06/04/2024
Morning

Aakash
Medical | IIT-JEE | Foundations

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6) **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A company has two plants *A* and *B* to manufacture motorcycles. 60% motorcycles are manufactured at plant *A* and the remaining are manufactured at plant *B*. 80% of the motorcycles manufactured at plant *A* are rated of the standard quality, while 90% of the motorcycles manufactured at plant *B* are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If *p* is the probability that it was manufactured at plant *B*, then 126*p* is

(1) 64 (2) 56

(3) 54 (4) 66

Answer (3)

Sol. $P(\text{standard automobile from } A) = \frac{6}{10}\times\frac{8}{10} = \frac{12}{25}$

$P(\text{standard automobile from } B) = \frac{4}{10}\times\frac{9}{10} = \frac{9}{25}$

Required Probability $\frac{\frac{9}{25}}{\frac{12}{25}+\frac{9}{25}}$

$P = \frac{9}{21} = \frac{3}{7}$

So, $126P = 126\times\frac{3}{7} = 54$

2. Let $y = y(x)$ be the solution of the differential equation $(2x\log_e x)\frac{dy}{dx} + 2y = \frac{3}{x}\log_e x$, $x > 0$ and $y(e^{-1}) = 0$. Then, $y(e)$ is equal to

(1) $-\frac{3}{e}$ (2) $-\frac{2}{3e}$

(3) $-\frac{3}{2e}$ (4) $-\frac{2}{e}$

Answer (1)

Sol. $(2x\log x)\frac{dy}{dx} + 2y = \frac{3}{x}\log x$

$\frac{dy}{dx} + \frac{y}{x\log x} = \frac{3}{2x^2}$

$IF = e^{\int\frac{1}{x\log x}} = e^{\log(\log x)} = \log x$

$y\times\log x = \int\log x\times\frac{3}{2x^2}dx + C$

$y\log x = \frac{3}{2}\left[\frac{-\log x}{x} - \frac{1}{x}\right] + C$

$y(e^{-1}) = 0$

$\Rightarrow 0 = \frac{3}{2}[e - e] + C$

$C = 0$

$y\log x = \frac{-3}{2}\left[\frac{\log x + 1}{x}\right]$

$y(e) \to y(e)\times 1 = \frac{-3}{2}\left[\frac{1+1}{e}\right] = -\frac{3}{e}$

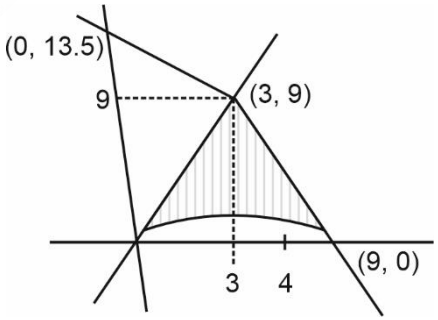
3. Let the area of the region enclosed by the curves $y = 3x$, $2y = 27 - 3x$ and $y = 3x - x\sqrt{x}$ be *A*. Then $10A$ is equal to

(1) 162 (2) 184

(3) 172 (4) 154

Answer (1)

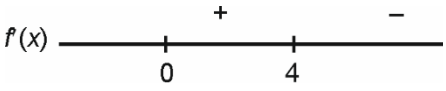
Sol.



$y = 3x - x^{3/2}$

$$y' = 3 - \frac{3}{2}\sqrt{x}$$

$$y' = 3\left(1 - \frac{\sqrt{x}}{2}\right)$$



$$\text{Area} = \int_0^3 \left(3x - \left(3x - x^{3/2}\right)\right)dx + \int_3^9 \frac{27-3x}{2} - \left(3x - x^{3/2}\right)dx$$

$$= \int_0^3 x^{3/2}dx + \frac{1}{2}\int_3^9 \left(27 - 9x + 2x^{3/2}\right)dx$$

$$= \left[\frac{2}{5}x^{5/2}\right]_0^3 + \frac{1}{2}\left[27x - \frac{9x^2}{2} + 2\times\frac{2}{5}x^{5/2}\right]_3^9$$

$$= \frac{2}{5}\times 9\sqrt{3} + \frac{1}{2}\left[243 - \frac{729}{2} + \frac{4}{5}\times 81\times 3 - 81 + \frac{81}{2} - \frac{4}{5}\times 9\sqrt{3}\right]$$

$$= \frac{1}{2}\left[\frac{486-729-81}{2} + \frac{972}{5}\right] = \frac{81}{5}$$

$$A = \frac{81}{5}$$

$$\therefore 10A = 10\times\frac{81}{5} = 162$$

4. Let $f : (-\infty, \infty) - \{0\} \to \mathbb{R}$ be a differentiable function such that $f'(1) = \lim\limits_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$. Then $\lim\limits_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\log_e a$ is equal to

(1) $\frac{3}{8} + \frac{\pi}{4}$ (2) $\frac{5}{2} + \frac{\pi}{8}$

(3) $\frac{3}{2} + \frac{\pi}{4}$ (4) $\frac{3}{4} + \frac{\pi}{8}$

Answer (Bonus)

Sol. Let $f'(1) = k$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)}{x^2} = k \qquad \left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{f'(x)}{2x} = \lim_{x\to 0}\frac{f''(x)}{2} = k$$

$$\Rightarrow f''(0) = 2k$$

Given information is not complete.

5. Let the relations R_1 and R_2 on the set $X = \{1, 2, 3, \ldots, 20\}$ be given by $R_1 = \{(x, y): 2x - 3y = 2\}$ and $R_2 = \{(x, y): -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then $M + N$ equals

(1) 12 (2) 8

(3) 10 (4) 16

Answer (3)

Sol. $R_1 = \{(x, y) : 2x - 3y = 2\}$

$R_2 = \{(x, y) : -5x + 4y = 0\}$

$2x - 3y = 2$

So $2x$ and $3y$ both has to be even or odd simultaneously and $2x$ can't be odd so $2x$ and $3y$ both will be even

$R_1 = \{(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\}$

For symmetric we need to add 6 elements as

(2, 4), (4, 7), (6, 10), (8, 13), (10, 16), (12, 19)

$M = 6$

For R_2 $-5x + 4y = 0$

$5x$ and $4y$ has to be equal $4y$ is always even so $5x$ will also be even

$R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\}$

For symmetric we need to add 4 element as

(5, 4) (10, 8) (15, 12) (20, 16)

$N = 4$

$M + N = 6 + 4 = 10$

6. The shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ is

(1) $4\sqrt{3}$ (2) $8\sqrt{3}$

(3) $5\sqrt{3}$ (4) $6\sqrt{3}$

Answer (1)

Sol. Given two lines are

$$\frac{x-3}{2}=\frac{y-(-15)}{-7}=\frac{z-9}{5} \text{ and}$$

$$\frac{x-(-1)}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$

Shortest distance between two lines are

$$d=\frac{\left|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$$

$$\therefore \quad \vec{a}_2-\vec{a}_1=-4\hat{i}+16\hat{j}+0\hat{k}$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-7&5\\2&1&-3\end{vmatrix}=16\hat{i}+16\hat{j}+16\hat{k}$$

$$\therefore \quad d=\frac{|-64+16\times16|}{16\sqrt{3}}=4\sqrt{3}$$

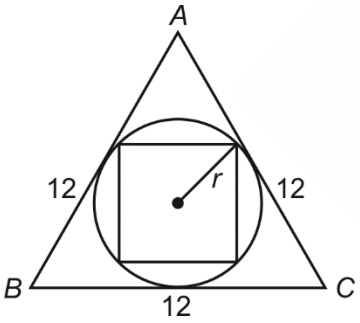
7. A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively, then $m + n^2$ is equal to

(1) 408 (2) 396

(3) 312 (4) 414

Answer (1)

Sol.



Inradius of $\Delta ABC = r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}}{4}\times(12)^2}{18}$

$r = 2\sqrt{3}$

Side length of square is a, then

$a^2 = 2r^2$

$\Rightarrow \quad a^2 = 24$

Area of square, $m = 24$

Perimeter of square, $n = 4\sqrt{24}$

$\Rightarrow \quad m + n^2 = 24 + 384$

$= 408$

8. For $\alpha, \beta \in \mathbb{R}$ and a natural number n, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $2A_{10} - A_8$ is

(1) 0 (2) $4\alpha + 2\beta$

(3) $2n$ (4) $2\alpha + 4\beta$

Answer (2)

Sol. $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

$$A_r = 2\begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ r & 1 & \frac{n^2}{2}-\frac{\beta}{2} \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$= 2\begin{vmatrix} 0 & 0 & \alpha+\frac{\beta}{2} \\ r & 1 & \frac{n^2}{2}-\frac{\beta}{2} \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$= 2\left(\alpha+\frac{\beta}{2}\right)(3r-3r+2)$$

$A_r = 4\alpha + 2\beta$

$2A_{10} - A_8 = 4\alpha + 2\beta$

9. If $A(3, 1, -1)$, $B\left(\frac{5}{3}, \frac{7}{3}, \frac{1}{3}\right)$, $C(2, 2, 1)$ and $D\left(\frac{10}{3}, \frac{2}{3}, \frac{-1}{3}\right)$ are the vertices of a quadrilateral $ABCD$, then its area is

(1) $\frac{2\sqrt{2}}{3}$ (2) $2\sqrt{2}$

(3) $\frac{5\sqrt{2}}{3}$ (4) $\frac{4\sqrt{2}}{3}$

Answer (4)

Sol. $A(3,1,-1), B\left(\frac{5}{3}, \frac{7}{3}, \frac{1}{3}\right), C(2,2,1), D\left(\frac{10}{3}, \frac{2}{3}, \frac{-1}{3}\right)$

are vertices of a quadrilateral

$\overrightarrow{AC} = (2\hat{i} + 2\hat{j} + \hat{k}) - (3\hat{i} + \hat{j} - \hat{k})$

$= -\hat{i} + \hat{j} + 2\hat{k}$

$\overrightarrow{BD} = \left(\frac{10}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}\right) - \left(\frac{5}{3}\hat{i} + \frac{7}{3}\hat{j} + \frac{1}{3}\hat{k}\right)$

$\overrightarrow{BD} = \frac{5}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{2}{3}\hat{k}$

Area $= \frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}|$

$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ \frac{5}{3} & \frac{-5}{3} & \frac{-2}{3} \end{vmatrix}$

$= \frac{1}{2}\sqrt{\left(\frac{8}{3}\right)^2 + \left(\frac{8}{3}\right)^2}$ $\left[\because \overrightarrow{AC} \times \overrightarrow{BD} = \frac{8}{3}\hat{i} + \frac{8}{3}\hat{j}\right]$

$= \frac{1}{2} \times \frac{8}{3} \times \sqrt{2} = \frac{4\sqrt{2}}{3}$

10. The function $f(x) = \frac{x^2 + 2x - 15}{x^2 - 4x + 9}, x \in \mathbb{R}$ is

(1) Neither one-one nor onto.

(2) Both one-one and onto.

(3) Onto but not one-one.

(4) One-one but not onto.

Answer (1)

Sol. $f(x) = \frac{x^2 + 2x - 15}{x^2 - 4x + 9} = \frac{(x-3)(x+5)}{x^2 - 4x + 9}$

For $x = 3$ and $x = -5$, $f(x) = 0$

So $f(x)$ is not one-one

Range of $f(x)$ is $[-2, 1.6]$

So $f(x)$ is not onto function

11. Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}|x|$. Then, which one of the following points lies on the circle C?

(1) (1, 2)

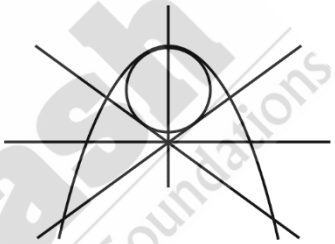
(2) (2, 4)

(3) (1, 1)

(4) (2, 2)

Answer (2)

Sol. Let centre be $(0, k)$



Now radius is $r = 6 - k$

Also, $6 - k = \left|\frac{k}{2}\right|$

$\Rightarrow 6 - k = \frac{k}{2}$

$\Rightarrow 12 - 2k = k$

$\Rightarrow k = 4$

Radius, $r = 6 - 4 = 2$

So circle will be

$(x)^2 + (y - k)^2 = 4$

$x^2 + (y - 4)^2 = 4$

(2, 4) satisfies this equation.

12. Let $y = y(x)$ be the solution of the differential equation $\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$, $y(1) = 0$. Then $y(0)$ is

(1) $\frac{1}{4}\left(e^{\pi/2}-1\right)$ (2) $\frac{1}{4}\left(1-e^{\pi/2}\right)$

(3) $\frac{1}{2}\left(1-e^{\pi/2}\right)$ (4) $\frac{1}{2}\left(e^{\pi/2}-1\right)$

Answer (3)

Sol. $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$

$$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^{\tan^{-1}x}}{1+x^2}$$

I.F. $= e^{\int\frac{1}{1+x^2}\cdot dx} = e^{\tan^{-1}x}$

So, $y\cdot e^{\tan^{-1}x}=\int e^{\tan^{-1}x}\cdot\frac{e^{\tan^{-1}x}}{1+x^2}dx$

$$y\cdot e^{\tan^{-1}x}=\int\frac{e^{2\tan^{-1}x}}{1+x^2}dx$$

Put $\tan^{-1}x = t$

$$\frac{1}{1+x^2}dx=dt$$

So $I=\int e^{2t}dt=\frac{e^{2t}}{2}+\frac{c}{2}$

$$y\cdot e^{\tan^{-1}x}=\frac{e^{2\tan^{-1}x}}{2}+\frac{c}{2}$$

$y(1) = 0$

So, $0=e^{2\tan^{-1}1}+c$

$0=e^{2\cdot\pi/4}+c$

$\Rightarrow \quad c=-e^{\pi/2}$

$$y\cdot e^{\tan^{-1}x}=\frac{e^{2\tan^{-1}x}-e^{\pi/2}}{2}$$

$$y(0)=y\cdot 1=\frac{1-e^{\pi/2}}{2}$$

$$\Rightarrow \quad y=\frac{1}{2}(1-e^{\pi/2})$$

13. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is

(1) 24

(2) 16

(3) 48

(4) 56

Answer (3)

Sol.

Total ways to select one point out of 8 points

$= {}^8C_1$ ways

Now total ways of selecting 2 non-consecutive points out of remaining 5 points = $4C_2$ ways

Total required triangles = ${}^8C_1 \times {}^4C_2$

$= 48$

14. If $f(x)=\begin{cases}x^3\sin\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$, then

(1) $f''\left(\frac{2}{\pi}\right)=\frac{12-\pi^2}{2\pi}$ (2) $f''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$

(3) $f''(0)=0$ (4) $f''(0)=1$

Answer (2)

Sol. $f(x)=\begin{cases}x^3\sin\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$

$$f'(x)=3x^2\sin\frac{1}{x}-x\cos\frac{1}{x}$$

$$f''(x)=6x\sin\left(\frac{1}{x}\right)-3x\cos\left(\frac{1}{x}\right)-\cos\frac{1}{x}-\frac{1}{x}\sin\frac{1}{x}$$

$\Rightarrow \quad f''\left(\frac{2}{\pi}\right)=\frac{12}{\pi}-\frac{\pi}{2}=\frac{24-\pi^2}{2\pi}$ and $f'(0)$ is not defined

15. Let α, β be the distinct roots of the equation $x^2 - (t^2 - 5t + 6)x + 1 = 0$, $t \in \mathbb{R}$ and $a_n = \alpha^n + \beta^n$.

Then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is

(1) 1/4

(2) –1/4

(3) –1/2

(4) 1/2

Answer (2)

Sol. $x^2 - (t^2 - 5t + 6)x + 1 = 0$

$\therefore a_{2025} - (t^2 - 5t + 6)a_{2024} + a_{2023} = 0$

$$\Rightarrow \frac{a_{2025} + a_{2023}}{a_{2024}} = t^2 - 5t + 6$$

$$= \left(t + \frac{5}{2}\right)^2 + \left(\frac{-1}{4}\right)$$

Minimum value $= \frac{-1}{4}$

16. The interval in which the function $f(x) = x^x$, $x > 0$, is strictly increasing is

(1) $\left[\frac{1}{e^2}, 1\right)$ (2) $\left[\frac{1}{e}, \infty\right)$

(3) $(0, \infty)$ (4) $\left(0, \frac{1}{e}\right]$

Answer (2)

Sol. $f(x) = x^x$

$f'(x) = x^x(\log x + 1)$

$f'(x) \geq 0$

$\Rightarrow 1 + \log x \geq 0$

$\Rightarrow \log x \geq -1$

$\Rightarrow x \geq e^{-1}$

$\therefore x \in \left[\frac{1}{e}, \infty\right)$

17. $\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{(\cos^3 x + \sin^3 x)} dx$ is equal to

(1) 1/9 (2) 1/3

(3) 1/6 (4) 1/12

Answer (3)

Sol. $\int_0^{\pi/4} \frac{\cos^2 x.\sin^2 x}{(\cos^3 x + \sin^3 x)^2} dx$

$$= \int_0^{\pi/4} \frac{\tan^2 x.\sec^2 x}{(1 + \tan^3 x)^2} dx$$

Let $\tan x = t$

$$\int_0^1 \frac{t^2 dt}{(1+t^3)^2}$$

Let $1 + t^3 = z$

$3t^2 dt = dz$

$$\frac{1}{3}\int_1^2 \frac{dz}{z^2} = \frac{1}{3}\left(-\frac{1}{z}\right)\Big|_1^2$$

$$= -\frac{1}{3}\left(\frac{1}{2} - 1\right) = \frac{1}{6}$$

18. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is

(1) $\sqrt{3.86}$ (2) 1.94

(3) $\sqrt{3.96}$ (4) 1.8

Answer (3)

Sol. mean = 10, $\sigma = 2$, $n = 20$

$x_1 + x_2 + x_3 + \ldots\ldots + x_{20} = 200$

correct mean $= \frac{200 - 8 + 12}{20} = \frac{204}{20} = 10.2$

$x_1^2 + x_2^2 + x_3^2 + ...x_{20}^2 = (\sigma^2 + \bar{x}^2) \times n$

= (4 + 100)20 = 104 × 20 = 2080

Corrected $x_1^2 + x_2^2 ..... + x_{20}^2 = 2080 - 64 + 144 = 2160$

$\therefore$ corrected $\sigma^2 = \frac{2160}{20} - (10.2)^2$

= 108 – 104.04

= 3.96

$\therefore \ \sigma = \sqrt{3.96}$

19. Let a variable line of slope $m > 0$ passing through the point (4, –9) intersect the coordinate axes at the points

(1) 15

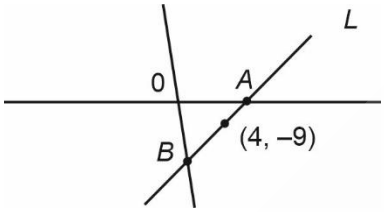
(2) 30

(3) 25

(4) 10

Answer (3)

Sol.



$L : (y + 9) = m(x - 4)$

$A : \left(4 + \frac{9}{m}, 0\right)$

$B : (0, -9 - 4m)$

$OA + OB]_{min}$

$\Rightarrow E_{min} = 4 + \frac{9}{m} + 9 + 4m$

$E = 13 + \frac{9}{m} + 4m$

$\frac{dE}{dM} = 0 \Rightarrow -\frac{9}{m^2} + 4 = 0 \Rightarrow m = \pm\frac{3}{2}$

$\frac{d^2E}{dM^2} = \frac{18}{m^3} > 0$ for $m = \frac{3}{2}$

$\therefore E_{min} = 4 + 6 + 9 + 6 = 25$

20. Let $A = \{n \in [100, 700] \cap \mathbb{N} : n$ is neither a multiple of 3 nor a multiple of 4$\}$. Then the number of elements in A is

(1) 290

(2) 300

(3) 280

(4) 310

Answer (2)

Sol. $n \in [100, 700]$

n(A) = Total – (multiple of 3 + multiple of 4) + (multiple of 12)

Total = 601

Multiple of 3 = 102, 105, ..., 699

$n = 699 = 102 + (n - 1)3$

$\Rightarrow n = 200$

Multiple of 4 = 100, 104...., 700

$n = 700 = 100 + (n - 1)4$

$\frac{600}{4} + 1 = n$

$\Rightarrow n = 151$

Multiple of 12 = 108, 120.....696

$n = 696 = 108 + (n - 1)12$

$n = 50$

$\therefore n(A) = 601 - (200 + 151) + 50$

= 300

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $\alpha\beta\gamma = 45$; $\alpha, \beta, \gamma \in \mathbb{R}$. If $x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0)$ for some $x, y, z \in R$, $xyz \neq 0$, then $6\alpha + 4\beta + \gamma$ is equal to _____

Answer (55)

Sol. $\alpha x + y + 2z = 0$

$x + \beta y + 3z = 0$

$2x + 2y + \gamma z = 0$

$\alpha\beta\gamma = 45$

$\because xyz \neq 0$

$$\Rightarrow \begin{vmatrix} \alpha & 1 & 2 \\ 1 & \beta & 3 \\ 2 & 2 & \gamma \end{vmatrix} = 0$$

$\Rightarrow \alpha\beta\gamma - 6\alpha - 4\beta - \gamma + 10 = 0$

$45 + 10 = 6\alpha + 4\beta + \gamma$

$\Rightarrow 6\alpha + 4\beta + \gamma = 55$

22. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3T_{r-1} + 6^r$, $r = 2, 3, \ldots, n$. If the sum of the first n terms of this series is $\frac{1}{5}\left(n^2 - 12n + 39\right)\left(4 \cdot 6^n - 5 \cdot 3^n + 1\right)$, then n is equal to _____

Answer (06)

Sol. $T_r = 3T_{r-1} + 6^r$

$\Rightarrow$ solving homogenous part

$T_r = 3T_{r-1}$

$\Rightarrow x = 3$ is the root

$\therefore T_r = a.3^r$

Solving for particular part

$T_r = b.6^r$

$b.6^r = 3b6^{r-1} + 6^r$

$\Rightarrow 6b = 3b + 6$

$\Rightarrow 3b = 6$

$\Rightarrow b = 2$

$T_r = a^n + a^p$

$T_r = a3^{br} + 2.6^r$ ... (i)

$T_r = 3T_{r-1} + 6^r$

Putting $r = 2$

$T_2 = 18 + 36 = 54$ … (ii)

Using equation (i) and (ii)

$54 = 9a + 72 \Rightarrow -18 = 9a \Rightarrow a = -2$

$\therefore T_r = 2 \cdot 6^r - 2 \cdot 3^r = 2\left(6^r - 3^r\right)$

$$\sum_{r=1}^{n} T_r = 2\sum 6^r - 2\sum 3^r$$

$$= 2 \cdot 6\frac{\left(6^n - 1\right)}{5} - 2 \cdot 3\frac{\left(3^n - 1\right)}{2}$$

$$= \frac{3}{5}\left(4 \cdot 6^n - 5 \cdot 3^n + 1\right)$$

$\therefore n^2 - 12n + 39 = 3$

$n^2 - 12n + 36 = 0$

$(n - 6)^2 = 0$

$\therefore n = 6$

23. Let L_1, L_2 be the lines passing through the point $P(0, 1)$ and touching the parabola $9x^2 + 12x + 18y - 14 = 0$. Let Q and R be the points on the lines L_1 and L_2 such that the ΔPQR is an isosceles triangle with base QR. If the slopes of the liens QR are m_1 and m_2, then $16\left(m_1^2 + m_2^2\right)$ is equal to _____ .

Answer (68)

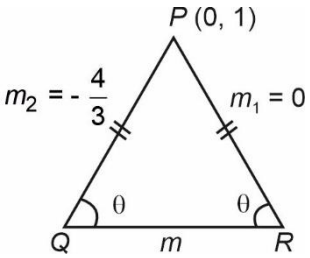
Sol. $9x^2 + 12x + 18y - 14 = 0$

$$\left(x+\frac{2}{3}\right)^2 = -2(y-1)...(1)$$

Equation of tangent to (1)

$t\left(x+\frac{2}{3}\right) = -(y-1)+\frac{1}{2}t^2$ passes through (0, 1)

$$\Rightarrow \frac{2}{3}t = \frac{1}{2}t^2 \Rightarrow t = 0, \frac{4}{3} \Rightarrow m = 0, m = -\frac{4}{3}$$



$$\Rightarrow \left|\frac{m+\frac{4}{3}}{1-\frac{4}{3}m}\right| = |m| \Rightarrow \left|\frac{3m+4}{3-4m}\right| = |m|$$

$\Rightarrow 3m + 4 = m(3 - 4m)$ or $3m + 4 = -m(3 - 4m)$

$3m + 4 = 3m - 4m^2$ or $3m + 4 = -3m + 4m^2$

$4m^2 + 4 = 0$ (not possible) or $4m^2 - 6m - 4 = 0$

$$m_1 + m_2 = \frac{3}{2}, m_1 m_2 = -1$$

$$\Rightarrow m_1^2 + m_2^2 = \frac{17}{4}$$

$$16\left(m_1^2 + m_2^2\right) = 68$$

24. Let $r_k = \frac{\int_0^1 (1-x^7)^k dx}{\int_0^1 (1-x^7)^{k+1} dx}$, $k \in \mathbb{N}$. Then the value of $\sum_{k=1}^{10} \frac{1}{7(r_k - 1)}$ is equal to _____

Answer (65)

Sol. $r_k = \frac{I_a}{I_b}$, where $I_a = \int_0^1 \left(1-x^7\right)^k dx$

and $I_b = \int_0^1 \underset{\text{(I)}\downarrow}{\left(1-x^7\right)^{k+1}} . \underset{\text{(II)}\downarrow}{I}\ dx$

$$= \left(1-x^7\right)^{k+1}.x\Big]_0^1 - \int_0^1 (k+1)\left(1-x^7\right)^k \left(-7x^6\right).x\,dx$$

$$= -7(k+1)\int_0^1 \left(1-x^7\right)^k \left(-1+1-x^7\right)$$

$$I_b = -7(k+1)\left[-I_a + I_b\right]$$

$$\Rightarrow r_k = \frac{I_a}{I_b} = \frac{7k+8}{7k+7} = 1 + \frac{1}{7(k+1)}$$

$$\frac{1}{7(r_k - 1)} = (k+1)$$

$$\Rightarrow \sum_{r=-1}^{10} \frac{1}{7(r_K - 1)} = \sum_{r=1}^{10} (K+1) = \frac{11.12}{2} - 1 = 65$$

25. If the second, third and fourth terms in the expansion of $(x + y)^n$ are 135, 30 and $\frac{10}{3}$, respectively, then $6(n^3 + x^2 + y)$ is equal to _____

Answer (806)

Sol. $T_2 = {}^nC_1\, y^1.\, x^{n-1} = 135$

$T_3 = {}^nC_2\, y^2.\, x^{n-2} = 30$

$$T_4 = {}^nC_3 y^3 x^{n-3} = \frac{10}{3}$$

$$\Rightarrow \frac{135}{30} = \left(\frac{x}{y}\right)\frac{n.2}{n(n-1)} = \left(\frac{2}{n-1}\right)\left(\frac{x}{y}\right) \quad ...(i)$$

$$\frac{30}{\frac{10}{3}} = \frac{n(n-1)}{2}\,\frac{3!}{n(n-1)(n-2)}\left(\frac{x}{y}\right)$$

$$9 = \left(\frac{3}{n-2}\right)\left(\frac{x}{y}\right)$$

$\Rightarrow \quad 3(n-2)=\frac{135}{60}(n-1) \Rightarrow n=5$

$\Rightarrow \quad x = 9y \qquad \ldots(i)$

$y.\, x^4 = 27 \Rightarrow \frac{x}{9}.\, x^4 = 3^3$

$\Rightarrow \quad x^5 = 3^5 \Rightarrow x = 3 \;\; y=\frac{1}{3}$

$\Rightarrow \quad 6\left(5^3+3^2+\frac{1}{3}\right)=6\left(125+9+\frac{1}{3}\right)$

$= 6(134) + 2 = 806$

26. For $n \in N$, if $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}n = \frac{\pi}{4}$, then n is equal to _____ .

Answer (47)

Sol. $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}(n) = \frac{\pi}{4}$

$\Rightarrow \quad \cot^{-1}\left(\frac{3\times4-1}{3+4}\right)+\cot^{-1}\left(\frac{5\times n-1}{5+n}\right)=\frac{\pi}{4}$

$\cot^{-1}\left(\frac{11}{7}\right)+\cot^{-1}\left(\frac{5n-1}{5+n}\right)=\frac{\pi}{4}$

$\cot^{-1}\frac{\left(\frac{11}{7}\times\left(\frac{5n-1}{5+n}\right)-1\right)}{\frac{11}{7}+\frac{5n-1}{5+n}}=\frac{\pi}{4}$

$\Rightarrow \quad \frac{11}{7}\left(\frac{5n-1}{5+n}\right)-1=\frac{11}{7}+\frac{5n-1}{5+n}$

$\Rightarrow \quad 55n - 11 - 35 - 7n = 55 + 11n + 35n - 7$

$\Rightarrow \quad 2n = 94$

$\Rightarrow \quad n = 47$

27. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{b}=3\hat{i}+4\hat{j}-5\hat{k}$ and a vector $\vec{c}$ be such that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times\vec{c}=\hat{i}+8\hat{j}+13\hat{k}$. If $\vec{a}\cdot\vec{c}=13$, then $(24-\vec{b}\cdot\vec{c})$ is equal to _____.

Answer (46)

Sol. Let $\hat{i}+8\hat{j}+13\hat{k}=\vec{u}$

Given $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times\vec{c}=\vec{u}$

$\Rightarrow \quad \vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c}=\vec{u}$

$(\vec{a}+\vec{b})\times c=\vec{u}-\vec{a}\times\vec{b}$

Taking cross product with $\vec{a}$ on both sides

$\vec{a}\times\left((\vec{a}+\vec{b})\times\vec{c}\right)=\vec{a}\times(\vec{u}-\vec{a}\times\vec{b})$

$\Rightarrow \quad \vec{c}.(\vec{a}^2+\vec{a}.\vec{b})=13(\vec{a}+\vec{b})-\vec{a}\times\vec{u}$

$+(\vec{a}.\vec{b}).\vec{a}-\vec{a}^2\,\vec{b}$

$\{\because \vec{a}.\vec{c}=13\}$

Putting the values, $\vec{c}=(-1,-1,3)$

$\vec{b}.\vec{c}=-22$

$\Rightarrow \quad 24-\vec{b}.\vec{c}=46$

28. Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $(4+x_1^2)(4+x_2^2)(4+x_3^2)(4+x_4^2)=\frac{125}{16}m$. Then the value of m is _____ .

Answer (221)

Sol. $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$

$(x + 1)\,(4x^3 + 4x^2 - 21x + 9) = 0$

$(x + 1)\,(x + 3)\,(4x^2 - 8x + 3) = 0$

$(x + 1)\,(x + 3)\,(4x^2 - 6x - 2x + 3) = 0$

$(x + 1)\,(x + 3)\,(2x(2x - 3) - 1\,(2x - 3)) = 0$

$\underbrace{(x+1)}_{\boxed{x=-1}}\;\underbrace{(x+3)}_{\boxed{x=-3}}\;\underbrace{(2x-1)}_{\boxed{x=\frac{1}{2}}}\;\underbrace{(2x-3)}_{\boxed{x=\frac{3}{2}}}=0$

$$(4+1)(4+9)\left(4+\frac{1}{4}\right)\left(4+\frac{9}{4}\right)=\frac{125}{16}m$$

$$5\times13\times\left(\frac{17}{4}\right)\times\left(\frac{25}{4}\right)=\frac{125}{16}m$$

$$\frac{125}{16}\times[13\times17]=\frac{125}{16}m$$

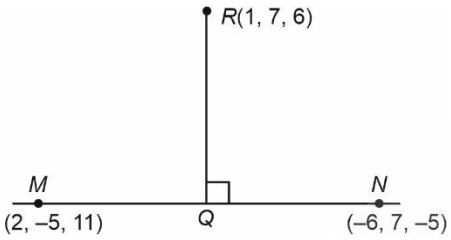
$m = 13 \times 17$

$\boxed{m=221}$

29. Let P be the point (10, –2, –1) and Q be the foot of the perpendicular drawn from the point R(1, 7, 6) on the line passing through the points (2, –5, 11) and (–6, 7, –5). Then the length of the line segment PQ is equal to _____ .

Answer (13)

Sol.



P(10, –2, –1)

$$MN:\frac{x-2}{8}=\frac{y+5}{-12}=\frac{z-11}{16}$$

General point

(8k+2, –12k – 5, 16k + 11)

$$\overrightarrow{RQ}=(8k+2-1)\hat{i}+(-12k-5-7)\hat{j}+(16k+11-6)\hat{k}$$

$$\overrightarrow{RQ}=(8k+1)\hat{i}-(12k+12)\hat{j}+(16k+5)\hat{k}$$

$\overrightarrow{RQ}.\overrightarrow{MN}=0$ (as both are perpendicular)

8(8k + 1) + 12 (12k + 12) + 16(16k + 5) = 0

64k +8 + 144k + 144 + 256k + 80 = 0

464k = –232

$$\boxed{k=\frac{-232}{464}=\frac{-1}{2}}$$

Q(–4 + 2, 6 – 5, –8+11)

Q(–2, 1, 3)

$$PQ=\sqrt{(10+2)^2+(-3)^2+(4)^2}$$

$$PQ=\sqrt{12^2+3^2+4^2}$$

$$\boxed{PQ=\sqrt{169}=13}$$

30. Let a conic C pass through the point (4, –2) and $P(x, y)$, $x \geq 3$, be any point on C. Let the slope of the line touching the conic C only at the single point P be half the slope of the line joining the points P and (3, –5). If the focal distance of the point (7, 1) on C is d, then 12d equals _____.

Answer (75)

Sol. As per given condition

$$\frac{dy}{dx}=\frac{y+5}{2(x-3)}$$

$$\Rightarrow \ln(y+5)=\frac{1}{2}\ln(x-3)+c$$

Passes through (4, –2) $\Rightarrow \ln 3=\frac{1}{2}\ln 1+c$

$\Rightarrow c = \ln 3$

$\Rightarrow$ Curve is $(y + 5)^2 = 9(x – 3)$

Focal distance of (7, 1) = $\frac{9}{4}+4=\frac{25}{4}=d$

12d = 75

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. While measuring diameter of wire using screw gauge the following readings were noted. Main scale reading is 1 mm and circular scale reading is equal to 42 divisions. Pitch of screw gauge is 1 mm and it has 100 divisions on circular scale. The diameter of the wire is $\frac{x}{50}$ mm. The value of x is

(1) 142 (2) 71

(3) 42 (4) 21

Answer (2)

Sol. $LC = \frac{1}{100}$ mm

Reading = MSR + CSR × LC

= 1 mm + 42 × 0.01 mm

= 1.42 mm = $\frac{x}{50}$ mm $\Rightarrow$ $\boxed{x = 71}$

32. A train starting from rest first accelerates uniformly up to a speed of 80 km/h for time t, then it moves with a constant speed for time $3t$. The average speed of the train for this duration of journey will be (in km/h)

(1) 30 (2) 70

(3) 40 (4) 80

Answer (2)

Sol. $V_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{\frac{1}{2} \times 80 \times t + 80 \times 3t}{4t}$

$\Rightarrow V_{avg} = 70$ km/h

33. A small ball of mass m and density ρ is dopped in a viscous liquid of density ρ_0. After sometime, the ball falls with constant velocity. The viscous force on the ball is

(1) $mg\left(1+\frac{\rho}{\rho_0}\right)$ (2) $mg\left(1-\frac{\rho_0}{\rho}\right)$

(3) $mg\left(\frac{\rho_0}{\rho}-1\right)$ (4) $mg(1-\rho\rho_0)$

Answer (2)

Sol. $F_V = (mg - F_B)\frac{mg}{mg} = \left(\frac{\rho V_g - \rho_0 V_g}{\rho V_g}\right)mg$

$F_V = mg\left(1-\frac{\rho_0}{\rho}\right)$

34. The specific heat at constant pressure of a real gas obeying $PV^2 - RT$ equation is

(1) R (2) $\frac{R}{3}+C_V$

(3) $C_V + R$ (4) $C_V + \frac{R}{2V}$

Answer (4)

Sol. $\because$ $PV^2 = RT$

$P(2vdv) + V^2(dP) = R\,dT$

at P = const.

$Pdv = \frac{RdT}{2V}$ …(i)

Now, for $n = 1$

$d\theta = dv + dw$

$C_P\,dT = C_V dT + PdV$ …(ii)

from (i) and (ii)

$C_P = C_V + \frac{R}{2V}$

35. Match **List I** with **List II**

List I		List II	
A.	Torque	I.	$[M^1L^1T^{-2}A^{-2}]$
B.	Magnetic field	II.	$[L^2A^1]$
C.	Magnetic moment	III.	$[M^1T^{-2}A^{-1}]$
D.	Permeability of free space	IV.	$[M^1L^2T^{-2}]$

Choose the **correct** answer from the options given below:

(1) A-I, B-III, C-II, D-IV (2) A-III, B-I, C-II, D-IV

(3) A-IV, B-II, C-III, D-I (4) A-IV, B-III, C-II, D-I

Answer (4)

Sol. $\tau = F.d$, $[\tau] = [M^1L^2T^{-2}]$

$F = qVB$, $[B] = [M^1T^2A^{-1}]$

$M_1 = iA$, $[M_1] = [L^2A^1]$

36. Given below are two statements :

Statement I : In an LCR series circuit, current is maximum at resonance.

Statement II : Current in a purely resistive circuit can never be less than that in a series LCR circuit when connected to same voltage source.

In the light of the above statements, choose the **correct** from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

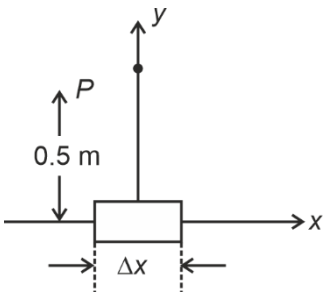
Answer (4)

Sol. Statement I : True

Statement II : True

Current in purely resistive circuit is equal to current in LCR circuit with same resistance at resonance, otherwise more.

37. An element $\Delta l = \Delta x\hat{i}$ is placed at the origin and carries a large current I = 10 A. The magnetic field on the y-axis at a distance of 0.5 m from the elements Δx of 1 cm length is:



(1) 4×10^{-8} T (2) 10×10^{-8} T

(3) 12×10^{-8} T (4) 8×10^{-8} T

Answer (1)

Sol. $B = \frac{u_0}{4\pi}\frac{idl\sin\theta}{r^2}$

$\Rightarrow B = \frac{10^{-7} \times 10 \times 10^{-2} \times 1}{\frac{1}{4}} = 4 \times 10^{-8}$ T

38. Which of the following phenomena does not explain by wave nature of light?

A. reflection

B. diffraction

C. photoelectric effect

D. interference

E. polarization

Choose the **most appropriate** answer from the options given below:

(1) C only (2) E only

(3) A, C only (4) B, D only

Answer (1)

Sol. Photoelectric effect explains the particle nature of light.

39. Electromagnetic waves travel in a medium with speed of 1.5×10^8 m s^{-1}. The relative permeability of the medium is 2.0. The relative permittivity will be:

(1) 5 (2) 1

(3) 2 (4) 4

Answer (3)

Sol. $v = \dfrac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \cdot \varepsilon_r}}$

$\Rightarrow \quad 1.5 \times 10^8 = \dfrac{3 \times 10^8}{\sqrt{2 \cdot \varepsilon_r}} \Rightarrow \varepsilon_r = 2$

40. A sample contains mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen in the sample, is:

(1) $\dfrac{2\sqrt{2}}{1}$ (2) $\dfrac{1}{32}$

(3) $\dfrac{1}{2\sqrt{2}}$ (4) $\dfrac{1}{4}$

Answer (1)

Sol. $v = \sqrt{\dfrac{3RT}{M}}$

$\Rightarrow \quad \dfrac{v_{H_e}}{v_{O_2}} = \sqrt{\dfrac{M_{O_2}}{M_{H_e}}} = \dfrac{2\sqrt{2}}{1}$

41. A bullet of mass 50g is fired with a speed 100 m/s on a plywood and emerges with 40 m/s. The percentage loss of kinetic energy is :

(1) 16% (2) 44%

(3) 32% (4) 84%

Answer (4)

Sol. % loss in $KE = \dfrac{KE_i - KE_f}{KE_i} \times 100$

$= 84\%$

42. To project a body of mass m from earth's surface to infinity, the required kinetic energy is (assume, the radius of earth is R_E, g = acceleration due to gravity on the surface of earth):

(1) mgR_E (2) $2mgR_E$

(3) $1/2mgR_E$ (4) $4mgR_E$

Answer (1)

Sol. $KE = \dfrac{1}{2} \times m\left(\dfrac{2GM}{R_E}\right) = \left(\dfrac{GM}{R_E^2}\right) m \cdot R_E = mgR_E$

43. Four particles A, B, C, D of mass $\dfrac{m}{2}$, m, $2m$, $4m$, have same momentum respectively. The particle with maximum kinetic energy is

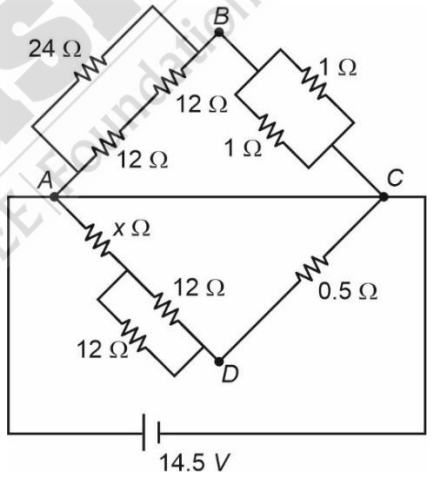
(1) C (2) D

(3) B (4) A

Answer (4)

Sol. $KE = \dfrac{P^2}{2m} \Rightarrow KE \propto \dfrac{1}{m}$

44. The value of unknown resistance (x) for which the potential difference between B and D will be zero in the arrangement shown, is



(1) 6 Ω (2) 9 Ω

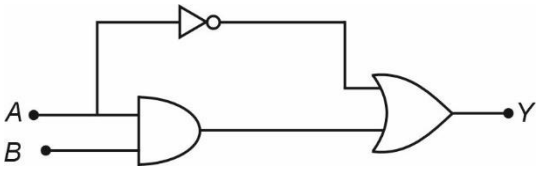
(3) 9 Ω (4) 42 Ω

Answer (1)

Sol. Balanced wheatstone bridge.

$\Rightarrow \quad 12 \times 0.5 = (x + 6) \times \dfrac{1}{2} \Rightarrow \boxed{x = 6\ \Omega}$

45. The correct truth table for the following logic circuit is



(1)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(3)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

Answer (4)

Sol. NOT, OR and AND gate.

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

46. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (where $m_2 > m_1$). If the acceleration of the system is $\frac{g}{\sqrt{2}}$, then the ratio of the masses $\frac{m_1}{m_2}$ is

(1) $\frac{\sqrt{3}+1}{\sqrt{2}-1}$ (2) $\frac{1+\sqrt{5}}{\sqrt{5}-1}$

(3) $\frac{1+\sqrt{5}}{\sqrt{2}-1}$ (4) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

Answer (4)

Sol. $a = \frac{(m_2 - m_1)g}{m_1 + m_2} = \frac{g}{\sqrt{2}}$

$\Rightarrow \sqrt{2}(m_2 - m_1) = m_1 + m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

47. In photoelectric experiment energy of 2.48 eV irradiates a photo sensitive material. The stopping potential was measured to be 0.5 V. Work function of the photo sensitive material is

(1) 2.48 eV (2) 0.5 eV

(3) 1.68 eV (4) 1.98 eV

Answer (4)

Sol. $E = \phi + eV_0$

$2.48 = \phi + 0.5$

$\Rightarrow \phi = 1.98$ eV

48. σ is the uniform surface charge density of a thin spherical shell of radius R. The electric field at any point on the surface of the spherical shell is

(1) $\sigma / 4 \in_0$ (2) $\sigma / \in_0$

(3) $\sigma / \in_0 R$ (4) $\sigma / 2 \in_0$

Answer (2)

Sol. $E = \frac{1}{4\pi \in_0} \cdot \frac{\sigma \cdot 4\pi R^2}{R^2} = \frac{\sigma}{\in_0}$

49. To find the spring constant (k) of a spring experimentally, a student commits 2% positive error in the measurement of time and 1% negative error in measurement of mass. The percentage error ion determining value of k is:

(1) 5% (2) 4%

(3) 3% (4) 1%

Answer (1)

Sol. $k = \frac{4\pi^2 m}{T^2}$

$\Rightarrow \frac{dk}{k} \times 100 = \frac{dm}{m} \times 100 + \frac{2dT}{T} \times 100$

= 1 + 2 × 2 = 5%

50. The ratio of the shortest wavelength of Balmer series to the shortest wavelength of Lyman series for hydrogen atom is

(1) 2 : 1 (2) 1 : 4

(3) 1 : 2 (4) 4 : 1

Answer (4)

Sol. $\frac{1}{\lambda_B} = Rz^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

$\frac{1}{\lambda_L} = Rz^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

$\frac{\lambda_B}{\lambda_L} = \frac{4}{1}$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

51. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of 1000 droplets to that of energy of big drop is $\frac{10}{x}$.

The value of x is _____.

Answer (1)

Sol. $R_{big} = 10\, R_{small}$

$\Rightarrow \frac{E_{1000}}{E_{big}} = \frac{1000 \times T \times 4\pi\left[\frac{R_{big}}{10}\right]^2}{T \times 4\pi R_{big}^2} = \frac{10}{1}$

52. For three vectors

$\vec{A} = \left(-x\hat{i} - 6\hat{j} - 2\hat{k}\right), \vec{B} = \left(-\hat{i} + 4\hat{j} + 3\hat{k}\right)$

and $\vec{C} = \left(-8\hat{i} - \hat{j} + 3\hat{k}\right)$, if $\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = 0$, then value of x is _____.

Answer (4)

Sol. $\left(\vec{B} \times \vec{C}\right) = 15\hat{i} - 21\hat{j} + 33\hat{k}$

$\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = 0 \Rightarrow -15x + 126 - 66 = 0$

$\Rightarrow \boxed{x = 4}$

53. The refractive index if prism is $\mu = \sqrt{3}$ and the ratio of the angle of minimum deviation to the angle of prism is one. The value of angle of prism is _____°.

Answer (60)

Sol. $\sqrt{3} = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \Rightarrow \cos\left(\frac{A}{2}\right) = \frac{\sqrt{3}}{2} \Rightarrow \boxed{A = 60°}$

54. Radius of a certain orbit of hydrogen atom is 8.48 Å. If energy of electron in this orbit is E/x, then x = _____.

(Given a_0 = 0.529 Å, E = energy of electron in ground state).

Answer (16)

Sol. $8.48 = 0.53\frac{n^2}{1} \Rightarrow n = 4$

$E = -13.6 \times \frac{1^2}{4^2} \Rightarrow \boxed{x = 16}$

55. When a *dc* voltage of 100 V is applied to an inductor, a *dc* current of 5 A flows through it. When an *ac* voltage of 200 V peak value is connected to inductor, its inductive reactance is found to be $20\sqrt{3}\Omega$. The power dissipated in the circuit is _____W.

Answer (250)

Sol. $R = \frac{100}{5} = 20\Omega$

$$Z = \sqrt{R^2 + X_L^2} = 40\Omega, I_0 = \frac{V_0}{Z} = \frac{200}{40} = 5 \text{ A}$$

$$P = V_{rms} I_{rms} \cos\phi = \frac{V_0 I_0}{2} \times \frac{R}{Z} = 250 \text{ W}$$

56. A circular coil having 200 turns, 2.5 $\times 10^{-4}$ m^2 area and carrying 100 μA current is placed in a uniform magnetic field of 1 T. Initially the magnetic dipole moment $(\vec{M})$ was directed along $\vec{B}$. Amount of work, required to rotate the coil through 90° from its initial orientation such that $\vec{M}$ becomes perpendicular to $\vec{B}$, is _____μJ.

Answer (5)

Sol. $W = U_f - U_i = (-MB\cos 90) - (-MB\cos 0)$

$\Rightarrow$ $W = MB = NiAB = 5$ μJ

57. A wire of resistance R and radius r is stretched till its radius became $r/2$. If new resistance of the stretched wire is x R, then value of x is _____.

Answer (16)

Sol. $R_1 = \frac{\rho . l}{\pi r^2}$, $R_2 = \frac{\rho . 4l}{\pi\left(\frac{r}{2}\right)^2} = 16R_1$

58. If the radius of earth is reduced to three-fourth of its present value without change in its mass then value of duration of the day of earth will be _____ hours 30 minutes.

Answer (13)

Sol. $\tau_{ext} = 0 \Rightarrow$ Angular momentum is conserved.

$$\Rightarrow \frac{2}{5}MR^2 \cdot \omega_i = \frac{2}{5}M\left(\frac{3R}{4}\right)^2 . \omega_f \Rightarrow \omega_f = \frac{16}{9}\omega$$

$$\because T = \frac{2\pi}{\omega} \Rightarrow T_1 = \frac{9}{16} \times 24 \text{ hours}$$

= 13 hours 30 min.

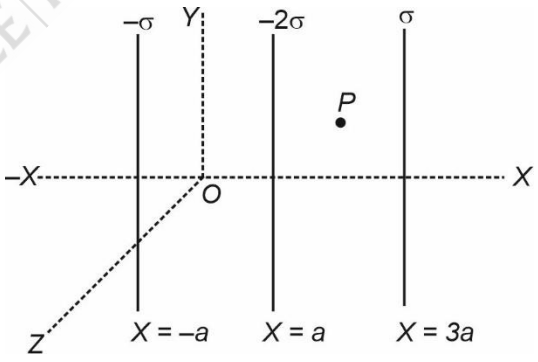
59. A particle is doing simple harmonic motion of amplitude 0.06 m and time period 3.14 s. The maximum velocity of the particle is _____ cm/s.

Answer (12)

Sol. $v_{max} = A\omega = \frac{A \cdot 2\pi}{T} = 0.06 \times \frac{2\pi}{3.14} = 0.12$ m/s

$\Rightarrow$ $v_{max} = 12$ cm/s

60. Three infinitely long charged thin sheets are placed as shown in figure. The magnitude of electric field at the point P is $\frac{x\sigma}{\epsilon_0}$. The value of x is _____ (all quantities are measured in SI units).



Answer (2)

Sol. $E_P = \frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{2\sigma}{\epsilon_0}$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. The electron affinity value are negative for

A. $Be \rightarrow Be^-$

B. $N \rightarrow N^-$

C. $O \rightarrow O^{2-}$

D. $Na \rightarrow Na^-$

E. $Al \rightarrow Al^-$

Choose the most appropriate answer from the options given below :

(1) A and D only (2) D and E only

(3) A, B and C only (4) A, B, D and E only

Answer (2)

Sol. In process D and E, energy is released.

62. Match **List I** with **List II**.

	List I (Molecule/species)		List II (Property/Shape)
A.	SO_2Cl_2	I.	Paramagnetic
B.	NO	II.	Diamagnetic
C.	NO_2^-	III.	Tetrahedral
D.	I_3^-	IV.	Linear

Choose the correct answer from the options given below :

(1) A-III, B-IV, C-II, D-I (2) A-III, B-I, C-II, D-IV

(3) A-II, B-III, C-I, D-IV (4) A-IV, B-I, C-III, D-II

Answer (2)

Sol. A → Tetrahedral (III)

B → Paramagnetic (I)

C → Diamagnetic (II)

D → Linear (IV)

63. Which of the following statements are correct?

A. Glycerol is purified by vacuum distillation because it decomposes at its normal boiling point.

B. Aniline can be purified by steam distillation as aniline is miscible in water.

C. Ethanol can be separated from ethanol water mixture by azeotropic distillation because it forms azeotrope.

D. An organic compound is pure, if mixed M.P. is remained same.

Choose the **most appropriate** answer from the options given below :

(1) A, C, D only (2) A, B, C only

(3) A, B, D only (4) B, C, D only

Answer (1)

Sol. Statements A, C and D are correct.

64. Which of the following material is not a semiconductor?

(1) Germanium (2) Graphite

(3) Copper oxide (4) Silicon

Answer (2)

Sol. Graphite is not a semiconductor.

65. The number of element from the following that do not belong to lanthanoids is

Eu, Cm, Er, Tb, Yb and Lu

(1) 3 (2) 1

(3) 4 (4) 5

Answer (2)

Sol. Cm belongs to actinoid series.

66. Match **List I** with **List II**.

	List I (Compound/species)		List II (Shape/Geometry)
A.	SF_4	I.	Tetrahedral
B.	BrF_3	II.	Pyramidal
C.	BrO_3^-	III.	See saw
D.	NH_4^+	IV.	Bent T-shape

Choose the **correct** answer from the options given below :

(1) A-II, B-III, C-I, D-IV (2) A-II, B-IV, C-III, D-I

(3) A-III, B-II, C-IV, D-I (4) A-III, B-IV, C-II, D-I

Answer (4)

Sol. A → Sea-saw (III)

B → Bent T-shape (IV)

C → Pyramidal (II)

D → Tetrahedral (I)

67. Consider the following complexes

$[CoCl(NH_3)_5]^{2+}$ $[Co(CN)_6]^{3-}$

(A) (B)

$[Co(NH_3)_5(H_2O)]^{3+}$ $Cu(H_2O)_4]^{2+}$

(C) (D)

The correct order of A, B, C and D in terms of *wavenumber* of light absorbed is

(1) B < C < A < D (2) D < A < C < B

(3) A < C < B < D (4) C < D < A < B

Answer (2)

Sol. Wavenumber $= \frac{1}{\lambda} \propto$ Frequency $\propto \Delta_0$

Wavenumber order : D < A < C < B

68. At –20 °C and 1 atm pressure, a cylinder is filled with equal number of H_2, I_2 and HI molecules for the reaction

$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$, the K_p for the process is $x \times 10^{-1}$.

x = _____.

[Given : R = 0.082 L atm K^{-1} mol^{-1}]

(1) 0.01 (2) 2

(3) 1 (4) 10

Answer (4)

Sol. $K_P = K_C = \frac{[HI]^2}{[H_2][I_2]} = 1 = 10 \times 10^{-1}$

69. Match **List I** with **List II**.

	List I (Precipitating reagent and conditions)		List II (Cation)
A.	$NH_4Cl + NH_4OH$	I.	Mn^{2+}
B.	$NH_4OH + Na_2CO_3$	II.	Pb^{2+}
C.	$NH_4OH + NH_4Cl + H_2S$ gas	III.	Al^{3+}
D.	dilute HCl	IV.	Sr^{2+}

Choose the **correct** answer from the options given below :

(1) A-IV, B-III, C-II, D-I (2) A-IV, B-III, C-I, D-II

(3) A-III, B-IV, C-I, D-II (4) A-III, B-IV, C-II, D-I

Answer (3)

Sol. A → Al^{3+} (III)

B → Sr^{2+} (IV)

C → Mn^{2+} (I)

D → Pb^{2+} (II)

70. Match **List I** with **List II**.

	List I (Hybridization)		List II (Orientation in Space)
A.	sp^3	I.	Trigonal bipyramidal
B.	dsp^2	II.	Octahedral
C.	sp^3d	III.	Tetrahedral
D.	sp^3d^2	IV.	Square planar

Choose the **correct** answer from the options given below :

(1) A-II, B-I, C-IV, D-III

(2) A-III, B-IV, C-I, D-II

(3) A-IV, B-III, C-I, D-II

(4) A-III, B-I, C-IV, D-II

Answer (2)

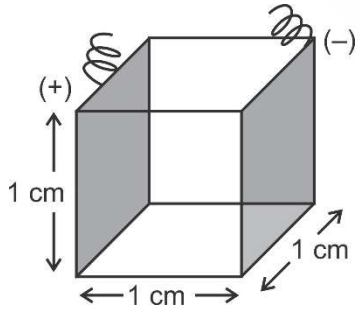
Sol. A $\rightarrow$ Tetrahedral (III)

B $\rightarrow$ Square planar (IV)

C $\rightarrow$ Trigonal bipyramidal (I)

D $\rightarrow$ Octahedral (II)

71. A conductivity cell with two electrodes (dark side) are half filled with infinitely dilute aqueous solution of a weak electrolyte. If volume is doubled by adding more water at constant temperature, the molar conductivity of the cell will



(1) remain same or cannot be measured accurately

(2) decrease sharply

(3) depend upon type of electrolyte

(4) increase sharply

Answer (1)

Sol. $\wedge_m = \frac{K \times 1000}{M}$

K will become half and M will also become half.

λ_m will remain same or can not be measure sharply.

For infinitely dilute solute $\wedge_m \approx \wedge_m^0$ = constant.

72. The density of 'x' M solution ('x' molar) of NaOH is 1.12 g mL^{-1}, while in molality, the concentration of the solution is 3 m (3 molal). Then x is

(Given : Molar mass of NaOH is 40 g/mol)

(1) 3.0

(2) 3.8

(3) 2.8

(4) 3.5

Answer (1)

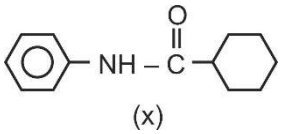
Sol. $3 = \frac{1000x}{1120 - 40x}$

$3360 - 120x = 1000x$

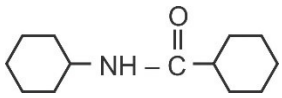
$3360 = 1120x$

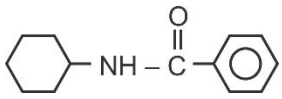
$x = 3$

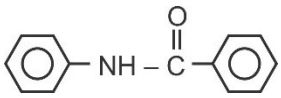
73. Which of the following is metamer of the given compound (X)?

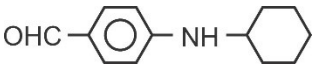


(x)

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. Compound given is option (2) is metamer of compound "X".

74. Given below are two statements :

Statement I : Picric acid is 2, 4, 6-trinitrotoluene

Statement II : Phenol-2,4-disulphonic acid is treated with Conc. HNO_3 to get picric acid.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

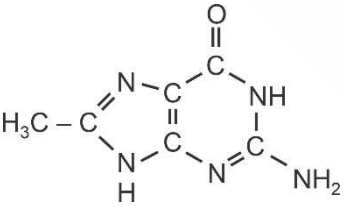
(4) Statement I is incorrect but Statement II is correct

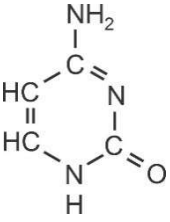
Answer (4)

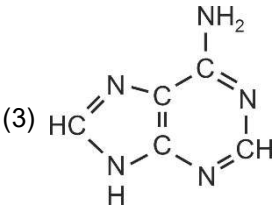
Sol. S-I is incorrect as picric acid is 2,4,6-trinitrophenol

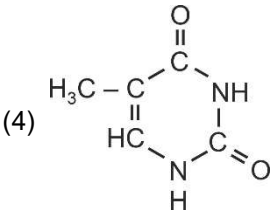
S-II is correct

75. DNA molecule contains 4 bases whose structure are shown below. One of the structures is not correct, identify the **incorrect** base structure.

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. Compound given in option (1) is not present in DNA.

76. Given below are two statements :

Statement I : Gallium is used in the manufacturing of thermometers.

Statement II : A thermometer containing gallium is useful for measuring the freezing point (256 K) of brine solution

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

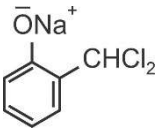
(4) Statement I is false but Statement II is true

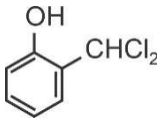
Answer (2)

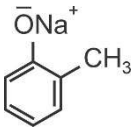
Sol. S-1 is correct as gallium is used in manufacturing of thermometers.

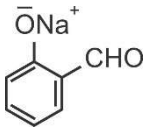
S-2 is incorrect

77. In Reimer-Tiemann reaction, phenol is converted into salicylaldehyde through an intermediate. The structure of intermediate is _____.

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. Carbene is intermediate.

Intermediate given in option (1) is produced.

78. Which among the following aldehydes is most reactive towards nucleophilic addition reactions?

(1) $CH_3 - \overset{\overset{O}{\|}}{C} - H$

(2) $H - \overset{\overset{O}{\|}}{C} - H$

(3) $C_3H_7 - \overset{\overset{O}{\|}}{C} - H$

(4) $C_2H_5 - \overset{\overset{O}{\|}}{C} - H$

Answer (2)

Sol. Formaldehyde is more reactive due to least hindrance.

79. Functional group present in sulphonic acids is

(1) $-SO_3H$

(2) $-SO_2$

(3) $-SO_4H$

(4) $-\overset{\overset{O}{\|}}{S} - OH$

Answer (1)

Sol. $-SO_3H$ is functional group in sulphonic acids.

80. Match **List I** with **List II**.

	List I (Compound)		**List II (Uses)**
A.	Iodoform	I.	Fire extinguisher
B.	Carbon tetrachloride	II.	Insecticide
C.	CFC	III.	Antiseptic
D.	DDT	IV.	Refrigerants

Choose the **correct** answer from the options given below :

(1) A-II, B-IV, C-I, D-III

(2) A-III, B-I, C-IV, D-II

(3) A-I, B-II, C-III, D-IV

(4) A-III, B-II, C-IV, D-I

Answer (2)

Sol. A $\rightarrow$ Antiseptic (III)

B $\rightarrow$ Fire extinguisher (I)

C $\rightarrow$ Refrigerants (IV)

D $\rightarrow$ Insecticide (II)

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

81. The difference in the 'spin-only' magnetic moment values of $KMnO_4$ and the manganese product formed during titration of $KMnO_4$ against oxalic acid in acidic medium is _____ BM. (nearest integer)

Answer (6)

Sol. Mn^{7+} ($\mu = 0$)

Mn^{2+} ($\mu = 5.91$)

Difference = 5.91

≈ 6 (Nearest integer)

82. Frequency of the de-Broglie wave of electron in Bohr's first orbit of hydrogen atom is _____ × 10^{13} Hz (nearest integer).

[Given : R_H (Rydberg constant) = 2.18×10^{-18} J, *h* (Plank's constant) = 6.6×10^{-34} J.s.]

Answer (661)

Sol. $\lambda = \frac{h}{mv}$

$\lambda \cdot v = \frac{h}{m}$

$\frac{\lambda \cdot v^2}{v} = \frac{h}{m}$

$\frac{v^2}{\text{Frequency}} = \frac{h}{m}$

$\text{Frequency} = \frac{mv^2}{h} = \frac{2R_H}{h}$

$= \frac{2 \times 2.18 \times 10^{-18}}{6.6 \times 10^{-34}}$

$= 660.6 \times 10^{13}$ Hz

Nearest integer = 661

If we take h = 6.626×10^{-34}

Then

$\text{Frequency} = \frac{2R_H}{h}$

$= 658.01 \times 10^{13}$ Hz

But if value of h is given as 6.6×10^{-34} correct answer will be 661.

83. Time required for 99.9% completion of a first order reaction is _____ times the time required for completion of 90% reaction. (nearest integer)

Given 3

Answer (3)

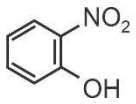
Sol. $\frac{t_{99.9}}{t_{90}} = 3$

For first order reaction

$t_{99.9} = \frac{2.303}{k} \log 1000$

$t_{90} = \frac{2.303}{k} \log 10$

84. Number of molecules from the following which can exhibit hydrogen bonding is _____, (nearest integer)

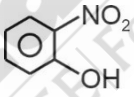
CH_3OH, H_2O, C_2H_6, C_6H_6,  , HF, NH_3

Answer (5)

Sol. compounds

CH_3OH

H_2O



HF

NH_3

Can show H-Bonding.

85. Consider the dissociation of the weak acid HX as given below

$HX(aq) \rightleftharpoons H^+(aq) + X^-(aq)$, $Ka = 1.2 \times 10^{-5}$

[K_a : dissociation constant]

The osmotic pressure of 0.03 M aqueous solution of HX at 300 K is _____ × 10^{-2} bar (nearest integer).

[Given : R = 0.083 L bar mol^{-1} K^{-1}]

Answer (76)

Sol. $Ka = \frac{C\alpha^2}{1-\alpha}$

$1.2 \times 10^{-5} = (0.03)(\alpha^2)$

$\alpha = 0.02$

$\pi = iCRT$

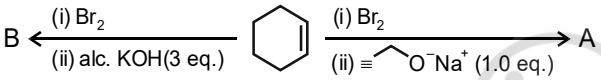
$= (1.02)(0.03)(0.083)(300)$

$= 0.76194$

$= 76.194 \times 10^{-2}$ bar

Nearest integer = 76

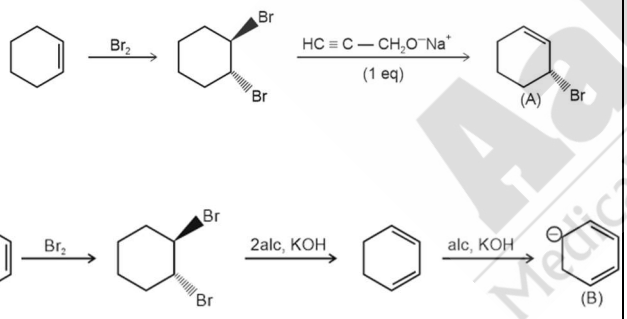
86. The major products from the following reaction sequence are product A and product B.



The total sum of π electrons in product A and product B are _____ (nearest integer)

Answer (8)

Sol.



Number of π electron in A = 2

Number of π electron in B = 6

Total Number of π electron in (A) and (B) = 8

87. 9.3 g of pure aniline upon diazotisation followed by coupling with phenol gives an orange dye. The mass of orange dye produced (assume 100% yield/conversion) is _____ g. (nearest integer)

Answer (20)

Sol. Moles of aniline = 0.1

Moles of dye = 0.1

Mass of dye = 0.1 × 198 = 19.8 gm

≈ 20

88. Among CrO, Cr_2O_3 and CrO_3, the sum of spin-only magnetic moment values of basic and amphoteric oxides is _____ 10^{-2} BM (nearest integer).

(Given atomic number of Cr is 24)

Answer (877)

Sol. CrO is Basic

Cr^{2+} : μ = 4.89 BM

Cr_2O_3 is amphoteric

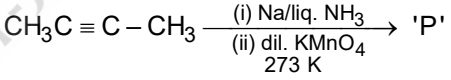
Cr^{3+} μ = 3. 87 BM

Sum = 8.771

$= 877.1 \times 10^{-2}$ BM

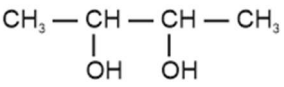
Nearest integer = 877

89. The major product of the following reaction is P.



Number of oxygen atoms present in product 'P' is _____. (nearest integer)

Answer (2)

Sol. Product is 

Number of oxygen atoms = 2

90. An ideal gas, $\bar{C}_V = \frac{5}{2}R$, is expanded adiabatically against a constant pressure of 1 atm until it doubles in volume. If the initial temperature and pressure is 298 K and 5 atm, respectively then the final temperature is _____ K (nearest integer).

[$\bar{C}_V$ is the molar heat capacity at constant volume]

Answer (274)

Sol. $-1(2V_1 - V_1) = n \times \frac{5R}{2}(T_2 - T_1)$

$-V_1 = \frac{5}{2}(nRT_2 - 5V_1)$

$-V_1 = 2.5(nRT_2) - 12.5V_1$

$11.5\ V_1 = 2.5\ (nRT_2)$

$11.5 \times \frac{nRT_1}{P_1} = 2.5 \times (nRT_2)$

$T_2 = 274.16$ k

≈ 274 (Nearest integer)

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