08/04/2024
Morning

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6) **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let $y = y(x)$ be the solution of the differential equation $(1 + y^2)e^{\tan x}\, dx + \cos^2 x(1 + e^{2\tan x})dy = 0$, $y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{2}{e^2}$ (2) $\frac{1}{e^2}$

(3) $\frac{1}{e}$ (4) $\frac{2}{e}$

Answer (3)

Sol. $(1+y^2)e^{\tan x}dx + \cos^2 x\left(1+e^{2\tan x}\right)dy = 0$

$$\frac{dy}{1+y^2} = -\frac{e^{\tan x}\cdot\sec^2 x\,dx}{1+e^{2\tan x}}$$

$$\int\frac{dy}{1+y^2} = -\int\frac{e^{\tan x}\cdot\sec^2 x\,dx}{1+e^{2\tan x}}$$

Let $e^{\tan x} = t$

$e^{\tan x}\cdot\sec^2 x\,dx = dt$

$$\int\frac{dy}{1+y^2} = -\int\frac{dt}{1+t^2}$$

$\tan^{-1}y = -\tan^{-1}t + c$

$\tan^{-1}y = -\tan^{-1}(e^{\tan x}) + c$

at $x = 0$, $y = 1$

$\tan^{-1}(1) = -\tan^{-1}(1) + c$

$$\frac{\pi}{4} = -\frac{\pi}{4} + c$$

$$c = \frac{\pi}{2}$$

$$\tan^{-1}y = -\tan^{-1}\left(e^{\tan x}\right) + \frac{\pi}{2}$$

Now, at $x = \frac{\pi}{4}$

$$\tan^{-1}y = -\tan^{-1}(e) + \frac{\pi}{2}$$

$$\tan^{-1}y = \cot^{-1}e = \tan^{-1}\frac{1}{e}$$

$$\Rightarrow\ y = \frac{1}{e}$$

2. The number of critical points of the function $f(x) = (x - 2)^{2/3}(2x + 1)$ is

(1) 0 (2) 1

(3) 3 (4) 2

Answer (4)

Sol. $f(x) = (x-2)^{2/3}(2x+1)$

$$f'(x) = 2(x-2)^{2/3} + (2x+1)\times\frac{2}{3}(x-2)^{-1/3}$$

$$f'(x) = 2(x-2)^{2/3} + \frac{2(2x+1)}{3(x-2)^{1/3}}$$

$$f'(x) = \frac{6(x-2)+2(2x+1)}{3(x-2)^{1/3}}$$

$$f'(x) = \frac{10x-10}{3(x-2)^{1/3}} = \frac{10(x-1)}{3(x-2)^{1/3}}$$

Critical points 1, 2

$\Rightarrow$ 2 critical points

3. Let $f(x)$ be a positive function such that the area bounded by $y = f(x)$, $y = 0$ from $x = 0$ to $x = a > 0$ is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is

(1) $\left(8e^x + 1\right)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(2) $\left(8e^x + 1\right)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

(3) $\left(8e^x - 1\right)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

(4) $\left(8e^x - 1\right)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Answer (1)

Sol. $\int_0^a f(x)dx = e^{-a} + 4a^2 + a - 1$

Differentiate equation w.r.t. 'a'

$f(a) = -e^{-a} + 8a + 1$

$\Rightarrow f(x) = -e^{-x} + 8x + 1$

And $y = c_1 f(x) + c_2$

$y = c_1(-e^{-x} + 8x + 1) + c_2$

$$y' = c_1\left(e^{-x} + 8\right) \Rightarrow c_1 = \frac{y'}{e^{-x} + 8}$$

$y'' = -c_1 e^{-x}$ put value of c_1

$$\frac{d^2y}{dx^2} = \frac{-\frac{dy}{dx}\cdot e^{-x}}{\left(e^{-x}+8\right)} = \frac{\frac{dy}{dx}}{\left(1+8e^{x}\right)}$$

$$\Rightarrow \left(1+8e^{x}\right)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 1$$

4. Let the circles $C_1 : (x - \alpha)^2 + (y - \beta)^2 = r_1^2$ and $C_2 : (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point (6, 6). If the point (6, 6) divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio 2 : 1, then $\alpha + \beta + 4\left(r_1^2 + r_2^2\right)$ equals

(1) 145 (2) 125

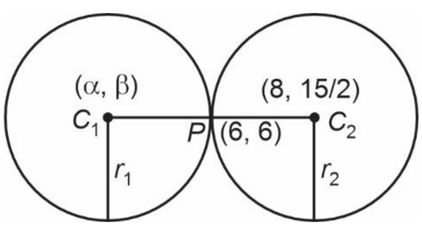
(3) 110 (4) 130

Answer (4)

Sol. $C_1 \rightarrow (x-\alpha)^2 + (y-\beta)^2 = r_1^2$

$$C_2 \rightarrow (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$$

Point P divide the line segment internally C_1C_2 in the ratio 2 : 1



$$\frac{\alpha\times 1 + 8\times 2}{1+2} = 6, \ \boxed{\alpha = 2}$$

$$\frac{\beta\times 1 + \frac{15}{\alpha}\times 2}{1+2} = 6, \ \boxed{\beta = 3}$$

$$r_1 = \sqrt{(6-2)^2 + (6-3)^2} = \sqrt{25} = 5$$

$$r_2 = \sqrt{(8-6)^2 + \left(\frac{15}{\alpha} - 6\right)^2} = \frac{5}{2}$$

$$\alpha + \beta + 4\left(r_1^2 + r_2^2\right) = 2 + 3 + 4\left(5^2 + \left(\frac{5}{2}\right)^2\right)$$

$$= 5 + 4\left(\frac{125}{4}\right)$$

$= 130$

5. The sum of all the solutions of the equation $(8)^{2x} - 16\cdot(8)^x + 48 = 0$ is :

(1) $\log_8(6)$ (2) $1 + \log_8(6)$

(3) $1 + \log_6(8)$ (4) $\log_8(4)$

Answer (2)

Sol. Given equation, $8^{2x} - 16.8^x + 48 = 0$

Let $8^x = t$

$\therefore t^2 - 16t + 48 = 0$

$\Rightarrow t = 4, 12$

$\Rightarrow 8^x = 4, 12$

$\Rightarrow x = \log_8 4, \log_8 12$

$\therefore$ Sum of solution

$= \log_8 4 + \log_8 12$

$= \log_8(48)$

$= 1 + \log_8(6)$

6. The set of all α, for which the vectors $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$, is

(1) [0, 1) (2) (–2, 0]

(3) $\left(-\frac{4}{3}, 1\right)$ (4) $\left(-\frac{4}{3}, 0\right]$

Answer (4)

Sol. Given $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$

and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$

angle between $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

We have, $\cos\theta < 0$ ($\because$ angle between $\vec{a}$ and $\vec{b}$ is obtuse)

$\Rightarrow \quad \vec{a}\cdot\vec{b} < 0$

$\Rightarrow \quad \alpha t^2 - 12 + 6\alpha t < 0 \ \forall t \in \mathbb{R}$

If $\alpha = 0$, then $-12 < 0$ (condition holds)

If $\alpha \neq 0 \ \Rightarrow \ \alpha < 0$ ...(i)

And maximum value of $\alpha t^2 + 6\alpha t - 12 < 0$

$\Rightarrow \quad \frac{-D}{4a} < 0$ (where D is discriminant and $a = \alpha$)

$\Rightarrow \quad \frac{36\alpha^2 + 48\alpha}{4\alpha} > 0$

$\Rightarrow \quad \alpha > \frac{-4}{3}$

$\therefore \quad \alpha \in \left(\frac{-4}{3}, 0\right]$

7. Let $[t]$ be the greatest integer less than or equal to t. Let A be the set of all prime factors of 2310 and $f : A \to \mathbb{Z}$ be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$.

The number of one-to-one function from A to the range of f is

(1) 120 (2) 24

(3) 25 (4) 20

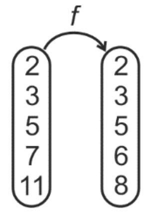
Answer (1)

Sol. $A = \{2, 3, 5, 7, 11\}$

$$f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$$

Ranges $f(x) = \{2, 3, 5, 6, 8\}$

Number of one-one $A \to R_f$



$5 \times 4 \times 3 \times 2 \times 1 = 120$

8. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to

(1) 18 (2) 19

(3) 108 (4) 109

Answer (4)

Sol. $\sin x = -\frac{3}{5}$ where $\pi < x < \frac{3\pi}{2}$

$$\tan x = \frac{3}{4}, \cos x = \frac{-4}{5}$$

$\therefore \quad 80(\tan^2 x - \cos x)$

$= 80\left(\frac{9}{16} + \frac{4}{5}\right)$

$= 80\left(\frac{45 + 64}{80}\right)$

$= 109$

9. Let $I(x) = \int \frac{6}{\sin^2 x(1 - \cot x)^2} dx$. If $I(0) = 3$, then $I\left(\frac{\pi}{12}\right)$ is equal to

(1) $6\sqrt{3}$ (2) $\sqrt{3}$

(3) $3\sqrt{3}$ (4) $2\sqrt{3}$

Answer (3)

Sol. $I(x) = \int \frac{6}{\sin^2 x(1 - \cot x)^2} dx$

$I(x) = \int \frac{6}{(\sin x - \cos x)^2} dx$

$= \int \frac{6\sec^2 x}{(\tan x - 1)^2} dx$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$= \int \frac{6dt}{(t-1)^2}$$

$$= -\frac{6}{(t-1)} + c$$

$$= \frac{-6}{(\tan x - 1)} + c$$

$$I(x) = \frac{6}{1-\tan x} + c$$

$I(0) = 3$

$$\frac{6}{1-\tan 0} + c = 3$$

$c = -3$

$$I(x) = \frac{6}{1-\tan x} - 3$$

$$I\left(\frac{\pi}{12}\right) = \frac{6}{1-\tan\left(\frac{\pi}{12}\right)} - 3$$

$$= \frac{6}{1-(2-\sqrt{3})} - 3$$

$$= \frac{6}{\sqrt{3}-1} - 3$$

$$= \frac{6-3\sqrt{3}+3}{\sqrt{3}-1}$$

$$= \frac{9-3\sqrt{3}}{\sqrt{3}-1}$$

$$= \frac{3\sqrt{3}(\sqrt{3}-1)}{\sqrt{3}-1}$$

$$= 3\sqrt{3}$$

10. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest possible product, is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $n - m$ equals

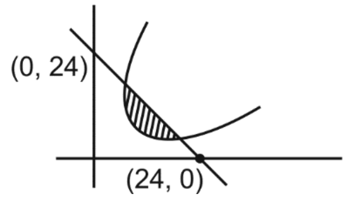
(1) 9 (2) 10

(3) 11 (4) 8

Answer (2)

Sol. Take two numbers as a and b

$a + b = 24$



For product to be maximum

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$144 > ab$

Maximum product is 144

Now, $ab \geq \frac{3}{4} \cdot 144 = 108$

Sample space = {(23, 1), (22, 2), …}

Integer points on line in shaded region

{(6, 18), (7, 17), (8, 16), … (18, 6)}

$$P(E) = \frac{n(E)}{n(S)} = \frac{13}{23} = \frac{m}{n} \Rightarrow n - m = 10$$

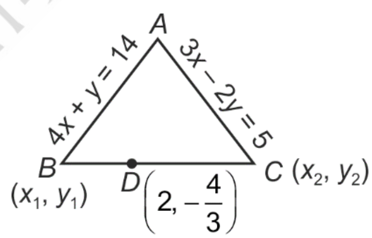
11. The equations of two sides AB and AC of a triangle ABC are $4x + y = 14$ and $3x - 2y = 5$, respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio 2 : 1, the equation of the side BC is

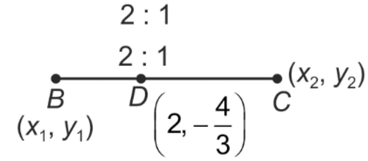
(1) $x + 6y + 6 = 0$ (2) $x - 6y - 10 = 0$

(3) $x - 3y - 6 = 0$ (4) $x + 3y + 2 = 0$

Answer (4)

Sol.





$$2 = \frac{2x_2 + x_1}{3}, \frac{-4}{3} = \frac{2y_2 + y_1}{3}$$

$2x_2 + x_1 = 6$, $2y_2 + y_1 = -4$

$x_1 = 6 - 2x_2$ ...(1)

$y_1 = -4 - 2y_2$ ...(2)

$4x_1 + y_1 = 14$ ...(3)

$3x_2 - 2y_2 = 5$ ...(4)

From here, $x_2 = 1$, $y_2 = -1$, $x_1 = 4$, $y_1 = -2$

$B(4, -2)$ $C(1, -1)$

$$y + 2 = \frac{-1+2}{1-4}(x - 4)$$

$-3y - 6 = x - 4$

$x + 3y + 2 = 0$

12. If the shortest distance between the lines

$$L_1 : \vec{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k},\ \lambda \in \mathbb{R}$$

$$L_2 : \vec{r} = 2(1+\mu)\hat{i} + 3(1+\mu)\hat{j} + (5+\mu)\hat{k},\ \mu \in \mathbb{R}$$

is $\frac{m}{\sqrt{n}}$, where gcd(m, n) = 1, then the value of m + n equals

(1) 377 (2) 384

(3) 390 (4) 387

Answer (4)

Sol. $L_1 : \vec{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k}$

$$L_1 = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda(\hat{i} - 3\hat{j} + 4\hat{k})$$

$$L_2 : \vec{r} = 2\hat{i} + 3\hat{j} + 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

$$\vec{a}_1 = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{a}_2 = 2\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 2\hat{j} + 2\hat{k}$$

$$\vec{b}_1 = \hat{i} - 3\hat{j} + 4\hat{k},\ \vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 4 \\ 2 & 3 & 1 \end{vmatrix}$$

$$\hat{i}(-3-12) - \hat{j}(1-8) + \hat{k}(3+6)$$

$$= -15\hat{i} + 7\hat{j} + 9\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{225 + 49 + 81}$$

$$\left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \frac{14+18}{\sqrt{355}} = \frac{32}{\sqrt{355}}$$

$m + n = 387$

13. If the set $R = \{(a, b) : a + 5b = 42, a, b \in \mathbb{N}\}$ has m elements and $\sum_{n=1}^{m}(1 - i^{n!}) = x + iy$, where $i = \sqrt{-1}$, then the value of $m + x + y$ is

(1) 5 (2) 12

(3) 4 (4) 8

Answer (2)

Sol. $R = \{(a, b) : a + 5b = 42\}$

Then R = {(2, 8), (7, 7), (12, 6), (17, 5), (22, 4), (27, 3), (32, 2), (37, 1)}

$m = 8$

and $\sum_{n=1}^{m}(1 - i^{n!}) = x + iy$

$$\therefore \sum_{n=1}^{8}(1 - i^{n!}) = 8 - (i + i^2 + i^6 + 1 + 1 + 1 + 1 + 1)$$

$= 5 - i$

$\therefore x = 5, y = -1$

$x + y + m = 5 - 1 + 8 = 12$

14. Let $H : \frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6)$, $\alpha > 0$ lies on H. If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to

(1) 171 (2) 172

(3) 169 (4) 170

Answer (1)

Sol. $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{3}$

$\Rightarrow 1 + \frac{a^2}{b^2} = 3$

$\Rightarrow \frac{a^2}{b^2} = 2$ …(1)

$\frac{2a^2}{b} = 4\sqrt{3}$

Using equation (1)

$\frac{4b^2}{b} = 4\sqrt{3}$

$\Rightarrow b = \sqrt{3}$

$a = \sqrt{6}$

$H: \frac{x^2}{6} - \frac{y^2}{3} = -1$

$\frac{\alpha^2}{6} - 12 = -1$

$\frac{\alpha^2}{6} = 11$

$\alpha^2 = 66$

Focus : $(0, bc)$ $(0, -bc)$

$(0, 3), (0, -3)$

$\beta = \sqrt{\alpha^2 + 9} \times \sqrt{\alpha^2 + 81}$

$\beta = 105$

$\alpha^2 + \beta = 66 + 105$

$= 171$

15. Let $P(x, y, z)$ be a point in the first octant, whose projection in the xy-plane is the point Q. Let $OP = \gamma$; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where O is the origin. Then the distance of P from the x-axis is

(1) $\gamma\sqrt{1 + \cos^2\theta \sin^2\phi}$ (2) $\gamma\sqrt{1 + \cos^2\phi \sin^2\theta}$

(3) $\gamma\sqrt{1 - \sin^2\theta \cos^2\phi}$ (4) $\gamma\sqrt{1 - \sin^2\phi \cos^2\theta}$

Answer (4)

Sol. $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

$\overrightarrow{OQ} = x\hat{i} + y\hat{j}$

$|OP| = \gamma = \sqrt{x^2 + y^2 + z^2}$

$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}} \Rightarrow \cos^2\theta = \frac{x^2}{\gamma^2 - z^2} = \frac{x^2}{\gamma^2 - \gamma^2\cos^2\phi}$

$\cos\phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{z}{\gamma}$

Distance of P from x-axis = $\sqrt{y^2 + z^2}$

$d = \sqrt{\gamma^2 - x^2}$

$\Rightarrow x^2 = \gamma^2\sin^2\phi\cos^2\theta$

$\Rightarrow d = \sqrt{\gamma^2 - \gamma^2\sin^2\phi\cos^2\theta}$

$= \gamma\sqrt{1 - \sin^2\phi\cos^2\theta}$

16. Let $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is

(1) 2 (2) 3

(3) 1 (4) 4

Answer (1)

Sol. $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$

$f'(x) = 12\cos^2 x.(-\sin x) + 6\sqrt{3}\cos x.(-\sin x) = 0$

$= -6\sqrt{3}\cos x.\sin x\left(1 + \frac{2}{\sqrt{3}}\cos x\right) = 0$

$\cos x = 0, \sin x = 0, \cos x = \frac{-\sqrt{3}}{2}$

Sign of $f'(x)$

	–ve	+ve	–ve	+ve	–ve	+ve
0	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	2π

$\therefore$ Maxima at $\frac{5\pi}{6}, \frac{7\pi}{6}$

17. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where I is the identity matrix of order 3 × 3, then $2a + 3b$ is equal to

(1) – 9 (2) –13

(3) –12 (4) –10

Answer (2)

Sol. $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & a & 0 \\ 1 & 3-\lambda & 1 \\ 0 & 5 & b-\lambda \end{vmatrix} = 0$$

$(2 - \lambda)[(3 - \lambda)(b - \lambda) - 5] - a[b - \lambda - 0] + 0 = 0$

$(2 - \lambda)[3b - 3\lambda - b\lambda + \lambda^2 - 5] - ab + a\lambda = 0$

$\lambda^3 - (b + 5)\lambda^2 + (1 - a + 5b)\lambda + (10 - 6b + ab) = 0$

$A^3 - (b + 5)A^2 + (1 - a + 5b)A + (10 - 6b + ab)I = 0$

$\Rightarrow b + 5 = 4,\ 1 - a + 5b = 1,\ 10 - 6b + ab = 21$

$\Rightarrow a = -5,\ b = -1$

$\Rightarrow 2a + 3b = -13$

18. For the function $f(x) = (\cos x) - x + 1$, $x \in \mathbb{R}$, between the following two statements

(S1) $f(x) = 0$ for only one value of x in $[0, \pi]$.

(S2) $f(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$.

(1) Only (S1) is correct.

(2) Both (S1) and (S2) are incorrect.

(3) Only (S2) is correct.

(4) Both (S1) and (S2) are correct.

Answer (1)

Sol. $f(x) = \cos x - x + 1$

$f'(x) = -\sin x - 1 \le 0$

$\therefore f(x)$ is decreasing function

$f(0) = 2$

$f(\pi) = -\pi$

$\therefore$ Only one root in $[0, \pi]$

S_1 is correct

S_2 is incorrect

19. The value of $k \in \mathbb{N}$ for which the integral $I_n = \int_0^1 (1 - x^k)^n dx$, $n \in \mathbb{N}$, satisfies $147I_{20} = 148I_{21}$ is

(1) 10 (2) 7

(3) 14 (4) 8

Answer (2)

Sol. $I(21) = \int_0^1 \left(1 - x^k\right)^{21} dx$

$= \int_0^1 \left(1 - x^k\right)\left(1 - x^k\right)^{20} dx$

$= \int_0^1 \left(1 - x^k\right)^{20} dx - \int_0^1 x^k \left(1 - x^k\right)^{20} dx$

$I(21) = I(20) - \int_0^1 x^k \left(1 - x^k\right)^{20} dx$

$= I(20) - \int_0^1 x \cdot \underbrace{x^{K-1}\left(1 - x^k\right)^{20}} dx$

$I(21) = I(20) - \left[\frac{\left(1 - x^k\right)^{21}}{-21k} x - \int_0^1 \frac{\left(1 - x^k\right)^{21}}{-21k} dx\right]$

$I(21) = I(20) - \frac{1}{21k} I(20)$

$\Rightarrow [I(21)](21k + 1) = 21KI(20)$

$\Rightarrow 21K = 147 \Rightarrow K = 7$

20. Let z be a complex number such that $|z + 2| = 1$ and $\text{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value $\left|\text{Re}\left(\overline{z+2}\right)\right|$ is

(1) $\frac{2\sqrt{6}}{5}$ (2) $\frac{1+\sqrt{6}}{5}$

(3) $\frac{24}{5}$ (4) $\frac{\sqrt{6}}{5}$

Answer (1)

Sol. $|z+2| = 1$

$$I_m\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$$

$$\left|\text{Re}\left(\overline{z+2}\right)\right| = ?$$

Let $z = x + iy$

$$\because |z+2| = 1 \Rightarrow (x+2)^2 + y^2 = 1 \quad ...(1)$$

$$I_m\left(\frac{z+1}{z+2}\right) = \frac{1}{5} \Rightarrow I_m\left(\frac{x+iy+1}{x+iy+2}\right) = \frac{1}{5}$$

$$\Rightarrow I_m\left[\frac{\left[(x+1)+iy\right]\left[(x+2)-iy\right]}{(x+2)^2+y^2}\right] = \frac{1}{5}$$

$$\frac{y(x+2)-y(x+1)}{(x+2)^2+y^2} = \frac{1}{5} \quad ...(2)$$

$$\Rightarrow y = \frac{1}{5}$$

Substituting in equation (1)

$$(x+2)^2 + \frac{1}{25} = 1$$

$$(x+2)^2 = \frac{24}{25}$$

$$\Rightarrow x = -2 \pm \frac{\sqrt{24}}{5}$$

$$\left|\text{Re}\left(\overline{x+iy+2}\right)\right|$$

$$= x+2 = \pm\frac{\sqrt{24}}{5} = \frac{2\sqrt{6}}{5}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$, $\vec{b} = 3i + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then $\dfrac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$ is equal to ______.

Answer (569)

Sol. $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$

$$\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$$

$$\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{r} \times \vec{a} = \left(\vec{b} + \vec{c}\right) \times \vec{a}$$

$$\left(\vec{r} - \left(\vec{b} + \vec{c}\right)\right) \times \vec{a} = 0$$

$$\Rightarrow \vec{r} = \left(\vec{b} + \vec{c}\right) + \lambda\vec{a}$$

$$\vec{r} = \left(20\hat{i} + 5\hat{j} - 12\hat{k}\right) + \lambda\left(9\hat{i} - 13\hat{j} + 25\hat{k}\right)$$

$$= (20 + 9\lambda)\hat{i} + (5 - 13\lambda)\hat{j} + (25\lambda - 12)\hat{k}$$

Now $\vec{r}.\left(\vec{b} - \vec{c}\right) = 0$

$$\vec{r}.\left(-14\hat{i} + 9\hat{j} - 14\hat{k}\right) = 0$$

Now

$$-14(20 + 9\lambda) + 9(5 - 13\lambda) - 14(25\lambda - 12) = 0$$

$$-593\lambda - 67 = 0$$

$$\lambda = -\frac{67}{593}$$

$$\therefore \vec{r} = \left(\vec{b} + \vec{c}\right) - \frac{67}{593}\vec{a}$$

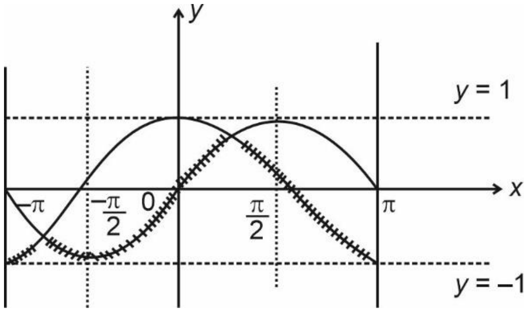
$$\frac{|593\vec{r} + 67\vec{a}|^2}{|593|^2} = |\vec{b} + \vec{c}|^2 = \left|20\hat{i} + 5\hat{j} - 12\hat{k}\right|^2$$

= 569

22. Let the area of the region enclosed by the curve y = min {sin x, cos x} and the x-axis between $x = -\pi$ to $x = \pi$ be A. Then A^2 is equal to _____.

Answer (16)

Sol. $y = f(x) = \min\{\sin x, \cos x\}$



$$A = -\int_{-\pi}^{-\frac{3\pi}{4}} \cos x\,dx - \int_{-\frac{3\pi}{4}}^{0} \sin x\,dx + \int_{0}^{\frac{\pi}{4}} \sin x\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\,dx$$

$$-\int_{\frac{\pi}{2}}^{\pi} \cos x\,dx$$

$A = 4$

$A^2 = 16$

23. Let the positive integers be written in the form

1

2 3

4 5 6

7 8 9 10

. . . . . .

If the k^{th} row contains exactly k, numbers for every natural number k, then the row in which the number 5310 will be, is _____.

Answer (103)

Sol. Let 5310 lies in k^{th} row

$\Rightarrow$ First element of k^{th} row is $\frac{(k-1)k}{2}+1$

Last element of k^{th} row is $\frac{k(k+1)}{2}$

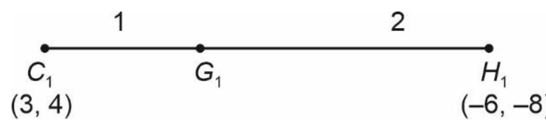
$$\Rightarrow \frac{(k-1)k}{2}+1 \le 5310 \le \frac{k(k+1)}{2}$$

$\Rightarrow k = 103$

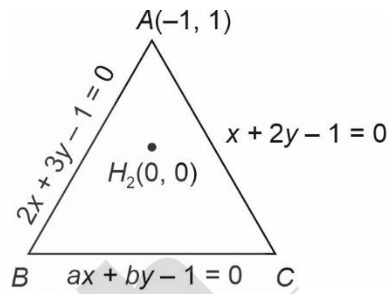
24. If the orthocentre of the triangle formed by the lines $2x + 3y - 1$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumcentre and orthocentre respectively are (3, 4) and (–6, –8), then the value of $|a - b|$ is _____.

Answer (16)

Sol. Let circumcentre, orthocentre and centroid of a triangle PQR are C_1, H_1 and G_1 respectively



$\Rightarrow G_1 \equiv (0, 0)$ orthocentre of ΔABC is (0, 0)



$m_{AH_2} = +\frac{b}{a} \Rightarrow \boxed{a+b=0}$

eq^n of lines H_2C is $y = \frac{3}{2}x$

$\Rightarrow$ point $C \equiv \left(\frac{1}{4}, \frac{3}{8}\right)$ lies on $ax + by - 1 = 0$

$$\Rightarrow \frac{a}{4}+\frac{3}{8}b-1=0 \Rightarrow \frac{a}{4}-\frac{3}{8}a-1=0$$

$\Rightarrow a = -8$, $b = 8$

$|a - b| = 16$

25. Let $\alpha = \sum_{r=0}^{n} (4r^2 + 2r + 1)\,{}^nC_r$ and

$\beta = \left(\sum_{r=0}^{n} \frac{{}^nC_r}{r+1}\right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$, then the value of n is _____.

Answer (5)

Sol. $\alpha = \sum_{r=0}^{n} (4r^2 + 2r + 1)\ {}^nC_r$

$$= 4\sum_{r=0}^{n} r^2\ {}^nC_r + 2\sum_{r=0}^{n} r.\ {}^nC_r + \sum_{r=0}^{n} {}^nC_r$$

$= 4n(n + 1)2^{n-2} + 2.\ n.\ 2^{n-1} + 2^n$

$= 2^n(n(n + 1) + n + 1) = 2^n(n + 1)^2$

$$\beta = \sum_{r=0}^{n}\left(\frac{{}^nC_r}{r+1}\right) + \left(\frac{1}{n+1}\right)$$

$$(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$$

$$\int_0^1 (1+x)^n dx = \sum_{r=0}^{n} \frac{{}^nC_r x^{r+1}}{r+1}\Bigg|_0^1 = \sum_{r=0}^{n} \frac{{}^nC_r}{r+1}$$

$$\frac{(1+x)^{n+1}}{n+1}\Bigg|_0^1 = \frac{2^{n+1}-1}{n+1}$$

$$\Rightarrow \beta = \frac{2^{n+1}-1+1}{(n+1)} = \frac{2^{n+1}}{n+1}$$

$$\Rightarrow \frac{2\alpha}{\beta} = \frac{2^{n+1}(n+1)^2}{\left(\frac{2^{n+1}}{n+1}\right)} = (n+1)^3 \in (140, 281)$$

$\Rightarrow (n + 1)^3 = 216$

$\Rightarrow n + 1 = 6 \Rightarrow n = 5$

26. The value of

$$\lim_{x\to 0} 2\left(\frac{1-\cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x}\ldots\sqrt[10]{\cos 10x}}{x^2}\right)$$ is

_____.

Answer (55)

Sol. $$\lim_{x\to 0} 2\left(\frac{1-\cos x\,(\cos 2x)^{\frac{1}{2}}\,(\cos 3x)^{\frac{1}{3}}\ldots(\cos 10x)^{\frac{1}{10}}}{x^2}\right)$$

$\left(\frac{0}{0}\text{ form}\right)$

Using L' hospital

$$2\lim_{x\to 0} \frac{\sin x(\cos 2x)^{\frac{1}{2}}\ldots(\cos 10x)^{\frac{1}{10}} + \ldots(\sin 2x)\,(\cos x)\,(\cos 3x)^{\frac{1}{3}} + \ldots}{2x}$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{\sin x}{x} + \frac{\sin 2x}{x} + \ldots + \frac{\sin 10x}{x}\right)$$

$= 1 + 2 + \ldots + 10 = 55$

27. If the range of $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}, \theta \in \mathbb{R}$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to _____.

Answer (96)

Sol. $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}, \theta \in R$

$$= 1 + \frac{2\cos^2\theta}{\sin^4\theta + \cos^2\theta} = 1 + \frac{2\cos^2\theta}{\cos^4\theta - \cos^2\theta + 1}$$

$$f(\theta) = 1 + \frac{2}{\left(\cos^2\theta + \frac{1}{\cos^2\theta} - 1\right)}, \cos\theta \neq 0$$

$$\cos^2\theta + \frac{1}{\cos^2\theta} \geq 2 \Rightarrow \cos^2\theta + \frac{1}{\cos^2\theta} - 1 \in [1, \infty)$$

$$\frac{1}{\cos^2\theta + \frac{1}{\cos^2\theta} - 1} \in (0, 1]$$

$f(\theta) \in (1, 3]$

When $\cos\theta = 0$; $f(\theta) = 1$

$\Rightarrow f(\theta) \in [1, 3] \Rightarrow \beta = 3, \alpha = 1 \Rightarrow \frac{\alpha}{\beta} = \frac{1}{3}$

Sum of infinite G.P. $= \left(\frac{64}{1-\frac{1}{3}}\right) = 64 \times \frac{3}{2} = 96$

28. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n, then n is equal to _____.

Answer (7)

Sol. $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix} = 3\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$A^4 = 9\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$$

$$A^8 = 81\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A^{12} = 729\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{13} = 729\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^{13} = \begin{bmatrix} 1458 & -729 \\ 729 & 729 \end{bmatrix}$$

Sum = 2187 = 3^n

$3^7 = 3^n$

$\boxed{n = 7}$

29. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and yellow balls. If $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively, then $7\bar{X} + 4\bar{Y}$ is equal to ____.

Answer (17)

Sol.

X	3	2	1	0
Y	0	1	2	3

$$\bar{X} = \sum X\, p(X)$$

$$\bar{Y} = \sum Y p(Y)$$

$$P(X=3) = P(Y=0) = \frac{{}^5C_3 \cdot {}^4C_0}{{}^9C_3} = \frac{{}^5C_2}{{}^9C_3} = \frac{5}{42}$$

$$P(X=2) = P(Y=1) = \frac{{}^5C_2 \cdot {}^4C_1}{{}^9C_3} = \frac{10}{21}$$

$$P(X=1) = P(Y=2) = \frac{{}^5C_1 \cdot {}^4C_2}{{}^9C_3} = \frac{5}{14}$$

$$P(X=0) = P(Y=3) = \frac{{}^5C_0 \cdot {}^4C_3}{{}^9C_3} = \frac{4}{84} = \frac{1}{21}$$

$$\bar{X} = 3\times\frac{5}{42} + 2\times\frac{10}{21} + \frac{5}{14} + 0\times\frac{1}{21} = \frac{15+40+15}{42} = \frac{70}{42}$$

$$\bar{Y} = 0\times\frac{5}{42} + 1\times\frac{10}{21} + 2\times\frac{5}{14} + 3\times\frac{1}{21} = \frac{20+30+6}{42} = \frac{56}{42}$$

$$\Rightarrow \quad 7\bar{X} + 4\bar{Y} = 17$$

30. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to ____.

Answer (36.00)

Sol. Possible triplets for which number is divisible by 3

(2, 3, 7), (2, 3, 4), (3, 5, 7), (3, 5, 4)

$\therefore$ Number of required numbers = ${}^5C_3 \cdot 3! - 4 \times 3!$

= 3! × (6)

= 6 × 6 = 36

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. Young's modulus is determined by the equation given by $Y = 49000\frac{m}{l}\frac{\text{dyne}}{\text{cm}^2}$ where M is the mass and l is the extension of wire used in the experiment. Now error in Young modules (Y) is estimated by taking data from M-l plot in graph paper. The smallest scale divisions are 5 g and 0.02 cm along load axis and extension axis respectively. If the value of M and l are 500 g and 2 cm respectively then percentage error of Y is

(1) 0.2% (2) 0.02%

(3) 0.5% (4) 2%

Answer (4)

Sol. $\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta l}{l}$

$\frac{\Delta Y}{Y} = \frac{5}{500} + \frac{.02}{2}$

% age = 1 + 1 = 2%

32. A LCR circuit is at resonance for a capacitor C, inductance L and resistance R. Now the value of resistance is halved keeping all other parameters same. The current amplitude at resonance will be now

(1) Halved (2) Same

(3) Double (4) Zero

Answer (3)

Sol. At resonance $i = \frac{v}{R}$

If $R \to \frac{R}{2}$

$\Rightarrow \quad i \to 2i$

33. Two planets A and B having masses m_1 and m_2 move around the sun in circular orbits of r_1 and r_2 radii respectively. If angular momentum of A is L and that of B is $3L$, the ratio of time period $\left(\frac{T_A}{T_B}\right)$ is

(1) $\frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$ (2) $27\left(\frac{m_1}{m_2}\right)^3$

(3) $\left(\frac{r_1}{r_2}\right)^3$ (4) $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$

Answer (1)

Sol. $\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$ ...(i)

$m_1 v_1 r_1 = h$

$m_2 v_2 r_2 = 32$

$\Rightarrow \quad \frac{v_1}{v_2} = \frac{1}{3}\frac{m_2 r_2}{m_1 r_1}$ ...(ii)

From (i) & (ii)

$\sqrt{\frac{r_2}{r_1}} = \frac{1}{3}\frac{m_2}{m_1}\frac{r_2}{r_1}$

$\frac{3m_1}{m_2} = \sqrt{\frac{r_2}{r_1}}$

$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{m_2}{3m_1}\right)^3 = \frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$

34. Correct Bernoulli's equation is (symbols have their usual meaning)

(1) $P + \rho gh + \rho v^2 = \text{constant}$

(2) $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

(3) $P + mgh + \frac{1}{2}mv^2 = \text{constant}$

(4) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Answer (4)

Sol. $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

35. Binding energy of a certain nucleus is 18×10^8 J. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus:

(1) 2 μg

(2) 0.2 μg

(3) 10 μg

(4) 20 μg

Answer (4)

Sol. $\Delta m = \frac{BE}{C^2} = \frac{18 \times 10^8}{9 \times 10^{16}} = 2 \times 10^{-8}$ kg

$= 2 \times 10^{-5}$ g

$= 20$ μg

36. In an expression $a \times 10^b$:

(1) b is order of magnitude of $a \le 5$

(2) a is order of magnitude for $b \le 5$

(3) b is order of magnitude for $a \ge 5$

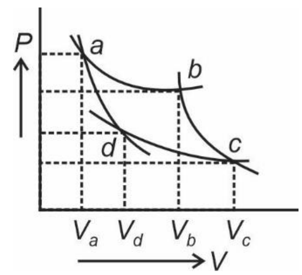
(4) b is order of magnitude for $5 < a \le 10$

Answer (1)

Sol. In expression $a \times 10^b$, If $a \le 5$; $a \approx 1$ by round off

$\Rightarrow$ Order B

37. Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram. The relation between the ratio $\frac{V_a}{V_d}$ and the ratio $\frac{V_b}{V_c}$ is



(1) $\frac{V_a}{V_d} \ne \frac{V_b}{V_c}$

(2) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^2$

(3) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$

(4) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$

Answer (4)

Sol. (1) $P_aV_a = P_bV_b$ …(i)

$P_cV_c = P_dV_d$ …(ii)

(2) $P_aV_a^{\gamma-1} = P_dV_d^{\gamma-1}$ …(iii)

$P_bV_b^{\gamma-1} = P_cV_c^{\gamma-1}$ …(iv)

(i) ÷ (iii) $\Rightarrow \frac{V_a}{V_a^{\gamma-1}} = \frac{P_bV_b}{P_dV_d^{\gamma-1}}$ (v)

(ii) ÷ (iv) $\Rightarrow \frac{V_c}{V_c^{\gamma-1}} = \frac{P_dV_d}{P_bV_b^{\gamma-1}}$ (vi)

(v) × (vi) $\Rightarrow \frac{V_a}{V_a^{\gamma-1}}\frac{V_c}{V_c^{\gamma-1}} = \frac{P_bV_b}{P_dV_d^{\gamma-1}} \times \frac{P_dV_d}{P_bV_b^{\gamma-1}}$

$\Rightarrow V_a^{\gamma-2}V_c^{\gamma-2} = V_d^{\gamma-1}V_b^{\gamma-2}$

$\Rightarrow \frac{V_a}{V_d} = \frac{V_b}{V_c}$

38. Paramagnetic substances:

A. Align themselves along the directions of external magnetic field.

B. Attract strongly towards external magnetic field.

C. Has susceptibility little more than zero.

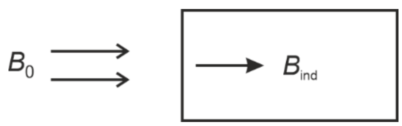
D. Move from a region of strong magnetic field to weak magnetic field.

Choose the **most appropriate** answer from the options given below:

(1) B, D only (2) A, B, C, D

(3) A, B, C only (4) A, C only

Answer (4)

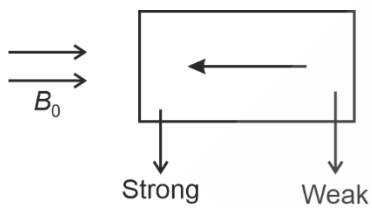
Sol. (1) 

Magnetic Dipole align

$B_{ind} << B_0$

(2) Paramagnetic substance are attracted weakly.

(3) $0 < \chi << 1$

(4) 

39. The diameter of a sphere is measured using a vernier calliper whose 9 divisions of main scale are equal to 10 divisions of vernier scale. The shortest division on the main scale is equal to 1 mm. The main scale reading is 2 cm and second division of vernier scale coincides with a division on main scale. If mass of the sphere is 8.635 g, the density of the sphere is

(1) 2.0 g/cm^3 (2) 1.7 g/cm^3

(3) 2.5 g/cm^3 (4) 2.2 g/cm^3

Answer (1)

Sol. $LC = \frac{1}{10}$mm

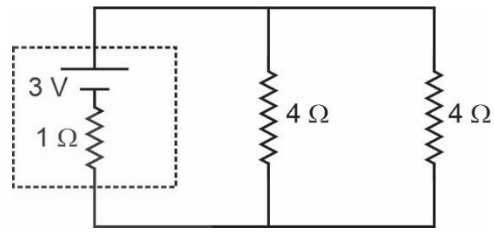
Reading = 2 cm + $2 \times \frac{1}{10}$ mm = 2.02 cm

r = 1.01 cm

$$r = \frac{8.635}{\frac{4}{3}\pi(1.01)^3} = 2.0018$$

$r \approx 2.0$

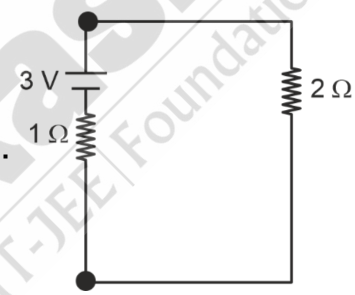
40. In the given circuit, the terminal potential difference of the cell is:



(1) 3 V (2) 4 V

(3) 2 V (4) 1.5 V

Answer (3)

Sol. 

$$V_T = \left(\frac{3}{1+2}\right) \times 2 = 2\text{ V}$$

41. Average force exerted on a non-reflecting surface at normal incidence is 2.4 × 10^{-4} N. If 360 W/cm^2 is the light energy flux during span of 1 hour 30 minutes, then the area of the surface is:

(1) 0.1 m^2 (2) 0.2 m^2

(3) 0.02 m^2 (4) 20 m^2

Answer (3)

Sol. $P = \frac{I}{C}$

$F = PA = \frac{I}{C} \times A$

$A = \frac{FC}{I} = \frac{2.4 \times 10^{-4} \times 3 \times 10^{8}}{360} \times 10^{-4}$

$= \frac{72}{3600} = 0.02\,\text{m}^2$

42. A mixture of one mole of monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature (27°C). the ratio of specific heat of gases at constant volume respectively is:

(1) $\frac{5}{3}$ (2) $\frac{3}{5}$

(3) $\frac{3}{2}$ (4) $\frac{7}{5}$

Answer (2)

Sol. $C_V = \frac{3}{2}R$ for monoatomic

$C_V = \frac{5}{2}R$ for diatomic

$\text{Ratio} = \frac{3}{5}$

43. Three bodies A, B and C have equal kinetic energies and their masses are 400 g, 1.2 kg and 1.6 kg respectively. The ratio of their linear momenta is:

(1) $\sqrt{2} : \sqrt{3} : 1$ (2) $1 : \sqrt{3} : 2$

(3) $\sqrt{3} : \sqrt{2} : 1$ (4) $1 : \sqrt{3} : \sqrt{2}$

Answer (2)

Sol. $P = \sqrt{2mk}$

$P_1 : P_2 : P_3 = \sqrt{0.4} : \sqrt{1.2} : \sqrt{1.6}$

for same kinetic energy

$= 2 : 2\sqrt{3} : 4$

$1 : \sqrt{3} : 2$

44. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies ($K_B : K_A$) is

(1) 1 : 1 (2) $m_Bv_B : m_Av_A$

(3) $m_B : m_A$ (4) $v_B : v_A$

Answer (4)

Sol. $K = \frac{1}{2}mv^2 = \frac{1}{2}PV$

$\Rightarrow \frac{K_1}{K_2} = \frac{v_1}{v_2}$ for same momentum

45. A clock has 75 cm, 60 cm long second hand and minute hand respectively. In 30 minutes duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly (Take π = 3.14):

(1) 220.0

(2) 118.9

(3) 140.5

(4) 139.4

Answer (4)

Sol. 30 minutes $\equiv$ 30 round of second hand

$\equiv \frac{1}{2}$ round of minute hand

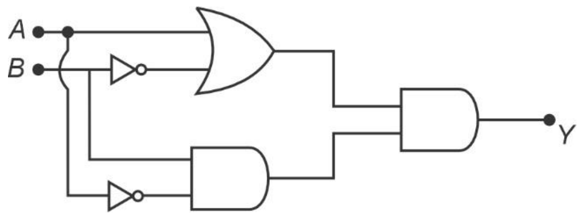
$d_s - d_m = 30\{2\pi r_s\} - \frac{1}{2}\{2\pi r_m\}$

$\pi\{60 \times 75 - 60\}$

$= \pi \times 60 \times 74 = 13941.6$ cm

≈ 139.4 m

46. The output Y of following circuit for given inputs is:



(1) $A \cdot B$ (2) $\Lambda \cdot B(\Lambda + B)$

(3) 0 (4) $\bar{A} \cdot B$

Answer (3)

Sol. $Y = (A + \bar{B}) \cdot (\bar{A} \cdot B)$

$= A \cdot \bar{A} \cdot B + \bar{B} \cdot \bar{A} \cdot B$

= 0 + 0 = 0

47. A player caught a cricket ball of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is:

(1) 150 N (2) 300 N

(3) 30 N (4) 3 N

Answer (3)

Sol. $F = \frac{\Delta P}{\Delta t} = \frac{150}{1000} \times \frac{20}{0.1} = 30$ N

48. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is:

(1) $\sqrt{ab}$ (2) ab

(3) $\frac{a}{b}$ (4) $\frac{b}{a}$

Answer (3)

Sol. $V_A = V_B = \frac{Q_A}{4\pi \epsilon_0 a} = \frac{Q_B}{4\pi \epsilon_0 b}$

$\Rightarrow \frac{Q_A}{Q_B} = \frac{a}{b}$

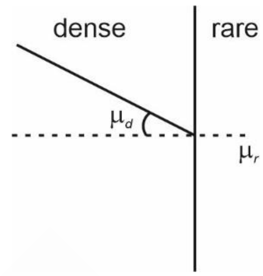
49. Critical angle of incidence for a pair of optical media is 45°. The refractive indices of first and second media are in the ratio

(1) $1:\sqrt{2}$ (2) 2 : 1

(3) $\sqrt{2}:1$ (4) 1 : 2

Answer (3)

Sol.



$\sin\theta_C = \frac{1}{\sqrt{2}} = \frac{\mu_d}{\mu_r}$

$\frac{\mu_d}{\mu_r} = \sqrt{2}$

50. A proton and an electron are associated with same de-Broglie wavelength. The ratio of their kinetic energies is:

(Assume $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.0 \times 10^{-31}$ kg and m_p = 1836 times m_e)

(1) $1:\frac{1}{1836}$ (2) $1:\frac{1}{\sqrt{1836}}$

(3) 1 : 1836 (4) $1:\sqrt{1836}$

Answer (3)

Sol. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk}}$

$\Rightarrow m_e k_e = m_p k_p$

$\frac{m_e}{m_p} = \frac{k_p}{k_e}$

$\Rightarrow \frac{k_p}{k_e} = \frac{1}{1836}$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

51. In an alpha particle scattering experiment distance of closest approach for the α particle is 4.5 × 10^{-14} m. If target nucleus has atomic number 80, then maximum velocity of α-particle is ________ × 10^5 ms approximately.

($\frac{1}{4\pi\in_0} = 9\times10^9$ SI unit, mass of α particle = 6.75 × 10^{-27} kg)

Answer (156)

Sol. $\frac{1}{2}mv_0^2 = \frac{1}{4\pi\in_0}\frac{z(e)}{r}$

$$\frac{1}{2}\times6.72\times10^{-27}v_0^2 = 9\times10^9\times\frac{80\times2\times1.6\times1.6\times10^{-19}\times10^{-19}}{4.5\times10^{-14}}$$

$$v_0^2 = \frac{2\times9\times80\times1.6\times1.6\times2}{6.72\times4.5}\times10^{-38+14+27+9}$$

$$v_0^2 = 243.8\times10^{12}$$

v_0 = 15.6 × 10^6

v_0 = 156 × 10^5

52. A closed and open organ pipe have same lengths. If the ratio of frequencies of their seventh orvertones is $\left(\frac{a-1}{a}\right)$, then the value of a is

Answer (16)

Sol. $f_o = \frac{v}{2l} \quad\Rightarrow\quad f_{o_7} = 8\frac{v}{2l}$

$f_c = \frac{v}{4l} \quad\Rightarrow\quad f_{c_7} = 15\frac{v}{4l}$

$$\frac{f_{c_7}}{f_{o_7}} = 15\frac{v}{4l}\frac{2l}{8v} = \frac{30}{32} = \frac{15}{16}$$

53. An electron with kinetic energy 5 eV enters a region of uniform magnetic field of 3 μT perpendicular to its direction. An electric field E is applied perpendicular to the direction of velocity and magnetic field. The value of E, so that electron moves along the same path, is ______ NC^{-1}.

(Given, mass of electron = 9 × 10^{-31} kg, electric charge = 1.6 × 10^{-19} C)

Answer (4)

Sol. $E = VB$

$$E = \sqrt{\frac{2k}{m}}\times B$$

$$= \sqrt{\frac{2\times5\times1.6\times10^{-19}}{9\times10^{-31}}}\times3\times10^{-6}$$

$$= \sqrt{\frac{16}{9}\times10^{12}}\times3\times10^{-6}$$

$$= \frac{4}{3}\times10^6\times3\times10^{-6}$$

= 4 N/C

54. Resistance of a wire at 0 °C, 100 °C and t °C is found to be 10 Ω, 10.2 Ω and 10.95 Ω respectively. The temperature t in Kelvin scale is ______.

Answer (748)

Sol. From thermometry

$$\frac{t°-0}{100-0} = \frac{10.95-10.00}{10.2-10.00}$$

$$\frac{t°}{100} = \frac{0.95}{0.20} = \frac{19}{4}$$

$$t° = \frac{19}{4}\times100 = 19\times25 = 475$$

k = 748

55. A liquid column of height 0.04 cm balances excess pressure of a soap bubble of certain radius. If density of liquid is 8×10^3 kg m^{-3} and surface tension of soap solution is 0.28 Nm^{-1}, then diameter of the soap bubble is _______ cm.

(if $g = 10$ m s^{-2})

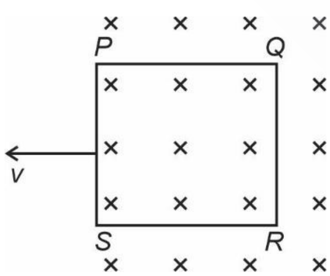
Answer (7)

Sol. $\frac{4S}{r} = h\rho g$

$$\frac{4 \times 0.28}{\left(\frac{d}{2}\right)} = 4 \times 10^{-4} \times 8 \times 10^3 \times 10$$

$$\frac{8 \times 0.28}{32} = d = \frac{8 \times 28}{32} \text{ cm} = 7 \text{ cm}$$

56. A square loop $PQRS$ having 10 turns, area 3.6×10^{-3} m^2 and resistance 100 Ω is slowly and uniformly being pulled out of a uniform magnetic field of magnitude $B = 0.5$ T as shown. Work done in pulling the loop out of the field in 1.0 s is ________ $\times 10^{-6}$ J.



Answer (3)

Sol. $A = 36 \times 10^{-4}$ m^2

$l = 6 \times 10^{-2}$ m

$= 6$ cm

$$v = \frac{6 \text{ cm}}{1 \text{ sec}} = 6 \text{ cm/s}$$

$$\varepsilon = Blvn^2 = 0.5 \times \frac{6}{100} \times \frac{6}{100}$$

$= 18 \times 10^{-4}$ V.

$$E = \frac{n^2\varepsilon^2}{R}t = 100 \times \frac{18 \times 18 \times 10^{-4} \times 10^{-4}}{10^2} \times 1$$

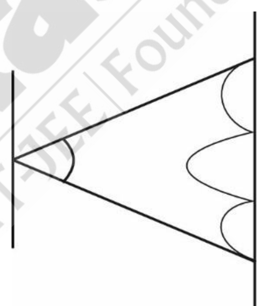
$= 324 \times 10^{-10} \times 10^2$

$= 3.24 \times 10^{-6}$

57. A parallel beam of monochromatic light of wavelength 600 nm passes through single slit of 0.4 mm width. Angular divergence corresponding to second order minima would be ______ $\times 10^{-3}$ rad.

Answer (6)

Sol.

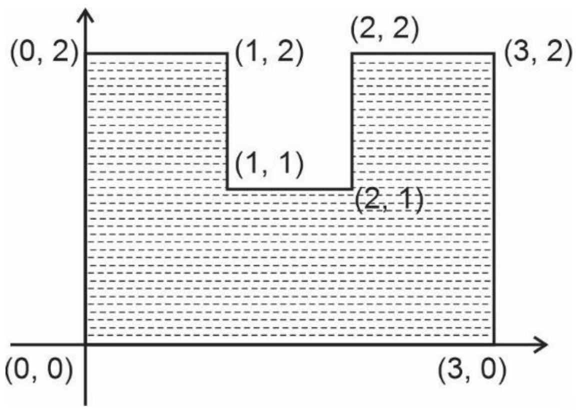


$$\theta = (2)\left(\frac{2\lambda}{a}\right)$$

$$= \frac{4\lambda}{a} = \frac{4 \times 600 \times 10^{-9}}{0.4 \times 10^{-3}}$$

$= 6 \times 10^{-3}$

58. A uniform thin metal plate of mass 10 kg with dimensions is shown. The ratio of x and y coordinates of center of mass of plate in $\frac{n}{9}$. The value of n is _______.



Answer (15)

Sol. $x_{cm} = 1.5$

$M_{+} = 6\ \sigma$ $\quad y_{+} = 1$

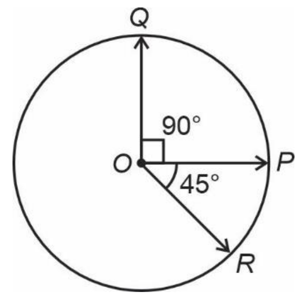
$M_{-} = -\ \sigma$ $\quad y_{-} = 1.5$

$$y_{cm} = \frac{6\sigma \times 1 + (-\sigma) \times 1.5}{6\sigma - \sigma}$$

$$= \frac{6 - 1.5}{5} = \frac{4.5}{5} = 0.9$$

$$\frac{x}{y} = \frac{1.5}{0.9} = \frac{15}{9}$$

59. Three vectors $\overrightarrow{OP}, \overrightarrow{OQ}$ and $\overrightarrow{OR}$ each of magnitude A are acting as shown in figure. The resultant of the three vectors is $A\sqrt{x}$. The value of x is ________.



Answer (3)

Sol. $\left(\vec{P} + \vec{Q}\right) = \sqrt{2}A$

$R = A$

$\theta = 90°$

Resultant $= \sqrt{3}A$

60. An electric field, $\vec{E} = \frac{2\hat{i} + 6\hat{j} + 8\hat{k}}{\sqrt{6}}$ passes through the surface of 4 m² area having unit vector $\hat{n} = \left(\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}\right)$. The electric flux for that surface is _______ V m.

Answer (12)

Sol. $Q = \vec{E} \cdot \vec{A}$

$$= 4\left(\frac{4 + 6 + 8}{6}\right) = \frac{18 \times 4}{6} = 12$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Match **List I** with **List II**

LIST I (Name of the test)		List II (Reaction sequence involved) [M is metal]	
A.	Borax bead test	I.	$MCO_3 \rightarrow MO \xrightarrow[+\Delta]{Co(NO_3)_2} CoO \cdot MO$
B.	Charcoal cavity test	II.	$MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$
C.	Cobalt nitrate test	III.	$MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$
D.	Flame test	IV.	$MSO_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II (2) A-III, B-I, C-II, D-IV

(3) A-III, B-I, C-IV, D-II (4) A-III, B-II, C-IV, D-I

Answer (1)

Sol. (A) Borax bead test

$$MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$$

(B) Charcoal cavity test

$$MSO_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$$

(C) Cobalt nitrate test

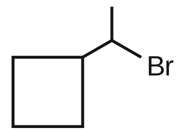
$$MCO_3 \rightarrow MO \xrightarrow[+\Delta]{CO(NO_3)_2} CoO \cdot MO$$

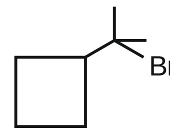
(D) Flame test

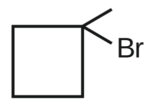
$$MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$$

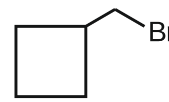
So, A $\rightarrow$ (III), B $\rightarrow$ (IV), C $\rightarrow$ (I), D $\rightarrow$ (II).

62. Which among the following compounds will undergo fastest S_N2 reaction.

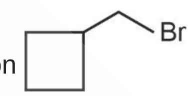
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. Rate of S_N2 reaction depends on steric hindrance, less the steric hindrance more will be rate of reaction  , it will undergo S_N2 reaction fastest.

63. Number of Complexes with even number of electrons in t_{2g} orbitals is

$[Fe(H_2O)_6]^{2+}$, $[Co(H_2O)_6]^{2+}$, $[Co(H_2O)_6]^{3+}$, $[Cu(H_2O)_6]^{2+}$, $[Cr(H_2O)_6]^{2+}$

(1) 2 (2) 3

(3) 5 (4) 1

Answer (2)

Sol. $[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} \Rightarrow 3d^6 \Rightarrow sp^3d^2 \Rightarrow t_{2g}^4e_g^2$

$[Co(H_2O)_6]^{2+} \Rightarrow Co^{2+} \Rightarrow 3d^7 \Rightarrow sp^3d^2 \Rightarrow t_{2g}^5e_g^2$

$[Co(H_2O)_6]^{3+} \Rightarrow Co^{3+} \Rightarrow 3d^6 \Rightarrow d^2sp^3 \Rightarrow t_{2g}^6e_g^0$

$[Cu(H_2O)_6]^{2+} \Rightarrow Cu^{2+} \Rightarrow 3d^9 \Rightarrow sp^3d^2 \Rightarrow t_{2g}^6e_g^3$

$[Cr(H_2O)_6]^{2+} \Rightarrow Cr^{2+} \Rightarrow 3d^4 \Rightarrow sp^3d^2 \Rightarrow t_{2g}^3e_g^1$

Three complexes having even number of electrons in t_{2g}.

64. Match **List I** with **List II**

LIST I (Compound)		LIST II (Colour)	
A.	$Fe_4[Fe(CN)_6]_3 \cdot xH_2O$	I.	Violet
B.	$[Fe(CN)_5NOS]^{4-}$	II.	Blood Red
C.	$[Fe(SCN)]^{2+}$	III.	Prussian Blue
D.	$(NH_4)_3PO_4 \cdot 12MoO_3$	IV.	Yellow

Choose the **correct** answer from the options given below.

(1) A-III, B-I, C-II, D-IV (2) A-I, B-II, C-III, D-IV

(3) A-II, B-III, C-IV, D-I (4) A-IV, B-I, C-II, D-III

Answer (1)

Sol. $Fe_4[Fe(CN)_6]_3 \cdot xH_2O \Rightarrow$ Prussian blue

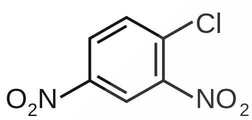
$[Fe(CN)_5NOS]^{4-} \Rightarrow$ Violet

$[Fe(SCN)]^{2+} \Rightarrow$ Blood red

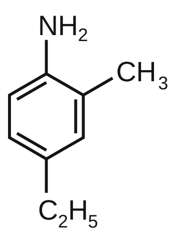
$[NH_4]_3PO_4 \cdot 12MoO_3 \Rightarrow$ Yellow

So, A $\rightarrow$ (III), B $\rightarrow$ (I), C $\rightarrow$ (II), D $\rightarrow$ (IV).

65. Given below are two statements:



Statement I: Compound A IUPAC name of Compound A is 4-chloro-1,3-dinitrobenzene.



Statement II: Compound B IUPAC name of Compound B is 4-ethyl-2-methylaniline.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

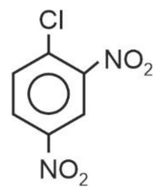
(1) Statement I is correct but Statement II is incorrect.

(2) Both Statement I and Statement II are incorrect.

(3) Statement I is incorrect but Statement II is correct.

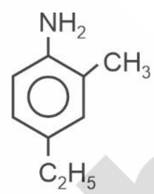
(4) Both Statement I and Statement II are correct.

Answer (3)

Sol. 

Correct name is 1-chloro-2,4-dinitro benzene

Statement-I is incorrect.



4-ethyl-2-methyl aniline

Statement-II is correct.

66. For the given hypothetical reactions, the equilibrium constants are as follows:

$X \rightleftharpoons Y; \; K_1 = 1.0$

$Y \rightleftharpoons Z; \; K_2 = 2.0$

$Z \rightleftharpoons W; \; K_3 = 4.0$

The equilibrium constant for the reaction $X \rightleftharpoons W$ is

(1) 8.0 (2) 7.0

(3) 12.0 (4) 6.0

Answer (2)

Sol. $X \rightleftharpoons Y; \; K_1 = 1.0$

$Y \rightleftharpoons Z; \; K_2 = 2.0$

$Z \rightleftharpoons W; \; K_3 = 4.0$

For the equilibrium constant value of

$X \rightleftharpoons W; \; K = K_1 \cdot K_2 \cdot K_3$

$K = 1 \times 2 \times 4 = 8$

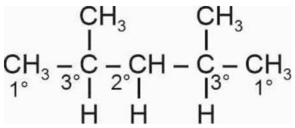
67. In the given compound, the number of 2º carbon atom/s is ________.

$CH_3 - C(CH_3) - CH - C(CH_3) - CH_3$ (with H on each of the C(CH₃), CH and C(CH₃) carbons)

(1) One (2) Three

(3) Four (4) Two

Answer (1)

Sol. 

Number of 2° carbon atom = 1

68. An octahedral complex with the formula $CoCl_3 \cdot nNH_3$ upon reaction with excess of $AgNO_3$ solution gives 2 moles of AgCl. Consider the oxidation state of Co in the complex is 'x'. The value of "x + n" is

(1) 5 (2) 6

(3) 8 (4) 3

Answer (3)

Sol. $CoCl_3 \cdot nNH_3 \xrightarrow[\text{excess}]{AgNO_3} 2$ mol of AgCl

it means 2 $Cl^{\ominus}$ ions will present in ionisation sphere.

So coordination compound should be

$[Co(NH_3)_5Cl]Cl_2$

Oxidation state of Cobalt is +3

x = 3

n = 5

(x + n) = 8

69. Among the following halogens

F_2, Cl_2, Br_2 and I_2

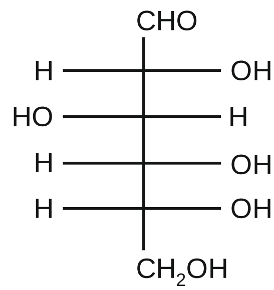
Which can undergo disproportionation reactions?

(1) F_2 and Cl_2 (2) Only I_2

(3) Cl_2, Br_2 and I_2 (4) F_2, Cl_2 and Br_2

Answer (3)

Sol. As fluorine shows an oxidation of only –1 and has tendency to reduce itself only, it cannot undergo disproportionation reaction, other Cl_2, Br_2 and I_2 can undergo disproportionation reaction.

70. 

The **incorrect** statement regarding the given structure is

(1) has 4 asymmetric carbon atom

(2) despite the presence of –CHO does not give Schiff's test

(3) will coexist in equilibrium with 2 other cyclic structure

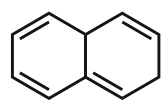
(4) can be oxidized to a dicarboxylic acid with Br_2 water

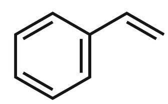
Answer (4)

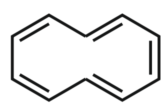
Sol. The given structure is of glucose.

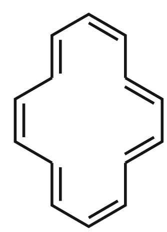
Glucose does not give Schiff test due to absence of aldehyde group in ring structure, glucose on reaction with Br_2/H_2O give gluconic acid which is mono carboxylic acid.

71. Which of the following are aromatic?

A. 

B. 

C. 

D. 

(1) A and C only (2) C and D only

(3) B and D only (4) A and B only

Answer (3)

Sol. Compound (C) is non planer due to trans annular hydrogen interaction and hence non aromatic.

Compound (B) and (D) are only aromatic compounds.

72. Combustion of glucose ($C_6H_{12}O_6$) produces CO_2 and water. The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is:

[Molar mass of glucose in g mol^{-1} = 180]

(1) 480 (2) 32

(3) 800 (4) 960

Answer (4)

Sol. $C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$

Mole of glucose $= \dfrac{900\text{ g}}{18\text{ g/mol}} = 5\text{ mol}$

1 mol glucose reacts with 6 mol of O_2, so 5 mol glucose will react with 30 mol O_2.

Mole of O_2 required = 30 mol

Mass of O_2 required = 32 × 30 g

= 960 g

73. Given below are two statements:

Statement I: $N(CH_3)_3$ and $P(CH_3)_3$ can act as ligands to form transition metal complexes.

Statement II: As N and P are from same group, the nature of bonding of $N(CH_3)_3$ and $P(CH_3)_3$ is always same with transition metals.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Statement I is correct but Statement II is incorrect.

(2) Both Statement I and Statement II are correct.

(3) Both Statement I and Statement II are incorrect.

(4) Statement I is incorrect but Statement II is correct.

Answer (1)

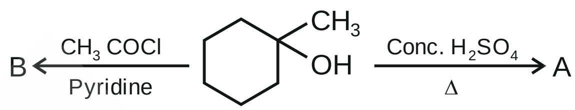
Sol. $N(CH_3)_3$ and $P(CH_3)_3$ both can acts as ligand as both have lone pair of electron.

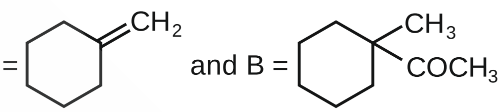
But bonding of both are different as $P(CH_3)_3$ has vacant d-orbital hence acts as sigma donor as well as π-acceptor but $N(CH_3)_3$ can acts only as σ-donor.

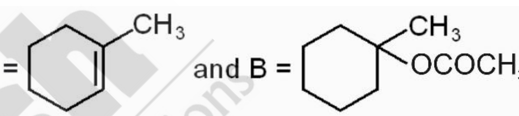
Statement – I is correct

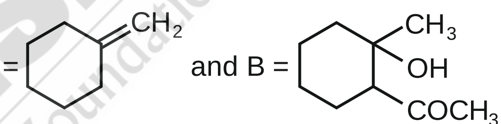
Statement – II is incorrect

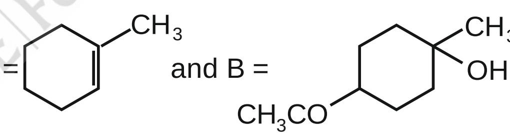
74. Identify the major products A and B respectively in the following set of reactions.



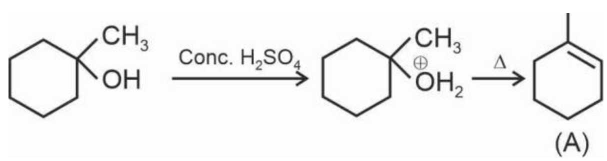
(1) A =  and B =

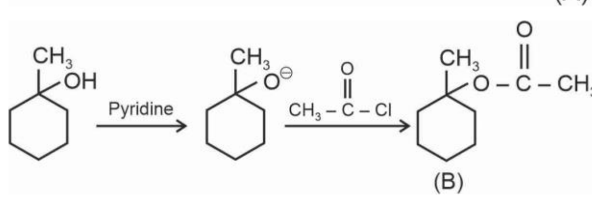
(2) A =  and B =

(3) A =  and B =

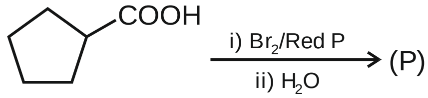
(4) A =  and B =

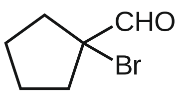
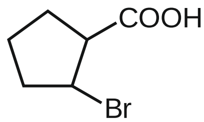
Answer (2)

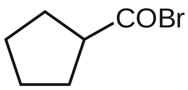
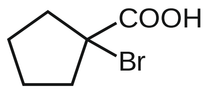
Sol. 



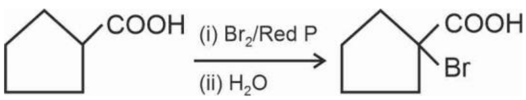
75. Identify the product (P) in the following reaction:



(1)  (2) 

(3)  (4) 

Answer (4)

Sol. 

Given reaction is HVZ reaction.

76. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A: The stability order of +1 oxidation state of Ga, In and Tl is Ga < In < Tl.

Reason R: The inert pair effect stabilizes the lower oxidation state down the group.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) **A** is false but **R** is true.

(2) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(4) **A** is true but **R** is false.

Answer (3)

Sol. Stability of +1 oxidation state increase down the group in group 13, due to inert pair effect.

77. Iron (III) catalyses the reaction between iodide and persulphate ions, in which

A. Fe^{3+} oxidises the iodide ion

B. Fe^{3+} oxidises the persulphate ion

C. Fe^{2+} reduces the iodide ion

D. Fe^{2+} reduces the persulphate ion

Choose the **most appropriate** answer from the options given below:

(1) A only (2) B only

(3) A and D only (4) B and C only

Answer (3)

Sol. $2I^{\ominus} + S_2O_8^{2-} \xrightarrow{Fe^{3+}} I_2 + 2SO_4^{2-}$

Mechanism

$2I^{\ominus} + Fe^{+3} \xrightarrow{Fe^{3+}} I_2 + Fe^{+2}$

$S_2O_8^{2-} + 2Fe^{2+} \longrightarrow 2Fe^{3+} + 2SO_4^{2-}$

Fe^{3+} oxidises iodide to iodine

Fe^{2+} reduces persulphate ion

A and D are correct

78. Thiosulphate reacts differently with iodine and bromine in the reactions given below:

$2S_2O_3^{2-} + I_2 \rightarrow S_4O_6^{2-} + 2I^-$

$S_2O_3^{2-} + 5Br_2 + 5H_2O \rightarrow 2SO_4^{2-} + 4Br^- + 10H^+$

Which of the following statement justifies the above dual behaviour of thiosulphate?

(1) Bromine undergoes oxidation and iodine undergoes reduction in these reactions

(2) Bromine is a weaker oxidant than iodine

(3) Bromine is a stronger oxidant than iodine

(4) Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reactions

Answer (3)

Sol. Br_2 oxidizes thiosulphate more than I_2 oxidises so, Br_2 is stronger oxidising agent or oxidant than I_2.

79. Match **List I** with **List II**

LIST I (Elements)		LIST II (Properties in their respective groups)	
A.	Cl, S	I.	Elements with highest electronegativity
B.	Ge, As	II.	Elements with largest atomic size
C.	Fr, Ra	III.	Elements which show properties of both metals and non-metal
D.	F, O	IV.	Elements with highest negative electron gain enthalpy

Choose the **correct** answer from the options given below:

(1) A-II, B-III, C-IV, D-I (2) A-IV, B-III, C-II, D-I

(3) A-II, B-I, C-IV, D-III (4) A-III, B-II, C-I, D-IV

Answer (2)

Sol. Cl, S $\Rightarrow$ Elements with highest negative electron gain enthalpy

Ge, As $\Rightarrow$ Elements which show property of both metals and non metals.

Fr, Ra $\Rightarrow$ Elements with largest atomic size.

F, O $\Rightarrow$ Elements with highest electronegativity

A-IV, B-III, C-II, D-I

80. Match **List I** with **List II**

LIST I (Molecule)		LIST II (Shape)	
A.	NH_3	I.	Square pyramid
B.	BrF_5	II.	Tetrahedral
C.	PCl_5	III.	Trigonal pyramidal
D.	CH_4	IV.	Trigonal bipyramidal

Choose the **correct** answer from the options given below:

(1) A-II, B-IV, C-I, D-III (2) A-III, B-I, C-IV, D-II

(3) A-IV, B-III, C-I, D-II (4) A-III, B-IV, C-I, D-II

Answer (2)

Sol. (A) $NH_3 \Rightarrow sp^3 \Rightarrow$ Trigonal pyramidal

(B) $BrF_5 \Rightarrow sp^3d^2 \Rightarrow$ Square pyramidal

(C) $PCl_5 \Rightarrow sp^3d \Rightarrow$ Trigonal bipyramidal

(D) $CH_4 \Rightarrow sp^3 \Rightarrow$ Tetrahedral

A $\rightarrow$ III, B $\rightarrow$ I, C $\rightarrow$ IV, D $\rightarrow$ II

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

81. Number of molecules from the following which are exceptions to octet rule is ______.

CO_2, NO_2, H_2SO_4 · BF_3. CH_4, SiF_4, ClO_2, PCl_5, BeF_2, C_2H_6, $CHCl_3$, CBr_4

Answer (6)

Sol. NO_2, H_2SO_4, BF_3, ClO_2, PCl_5, BeF_2

These are exception of octet rule

82. A solution containing 10 g of an electrolyte AB_2 in 100 g of water boils at 100.52°C. The degree of ionization of the electrolyte (α) is ____ $\times 10^{-1}$. (nearest integer)

[Given: Molar mass of AB_2 = 200 g mol^{-1}, K_b (molal boiling point elevation const. of water) = 0.52 K kg mol^{-1}, boiling point of water = 100°C : AB_2 ionises as $AB_2 \rightarrow A^{2+} 2B^-$]

Answer (5)

Sol. $\Delta T_b = 0.52°C$

$\Delta T_b = i \times k_b \times m$

$0.52 = i \times 0.52 \times \frac{10}{200 \times 0.1}$

$i = 2$

$\alpha = \frac{i-1}{n-1}$

$AB_2 \rightarrow A^{2+} + 2B^-$

$n = 3$

$\alpha = \frac{2-1}{3-1}$

$\alpha = \frac{1}{2} = 0.5$

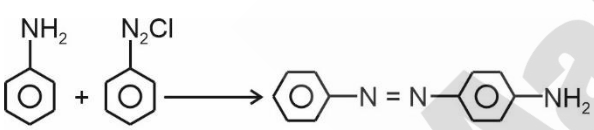
$= 5 \times 10^{-1}$

83. If 279 g of aniline is reacted with one equivalent of benzenediazonium chloride, the maximum amount of aniline yellow formed will be integer) ______ g. (nearest integer)

(consider complete conversion).

Answer (591)

Sol.



p-Aminoazobenene
(yellow dye)
(aniline yellow)

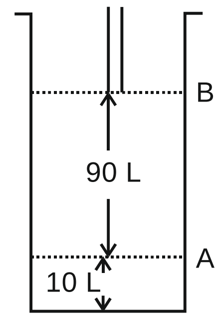
Mole of aniline = $\frac{279}{93}$ = 3 mol

Mole of aniline yellow formed = 3 mol

Mass = 3 × 197

= 591 g

84.



Consider the figure provided.

1 mol of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at 18° C. If the piston is moved to position B, keeping the temperature unchanged, then 'x' L atm work is done in this reversible process.

x = ________ L atm. (nearest integer)

[Given: Absolute temperature = °C + 273.15, R = 0.08206 L atm mol^{-1} K^{-1}]

Answer (55)

Sol. $V_1 = 100$ L

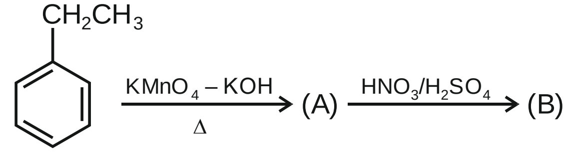
$V_2 = 10$ L

$W = -nRT\ln\frac{V_2}{V_1}$

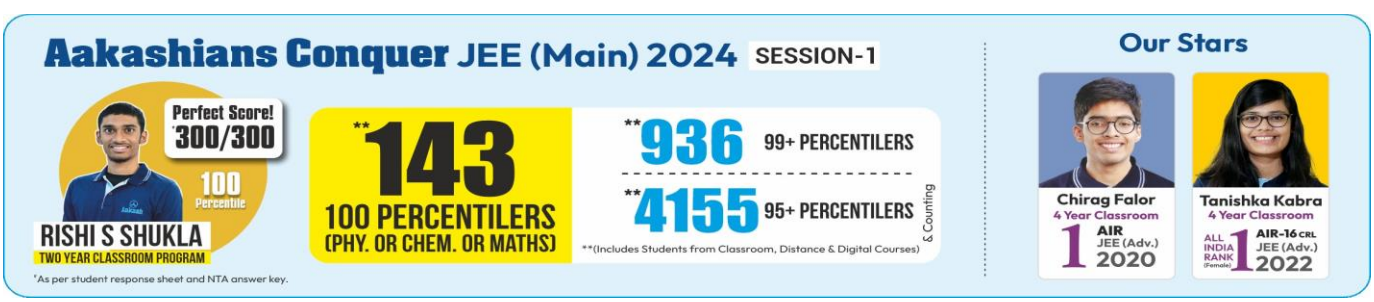
$= -1 \times 0.08206 \times 291.15 \times 2.303 \log\frac{10}{100}$

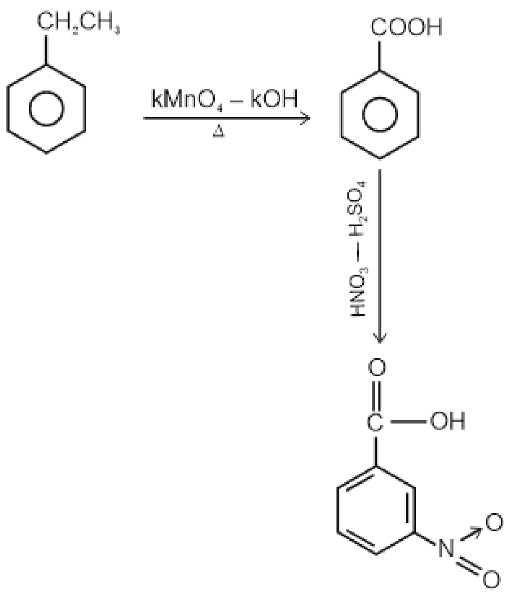
= 55 L atm

85. Major product B of the following reaction has ______ π-bond.



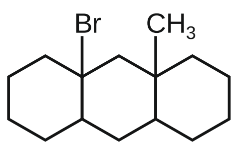
Answer (5)



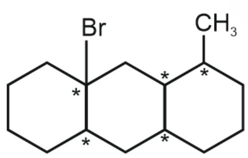
Sol. 

Number of π bonds in B = 5

86. The number of optical isomers in following compound is: __________



Answer (32)

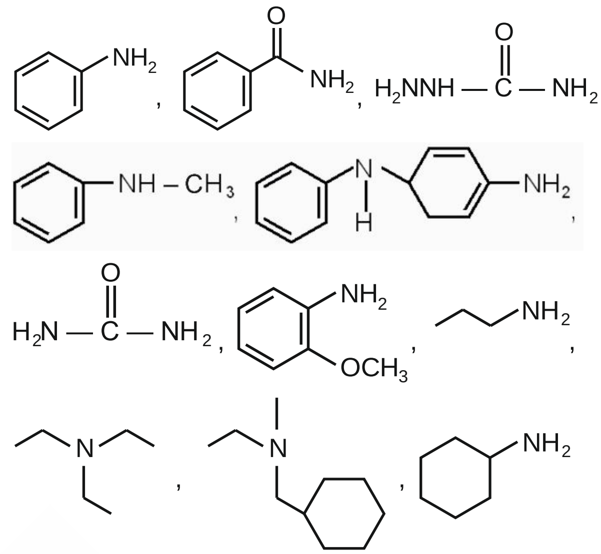
Sol. 

Total stereogenic centre = 5

Total optical isomer = 2^n

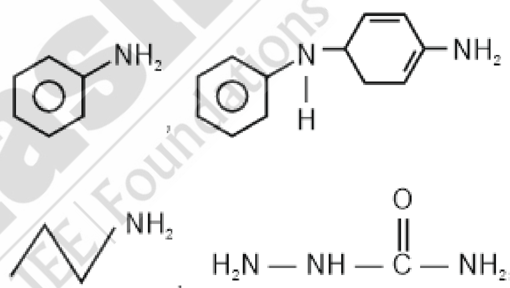
$= 2^5 = 32$

87. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsberg's reagent is _______.

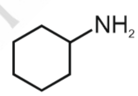


Answer (5)

Sol. $-NH_2$ group containing compound can give solid with Hinsberg's reagent, which is soluble in NaOH solution



and

 can give solid with Heinsberg reagent, which is soluble in NaOH solution

88. The 'spin only' magnetic moment value of MO_4^{2-} is ______ BM. (Where M is a metal having least metallic radii. among Sc, Ti, V, Cr, Mn and Zn).

(Given atomic number: Sc = 21, Ti = 22, V = 23, Cr = 24, Mn = 25 and Zn = 30)

Answer (5)

Sol. Chromium has least metallic radii among given metals.

$CrO_4^{2-} \Rightarrow Cr^{6+} \Rightarrow 3d^0$

Number of unpaired electron = 0

Magnetic moment = 0

89. Consider the following reaction

$A + B \rightarrow C$

The time taken for A to become 1/4th of its initial concentration is twice the time taken to become 1/2 of the same. Also, when the change of concentration of B is plotted against time, the resulting graph gives a straight line with a negative slope and a positive intercept on the concentration axis.

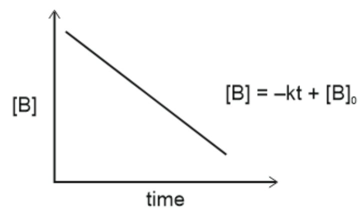
The overall order of the reaction is ________.

Answer (1)

Sol. $A + B \rightarrow C$

$$A \xrightarrow{t = x} \frac{A}{2} \xrightarrow{t = 2x} \frac{A}{4}$$

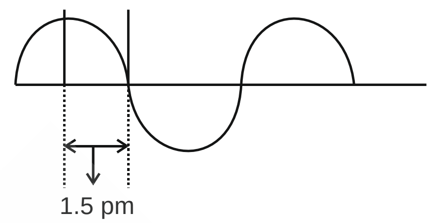
Order w.r.t. A = 1



Order w.r.t. B = zero order

Overall order = 1 + 0 = 1

90. A hypothetical electromagnetic wave is show below.



The frequency of the wave is x × 10^{19} Hz.

x = ___________ (nearest integer).

Answer (5)

Sol. $\lambda = 1.5 \times 4 = 6$ pm

$$\nu = \frac{C}{\lambda}$$

$$\nu = \frac{3 \times 10^8}{6 \times 10^{-12}}$$

$= 0.5 \times 10^{20}$

$\nu = 5 \times 10^{19}$

x = 5

❐ ❐ ❐