09/04/2024

Evening

Aakash

Medical | IIT-JEE | Foundations

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

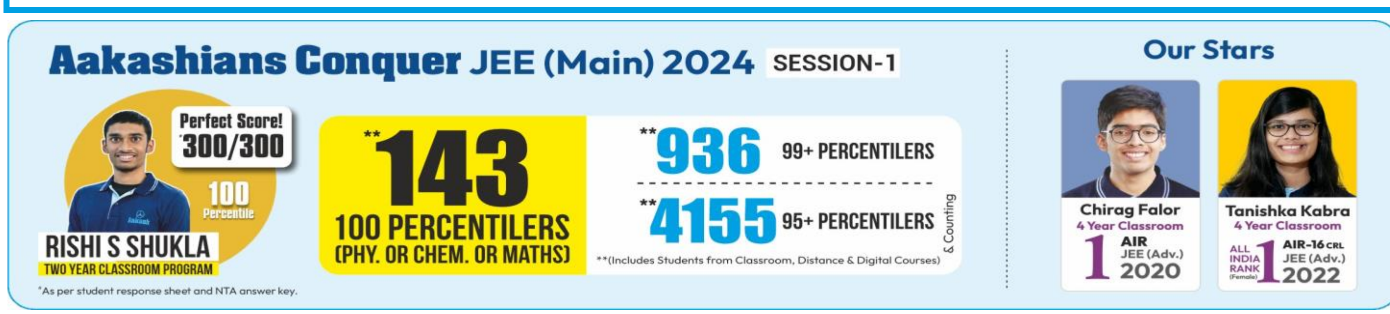
(3) This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6) **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.



MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\int_{x^3}^{(\pi/2)^3}(\sin(2t^{1/3})+\cos(t^{1/3}))dt}{\left(x-\frac{\pi}{2}\right)^2}\right)$ is equal to

(1) $\dfrac{9\pi^2}{8}$ (2) $\dfrac{3\pi^2}{2}$

(3) $\dfrac{5\pi^2}{9}$ (4) $\dfrac{11\pi^2}{10}$

Answer (1)

Sol. $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\int_{x^3}^{(\pi/2)^3}(\sin(2t^{1/3})+\cos(t^{1/3}))dt}{\left(x-\frac{\pi}{2}\right)^2}\right)$

Using Newton Leibniz theorem

$$=\lim_{x\to\frac{\pi}{2}}\left[\frac{\sin\left(2\times\frac{\pi}{2}\right)\cdot 0-\sin(2x)\cdot 3x^2+\left(\cos\frac{\pi}{2}\right)\cdot 0-\cos x\cdot 3x^2}{2\left(x-\frac{\pi}{2}\right)}\right]$$

$$=\lim_{x\to\frac{\pi}{2}}\frac{-3x^2\sin 2x-3x^2\cos x}{2\left(x-\frac{\pi}{2}\right)}\left(\frac{0}{0}\right)\text{ form}$$

$$=\lim_{x\to\frac{\pi}{2}}\left[\frac{-6x\sin 2x-6x^2\cos 2x-6x\cos x+3x^2\sin x}{2}\right]$$

$$=\frac{6\times\frac{\pi^2}{4}+3\times\frac{\pi^2}{4}}{2}$$

$$=\frac{9\pi^2}{8}$$

2. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$ is

(1) 63/16 (2) 21/4

(3) 69/16 (4) 19/4

Answer (2)

Sol. $T_{r+1}={}^9C_r\left(\dfrac{x^{-2/5}}{2}\right)^r\left(x^{2/3}\right)^{9-r}$

$$={}^9C_r\frac{1}{2^r}x^{\frac{2}{3}(9-r)+\left(\frac{-2r}{5}\right)}$$

$$={}^9C_r\cdot\frac{1}{2^r}\cdot x^{6-\frac{16r}{15}}$$

For coefficient of $x^{2/3}$ $\Rightarrow 6-\dfrac{16r}{15}=\dfrac{2}{3}$

$\Rightarrow 90-16r=10$

$\Rightarrow r=5$

For coefficient of $x^{-2/5}$ $\Rightarrow 6-\dfrac{16r}{15}=\dfrac{-2}{5}$

$\Rightarrow 90-16r=-6$

$\Rightarrow r=6$

Sum of coefficient of $x^{2/3}$ & $x^{-2/5}$

$$={}^9C_5\cdot\frac{1}{2^5}+{}^9C_6\cdot\frac{1}{2^6}$$

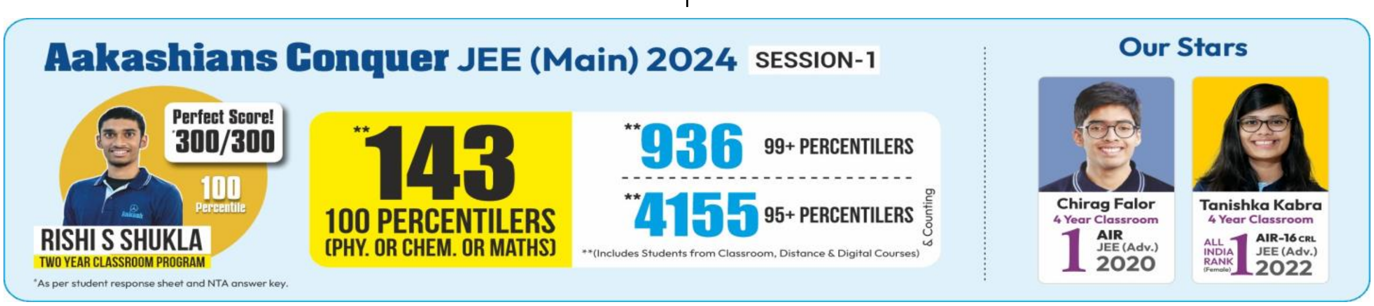
$$=\frac{9!}{5!\,4!}\left(\frac{1}{2^5}\right)+\frac{9!}{6!\,3!}\left(\frac{1}{2^6}\right)=\frac{21}{4}$$

3. The area (in square units) of the region enclosed by the ellipse $x^2+3y^2=18$ in the first quadrant below the line $y=x$ is

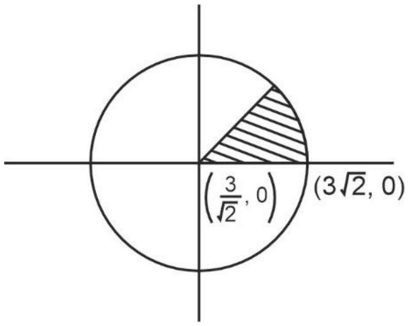
(1) $\sqrt{3}\pi+\dfrac{3}{4}$ (2) $\sqrt{3}\pi+1$

(3) $\sqrt{3}\pi$ (4) $\sqrt{3}\pi-\dfrac{3}{4}$

Answer (3)



Sol.



$$\text{Area} = \int_0^{3\sqrt{2}} x\,dx + \int_{3/\sqrt{2}}^{3\sqrt{2}} \sqrt{\frac{18-x^2}{3}}\,dx$$

$$= \frac{1}{2}(x^2)_0^{3/\sqrt{2}} + \frac{1}{\sqrt{3}}\left[\frac{x}{2}\sqrt{18-x^2} + 9\sin^{-1}\left(\frac{x}{3\sqrt{2}}\right)\right]_{\frac{3}{\sqrt{2}}}^{3\sqrt{2}}$$

$$= \frac{1}{2}\left(\frac{9}{2}\right) + \frac{1}{\sqrt{3}}\left[9\sin^{-1}(1) - \frac{3}{2\sqrt{2}}\cdot\frac{3\sqrt{3}}{\sqrt{2}} - 9\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= \frac{9}{4} + \frac{1}{\sqrt{3}}\left(\frac{9\pi}{2} - \frac{9\sqrt{3}}{4} - \frac{9\pi}{6}\right) = \sqrt{3}\pi$$

4. Let $\alpha, \beta;\ \alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $P_n = \alpha^n - \beta^n, n \in \mathbb{N}$. Then $(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$ is equal to

(1) $10\sqrt{3}P_9$ (2) $11\sqrt{2}P_9$

(3) $10\sqrt{2}P_9$ (4) $11\sqrt{3}P_9$

Answer (1)

Sol. $x^2 - \sqrt{2}x - \sqrt{3} = 0$

$P_n = \alpha^n - \beta^n$

α and β are the roots of the equation

Using Newton's theorem

$P_{n+2} - \sqrt{2}P_{n+1} - \sqrt{3}P_n = 0$

Put $n = 10$

$P_{12} - \sqrt{2}P_{11} - \sqrt{3}P_{10} = 0$

$\boxed{P_{12} = \sqrt{2}P_{11} + \sqrt{3}P_{10}}$

Put $n = 9$

$P_{11} - \sqrt{2}P_{10} - \sqrt{3}P_9 = 0$

$\boxed{P_{11} = \sqrt{2}P_{10} + \sqrt{3}P_9}$

$(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$

Put the value of P_{12} & P_{12} in above equation.

$$= (11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)(\sqrt{2}P_{10} + \sqrt{3}P_9) - 11(\sqrt{2}P_{11} + \sqrt{3}P_{10})$$

$$= 11\sqrt{3}P_{10} - 10\sqrt{2}P_{10} + 22P_{10} + 10\sqrt{2}P_{10} + 11\sqrt{6}P_9 + 10\sqrt{3}P_9 - 11\sqrt{2}P_{11} - 11\sqrt{3}P_{10}$$

$$= 22P_{10} + 11\sqrt{6}P_9 + 10\sqrt{3}P_9 - 11\sqrt{2}(\sqrt{2}P_{10} + \sqrt{3}P_9)$$

$$= 22P_{10} + 11\sqrt{6}P_9 + 10\sqrt{3}P_9 - 22P_{10} - 11\sqrt{6}P_9$$

$$= 10\sqrt{3}P_9$$

5. Let $\vec{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = \beta\hat{j} - \hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let the values of the ordered pair (α, β), for which the area of the parallelogram of diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2). Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$ is equal to

(1) 19 (2) 17

(3) 24 (4) 21

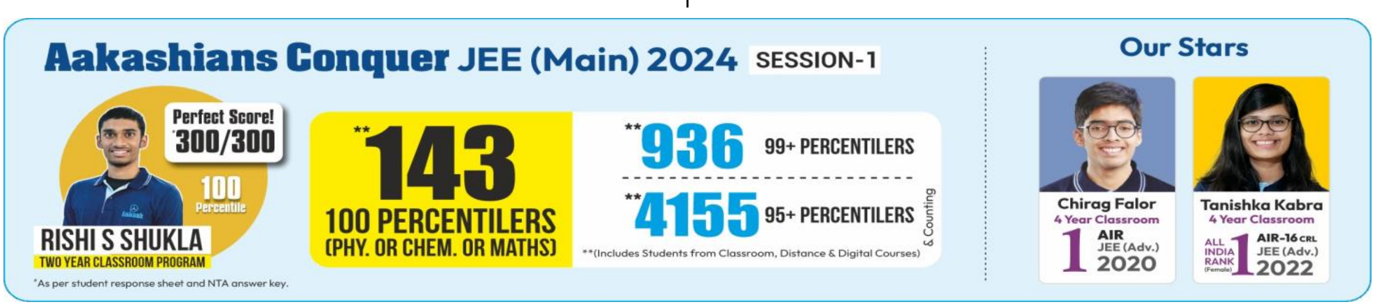
Answer (1)

Sol. Area of parallelogram whose diagonals are $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is

$$= \frac{1}{2}\left|(\vec{a} + \vec{b}) \times (\vec{b} + \vec{c})\right|$$

$$= \frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c}\right|$$

$$= \frac{1}{2}\left|-2\beta\hat{i} - 2\hat{j} + (\alpha + \beta)\hat{k}\right|$$



$= \frac{1}{2}\sqrt{4\beta^2 + 4 + (\alpha+\beta)^2}$

Which is given $\frac{\sqrt{21}}{2}$

$\therefore$ $4\beta^2 + 4 + (\alpha + \beta)^2 = 21$

$\Rightarrow$ $(\alpha + \beta)^2 + 4\beta^2 = 17$

$\Rightarrow$ $\alpha^2 + 5\beta^2 + 2\alpha\beta = 17$

$\Rightarrow$ $\alpha^2 + 5\beta^2 = 29$

$\therefore$ $(\alpha, \beta) \in \{(3,2),(-3,-2),(-3,2),(3,-2)\}$

$\because$ $\alpha\beta = -6$

$\therefore$ $(\alpha, \beta) \in \{(-3,2),(3,-2)\}$

$\therefore$ $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$

$=$ $9 + 4 - (-6) = 19$

6. Between the following two statements:

Statement I: Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - \hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a} \times \vec{r} = \vec{a} \times \vec{b}$ and $\vec{a} \cdot \vec{r} = 0$ is of magnitude $\sqrt{10}$.

Statement II: In a triangle *ABC*,

$\cos 2A + \cos 2B + \cos 2C \ge -\frac{3}{2}$

(1) **Statement I** is correct but **Statement II** is incorrect

(2) **Statement I** is incorrect but **Statement II** is correct

(3) Both **Statement I** and **Statement II** are correct

(4) Both **Statement I** and **Statement II** are incorrect

Answer (2)

Sol. $\because$ $\forall$ two vectors $\vec{c}$ & $\vec{d}$

$$|\vec{c} \times \vec{d}|^2 = |\vec{c}|^2 |\vec{d}|^2 - (\vec{c} \cdot \vec{d})^2$$

replacing $\vec{c} = \vec{a}$ & $\vec{d} = \vec{r}$

$\Rightarrow$ $|\vec{a} \times \vec{r}| = |\vec{a}|^2 |\vec{r}|^2 - (\vec{a} \cdot \vec{r})^2$

$\Rightarrow$ $|\vec{a} \times \vec{b}| = |\vec{a}|^2 |\vec{r}|^2$ ($\because$ $\vec{a} \times \vec{r} = \vec{a} \times \vec{b}$ and $\vec{a} \cdot \vec{r} = 0$)

$\Rightarrow$ $35 = 14|\vec{r}|^2$

$\Rightarrow$ $|\vec{r}| = \sqrt{\frac{35}{14}} = \sqrt{\frac{5}{2}} \ne \sqrt{10}$

$\therefore$ Statement I is incorrect

Statement II is correct

(*i.e.*, $\cos 2A + \cos 2B + \cos 2C \ge -\frac{3}{2}$)

Proof: $\because$ $(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}) \ge 0$ ...(1)

and $|\overrightarrow{OA}|^2 = |\overrightarrow{OB}|^2 = |\overrightarrow{OC}|^2 = R^2$ ...(2)

Now, using (1), we get

$$|\overrightarrow{OA}|^2 + |\overrightarrow{OB}|^2 + |\overrightarrow{OC}|^2$$

$$+2(\overrightarrow{OA} \cdot \overrightarrow{OB} + \overrightarrow{OB} \cdot \overrightarrow{OC} + \overrightarrow{OC} \cdot \overrightarrow{OA}) \ge 0$$

$\Rightarrow$ $3R^2 + 2R^2(\cos 2A + \cos 2B + \cos 2C) \ge 0$

$\Rightarrow$ $\cos 2A + \cos 2B + \cos 2C \ge -\frac{3}{2}$

7. Let *z* be a complex number such that the real part of $\frac{z - 2i}{z + 2i}$ is zero. Then, the maximum value of $|z - (6 + 8i)|$ is equal to

(1) 10
(2) 12
(3) 8
(4) ∞

Answer (2)

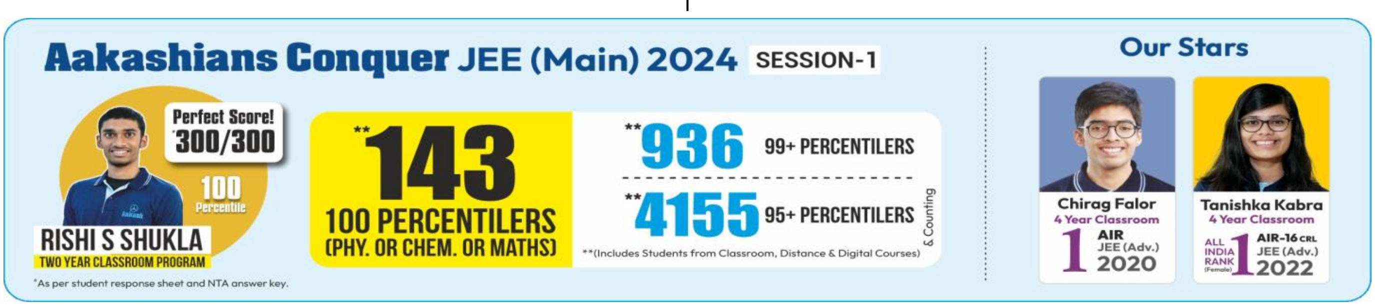
Sol. $n = \frac{z - 2i}{z + 2i}$

Let $z = x + iy$

$$n = \frac{x + (y-2)i}{x + (y+2)i} \times \left(\frac{x - (y+2)i}{x - (y+2)i}\right)$$

$$\text{Re}(n) = \frac{x^2 + (y-2)(y+2)}{x^2 + (y+2)^2} = 0$$

$\Rightarrow x^2 + (y-2)(y+2) = 0$



$\Rightarrow x^2 + y^2 - 4 = 0$

$\Rightarrow x^2 + y^2 = 4$

also, $|z-(6+8i)| \le |z| + |-6-8i|$

$|z - (6 + 8i)| \le 2 + 10 = 12$

Hence, Maximum value of $|z-(6+8i)|$ is 12.

8. $\lim\limits_{x\to 0} \dfrac{e-(1+2x)^{\frac{1}{2x}}}{x}$ is equal to

(1) 0 (2) e

(3) $e - e^2$ (4) $\dfrac{-2}{e}$

Answer (2)

Sol. $\lim\limits_{x\to 0} \dfrac{e-(1+2x)^{\frac{1}{2x}}}{x}$

Using expansion

$$= \lim_{x\to 0} \frac{e - e\left[1 - \frac{2x}{2} + \frac{11\times 4x^2}{24} + \dots\right]}{x}$$

$$= \lim_{x\to 0}\left(e - \frac{11x}{6}e + \dots\right) = e$$

9. Let the foci of a hyperbola H coincide with the foci of the ellipse $E : \dfrac{(x-1)^2}{100} + \dfrac{(y-1)^2}{75} = 1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2 + 2\beta^2$ is equal to

(1) 205 (2) 225

(3) 242 (4) 237

Answer (2)

Sol. $E : \dfrac{(x-1)^2}{100} + \dfrac{(y-1)^2}{75} = 1$

Eccentricity of ellipse, $e_E = \sqrt{1 - \dfrac{b^2}{a^2}}$

$= \sqrt{1 - \dfrac{75}{100}}$

$e_E = \dfrac{1}{2}$

$\therefore e_H = 2$ [as eccentricity of hyperbola is reciprocal of eccentricity of ellipse]

Transverse axis of hyperbola = α

Conjugate axis of hyperbola = β

Also, foci of ellipse $(1 \pm ae, 1)$

$= \left(1 \pm \left(10 \times \dfrac{1}{2}\right), 1\right)$

$= (1 \pm 5, 1)$

= (6, 1) and (–4, 1)

Distance between foci = 10

$2ae = 10$

$\Rightarrow \boxed{a = \dfrac{5}{2}}$

also, $e^2 = 1 + \dfrac{b^2}{a^2}$

$4 = 1 + \dfrac{4b^2}{25}$

$b^2 = \dfrac{75}{4}$

$b = \dfrac{\sqrt{75}}{2}$

$\Rightarrow \alpha = 5$

and $\beta = \sqrt{75}$

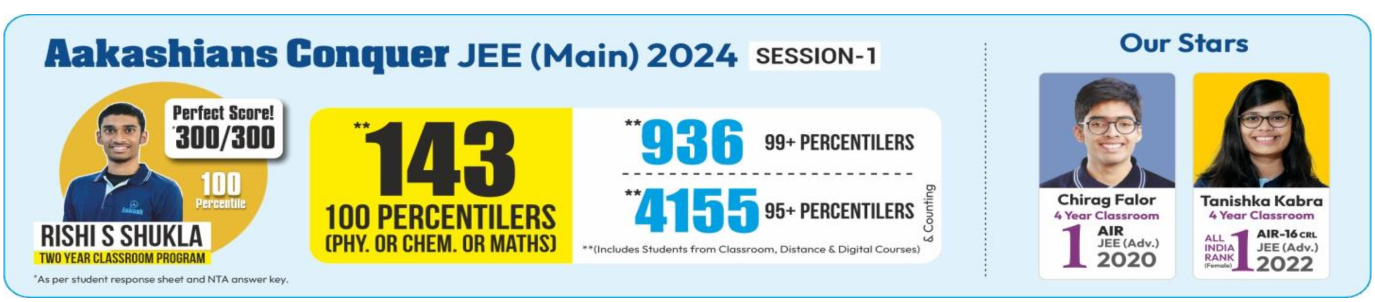
$3\alpha^2 + 2\beta^2 = 3(5)^2 + 2(75) = 225$

10. The integral $\displaystyle\int_{1/4}^{3/4} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$ is equal to

(1) 1/2 (2) –1/2

(3) –1/4 (4) 1/4

Answer (3)



Sol. $\int_{\frac{1}{4}}^{\frac{3}{4}} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$

$x = \cos 2\theta$

$\Rightarrow dx = (-2\sin 2\theta d\theta)$

Take limit as α and β

$$-2\int_{\alpha}^{\beta}\cos 2\theta \cdot \sin 2\theta d\theta$$

$$= \int_{\alpha}^{\beta}\sin 4\theta d\theta$$

$$= \frac{-\cos 4\theta}{4}\Big|_{\alpha}^{\beta}$$

$$= -\frac{1}{4}\left(2\cdot(x^2)-1\right)\Big|_{1/4}^{3/4}$$

$$= -\frac{1}{4}\left(2x^2-1\right)\Big|_{1/4}^{3/4}$$

$$= -\frac{1}{4}\left(\frac{18}{16}-1-\frac{2}{16}+1\right)$$

$$= -\frac{1}{4}$$

11. If $\log_e y = 3\sin^{-1}x$, then $(1-x^2)y'' - xy'$ at $x = \frac{1}{2}$ is equal to

(1) $3e^{\pi/6}$ (2) $3e^{\pi/2}$

(3) $9e^{\pi/6}$ (4) $9e^{\pi/2}$

Answer (4)

Sol. $\log_e y = 3\sin^{-1}x$

$$y = e^{3\sin^{-1}x}$$

$$\frac{dy}{dx} = e^{3\sin^{-1}x}\cdot\frac{3}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = 3y$$

Again differentiate

$$\sqrt{1-x^2}\cdot y'' - \frac{2x}{2\sqrt{1-x^2}}y' = 3y'$$

$$(1-x)^2y'' - xy' = 3y'\left(\sqrt{1-x^2}\right)$$

So value of $3y'\left(\sqrt{1-x^2}\right)$ at $x = \frac{1}{2}$

$$3\cdot\frac{3}{\sqrt{1-x^2}}e^{\sin^{-1}x}\left(\sqrt{1-x^2}\right)$$

$$= 9e^{3\frac{\pi}{6}} = 9e^{\frac{\pi}{2}}$$

12. If the variance of the frequency distribution

x	c	$2c$	$3c$	$4c$	$5c$	$6c$
f	2	1	1	1	1	1

Is 160, then the value of $c \in \mathbb{N}$ is

(1) 6 (2) 7

(3) 5 (4) 8

Answer (2)

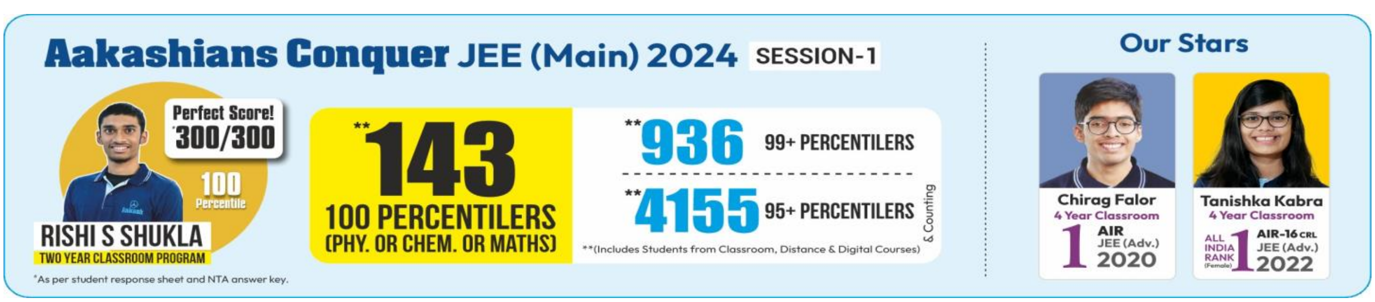
Sol.

x_i	$f(x_i)$	$x(f(x)$	$x^2f(x)$
C	2	$2C$	$2C^2$
$2C$	1	$2C$	$4C^2$
$3C$	1	$3C$	$9C^2$
$4C$	1	$4C$	$16C^2$
$5C$	1	$5C$	$25C^2$
$6C$	1	$6C$	$36C^2$

$$\sigma^2 = E(x^2) - [E(x)], \sum f(x_i) = 7$$

$$E(x) = \sum x\,f(x) = 22C$$

$$E(x^2) = \sum x^2 f(x) = 92C^2$$



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$$\sigma^2 = 160 = \frac{92C^2}{7} - \left(\frac{22C}{7}\right)^2$$

$\Rightarrow C = \pm 7$ but $C \in N$

$\Rightarrow \quad C = 7$

13. Let the range of the function $f(x) = \dfrac{1}{2 + \sin 3x + \cos 3x}, x \in \mathbb{R}$ be $[a, b]$. If α and β are respectively the A.M. and the G.M. of a and b, then $\dfrac{\alpha}{\beta}$ is equal to

(1) $\sqrt{2}$ (2) π

(3) $\sqrt{\pi}$ (4) 2

Answer (1)

Sol. $F(x) = \dfrac{1}{2 + \sin 3x + \cos 3x}, x \in \mathbb{R}$

$\sin 3x + \cos 3x \in \left[-\sqrt{2}, \sqrt{2}\right]$

$2 + \sin 3x + \cos 3x \in \left[2 - \sqrt{2}, 2 + \sqrt{2}\right]$

$$\Rightarrow \quad \frac{1}{2 + \sin 3x + \cos 3x} \in \left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right]$$

$$\Rightarrow \quad a = \frac{1}{2+\sqrt{2}}, b = \frac{1}{2-\sqrt{2}}$$

$$\alpha = \frac{a+b}{2} = \frac{\frac{1}{2+\sqrt{2}} + \frac{1}{2-\sqrt{2}}}{2}$$

$$= \frac{4}{2 \times 2} = 1$$

$$\beta = \sqrt{ab} = \sqrt{\left(\frac{1}{2+\sqrt{2}}\right) \times \left(\frac{1}{2-\sqrt{2}}\right)}$$

$$= \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{\alpha}{\beta} = \sqrt{2}$$

14. Let $B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$ and A be a 2 × 2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4 + \alpha C^2 + \beta I = O$, then $2\beta - \alpha$ is equal to

(1) 16 (2) 8

(3) 2 (4) 10

Answer (4)

Sol. $B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$

$AB^{-1} = A^{-1}$

$\Rightarrow \quad A^2 = B$

Also, $BCB^{-1} = A$

$\Rightarrow \quad C = B^{-1}AB$

$\Rightarrow \quad C^4 = (B^{-1}AB)(B^{-1}AB)(B^{-1}AB)(B^{-1}AB)$

$= B^{-1}A^4B$

$= B^{-1}B^2B$

$\Rightarrow \quad C^4 = B^2$

Also, $C^2 = (B^{-1}AB)(B^{-1}AB)$

$= B^{-1}A^2B$

$= B^{-1}BB$

$\Rightarrow \quad C^2 = B$

$\Rightarrow \quad C^4 + \alpha C^2 + \beta I = 0$

$\Rightarrow \quad B^2 + \alpha B + \beta I = 0$

$$B^2 = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 18 \\ 6 & 28 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4 & 18 \\ 6 & 28 \end{bmatrix} + \alpha \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} \beta & 0 \\ 0 & \beta \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

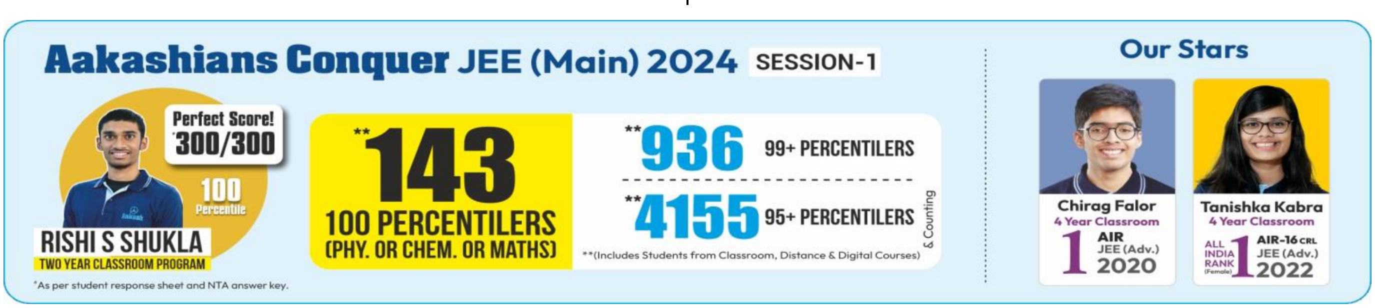
$\Rightarrow \quad 4 + \alpha + \beta = 0$

and $18 + 3\alpha = 0$

$\Rightarrow \quad \alpha = -6$

$\Rightarrow \quad \beta = 2$

$\Rightarrow \quad 2\beta - \alpha = 10$



15. The value of the integral $\int_{-1}^{2}\log_e\left(x+\sqrt{x^2+1}\right)dx$ is

(1) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(2) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(3) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(4) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

Answer (3)

Sol. $\int_{-1}^{2}\log_e\left(x+\sqrt{x^2+1}\right)dx$

$$=\left[x\log_e\left(x+\sqrt{x^2+1}\right)\right]_{-1}^{2}-\int_{-1}^{2}\frac{x}{\left(x+\sqrt{x^2+1}\right)}\left(1+\frac{x}{\sqrt{x^2+1}}\right)dx$$

$$=2\log_2\left(2+\sqrt{5}\right)+\log_e\left(\sqrt{2}-1\right)-\int_{-1}^{2}\frac{x}{\sqrt{x^2+1}}dx$$

$$=\log_e\left[\left(2+\sqrt{5}\right)^2\left(\sqrt{2}-1\right)\right]-\left[\sqrt{x^2+1}\right]_{-1}^{2}$$

$$=\log_e\left[\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right]-\sqrt{5}+\sqrt{2}$$

$$=\sqrt{2}-\sqrt{5}+\log_e\left[\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right]$$

16. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll, $i = 2, 3$, is equal to

(1) 1/54 (2) 5/54

(3) 2/54 (4) 3/54

Answer (2)

Sol. Let the outcome in Ist, IInd and IIIrd roll be a, b, c

Given: $a < b < c$

Probability = $\frac{^6C_3}{6^3}=\frac{5}{54}$

17. Let $a, ar, ar^2, \ldots$ be an infinite G.P. If $\sum_{n=0}^{\infty}ar^n=57$ and $\sum_{n=0}^{\infty}a^3r^{3n}=9747$, then $a + 18r$ is equal to

(1) 46 (2) 38

(3) 27 (4) 31

Answer (4)

Sol. $\sum_{n=0}^{\infty}ar^n=57 \quad\Rightarrow\quad \frac{a}{1-r}=57 \quad \ldots(i)$

$\sum_{n=0}^{\infty}a^3r^{3n}=9747 \quad\Rightarrow\quad \frac{a^3}{1-r^3}=9747 \quad \ldots(ii)$

$$\frac{\left(1-r^3\right)}{\left(1-r\right)^3}=\frac{(57)^3}{9747}=19$$

$$\Rightarrow \frac{(1-r)\left(1+r+r^2\right)}{(1-r)^3}=19$$

$\Rightarrow 18r^2-39r+18=0$

$\Rightarrow r=\frac{2}{3}, \frac{3}{2}$ (rejected)

$\therefore$ $a = 19$

$a + 18r$

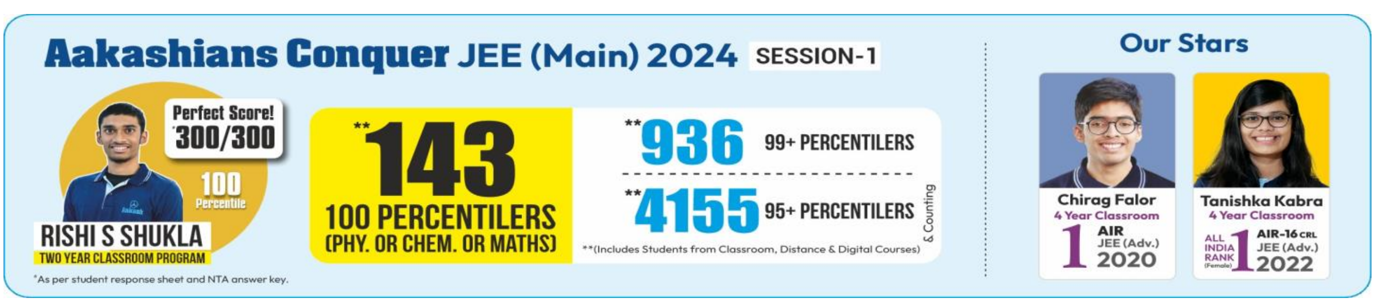
$= 19 + 12 = 31$

18. Two vertices of a triangle ABC are $A(3, -1)$ and $B(-2, 3)$, and its orthocentre is $P(1, 1)$. If the coordinates of the point C are (α, β) and the centre of the of the circle circumscribing the triangle PAB is (h, k), then the value of $(\alpha+\beta)+2(h+k)$ equals

(1) 5 (2) 51

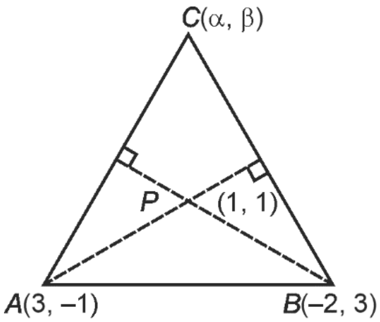
(3) 15 (4) 81

Answer (1)



Sol. $m_{PA} = \frac{2}{-2} = -1$

$\therefore \quad m_{BC} = 1$



$BC : y = x + 5$

$m_{BP} = \frac{2}{-3} = \frac{-2}{3}$

$\therefore \quad m_{AC} = \frac{3}{2}$

$AC : y = \frac{3}{2}x - \frac{11}{2} \qquad \Rightarrow \quad 2y = 3x - 11$

$\therefore \quad C : (21, 26)$

Let the circumcentre be (h, k)

$(h-1)^2 + (k-1)^2 = (h+2)^2 + (k-3)^2 \quad \ldots(i)$

$(h-1)^2 + (k-1)^2 = (h-3)^2 + (k+1)^2 \quad \ldots(ii)$

Solving (i) and (ii)

$h = \frac{-19}{2}, k = \frac{-23}{2}$

$\alpha + \beta + 2(h + k)$

$= 21 + 26 - 19 - 23$

$= 2 + 3 = 5$

19. Let $\int_0^x \sqrt{1-(y'(t))^2}\,dt = \int_0^x y(t)\,dt$,

$0 \le x \le 3, y \ge 0, y(0) = 0$. Then at $x = 2$, $y'' + y + 1$ is equal to

(1) 1 (2) 2

(3) $\sqrt{2}$ (4) $1/2$

Answer (1)

Sol. $\int_0^x \sqrt{1-(y'(t))^2}\,dt = \int_0^x y(t)\,dt$

Differentiating both side

$\sqrt{1-(y'(x))^2} = y(x)$

$\left(\frac{dy}{dx}\right)^2 + y^2 = 1$

$y'^2 + y^2 = 1$

$2y'y'' + 2yy' = 0$

$y'' + y = 0$

$\therefore \quad \underbrace{y'' + y}_{0} + 1 = 1$

20. Consider the line L passing through the points (1, 2, 3) and (2, 3, 5). The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to

(1) 4 (2) 6

(3) 5 (4) 3

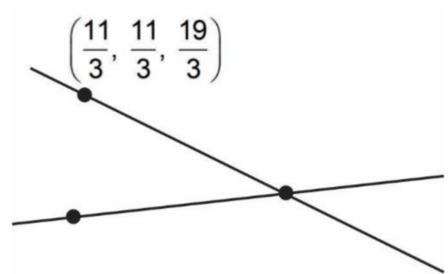
Answer (4)

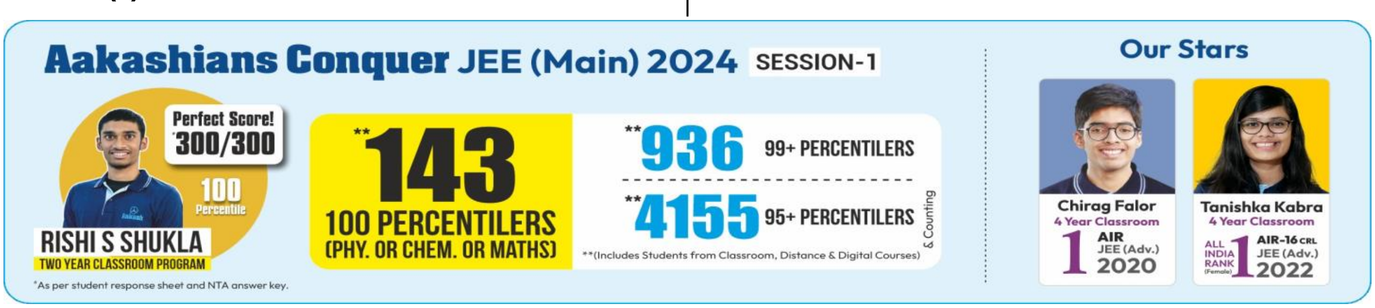
Sol. $L : \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2} = \mu$

Measured along $L_2 : \frac{x-\frac{11}{3}}{\frac{2}{3}} = \frac{y-\frac{11}{3}}{\frac{1}{3}} = \frac{z-\frac{19}{3}}{\frac{2}{3}} = \lambda$

Any point on L_1 : $(\mu + 1, \mu + 2, 2\mu + 3)$

Any point on $L_2 \left(\frac{2}{3}\lambda + \frac{11}{3}, \frac{\lambda}{3} + \frac{11}{3}, \frac{2}{3}\lambda + \frac{19}{3}\right)$





Now

$$\mu+1=\frac{2}{3}\lambda+\frac{11}{3}$$

$$\mu+2=\frac{\lambda}{3}+\frac{11}{3}$$

$$\lambda=-3$$

$$\mu=\frac{2}{3}$$

Point on $L=\left(\frac{5}{3},\ \frac{8}{3},\ \frac{13}{3}\right)$

$$d=\sqrt{\left(\frac{11}{3}-\frac{5}{3}\right)^2+\left(\frac{8}{3}-\frac{11}{3}\right)^2+\left(\frac{19}{3}-\frac{13}{3}\right)^2}$$

$$d=\sqrt{4+1+4}$$

$d = 3$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\ \sin^{-1}x+3\ \cos^{-1}x=\frac{2\pi}{5}$, is ________

Answer (0)

Sol. $2\sin^{-1}x+3\cos^{-1}x=\frac{2\pi}{5}$

$$\frac{\pi}{2}+\cos^{-1}x=\frac{2\pi}{5}$$

$$\cos^{-1}x=\frac{2\pi}{5}-\frac{\pi}{2}$$

$$\cos^{-1}x=\frac{-\pi}{10}$$

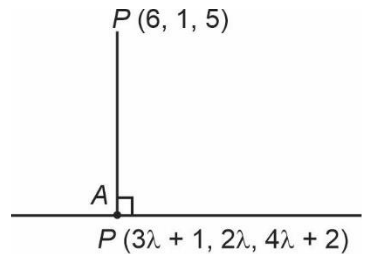
Which is not possible as $\cos^{-1}x\in[0,\ \pi]$

$\therefore$ No solution

22. The square of the distance of the image of the point (6, 1, 5) in the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z-2}{4}$, from the origin is ______

Answer (62)

Sol.



$$\overrightarrow{PA}=(3\lambda-5)\hat{i}+(2\lambda-1)\hat{j}+(4\lambda-3)\hat{k}$$

$(3\lambda-5)3+(2\lambda-1)2+(4\lambda-3)4=0$

$\Rightarrow\ 9\lambda-15+4\lambda-2+16\lambda-12=0$

$\Rightarrow\ 29\lambda=29$

$\therefore\ \lambda=1$

$\therefore\ A(4, 2, 6)$

$\therefore\ P'$: mirror image of P

$\Rightarrow\ P'(2, 3, 7)$

$(OP')^2 = 4 + 9 + 49$

$= 62$

23. Consider the matrices: $A=\begin{bmatrix}2 & -5\\ 3 & m\end{bmatrix}$, $B=\begin{bmatrix}20\\ m\end{bmatrix}$ and $X=\begin{bmatrix}x\\ y\end{bmatrix}$. Let the set of all m, for which the system of equations $AX = B$ has a negative solution (*i.e.*, $x < 0$ and $y < 0$), be the interval (a, b). Then $8\int_a^b|A|dm$ is equal to ______

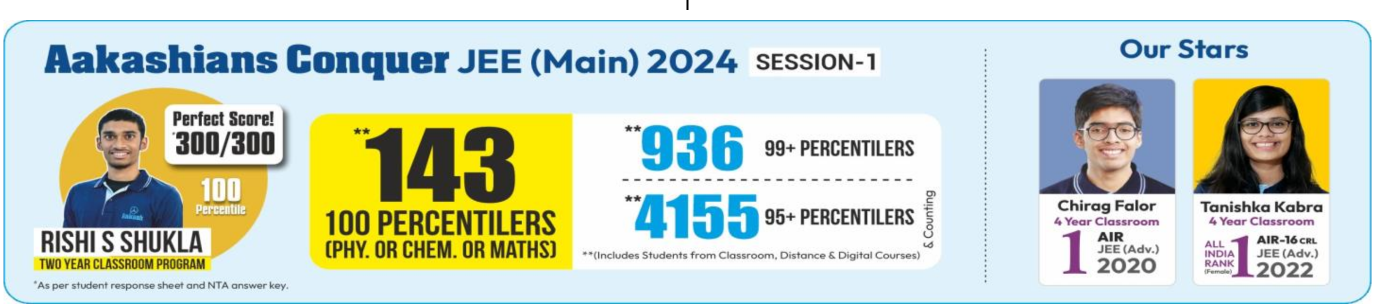
Answer (450)

Sol. $AX = B$

$2x - 5y = 20$

$3x + my = m$

$$\Rightarrow\ 3\left(\frac{20+5y}{2}\right)+my=m$$



$\Rightarrow \quad 30 + \frac{15}{2}y + my = m$

$\Rightarrow \quad y\left(\frac{15}{2} + m\right) = m - 30$

$\Rightarrow \quad y = \frac{m-30}{\frac{15}{2}+m} < 0 \;\Rightarrow\; m \in \left(-\frac{15}{2},\ 30\right)$

Similarly : $3x + m\left(\frac{2x-20}{5}\right) = m$

$\Rightarrow \quad 3x + \frac{2mx}{5} - \frac{20m}{5} = m$

$\Rightarrow \quad \frac{15x + 2mx}{5} = 5m \;\Rightarrow\; x = \frac{25m}{15+2m}$

$x < 0 \;\Rightarrow\; \frac{25m}{15+2m} < 0 \;\Rightarrow\; m \in \left(-\frac{15}{2},\ 0\right)$

$\therefore \quad m \in \left(-\frac{15}{2},\ 0\right)$

$a = -\frac{15}{2},\ b = 0$

$8\int_{-\frac{15}{2}}^{0} (2m+15)dm = 450$

24. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is ____

Answer (70)

Sol. Number in this range will be 3-digit number.

$N = \overline{abc}$ such that $a + b + c = 14$

Also, $a \ge 1, \quad a, b, c \in \{0, 1, 2, ...9\}$

Case I

All 3-digit same

$\Rightarrow 3a = 14$ not possible

Case II

Exactly 2 digit same:

$\Rightarrow 2a + c = 14$

$(a, c) \in \{(3, 8), (4, 6), (5, 4), (6, 2), (7, 0)\}$

$\Rightarrow \left(\frac{3!}{2!}\right)$ ways $\Rightarrow 5 \times 3 - 1$

$= 15 - 1 = 14$

Case III

All digits are distinct

$a + b + c = 14$

without losing generality $a > b > c$

$$(a, b, c) \in \begin{cases} (9, 5, 0), (9, 4, 1), (9, 3, 2) \\ (8, 6, 0), (8, 5, 1), (8, 4, 2) \\ (7, 6, 1), (7, 5, 2), (7, 4, 3) \\ (6, 5, 3) \end{cases}$$

$\Rightarrow 8 \times 3! + 2(3! - 2!) = 48 + 8 = 56$

$= 0 + 14 + 56 = 70$

25. If $\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + ... + \frac{1}{\alpha+1012}\right)$
$-\left(\frac{1}{2\cdot1} + \frac{1}{4\cdot3} + \frac{1}{6\cdot5} + ... + \frac{1}{2024\cdot2023}\right) = \frac{1}{2024}$, then

α is equal to ______

Answer (1011)

Sol. $\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + ... + \frac{1}{\alpha+2012} -$

$\left(\frac{1}{2\times21} + \frac{1}{4\times3} + ... + \frac{1}{2024}\cdot\frac{1}{2023}\right) = \frac{1}{2024}$

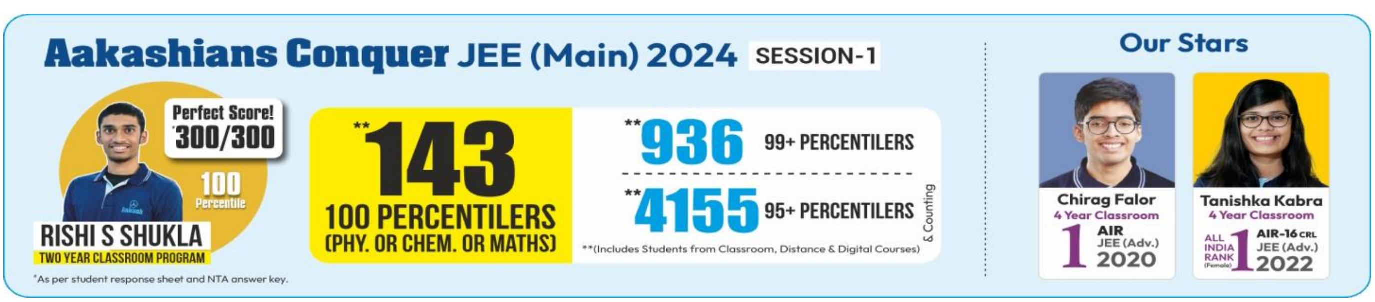
$\sum_{r=1}^{1012} \frac{1}{2r(2r-1)} = \sum_{r=1}^{1012}\left(\frac{1}{2r-1} - \frac{1}{2r}\right)$

$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + ... + \left(\frac{1}{2023} - \frac{1}{2024}\right)$

$= \left(1 + \frac{1}{3} + \frac{1}{5} + ... + \frac{1}{2023}\right)$

$-\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + ... + \frac{1}{2024}\right)$

$= \left(1 + \frac{1}{3} + ... + \frac{1}{2023}\right) - \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{1012}\right)$



$$= \left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2023}\right)-\frac{1}{2}\left(1+\frac{1}{2}+...+\frac{1}{1011}\right)$$

$$\frac{-1}{2}\left(1+\frac{1}{2}+...+\frac{1}{1012}\right)$$

$$= \frac{1}{1012}+\frac{1}{1013}+...+\frac{1}{2023}-\frac{1}{2024}$$

$$\Rightarrow \alpha + 1012 = 2023$$

$$\Rightarrow \alpha = 1011$$

26. Let the set of all values of p, for which

$f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$

does not have any critical point, be the interval (a, b). Then $16ab$ is equal to ______

Answer (252)

Sol. $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x)$

$$+2(2-p)x+7$$

$$f(x) = -\cos 4x(p^2 - 6p + 8) + 2(2-p)x + 7$$

$$f'(x) = 4\sin 4x(p^2 - 6p + 8) + 2(2-p) \neq 0$$

$$2(2-p) + \left[-4(p^2 - 6p + 8),\ 4(p^2 - 6p + 8)\right]$$

$$\Rightarrow [-4p^2 + 24p - 32, 4p^2 - 24p + 32] + (4 - 2p)$$

$$\left[-4p^2 + 22p - 28,\ 4p^2 - 26p + 36\right]$$

$$[(p-2)(-4p+14),\ (p-2)(4p-18)]$$

$$\Rightarrow (p-2)\left[(-4p+14), 4p-18\right] \Rightarrow p \in \left(\frac{7}{2}, \frac{9}{2}\right)$$

$$\Rightarrow a = \frac{7}{2}, b = \frac{9}{2}$$

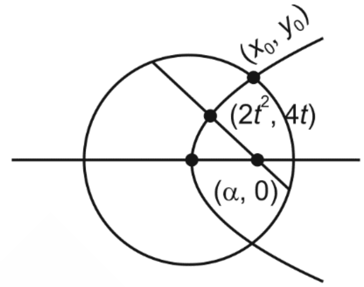
$$\Rightarrow 16ab = 4 \times 63 = 252$$

27. Consider the circle C: $x^2 + y^2 = 4$ and the parabola P: $y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to

______

Answer (80)

Sol.



Chord with the middle point $(\alpha, 0)$

$\Rightarrow$ $T = S_1$

$\Rightarrow$ $yy_1 - 4(x + x_1) = y_1^2 - 8x_1$

$\Rightarrow$ $-4(x + \alpha) = 0 - 8\alpha$

$\Rightarrow$ $x + \alpha = 2\alpha \Rightarrow x = \alpha$

For circle chord with $(2t^2, 4t)$ as mid point

$\Rightarrow$ $T = S_1$

$\Rightarrow$ $xx_1 + yy_1 - 4 = x_1^2 + y_1^2 - 4$

$\Rightarrow$ $2t^2x + 4ty = 4t^4 + 16t^2$

Passes through $(\alpha, 0)$

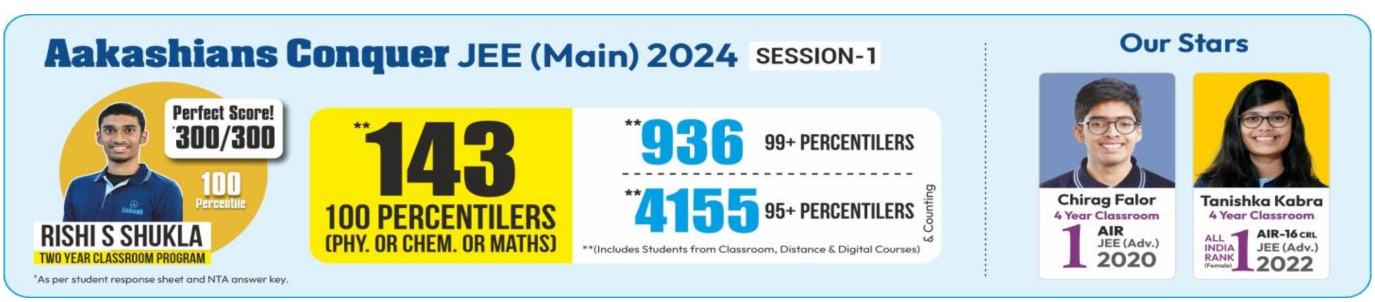
$\Rightarrow$ $2t^2\alpha = 4t^4 + 16t^2$

$\Rightarrow$ $2\alpha = 4t^2 + 16 \Rightarrow \alpha = 2t^2 + 8 = x_0 + 8$

$x^2 + y^2 = 4$ and $y^2 = 8x$

$\Rightarrow$ $x^2 + 8x - 4 = 0$ $\Rightarrow$ $x_0 = \dfrac{-8+\sqrt{80}}{2}$

$\Rightarrow$ $p = 8$ and $q = 4 + \dfrac{\sqrt{80}}{2}$ $\Rightarrow (2q - p)^2 = 80$



28. For a differentiable function $f : \mathbb{R} \to \mathbb{R}$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in \mathbb{R}$, $f(0) = 1$ and $\lim_{x\to-\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to ____.

Answer (61)

Sol. $f'(x) = 3f(x) + \alpha$

$$\Rightarrow \frac{dy}{3y+\alpha} = dx$$

$$\Rightarrow \frac{1}{3}\ln(3y+\alpha) = x + C$$

$$y(0) = 1 \Rightarrow C = \frac{1}{3}\ln(3+\alpha)$$

$$\frac{1}{3}\ln\left(\frac{3y+\alpha}{3+\alpha}\right) = x$$

$$\Rightarrow y = \frac{1}{3}\left((3+\alpha)e^{3x} - \alpha\right) = f(x)$$

$$\lim_{x\to-\infty} f(x) = 7 \Rightarrow \alpha = -21$$

$$\Rightarrow f(x) = 7 - 6e^{3x}$$

$$9f(-\ln 3) = 61$$

29. Let $A = \{(x, y): 2x + 3y = 23,\ x, y \in \mathbb{N}\}$ and $B = \{x: (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to

Answer (24)

Sol. $A = \{(x,y); 2x + 3y = 23, x, y \in N\}$

$$\boxed{A = \{(1,7),(4,5),(7,3),(10,1)\}}$$

$$B = \{x : (x,y) \in A\}$$

$$\boxed{B = \{1,4,7,10\}}$$

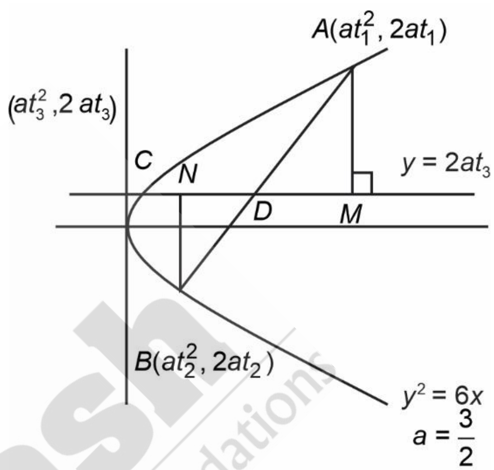
So, total number of one-one functions from A to B is 4! = 24

30. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L. Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to ________

Answer (36)

Sol. Equation of AB

$$y(t_1 + t_2) = 2x + 2at_1t_2$$



For D, $y = 2at_3$

$$\Rightarrow x = a(t_1t_3 + t_2t_3 - t_1t_2)$$

$$CD = |a(t_1t_3 + t_2t_3 - t_1t_3) - at_3^2|$$

$$AM = |2at_1 - 2at_3|$$

$$BN = |2at_3 - 2at_2|$$

$$\left(\frac{AM \cdot BN}{CD}\right)^2 = \left(\frac{4a^2(t_1 - t_3)(t_3 - t_2)}{a(t_1t_3 + t_2t_3 - t_1t_3 - t_3^2)}\right)^2$$

$$= \left(\frac{4a^2(t_1 - t_3)(t_3 - t_2)}{a(t_1 - t_3)(t_2 - t_3)}\right)^2$$

$$= 16a^2 = 16 \cdot \left(\frac{3}{2}\right)^2 = 36$$

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PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. The temperature of a gas is –78°C and the average translational kinetic energy of its molecules is K. The temperature at which the average translational kinetic energy of the molecules of the same gas becomes $2K$ is

(1) 127°C

(2) 117°C

(3) –78°C

(4) –39°C

Answer (2)

Sol. $K \propto T$

$\therefore \quad T' = 2T$

$= 2 \times (273 + 78)$

$= 390$ K

$= 117$°C

32. A hydrogen atom in ground state is given an energy of 10.2 eV. How many spectral lines will be emitted due to transition of electrons?

(1) 3 (2) 6

(3) 1 (4) 10

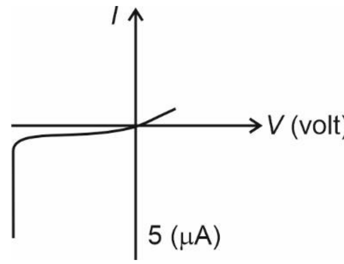
Answer (3)

Sol. $\Delta E = 10.2$ eV

$\therefore \quad n = 2$

Number of lines = ${}^2C_2 = 1$

33. The I-V characteristics of an electronic device shown in the figure. The device is



(1) A transistor which can be used as an amplifier

(2) A diode which can be used as a rectifier

(3) A solar cell

(4) A Zener diode which can be used as a voltage regulator

Answer (4)

Sol. As this is a reverse bias characteristic. It should be for Zener diode working as voltage regulator.

34. The de-Broglie wavelength associated with a particle of mass m and energy E is $h/\sqrt{2mE}$. The dimensional formula for Planck's constant is

(1) $[M^2L^2T^{-2}]$

(2) $[MLT^{-2}]$

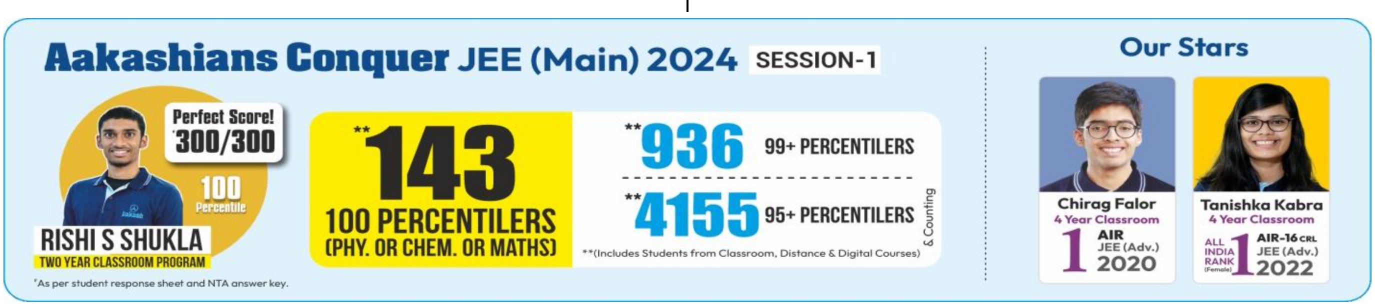
(3) $[ML^{-1}T^{-2}]$

(4) $[ML^2T^{-1}]$

Answer (4)

Sol. $h = \dfrac{E}{f}$

$$= \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$



35. Two cars are travelling towards each other at speed of 20 m s^{-1} each. When the cars are 300 m apart, both the drivers apply brakes and the cars retard at the rate of 2 m s^{-2}. The distance between them when they come to rest is

(1) 25 m (2) 200 m

(3) 100 m (4) 50 m

Answer (3)

Sol. $v^2 - u^2 = 2aS$

$\Rightarrow 0 - (20)^2 = 2(-2)S$

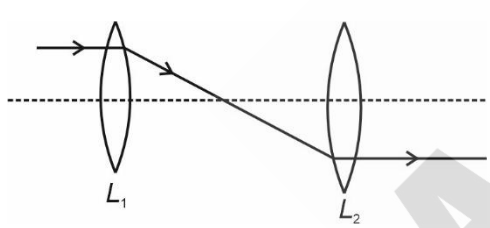
$\Rightarrow S = 100$

$\therefore S_{net} = 2S = 200$

$d = 300 - S_{net}$

$= 100$ m

36. The following figure represents two biconvex lenses L_1 and L_2 having focal length 10 cm and 15 cm respectively. The distance between L_1 and L_2 is



(1) 10 cm (2) 25 cm

(3) 15 cm (4) 35 cm

Answer (2)

Sol. Parallel rays are focussed by convex lens and vice-versa.

$\therefore L_1L_2 = f_1 + f_2$

$= 10 + 15 = 25$ cm

37. The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. The ratio between the volume of the first and the second bubble is

(1) 1 : 81 (2) 1 : 3

(3) 1 : 9 (4) 1 : 27

Answer (4)

Sol. $\Delta P = \frac{4T}{r}$

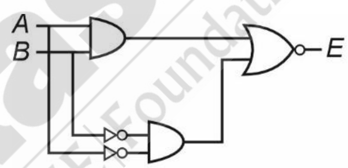
$\Rightarrow \frac{P_1}{P_2} = \frac{r_2}{r_1} = 3$

$\Rightarrow r_2 = 3r_2$

and, $V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{V_1}{V_2} = \frac{1}{27}$

38.



A	B	E
0	0	0
0	1	x
1	0	y
1	1	0

In the truth table of the above circuit the value of *X* and *Y* are

(1) 1, 0 (2) 0, 0

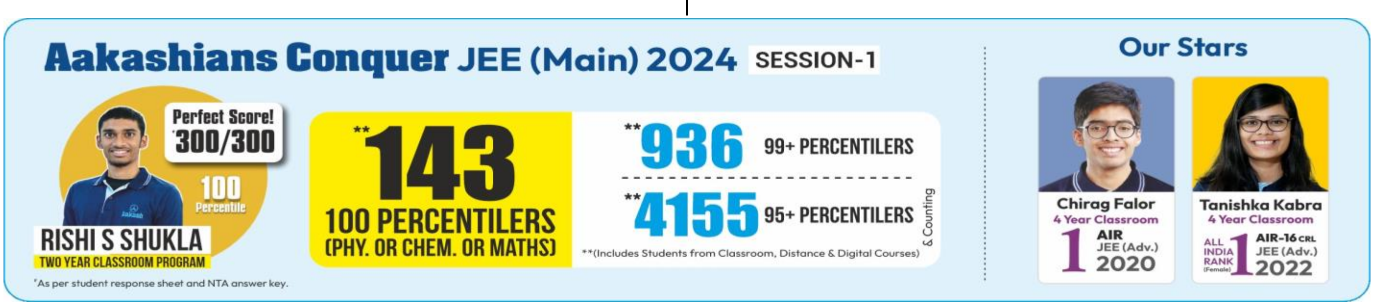
(3) 1, 1 (4) 0, 1

Answer (3)

Sol. $Y = \overline{AB + \overline{A}\,\overline{B}}$

for $A = 0$, $B = 1$, $Y = 1$

for $A = 1$, B = 0, $Y = 1$



39. The magnetic field in a plane electromagnetic wave is

$B_y = (3.5 \times 10^{-7}) \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)$T. The corresponding electric field will be

(1) $E_y = 10.5 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)\text{Vm}^{-1}$

(2) $E_y = 1.17 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)\text{Vm}^{-1}$

(3) $E_z = 1.17 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)\text{Vm}^{-1}$

(4) $E_z = 105 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)\text{Vm}^{-1}$

Answer (None)

Sol. $E_0 = B_0 C$

$$C = \frac{\omega}{K} = \frac{10^8}{3}$$

$$\therefore \quad E_0 = 3.5 \times 10^{-7} \times \frac{10^8}{3}$$

$$= 11\cdot 7$$

$$\therefore \quad E_z = 11.7 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)\ \text{Vm}^{-1}$$

None of the options are correct.

40. UV light of 4.13 eV is incident on a photosensitive metal surface having work function 3.13 eV. The maximum kinetic energy of ejected photoelectrons will be

(1) 1 eV (2) 3.13 eV

(3) 4.13 eV (4) 7.26 eV

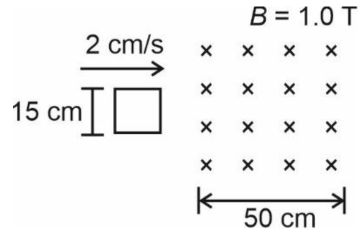
Answer (1)

Sol. $KE_{max} = h\nu - \phi_0$

$= 4.13 - 3.13$

$= 1$ eV

41. A square loop of side 15 cm being moved towards right at a constant speed of 2 cm/s shown in figure. The front edge enters the 50 cm wide magnetic field at $t = 0$. The value of induced emf in the loop at $t = 10$ s will be



(1) Zero (2) 4.5 mV

(3) 3 mV (4) 0.3 mV

Answer (1)

Sol. Time taken to cross the field region

$$= \frac{50}{2} = 25 \text{ s}$$

At 10 s the loop is inside field and flux is not changing.

$\therefore \quad \varepsilon_{induced} = 0$

42. A proton and a deutron (q = +e, m = 2.0 u) having same kinetic energies enter a region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. The ratio of the radius r_d of deutron path to the radius r_p of the proton path is

(1) $\sqrt{2}:1$ (2) 1 : 2

(3) 1 : 1 (4) $1:\sqrt{2}$

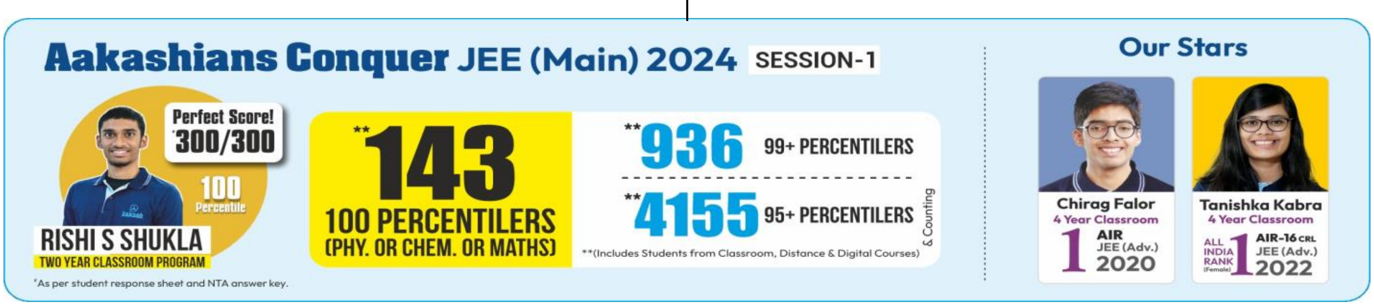
Answer (1)

Sol. $r = \dfrac{mv}{Bq}$

and $mv = \sqrt{2km}$

$\Rightarrow \quad r \propto \sqrt{m}$

$$\therefore \quad \frac{r_d}{r_p} = \sqrt{2}$$



43. A satellite of 10^3 kg mass is revolving in circular orbit of radius $2R$. If $\frac{10^4 R}{6}$ J energy is supplied to the satellite, it would revolve in a new circular orbit of radius :

(use g = 10 m/s^2, R = radius of earth)

(1) 4 R (2) 6 R

(3) 3 R (4) 2.5 R

Answer (2)

Sol. $E = -\frac{GMm}{2r}$

$$E_1 = -\frac{GMm}{4R}$$

and $E_2 = -\frac{GMm}{4R} + \frac{10^4 R}{6}$

$$\Rightarrow \frac{-GMm}{2r'} = -\frac{GMm}{4R} + \frac{10^4 R}{6}$$

$$\Rightarrow \frac{1}{2r'} = \frac{1}{4R} - \frac{1}{6R}$$

$$\Rightarrow r' = 6R$$

44. The energy released in the fusion of 2 kg of hydrogen deep in the sun is E_H and the energy released in the fission of 2 kg of ^{235}U is E_U. The ratio of $\frac{E_H}{E_U}$ is approximately:

(Consider the fusion reaction as $4\,{}^1_1\text{H} + 2e^- \rightarrow {}^4_2\text{He} + 2\nu + 6\gamma + 26.7$ MeV, energy released in the fission reaction of ^{235}U is 200 MeV per fission nucleus and $N_A = 6.023 \times 10^{23}$)

(1) 15.04

(2) 25.6

(3) 7.62

(4) 9.13

Answer (3)

Sol. For fusion

$$\Delta E = 26.7$$

Number of moles in 2 kg of H = $\frac{2000}{1}$

$$\therefore \quad E_H = 26.7 \times \frac{2000}{4} N_A$$

and, $E_U = 200 \times \frac{2000}{235} N_A$

$$\frac{E_H}{E_U} = \frac{26.7}{200} \times \frac{235}{4} \approx 7.62$$

45. A spherical ball of radius 1×10^{-4} m and density 10^5 kg/m^3 falls freely under gravity through a distance h before entering a tank of water, If after entering in water the velocity of the ball does not change, then the value of h is approximately:

(The coefficient of viscosity of water is 9.8×10^{-6} N s/m^2)

(1) 2518 m (2) 2296 m

(3) 2249 m (4) 2396 m

Answer (1)

Sol. $v_t = \sqrt{2gh}$

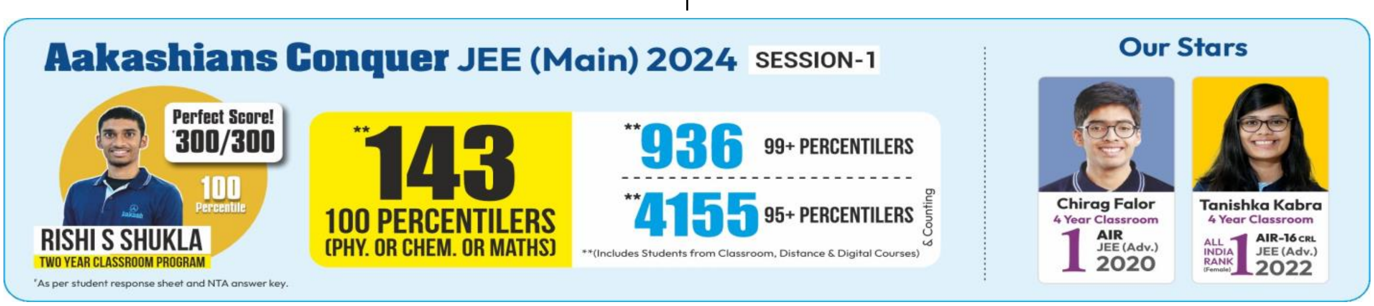
$$v_t = \frac{2}{9} r^2 \frac{(\rho - \sigma) g}{\eta}$$

$$= \frac{2}{9} \times \frac{10^{-8} \times 99 \times 10^3 \times 10}{9.8 \times 10^{-6}}$$

$$\approx \frac{2}{9} \times 10^3$$

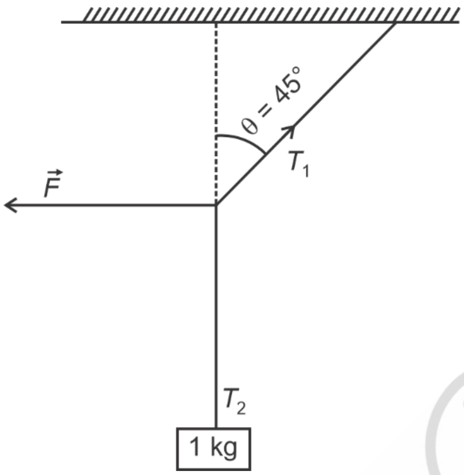
$$\Rightarrow \frac{2}{9} \times 10^3 = \sqrt{20h}$$

$$\Rightarrow h \approx 2518 \text{m}$$



46. A 1 kg mass is suspended from the ceiling by a rope of length 4m. A horizontal force 'F' is applied at the mid point of the rope so that the rope makes an angle of 45° with respect to the vertical axis as shown in figure. The magnitude of F is

(Assume that the system is in equilibrium and $g = 10$ m/s^2)



(1) $\frac{1}{10\times\sqrt{2}}$ N

(2) 10 N

(3) $\frac{10}{\sqrt{2}}$ N

(4) 1 N

Answer (2)

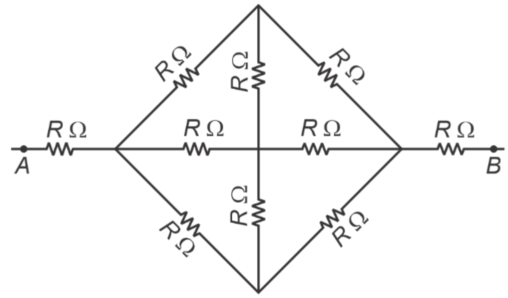
Sol. $T_1 \cos 45 = F$

and, $T_1 \sin 45 = mg$

$\therefore \quad F = mg$

$\Rightarrow \quad F = 10$

47. The effective resistance between A and B, if resistance of each resistor is R, will be



(1) $\frac{5R}{3}$

(2) $\frac{4R}{3}$

(3) $\frac{8R}{3}$

(4) $\frac{2}{3}R$

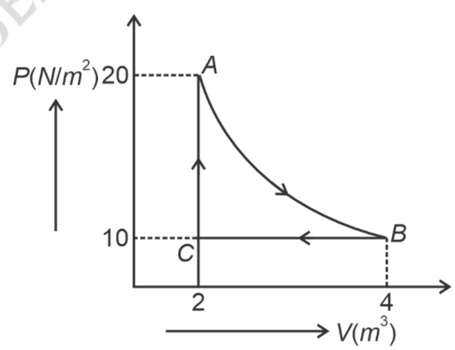
Answer (3)

Sol. Removing the resistors along the line of symmetry

$$R_{eq} = R + \frac{2R}{3} + R$$

$$= \frac{8R}{3}$$

48. A real gas within a closed chamber at 27°C undergoes the cyclic process as shown in figure. The gas obeys $PV^3 = RT$ equation for the path A to B. The net work done in the complete cycle is (assuming R = 8 J/mol K)



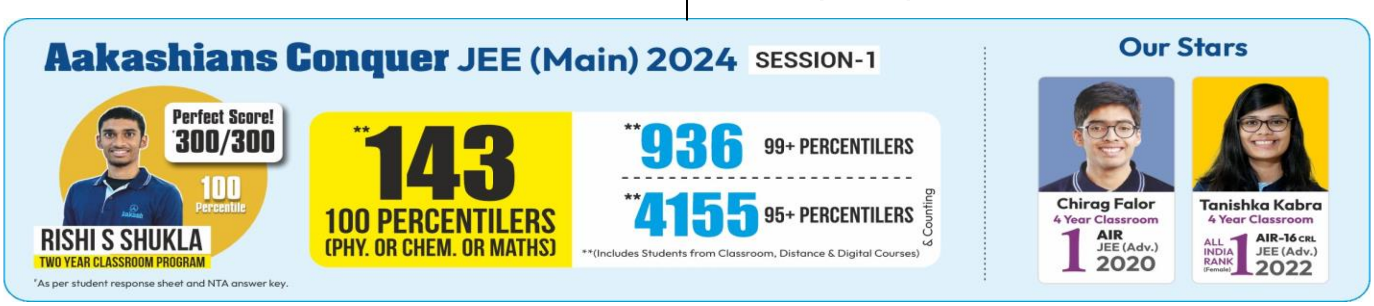
(1) 20 J

(2) 205 J

(3) –20 J

(4) 225 J

Answer (Bonus)



Sol. $PV^3 = RT$

$V^2 (nRT) = RT$

V^2 = Constant

$A \to B$ comes out to be isochoric

So, incorrect question.

49. A nucleus at rest disintegrates into two smaller nuclei with their masses in the ratio of 2 : 1. After disintegration they will move

(1) In opposite directions with speed in the ratio of 2 : 1 respectively.

(2) In the same direction with same speed.

(3) In opposite directions with the same speed.

(4) In opposite directions with speed in the ratio of 1 : 2 respectively.

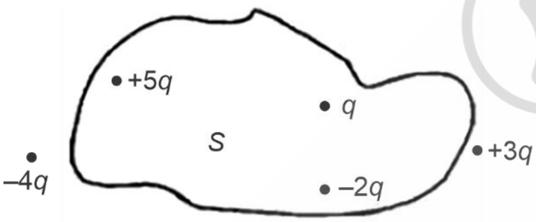
Answer (4)

Sol. According to conservation of momentum

$0 = m_1\vec{v_1} + m_2\vec{v_2}$

$\vec{v_2} = -2\,\vec{v_1}$

50. Five charges +*q*, +5*q*, –2*q*, + 3*q* and –4*q* are situated as shown in the figure. The electric flux due to this configuration through the surface *S* is



(1) $\frac{5q}{\epsilon_0}$ (2) $\frac{4q}{\epsilon_0}$

(3) $\frac{3q}{\epsilon_0}$ (4) $\frac{q}{\epsilon_0}$

Answer (2)

Sol. $\phi = \frac{q_{en}}{\epsilon_0}$

$= \frac{(5+1-2)q}{\epsilon_0}$

$= \frac{4q}{\epsilon_0}$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

51. A circular disc reaches from top to bottom of an inclined plane of length *l*. When it slips down the plane, if takes *t* s. When it rolls down the plane then it takes $\left(\frac{\alpha}{2}\right)^{1/2} t$ s, where α is _________.

Answer (3)

Sol. During slipping

$a_1 = g\sin\theta$

During rolling

$Rmg\sin\theta = \left(\frac{MR^2}{2} + MR^2\right)\alpha$

$\therefore\quad a_2 = \frac{2g\sin\theta}{3}$

and $t = \sqrt{\frac{2s}{a}}$

$\therefore\quad t_2 = t_1\sqrt{\frac{a_1}{a_2}}$

$= t\sqrt{\frac{3}{2}}$

$\Rightarrow\quad \alpha = 3$

52. At room temperature (27°C), the resistance of a heating element is 50 Ω. The temperature coefficient of the material is 2.4 × 10^{-5} °C^{-1}. The temperature of the element, when its resistance is 62 Ω is _________ °C.

Answer (1027)

Sol. $R = R_0 (1 + \alpha \Delta T)$

$\Rightarrow$ $62 = 50 (1 + 2.4 \times 10^{-4} \Delta T)$

$\Rightarrow$ $0.24 = 2.4 \times 10^{-4} \Delta T$

$\Rightarrow$ $\Delta T = 1000$

$\therefore$ $T = 1000 + 27$

$= 1027$ °C

53. A force $(3x^2 + 2x - 5)$ N displaces a body from $x = 2$ m to $x = 4$ m. Work done by this force is _______ J.

Answer (58)

Sol. $W = \int F \cdot dx$

$$W = \left[x^3 + x^2 - 5x\right]_2^4$$

$= 58$

54. A straight magnetic strip has a magnetic moment of 44 Am^2. If the strip is bent in a semicircular shape, its magnetic moment will be _________ Am^2.

$\left(\text{given } \pi = \frac{22}{7}\right)$

Answer (28)

Sol. In semicircular shape

Effective length = $2R$

$\therefore$ $l' = 2R$

$l = \pi R$

$$l' = \frac{2}{\pi} l$$

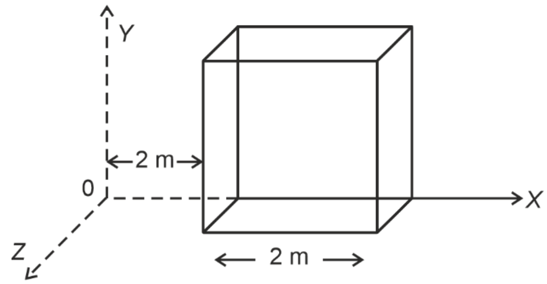
$M \propto l$

$$\therefore \quad M' = \frac{2}{\pi} M$$

$$= \frac{2}{22} \times 7 \times 44$$

$= 28$

55. An electric field $\vec{E} = \left(2x\hat{i}\right) NC^{-1}$ exists in space. A cube of side 2 m is placed in the space as per figure given below. The electric flux through the cube is ________ Nm^2/C.



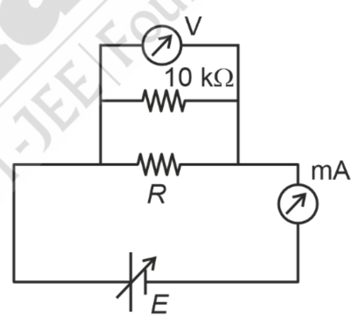
Answer (16)

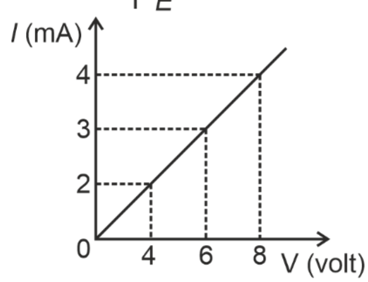
Sol. Flux will only be due to surfaces having area vector parallel to x – axis

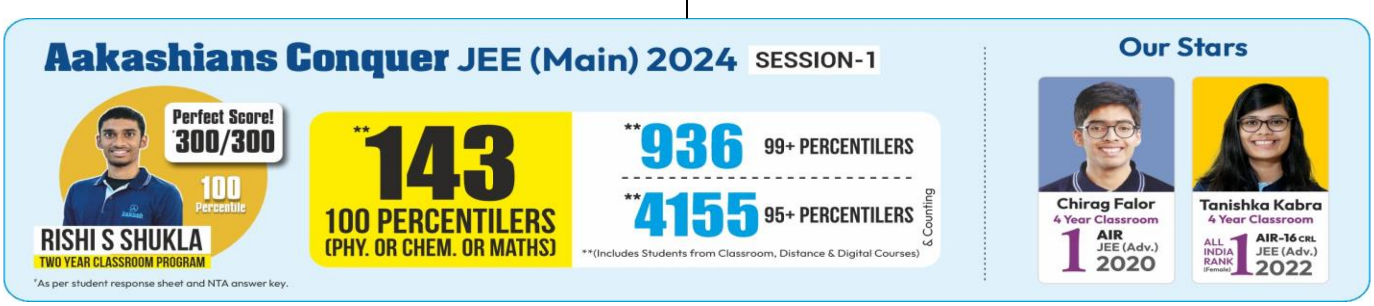
$\therefore$ $\phi_{net} = A [8 - 4]$

$= 4A = 4 \times 4 = 16$

56. To determine the resistance (R) of a wire, a circuit is designed below. The V-I characteristic curve for this circuit is plotted for the voltmeter and the ammeter readings as shown in figure. The value of R is _________ Ω.







Answer (2500)

Sol. $R_{net} = \frac{1}{\text{Slope}} = \frac{8}{4} \times 10^3 = 2 \times 10^3$

$\Rightarrow \ 2 = \frac{10R}{10 + R}$

$\Rightarrow$ 20 + 2R = 10R

$\Rightarrow$ 8R = 20

$\Rightarrow$ R = 2.5 kΩ

= 2500 Ω

57. A particle of mass 0.50 kg executes simple harmonic motion under force $F = -50\ (\text{Nm}^{-1})\ x$. The time period of oscillation is $\frac{x}{35}$ s. The value of x is ________.

$\left(\text{Given } \pi = \frac{22}{7}\right)$

Answer (22)

Sol. $F = -50x = ma$

$\Rightarrow \ a = \frac{-50x}{m} = -100x$

$\Rightarrow \ a = -\omega^2 x$

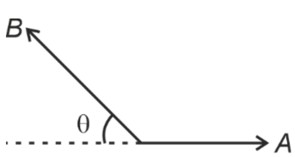
$\Rightarrow \ \omega = 10$

$\therefore \ T = \frac{2\pi}{\omega} = \frac{\pi}{5} = \frac{22}{7 \times 5} = \frac{22}{35}\text{s}$

58. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$ and its magnitude is half that of $\vec{B}$. The angle between vectors $\vec{A}$ and $\vec{B}$ is _______°.

Answer (150)

Sol.



$B\cos\theta = A$

$B\sin\theta = \frac{B}{2}$

$\therefore \ \sin\theta = \frac{1}{2}$

$\Rightarrow \ \theta = 30°$

Angle between vectors = 180 – θ = 150°

59. Monochromatic light of wavelength 500 nm is used in Young's double slit experiment. An interference pattern is obtained on a screen. When one of the slits is covered with a very thin glass plate (refractive index = 1.5), the central maximum is shifted to a position previously occupied by the 4th bright fringe. The thickness of the glass-plate is ________ μm.

Answer (4)

Sol. $\Delta x = 4\lambda$

and, $\Delta x = (\mu - 1)t$

$\Rightarrow \ (\mu - 1)t = 4 \times 500 \times 10^{-9}$

$\Rightarrow \ \frac{t}{2} = 2\ \mu\text{m}$

$\Rightarrow \ t = 4\ \mu\text{m}$

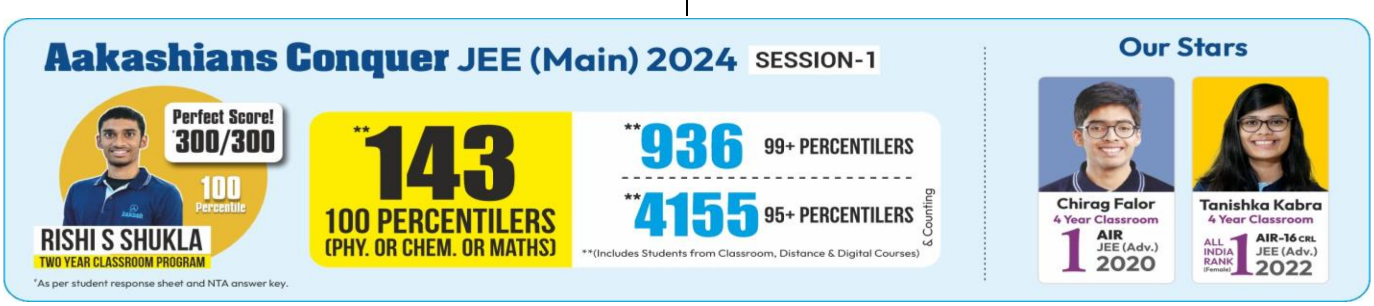
60. A capacitor of reactance $4\sqrt{3}\ \Omega$ and a resistor of resistance 4 Ω are connected in series with an ac source of peak value $8\sqrt{2}$ V. The power dissipation in the circuit is _______ W.

Answer (4)

Sol. $Z = \sqrt{X_c^2 + R^2} = 8\ \Omega$

$\therefore \ i = \frac{V_0}{\sqrt{2}Z} = 1$

$\Rightarrow \ P = i^2R = 1 \times 4 = 4$ W

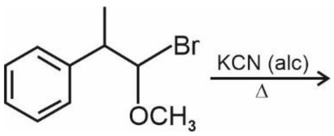


CHEMISTRY

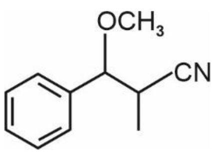
SECTION - A

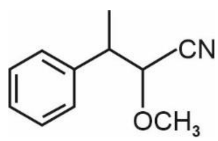
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

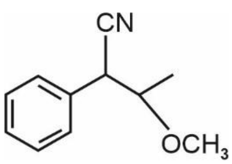
Choose the correct answer :

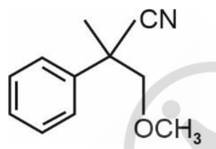
61.  Major Product 'P'

In the above reaction product 'P' is

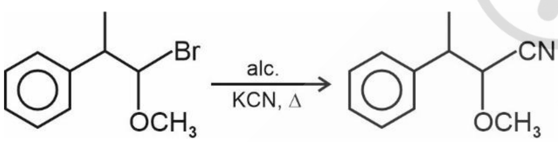
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

Nucleophilic substitution reaction will occur.

62. Which out of the following is a correct equation to show change in molar conductivity with respect to concentration for a weak electrolyte, if the symbols carry their usual meaning :

(1) $\Lambda_m^2 C - K_a \Lambda_m^{o2} + K_a \Lambda_m \Lambda_m^o = 0$

(2) $\Lambda_m - \Lambda_m^o + AC^{\frac{1}{2}} = 0$

(3) $\Lambda_m - \Lambda_m^o - AC^{\frac{1}{2}} = 0$

(4) $\Lambda_m^2 C + K_a \Lambda_m^{o2} - K_a \Lambda_m \Lambda_m^o = 0$

Answer (1)

Sol. For weak electrolyte

$$K_a = \frac{C\lambda^2}{1-\lambda}$$

$$\lambda = \frac{\Lambda_m}{\Lambda_m^o}$$

$$K_a = \frac{C\left(\frac{\Lambda_m}{\Lambda_m^o}\right)^2}{1-\frac{\Lambda_m}{\Lambda_m^o}}$$

$$K_a = \frac{C\Lambda_m^2}{\Lambda_m^o\left(\Lambda_m^o - \Lambda_m\right)}$$

$$K_a \Lambda_m^{o2} - K_a \Lambda_m^o \Lambda_m = C\Lambda_m^2$$

$$C\Lambda_m^2 - K_a \Lambda_m^{o2} + K_a \Lambda_m \Lambda_m^o = 0$$

63. The coordination environment of Ca^{2+} ion in its complex with $EDTA^{4-}$ is

(1) square planar

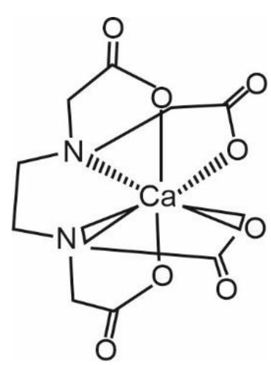
(2) tetrahedral

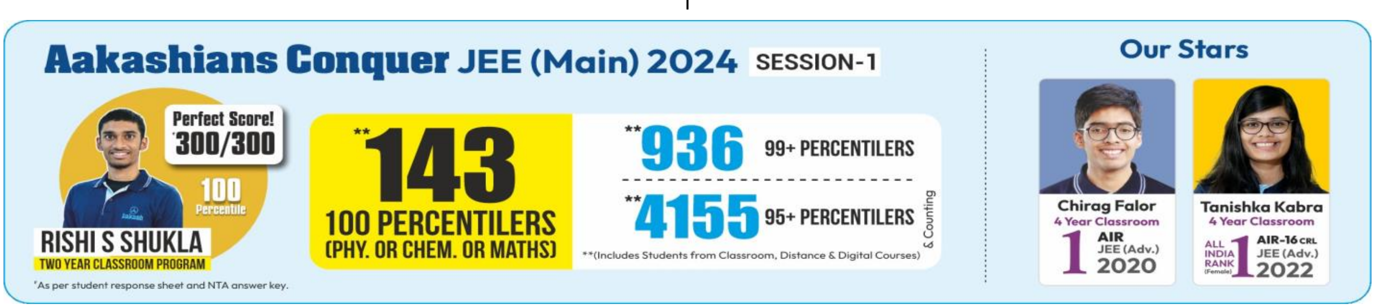
(3) octahedral

(4) trigonal prismatic

Answer (3)

Sol. Octahedral





64. The electronic configuration of Einsteinium is :

(Given atomic number of Einsteinium = 99)

(1) $[Rn]5f^{13}6d^{0}7s^{2}$ (2) $[Rn]5f^{12}6d^{0}7s^{2}$

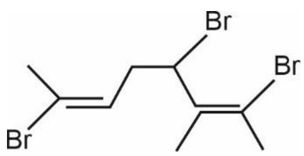
(3) $[Rn]5f^{11}6d^{0}7s^{2}$ (4) $[Rn]5f^{10}6d^{0}7s^{2}$

Answer (3)

Sol. Es atomic number is $\Rightarrow$ 99

Es electronic configuration = $[Rn]5f^{11}6d^{0}7s^{2}$

65. Total number of stereo isomers possible for the given structure :

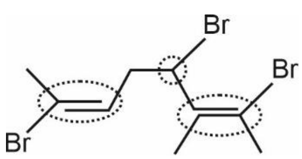


(1) 4 (2) 8

(3) 2 (4) 3

Answer (2)

Sol.



3 stereocentres

$\therefore$ Total Stereoisomerism = $2^3 = 8$

66. The **incorrect** statement about Glucose is :

(1) Glucose is soluble in water because of having aldehyde functional group

(2) Glucose is an aldohexose

(3) Glucose is one of the monomer unit in sucrose

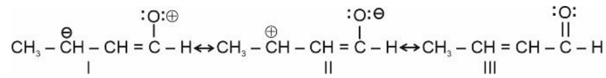
(4) Glucose remains in multiple in isomeric form in its aqueous solution

Answer (1)

Sol. The reason glucose dissolves readily in water is because it has lots of polar hydroxyl groups which can H-bond with water and not because of aldehyde functional group.

$\therefore$ The incorrect statement is (1)

67. The correct stability order of the following resonance structures of $CH_3 - CH = CH - CHO$ is



(1) II > I > III (2) III > II > I

(3) II > III > I (4) I > II > III

Answer (2)

Sol. The correct stability order is III > II > I.

The reason is $\rightarrow$ III$\rightarrow$ No charge, neutral structures are more stable than charged.

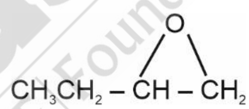
$\rightarrow$ In II, negative charge on more electronegative atom and positive on less electronegative atom.

$\rightarrow$ In I, negative charge on less electronegative atom to less stable.

68. Which of the following compound can give positive iodoform test when treated with aqueous KOH solution followed by potassium hypoiodite?

(1) $CH_3CH_2CH_2CHO$

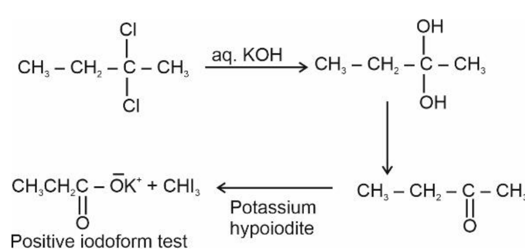
(2) $CH_3CH_2 - \overset{O}{\overset{||}{C}} - CH_2CH_3$

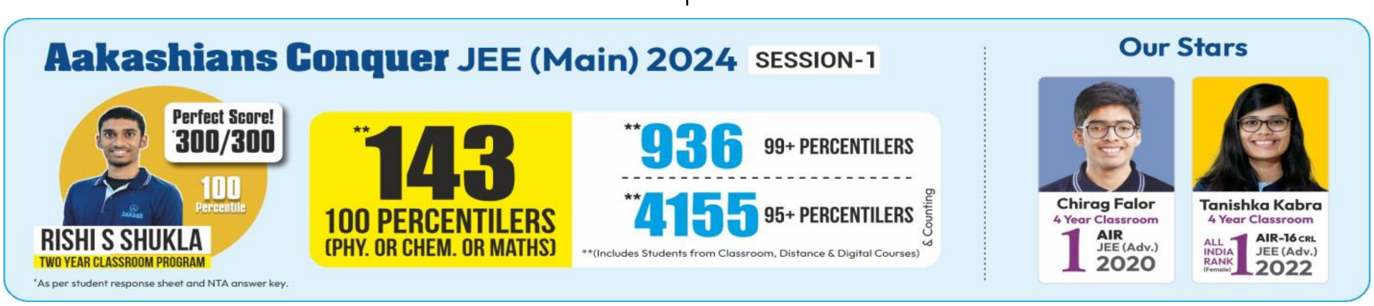
(3) 

(4) $CH_3CH_2 - C(Cl)_2 - CH_3$

Answer (4)

Sol.





69. Match List I with List II

	List I (Test)		**List II (Observation)**
A.	Br_2 water test	I.	Yellow orange or orange red precipitate formed
B.	Ceric ammonium nitrate test	II.	Reddish orange colour disappears
C.	Ferric chloride test	III.	Red colour appears
D.	2, 4-DNP test	IV.	Blue, Green, Violet or Red colour appear

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV (2) A-III, B-IV, C-I, D-II

(3) A-II, B-III, C-IV, D-I (4) A-IV, B-I, C-II, D-III

Answer (3)

Sol. A-II, B-III, C-IV, D-I

A.	Br_2 water test	II.	Reddish orange colour disappears
B.	Ceric ammonium nitrate test	III.	Red colour appears
C.	Ferric chloride test	IV.	Blue, Green, Violet or Red colour appear
D.	2, 4-DNP test	I.	Yellow orange or orange red precipitate formed

70. Match List I with List II

	List I		**List II**
A.	Melting Point [K]	I.	Tl > In > Ga > Al > B
B.	Ionic Radius [M^{+3}/pm]	II.	$B > Tl > Al \approx Ga > In$
C.	$\Delta_i H_1$ [kJ mol^{-1}]	III.	Tl > In > Al > Ga > B
D.	Atomic Radius [pm]	IV.	B > Al > Tl > In > Ga

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III (2) A-II, B-III, C-IV, D-I

(3) A-III, B-IV, C-I, D-II (4) A-I, B-II, C-III, D-IV

Answer (1)

Sol.

Melting point order of group 13

$\Rightarrow$ B > Al > Tl > In > Ga

Ionic radius (M^{3+}) order or group 13

$\Rightarrow$ Tl > In > Ga > Al > B

$\Delta_i H_1$ [kJ mol^{-1}] order of gr. 13

$\Rightarrow B > Tl > Al \approx Ga > In$

Atomic Radius [pm] order of group13

$\Rightarrow$Tl > In > Al > Ga > B

71. Which of the following compounds will give silver mirror with ammoniacal silver nitrate?

A. Formic acid

B. Formaldehyde

C. Benzaldehyde

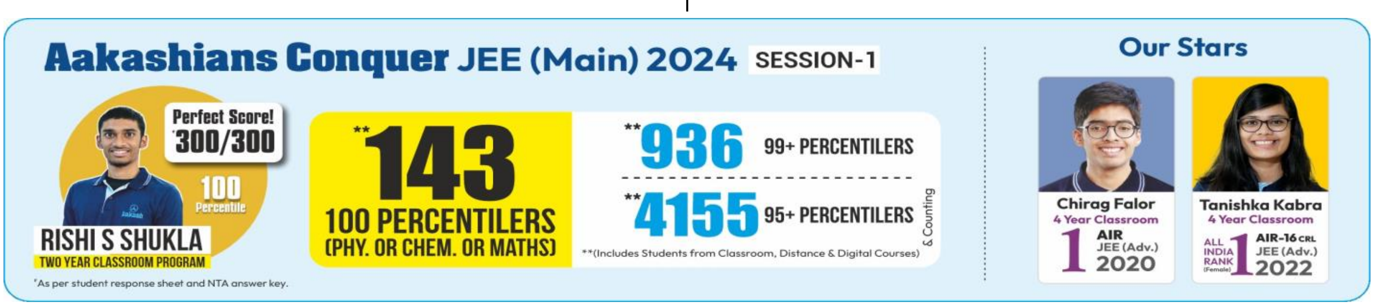
D. Acetone

Choose the correct answer from the options given below:

(1) B and C only (2) C and D only

(3) A, B and C only (4) A only

Answer (3)



Sol. Aldehydes reacts with Tollen's reagent to form a silver mirror aldehydes themselves are oxidised to carboxylate ions and silver nitrate is reduced to block silver metal.

$$R-CHO+[Ag(NH_3)_2]^{+}OH^{-} \rightarrow RCOO^{-}+Ag\downarrow+NH_4^{\oplus}$$

Formaldehyde and benzaldehyde gives silver mirror with ammoniacal silver nitrate as they contain an aldehyde group. Formic acid has the aldehydic H along with –COOH group it also gives silver mirror with ammoniacal silver nitrate. Acetone does not exhibit silver mirror test as they do not contain an aldehyde group.

72. The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 'A' × 10^{12} hertz and that has a radiant intensity in that direction of $\frac{1}{\text{'B'}}$ watt per steradian.

'A' and 'B' are respectively

(1) 450 and $\frac{1}{683}$ (2) 450 and 683

(3) 540 and $\frac{1}{683}$ (4) 540 and 683

Answer (4)

Sol. The candela is the luminous intensity, in a given direction of a source that emits monochromatic radiation of frequency 540 × 10^{12} hertz and that has a radiant intensity in that direction of $\frac{1}{683}$ watt per steradian.

73. Match List-I with List-II

List-I (Element)		List-II (Electronic Configuration)	
(A)	N	(I)	[Ar] $3d^{10}\ 4s^2\ 4p^5$
(B)	S	(II)	[Ne] $3s^2\ 3p^4$
(C)	Br	(III)	[He] $2s^2\ 2p^3$
(D)	Kr	(IV)	[Ar] $3d^{10}\ 4s^2\ 4p^6$

Choose the correct answer from the options given below:

(1) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

(2) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

(3) (A) – (I), (B) – (IV), (C) – (III), (D) – (II)

(4) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

Answer (1)

Sol.

	Element		**Electronic configuration**
(A)	N (Z = 7)	(III)	$[He]2s^22p^3$
(B)	S (Z = 16)	(II)	$[Ne]3s^23p^4$
(C)	Br (Z = 35)	(I)	$[Ar]3d^{10}4s^24p^5$
(D)	Kr (Z = 36)	(IV)	$[Ar]3d^{10}4s^24p^6$

74. Match List-I with List-II

List-I		List-II	
(A)	$K_2[Ni(CN)_4]$	(I)	sp^3
(B)	$[Ni(CO)_4]$	(II)	sp^3d^2
(C)	$[Co(NH_3)_6]Cl_3$	(III)	dsp^2
(D)	$Na_3[CoF_6]$	(IV)	d^2sp^2

Choose the correct answer from the options given below:

(1) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

(2) (A) – (III), (B) – (I), (C) – (II), (D) – (IV)

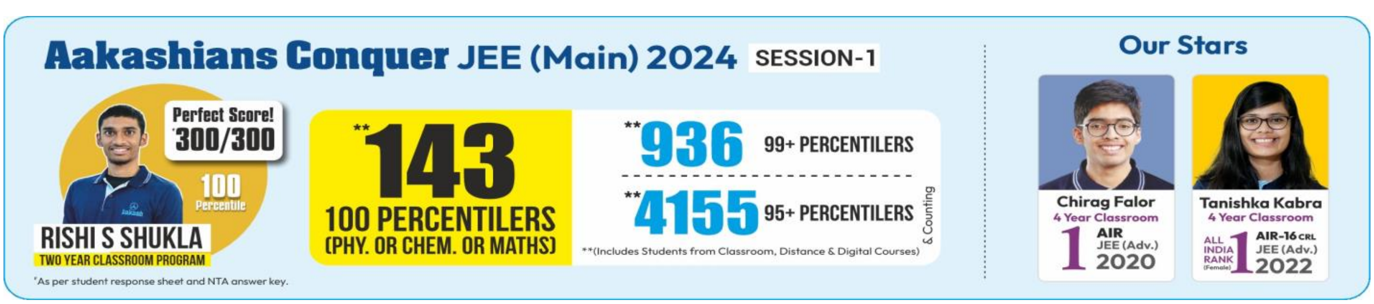
(3) (A) – (I), (B) – (III), (C) – (II), (D) – (IV)

(4) (A) – (III), (B) – (II), (C) – (IV), (D) – (I)

Answer (1)

Sol. $[Ni(CN)_4]^{2-}$ is dsp^2 hybridized as Ni is present in +2 oxidation state and CN^- is a strong field ligand whereas Ni in $[Ni(CO)_4]$ exists in zero oxidation state and sp^3 hybridized.

For $[Co(NH_3)_6]Cl_3$, Co is d^2sp^3 hybridized as NH_3 for Co^{3+} acts as strong field ligand whereas F^- acts as weak field ligand thus, $Na_3[CoF_6]$ is sp^3d^2 hybridized.



75. For a sparingly soluble salt AB_2, the equilibrium concentrations of A^{2+} ions and B^- ions are 1.2×10^{-4} M and 0.24×10^{-3} M, respectively. The solubility product of AB_2 is

(1) 0.069×10^{-12} (2) 6.91×10^{-12}

(3) 0.276×10^{-12} (4) 27.65×10^{-12}

Answer (2)

Sol. For $AB_2 \rightleftharpoons A^{2+} + 2B^-$

$K_{sp} = [A^{2+}][B^-]$

$= (1.2 \times 10^{-4})(0.24 \times 10^{-3})^2$

$= 0.0691 \times 10^{-10}$

$K_{sp} = 6.91 \times 10^{-12}$

76. The correct increasing order for bond angles among BF_3, PF_3 and ClF_3 is

(1) $ClF_3 < PF_3 < BF_3$ (2) $PF_3 < BF_3 < ClF_3$

(3) $BF_3 < PF_3 < ClF_3$ (4) $BF_3 = PF_3 < ClF_3$

Answer (1)

Sol. BF_3 is trigonal planar with an angle of 120°, PF_3 has a lone pair of electrons. The angle here is around 96°. ClF_3 has 2 lone pairs of electrons and has a bond angle of 90°- it is T-shaped.

77. Match List-I with List-II

List-I (Cell)		**List-I (Use/Property/Reaction)**	
(A)	Leclanche Cell	(I)	Converts energy of combustion into electrical energy
(B)	Ni -Cd cell	(II)	Does not involve any ion in solution and is used in hearing aids
(C)	Fuel cell	(III)	Rechargeable
(D)	Mercury cell	(IV)	Reaction at anode $Zn \rightarrow Z^{2+} + 2e^-$

Choose the correct answer from the options given below:

(1) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

(2) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

(3) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

(4) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

Answer (2)

Sol. Leclanche cell is the dry cell. It consists of a zinc container that acts as anode and cathode is a carbon rod.

The electrode reactions are:

Anode: $Zn(s) \rightarrow Zn^{2+} + 2e^-$

Cathode: $MnO_2 + NH_4^+ + e^- \rightarrow MnO(OH) + NH_3$

Nickel-cadmium cell is rechargeable. Galvanic cells that are designed to convert the energy of combustion of fuels like hydrogen, methane etc directly into electrical energy are called fuel cells. Mercury cell is suitable for low current devices like hearing ards.

78. The incorrect **statements** regarding ethyne is

(1) The corbon – carbon bonds in ethyne is weaker than that in ethene

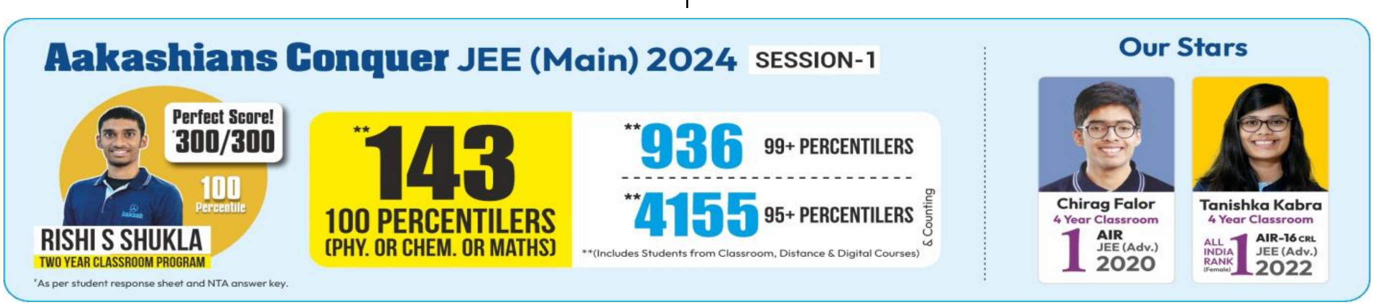
(2) Both carbons are sp hybridised

(3) The C – C bond in ethyne is shorter than that in ethane

(4) Ethyne is linear

Answer (1)

Sol. In Ethyne $HC \equiv CH$, both carbons are *sp* hybridised and carbon – carbon bonds in ethyne are shorter (~ 120 pm) than that in ethene (~133 pm) and the strength of $C \equiv C$ bond (bond enthalpy ~823 kJ mol^{-1}) is more than those of C = C bond (bond enthalpy ~681 kJ mol^{-1}). Electron cloud between two carbon atoms in cylindrically symmetrical about the internuclear axis. Thus, ethyne is a linear molecule.



79. Give below are two statements:

Statement I : The higher oxidation states are more stable down the group among transition elements unlike p-block elements.

Statement II : Copper can not liberate hydrogen from weak acids

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

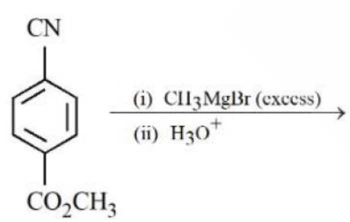
Answer (4)

Sol. In transition elements the higher oxidation state are more stable for heavier elements in a group e.g. Mo(VI) and W(VI) are more stable than Cu(VI) in group 6

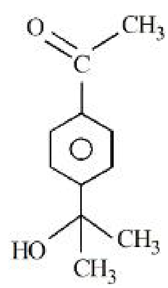
Standard oxidation potential of copper is less than that of hydrogen. This means in standard conditions oxidation of H_2 to $2H^+$ is more favourable than oxidation of Cu to Cu^{2+}. So Cu cannot displace or reduce H^+ from dilute acids as H_2 gas.

Therefore, both statement I and Statement II are true.

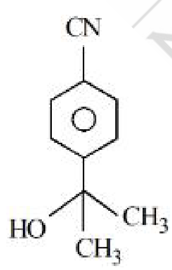
80. Major product of the following reaction is



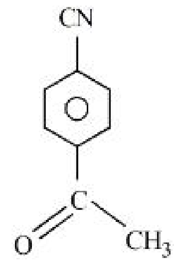
(1)



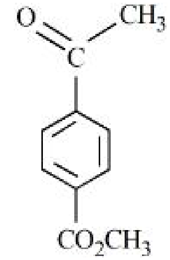
(2)



(3)

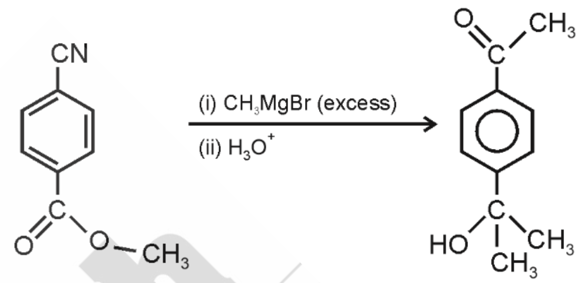


(4)



Answer (1)

Sol. Grignard reagent is a strong nucleophile. It undergoes nucleophilic addition reaction when it reacts with ester and produces ketone which on further addition gives tertiary alcohol and nitrile on reaction with Grignard gives ketone.



SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

81. When ΔH_{vap} = 30 kJ/mol and ΔS_{vap} = 75 J mol^{-1} K^{-1}, then the temperature of vapour, of one atmosphere is _______ K.

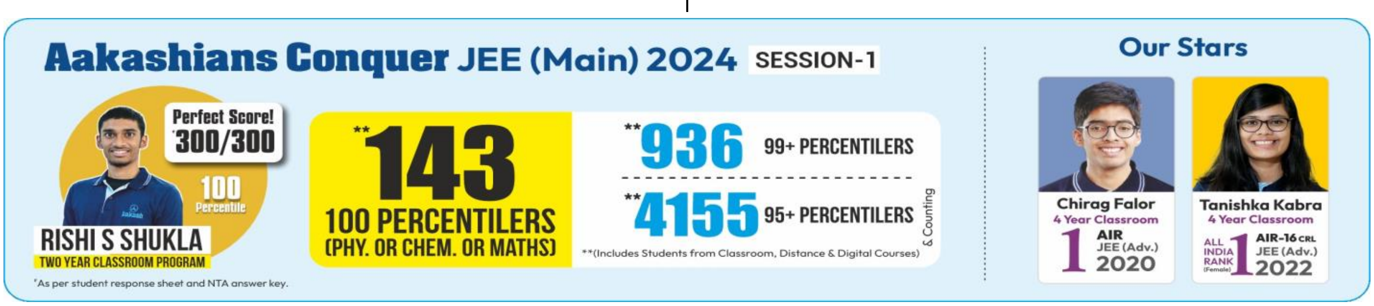
Answer (400)

Sol. $\Delta H_{vap} = T\Delta S_{vap}$

$$\therefore \quad T = \frac{\Delta H_{vap}}{\Delta S_{vap}} = \frac{30 \times 1000}{75} = 400 \text{ K}$$

82. Number of compounds from the following which **cannot** undergo Friedel-Crafts reactions is : ______

Toluene, nitrobenzene, xylene, cumene aniline, chlorobenzene, *m*-nitroaniline, *m*-dinitrobenzene



Answer (4)

Sol. Nitrobenzene does not undergo Friedel-Crafts reaction because the nitro group is strongly electron withdrawing and deactivates the ring. Aniline also does not undergo Friedel-Crafts reaction as in the presence of $AlCl_3$, aniline and $AlCl_3$ forms a complex which deactivates the ring.

Toluene, xylene, cumene and chlorobenzene undergo Friedel-Crafts reaction whereas nitrobenzene, aniline, *m*-nitroaniline and *m*-dinitrobenzene do not undergo Friedel-Crafts reactions.

83. Based on Heisenberg's uncertainty principle, the uncertainty in the velocity of the electron to be found within an atomic nucleus of diameter 10^{-15} m is _______ × 10^9 ms–1 (nearest integer)

[Given : mass of electron = 9.1 × 10^{-31} kg. Plank's constant (h) = 6.626 × 10^{-34} Js]

(Value of π = 3.14)

Answer (58)

Sol. According to Heisenberg's uncertainty formula

$$\Delta x \cdot m\Delta v = \frac{h}{4\pi}$$

$$\therefore \quad \Delta v = \frac{h}{4\pi \times m \cdot \Delta x}$$

$$\Delta v = \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times (10^{-15}) \times 9.1 \times 10^{-31}}$$

$$\Delta v = 0.0579 \times 10^{12} \ \ ms^{-1}$$

or $\Delta v = 58 \times 10^{9}$ ms^{-1}

84. A transition metal 'M' among Sc, Ti, V, Cr, Mn and Fe has the highest second ionisation enthalpy. The spin-only magnetic moment value of M^+ ion is _____ BM (Near Integer)

(Given atomic number Sc : 21, Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26)

Answer (6)

Sol. Cr^+ : [Ar]$3d^5$ have stable half filled configuration so it has highest second ionization energy

Spin only magnetic moment, $\mu = \sqrt{n(n+2)}$ BM.

$$= \sqrt{5(7)} = \sqrt{35} \text{ BM.}$$

$$= 5.91 \text{ BM.}$$

$$\simeq 6 \text{ BM.}$$

85. Consider the following first order gas phase reaction at constant temperature

A(g) → 2B(g) + C(g)

If the total pressure of the gases is found to be 200 torr after 23 sec. and 300 torr upon the complete decomposition of A after a very long time, then the rate constant of the given reaction is _________ × 10^{-2} s^{-1} (nearest integer)

[Given : $\log_{10}(2)$ = 0.301]

Answer (3)

Sol. Given reaction is

	A(g) →	2B(g) +	C(g)	
At t = 0	p_i	0	0	
t = 23 s	$p_i - p$	2p	p	p_{Total} = 200 torr
t = very long time	0	$2p_i$	p_i	p_{Total} = 300 torr

$\therefore$ $2p_i + p_i = 300 \Rightarrow p_i = 100$ torr

Now, for first order reaction

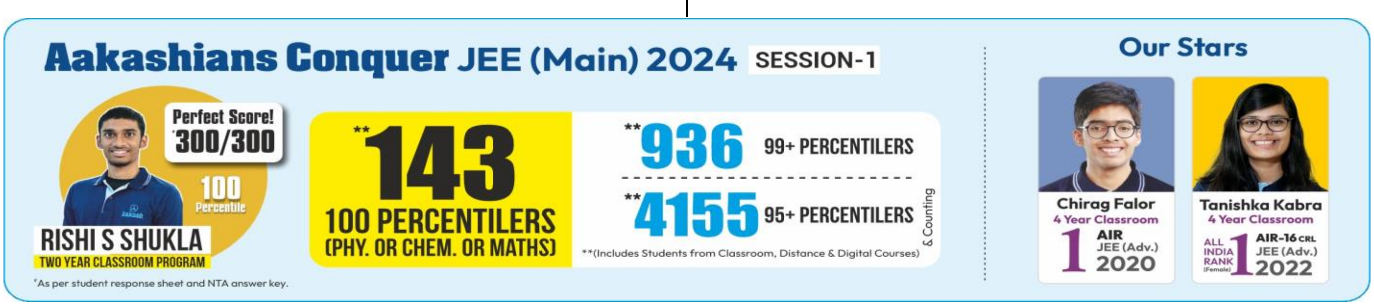
$$k = \frac{2.303}{t}\log\frac{p_i}{p_i - p}$$

For t = 23 s

$$k = \frac{2.303}{23}\log\frac{2p_i}{3p_i - 300}$$

$$k = \frac{2.303}{23}\log 2 = 0.0301$$

k = 3.01 × 10^{-2} s^{-1}



86. Total number of electrons present in (π^*) molecular orbitals of O_2, O_2^+ and O_2^- is________.

Answer (6)

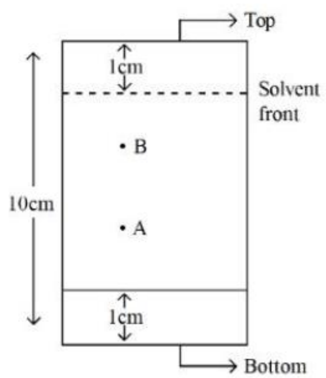
Sol. $O_2 \Rightarrow 2\pi^*$ electrons

$O_2^+ \Rightarrow 1\pi^*$ electrons

$O_2^- \Rightarrow 3\pi^*$ electrons

Total $\Rightarrow 6\pi^*$ electrons are present

87.



In the given TLC, the distance of spot A & B are 5 cm & 7 cm, from the bottom of TLC plate, respectively.

R_f value of B is x × 10^{-1} time more than A. The value of x is _____.

Answer (15)

Sol. $R_f = \dfrac{\text{Distance travelled by sample}}{\text{Distance travelled by solvent}}$

R_f for $A = \dfrac{4\text{ cm}}{8\text{ cm}} = 0.5$

R_f for $B = \dfrac{6\text{ cm}}{8\text{ cm}} = \dfrac{3}{4}$

$\therefore$ R_f value of B is 15 × 10^{-1} times of R_f value of A.

The value of x is 15.

88. Consider the following test for a group-IV cation.

$M^{2+} + H_2S \rightarrow$ A (Black precipitate) + byproduct

A + aqua regia $\rightarrow$ B + NOCl + S + H_2O

B + KNO_2 + CH_3COOH $\rightarrow$ C + byproduct

The spin-only magnetic moment value of the metal complex C is _______ BM (Nearest integer)

Answer (0)

Sol. $Co^{2+} + H_2S \rightarrow \underset{\text{(Black ppt (A))}}{CoS} + \text{byproduct}$

$$CoS + \underset{\text{(aqua regia)}}{HNO_3 + 3HCl} \rightarrow \underset{\text{(B)}}{CoCl_2} + NOCl + S + 2H_2O$$

$$CoCl_2 + 7KNO_2 + 2CH_3COOH \rightarrow \underset{\text{Potassium hexanitrito cobalt (III)}}{K_3[Co(NO_2)_6]}$$

$$+ KCl + 2CH_3COOK + NO + H_2O$$

The spin only magnetic moment,

$\mu_{spin} = \sqrt{n(n+2)}$ BM

For $K_3[Co(NO_2)_6]$ complex, number of unpaired electron is zero. Therefore $\mu_{spin} = 0$

89. The vapor pressure of benzene and methyl benzene at 27ºC is given as 80 Torr and 24 Torr, respectively. The mole fraction of methyl benzene in vapor phase, in equilibrium with an equimolar mixture of those two liquids (ideal solution) at the same temperature is _______ × 10^{-2} (nearest integer)

Answer (23)

Sol. $y_A = \dfrac{24}{80+24} = 0.2307$

$= 23.07 \times 10^{-2}$

$= 23$

90. Number of oxygen atoms present in chemical formula of fuming sulphuric acid is_______.

Answer (7)

Sol. Fuming sulphuric acid is $\underbrace{H_2SO_4 + SO_3}_{\text{oleum}}$ vapours.

$H_2S_2O_7 \Rightarrow$ oleum $\Rightarrow$ fuming sulphuric acid

$\Rightarrow$ 7 oxygen atoms are present.

❑ ❑ ❑