n+3)r^2-(2+3n)r+2n$

We will take term with the greatest power of n

$= \frac{-1}{4}n^4 + \frac{1}{3}n^4 = \frac{1}{12}n^4$

Denominator = $\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2$

$= \left(\frac{n(n+1)}{2}\right)^2 - \left(\frac{n(n+1)(2n+1)}{6}\right)$

Greatest power of n is $\frac{n^4}{4}$

$$\lim_{n\to\infty} \frac{\frac{1}{12}n^4}{\frac{n^4}{4}} = \frac{1}{3}$$

15. Let $\vec{a} = 6\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $|\vec{c}| \le 6, \vec{a}\cdot\vec{c} = 6|\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 60°, then $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|$ is equal to

(1) $\frac{3}{2}\sqrt{3}$ (2) $\frac{9}{2}\left(6-\sqrt{6}\right)$

(3) $\frac{9}{2}\left(6+\sqrt{6}\right)$ (4) $\frac{3}{2}\sqrt{6}$

Answer (3)

Sol. $\left|(\vec{a}\times\vec{b})\times\vec{c}\right| = |\vec{a}\times\vec{b}||\vec{c}|\sin 60°$

$$\begin{vmatrix} i & j & k \\ 6 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = i(1) - j(1) + k(5)$$

$= i - j + 5k$

$|\vec{a}\times\vec{b}| = \sqrt{1+1+25} = \sqrt{27}$

$|\vec{c} - \vec{a}| = 2\sqrt{2}$

$c^2 + a^2 - 2ac = 8$

$c^2 - 12c + 30 = 0$

$c = \frac{12 + \sqrt{24}}{2} = 6 + \sqrt{6}$

$\Rightarrow \left|(\vec{a}\times\vec{b})\times\vec{c}\right| = \sqrt{27}\times\left(6+\sqrt{6}\right)\times\frac{\sqrt{3}}{2}$

$= \frac{a}{2}\left(6+\sqrt{6}\right)$

16. If the function $f(x)=\left(\frac{1}{x}\right)^{2x}; x>0$ attains the maximum value at $x=\frac{1}{e}$ then

(1) $(2e)^{\pi} > \pi^{(2e)}$ (2) $e^{2\pi} < (2\pi)^{e}$

(3) $e^{\pi} > \pi^{e}$ (4) $e^{\pi} < \pi^{e}$

Answer (3)

Sol. $f\left(\frac{1}{\pi}\right) < f\left(\frac{1}{e}\right)$ as $\frac{1}{\pi} < \frac{1}{e}$

$\Rightarrow \left(\frac{1}{\frac{1}{\pi}}\right)^{\frac{2}{\pi}} < \left(\frac{1}{\frac{1}{e}}\right)^{\frac{2}{e}}$

$\Rightarrow (\pi)^{\frac{2}{\pi}} < (e)^{\frac{2}{e}}$

$\Rightarrow \pi^{e} < e^{\pi}$

17. Suppose for a differentiable function h, $h(0) = 0$, $h(1) = 1$ and $h'(0) = h'(1) = 2$. If $g(x) = h(e^x)e^{h(x)}$, then $g'(0)$ is equal to

(1) 4 (2) 3

(3) 8 (4) 5

Answer (1)

Sol. $g(x) = h(e^x)\cdot e^{h(x)}$

$g'(x) = h'(e^x)\cdot e^x \cdot e^{h(x)} + h(e^x)\cdot e^{h(x)}\cdot h'(x)$

$g'(0) = h'(1)\cdot e^{h(0)} + h(1)\cdot e^{h(0)}\cdot h'(0)$

$= 2\cdot e^0 + 1\cdot e^0 \cdot 2$

$= 2 + 2 = 4$

18. Let $A = \{1, 2, 3, 4, 5\}$. Let R be a relation of A defined by xRy if and only if $4x \le 5y$. Let m be the number of elements in R and n be the minimum number of elements from $A \times A$ that are required to be added to R to make it a symmetric relation. Then $m + n$ is equal to

(1) 26 (2) 24

(3) 23 (4) 25

Answer (4)

Sol. $A = \{1, 2, 3, 4, 5\}$

$xRy \Leftrightarrow 4x \le 5y$

$4x \le 5y \Rightarrow \frac{x}{y} \le \frac{5}{4} \Rightarrow \frac{x}{y} \le 1.25$

R = {(1,2), (1,3), (1,4), (1,5), (1,1), (2,2), (2,3), (2,4), (2,5), (3,3), (3,4), (3,5), (4,4), (4,5), (5,4), (5,5)}

$\therefore$ $n(R) = m = 16$

Elements to be added to R to make it symmetric

$(1, 2) \in R \Rightarrow$ (2, 1) should be added

Similarly, (3,1), (4,1), (5,1), (3,2), (4,2), (5,2), (4,3), (5,3)

$\therefore$ 9 elements should be added

$\therefore$ $n = 9$

$\therefore$ $m + n = 25$

19. If the area of the region

$\left\{(x,y): \frac{a}{x^2} \le y \le \frac{1}{x}, 1 \le x \le 2, 0 < a < 1\right\}$ is

$(\log_e 2) - \frac{1}{7}$ then the value of $7a - 3$ is equal to

(1) 2 (2) 0

(3) 1 (4) –1

Answer (4)

Sol. $\left\{(x, y): \frac{a}{x^2} \le y \le \frac{1}{x}, 1 \le x \le 2, 0 < a < 1\right\}$

$\Rightarrow \int_1^2 \left(\frac{1}{x} - \frac{a}{x^2}\right)dx = \left|\ln|x| + \frac{a}{x}\right|_1^2$

$\left(\ln 2 + \frac{a}{2}\right) - (\ln 1 + a) = \ln 2 - \frac{a}{2}$

$\Rightarrow \frac{a}{2} = \frac{1}{7}$

$\Rightarrow a = \frac{2}{7}$

$7a - 3 = \frac{2}{7}\times 7 - 3 = -1$

20. If all the words with or without meaning made using all the word "NAGPUR" are arranged as in a dictionary, then the word at 315th position in this arrangement is

(1) NRAPUG (2) NRAGUP

(3) NRAGPU (4) NRAPGU

Answer (4)

Sol. NAGPUR

Word at 315th position

A…….. = 5! = 120

G…….. = 5! = 120

NA…… = 4! = 24

NG…… = 4! = 24

NP…… = 4! = 24

….. Till 312 words

313th word = NRAGPU

314th word = NRAGUP

315th word = NRAPGU

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If the shortest distance between the lines $\frac{x-\lambda}{3}=\frac{y-2}{-1}=\frac{z-1}{1}$ and $\frac{x+2}{-3}=\frac{y+5}{2}=\frac{z+4}{4}$ is $\frac{44}{\sqrt{30}}$, then the largest possible value of $|\lambda|$ is

Answer (43)

Sol. $L_1 : \frac{x-\lambda}{3}=\frac{y-2}{-1}=\frac{z-1}{1}$

$L_2 : \frac{x+2}{-3}=\frac{y+5}{2}=\frac{z-4}{4}$

$$n_1 \times n_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$= -6\hat{i} - 15\hat{j} + 3\hat{k}$

$$d = \left| \frac{[(\lambda+2)\hat{i} + 7\hat{j} - 3\hat{k}][-6\hat{i} - 15\hat{j} + 3\hat{k}]}{|-6\hat{i} - 15\hat{j} + 3\hat{k}|} \right| = \frac{44}{\sqrt{30}}$$

$$\left| \frac{-6\lambda - 12 - 105 - 9}{\sqrt{270}} \right| = \frac{44}{\sqrt{30}}$$

$|6\lambda + 126| = 132$

$|\lambda + 21| = 22$

$\lambda + 21 = \pm 22$

$|\lambda|_{max} = 43$

22. If the system of equations

$2x + 7y + \lambda z = 3$

$3x + 2y + 5z = 4$

$x + \mu y + 32z = -1$

has infinitely many solutions, then $(\lambda - \mu)$ is equal to _____.

Answer (38)

Sol. By Cramer's rule

$\Delta = -2\lambda + 3\lambda\mu - 10\mu - 509$

$\Delta_1 = 2\lambda + 3\lambda\mu - 15\mu - 739$

$\Delta_2 = -7\lambda - 7$

$\Delta_3 = \mu + 39$

For infinitely many solutions

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$\Rightarrow \lambda = -1, \mu = -39$

$\Rightarrow \lambda - \mu = 38$

23. If the solution $y(x)$ of the given differential equation $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$ passes through the point $\left(\frac{\pi}{2}, 0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to _____.

Answer (3)

Sol. $(e^y + 1)\cos x dx + e^y \sin x dy = 0$

$d(e^y \sin x) + \cos x dx = 0$

$\Rightarrow$ $e^y \sin x + \sin x = c$ passes through $\left(\frac{\pi}{2}, 0\right)$

$1 + 1 = C \Rightarrow C = 2$

$\Rightarrow$ $e^y \sin x + \sin x = 2$

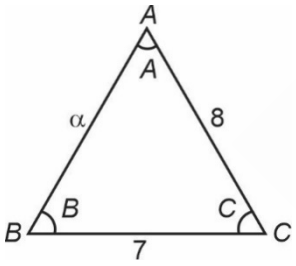
$\Rightarrow$ $\left(e^{y\left(\frac{\pi}{6}\right)} + 1\right)\cdot\frac{1}{2} = 2$

$e^{y\left(\frac{\pi}{6}\right)} = 3$

24. In a triangle ABC, $BC = 7$, $AC = 8$, $AB = \alpha \in N$ and $\cos A = \frac{2}{3}$. If $49\cos(3C) + 42 = \frac{m}{n}$, where $gcd(m, n) = 1$, then $m + n$ is equal to _____.

Answer (39)

Sol.



$\cos A = \frac{2}{3} = \frac{\alpha^2 + 8^2 - 7^2}{2\cdot\alpha\cdot 8}$

$\Rightarrow$ $(\alpha^2 + 15)3 = 32\alpha$

$3\alpha^2 - 32\alpha + 45 = 0$

$\Rightarrow$ $\alpha = \frac{5}{3}, 9$

$\because$ $\alpha \in N \Rightarrow \alpha = 9$

$\cos C = \frac{7^2 + 8^2 - 9^2}{2\cdot 7\cdot 8} = \frac{2}{7}$

$\cos 3C = 4\cos^3 C - 3\cos C$

$= \frac{4\times 8}{7^3} - \frac{6}{7}$

$49\cos 3C = \frac{32}{7} - 42$

$\Rightarrow$ $49\cos 3C + 42 = \frac{32}{7}$

$\Rightarrow$ $m + n = 39$

25. Let $[t]$ denote the greatest integer less than or equal to t. Let $f : [0, \infty) \to \mathbf{R}$ be function defined by $f(x) = \left[\frac{x}{3} + 3\right] - \left[\sqrt{x}\right]$. Let S be the set of all points in the interval $[0, 8]$ at which f is not continuous. Then $\sum_{a\in S} a$ is equal to _____.

Answer (17)

Sol. $f(x) = \left[\frac{x}{3} + 3\right] - \left[\sqrt{x}\right]$

$= \left[\frac{x}{2}\right] - \left[\sqrt{x}\right] + 3$

Critical points where $f(x)$ might change behaviours when $\frac{x}{2} \in$ integer and $\sqrt{x} \in$ integer

$\Rightarrow$ Critical points,

$f(0) = 3$

$f(0^+) = 3$

$f(1^-) = 3$

$f(1^+) = 2$

$f(2^-) = 2$

$f(2^+) = 3$

$f(3^+) = f(3^-) = 3 = f(4^+) = f(4^-) = f(5^+) = f(5^-)$

$f(6^-) = 3$

$f(6^+) = 4$

$f(7^-) = f(7^+) = 4$

$f(8^-) = 4$

$f(8) = 5$

$\Rightarrow$ $f(x)$ is not continuous at $x = 1, 2, 6, 8$

$\Rightarrow$ $\sum_{a\in s} a = 1 + 2 + 6 + 8$

$= 17$

26. Let α, β be roots of $x^2+\sqrt{2}x-8=0$. If $U_n=\alpha^n+\beta^n$, then $\frac{U_{10}+\sqrt{2}U_9}{2U_8}$ is equal to _____.

Answer (4)

Sol. $x^2+\sqrt{2}x-8=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$ $\Rightarrow \alpha^2+\sqrt{2}\alpha=8$

$\alpha+\beta=\sqrt{2}, \quad \alpha\beta=-8, \quad \Rightarrow \alpha+\sqrt{2}=\frac{8}{\alpha}$

$\Rightarrow \beta+\sqrt{2}=\frac{8}{\beta}$

$$\frac{U_{10}+\sqrt{2}U_9}{2U_8}=\frac{\alpha^{10}+\beta^{10}+\sqrt{2}\alpha^9+\sqrt{2}\beta^9}{2\alpha^8+2\beta^8}$$

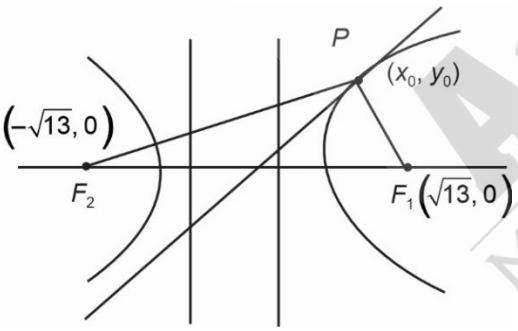
$$=\frac{\alpha^9(\alpha+\sqrt{2})+\beta^9(\beta+\sqrt{2})}{2(\alpha^8+\beta^8)}$$

$$=\frac{\alpha^9\cdot\left(\frac{8}{\alpha}\right)+\beta^9\left(\frac{8}{\beta}\right)}{2(\alpha^8+\beta^8)}=\frac{8}{2}=4$$

27. The length of the latus rectum and directrices of a hyperbola with eccentricity e and 9 and $x=\pm\frac{4}{\sqrt{3}}$, respectively. Let the line $y-\sqrt{3}x+\sqrt{3}=0$ touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of the point (x_0, y_0), then $4e^2+m$ is equal to _____.

Answer (61)

Sol.



$y=mx\pm\sqrt{a^2m^2-b^2}$ is the tangent

$\Rightarrow m=\sqrt{3} \quad \Rightarrow a^2m^2-b^2=3$

$\Rightarrow 3a^2-b^2=3$

$\frac{2b^2}{a}=9 \quad \Rightarrow b^2=\frac{9a}{2} \quad \Rightarrow 3a^2-\frac{9a}{2}=3$

$\Rightarrow a^2-\frac{3}{2}a-1=0$

$\Rightarrow$ $a = 2$ or -0.5 (ignore)

$\Rightarrow$ $b = 3$

$\Rightarrow \frac{x^2}{4}-\frac{y^2}{9}=1$

$\Rightarrow$ Solving hyperbola and tangent $y=\sqrt{3x}-\sqrt{3}$

$x_0=4, \; y_0=3\sqrt{3}, \; e=\sqrt{1+\frac{9}{4}}=\frac{\sqrt{13}}{2}$

$PF_1 \cdot PF_2 =$

$\sqrt{(4-\sqrt{13})^2+(3\sqrt{3})^2}\sqrt{(4+\sqrt{13})^2+(3\sqrt{3})^2}$

$=\sqrt{(56-8\sqrt{13})(56+8\sqrt{13})}$

$=\sqrt{2304}=48=m$

$\Rightarrow 4e^2+m=13+48=61$

28. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$, where $gcd(m, n)=1$, then $n-m$ is equal to _____.

Answer (71)

Sol. Given a lot of 12 items, 3 are defective.

Good items, $12-3=9$

Let X denote the number of defective items.

So, value of $X = 0, 1, 2, 3$

A sample of S items is drawn.

$P(X=0)=GGGGG$

(here G is good item and d is defective)

$$\frac{9}{12}\cdot\frac{8}{11}\cdot\frac{7}{10}\cdot\frac{6}{9}\cdot\frac{5}{8}=\frac{21}{132}=\frac{7}{44}$$

$$P(X=1)=5\left[\frac{9\cdot8\cdot7\cdot6\cdot3}{12\cdot11\cdot10\cdot9\cdot8}\right]=\frac{21}{44}$$

$$P(X=2)=5\left[\frac{9\cdot8\cdot7\cdot3\cdot2}{12\cdot11\cdot10\cdot9\cdot8}\right]=\frac{14}{44}$$

$$P(X=3)=5\left[\frac{3\cdot2\cdot1\cdot9\cdot8}{12\cdot11\cdot10\cdot9\cdot8}\right]=\frac{2}{44}$$

$P(X=4)=0$

$P(X=5)=0$

X	0	1	2	3	4	5
$P(X)$	$\frac{7}{44}$	$\frac{21}{44}$	$\frac{14}{44}$	$\frac{2}{44}$	0	0
$XP(X)$	0	$\frac{21}{44}$	$\frac{28}{44}$	$\frac{6}{44}$	0	0
$X^2P(X)$	0	$\frac{21}{44}$	$\frac{56}{44}$	$\frac{18}{44}$	0	0

$$\sigma_x^2=\sum X^2P(x)-\left(\sum x\,P(x)\right)^2$$

$$=\frac{95}{44}-\left(\frac{55}{44}\right)^2$$

$$=\frac{4180-3025}{1936}=\frac{1155}{1936}=\frac{105}{176}=\frac{m}{n}$$

$= n - m = 71$

29. If $S(x) = (1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + \ldots + 60(1 + x)^{60}$, $x \neq 0$, and $(60)^2S(60) = a(b)^b + b$, where $a, b \in \mathbf{N}$, then $(a + b)$ equal to _____.

Answer (??)

Sol.

$$S(x)=(1+x)+2(1+x)^2+3(1+x)^3+\ldots.+60(1+x)^{60}$$

$$(1+x)S(x)\quad=1(1+x)^2+2(1+x)^3+\ldots.+59(1+x)^{60}+60(1+x)^{61}$$

$$S(x)[1-1-x]=[(1+x)+(1+x)^2+\ldots+(1+x)^{60}]-60(1+x)^{61}$$

$$(-x)(S(x))=\frac{(1+x)[(1+x)^{60}-1]}{(1+x-1)}-60(1+x)^{61}$$

$$(-x)S(x)=\frac{(1+x)[(1+x)^{60}-1]}{x}-60(1+x)^{61}$$

$$xS(x)=60(1+x)^{61}-\frac{(1+x)[(1+x)^{60}-1]}{x}$$

Multiplying x on both side,

$$x^2S(x)=60x(1+x)^{61}-(1+x)[(1+x)^{60}-1]$$

Putting $x = 60$

$$(60)^2S(60)=60\times60(61)^{61}-(61)[61^{60}-1]$$

$$=60\times60(61)^{61}-(61)\cdot61^{60}+61$$

$$=(61)^{61}[60\times60-1]+61$$

$$=(3600-1)\cdot61^{61}+61$$

$a = 3600 - 1, \quad b = 61 \Rightarrow a + b = 3660$

30. Let $[t]$ denote the largest integer less than or equal to t.

If $\int_0^3\left(\left[x^2\right]+\left[\frac{x^2}{2}\right]\right)dx=a+b\sqrt{2}-\sqrt{3}-\sqrt{5}+c\sqrt{6}-\sqrt{7}$, where $a, b, c \in \mathbf{Z}$, then $a + b + c$ is equal to _____.

Answer (23.00)

Sol. $\int_0^3\left(\left[x^2\right]+\left[\frac{x^2}{2}\right]\right)dx=\int_0^1 0dx+\int_1^{\sqrt{2}}1dx+\int_{\sqrt{2}}^{\sqrt{3}}3dx+\int_{\sqrt{3}}^{2}4dx+\int_2^{\sqrt{5}}6dx+\int_{\sqrt{5}}^{\sqrt{6}}7dx+\int_{\sqrt{6}}^{\sqrt{7}}9dx+\int_{\sqrt{7}}^{\sqrt{8}}10dx+\int_{\sqrt{8}}^{3}12dx$

$$=31-6\sqrt{2}-\sqrt{3}-\sqrt{5}-\sqrt{7}-2\sqrt{6}$$

$\Rightarrow a = 31, b = -6, c = -2$

$\Rightarrow a + b + c = 23$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. In finding out refractive index of glass slab the following observations were made through travelling microscope 50 vernier scale division = 49 MSD; 20 divisions on main scale in each cm

For mark on paper

MSR = 8.45 cm, VC = 26

For mark on paper seen through slab

MSR = 7.12 cm, VC = 41

For power particle on the top surface of the glass slab

MSR = 4.05 cm, VC = 1

(MSR = Main Scale Reading, VC = Vernier Coincidence)

Refractive index of the glass slab is

(1) 1.24 (2) 1.35

(3) 1.42 (4) 1.52

Answer (3)

Sol. $\text{L.C.} = \left(1-\frac{49}{50}\right)\left(\frac{1}{20}\right)\text{cm}$

$= \frac{1}{1000}\text{cm}$

$t_1 = 8.45 + 26\left(\frac{1}{1000}\right) = 8.4526\text{ cm}$

$t_2 = 4.05 + \frac{1}{1000} = 4.0501\text{ cm}$

Δt = thickness = 4.4025 cm

Also

$\text{shift} = 8.4526 - \left(7.12 + \frac{41}{1000}\right)$

$= 1.3285$

$\because \quad \text{shift} = \left(1-\frac{1}{\mu}\right)(\text{thickness})$

$1.3285 = \left(1-\frac{1}{\mu}\right)(4.4025)$

$\therefore \quad \boxed{\mu = 1.432}$

32. The number of electrons flowing per second in the filament of a 110 W bulb operating at 220 V is

(Given $e = 1.6 \times 10^{-19}$ C)

(1) 31.25×10^{17} (2) 6.25×10^{17}

(3) 1.25×10^{19} (4) 6.25×10^{18}

Answer (1)

Sol. $P = v \times i \Rightarrow i = \frac{110}{220} = \frac{1}{2}\text{ A}$

$i = n e \Rightarrow n = \frac{1}{2 \times 1.6 \times 10^{-19}} = 31.25 \times 10^{17}$

33. In a coil, the current changes from –2 A to +2 A in 0.2 s and induces an emf of 0.1 V. The self inductance of the coil is

(1) 5 mH (2) 1 mH

(3) 4 mH (4) 2.5 mH

Answer (1)

Sol. $E = \left|L \cdot \frac{di}{dt}\right|$

$0.1 = L \cdot \frac{4}{0.2} \Rightarrow L = 5 \times 10^{-3}\text{ H}$

$L = 5\text{ mH}$

34. Assuming the earth to be a sphere of uniform mass density, a body weighed 300 N on the surface of earth. How much it would weigh at $R/4$ depth under surface of earth?

(1) 75 N (2) 375 N

(3) 300 N (4) 225 N

Answer (4)

Sol. $W = mg$

$$\Rightarrow \frac{W}{300} = \frac{g\left(1-\frac{R}{4R}\right)}{g} \Rightarrow W = 225\ \text{N}$$

35. The longest wavelength associated with Paschen series is

(Given $R_H = 1.097 \times 10^7$ SI unit)

(1) 1.094×10^{-6} m

(2) 3.646×10^{-6} m

(3) 1.876×10^{-6} m

(4) 2.973×10^{-6} m

Answer (3)

Sol. $\frac{1}{\lambda} = Rz^2\left(\frac{1}{3^2}-\frac{1}{4^2}\right); n = 4 \rightarrow n = 3$

$\Rightarrow \lambda = 1.876 \times 10^{-6}$ m

36. In the given electromagnetic wave $E_y = 600 \sin(\omega t - kx)$ Vm^{-1}, intensity of the associated light beam is

$\left(\text{in W/m}^2 : \left(\text{Given } \in_0 = 9\times10^{-12}C^2N^{-1}m^{-2}\right)\right)$

(1) 729 (2) 243

(3) 972 (4) 486

Answer (4)

Sol. $I = \frac{1}{2}\in_0 E_0^2 \times C$

$$\Rightarrow I = \frac{1}{2}\times9\times10^{-12}\times600^2\times3\times10^8 = 486\ \text{w/m}^2$$

37. When *UV* light of wavelength 300 nm is incident on the metal surface having work function 2.13 eV, electron emission takes place. The stopping potential is

(Given hc = 1240 eV nm)

(1) 4 V (2) 1.5 V

(3) 2 V (4) 4.1 V

Answer (3)

Sol. $\frac{hc}{\lambda} = \phi + eV_0$

$$\Rightarrow \frac{1240}{300} = 2.13 + V_0 \Rightarrow V_0 = 2V$$

38. Given below are two statements:

Statement (I) : Dimensions of specific heat is $[L^2T^{-2}K^{-1}]$.

Statement (II) : Dimensions of gas constant is $[M\,L^2\,T^{-1}K^{-1}]$.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **statement (I)** and **statement (II)** are incorrect

(2) Both **statement (I)** and **statement (II)** are correct

(3) **Statement (I)** is incorrect but **statement (II)** is correct

(4) **Statement (I)** is correct but **statement (II)** is incorrect

Answer (4)

Sol. Dimension of gas constant : $[M^1L^2T^{-2}K^{-1}]$

$$[\text{Specific heat}] = \left[\frac{ML^2T^{-2}}{MK}\right]$$

$$= [L^2\,T^{-2}\,K^{-1}]$$

39. Pressure inside a soap bubble is greater than the pressure outside by an amount

(given: R = Radius of bubble, S = Surface tension of bubble)

(1) $\frac{4R}{S}$ (2) $\frac{4S}{R}$

(3) $\frac{2S}{R}$ (4) $\frac{S}{R}$

Answer (2)

Sol. $\Delta P_{soap} = \frac{4S}{R}$

40. When kinetic energy of a body becomes 36 times of its original value, the percentage increase in the momentum of the body will be

(1) 500% (2) 600%

(3) 60% (4) 6%

Answer (1)

Sol. $k_1 = \frac{P_1^2}{2m}$

$36k_1 = \frac{P_2^2}{2m} \Rightarrow P_2 = 6P_1$ (Increase by 500%)

41. A body of weight 200 N is suspended from a tree branch through a chain of mass 10 kg. The branch pulls the chain by a force equal to (if g = 10 m/s^2)

(1) 200 N (2) 100 N

(3) 300 N (4) 150 N

Answer (3)

Sol. T = 200 N + 100 N = 300 N

By action-reaction, branch pulls the chain by 300 N

42. In a vernier calliper, when both jaws touch each other, zero of the vernier scale shifts towards left and its 4th division coincides exactly with a certain division on main scale. If 50 vernier scale divisions equal to 49 main scale divisions and zero error in the instrument is 0.04 mm then how many main scale divisions are there in 1 cm?

(1) 40 (2) 20

(3) 5 (4) 10

Answer (2)

Sol. 0.04 = 4(L. C.)

$\Rightarrow$ L.C = 0.01 mm

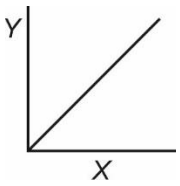
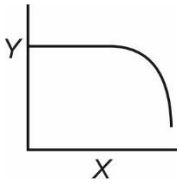
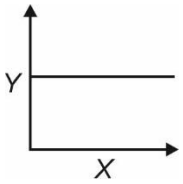
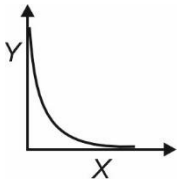
$1\,\text{MSD} - \frac{49}{50}\text{MSD} = 0.01\,\text{mm}$

$\Rightarrow$ 1 MSD = 50 × 0.01 mm

= 0.5 mm

$\Rightarrow$ 1 cm = 20(0.5 mm)

43. Match **List-I** with **List-II**

	List-I		**List-II**
	Y vs X		Shape of Graph
(A)	Y = magnetic susceptibility X = magnetising field	(I)	
(B)	Y = magnetic field X = distance from centre of a current carrying wire for $x < a$ (where a radius of wire)	(II)	
(C)	Y = magnetic field X = distance from centre of a current carrying wire for $x > a$ (where a = radius of wire)	(III)	
(D)	Y = magnetic field inside solenoid X = distance from centre	(IV)	

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(4) (A)-(1), (B)-(III), (C)-(II), (D)-(IV)

Answer (2)

Sol. In para and dia materials, susceptibility is independent of magnetic field.

Inside a current carrying wire, $B \propto x$

Outside a current carrying wire, $B \propto \frac{1}{x}$

Inside a solenoid, B is constant upto ends.

44. A car of 800 kg is taking turn on a banked road of radius 300 m and angle of banking 30°. If coefficient of static friction is 0.2 then the maximum speed with which car can negotiate the turn safely

(g = 10 m/s^2, $\sqrt{3}$ = 1.73)

(1) 70.4 m/s (2) 264 m/s

(3) 102.8 m/s (4) 51.4 m/s

Answer (4)

Sol. $v_{max} = \sqrt{\frac{Rg(\mu + \tan\theta)}{1 - \mu\tan\theta}} = 51.4 \text{ m/s}$

45. Energy of 10 non rigid diatomic molecules at temperature T is

(1) $\frac{7}{2}RT$

(2) 35 $K_B T$

(3) 70 $K_B T$

(4) 35 RT

Answer (2)

Sol. $E = 10 \times \frac{7}{2} K_B T = 35\, K_B T$

46. A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1. If it is projected vertically downwards from the same point with the same speed, it reaches the ground in t_2. Time required to reach the ground, if it is dropped from the top of the tower, is

(1) $\sqrt{\frac{t_1}{t_2}}$ (2) $\sqrt{t_1 + t_2}$

(3) $\sqrt{t_1 t_2}$ (4) $\sqrt{t_1 - t_2}$

Answer (3)

Sol. $h = ut_1 - \frac{1}{2} g\, t_1^2$

$h = ut_2 + \frac{1}{2} g\, t_2^2$

$h(t_1 + t_2) = \frac{1}{2} g\, t_1 t_2 (t_1 + t_2)$

$h = \frac{1}{2} g\, t_1 t_2$ ...(i)

$h = \frac{1}{2} g\, t^2$ ...(ii)

$t = \sqrt{t_1 t_2}$

47. The acceptor level of a p-type semiconductor is 6 eV. The maximum wavelength of light which can create a hole would be

Given hc1242 eV nm.

(1) 103.5 nm (2) 207 nm

(3) 414 nm (4) 407 nm

Answer (2)

Sol. E = 6eV

$\therefore \quad \lambda = \frac{hc}{E} = \frac{1240}{6} \text{ nm} \simeq 207 \text{ nm}$

48. A total of 48 J heat is given to one mole of helium kept in a cylinder. The temperature of helium increases by 2°C. The work done by the gas is

Given, R = 8.3 J K^{-1} mol^{-1}.

(1) 24.9 J (2) 72.9 J

(3) 23.1 J (4) 48 J

Answer (3)

Sol. $Q = \Delta U + W$

$$48 = 1\frac{3R}{2} \times 2 + W$$

$$\Rightarrow W = 23.1 \text{ J}$$

49. For the thin convex lens, the radii of curvature are at 15 cm and 30 cm respectively. The focal length the lens is 20 cm. The refractive index of the material is

(1) 1.4 (2) 1.5

(3) 1.8 (4) 1.2

Answer (2)

Sol. $\frac{1}{F} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{20} = (\mu - 1)\left(\frac{1}{15} - \frac{1}{-30}\right)$$

$$\Rightarrow \mu = 1.5$$

50. Two identical conducting spheres P and S with charge Q on each, repel each other with a force 16 N. A third identical uncharged conducting sphere R is successively brought in contact with the two spheres. The new force of repulsion between P and S is

(1) 12 N

(2) 4 N

(3) 1 N

(4) 6 N

Answer (4)

Sol. $F_1 = \frac{KQ^2}{r^2} = 16 \text{ N}$

$$F_2 = \frac{K\left(\frac{Q}{2}\right)\left(\frac{3Q}{4}\right)}{r^2} = \frac{3}{8} \times 16 = 6 \text{ N}$$

Final charges on spheres are $\frac{Q}{2}$ and $\frac{3Q}{4}$.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

51. In Franck-Hertz experiment, the first dip in the current-voltage graph for hydrogen is observed at 10.2 V. The wavelength of light emitted by hydrogen atom when excited to the first excitation level is _____ nm.

(Given hc 1245 eV nm, e = 1.6 × 10^{-19}C).

Answer (122)

Sol. $\lambda = \frac{hc}{E} = \frac{1240}{10.2} \text{ nm} = 122 \text{ nm}$

52. Three balls of masses 2 kg, 4 kg and 6 kg respectively are arranged at centre of the edges of an equilateral triangle of side 2 m. The moment of intertia of the system about an axis through the centroid and perpendicular to the plane of triangle, will be _____ kg m^2.

Answer (4)

Sol. $I = 2 \times \left(\frac{a}{2\sqrt{3}}\right)^2 + 4 \times \left(\frac{a}{2\sqrt{3}}\right)^2 + 6\left(\frac{a}{2\sqrt{3}}\right)^2$

I = 4 kg m^2

Distance between centroid and midpoint of sides

$$= \frac{a}{2\sqrt{3}}$$

53. A particle moves in a straight line so that its displacement x at any time t is given by $x^2 = 1 + t^2$. Its acceleration at any time t is x^{-n} where n = _____.

Answer (3)

Sol. $x^2 = 1 + t^2$

$\Rightarrow 2x \cdot v = 2t$

$\Rightarrow x \cdot a + v^2 = 1$

$\Rightarrow a = \frac{1-v^2}{x} = \frac{x^2-t^2}{x^3} = \frac{1}{x^3}$

$\Rightarrow \boxed{a = x^{-3}}$

54. For a given series LCR circuit it is found that maximum current is drawn when value of variable capacitance is 2.5 nF. If resistance of 200 Ω and 100 mH inductor is being used in the given circuit. The frequency of ac source is _____ $\times 10^3$ Hz. (given $\pi^2 = 10$)

Answer (10)

Sol. $\omega^2 = \frac{1}{LC}$

$\Rightarrow 4\pi^2 f^2 = \frac{1}{10^{-1} \times 2.5 \times 10^{-9}}$

$\Rightarrow f = 10 \times 10^3$ Hz

55. A coil having 100 turns, area of 5×10^{-3} m^2, carrying current of 1 mA is placed in uniform magnetic field of 0.20 T such a way that plane of coil is perpendicular to the magnetic field. The work done in turning the coil through 90° is _____ μJ.

Answer (100)

Sol. $W = U_f - U_i = (-MB\cos 90) - (-MB\cos 0)$

$W = MB = NiAB = 100$ μJ

56. Two coherent monochromatic light beams of intensities I and $4I$ are superimposed. The difference between maximum and minimum possible intensities in the resulting beam is xI. The value of x is _____.

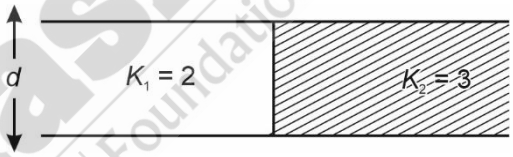
Answer (8)

Sol. $I_{max} = \sqrt{4I} + \sqrt{I} + 2\sqrt{4I^2}$

and $I_{min} = \sqrt{4I} + \sqrt{I} - 2\sqrt{4I^2}$

$\Rightarrow I_{max} - I_{min} = 8I$

57. A capacitor of 10 μF capacitance whose plates are separated by 10 mm through air and each plate has area 4 cm^2 is now filled equally with two dielectric media of $K_1 = 2$, $K_2 = 3$ respectively as shown in figure. If new force between the plates is 8 N. The supply voltage is _____ V.



Answer (None)

Sol. $C_1 = 10$ μF $\qquad C_2 = 15$ μF

$F = F_1 + F_2$

$F = \frac{\left(10 \times 10^{-6} V\right)^2}{2 \cdot \frac{A}{2} \cdot \epsilon_0} + \frac{\left(15 \times 10^{-6} V\right)^2}{2 \cdot \frac{A}{2} \cdot \epsilon_0} = 8$

$V^2 = 0.9 \times 10^{-4}$

$V = 0.95 \times 10^{-2}$ V

Data in consistent. Answer not matching.

58. Two open organ pipes of lengths 60 cm and 90 cm resonate at 6th and 5th harmonics respectively. The difference of frequencies for the given modes is _____ Hz.

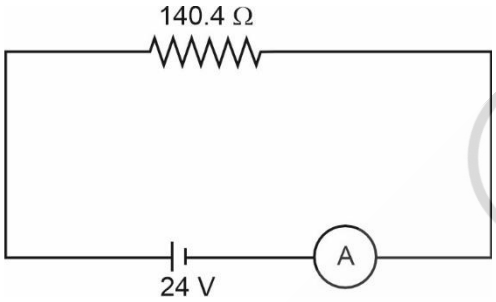
(Velocity of sound in air = 333 m/s)

Answer (740)

Sol. $\Delta F = \frac{6V}{2l_1} - \frac{5V}{2l_2} = V\left(\frac{6}{2\times 0.6} - \frac{5}{2\times 0.9}\right)$

$\Rightarrow \quad \Delta F \simeq 740$ Hz

59. In the given figure an ammeter A consists of a 240 Ω coil connected in parallel to a 100 Ω shunt. The reading of the ammeter is _____ mA.



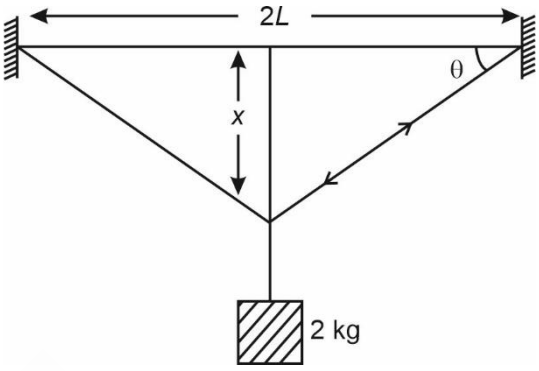
Answer (160)

Sol. $i = \frac{24}{140\cdot 4 + r_A} = \frac{24}{140\cdot 4 + 9\cdot 6} = 0.16\,\text{A}$

i = 160 mA

60. A wire of cross sectional area A, modulus of elasticity 2 × 10^{11} Nm^{-2} and length 2 m is stretched between two vertical rigid supports. When a mass of 2 kg is suspended at the middle it sags lower from its original position making angle $\theta = \frac{1}{100}$ radian on the points of support. The value of A is _____ × 10^{-4} m^2 (consider $x << L$).

(Given : g = 10 m/s^2)

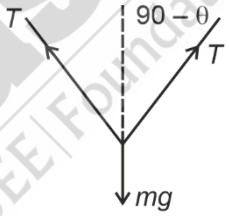


Answer (1)

Sol. $\Delta l_{spring} = \left(\sqrt{L^2 + x^2}\right) - L$

$= \frac{x^2}{2L}$

$T = k\Delta l = \frac{kn^2}{2L}$



$\Rightarrow \quad 2T\theta = mg$

$2\left(\frac{YA}{l}\right)\frac{x^2}{2l}\theta = mg$

$A = \frac{mg}{\theta^3 Y}$

$= \frac{20}{10^{-6}\times 2\times 10^{11}}$

$= 10 \times 10^{-5}$

$= 1 \times 10^{-4}$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Evaluate the following statements related to group 14 elements for their correctness.

(A) Covalent radius decreases down the group from C to Pb in a regular manner.

(B) Electronegativity decreases from C to Pb down the group gradually.

(C) Maximum covalance of C is 4 whereas other elements can expand their covalance due to presence of d orbitals.

(D) Heavier elements do not form $p\pi$-$p\pi$ bonds.

(E) Carbon can exhibit negative oxidation states.

Choose the **correct** answer from the options given below :

(1) (C) and (D) Only (2) (A), (B) and (C) Only

(3) (A) and (B) Only (4) (C), (D) and (E) Only

Answer (4)

Sol. (A) Covalent radius of group 14 elements increases from C to Pb.

(B) Electronegativity decreases from C to Si and thereafter it remains more or less constant.

(C) Maximum covalency of C is 4 as the outermost shell of C is 2^{nd} shell and it cannot expand its octet. But other elements can expand their covalency due to the availability of vacant d-orbitals.

(D) Heavier elements do not form $p\pi$-$p\pi$ bonds due to their larger atomic size.

(E) Carbon can show negative oxidation states when it is covalently bonded to less electronegative elements like CH_4.

62. Molality (m) of 3 M aqueous solution of NaCl is :

(Given : Density of solution = 1.25 g mL^{-1}, Molar mass in g mol^{-1} : Na-23, Cl-35.5)

(1) 1.90 m (2) 2.90 m

(3) 2.79 m (4) 3.85 m

Answer (3)

Sol. Molarity of aq. NaCl solution = 3M

Density of solution = 1.25 g mL^{-1}

Mass of 1 L solution = 1250 g

Mass of NaCl in 1 L solution = 175.5 g

Mass of solvent in 1 L solution = 1250–175.5

= 1074.5 g

Molality of given solution = $\frac{3\times1000}{1074.5}$

= 2.79 m

63. How can an electrochemical cell be converted into an electrolytic cell?

(1) Applying an external opposite potential greater than E^{o}_{cell}.

(2) Reversing the flow of ions in salt bridge.

(3) Applying an external opposite potential lower than E^{o}_{cell}.

(4) Exchanging the electrodes at anode and cathode.

Answer (1)

Sol. An electrochemical cell can be converted into an electrolytic cell by applying an external potential greater than E^{o}_{cell}. The entire cell reaction will be reversed. The oxidised species will be reduced and the reduced species will be oxidised.

64. Given below are two statements :

Statement I : PF_5 and BrF_5 both exhibit sp^3d hybridisation.

Statement II : Both SF_6 and $[Co(NH_3)_6]^{3+}$ exhibit sp^3d^2 hybridisation.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement I** is false but **Statement II** is true

(2) Both **Statement I** and **Statement II** are false

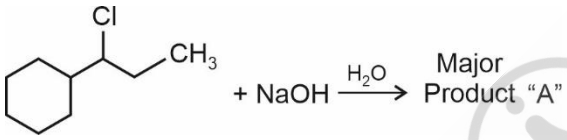
(3) Both **Statement I** and **Statement II** are true

(4) **Statement I** is true but **Statement II** is false

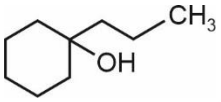
Answer (2)

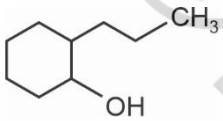
Sol. Hybridisation of P in PF_5 is sp^3d but the hybridisation of Br in BrF_5 is sp^3d^2. So, statement-I is false.

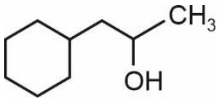
Hybridisation of S in SF_6 is sp^3d^2 but the hybridisation of Co^{3+} in $[Co(NH_3)_6]^{3+}$ is d^2sp^3 as NH_3 is a strong field ligand forcing the unpaired electrons to pair up. So, statement-II is false.

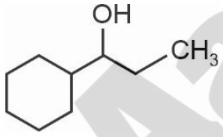
65. 

Consider the above chemical reaction. Product "A" is :

(1) 

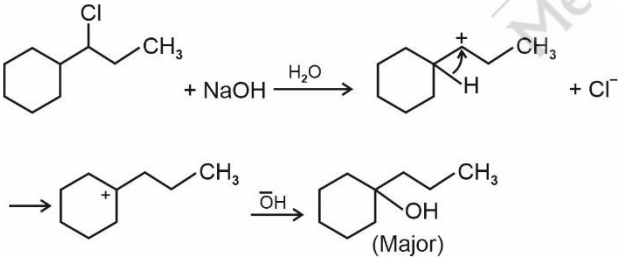
(2) 

(3) 

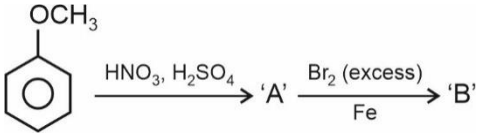
(4) 

Answer (1)

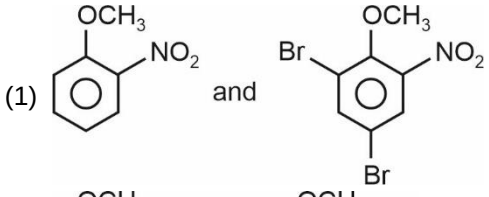
Sol. In aqueous medium the reaction follows S_N1 mechanism majorly

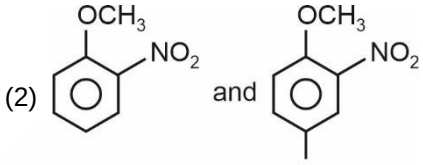


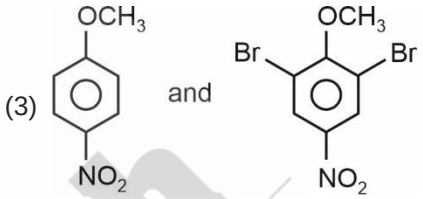
66. The major products formed :

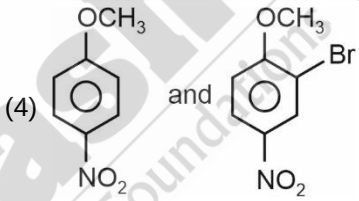


A and B respectively are :

(1) 

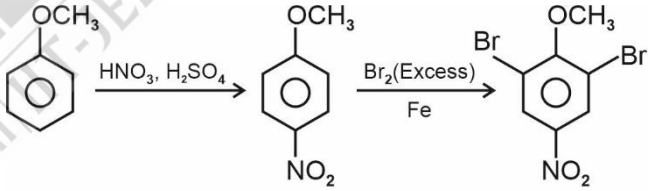
(2) 

(3) 

(4) 

Answer (3)

Sol.



Methoxy group is o-, p-directing. Nitration of methoxy benzene will produce p-isomer as the major product due to steric crowding in o-isomers. Bromination of p-nitroanisole (A) with excess of Br_2 will result in disubstitution to give 2, 5-dibromo-4-nitroanisole.

(B) as the major product.

67. The ratio $\frac{K_P}{K_C}$ for the reaction :

$CO_{(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons CO_{2(g)}$ is

(1) $(RT)^{\frac{1}{2}}$ (2) 1

(3) RT (4) $\frac{1}{\sqrt{RT}}$

Answer (4)

Sol. $CO(g) + \frac{1}{2}O_2(g) \rightleftharpoons CO_2(g)$

K_P and K_C are related as

$K_P = K_C(RT)^{\Delta n_g}$

Δn_g for the given reaction = $-\frac{1}{2}$

$\therefore \quad \frac{K_P}{K_C} = \frac{1}{\sqrt{RT}}$

68. During the detection of acidic radical present in a salt, a student gets a pale yellow precipitate soluble with difficulty in NH_4OH solution when sodium carbonate extract was first acidified with dil. HNO_3 and then $AgNO_3$ solution was added. This indicates presence of :

(1) CO_3^{2-} (2) I^-

(3) Cl^- (4) Br^-

Answer (4)

Sol. The Br^- ion present in the salt gives pale yellow ppt. of AgBr with $AgNO_3$, insoluble in dil HNO_3 but partially soluble in aq. NH_4OH.

$Br^- + AgNO_3 \rightarrow \underset{\text{Pale yellow}}{AgBr\downarrow} + NO_3^-$

$AgBr + 2NH_4OH \rightarrow \underset{\text{Partially soluble}}{[Ag(NH_3)_2]Br} + 2H_2O$

69. The correct statement among the following, for a "chromatography" purification method is

(1) Organic compounds run faster than solvent in the thin layer chromatographic plate.

(2) Non-polar compounds are retained at top and polar compounds come down in column chromatography.

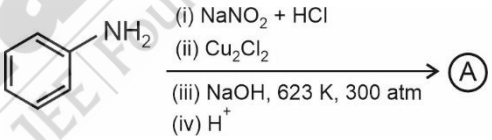
(3) R_f is an integral value.

(4) R_f of a polar compound is smaller than that of a non-polar compound.

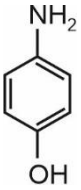
Answer (4)

Sol. Organic compounds run slower than solvent in thin layer chromatography. R_f is not an integral value. It is the ratio of distance travelled by the organic compound to that of solvent. R_f value of a polar compound is smaller than that of non-polar compound as the polar compound is retained more by the adsorbent than non-polar compound.

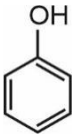
70. Identify the product (A) in the following reaction.



(1) 

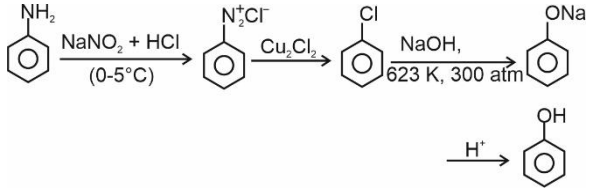
(2) 

(3) 

(4) 

Answer (4)

Sol.



Conversion of diazonium chloride to chlorobenzene with Cu_2Cl_2 is Sandmeyer reaction and the fusion of chlorobenzene to sodium phenoxide in presence of NaOH at high temperature and high pressure is Dow's process.

71. Match **List-I** with **List-II**.

	List-I Tetrahedral Complex		**List-II Electronic configuration**
(A)	$TiCl_4$	(I)	e^2, t_2^0
(B)	$[FeO_4]^{2-}$	(II)	e^4, t_2^3
(C)	$[FeCl_4]^-$	(III)	e^0, t_2^0
(D)	$[CoCl_4]^{2-}$	(IV)	e^2, t_2^3

Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(3) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Answer (4)

Sol. (A) $TiCl_4 : Ti^{4+} : 3d^0 4s^0$ or $e^0 t_2^0$

(B) $[FeO_4]^{2-} : Fe^{6+} : 3d^2 4s^0$ or $e^2 t_2^0$

(C) $[FeCl_4]^- : Fe^{3+} : 3d^5 4s^0$ or $e^2 t_2^3$

(D) $[CoCl_4]^{2-} : Co^{2+} : 3d^7 4s^0$ or $e^4 t_2^3$

∴ Correct matching of tetrahedral complexes given in List-I with their electronic configuration given in List-II is

(A)-(III); (B)-(I); (C)-(IV); (D)-(II)

72. Consider the given reaction, identify the major product P.

$$CH_3 - COOH \xrightarrow[(iv)\ H_2O/\bar{O}H,\ \Delta]{(i)\ LiAlH_4\ (ii)\ PCC\ (iii)\ HCN/\bar{O}H} \text{“P”}$$

(1) $CH_3 - \overset{\overset{O}{\|}}{C} - CH_2CH_3$

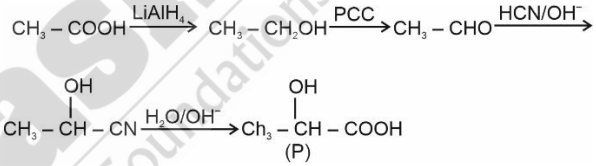
(2) $CH_3 - CH_2 - CH_2 - OH$

(3) $CH_3 - CH_2 - \overset{\overset{O}{\|}}{C} - NH_2$

(4) $CH_3 - \overset{\overset{OH}{|}}{CH} - COOH$

Answer (4)

Sol.



The major product (P) formed in the given reaction is 2-hydroxy propanoic acid.

73. The number of ions from the following that are expected to behave as oxidising agent is :

Sn^{4+}, Sn^{2+}, Pb^{2+}, Tl^{3+}, Pb^{4+}, Tl^+

(1) 4 (2) 3

(3) 2 (4) 1

Answer (3)

Sol. Due to inert pair effect Pb^{2+} is more stable than Pb^{4+} and Tl^+ is more stable than Tl^{3+}. Therefore, Pb^{4+} and Tl^{3+} will function as oxidising agents and easily get reduced to Pb^{2+} and Tl^+ respectively.

74. The correct IUPAC name of $[PtBr_2(PMe_3)_2]$ is :

(1) dibromobis(trimethylphosphine)platinum(II)

(2) dibromodi(trimethylphosphine)platinum(II)

(3) bis(trimethylphosphine)dibromoplatinum(II)

(4) bis[bromo(trimethylphosphine)]platinum(II)

Answer (1)

Sol. The correct IUPAC name of $[PtBr_2(PMe_3)_2]$ is dibromobis(trimethylphosphine)platinum(II).

75. Match **List-I** with **List-II**.

	List-I Alkali Metal		**List-II Emission Wavelength in nm**
(A)	Li	(I)	589.2
(B)	Na	(II)	455.5
(C)	Rb	(III)	670.8
(D)	Cs	(IV)	780.0

Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(2) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(4) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

Answer (1)

Sol. The emission wavelengths in nm as given in NCERT (XI) Part-II, page 292 are

(A) Li 670.8

(B) Na 589.2

(C) Rb 780.0

(D) Cs 455.5

$\therefore$ Correction match of List-I and List-II is

(A)-(III), (B)-(I), (C)-(IV), (D)-(II)

76. The incorrect statements regarding enzymes are :

(A) Enzymes are biocatalysts.

(B) Enzymes are non-specific and can catalyse different kinds of reactions.

(C) Most Enzymes are globular proteins.

(D) Enzyme-oxidase catalyses the hydrolysis of maltose into glucose.

Choose the **correct** answer from the option given below :

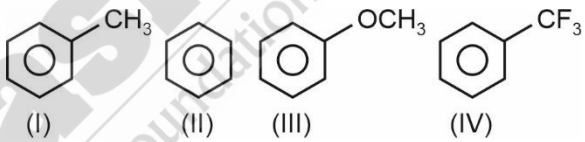
(1) (A), (B) and (C) (2) (B), (C) and (D)

(3) (B) and (D) (4) (B) and (C)

Answer (3)

Sol. Enzymes are biocatalysts and are very specific for a particular reaction and for a particular substrate. Almost all enzymes are globular proteins. Enzyme maltase hydrolyses maltose into glucose.

$$\text{Maltose} \xrightarrow{\text{Maltase}} \text{Glucose}$$

77. 

The **correct** arrangement for decreasing order of electrophilic substitution for above compounds is :

(1) (III) > (IV) > (II) > (I)

(2) (IV) > (I) > (II) > (III)

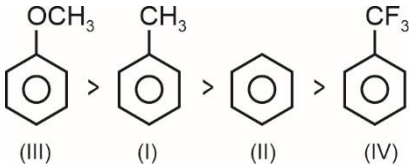
(3) (II) > (IV) > (III) > (I)

(4) (III) > (I) > (II) > (IV)

Answer (4)

Sol. Methoxy group strongly activates benzene ring towards electrophilic aromatic substitution due to +R effect. Methyl group weakly activates benzene ring due to its +I and +H effects. CF_3 group strongly deactivates the benzene ring due to –I and –H effects.

Therefore, the correct order of the given compounds towards electrophilic aromatic substitution is



78. The incorrect statement regarding the geometrical isomers of 2-butene is :

(1) cis-2-butene and trans-2-butene are stereoisomers.

(2) trans-2-butene is more stable than cis-2-butene.

(3) cis-2-butene has less dipole moment than trans-2-butene.

(4) cis-2-butene and trans-2-butene are not interconvertible at room temperature.

Answer (3)

Sol. cis-2-butene are trans-2-butene are stereoisomers.

trans-2-butene is more stable than cis-2-butene.

cis-2-butene has more dipole moment than trans-2-butene.

cis-2-butene are trans-2-butene are interconvertible at room temperature.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

79. Arrange the following elements in the increasing order of number of unpaired electrons in it.

(A) Sc (B) Cr

(C) V (D) Ti

(E) Mn

Choose the **correct** answer from the options given below :

(1) (A) < (D) < (C) < (B) < (E)

(2) (B) < (C) < (D) < (E) < (A)

(3) (A) < (D) < (C) < (E) < (B)

(4) (C) < (E) < (B) < (A) < (D)

Answer (3)

Sol. Electronic configuration of the given metals

(A) Sc $\Rightarrow 3d^1 4s^2 \Rightarrow n = 1$

(B) Cr $\Rightarrow 3d^5 4s^1 \Rightarrow n = 6$

(C) V $\Rightarrow 3d^3 4s^2 \Rightarrow n = 3$

(D) Ti $\Rightarrow 3d^2 4s^2 \Rightarrow n = 2$

(E) Mn $\Rightarrow 3d^5 4s^2 \Rightarrow n = 5$

$\therefore$ Correct order of increasing number of unpaired electrons.

A < D < C < E < B

80. Match **List-I** with **List-II**.

	List-I Reaction		**List-II Type of redox reaction**
(A)	$N_{2(g)} + O_{2(g)} \rightarrow 2NO_{(g)}$	(I)	Decomposition
(B)	$2Pb(NO_3)_{2(s)} \rightarrow 2PbO_{(s)} + 4NO_{2(g)} + O_{2(g)}$	(II)	Displacement
(C)	$2Na_{(s)} + 2H_2O_{(l)} \rightarrow 2NaOH_{(aq.)} + H_{2(g)}$	(III)	Disproportionation
(D)	$2NO_{2(g)} + 2OH^-_{(aq.)} \rightarrow NO^-_{2(aq.)} + NO^-_{3(aq.)} + H_2O_{(l)}$	(IV)	Combination

Choose the **correct** answer from the options given below :

(1) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(3) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Answer (2)

Sol. (A) $\overset{o}{N}_2 + \overset{o}{O}_2 \longrightarrow 2NO_{(g)}$

Combination reaction

(B) $2Pb(NO_3)_{2(s)} \longrightarrow 2PbO + 4NO_2 + O_2$

Decomposition reaction

(C) $2Na_{(s)} + 2H_2O(l) \longrightarrow 2NaOH_{(aq)} + H_{2(g)}$

Displacement reaction

(D) $2NO_{2(g)} + 2OH^-_{(g)} \longrightarrow NO^-_{2(aq)} + NO^-_{3(aq)} + H_2O$

Nitrogen oxidises and reduces both. So it is a disproportionation reaction.

A→(IV), B→(I), C→(II), D→(III)

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

81. Among VO_2^+, MnO_4^- and $Cr_2O_7^{2-}$, the spin-only magnetic moment value of the species with least oxidising ability is _____ BM (Nearest integer).

(Given atomic member V = 23, Mn = 25, Cr = 24)

Answer (0)

Sol. VO_2^+ has least oxidising ability

$VO_2^+ \Rightarrow V^{+5} \Rightarrow 3d^0 4s^0$

There is no unpaired electron present. So, magnetic moment = 0

82. Consider the following reactions

$$NiS + HNO_3 + HCl \rightarrow A + NO + S + H_2O$$

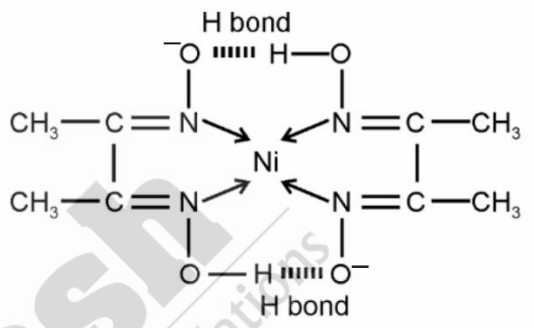
$$A + NH_4OH + H_3C - \underset{\underset{H_3C - \underset{|}{C} = N - OH}{|}}{C} = N - OH \rightarrow B + NH_4Cl + H_2O$$

The number of protons that do not involve in hydrogen bonding in the product B is _____.

Answer (12)

Sol. $NiS + HNO_3 + HCl \longrightarrow \underset{(A)}{NiCl_2} + S + NO + H_2O$

$$\underset{(A)}{NiCl_2} + NH_4OH + Dimethyl\,gyoxime \longrightarrow \underset{(B)}{Ni(dmg)_2} + NH_4Cl + H_2O$$



Total 12 proton do not involve in H-bond

83. For the reaction at 298 K, 2A + B → C. ΔH = 400 kJ mol^{-1} and ΔS = 0.2 kJ mol^{-1} K^{-1}. The reaction will become spontaneous above _____ K.

Answer (2000)

Sol. $\Delta G = \Delta H - T\Delta S$

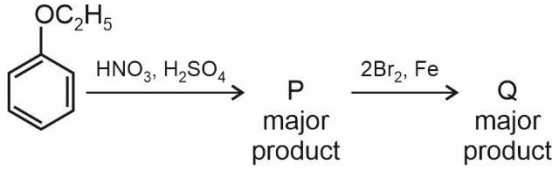
For reaction to be spontaneous ΔG = –ve

For limiting case $\Delta G = 0$

$\Delta H = T\Delta S$

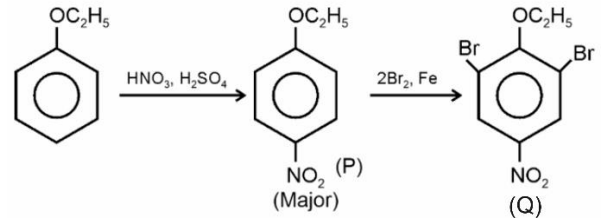
$$T = \frac{\Delta H}{\Delta S} = \frac{400\ kJ/mol}{0.2\ kJ/mol\text{-}K}$$

= 2000 K

84. 

The ratio of number of oxygen atoms to bromine atoms in the product Q is _____ × 10^{-1}.

Answer (15)

Sol. 

Number of O-atoms = 3

Number of Br-atoms = 2

Required ratio $= \frac{3}{2} = 1.5$

$= 15 \times 10^{-1}$

85. For hydrogen atom, energy of an electron in first excited state is –3.4 eV, K.E. of the same electron of hydrogen atom is *x* eV. Value of *x* is _____ × 10^{-1} eV. (Nearest integer)

Answer (34)

Sol. $E_2 = -3.4$ eV

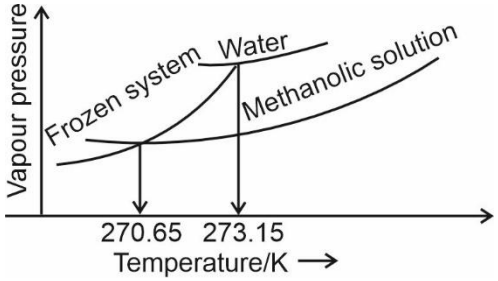
$KE = -E_2$

KE = 3.4 eV

KE = 34 × 10^{-1} eV

x = 34

86. When '*x*' × 10^{-2} mL methanol (molar mass = 32 g; density = 0.792 g/cm^3) is added to 100 mL water (density = 1 g/cm^3), the following diagram is obtained.



x = _____ (nearest integer).

[Given : Molal freezing point depression constant of water at 273.15 K is 1.86 K kg mol^{-1}]

Answer (543)

Sol. $\Delta T_f = 2.5^oC$

$\Delta T_f = i \times k_f \times m$

$2.5 = 1 \times 1.86 \times \frac{n_B}{0.1}$

$n_B = \frac{2.5 \times 0.1}{1.86} = 0.1344$ mol

mass of methanol = 0.1344 × 32 = 4.3 g

$d = \frac{m}{v}$

$v = \frac{m}{d}$

$v = \frac{4.3}{0.792}$ mL

= 5.43 mL

= 543 × 10^{-2} mL

x = 543

87. Total number of species from the following with central atom utilising sp^2 hybrid orbitals for bonding is _____.

NH_3, SO_2, SiO_2, $BeCl_2$, C_2H_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3, $C_2H_4Cl_2$

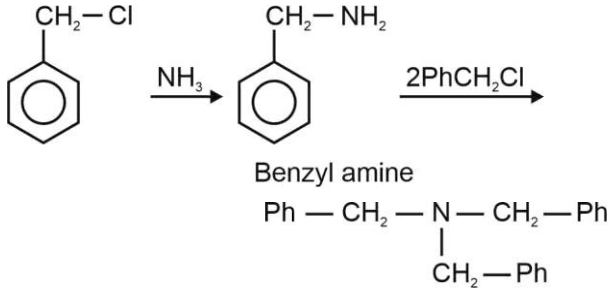
Answer (6)

Sol. SO_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3 are sp^2 hybridised central atom

88. An amine (X) is prepared by ammonolysis of benzyl chloride. On adding p-toluenesulphonyl chloride to it the solution remains clear. Molar mass of the amine (X) formed is _____ g mol^{-1}.

(Given molar mass in gmol^{-1} C : 12, H : 1, O : 16, N : 14)

Answer (287)

Sol. 

Benzyl amine

Molar mass of amine = 287 g/mol

89. Consider the two different first order reactions given below

A + B $\rightarrow$ C (Reaction 1)

P $\rightarrow$ Q (Reaction 2)

The ratio of the half life of Reaction 1 : Reaction 2 is 5 : 2. If t_1 and t_2 represent the time taken to complete 2/3rd and 4/5th of Reaction 1 and Reaction 2, respectively, then the value of the ratio $t_1 : t_2$ is _____ $\times 10^{-1}$ (nearest integer).

[Given : $\log_{10}(3) = 0.477$ and $\log_{10}(5) = 0.699$]

Answer (17)

Sol. A + B $\rightarrow$ C Reaction …(1)

P $\rightarrow$ Q Reaction …(2)

$$\frac{\left(t_{\frac{1}{2}}\right)_1}{\left(t_{\frac{1}{2}}\right)_2} = \frac{k_2}{k_1} = \frac{5}{2}$$

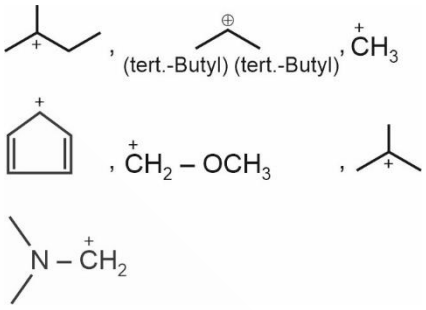
$$\frac{t_{\frac{2}{3}}}{t_{\frac{4}{5}}} = \frac{k_2}{k_1}\frac{\log 3}{\log 5}$$

$$= \left(\frac{5}{2}\right)\frac{0.477}{0.699} = 1.7$$

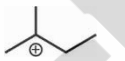
$= 17 \times 10^{-1}$

$= x = 17$

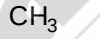
90. Number of carbocations from the following that are **not** stabilized by hyperconjugation is _____.



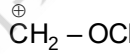
Answer (5)

Sol.  has 8 α – H

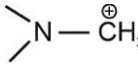
 no α – H

 no α – H

 no hyperconjugable H

 no α – H

 9 α – H

 no α – H

5 carbocations are not stabilised by hyperconjugation.

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