Aakash

Medical | IIT-JEE | Foundations

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Answers & Solutions

for

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

Time : 3 hrs.   M.M. : 300

IMPORTANT INSTRUCTIONS:

(1)   The test is of **3 hours** duration.

(2)   This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

(3)   This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B**.

(4)   **Section - A :** Attempt all questions.

(5)   **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6)   **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7)   **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let $\displaystyle\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$. Then e^{α} and $e^{-\alpha}$ are the roots of the equation:

   (1) $x^2 + 2x - 8 = 0$ (2) $2x^2 - 5x + 2 = 0$

   (3) $2x^2 - 5x - 2 = 0$ (4) $x^2 - 2x - 8 = 0$

Answer (2)

Sol. Let $\sqrt{e^x - 1} = t$

$e^x - 1 = t^2$

$e^x = 1 + t^2$

$e^x = 0 + 2t \cdot \dfrac{dt}{dx}$

$\dfrac{dt}{dx} = \dfrac{e^x}{2t} = \dfrac{t^2 + 1}{2t}$

$I = \int \dfrac{2t}{t(1 + t^2)} dt = 2\tan^{-1} t$

$\Rightarrow I = \displaystyle\int_{\alpha}^{\log 4} \frac{dx}{\sqrt{e^x - 1}}$

$I = 2\tan^{-1}\sqrt{e^x - 1}\Big|_{\alpha}^{\log 4}$

$= 2\left(\tan^{-1}\sqrt{3} - \tan^{-1}\sqrt{e^{\alpha} - 1}\right) = \dfrac{\pi}{6}$

$\Rightarrow \dfrac{\pi}{3} - \tan^{-1}(\sqrt{e^{\alpha} - 1}) = \dfrac{\pi}{12}$

$\tan^{-1}(\sqrt{e^{\alpha} - 1}) = \dfrac{\pi}{4}$

$\Rightarrow e^{\alpha} - 1 = 1$

$\boxed{e^{\alpha} = 2} \Rightarrow \boxed{e^{-\alpha} = \dfrac{1}{2}}$

$\therefore$ Quadratic equation whose roots are e^{α} & $e^{-\alpha}$ is

$x^2 - (e^{\alpha} + e^{-\alpha})x + e^{\alpha} \times e^{-\alpha} = 0$

$x^2 - \left(2 + \dfrac{1}{2}\right)x + 1 = 0$

$\boxed{2x^2 - 5x + 2 = 0}$

2. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b} + \vec{c}$. If $\vec{r} \cdot \vec{a} = 3$, then 3λ is equal to:

   (1) 21   (2) 30

   (3) 27   (4) 25

Answer (4)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{c} = 3\hat{i} - \hat{j} + \lambda\hat{k}$

$\vec{b} + \vec{c} = 5\hat{i} + 2\hat{j} + (\lambda - 5)\hat{k}$

$\vec{r}$ is a unit vector along $\vec{b} + \vec{c}$

$\therefore \vec{r} = \dfrac{5\hat{i} + 2\hat{j} + (\lambda - 5)\hat{k}}{\sqrt{25 + 4 + (\lambda - 5)^2}}$

Now, $\vec{r} \cdot \vec{a} = 3$

$\dfrac{1}{\sqrt{29 + (\lambda - 5)^2}}[5 + 4 + 3(\lambda - 5)] = 3$

Squaring both sides

$\Rightarrow \dfrac{1}{29 + (\lambda - 5)^2}[9 + 3(\lambda - 5)^2] = 9$

$\Rightarrow [3 + (\lambda - 5)]^2 = 29 + (\lambda - 5)^2$

$\Rightarrow 9 + (\lambda - 5)^2 + 6(\lambda - 5) = 29 + (\lambda - 5)^2$

$\Rightarrow 9 + 6(\lambda - 5) = 29$

$\Rightarrow \lambda = \dfrac{20}{6} + 5 = \dfrac{25}{3}$

$\therefore 3\lambda = 3 \times \dfrac{25}{3} = 25$

3. In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\dfrac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4th, 6th and 8th terms is equal to:

(1) 84
(2) 78
(3) 91
(4) 96

Answer (3)

Sol. Let the G.P. be $\dfrac{a}{r^3}, \dfrac{a}{r^2}, \dfrac{a}{r}, a, ar, ar^2, ar^3, ar^4$

Now, $\dfrac{a}{r^2} + ar^2 = \dfrac{70}{3}$

$\dfrac{a}{r} \times ar = 49 \Rightarrow a^2 = 49 \Rightarrow a = 7$ (G.P. is increasing)

Now, $\dfrac{7}{r^2} + 7r^2 = \dfrac{70}{3}$

$3r^4 - 10r^2 + 3 = 0$

$(3r^2 - 1)(r^2 - 3) = 0$

$\Rightarrow r^2 = \dfrac{1}{3}$ or $r^2 = 3$

As it is increasing, $\therefore r^2 = 3 \Rightarrow r = \sqrt{3}$

Now, $a + ar^2 + ar^4 = 7 + 7(3) + 7(9)$

$= 7 + 21 + 63 = 91$

4. If the line segment joining the points (5, 2) and (2, a) subtends an angle $\dfrac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of a is:

(1) 8
(2) 6
(3) 2
(4) 4

Answer (4)

Sol. $m_1 = \dfrac{2}{5}$, $m_2 = \dfrac{a}{2}$

angle between two line

$\tan\theta = \left| \dfrac{m_1 - m_2}{1 + m_1 m_2} \right|$

$\left| \dfrac{\dfrac{2}{5} - \dfrac{a}{2}}{1 + \dfrac{2}{5} \times \dfrac{a}{2}} \right| = \tan\dfrac{\pi}{4} = 1$

$\left| \dfrac{\dfrac{2}{5} - \dfrac{a}{2}}{1 + \dfrac{a}{5}} \right| = 1$

$\left| \dfrac{4 - 5a}{10 + 2a} \right| = 1$

$\dfrac{4 - 5a}{10 + 2a} = \pm 1$

$\dfrac{4 - 5a}{10 + 2a} = 1$ or $\dfrac{4 - 5a}{10 + 2a} = -1$

$4 - 5a = 10 + 2a$ or $4 - 5a = -10 - 2a$

$a = -\dfrac{6}{7}$ or $a = \dfrac{14}{3}$

We have to take absolute value of a

$a_1 \times a_2 = \dfrac{6}{7} \times \dfrac{14}{3} = 4$

5. The area of the region in the first quadrant inside the circle $x^2 + y^2 = 8$ and outside the parabola $y^2 = 2x$ is equal to:

(1) $\dfrac{\pi}{2} - \dfrac{1}{3}$
(2) $\dfrac{\pi}{2} - \dfrac{2}{3}$

(3) $\pi - \dfrac{1}{3}$
(4) $\pi - \dfrac{2}{3}$

Answer (4)

Sol. We have, $x^2 + y^2 = 8$ and $y^2 = 2x$
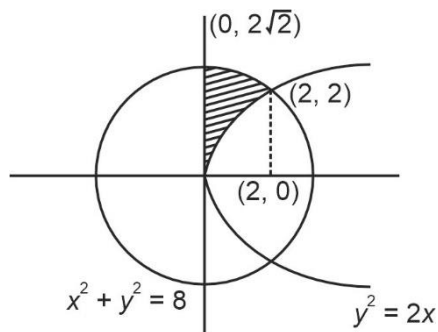
Area of shaded region

$$= \int_0^2 \frac{y^2}{2}\,dy + \int_2^{2\sqrt{2}} \sqrt{8 - y^2}\,dy$$

$$= \frac{1}{6}[y^3]_0^2 + \left[\frac{y}{2}\sqrt{8 - y^2} + 4\sin^{-1}\left(\frac{y}{2\sqrt{2}}\right)\right]_2^{2\sqrt{2}}$$

$$= \frac{4}{3} + \pi - 2 = \pi - \frac{2}{3}$$

6. Let $A = \{2, 3, 6, 8, 9, 11\}$ and $B = \{1, 4, 5, 10, 15\}$. Let be a relation on $A \times B$ defined by $(a, b)R(c, d)$ if and only if $3ad - 7bc$ is an even integer. Then the relation R is

   (1) Reflexive but not symmetric

   (2) Transitive but not symmetric

   (3) Reflexive and symmetric but not transitive

   (4) An equivalence relation

Answer (3)

Sol. $(a, b)\ R\ (c, d) \Rightarrow 3ad - 7bc \in$ even

For reflexive

$(a, b)\ R\ (a, b) \Rightarrow 3ab - 7ba = -4ab \in$ even

For symmetric

$(a, b)\ R\ (c, d)$

$= 3ad - 7bc = \underbrace{(7ad - 3bc)}_{\text{even}} + (-4ad - 4bc)$

then

$(c, d)\ R\ (a, b) = 3bc - 7ad$

$\in$ even

Now check for transitive

$(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$ then $(a, b)\ R\ (e, f)$

$3ad - 7bc = 2m \qquad \Rightarrow (2, 5)\ R\ (6, 8)$ and

$3cf - 7ed = 2n \qquad (6, 8)\ R\ (9, 4)$

then $3af - 7eb \ne$ even $\notin (2, 5)\ R\ (9, 4)$

$\Rightarrow$ Not transitive option (3)

7. For $a, b > 0$, let

$$f(x) = \begin{cases} \dfrac{\tan((a+1)x) + b\tan x}{x}, & x < 0 \\ 3 & x = 0 \\ \dfrac{\sqrt{ax + b^2 x^2} - \sqrt{ax}}{b\sqrt{ax}\sqrt{x}}, & x > 0 \end{cases}$$

be a continuous function at $x = 0$. Then $\dfrac{b}{a}$ is equal to

   (1) 4   (2) 8

   (3) 6   (4) 5

Answer (3)

Sol. $f(x) = \begin{cases} \dfrac{\tan((a+1)x) + b\tan x}{x}, & x < 0 \\ 3 & x = 0 \\ \dfrac{\sqrt{ax + b^2 x^2} - \sqrt{ax}}{b\sqrt{ax}\sqrt{x}}, & x > 0 \end{cases}$

$f(x)$ is continuous at $x = 0$

$\Rightarrow \lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$

$\lim_{x \to 0^-} f(x) = 3$

$\Rightarrow \lim_{x \to 0^-} \dfrac{\tan((a+1)x) + b + ax}{x} = 3$

$\Rightarrow a + 1 + b = 3$

$\Rightarrow a + b = 2 \qquad \dots (1)$

also, $\lim_{x \to 0^+} \dfrac{\sqrt{ax + b^2 x^2} - \sqrt{ax}}{b\sqrt{ax}\sqrt{x}} = 3$

$$= \lim_{x \to 0^+} \frac{\sqrt{ah + b^2 h^2} - \sqrt{ah}}{b\sqrt{a} \times h\sqrt{h}} = 3$$

$$= \lim_{h \to 0} \frac{\sqrt{a + b^2 h^2} - \sqrt{a}}{b\sqrt{a}h} \times \frac{\sqrt{a + b^2 h} + \sqrt{a}}{\sqrt{a + b^2 h} + \sqrt{a}} = 3$$

$$= \lim_{h \to 0} \frac{a + b^2 h - a}{b\sqrt{a}h\left(\sqrt{a + b^2 h} + \sqrt{a}\right)} = 3$$

$$\Rightarrow \frac{b^2}{b\sqrt{a}\left(2\sqrt{a}\right)} = 3$$

$$\Rightarrow \frac{b}{2a} = 3$$

$$\Rightarrow \frac{b}{a} = 6 \qquad\qquad \dots(2)$$

8. If the system of equations $x + 4y - z = \lambda$,

$7x + 9y + \mu z = -3$, $5x + y + 2z = -1$ has infinitely many solutions, then $(2\mu + 3\lambda)$ is equal to:

(1) 3
(2) −2
(3) −3
(4) 2

Answer (3)

Sol. $x + 4y - z = \lambda$

$7x + 9y + \mu z = -3$

$5x + y + 2z = -1$

$$\begin{bmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \lambda \\ -3 \\ -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} \lambda \\ -3 \\ -1 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$= \frac{adjA}{|A|} B$$

If $|A| = 0$ and $(adjA) \cdot B = 0$, system has infinitely many solutions.

$|A| = 18 - \mu - 4(14 - 5\mu) - 1(7 - 45) = 0$

$\Rightarrow 18 - \mu - 56 + 20\mu + 38 = 0$

$\Rightarrow 19\mu = 0$

$\Rightarrow \mu = 0$

Also $adjA = \begin{bmatrix} 18 & -9 & 9 \\ -14 & 7 & -7 \\ -38 & 19 & -19 \end{bmatrix}$

$(adjA) \cdot B = 0$

$$\begin{bmatrix} 18 & -9 & 9 \\ -14 & 7 & -7 \\ -38 & 19 & -19 \end{bmatrix} \begin{bmatrix} \lambda \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$18\lambda + 27 - 9 = 0$

$\Rightarrow 18\lambda = -18$

$\Rightarrow \boxed{\lambda = -1}$

$\Rightarrow 2\mu + 3\lambda = 3(-1) = -3$

9. The sum of all possible values of $\theta \in [-\pi, 2\pi]$, for which $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is purely imaginary is equal to:

(1) 3π
(2) 2π
(3) 5π
(4) 4π

Answer (1)

Sol. $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is purely imaginary

$$n = \frac{1 + i\cos\theta}{1 - 2i\cos\theta} \times \frac{1 + 2i\cos\theta}{1 + 2i\cos\theta} = \frac{1 + 3i\cos\theta - 2\cos^2\theta}{1 + 4\cos^2\theta}$$

$$n = \frac{1 - 2\cos^2\theta}{1 + 4\cos^2\theta} + i\left(\frac{3\cos\theta}{1 + 4\cos^2\theta}\right)$$

n is purely imaginary

$$\Rightarrow \frac{1 - 2\cos^2\theta}{1 + 4\cos^2\theta} = 0$$

$$\Rightarrow \cos^2\theta = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$$

θ can be $\frac{\pi}{4}, \frac{-\pi}{4}, \frac{3\pi}{4}, \frac{-3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Sum of all possible values of $\theta = 3\pi$

10. Let $f(x) = \begin{cases} -a & \text{if } -a \le x \le 0 \\ x+a & \text{if } 0 < x \le a \end{cases}$ where $a > 0$ and

$g(x) = \dfrac{(f(|x|) - |f(x)|)}{2}$. Then the function

$g : [-a, a] \to [-a, a]$ is

(1) Onto

(2) Both one-one and onto

(3) Neither one-one nor onto

(4) One-one

Answer (3)

Sol. $f(x) = \begin{cases} -a & \text{if } -a \le x \le 0 \\ x+a & \text{if } 0 < x \le a \end{cases}$
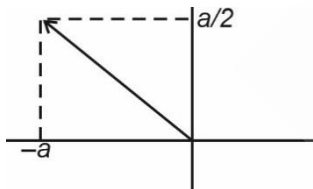
$f(|x|) = \begin{cases} -a & -a \le |x| \le 0 \\ |x|+a & \text{if } 0 < |x| \le a \end{cases}$

$|x| < 0$ is not possible, so

$f(|x|) = \begin{cases} x+a & -a \le x \le 0 \\ -x+a & 0 < x \le a \end{cases}$

$|f(x)| = \begin{cases} a & -a \le x \le 0 \\ x+a & 0 < x \le a \end{cases}$

$h(x) = \begin{cases} -\dfrac{x}{2} & -a \le x \le 0 \\ 0 & 0 < x \le a \end{cases}$

Neither one-one nor onto

11. Let $y = y(x)$ be the solution curve of the differential

equation $\sec y \dfrac{dy}{dx} + 2x \sin y = x^3 \cos y$, $y(1) = 0$.

Then $y(\sqrt{3})$ is equal to:

(1) $\dfrac{\pi}{12}$ (2) $\dfrac{\pi}{3}$

(3) $\dfrac{\pi}{4}$ (4) $\dfrac{\pi}{6}$

Answer (3)

Sol. $\sec y \dfrac{dy}{dx} + 2x \sin y = x^3 \cos y$

$\Rightarrow \sec^2 y \dfrac{dy}{dx} + 2x \tan y = x^3$

Let $z = \tan y$

$\dfrac{dz}{dx} = \sec^2 y \dfrac{dy}{dx}$

$\Rightarrow \dfrac{dz}{dx} + 2xz = x^3$

I.F. $= e^{x^2}$

$\Rightarrow z \cdot e^{x^2} = \int e^{x^2} \cdot x^3 \, dx + c$

$\Rightarrow \tan y \cdot e^{x^2} = \dfrac{1}{2}\left(x^2 e^{x^2} - e^{x^2}\right) + c$

$\Rightarrow \tan(0) \cdot e = \dfrac{1}{2}(1 \cdot e - e) + c$

$\Rightarrow c = 0$

$\Rightarrow \tan y = \dfrac{x^2 - 1}{2}$

$f(x) = \tan^{-1}\left(\dfrac{x^2 - 1}{2}\right) \Rightarrow f(\sqrt{3}) = \dfrac{\pi}{4}$

12. There are three bags X, Y and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability, that it came from bag Y, is:

(1) $\dfrac{1}{3}$ (2) $\dfrac{1}{4}$

(3) $\dfrac{1}{2}$ (4) $\dfrac{5}{12}$

Answer (1)

Sol. By Baye's theorem,

Probability (coin drawn from bag y)

$$= \frac{\dfrac{1}{3} \cdot \dfrac{4}{9}}{\dfrac{1}{3} \cdot \dfrac{5}{9} + \dfrac{1}{3} \cdot \dfrac{4}{9} + \dfrac{1}{3} \cdot \dfrac{3}{9}} = \frac{1}{3}$$

13. If the shortest distance between the lines $\dfrac{x-\lambda}{2} = \dfrac{y-4}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-2}{4} = \dfrac{y-4}{6} = \dfrac{z-7}{8}$ is $\dfrac{13}{\sqrt{29}}$, then a value of λ is:

(1) $-\dfrac{13}{25}$ (2) 1

(3) $\dfrac{13}{25}$ (4) -1

Answer (2)

Sol. $\vec{l}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{l}_2 = 4\hat{i} + 6\hat{j} + 8\hat{k}$
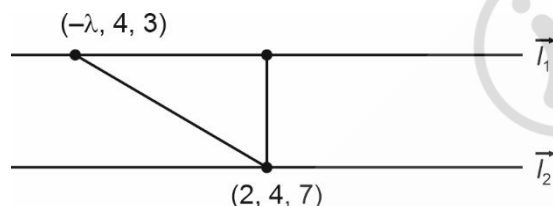
$$S.D. = \frac{\left| (2\hat{i} + 3\hat{j} + 4\hat{k}) \times ((\lambda - 2)\hat{i} - 4\hat{k}) \right|}{\left| 2\hat{i} + 3\hat{j} + 4\hat{k} \right|}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ \lambda - 2 & 0 & -4 \end{vmatrix} = -12\hat{i} - \hat{j}(-8 - 4\lambda + 8) + \hat{k}(6 - 3\lambda)$$

$$= -12\hat{i} + 4\lambda\hat{j} + (6 - 3\lambda)\hat{k}$$

$$\frac{\sqrt{144 + 16\lambda^2 + (6 - 3\lambda)^2}}{\sqrt{29}} = \frac{13}{\sqrt{29}}$$

$144 + 16\lambda^2 + (6 - 3\lambda)^2 = 169$

$\Rightarrow 16\lambda^2 + 9\lambda^2 + 36 - 36\lambda + 144 - 169 = 0$

$\Rightarrow 25\lambda^2 - 36\lambda + 11 = 0$

$\Rightarrow 25\lambda^2 - 25\lambda - 11\lambda + 11 = 0$

$\Rightarrow (25\lambda - 11)(\lambda - 1) = 0$

$\Rightarrow \lambda = 1, \dfrac{11}{25}$

14. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to:

(1) 179 (2) 181

(3) 175 (4) 177

Answer (1)

Sol. 2M

2A

2T

H, E, I, C, S

Case-I

2 Alike   2 Alike   1 Diff

$^3C_2 \times {}^6C_1 = 18$

Case-II

2 Alike   +   3 Diff

$^3C_1 \times {}^7C_3 = 105$

Case-III

All different

$^8C_5 = 56$

Total ways = 179

15. Let $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 11\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + \vec{b}) \times \vec{c} = \vec{c} \times (-2\vec{a} + 3\vec{b})$. If $(2\vec{a} + 3\vec{b}) \cdot \vec{c} = 1670$, then $|\vec{c}|^2$ is equal to:

(1) 1627 (2) 1618

(3) 1600 (4) 1609

Answer (2)

Sol. $\begin{aligned}\vec{a} &= 4\hat{i} - \hat{j} + \hat{k} \\ \vec{b} &= 11\hat{i} - \hat{j} + \hat{k}\end{aligned}\bigg]$ $\vec{a} \cdot \vec{b} = 44 + 1 + 1 = 46$

$|\vec{a}|^2 = 18, \ |\vec{b}^2| = 123$

$(\vec{a} + \vec{b}) \times \vec{c} = \vec{c} \times (-2\vec{a} + 3\vec{b})$

$(2\vec{a} + 3\vec{b}) \cdot \vec{c} = 1670$

$(\vec{a} + \vec{b}) \times c = (2\vec{a} - 3\vec{b}) \times \vec{c}$

$(-\vec{a} + 4\vec{b}) \times \vec{c} = 0$

$\vec{c} = \lambda(4\vec{b} - \vec{a})$

$(2\vec{a} + 3\vec{b}) \cdot \lambda(4\vec{b} - \vec{a}) = 1670$

$\lambda\left(5\vec{a} \cdot \vec{b} - 2|\vec{a}|^2 + 12|\vec{b}|^2\right) = 1670$

$\lambda = \dfrac{1670}{5 \times 46 - 2 \times 18 + 12 \times 123}$

$\lambda = 1$

$\vec{c} = 4\vec{b} - \vec{a}$

$= 4(11\hat{i} - \hat{j} + \hat{k}) - (4\hat{i} - \hat{j} + \hat{k})$

$= 40\hat{i} - 3\hat{j} + 3\hat{k}$

$|\vec{c}^2| = 1600 + 9 + 9$

$= 1618$

16. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, $a > 0$ has a local maximum at $x = \alpha$ and a local minimum at $x = \alpha^2$, then α and α^2 are the roots of the equation:

(1) $x^2 - 6x + 8 = 0$ (2) $8x^2 + 6x - 1 = 0$

(3) $x^2 + 6x + 8 = 0$ (4) $8x^2 - 6x + 1 = 0$

Answer (1)

Sol. $f(x) = 6x^2 - 18ax + 12a^2$

$= 6(x^2 - 3a + 2a^2)$

$= 6(x - a)(x - 2a) = 0$

$x = a, 2a$

$a = \alpha, \qquad 2a = \alpha^2 \qquad \Rightarrow \ \alpha = 0, 2$

$a > 0 \qquad \therefore \quad \alpha = 2$

$\qquad \qquad \qquad \alpha^2 = 4$

$\therefore \ x^2 - 6x + 8 = 0$ is the required quadratic equation.

17. If the image of the point $(-4, 5)$ in the line $x + 2y = 2$ lie on the circle $(x + 4)^2 + (y - 3)^2 = r^2$, then r is equal to:

(1) 4 (2) 1

(3) 2 (4) 3

Answer (4)

Sol. $\dfrac{x + 4}{1} = \dfrac{y - 5}{2} = \dfrac{-2(4)}{5}$

$\Rightarrow \ x = -4 - \dfrac{8}{5} = -\dfrac{28}{5}, \ y = 5 - \dfrac{16}{5} = \dfrac{9}{5}$

$\therefore$ Image is $\left(\dfrac{-28}{5}, \dfrac{9}{5}\right)$

Image lies on circle $(x + 4)^2 + (y - 3)^2 = r^2$

$\left(\dfrac{-28}{5} + 4\right)^2 + \left(\dfrac{9}{5} - 3\right)^2 = r^2$

$\Rightarrow \ \dfrac{64}{25} + \dfrac{36}{25} = r^2$

$\Rightarrow \ r = 2$

18. If $\alpha \neq a$, $\beta \neq b$, $\gamma \neq c$ and $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$, then

$\dfrac{a}{\alpha - a} + \dfrac{b}{\beta - b} + \dfrac{\gamma}{\gamma - c}$ is equal to:

(1) 2 (2) 3

(3) 1 (4) 0

Answer (4)

Sol. $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$

$R_1 \rightarrow R_1 - R_2,\ R_2 \rightarrow R_2 - R_3$

$$\Rightarrow \begin{vmatrix} \alpha - a & b - \beta & 0 \\ 0 & \beta - b & c - \gamma \\ a & b & \gamma \end{vmatrix} = 0$$

Take α-a, β-b, γ-c common from column-1, 2 and 3 respectively

$$(\alpha - a)(\beta - b)(\gamma - c)\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \dfrac{a}{\alpha - a} & \dfrac{b}{\beta - b} & \dfrac{\gamma}{\gamma - c} \end{vmatrix} = 0$$

$$\Rightarrow \quad \frac{\gamma}{\gamma - c} + \frac{b}{\beta - b} + \frac{a}{\alpha - a} = 0$$

19. If the term independent of x in the expansion of $\left(\sqrt{a}\, x^2 + \dfrac{1}{2x^3}\right)^{10}$ is 105, then a^2 is equal to :

(1) 2   (2) 4

(3) 6   (4) 9

Answer (2)

Sol. $\left(\sqrt{a}x^2 + \dfrac{1}{2x^3}\right)^{10}$

$$T_{r+1} = {}^{10}C_r \left(\sqrt{a}x^2\right)^{10-r}\left(\frac{1}{2x^3}\right)^r$$

Independent of $x \Rightarrow 20 - 2r - 3r = 0$

$r = 4$

Independent of x is ${}^{10}C_4 \left(\sqrt{a}\right)^6 \left(\dfrac{1}{2}\right)^4 = 105$

$$\frac{210}{2 \times 8} a^3 = 105$$

$\Rightarrow \quad a = 2$

$a^2 = 4$

20. If the value of $\dfrac{3\cos 36° + 5\sin 18°}{5\cos 36° - 3\sin 18°}$ is $\dfrac{a\sqrt{5} - b}{c}$, where a, b, c are natural numbers and

$gcd\ (a, c) = 1$, then $a + b + c$ is equal to :

(1) 40   (2) 52

(3) 50   (4) 54

Answer (2)

Sol. $\ell = \dfrac{3\cos 36° + 5\sin 18°}{5\cos 36° - 3\sin 18°}$

$\because \quad \sin 18° = \dfrac{\sqrt{5} - 1}{4}$

$\cos 36° = \dfrac{1 + \sqrt{5}}{4}$

$\therefore \quad \ell = \dfrac{3\left[\dfrac{1 + \sqrt{5}}{4}\right] + 5\left[\dfrac{\sqrt{5} - 1}{4}\right]}{5\left[\dfrac{1 + \sqrt{5}}{4}\right] - 3\left[\dfrac{\sqrt{5} - 1}{4}\right]}$

$= \dfrac{3 + 3\sqrt{5} + 5\sqrt{5} - 5}{3 + 5\sqrt{5} - 3\sqrt{5} + 3} = \dfrac{8\sqrt{5} - 2}{2\sqrt{5} + 8} = \dfrac{(4\sqrt{5} - 1)(4 - \sqrt{5})}{(\sqrt{5} + 4)(4 - \sqrt{5})}$

$\Rightarrow \quad \dfrac{16\sqrt{5} - 20 - 4 + \sqrt{5}}{16 - 5} = \dfrac{17\sqrt{5} - 24}{11}$

$\therefore \quad a = 17,\ b = 24,\ c = 11$

$\therefore \quad a + b + c = 17 + 24 + 11$

$= 52$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $\alpha|x| = |y|e^{xy - \beta}$, α, $\beta \in N$ be the solution of the differential equation $xdy - ydx + xy(xdy + ydx) = 0$, $y(1) = 2$. Then $\alpha + \beta$ is equal to _____

Answer (4)

Sol. $\alpha|x| = |y|e^{xy - \beta}$

$$\frac{xdy - ydx}{y^2} + \frac{xy(xdy + ydx)}{y^2} = 0$$

$$-d\left(\frac{x}{y}\right) + \frac{x}{y}d(xy) = 0$$

$$\int d(xy) = \int \frac{d\left(\frac{x}{y}\right)}{\frac{x}{y}}$$

$$xy = \ln\left|\frac{x}{y}\right| + \ln c$$

$$xy = \ln\left(\left|\frac{x}{y}\right| \cdot c\right)$$

$$\because \quad y(1) = 2$$

$$2 = \ln\left|\frac{1}{2}\right|c \implies c = 2e^2$$

$$\therefore \quad \text{solution } xy = \ln\left(\left|\frac{x}{y}\right| \cdot 2e^2\right)$$

$$e^{xy} = \frac{|x|}{|y|} \cdot 2e^2$$

$$2|x| = |y|e^{xy-2}$$

$$\implies \quad \alpha = 2, \beta = 2, \alpha + \beta = 4$$

22. Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x-axis. Let C be the circle with its centre at $A(\sqrt{6}, \sqrt{5})$ and passing through the point S. If O is the origin and SAB is a diameter of C, then the square of the area of the triangle OSB is equal to _____

Answer (40)

Sol. $\frac{x^2}{3} - \frac{y^2}{5} = 1$

$$5 = 3(e^2 - 1) \implies e = \sqrt{\frac{8}{3}}$$

$$S \equiv \left(2\sqrt{2}, 0\right)$$
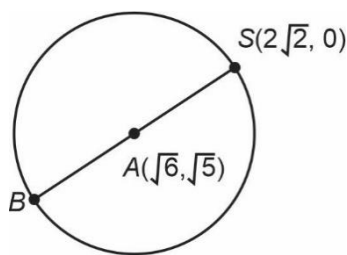
A is mid-point of BS

$$\implies \quad B\left(2\sqrt{6} - 2\sqrt{2}, \ 2\sqrt{5}\right)$$

$$\Delta(OSB) = \frac{1}{2}\left\|\begin{array}{ccc} 0 & 0 & 1 \\ 2\sqrt{2} & 0 & 1 \\ 2\sqrt{6} - 2\sqrt{2} & 2\sqrt{5} & 1 \end{array}\right\| = 2\sqrt{10}$$

$$(\Delta(OSB))^2 = 40$$

23. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(1, 6, 4)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to _____

Answer (11)

Sol.
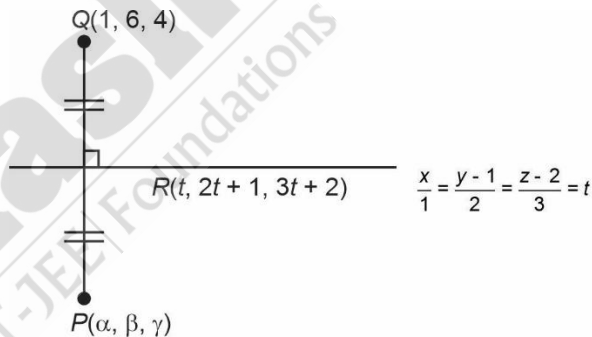
$$\overrightarrow{QR} \cdot \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 0$$

$$(t-1) + (2t-5) \times 2 + (3t-2) \times 3 = 0 \implies t = \frac{17}{14}$$

$$\implies R \equiv \left(\frac{17}{14}, \frac{48}{14}, \frac{79}{14}\right)$$

$$\implies \frac{\alpha+1}{2} = \frac{17}{14}, \frac{\beta+6}{2} = \frac{48}{14}, \frac{\gamma+4}{2} = \frac{79}{14}$$

$$2\alpha + \beta + \gamma = \frac{68}{14} - 2 + \frac{96}{14} - 6 + \frac{158}{14} - 4 = 11$$

24. The number of distinct real roots of the equation $|x+1||x+3| - 4|x+2| + 5 = 0$, is _____

Answer (2)

Sol. $|x+1||x+3| - 4|x+2| + 5 = 0$
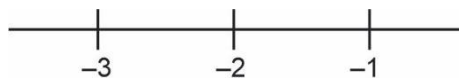
(I) If $x < -3$

$x^2 + 4x + 3 + 4x + 8 + 5 = 0$

$x^2 + 8x + 16 = 0 \Rightarrow x = -4 \Rightarrow$ one solution

(II) If $-3 \le x < -2$

$-x^2 - 4x - 3 + 4x + 8 + 5 = 0$

$x^2 - 10 = 0 \Rightarrow x = \pm\sqrt{10} \Rightarrow$ do not satisfy $-3 \le x < -2$

(III) If $-2 \le x < -1$

$-x^2 - 4x - 3 - 4x - 8 + 5 = 0$

$x^2 - 8x + 6 = 0$

$(x + 4)^2 = 10 \Rightarrow x = -4 \pm\sqrt{10} \Rightarrow$ do not satisfy $-2 \le x < -1$

(IV) If $x \ge -1$

$x^2 + 4x + 3 - 4x - 8 + 5 = 0$

$x^2 = 0$

$x = 0$ (One solution)

$\Rightarrow$ The number of distinct real roots are two

25. An arithmetic progression is written in the following way

```
                2
          5           8
      11        14        17
  20        23        26        29
------------------------------------
- - - - - - - - - - - - - - - - - - -
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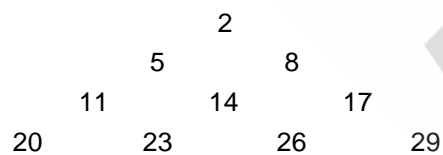
The sum of all the terms of the 10th row is_____.

Answer (1505)

Sol. First term is each row form pattern

2, 5, 11, 20

3   6   9 $\longrightarrow$ forms an A.P.
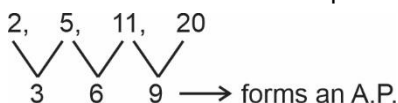
$\Rightarrow T_n = an^2 + bn + c$

$\Rightarrow T_1 = a + b + c = 2$

$\Rightarrow T_2 = 4a + 2b + c = 5$

$\Rightarrow T_3 = 9a + 3b + c = 11$

$\Rightarrow 3a + b = 3$

$5a + b = 6$

$\Rightarrow 2a = 3 \Rightarrow a = \dfrac{3}{2}, \quad b = \dfrac{-3}{2} \Rightarrow c = 2$

$\Rightarrow T_n = \dfrac{3}{2}n^2 - \dfrac{3(n)}{2} + 2 \Rightarrow \dfrac{3n^2 - 3n + 4}{2}$

$T_{10} = \dfrac{3 \times 100 - 3 \times 10 + 4}{2} = \dfrac{274}{2} = 137$

Terms in 10th row is 10 with 3 differences

$\Rightarrow 137, 140, 143\ldots$

$\Rightarrow S_{10} = \dfrac{10}{2}(2 \times 137 + (10 - 1) \times 3)$

$= 5(274 + 27) = 5 \times 301 = 1505$

26. If $\displaystyle\int \dfrac{1}{\sqrt[5]{(x-1)^4 (x+3)^6}} dx = A\left(\dfrac{\alpha x - 1}{\beta x + 3}\right)^{B} + C$, where C is the constant of integration, then the value of $\alpha + \beta + 20AB$ is _____.

Answer (7)

Sol. $I = \displaystyle\int \dfrac{1}{\sqrt[5]{(x-1)^4 (x+3)^6}} dx$

$\dfrac{x+3}{x-1} = t \Rightarrow dx = \dfrac{-4}{(t-1)^2}dt \Rightarrow x = \left(\dfrac{3+t}{t-1}\right)$

$\Rightarrow (x-1)^4(x+3)^6 = (x-1)^5(x+3)^5\left(\dfrac{x+3}{x-1}\right)$

$I = \displaystyle\int \dfrac{\dfrac{-4}{(t-1)^2}dt}{t^{1/5}\left(\dfrac{3+t}{t-1} - 1\right)\left(\dfrac{3+t}{t-3} + 3\right)}$

$I = \displaystyle\int \dfrac{-4\,dt}{t^{1/5}(16t)} = \dfrac{5}{4}\left(\dfrac{x-1}{x+3}\right)^{1/5} + c$

Comparing,

$$\Rightarrow A = \frac{5}{4}, \ B = \frac{1}{5}, \ \alpha = 1, \ \beta = 1$$

$$\Rightarrow \alpha + \beta + 20AB = 1 + 1 + 20 \times \frac{5}{4} \times \frac{1}{5} = 7$$

27. If $\alpha = \lim_{x \to 0^+} \left(\dfrac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}} \right)$ and

$\beta = \lim_{x \to 0} (1 + \sin x)^{\frac{1}{2}\cot x}$ are the roots of the

quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then

$12 \log_e(a + b)$ is equal to _____.

Answer (6)

Sol. $\alpha = \lim_{x \to 0^+} \dfrac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\left(\sqrt{\tan x} - \sqrt{x} \right)}$

$= \lim_{x \to 0} \dfrac{e^{\sqrt{x}}(e^{\sqrt{\tan x} - \sqrt{x}} - 1)}{\left(\sqrt{\tan x} - \sqrt{x} \right)} = 1$

$\beta = \lim_{x \to 0} (1 + \sin x)^{\frac{1}{2}\cot x} = \lim_{x \to 0} e^{(\sin x)\left(\frac{1}{2}\cot x\right)}$

$= \lim_{x \to 0} e^{\frac{1}{2}\cos x} = e^{1/2}$

Product of roots $= \sqrt{e} = \dfrac{-\sqrt{e}}{a} \Rightarrow a = -1$

Sum of roots $= \dfrac{-b}{a} = 1 + \sqrt{e}$

$= b = \sqrt{e} + 1$

$\Rightarrow 12\ln(a + b) = 12\ln\left(\sqrt{e} + 1 - 1\right) = 12\ln\left(e^{1/2}\right) = 6$

28. Let $a, b, c \in N$ and $a < b < c$. Let the mean, the mean deviation about the mean and the variance of the 5 observations 9, 25, a, b, c be 18, 4 and $\dfrac{136}{5}$, respectively. Then $2a + b - c$ is equal to _____

Answer (33)

Sol. $a, b, c \in N$

$a < b < c$

Mean = 18

$\dfrac{9 + 25 + a + b + c}{5} = 18$

$34 + a + b + c = 90$

$a + b + c = 56$

$\dfrac{|9 - 18| + |25 - 18| + |a - 18| + |b - 18| + |c - 18|}{5} = 4$

$9 + 7 + |a - 18| + |b - 18| + |c - 18| = 20$

$|a - 18| + |b - 18| + |c - 18| = 4$

$\dfrac{136}{5} = \dfrac{706 + a^2 + b^2 + c^2}{5} - (18)^2$

$\Rightarrow 136 = 706 + a^2 + b^2 + c^2 - 1620$

$\Rightarrow a^2 + b^2 + c^2 = 1050$

Consider $a < 19 < b < c$

Solving $a = 17$, $b = 19$, $c = 20$

$2a + b - c$

$34 + 19 - 20$

$= 33$

29. Let a ray of light passing through the point (3, 10) reflects on the line $2x + y = 6$ and the reflected ray passes through the point (7, 2). If the equation of the incident ray is $ax + by + 1 = 0$, then

$a^2 + b^2 + 3ab$ is equal to _____.

Answer (1)

Sol. Equation of incident ray : $ax + by + 1 = 0$

Using mirror image,

$$\frac{m-7}{2} = \frac{n-2}{1} = \frac{-2(14+2-6)}{5}$$

$$\frac{m-7}{2} = -4 \qquad \begin{array}{l} n-2 = -4 \\ n = -2 \end{array}$$

$$m = -8 + 7$$

$$m = -1$$
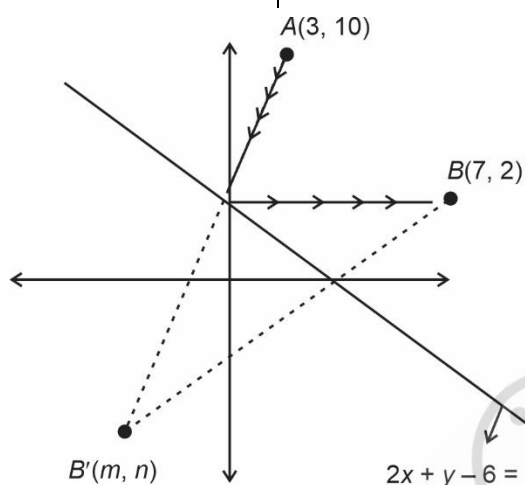
*Note: It can be observed from diagram A, P, B' are collinear.

Equation of Incident Ray,

Using two-point form,

$$(y-10) = \frac{10+2}{3+1}(x-3)$$

$$(y-10) = \frac{12}{4}(x-3)$$

$$y - 10 = +3(x-3)$$

$$y - 10 = +3x - 9$$

$$3x - y + 1 = 0$$

On comparing,

$$a = 3$$

$$b = -1$$

30. Let A be the region enclosed by the parabola $y^2 = 2x$ and the line $x = 24$. Then the maximum area of the rectangle inscribed in the region A is _____.

Answer (128)

Sol. $y^2 = 2x$

$$a = \left(\frac{1}{2}\right)$$
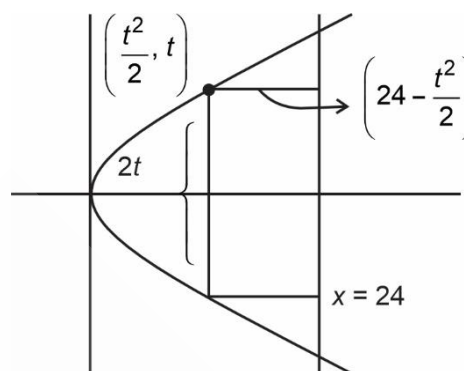
$$A(t) = 2t \times \left(24 - \frac{t^2}{2}\right)$$

$$A = 48t - t^3$$

$$\frac{dA}{dt} = 48 - 3t^2$$

$$48 - 3t^2 = 0$$

$$3t^2 = 48$$

$$t^2 = 16$$

$$t = \pm 4$$

$$\frac{d^2A}{dt^2} = 6t \quad \begin{array}{l} \text{at } 4 \rightarrow -24 \text{ \{Maxima\}} \\ \text{at } -4 \rightarrow 24 \text{ \{Minima\}} \end{array}$$

$$A(4) = 48 \times 4 - 4^3$$

$$= 192 - 64$$

$$A(4) = 128$$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. Least count of a vernier calliper is $\dfrac{1}{20N}$ cm. The value of one division on the main scale is 1 mm. Then the number of divisions of main scale that coincide with N divisions of vernier scale is

    (1) $\left(\dfrac{2N-1}{2}\right)$

    (2) $\left(\dfrac{2N-1}{20N}\right)$

    (3) $(2N-1)$

    (4) $\left(\dfrac{2N-1}{2N}\right)$

Answer (1)

Sol. Let m scale division of main scale coincides with N^{th} VSD.

$\Rightarrow \quad m\,\text{MSD} = N\,\text{V.S.D}$

$\dfrac{m}{N}\text{M.SD} = I\,\text{V.SD}$

L-C $= 1 - \dfrac{m}{N} = \dfrac{1}{20\,N} \times 10$

$1 - \dfrac{1}{2N} = \dfrac{m}{N}$

$\dfrac{2N-1}{2} = N$

32. A thin circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with angular velocity ω. If another disc of same dimensions but of mass $\dfrac{M}{2}$ is placed gently on the first disc co-axially, then the new angular velocity of the system is

    (1) $\dfrac{2}{3}\omega$

    (2) $\dfrac{3}{2}\omega$

    (3) $\dfrac{4}{5}\omega$

    (4) $\dfrac{5}{4}\omega$

Answer (1)

Sol. COAM

$\dfrac{MR^2}{2}\omega = \left(\dfrac{MR^2}{2} + \dfrac{M}{2}\dfrac{R^2}{2}\right)\omega'$

$\dfrac{\omega}{2} = \dfrac{3}{4}\omega'$

$\dfrac{2\omega}{3} = \omega'$

33. There are 100 divisions on the circular scale of a screw gauge of pitch 1 mm. With no measuring quantity in between the jaws, the zero of the circular scale lies 5 divisions below the reference line. The diameter of a wire is then measured using this screw gauge. It is found that 4 linear scale divisions are clearly visible while 60 divisions on circular scale coincide with the reference line. The diameter of the wire is

    (1) 4.65 mm

    (2) 4.55 mm

    (3) 3.35 mm

    (4) 4.60 mm

Answer (2)

Sol. 1 mm = 100 C.SD

$\Rightarrow$ 1 C.SD = $\dfrac{1}{100}$ mm

Zero error = $+5 \times \dfrac{1}{100}$ mm

Reading = 4 × 1 mm + 60 × $\dfrac{1}{100} - \dfrac{5}{100}$

= 4.55 mm

34. If ε_0 is the permittivity of free space and E is the electric field, then $\varepsilon_0 E^2$ has the dimensions

(1) $[M^0L^{-2}TA]$

(2) $[ML^2T^{-2}]$

(3) $[ML^{-1}T^{-2}]$

(4) $[M^{-1}L^{-3}T^4A^2]$

Answer (3)

Sol. $\varepsilon_0 E^2 \equiv$ Energy / Volume $\equiv \dfrac{ML^2T^{-2}}{L^3}$

= $ML^{-1}T^{-2}$

35. A coil of negligible resistance is connected in series with 90 Ω resistor across 120 V, 60 Hz supply. A voltmeter reads 36 V across resistance. Inductance of the coil is

(1) 0.286 H

(2) 0.91 H

(3) 2.86 H

(4) 0.76 H

Answer (4)

Sol. $i = \dfrac{V_R}{R} = \dfrac{36}{90} = \dfrac{2}{5}$ A

$V_L = \sqrt{120^2 - 36^2}$

$L\omega = \dfrac{V_L}{i} = \dfrac{12\sqrt{91}}{2/5}$

$L = \dfrac{30\sqrt{91}}{120\pi} = 0.76$

36. The position of the image formed by the combination of lenses is

f_1 = 10 cm f_2 = –10 cm f_3 = 30 cm

← 30 cm → ← 5 cm → ← 10 cm →
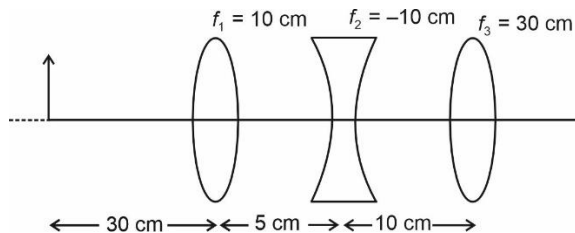
(1) 30 cm (right of third lens)

(2) 15 cm (left of second lens)

(3) 15 cm (right of second lens)

(4) 30 cm (left of third lens)

Answer (1)

Sol. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\dfrac{1}{v} - \dfrac{1}{-30} = \dfrac{1}{10}$

$\dfrac{1}{v} = \dfrac{1}{10} - \dfrac{1}{30} = \dfrac{3-1}{30} = \dfrac{1}{15}$

$v = 15$
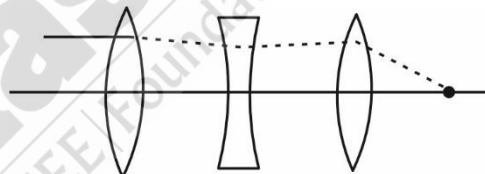
$\Rightarrow$ 30 cm right of third

37. A long straight wire of radius a carries a steady current I. The current is uniformly distributed across its cross section. The ratio of the magnetic field at $\dfrac{a}{2}$ and 2a from axis of the wire is

(1) 3 : 4

(2) 1 : 4

(3) 4 : 1

(4) 1 : 1

Answer (4)

Sol. (i) $\quad B_1 2\pi \dfrac{a}{2} = \mu_0 \hat{j}\pi \dfrac{a^2}{4}$

(ii) $\quad B_2 2\pi 2a = \mu_0 \hat{j}\pi a^2$

$\Rightarrow \quad B_1 = B_2$

38. The angle of projection for a projectile to have same horizonal range and maximum height is

(1) $\tan^{-1}(4)$ \qquad (2) $\tan^{-1}\left(\dfrac{1}{4}\right)$

(3) $\tan^{-1}\left(\dfrac{1}{2}\right)$ \qquad (4) $\tan^{-1}(2)$

Answer (1)

Sol. $\dfrac{u^2 \sin^2\theta}{g} = \dfrac{u^2 \sin^2\theta}{2g}$

$2\cos\theta = \dfrac{\sin\theta}{2}$

$\tan\theta = 4$

39. Two satellite A and B go round a planet in circular orbits having radii $4R$ and R respectively. If the speed of A is $3v$, the speed of B will be

(1) $12v$ \qquad (2) $\dfrac{4}{3}v$

(3) $6v$ \qquad (4) $3v$

Answer (3)

Sol. $T \propto a^{\frac{3}{2}}$

$\dfrac{a}{v} \propto a^{\frac{3}{2}}$

$\Rightarrow \quad v \propto \dfrac{1}{\sqrt{a}}$

$\dfrac{3v}{v'} = \dfrac{\sqrt{R}}{\sqrt{4R}}$

$v' = 6v$

40. A given object takes n times the time to slide down 45° rough inclined plane as it takes the time to slide down an identical perfectly smooth 45° inclined plane. The coefficient of kinetic friction between the object and the surface of inclined plane is

(1) $\sqrt{1 - \dfrac{1}{n^2}}$ \qquad (2) $\sqrt{1 - n^2}$

(3) $1 - n^2$ \qquad (4) $1 - \dfrac{1}{n^2}$

Answer (4)

Sol. $\dfrac{t_r}{t_s} = \dfrac{\sqrt{\dfrac{2l}{g(\sin\theta - \mu\cos\theta)}}}{\sqrt{\dfrac{2l}{g\sin\theta}}} = \dfrac{n}{1}$

$\dfrac{\sin\theta}{\sin\theta - \mu\cos\theta} = n^2$

$\dfrac{\sin\theta}{n^2} = \sin\theta - \mu\cos\theta$

$\mu\cos\theta = \sin\theta\left[1 - \dfrac{1}{n^2}\right]$

$\mu = 1 - \dfrac{1}{n^2}$

41.
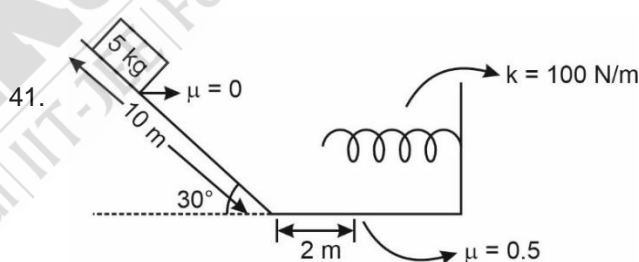
A block is simply released from the top of an inclined plane as shown in the figure above. The maximum compression in the spring when the block hits the spring is

(1) $\sqrt{6}$ m \qquad (2) $\sqrt{5}$ m

(3) 1 m \qquad (4) 2 m

Answer (4)

Sol. $mg5 = \mu mg(2 + x) + \dfrac{1}{2}kx^2$

$250 = 25(2 + x) + 50x^2$

$10 = 2 + x + 2x^2$

$2x^2 + x - 8 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 64}}{4}$

$x = 1.76 \approx 2$

42. Water boils in an electric kettle in 20 minutes after being switched on. Using the same main supply, the length of the heating element should be _____ to _____ times of its initial length if the water is to be boiled in 15 minutes.

    (1) increased, $\dfrac{4}{3}$

    (2) decreased, $\dfrac{3}{4}$

    (3) increased, $\dfrac{3}{4}$

    (4) decreased, $\dfrac{4}{3}$

Answer (2)

Sol. $\dfrac{V^2}{R_1} \times 20 = \dfrac{V^2}{R_2} \times 15$

$\dfrac{4}{3} = \dfrac{\rho \dfrac{l_1}{A}}{\rho \dfrac{l_2}{A}} = \dfrac{l_1}{l_2}$

$l_2 = \dfrac{3}{4}l_1$

43. A diatomic gas ($\gamma = 1.4$) does 100 J of work in an isobaric expansion. The heat given to the gas is

    (1) 250 J   (2) 490 J

    (3) 150 J   (4) 350 J

Answer (4)

Sol. $\omega = PV_2 - PV_1 = 100$

$Q = \dfrac{7}{2}\mu R\Delta T = \dfrac{7}{2} \times 100 = 350$

44. A proton and an electron have the same de Broglie wavelength. If K_p and K_e be the kinetic energies of proton and electron respectively, then choose the correct relation

    (1) $K_p = K_e$

    (2) $K_p > K_e$

    (3) $K_p < K_e$

    (4) $K_p = K_e^2$

Answer (3)

Sol. $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2m_e k_e}} = \dfrac{h}{\sqrt{2m_p k_p}}$

$\dfrac{K_p}{K_e} = \dfrac{m_e}{m_p} \ll 1$

$\Rightarrow$ 3

45. If M_0 is the mass of isotope $^{12}_{5}B$, M_p and M_n are the masses of proton and neutron, then nuclear binding energy of isotope is

    (1) $(M_0 - 5M_p)C^2$

    (2) $(5M_p + 7M_n - M_0)C^2$

    (3) $(M_0 - 12M_n)C^2$

    (4) $(M_0 - 5M_p - 7M_n)C^2$

Answer (2)

Sol. B.E. $= |\Delta M|C^2 = (M_r - M_p)C^2$

$= (5M_p + 7M_n - M_o)C^2$

46. A capacitor has air as dielectric medium and two conducting plates of area 12 cm^2 and they are 0.6 cm apart. When a slab of dielectric having area 12 cm^2 and 0.6 cm thickness is inserted between the plates, one of the conducting plates has to be moved by 0.2 cm to keep the capacitance same as in previous case. The dielectric constant of the slab is

(Given $\varepsilon_0 = 8.834 \times 10^{-12}$ F/m)

(1) 1
(2) 1.33
(3) 1.50
(4) 0.66

Answer (3)

Sol.
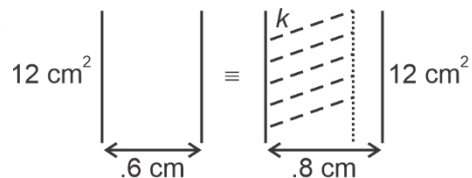
$$\frac{\in_0 A}{d} = \frac{\in_0 A}{\frac{d}{k} + \frac{0.2}{1}}$$

$$\Rightarrow \quad 0.6 - 0.2 = \frac{0.6}{k}$$

$$k = \frac{0.6}{0.4} = 1.5$$

47. In a hypothetical fission reaction

$$_{92}X^{236} \rightarrow {}_{56}Y^{141} + {}_{36}Z^{92} + 3R$$
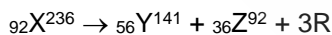
The identity of emitted particles (R) is

(1) Proton
(2) Neutron
(3) Electron
(4) γ-radiations

Answer (2)

Sol. $Z_{reactant} = Z_{product} \Rightarrow$ neutral

$3M_R = 236 - 141 - 92 = 3$

$M_R = 1 \Rightarrow$ neutron

48. A cube of ice floats partly in water and partly in kerosene oil. The ratio of volume of ice immersed in water to that in kerosene oil (specific gravity of Kerosene oil = 0.8, specific gravity of ice = 0.9)
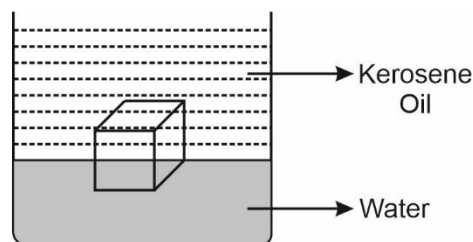
(1) 5 : 4
(2) 9 : 10
(3) 8 : 9
(4) 1 : 1

Answer (4)

Sol. $v_\omega(1) + v_k(0.8) = (v_\omega + v_k)(0.9)$

$$\Rightarrow \quad \frac{v_\omega}{v_k} = 1$$

49. A plane progressive wave is given by $y = 2\cos 2\pi(330t - x)$ m. The frequency of the wave is

(1) 165 Hz
(2) 660 Hz
(3) 340 Hz
(4) 330 Hz

Answer (4)

Sol. Comparing with $A\cos\left(2\pi\upsilon t - 2\pi\bar{\upsilon}x\right)$

$$\Rightarrow \quad \upsilon = 330$$

50. Given below are two statements :

    Statement (I) : The mean free path of gas molecules is inversely proportional to square of molecular diameter.

    Statement (II) : Average kinetic energy of gas molecules is directly proportional to absolute temperature of gas.

    In the light of the above statements, choose the **correct** answer from the options given below :

    (1) Both **Statement I** and **Statement II** are true

    (2) **Statement I** is false but **Statement II** is true

    (3) Both **Statement I** and **Statement II** are false

    (4) **Statement I** is true but **Statement II** is false

Answer (1)

Sol. As $\lambda = \dfrac{1}{\sqrt{2}\pi d^2 n}$

$<\varepsilon> = \dfrac{f}{2} kT$

$\Rightarrow 1$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

---

51. An alternating emf $E = 110\sqrt{2}\sin 100t$ volt is applied to a capacitor of $2\ \mu F$, the rms value of current in the circuit is _____ mA

Answer (22)

Sol. $\dfrac{V}{(X_c)} = i\dfrac{110}{\dfrac{1}{100 \times 2 \times 10^{-6}}} = 110 \times 2 \times 10^{-4} = 22$ mA.

52. An object of mass 0.2 kg executes simple harmonic motion along x axis with frequency of $\left(\dfrac{25}{\pi}\right)$ Hz . At the position x = 0.04 m the object has kinetic energy 0.5 J and potential energy 0.4 J. The amplitude of oscillation is _____ cm.

Answer (6)

Sol. $M = \dfrac{1}{2}m4\pi^2 v^2 A^2$

$0.9 = \dfrac{1}{2} \times .2 \times 4\pi^2 \times \dfrac{25}{\pi} \times \dfrac{25}{\pi} A^2$

$\dfrac{9}{4 \times 25 \times 25} = A^2$

In CGS $\dfrac{9 \times 4 \times 4}{4} = A^2 = 36 \Rightarrow A = 6$

(However, with kinetic energy, data is inconsistent)

53. Small water droplets of radius 0.01 mm are formed in the upper atmosphere and falling with a terminal velocity of 10 cm/s. Due to condensation, if 8 such droplets are coalesced and formed a larger drop, the new terminal velocity will be _____ cm/s.

Answer (40)

Sol. (1) $V_T \propto r^2$

(2) 8 drops of $r \equiv 1$ drop of R

$\Rightarrow R = 2r$.

(3) $V_T' = 40$ cm/s

54. The coercivity of a magnet is 5×10^3 A/m. The amount of current required to be passed in a solenoid of length 30 cm and the number of turns 150, so that the magnet gets demagnetised when inside the solenoid is _____ A.

Answer (10)

Sol. At demagnetization $B = \mu_0 H$

$H = ni$

$\dfrac{5 \times 10^3}{\dfrac{150}{30} \times 100} = i = 10$ A

55. A potential divider circuit is connected with a dc source of 20 V, a light emitting diode of glow in voltage 1.8 V and a Zener diode of breakdown voltage of 3.2 V. The length (PR) of the resistive wire is 20 cm. The minimum length of PQ to just glow the LED is _____ cm.
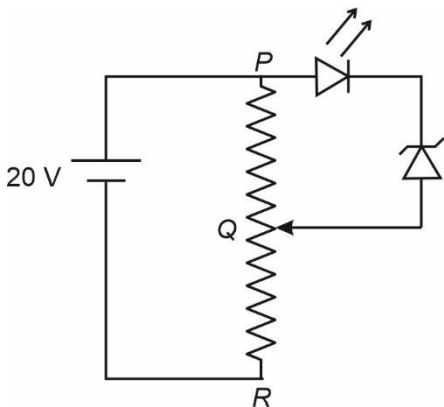
Answer (5)

Sol.
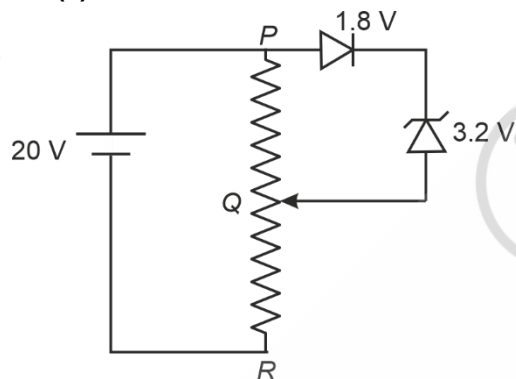
For minimum length of PQ,

$V_{diodes} = 1.8 + 3.2 = 5V$

$\therefore \quad I_{PQ} = \dfrac{20}{20} \times 5$

$= 5$ cm

56. A body of mass M thrown horizontally with velocity v from the top of the tower of height H touches the ground at a distance of 100 m from the foot of the tower. A body of mass $2M$ thrown at a velocity $\dfrac{v}{2}$ from the top of the tower of height $4H$ will touch the ground at a distance of _____ m.

Answer (100)
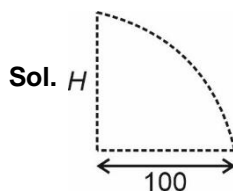
Sol.

$R = \sqrt{\dfrac{2H}{g}}\, v$

$R' = \sqrt{\dfrac{2(4H)}{g}}\, \dfrac{v}{2}$

$\Rightarrow \quad R = R'$

(Assume horizontal projectile in second case)

57. If the net electric field at point P along Y axis is zero, then the ratio of $\left|\dfrac{q_2}{q_3}\right|$ is $\dfrac{8}{5\sqrt{x}}$, where $x =$ _____.
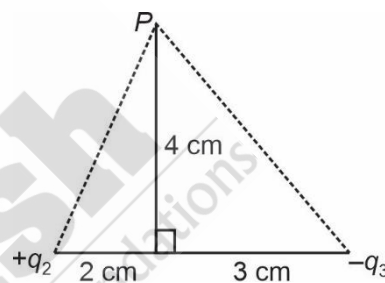
Answer (5)

Sol.
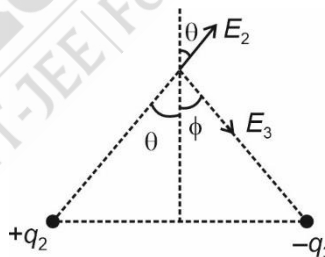
$\Rightarrow \quad E_2 \cos\theta = E_3 \cos\phi$

$\dfrac{q_2}{(2^2 + 4^2)} \dfrac{4}{\sqrt{2^2 + 4^2}} = \dfrac{q_3}{4^2 + 3^2} \dfrac{4}{\sqrt{4^2 + 3^2}}$

$\dfrac{q_2}{q_3} = \dfrac{20}{25} \dfrac{\sqrt{20}}{5} = \dfrac{4}{5} \times \dfrac{2\sqrt{5}}{5}$

$\Rightarrow \quad n = 5$

58. A circular table is rotating with an angular velocity of ω rad/s about its axis (see figure). There is a smooth groove along a radial direction on the table. A steel ball is gently placed at a distance of 1 m on the groove. All the surfaces are smooth. If the radius of the table is 3 m, the radial velocity of the ball w.r.t. the table at the time ball leaves the table is $x\sqrt{2}\omega$ m/s , where the value of x is _____.
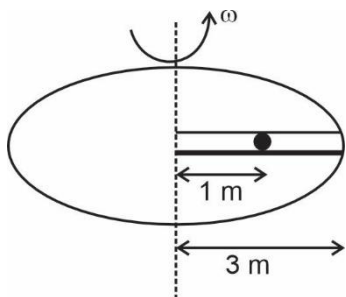
Answer (2)

Sol. $mv_r\dfrac{dv_r}{dr} = m\omega^2 r$

$\dfrac{v_v^2}{2} = \dfrac{\omega^2(r^2-1)}{2}$

$v_r = \omega\sqrt{8} = 2\omega\sqrt{2}$

59. Two slits are 1 mm apart and the screen is located 1 m away from the slits. A light of wavelength 500 nm is used. The width of each slit to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern is _____ × 10⁻⁴ m.

Answer (2)

Sol.
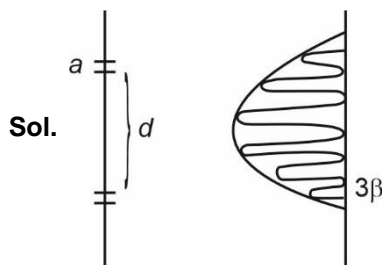
Width central maxima = 10 fringe width = $\dfrac{2\lambda\theta}{a} = 10\beta$

$\dfrac{2\lambda\theta}{a} = 10 \times \dfrac{\lambda D}{d}$

$a = \dfrac{d}{5} = \dfrac{10^{-3}}{5} = 2\times 10^{-4}$

60. A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected in combination with a resistance of 10 Ω and a resistance R, to a 100 V mains as shown in figure. For the heater to operate at 62.5 W, the value of R should be _____ Ω.

Answer (5)

Sol.
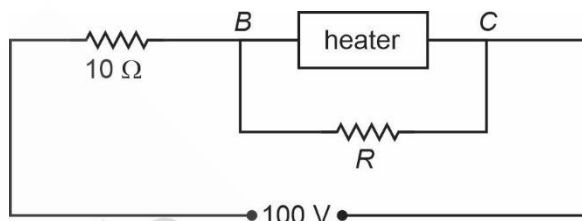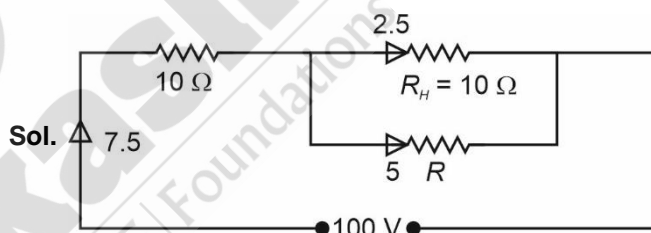
$R_H = \dfrac{100\times 100}{1000} = 10\ \Omega$

$i_H = \sqrt{\dfrac{62.5}{10}} = 2.5$ A

$V_H = 25$ V

$\Rightarrow i_b = \dfrac{75}{10} = 7.5$ A

$\Rightarrow i_R = 5$ A

$R = \dfrac{25}{5} = 5\ \Omega$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Given below are two statements:

    Statement (I): A Buffer solution is the mixture of a salt and an acid or a base mixed in any particular quantities.

    Statement (II): Blood is naturally occurring buffer solution whose pH is maintained by H_2CO_3/HCO_3^- concentrations.

    In the light of the above statements, choose the **correct** answer from the options given below:

    (1) Both Statement I and Statement II are true

    (2) Statement I is true but Statement II is false

    (3) Statement I is false but Statement II is true

    (4) Both statement I and statement II are false

Answer (3)

Sol. Buffer solution is the mixture of weak acid or weak base with their conjugate ions.

62. Identify the incorrect statements about group 15 elements:

    (A) Dinitrogen is a diatomic gas which acts like an inert gas at room temperature.

    (B) The common oxidation states of these elements are –3, +3 and +5.

    (C) Nitrogen has unique ability to form $p\pi - p\pi$ multiple bonds.

    (D) The stability of +5 oxidation states increases down the group

    (E) Nitrogen shows a maximum covalency of 6

Choose the correct answer from the options given below:

    (1) (A), (B), (D) only   (2) (D) and (E) only

    (3) (B), (D), (E) only   (4) (A), (C), (E) only

Answer (2)

Sol. The stability of +5 oxidation states decreases down the group due to inert pair effect.

    Nitrogen can show a maximum covalency of 4.

63. When Ψ_A and Ψ_B are the wave functions of atomic orbital, then σ^* is represent by:

    (1) $\Psi_A + 2\Psi_B$ (2) $\Psi_A - \Psi_B$

    (3) $\Psi_A - 2\Psi_B$ (4) $\Psi_A + \Psi_B$

Answer (2)

Sol. $\sigma^* = \psi_A - \psi_B$

64. Identify the correct statements about p-block elements and their compounds.

    (A) Non metals have higher electronegativity than metals.

    (B) Non metals have lower ionisation enthalpy than metals.

    (C) Compounds formed between highly reactive nonmetals and highly reactive metals are generally ionic.

    (D) The non-metal oxides are generally basic in nature.

    (E) The metal oxides are generally acidic or neutral in nature.

    (1) (B) and (E) only   (2) (B) and (D) only

    (3) (A) and (C) only   (4) (D) and (E) only

Answer (3)

Sol. Non metals have higher ionisation enthalpy than metals due to less electropositive nature.

    The non metal oxides are acidic in general.

    The metal oxides are generally basic.

65. Match List I with List - II.

List - I (Complex ion)	List-II (Spin only magnetic moment in B.M.)
(A) $[Cr(NH_3)_6]^{3+}$	(I) 4.90
(B) $[NiCl_4]^{2-}$	(II) 3.87
(C) $[CoF_6]^{3-}$	(III) 0.0
(D) $[Ni(CN)_4]^{2-}$	(IV) 2.83

Choose the correct answer from the options given below:

(1) (A)–(I), (B)–(IV), (C)–(II), (D)–(III)

(2) (A)–(II), (B)–(III), (C)–(I), (D)–(IV)

(3) (A)–(II), (B)–(IV), (C)–(I), (D)–(III)

(4) (A)–(IV), (B)–(III), (C)–(I), (D)–(II)

Answer (3)

Sol. $[Cr(NH_3)_6]^{3+}$: Cr^{3+} :

$\mu = \sqrt{3 \times 5} = 3.87$

$[NiCl_4]^{2-}$: Ni^{2+} :

$\mu = \sqrt{2 \times 4} = 2.83$

$[CoF_6]^{3-}$: Co^{3+} :

$\mu = \sqrt{4 \times 6} = 4.9$

$[NiCl_4]^{2-}$: Ni^{2+} :

$\mu = 0$

66. The equilibrium $Cr_2O_7^{2-} \rightleftharpoons 2CrO_4^{2-}$ is shifted to the right in

(1) A neutral medium

(2) A weakly acidic medium

(3) An acidic medium

(4) A basic medium

Answer (4)

Sol. $Cr_2O_7^{2-} + H_2O \rightleftharpoons 2CrO_4^{2-} + 2H^+$

Reaction will shift in forward direction in basic medium.

67. The emf of cell $Tl \left| \begin{array}{c} Tl^+ \\ (0.001M) \end{array} \right| \left| \begin{array}{c} Cu^+ \\ (0.01M) \end{array} \right| Cu$ is 0.83 V at 298K. It could be increased by

(1) increasing concentration of Tl^+ ions

(2) decreasing concentration of both Tl^+ and Cu^{2+} ions

(3) increasing concentration of Cu^{2+} ions

(4) increasing concentration of both Tl^+ and Cu^{2+} ions

Answer (3)

Sol. $Tl \longrightarrow Tl^+ + e^-$

$Cu^{2+} + 2e^- \longrightarrow Cu$

$2Tl + Cu^{2+} \longrightarrow 2Tl^+ + Cu$
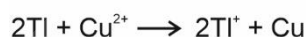
$E_{cell} = E_{cell}^\circ - \dfrac{0.0591}{2} \log \dfrac{\left[Tl^+ \right]^2}{\left[Cu^{2+} \right]}$

E_{cell} will increase by decrease in $[Tl^+]$ or increase in $[Cu^{2+}]$.

68. Match List-I with List-II

List-I (Reactions)

List-II (Products)
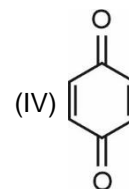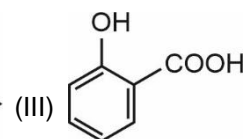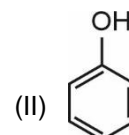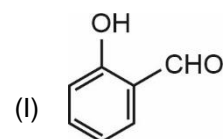
Choose the **correct** answer from the options given below:

(1) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(2) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(3) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)
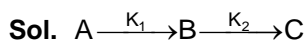
(4) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

Answer (1)

Sol.

69. For a reaction $A \xrightarrow{K_1} B \xrightarrow{K_2} C$
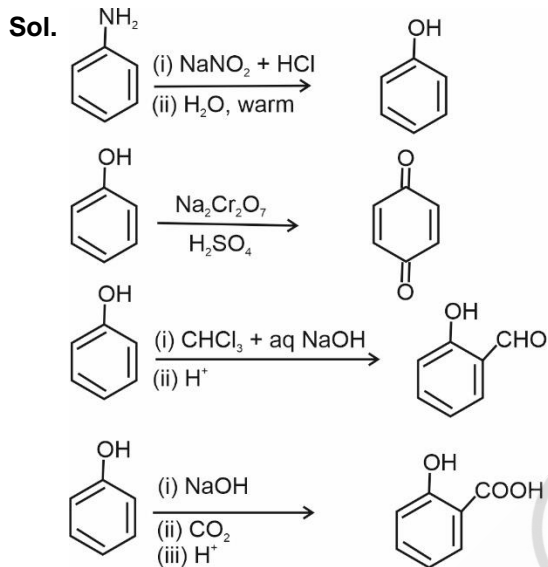
If the rate of formation of B is set to be zero then the concentration of B is given by:

(1) $(K_1 + K_2)[A]$

(2) $(K_1 - K_2)[A]$

(3) $K_1 K_2 [A]$

(4) $(K_1 / K_2)[A]$

Answer (4)

Sol. $A \xrightarrow{K_1} B \xrightarrow{K_2} C$

Net rate of formation of B

$$= \frac{d[B]}{dt} = K_1[A] - K_2[B] = 0$$

So, $[B] = \dfrac{K_1[A]}{K_2}$

70. Which one the following compounds will readily react with dilute NaOH?

(1) C_2H_5OH

(2) C_6H_5OH

(3) $(CH_3)_3COH$

(4) $C_6H_5CH_2OH$

Answer (2)

Sol. C_6H_5OH (Phenol) is more acidic than water hence reacts with NaOH.

71. Given below are two statements

Statement (I): Kjeldahl method is applicable to estimate nitrogen in pyridine

Statement (II): The nitrogen present in pyridine can easily be converted into ammonium sulphate in Kjeldahl method

In the light of the above statements, choose the **correct** answer from the options given below:

(1) **Statement I** is false but **statement II** is true

(2) Both **statement I** and **statement II** are true

(3) Both **statement I** and **statement II** are false

(4) **Statement I** is true but **statement II** is false

Answer (3)

Sol. Kjeldahl method is not applicable to compounds containing nitrogen in the ring as nitrogen of these compounds does not change to ammonium sulphate under these conditions.

72. Match **List-I** with **List-II**.

List-I	List-II
(Test)	(Identification)
(A) Bayer's test	(I) Phenol
(B) Ceric ammonium nitrate test	(II) Aldehyde
(C) Phthalein dye test	(III) Alcoholic-OH group
(D) Schiff's test	(IV) Unsaturation

Choose the **correct** answer from the options given below

(1) (A)–(IV), (B)–(III), (C)–(I), (D)–(II)

(2) (A)–(IV), (B)–(I), (C)–(II), (D)–(III)

(3) (A)–(III), (B)–(I), (C)–(IV), (D)–(II)

(4) (A)–(II), (B)–(III), (C)–(IV), (D)–(I)

Answer (1)

Sol. Bayer's test – Used to test unsaturation

Ceric ammonium nitrate test – gives red ppt with alcohols
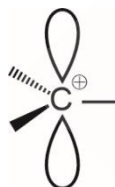
Phthalein dye test – used to test for phenol

Schiff's test – gives pink colour with aldehydes

73. The shape of carbocation is

(1) Diagonal

(2) Trigonal planar

(3) Diagonal pyramidal

(4) Tetrahedral

Answer (2)

Sol. Carbocation is sp^2 hybridised hence it's shape is trigonal planar

74. In qualitative test for identification of presence of phosphorous, the compound is heated with an oxidising agent. Which is further treated with nitric acid and ammonium molybdate respectively. The yellow coloured precipitate obtained is:

(1) $(NH_4)_3PO_4.12(NH_4)_2MoO_4$

(2) $MoPO_4.21NH_4NO_3$

(3) $Na_3PO_4.12MoO_3$

(4) $(NH_4)_3PO_4.12MoO_3$

Answer (4)

Sol. For test of phosphorus:

$$H_3PO_4 + 12(NH_4)_2MoO_4 + 21HNO_3 \rightarrow \underset{(\text{Yellow ppt})}{(NH_4)_3 \cdot 12MoO_3}$$
$$+ 21NH_4NO_3 + 12H_2O$$
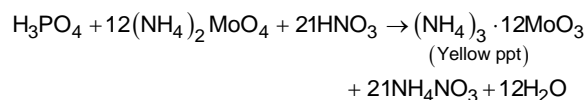
75. Given below are two statements:

Statement (I): All the following compounds react with p-toluenesulfonyl chloride.

$C_6H_5NH_2$ $(C_6H_5)_2NH$ $(C_6H_5)_3N$
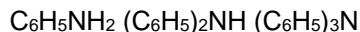
Statement (II) : Their products in the above reaction are soluble is aqueous NaOH.

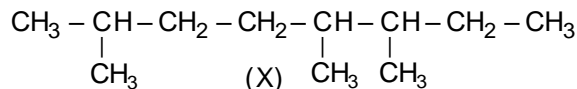In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is true but **Statement II** is false

Answer (3)

Sol. 1° and 2° amines only reacts with p-toluenesulfonyl chloride and product of 1° amine only is soluble in aqueous NaOH.

76. The correct sequence of acidic strength of the following aliphatic acids in their decreasing order is: CH_3CH_2COOH, CH_3COOH, $CH_3CH_2CH_2COOH$, $HCOOH$

(1) $HCOOH > CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH$

(2) $CH_3CH_2CH_2COOH > CH_3CH_2COOH > CH_3COOH > HCOOH$

(3) $HCOOH > CH_3CH_2CH_2COOH > CH_3CH_2COOH > CH_3COOH$

(4) $CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH > HCOOH$

Answer (1)

Sol. $HCOOH > CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH$

77. Given below are two statements:

Statement (I): S_N2 reactions are 'stereospecific', indicating that they result in the formation of only one stereo-isomer as the product.

Statement (II): S_N1 reactions generally result in formation of product as racemic mixtures. In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Answer (4)

Sol. S_N2 reactions are stereospecific and inversion of configuration is observed while via S_N1 generally racemisation is observed.

78. Given below are two statements:

Statement (I): Fusion of MnO_2 with KOH and an oxidising agent gives dark green K_2MnO_4.

Statement (II): Manganate ion on electrolytic oxidation in alkaline medium gives permanganate ion.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II is false

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Answer (4)

Sol. Dark green K_2MnO_4 is observed when an oxidising agent is added to alkaline MnO_2 and manganate ion oxidises to permanganate in alkaline medium.
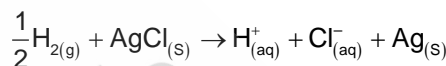
79. IUPAC name of following hydrocarbon(X) is:

$$CH_3 - CH - CH_2 - CH_2 - CH - CH - CH_2 - CH_3$$
$$\quad\quad\;\; | \quad\quad\quad\quad\quad\quad\quad | \quad\;\; |$$
$$\quad\quad\; CH_3 \quad\quad (X) \;\; CH_3 \; CH_3$$

(1) 2,5,6 – Trimethyloctane

(2) 2-Ethyl-3,6-dimethylheptane

(3) 2-Ethyl-2,6-diethylheptane

(4) 3,4,7-Trimethyloctane

Answer (1)

Sol.

$$\overset{1}{CH_3} - \overset{2}{CH} - \overset{3}{CH_2} - \overset{4}{CH_2} - \overset{5}{CH} - \overset{6}{CH} - \overset{7}{CH_2} - \overset{8}{CH_3}$$
$$\quad\quad | \quad\quad\quad\quad\quad\quad\quad\quad | \quad\;\; |$$
$$\quad\; CH_3 \quad\quad\quad\quad\quad\; CH_3 \; CH_3$$
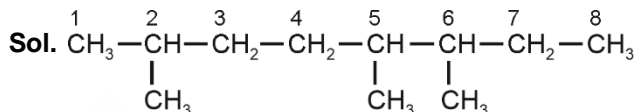
2, 5, 6-trimethyloctane

80. The reaction;

$$\frac{1}{2}H_{2(g)} + AgCl_{(S)} \rightarrow H^+_{(aq)} + Cl^-_{(aq)} + Ag_{(S)}$$

Occurs in which of the following galvanic cell:

(1) $Ag|AgCl_{(S)}|KCl_{(soln.)}|AgNO_{3(aq.)}|Ag$

(2) $Pt|H_{2(g)}|KCl_{(soln.)}|AgCl_{(S)}|Ag$

(3) $Pt|H_{2(g)}|HCl_{(soln.)}|AgCl_{(S)}|Ag$

(4) $Pt|H_{2(g)}|HCl_{(soln.)}|AgNO_{3(aq)}|Ag$

Answer (3)

Sol. The given reaction has the following half cell reaction:

at anode : $\frac{1}{2}H_2 \rightarrow H^+ + e^-$

at cathode : $AgCl + e^- \rightarrow Ag + Cl^-$
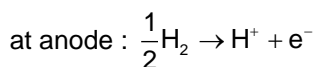
The cell representation is :
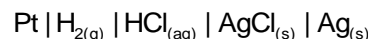
$Pt \,|\, H_{2(g)} \,|\, HCl_{(aq)} \,|\, AgCl_{(s)} \,|\, Ag_{(s)}$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

81. Molality of an aqueous solution of urea is 4.44m. Mole fraction of urea in solution is $x \times 10^{-3}$. Value of x is _____. (Integer answer)

Answer (74)

Sol. $\dfrac{X_B}{X_A} = \dfrac{m \cdot M_A}{1000}$

where m = molality; M_A = molar mass of solvent

$\dfrac{X_B}{1 - X_B} = \dfrac{4.44 \times 18}{1000} = 0.08$

$X_B = 0.074 = 74 \times 10^{-3}$

So, x = 74

82. The total number of carbon atoms present in tyrosine, an amino acid, is _____

Answer (9)

Sol. The structure of tyrosine is :

$$HO-\bigcirc-CH_2-CH-COOH$$
$$\underset{NH_2}{|}$$
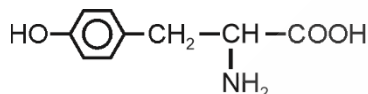
Number of C-atoms = 9

83. $\Delta_{vap}H°$ for water is +40.79 kJ mol^{-1} at 1 bar and 100°C. Change in internal energy for this vapourisation under same condition is _____ kJ mol^{-1}. (Given R=8.3 JK^{-1} mol^{-1})

Answer (38)

Sol. $\Delta H° = \Delta U° + \Delta n_g RT$

Or $\Delta U° = 40.79 - \dfrac{8.3 \times 373}{1000}$

$= 37.67 = 38$

84. Two moles of benzaldehyde and one mole of acetone under alkaline conditions using aqueous NaOH after heating gives x as the major product. The number of π bonds in the product x is

Answer (9)

Sol.

$$2Ph-CHO + CH_3-\overset{O}{\overset{||}{C}}-CH_3 \xrightarrow[\Delta]{aq.\ NaOH}$$

$$Ph-CH=CH-\overset{O}{\overset{||}{C}}-CH=CH-Ph$$
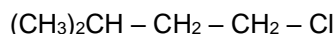$$(X)$$
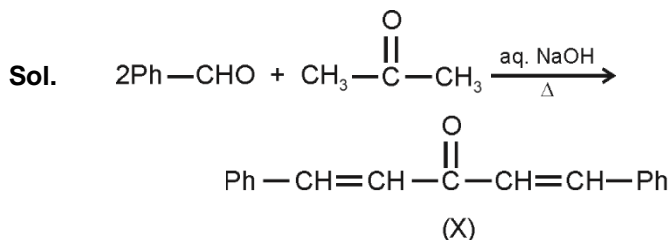
Total number of π bonds in X = 9

85. Number of molecules having bond order 2 from the following molecules is _____

C_2, O_2, Be_2, Li_2, Ne_2, He_2

Answer (2)

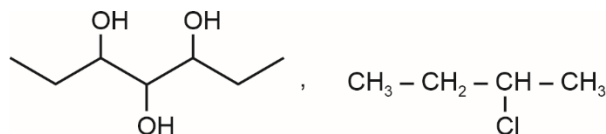**Sol.** The molecules having bond order 2 are: C_2 and O_2.

86. Total number of optically active compounds from the following is _____.

$$\begin{array}{c} CH_3 \\ | \\ H-C-OH \\ | \\ H-C-OH \\ | \\ CH_3 \end{array}$$, (structure with OH OH OH groups on a carbon chain)

$CH_3-CH_2-CH_2-CH_2-OH$,

$CH_3-CH_2-\underset{Cl}{\underset{|}{C}H}-CH_3$,

$CH_3-CH_2-CH_2-CH_2-Cl$,

$(CH_3)_2CH-CH_2-CH_2-Cl$
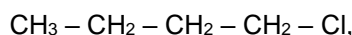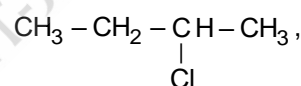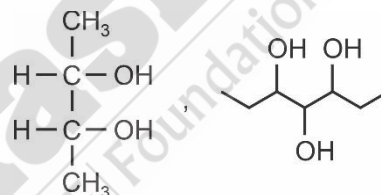
Answer (2)

Sol. The optically active compounds are:

(structure with OH OH OH groups), $CH_3-CH_2-\underset{Cl}{\underset{|}{CH}}-CH_3$

87. A solution is prepared by adding 1 mole ethyl alcohol in 9 mole water. The mass percent of solute in the solution is _____. (Integer answer)

(Given: Molar mass in g mol^{-1}, Ethyl alcohol: 46, water: 18)

Answer (22)

Sol. $\%m/m = \dfrac{1 \times 46}{(46 + 18 \times 9)} \times 100 = 22.11$

$\simeq 22$
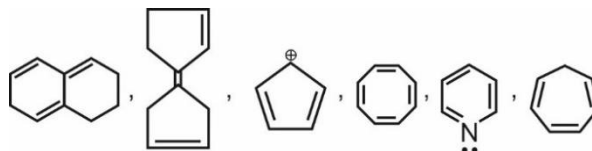
88. Total number of unpaired electrons in the complex ions $[Co(NH_3)_6]^{3+}$ and $[NiCl_4]^{2-}$ is

Answer (2)

Sol. $[Co(NH_3)_6]^{3+} : Co^{3+} : t_{2g}^6 e_g^0 : n = 0$

$[NiCl_4]^{2-} : Ni^{2+} : t_{2g}^6 e_g^2 : n = 2$

89. Total number of aromatic compounds among the following compounds is _____

Answer (1)

Sol. The aromatic compounds is only in the given compounds.

90. Wavenumber for a radiation having 5800 Å wavelength is $x \times 10$ cm^{-1}. The value of x is _____.

Answer (1724)

Sol. $\bar{v} = \dfrac{1}{\lambda} = \dfrac{1}{5800 \times 10^{-8} \text{ cm}} = 1724.1 \times 10 \text{ cm}^{-1}$

$x = 1724$

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