09/04/2024
Morning

Aakash
Medical | IIT-JEE | Foundations

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6) **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let α, β be the roots of the equation $x^2+2\sqrt{2}x-1=0$. The quadratic equation, whose roots are $\alpha^4+\beta^4$ and $\frac{1}{10}\left(\alpha^6+\beta^6\right)$, is:

(1) $x^2 - 180x + 9506 = 0$

(2) $x^2 - 190x + 9466 = 0$

(3) $x^2 - 195x + 9466 = 0$

(4) $x^2 - 195x + 9506 = 0$

Answer (4)

Sol. $x^2+2\sqrt{2x}-1=0$

$\alpha+\beta=-2\sqrt{2}$ and $\alpha\beta = -1$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 = -2\alpha\beta$

$= 8 + 2 = 10$

$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$

$= 100 - 2 = 98$

$\alpha^6 + \beta^6 = (\alpha^2 + \beta^2)^3 - 3\alpha^2\beta^2(\alpha^2 + \beta^2)$

$= 1000 - 3(10)$

$= 970$

$\therefore \quad \frac{1}{10}(\alpha^6+\beta^6)=97$

Equation whose roots are $\alpha^4+\beta^4$ and $\frac{1}{10}(\alpha^6+\beta^6)$ is

$x^2 - (98 + 97)x + 98 \times 97 = 0$

$x^2 - 195x + 9506 = 0$

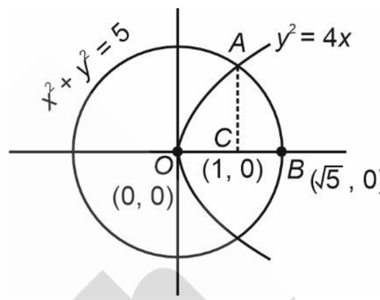
2. The parabola $y^2 = 4x$ divides the area of the circle $x^2 + y^2 = 5$ in two parts. The area of the smaller part is equal to:

(1) $\frac{2}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (2) $\frac{1}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(3) $\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (4) $\frac{1}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Answer (3)

Sol.



The point of intersection of $y^2 = 4x$ and $x^2 + y^2 = 5$ are (1, 2) and (1, –2).

$\because$ Area of smaller region bounded by $y^2 = 4x$ and $x^2 + y^2 = 5$

=2{area of *OACO* + area of *CABC*}

$$=2\left[\int_0^1 2\sqrt{x}\,dx+\int_1^{\sqrt{5}}\sqrt{5-x^2}\,dx\right]$$

$$=2\left[\left|\frac{4}{3}x^{\frac{3}{2}}\right|_0^1+\left(\frac{1}{2}x\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right)\right]_1^{\sqrt{5}}$$

$$=2\left[\left(\frac{4}{3}-0\right)+\left(0+\frac{5\pi}{4}\right)-\left(1+\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right)\right]$$

$$=2\left[\frac{1}{3}+\frac{5\pi}{4}-\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right]=\frac{2}{3}+\frac{5\pi}{2}-5\sin^{-1}\frac{1}{\sqrt{5}}$$

$$=\frac{2}{3}+5\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)=\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

3. Let

$$\int \frac{2-\tan x}{3+\tan x}dx = \frac{1}{2}\left(\alpha x + \log_e\left|\beta\sin x + \gamma\cos x\right|\right) + C,$$

where C is the constant of integration. Then $\alpha + \frac{\gamma}{\beta}$ is equal to

(1) 1 (2) 4

(3) 3 (4) 7

Answer (2)

Sol. $I = \int \frac{2-\tan x}{3+\tan x}dx = \int \frac{2\cos x - \sin x}{3\cos x + \sin x}dx$

Put, $2\cos x - \sin x = a(-3\sin x + \cos x) + b(3\cos x + \sin x)$

$a + 3b = 2$ ...(i)

$-3a + b = -1$ ...(ii)

From equation (i) and (ii) $a = b = \frac{1}{2}$

$$I = \frac{1}{2}\int \frac{-3\sin x + \cos x}{3\cos x + \sin x}dx + \frac{1}{2}\int \frac{3\cos x + \sin x}{3\cos x + \sin x}dx$$

$$I = \frac{1}{2}\ln\left|3\cos x + \sin x\right| + \frac{1}{2}x = \frac{1}{2}\left(dx + \log\left|\beta\sin x + \gamma\cos x\right|\right)$$

On comparing, we get

$$\Rightarrow \frac{1}{2}\left(x + \log\left(\left|3\cos x + \sin x\right|\right)\right) = \frac{1}{2}\left(dx + \log\left|\beta\sin x + \gamma\cos x\right|\right)$$

$\alpha = 1, \beta = 1, \gamma = 3$

$$\alpha + \frac{\gamma}{\beta} = 1 + \frac{3}{1} = 4$$

4. The shortest distance between the lines $\frac{x-3}{4} = \frac{y+7}{-11} = \frac{z-1}{5}$ and $\frac{x-5}{3} = \frac{y-9}{-6} = \frac{z+2}{1}$ is:

(1) $\frac{179}{\sqrt{563}}$ (2) $\frac{178}{\sqrt{563}}$

(3) $\frac{187}{\sqrt{563}}$ (4) $\frac{185}{\sqrt{563}}$

Answer (3)

Sol. Given lines are

$$\frac{x-3}{4} = \frac{y-(-7)}{-11} = \frac{z-1}{5} \text{ and}$$

$$\frac{x-5}{3} = \frac{y-9}{-6} = \frac{z-(-2)}{1}$$

Shortest distance between two lines,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right|}{\left|(\vec{b}_1 \times \vec{b}_2)\right|}$$

$\vec{a}_2 - \vec{a}_1 = 2\hat{i} + 16\hat{j} - 3\hat{k}$ and

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -11 & 5 \\ 3 & -6 & 1 \end{vmatrix} = 19\hat{i} + 11\hat{j} + 9\hat{k}$$

$$\therefore \quad d = \frac{187}{\sqrt{563}}$$

5. If the sum of the series $\frac{1}{1\cdot(1+d)} + \frac{1}{(1+d)(1+2d)} + \ldots + \frac{1}{(1+9d)(1+10d)}$ is equal to 5, then $50d$ is equal to:

(1) 15 (2) 10

(3) 5 (4) 20

Answer (3)

Sol. $\frac{1}{1\cdot(1+d)} + \frac{1}{(1+d)(1+2d)} + \ldots + \frac{1}{(1+9d)(1+10d)} = 5$

Multiply and divide by d

$$\frac{1}{d}\left[\frac{d}{1\times(1+d)} + \frac{d}{(1+d)(1+2d)} + \ldots + \frac{1}{(1+9d)(1+10d)}\right] = 5$$

$$\frac{1}{d}\left[\left(1 - \frac{1}{1+d}\right) + \left(\frac{1}{1+d} - \frac{1}{1+2d}\right) + \ldots + \left(\frac{1}{1+9d} - \frac{1}{1+10d}\right)\right] = 5$$

$$\frac{1}{d}\left[1 - \frac{1}{1+10d}\right] = 5$$

$$\frac{1}{d}\left[\frac{1+10d-1}{1+10d}\right] = 5$$

$$\frac{10}{1+10d}=5$$

$1 + 10d = 2$

$$d=\frac{1}{10}$$

$$50d=50\times\frac{1}{10}=5$$

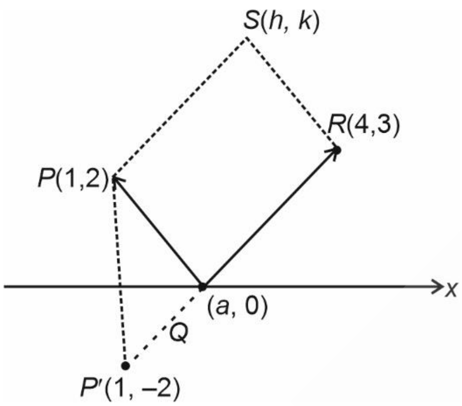
6. A ray of light coming from the point $P(1, 2)$ gets reflected from the point Q on the x-axis and then passes through the point $R(4, 3)$. If the point $S(h, k)$ is such that $PQRS$ is a parallelogram, then hk^2 is equal to:

(1) 90 (2) 80

(3) 70 (4) 60

Answer (3)

Sol.



$$P'R: y+2=\frac{5}{3}(x-1)$$

For Point $Q \Rightarrow y = 0$

$$\frac{6}{5}=a-1 \Rightarrow a=\frac{11}{5}$$

Now, $PQRS$ is parallelogram

$$\therefore \frac{h+a}{2}=\frac{4+1}{2} \Rightarrow h=5-\frac{11}{5}=\frac{14}{5}$$

and $\frac{2+3}{2}=\frac{k}{2} \Rightarrow K=5$

Now $hk^2=25\times\frac{14}{5}=14\times5=70$

7. Let the line L intersect the lines $x - 2 = -y = z - 1$, $2(x + 1) = 2(y - 1) = z + 1$ and be parallel to the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2}$. Then which of the following points lies on L?

(1) $\left(-\frac{1}{3},-1,-1\right)$ (2) $\left(-\frac{1}{3},-1,1\right)$

(3) $\left(-\frac{1}{3},1,1\right)$ (4) $\left(-\frac{1}{3},1,-1\right)$

Answer (4)

Sol. $L_1:\frac{x-2}{1}=\frac{y}{-1}=\frac{z-1}{1}=\lambda$

$$L_2:\frac{x+1}{(1/2)}=\frac{y-1}{(1/2)}=\frac{z+1}{1}=\mu$$

Any point of L_1 and L_2 will be $(\lambda + 2, -\lambda, \lambda + 1)$ and $\left(\frac{\mu}{2}-1, \frac{\mu}{2}+1, \mu-1\right)$

Now Dr of line $<\lambda-\frac{\mu}{2}+3, -\lambda-\frac{\mu}{2}-1, \lambda-\mu+2>$

Now $\frac{\lambda-\frac{\mu}{3}+3}{3}=\frac{-\lambda-\frac{\mu}{2}-1}{1}=\frac{\lambda-\mu+2}{2}$

$$\left.\begin{aligned}&\lambda-\frac{\mu}{3}+3=-3\lambda-\frac{3\mu}{2}-3 \quad ..(1)\\ &2\left(\lambda-\frac{\mu}{3}+3\right)=3(\lambda-\mu+2) \quad ...(2)\end{aligned}\right\}\lambda=\frac{-4}{3},\ \mu=\frac{-2}{3}$$

$\therefore$ Points will be $\left(\frac{2}{3},\frac{4}{3},\frac{-1}{3}\right)$ and $\left(\frac{-4}{3},\frac{2}{3},\frac{-5}{3}\right)$

$\therefore$ L will be $\frac{x-\frac{2}{3}}{3}=\frac{y-\frac{4}{3}}{1}=\frac{z+\frac{1}{3}}{2}$

$\therefore$ $\left(\frac{-1}{3},1,-1\right)$ will satisfy L

8. The coefficient of x^{70} in $x^2(1+x)^{98} + x^3(1+x)^{97}$ $+x^4(1+x)^{96} + \ldots + x^{54}(1+x)^{46}$ is ${}^{99}C_p - {}^{46}C_q$. Then a possible value of $p + q$ is:

(1) 55 (2) 61

(3) 68 (4) 83

Answer (4)

Sol. $x^2(1+x)^{98} + x^3(1+x)^{97} + \ldots + x^{54}(1+x)^{46}$

It is a G.P. with first term = $x^2(1 + x)^{98}$

and common ratio $= \dfrac{x}{1+x}$

sum of these term = $= x^2(1+x)^{98}\left(\dfrac{\left(\dfrac{x}{1+x}\right)^{53} - 1}{\dfrac{x}{1+x} - 1}\right)$

$= x^2(1+x)^{98}((1+x) - x^{53}(1+x)^{-52})$

$= x^2\underbrace{(1 + x)^{99}}_{\text{coeff of } x^{68}} - x^{55}\underbrace{(1 + x)^{46}}_{\text{coeff of } x^{15}}$

$= {}^{99}C_{68} - {}^{46}C_{15}$

$\Rightarrow$ $p = 68$, $q = 15$

$\Rightarrow$ $p + q = 83$

9. Let $\lambda, \mu \in \mathbf{R}$. If the system of equations

$3x + 5y + \lambda z = 3$

$7x + 11y - 9z = 2$

$97x + 155y - 189z = \mu$

has infinitely many solutions, then $\mu + 2\lambda$ is equal to:

(1) 24 (2) 27

(3) 25 (4) 22

Answer (3)

Sol. $3x + 5y + \lambda z = 3$

$7x + 11y - 9z = 2$

$97x + 155y - 189z = \mu$

$$\begin{bmatrix} 3 & 5 & \lambda \\ 7 & 11 & -9 \\ 97 & 155 & -189 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ \mu \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$X = \dfrac{\text{adj } A}{|A|}B$

For Infinitely many solution

$|A| = 0$ and $(\text{adj } A)B = 0$

$$\begin{vmatrix} 3 & 5 & \lambda \\ 7 & 11 & -9 \\ 97 & 155 & -189 \end{vmatrix} = 0$$

$-2052 + 2250 + 18\lambda = 0$

$\Rightarrow \lambda = -11$

$$\text{adj } A = \begin{bmatrix} -684 & -760 & 76 \\ 450 & 500 & -50 \\ 18 & 20 & -2 \end{bmatrix}$$

$$(\text{adj } A)B = \begin{bmatrix} 684 & -760 & 76 \\ 450 & 500 & -50 \\ 18 & 20 & -2 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ \mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 54 + 40 - 2\mu = 0$

$\Rightarrow 2\mu = 94$

$\Rightarrow \mu = 47$

$\Rightarrow \mu + 2\lambda = 25$

10. The frequency distribution of the age of students in a class of 40 students is given below.

Age	15	16	17	18	19	20
No. of students	5	8	5	12	x	y

If the mean deviation about the median is 1.25, then $4x + 5y$ is equal to:

(1) 47 (2) 46

(3) 44 (4) 43

Answer (3)

Sol.

Age	No. of Students	CF
15	5	5
16	8	13
17	5	18
18	12	30
19	x	$30 + x$
20	y	$30 + x + y$

$30 + x + y = 40$

$x + y = 10$

Median $= \left(\frac{n+1}{2}\right)^{\text{th}}$ observation

$= \frac{40+1}{2} = \frac{41}{2}$

Median = 18

Mean deviation about median

$5.3 + 8.2 + 5.1 + 12.0 + x{\cdot}1 + y{\cdot}2 = 1.25 \times 40$

$15 + 16 + 5 + x + 2y = 50$

$x + 2y = 14$

$x + y = 10$

$\Rightarrow x = 4$

$\Rightarrow y = 6$

$4x + 5y = 24 + 20$

$= 44$

11. A variable line L passes through the point (3, 5) and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is:

(1) 30

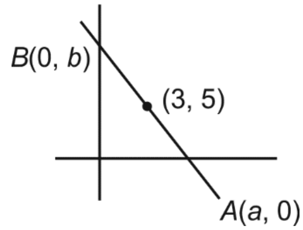
(2) 40

(3) 35

(4) 25

Answer (1)

Sol.



$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{3}{a} + \frac{5}{b} = 1$

$3b + 5a = ab$

$5a - ab = -3b$

$a(5 - b) = -3b$

$a = \frac{3b}{b-5}$

Area of triangle $= \left|\frac{1}{2} \times a \times b\right|$

$= \frac{1}{2} \times \frac{3b}{b-5} \times b$

$\Rightarrow f(b) = \frac{3b^2}{2b-10}$

$\Rightarrow f'(b) = 0, (2b - 10)6b - 2(3b^2) = 0$

$12b^2 - 60b - 6b^2 = 0$

$6b^2 - 60b = 0$

$b^2 - 10b = 0$

$b(b - 10) = 0$

$b = 0$ or $b = 10$

So for minimum area, $b = 10$

then $\frac{1}{2} \times a \times b = 30$

12. The solution curve, of the differential equation $2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx}$, passing through the point (0, 1) is a conic, whose vertex lies on the line:

(1) $2x + 3y = 9$

(2) $2x + 3y = -6$

(3) $2x + 3y = 6$

(4) $2x + 3y = -9$

Answer (1)

Sol. $2y\dfrac{dy}{dx}+3=5\dfrac{dy}{dx}$

$2y\,dy + 3dx = 5dy$

$y^2+3x=5y+c\,|_{(0,1)}$

$1 + 0 = 5 + c$

$c = -4$

$y^2 - 5y = -3x - 4$

$y^2-5y+\dfrac{25}{4}-\dfrac{25}{4}=-3x-4$

$\left(y-\dfrac{5}{2}\right)^2=-3x+\dfrac{9}{4}$

$\left(y-\dfrac{5}{2}\right)^2=-3\left(x-\dfrac{3}{4}\right)$

$\left(\dfrac{3}{4},\dfrac{5}{2}\right)$ satisfies by $2x + 3y = 9$

13. Let $f(x) = x^2$, $g(x)=\dfrac{x}{x-9}$ and $a = fog(10)$, $b = gof(3)$. If e and l denote the eccentricity and the length of the latus rectum of the ellipse $\dfrac{x^2}{a}+\dfrac{y^2}{b}=1$, then $8e^2 + l^2$ is equal to.

(1) 8 (2) 6

(3) 16 (4) 12

Answer (1)

Sol. $g(10) = 10$

$a = f(g(10)) = f(10) = 109$

$f(3) = 18$

$b = g(f(3)) = g(18) = 2$

$\dfrac{x^2}{109}+\dfrac{y^2}{2}=1$

$e=\sqrt{1-\dfrac{2}{109}}=\sqrt{\dfrac{107}{109}}$

$l=\dfrac{2b^2}{a}=\dfrac{2\times 2}{\sqrt{109}}$

$8e^2+l^2=\dfrac{8\times 107}{109}+\dfrac{16}{109}$

$= 8$

14. Let a circle passing through (2, 0) have its centre at the point (h, k). Let (x_c, y_c) be the point of intersection of the lines $3x + 5y = 1$ and $(2 + c)x + 5c^2y = 1$. If $h=\lim\limits_{c\to 1}x_c$ and $k=\lim\limits_{c\to 1}y_c$, then the equation of the circle is:

(1) $25x^2 + 25y^2 - 20x + 2y - 60 = 0$

(2) $5x^2 + 5y^2 - 4x + 2y - 12 = 0$

(3) $5x^2 + 5y^2 - 4x - 2y - 12 = 0$

(4) $25x^2 + 25y^2 - 2x + 2y - 60 = 0$

Answer (1)

Sol. $3x + 5y = 1$

$(2 + c)x + 5c^2y = 1$

$3c^2x + 5c^2y = c^2$

Subtracting

$(2 + c - 3c^2)x = 1 - c^2$

$x_c=\dfrac{1-c^2}{2+c-3c^2}=\dfrac{(1-c)(1+c)}{(1-c)(3c+2)}=\dfrac{c+1}{3c+2}$

$y=\dfrac{1-3x}{5}=\dfrac{1-3\left(\dfrac{c+1}{3c+2}\right)}{5}$

$=\dfrac{3c+2-3c-3}{5(3c+2)}$

$y_c=\dfrac{-1}{5(3c+2)}$

$\lim\limits_{c\to 1}x_c=\dfrac{2}{5}=h$

$\lim\limits_{c\to 1}y_c=\dfrac{-1}{25}=k$

Equation of circle is

$25x^2 + 25y^2 - 20x + 2y - 60 = 0$

15. If the domain of the function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is **R** – (α, β)**,** then 12αβ is equal to:

(1) 32

(2) 40

(3) 36

(4) 24

Answer (1)

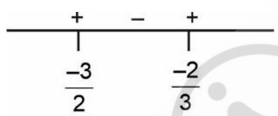
Sol. $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$

$-1 \le \frac{x-1}{2x+3} \le 1$

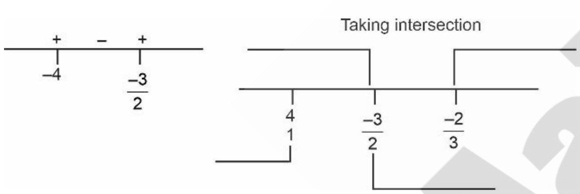
$\frac{x-1}{2x+3} - 1 \le 0$ $\qquad$ $\frac{x-1}{2x+3} + 1 \ge 0$

$\frac{x-1-2x-3}{2x+3} \le 0$ $\qquad$ $\frac{x-1+2x+3}{2x+3} \ge 0$

$\frac{-x-4}{2x+3} \le 0$ $\qquad$ $\frac{3x+2}{2x+3} \ge 0$



$\frac{x+4}{2x+3} \ge 0$



$x \in (-\infty, -4] \cup \left[\frac{-2}{3}, \infty\right)$

Domain : $R - \left(-4, \frac{-2}{3}\right)$

α = – 4

$\beta = \frac{-2}{3}$

$12 \times \alpha\beta = 12 \times 4 \times \frac{2}{3} = 32$

16. Let three vectors $\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ form a triangle such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is $5\sqrt{6}$. If α is a positive real number, then $|\vec{c}|^2$ is equal to:

(1) 10 $\qquad$ (2) 12

(3) 16 $\qquad$ (4) 14

Answer (4)

Sol. $\vec{a} - \vec{b} = \vec{c}$

α – 5 = *x*, 1 = *y*, –2 = *z*

$\Delta = 5\sqrt{6}$

$\frac{1}{2}|\vec{a} \times \vec{b}| = 5\sqrt{6} \Rightarrow |\vec{a} \times \vec{b}| = 10\sqrt{6}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 4 & 2 \\ 5 & 3 & 4 \end{vmatrix}$$

$= \hat{i}(10) - \hat{j}(4\alpha - 10) + \hat{k}(3\alpha - 20)$

$100 + (4\alpha - 10)^2 + (3\alpha - 20)^2 = 600$

$\Rightarrow 16\alpha^2 + 100 - 80\alpha + 9\alpha^2 + 400 - 120\alpha = 600 - 100$

$25\alpha^2 - 200\alpha = 0$

α = 8

$\therefore \vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$

$|\vec{c}|^2 = 9 + 1 + 4 = 14$

17. Let $\overrightarrow{OA} = 2\vec{a}, \overrightarrow{OB} = 6\vec{a} + 5\vec{b}$ and $\overrightarrow{OC} = 3\vec{b}$, where *O* is the origin. If the area of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is 15 sq. units, then the area (in sq. units) of the quadrilateral *OABC* is equal to:

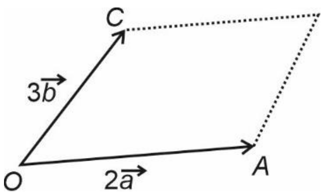
(1) 32 $\qquad$ (2) 35

(3) 38 $\qquad$ (4) 40

Answer (2)

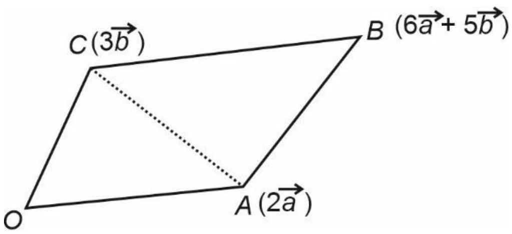
Sol. $6|\vec{a} \times \vec{b}| = 15$

$\Rightarrow |\vec{a} \times \vec{b}| = \frac{5}{2}$



Area of quadrilateral $OABC$

= area of ΔOAC + area of ΔABC



$= \frac{15}{2} + \frac{1}{2}|(\overrightarrow{AB} \times \overrightarrow{BC})|$

$= \frac{15}{2} + \frac{1}{2}|(4\vec{a} + 5\vec{b}) \times (6\vec{a} + 2\vec{b})|$

$= \frac{15}{2} + \frac{1}{2}|(22\vec{a} \times \vec{b})|$

$= \frac{15}{2} + 11 \times \frac{5}{2}$

$= \frac{70}{2} = 35$

18. Let $f(x) = ax^3 + bx^2 + cx + 41$ be such that $f(1) = 40$, $f'(1) = 2$ and $f''(1) = 4$. Then $a^2 + b^2 + c^2$ is equal to:

(1) 51 (2) 62

(3) 73 (4) 54

Answer (1)

Sol. $f(1) = a + b + c + 41 = 40 \Rightarrow a + b + c = -1$

$f'(1) = 3a + 2b + c = 2$

$f''(1) = 6a + 2b = 4$

$a = -1, b = 5, c = -5$

$\therefore a^2 + b^2 + c^2 = 1 + 25 + 25 = 51$

19. The solution of the differential equation $(x^2 + y^2)dx - 5xy\,dy = 0$, $y(1) = 0$, is:

(1) $|x^2 - 4y^2|^5 = x^2$

(2) $|x^2 - 4y^2|^6 = x$

(3) $|x^2 - 2y^2|^5 = x^2$

(4) $|x^2 - 2y^2|^6 = x$

Answer (1)

Sol. $(x^2 + y^2)dx - 5xydy = 0$

$\frac{dy}{dx} = \frac{x^2 + y^2}{5xy}$

Let $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$V + x\frac{dv}{dx} = \frac{1 + V^2}{5v}$

$x\frac{dv}{dx} = \frac{1 + v^2 - 5v^2}{5v}$

$\frac{1}{8}\int \frac{8 \times 5v\,dv}{1 - 4v^2} = \int \frac{dx}{x}$

$\frac{-5}{8}\ln|1 - 4v^2| = \ln|x| + \ln c$

$\Rightarrow \frac{5}{8}\ln\frac{|x^2 - 4y^2|}{x^2} + \ln|x| = \ln c$

$\left|\frac{x^2 - 4y^2}{x^2}\right|^{5/8} |x| = c$

$\frac{|x^2 - 4y^2|^{5/8}}{|x|^{\frac{1}{4}}} = c \because y(1) = 0 \Rightarrow c = 1$

$\Rightarrow |x^2 - 4y^2|^{5/8} = |x|^{\frac{1}{4}}$

$|x^2 - 4y^2|^5 = x^2$

20. Let $\left|\cos\theta\cos(60-\theta)\cos(60+\theta)\right| \le \frac{1}{8}, \theta \in [0, 2\pi]$.

Then, the sum of all $\theta \in [0, 2\pi]$, where $\cos 3\theta$ attains its maximum value, is:

(1) 9π

(2) 6π

(3) 15π

(4) 18π

Answer (2)

Sol. $\left|\cos\theta\cos(60-\theta)\cos(60+\theta)\right| \le \frac{1}{8}$

$\Rightarrow \frac{1}{4}|\cos 3\theta| \le \frac{1}{8}$

$\cos 3\theta$ is max if $\cos 3\theta = \frac{1}{2}$

$\therefore\ \theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$

$\sum \theta_i = 6\pi$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If a function f satisfies $f(m+n) = f(m) + f(n)$ for all $m, n \in \mathbf{N}$ and $f(1) = 1$, then the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda + k) \le (2022)^2$ is equal to _______.

Answer (1010)

Sol. $f(m+n) = f(m) + f(n)$

$f(x) = kx$

$\because f(1) = 1$

$\Rightarrow k = 1$

$\Rightarrow f(x) = x$

$$\sum_{k=1}^{2022} f(\lambda+k) = \sum_{k=1}^{2022} (\lambda+k) = \underbrace{\lambda+\lambda+\ldots+\lambda}_{2022} + (1+2+\ldots+2022)$$

$$= 2022\lambda + \frac{2022 \times 2023}{2} \le (2022)^2$$

$$\Rightarrow \lambda \le \frac{2021}{2}$$

largest $\lambda = 1010$

22. Let $\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}} - \frac{2n}{(n^2+1)\sqrt{n^4+1}} + \frac{n}{\sqrt{n^4+16}} - \frac{8n}{(n^2+4)\sqrt{n^4+16}} + \ldots + \frac{n}{\sqrt{n^4+n^4}} - \frac{2n\cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}}\right)$ be $\frac{\pi}{k}$, using only the principal values of the inverse trigonometric functions. Then k^2 is equal to _____.

Answer (32)

Sol. $\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}} + \frac{n}{\sqrt{n^4+16}} + \ldots \frac{n}{\sqrt{n^4+n^4}}\right)$

$$-\lim_{n\to\infty}\left(\frac{2n}{(n^2+1)\left(\sqrt{n^4+1}\right)}\right) + \frac{8n}{(n^2+4)\sqrt{n^4+1}} + \ldots \frac{2n\cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}}$$

$$= \lim_{n\to\infty}\sum_{n=1}^{n}\frac{1}{n\sqrt{1+\frac{r^4}{n^4}}} - \lim_{n\to\infty}\sum_{n=1}^{n}\frac{1}{n}\frac{2\cdot(r/n)^2}{\left(1+\left(\frac{r}{n}\right)^2\right)\sqrt{1+\frac{r^4}{n^4}}}$$

$$= \int_0^1 \frac{dx}{\sqrt{1+x^4}} - 2\int_0^1 \frac{x^2}{(1+x^2)\sqrt{1+x^4}}dx$$

$$= \int_0^1 \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}dx$$

$$\int_0^1 \frac{\left(\frac{1}{x^2}-1\right)dx}{\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}}$$

$$=\int_0^1 \frac{\left(\frac{1}{x^2}-1\right)dx}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}$$

$$x+\frac{1}{x}=t \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$$

$$-\int_\infty^2 \frac{dt}{t\sqrt{t^2-2}} = \int_2^\infty \frac{dt}{t\sqrt{t^2-2}} = \frac{-1}{\sqrt{2}}\sin^{-1}\frac{\sqrt{2}}{x}\Bigg|_2^\infty$$

$$=\frac{-1}{\sqrt{2}}\left(0-\frac{\pi}{4}\right)$$

$$=\frac{\pi}{2^{5/2}}$$

$\therefore k = 2^{5/2}$

$\therefore k^2 = 32$

23. Let the centre of a circle, passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$, be (h, k). Then for all possible values of the coordinates of the centre (h, k), $4(h^2 + k^2)$ is equal to ______.

Answer (9)

Sol. Circle will touch internally

$C_1C_2 = |r_1 - r_2|$

$$=\sqrt{h^2+k^2} = 3-\sqrt{h^2+k^2}$$

$$\Rightarrow 2\sqrt{h^2+k^2} = 3$$

$$\Rightarrow h^2+k^2 = \frac{9}{4}$$

$\therefore 4(h^2 + k^2) = 9$

24. Let $f:(0,\pi)\to \mathbf{R}$ be a function given by

$$f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0<x<\frac{\pi}{2}\\ a-8, & x=\frac{\pi}{2}\\ (1+|\cot x|)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2}<x<\pi\end{cases}$$

where $a, b \in \mathbf{Z}$. If f is continuous at $x=\frac{\pi}{2}$, then $a^2 + b^2$ is equal to _____.

Answer (81)

Sol. $\lim\limits_{x\to\frac{\pi}{2}^-} f(x) = f\left(\frac{\pi}{2}\right) = \lim\limits_{x\to\frac{\pi}{2}^+} f(x)$ for continuity at $x=\frac{\pi}{2}$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}^-}\left(\frac{8}{7}\right)^{\left(\frac{\tan 8x}{\tan 7x}\right)} \qquad \text{Let } x=\frac{\pi}{2}-h$$

$$\Rightarrow \lim_{h\to 0}\left(\frac{8}{7}\right)^{\frac{\tan(4\pi-8h)}{\tan\left(3\pi+\frac{\pi}{2}-7h\right)}} = \lim_{h\to 0}\left(\frac{8}{7}\right)^{\frac{\tan(-8h)}{\cot(7h)}} = \left(\frac{8}{7}\right)^0 = 1$$

$\Rightarrow a - 8 = 1 \Rightarrow a = 9$

$$\lim_{x\to\frac{\pi}{2}^+}(1+|\cot x|)^{\frac{b}{a}|\tan x|},\ x=\frac{\pi}{2}+h$$

$$\lim_{h\to 0}(1-\tan h)^{-\frac{b}{9}\cot h} = \lim_{h\to 0}(1-\tan h)^{-\frac{b}{9}\cot h}$$

$$=\lim_{h\to 0}(1-\tan h)^{\left(\frac{-1}{\tan h}\right)\cdot(-\tan h)\cdot\left(\frac{-b}{9}\cot h\right)}$$

$$=e^{\frac{b}{9}}=1 \qquad \Rightarrow b = 0$$

$\Rightarrow a^2 + b^2 = 81 + 0 = 81$

25. The remainder when 428^{2024} is divided by 21 is ______.

Answer (1)

Sol. $428 = 21 \times 20 + 8$

$\Rightarrow (428)^{2024} \equiv (20 \times 21 + 8)^{2024} \equiv 8^{2024} (\text{mod } 21)$

$8^2 = 21 \times 3 + 1$

$8^{2024} = (21 \times 3 + 1)^{1012}$

$\Rightarrow 8^{2024} \equiv (21 \times 3 + 1)^{1012} (\text{mod } 21)$

$\equiv 1^{2012} (\text{mod } 21)$

$428^{2024} \equiv 1 (\text{mod } 21)$

26. Let A be a non-singular matrix of order 3. If $\det(3\,\text{adj}(2\,\text{adj}((\det A)A))) = 3^{-13} \cdot 2^{-10}$ and $\det(3\,\text{adj}(2A)) = 2^m \cdot 3^n$, then $|3m + 2n|$ is equal to ______.

Answer (14)

Sol. $|3\text{adj}\,(2\text{adj}(|A|A))| = 3^{-13} \cdot 2^{-10}$

Let $|A|A = B \Rightarrow |B| = ||A|A| = |A|^3|A| = |A|^4$

$\Rightarrow \text{adj}(|A|A) = (\text{adj } B)$

$\Rightarrow 2\text{adj}(|A|A) = (2\text{adj } B) = C$ (say)

$|3\text{adj}(C)| = 3^3 \cdot |C|^2$

$|C| = |(2\text{adj } B)| = 2^3 |B|^2 = 2^3 \cdot |A^4|^2 = 2^3 \cdot |A|^8$

$\Rightarrow |3\text{adj}C| = 3^3 \cdot (2^3 |A|^8)^2 = 3^{-13} \cdot 2^{-10}$

$= 2^6 |A|^{16} = 3^{-16} \cdot 2^{-10}$

$\Rightarrow |A|^{16} = (3 \cdot 2)^{-16} = \left(\frac{1}{6}\right)^{16}$

$\Rightarrow |A| = \pm\frac{1}{6}$

$|3\text{adj } 2A| = 3^3 |2A|^2 = 3^3 \cdot (2^3|A|)^2 = 3^3 \cdot 2^6 |A|^2$

$= 3^3 \cdot 2^6 \cdot \frac{1}{36} = \frac{27 \times 64}{36} = 48$

$\Rightarrow 2^m \cdot 3^n = 2^4 \cdot 3^1 \Rightarrow m = 4$

$n = 1$

$\Rightarrow |3 \times 4 + 2 \times 1| = 14$

27. Let $A = \{2,3,6,7\}$ and $B = \{4,5,6,8\}$. Let R be a relation defined on $A \times B$ by $(a_1, b_1)\ R\ (a_2, b_2)$ if and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is ______.

Answer (25)

Sol. $A \times B = (a_1, b_1)\ R\ (a_2, b_2)$

Sum (a_1, a_2) (b_1, b_2)

$8\ \{(2, 6)$ $(4, 4) \Rightarrow 2 \times 1 = 2$

$$9\begin{cases}(2,7) & (4,5) & \Rightarrow & 2\times 2 \\ (3,6) & (4,5) & \Rightarrow & 2\times 2\end{cases} = 8$$

$$10\begin{cases}(3,7) & (6,4) & \Rightarrow & 2\times 2 \\ (3,7) & (5,5) & \Rightarrow & 2\times 1\end{cases} = 6$$

$$12\begin{cases}(6,6) & (6,6) & \Rightarrow & 1\times 1 \\ (6,6) & (8,4) & \Rightarrow & 1\times 2\end{cases} = 3$$

$$13\{(6,7) \quad (8,5) \quad \Rightarrow \quad 2\times 2 = 4$$

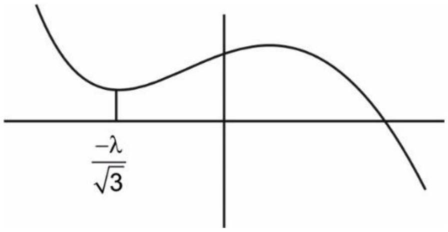
$$14\{(7,7) \quad (8,6) \quad \Rightarrow \quad 1\times 2 = 2$$

Total ways to form relation = 25

28. Let the set of all positive values of λ, for which the point of local minimum of the function $(1 + x(\lambda^2 - x^2))$ satisfies $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$, be (α, β). Then $\alpha^2 + \beta^2$ is equal to ______.

Answer (39)

Sol.



$f(x) = 1 + x(\lambda^2 - x^2)$

$f(x) = -x^3 + (\lambda^2 x + 1)$

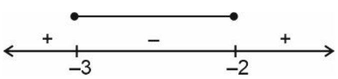
$\boxed{f'(x) = -3x^2 + \lambda^2}$

$\boxed{x = \pm\frac{\lambda}{\sqrt{3}}}$

$-\frac{\lambda}{\sqrt{3}}$ should satisfy the given condition

$\frac{x^2 + x + 2}{x^2 + 5x + 6} < 6$

$\frac{1}{(x+2)(x+3)} < 0$



$x \in (-3, -2)$

$-3 < -\frac{\lambda}{\sqrt{3}} < -2$

$-3\sqrt{3} < -\lambda < -2\sqrt{3}$

$\boxed{2\sqrt{3} < \lambda < 3\sqrt{3}}$

$\left.\begin{array}{l}\alpha = 2\sqrt{3}\\ \beta = 2\sqrt{3}\end{array}\right\}$

$(2\sqrt{3})^2 + (3\sqrt{2})^2$

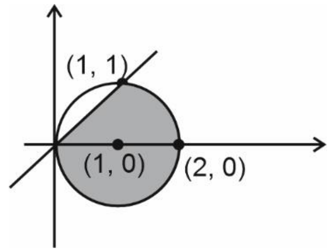
$12 + 27$

$\boxed{39}$

29. The sum of the square of the modulus of the elements in the set $\{z = a + ib : a, b \in \mathbf{Z}, z \in \mathbf{C}, |z-1| \le 1, |z-5| \le |z-5i|\}$ is ______.

Answer (9)

Sol.



z should be lying in the shaded region shown in adjacent figure.

Possible z are

$z = 0 + 0i$, $(1 + 0i)$, $(2 + 0i)$, $(1 - i)$, $(1 + i)$

Sum of squares of modulus

$= 0 + 1 + 4 + 2 + 2$

$= 9$

30. Let a, b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose four faces are marked 1, 2, 3, 4. If the probability that $ax^2 + bx + c = 0$ has all real roots is $\frac{m}{n}$, $gcd(m, n) = 1$, then $m + n$ is equal to ______.

Answer (19)

Sol. $ax^2 + bx + c = 0$

for real roots $b^2 - 4ac \ge 0$ …(i)

$a, b, c \in \{1, 2, 3, 4\}$ …(ii)

ordered triplet (a, b, c) satisfying (i) and (ii) are

(1, 2, 1), (1, 3, 1), (2, 3, 1), (1, 3, 2), (1, 4, 1), (1, 4, 2), (2, 4, 1), (2, 4, 2), (4, 4, 1), (1, 4, 4), (3, 4, 1), (1, 4, 3),

i.e., total 12 favourable outcomes.

Total number of outcomes = 4 × 4 × 4

Required probability = $\frac{12}{64} = \frac{3}{16} = \frac{m}{n}$

$m + n = 1$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. A proton, an electron and an alpha particle have the same energies. Their de-Broglie wavelengths will be compared as

(1) $\lambda_e > \lambda_\alpha > \lambda_p$ (2) $\lambda_\alpha < \lambda_p < \lambda_e$

(3) $\lambda_p > \lambda_e > \lambda_\alpha$ (4) $\lambda_p < \lambda_e < \lambda_\alpha$

Answer (2)

Sol. $\lambda = \frac{h}{mv}$

and $mv = \sqrt{2\,km}$

$\therefore \quad \lambda = \frac{h}{\sqrt{2\,km}}$

$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{m}}$

$\Rightarrow \quad \lambda_e > \lambda_p > \lambda_\alpha$

32. Given below are two statements :

Statement-I : When an object is placed at the centre of curvature of a concave lens, image is formed at the centre of curvature of the lens on the other side.

Statement-II : Concave lens always forms a virtual and erect image.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement-I** and **Statement-II** are false

(2) **Statement-I** is false but **Statement-II** is true

(3) **Statement-I** is true but **Statement-II** is false

(4) Both **Statement-I** and **Statement-II** are true

Answer (2)

Sol. If an object is at the centre of curvature of convex lens, image is formed at the centre of curvature of the lens on the other side.

It is not true for concave lens.

Concave lens always forms a virtual and erect image for real objects.

33. The volume of an ideal gas (γ = 1.5) is changed adiabatically from 5 litres to 4 litres. The ratio of initial pressure to final pressure is

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{8}{5\sqrt{5}}$

(3) $\frac{4}{5}$ (4) $\frac{16}{25}$

Answer (2)

Sol. $\because \quad Pv^{\gamma} = \text{constant}$

$\Rightarrow \quad P_1(5)^{1.5} = P_2(4)^{1.5}$

$\Rightarrow \quad \frac{P_1}{P_2} = \frac{4}{5}\sqrt{\frac{4}{5}}$

$= \frac{8}{5\sqrt{5}}$

34. A bulb and a capacitor are connected in series across an ac supply. A dielectric is then placed between the plates of the capacitor. The glow of the bulb

(1) increases

(2) becomes zero

(3) remains same

(4) decreases

Answer (1)

Sol. $z = \sqrt{R^2 + x_C^2}$

and $x_C = \dfrac{1}{\omega C}$

and due to dielectric C increases

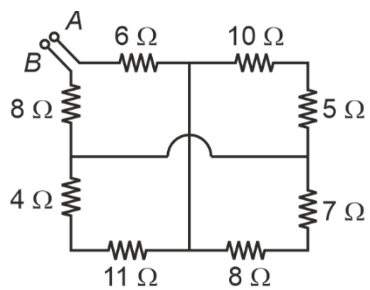
$\therefore$ x_C decreases

$\therefore$ Z decreases

$\Rightarrow$ $i = \dfrac{v}{z}$

$\therefore$ i increases

35. The equivalent resistance between A and B is



(1) 27 Ω

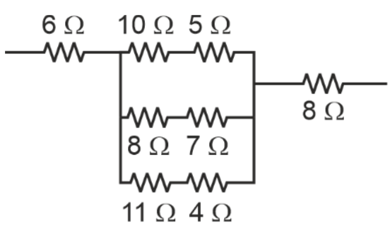
(2) 18 Ω

(3) 25 Ω

(4) 19 Ω

Answer (4)

Sol. Redrawing the circuit



$\Rightarrow$ $R_{eq} = 6 + 5 + 8 = 19\ \Omega$

36. A sample of 1 mole gas at temperature T is adiabatically expanded to double its volume. If adiabatic constant for the gas is $\gamma = \dfrac{3}{2}$, then the work done by the gas in the process is

(1) $RT\left[2-\sqrt{2}\right]$

(2) $\dfrac{R}{T}\left[2-\sqrt{2}\right]$

(3) $\dfrac{T}{R}\left[2+\sqrt{2}\right]$

(4) $RT\left[2+\sqrt{2}\right]$

Answer (1)

Sol. For adiabatic process

$W = -\Delta U$

$= -\dfrac{R}{(\gamma-1)}(T_2 - T)$

and $TV^{\gamma-1} = \text{constant}$

$\Rightarrow$ $T(V)^{\gamma-1} = T_2(2V)^{\gamma-1}$

$\Rightarrow$ $T_2 = \dfrac{T}{\sqrt{2}}$

$\therefore$ $W = -2R\left(\dfrac{T}{\sqrt{2}} - T\right)$

$= RT\left(2-\sqrt{2}\right)$

37. A plane EM wave is propagating along x direction. It has a wavelength of 4 mm. If electric field is in y direction with the maximum magnitude of 60 Vm^{-1}, the equation for magnetic field is :

(1) $B_x = 60\sin\left[\dfrac{\pi}{2}(x - 3\times10^8 t)\right]\hat{i}$ T

(2) $B_z = 2\times10^{-7}\sin\left[\dfrac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{k}$ T

(3) $B_z = 60\sin\left[\dfrac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{k}$ T

(4) $B_z = 2\times10^{-7}\sin\left[\dfrac{\pi}{2}\times10^3\left(x - 3\times10^8 t\right)\right]\hat{k}$ T

Answer (4)

Sol. Direction of propagation is x-axis

$\therefore \quad \hat{E} \times \hat{B} = \hat{i}$

$\therefore \quad \hat{B} = \hat{k}$

$$B_0 = \frac{E_0}{C}$$

$$= \frac{60}{3 \times 10^8} = 2 \times 10^{-7}$$

and, $\lambda = 4 \times 10^{-3}$

$$\Rightarrow \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{4 \times 10^{-3}} = \frac{\pi}{2} \times 10^3$$

$$\therefore \quad B_z = 2 \times 10^{-7} \sin\left[\frac{\pi}{2} \times 10^3 \left(x - 3 \times 10^8 t\right)\right] \hat{k} \text{ T}$$

38. A particle of mass m moves on a straight line with its velocity increasing with distance according to the equation $v = \alpha\sqrt{x}$, where α is a constant. The total work done by all the forces applied on the particle during its displacement from $x = 0$ to $x = d$, will be:

(1) $2m\alpha^2 d$

(2) $\dfrac{m\alpha^2 d}{2}$

(3) $\dfrac{md}{2\alpha^2}$

(4) $\dfrac{m}{2\alpha^2 d}$

Answer (2)

Sol. $\because \quad v = \alpha\sqrt{x}$

and, $W = \Delta K$

$$\Rightarrow \quad W = \frac{1}{2} m\left(v_f^2 - v_i^2\right)$$

$$= \frac{1}{2} m\, \alpha^2 d$$

39. On main scale division of a vernier caliper is equal to m units. If n^{th} division of main scale coincides with $(n + 1)^{\text{th}}$ division of vernier scale, the least count of the vernier caliper is :

(1) $\dfrac{m}{n(n+1)} p$ (2) $\dfrac{n}{(n+1)}$

(3) $\dfrac{1}{(n+1)}$ (4) $\dfrac{m}{(n+1)}$

Answer (4)

Sol. 1 MSD = m

n MSD = $(n + 1)$ VSD

$$\Rightarrow \quad 1 \text{ VSD} = \frac{n}{n+1} \text{ MSD}$$

$\therefore$ LC = 1 MSD – 1 VSD

$$= \left(1 - \frac{n}{n+1}\right) \text{MSD}$$

$$= \left(\frac{1}{n+1}\right) \text{MSD}$$

$$\Rightarrow \quad \text{LC} = \frac{m}{n+1}$$

40. A light unstretchable string passing over a smooth light pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is $\dfrac{g}{8}$, then the ratio of the masses $\dfrac{m_2}{m_1}$ is :

(1) 4 : 3 (2) 8 : 1

(3) 9 : 7 (4) 5 : 3

Answer (3)

Sol. $a = \dfrac{(m_2 - m_1)g}{(m_1 + m_2)} = \dfrac{g}{8}$

$$\Rightarrow \quad \frac{m_2}{m_1} = \frac{9}{7}$$

41. An astronaut takes a ball of mass m from earth of space. He throws the ball into a circular orbit about earth at an altitude of 318.5 km. From earth's surface to the orbit, the change in total mechanical energy of the ball is $x\dfrac{GM_e m}{21R_e}$. The value of x is (take R_e = 6370 km):

(1) 10 (2) 9

(3) 12 (4) 11

Answer (4)

Sol. At earth surface, $E_1 = -\dfrac{GM_m}{R_e}$

in the orbit, $E_2 = -\dfrac{GM_m}{2r}$

$$\Delta E = GM_m\left[\frac{1}{R_e} - \frac{1}{2r}\right]$$

$$= GM_m\left[\frac{1}{R_e} - \frac{1}{2.1R_e}\right]$$

$$= \frac{11}{21}\frac{GM_m}{R_e}$$

$\Rightarrow$ $x = 11$

42. Given below are two statements:

Statement I : When currents vary with time, Newton's third law is valid only if momentum carried by the electromagnetic field is taken into account.

Statement II : Ampere's circuital law does not depend on Biot-Savart's law.

In the light of the above statements, choose the correct answer from the options given below:

(1) **Statement I** is false but **Statement II** is true

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is true but **Statement II** is false

(4) Both **Statement I** and **Statement II** are false

Answer (3)

Sol. Ampere's circuital law is not independent of Blot-Savart's law.

43. A heavy iron bar, of weight W is having its one end on the ground and the other on the shoulder of a person. The bar makes an angle θ with the horizontal. The weight experienced by the person is

(1) $W \sin\theta$

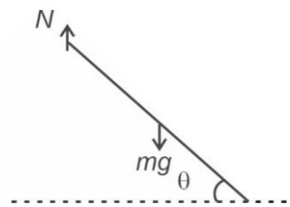
(2) $\dfrac{W}{2}$

(3) W

(4) $W\cos\theta$

Answer (2)

Sol.



$$mg \times \frac{L}{2}\cos\theta = N \times L \cos\theta$$

$$\Rightarrow N = \frac{mg}{2} = \frac{W}{2}$$

44. A light emitting diode (LED) is fabricated using GaAs semiconducting material whose band gap 1.42 eV. The wavelength of light emitted from the LED is

(1) 1243 nm

(2) 875 nm

(3) 650 nm

(4) 1400 nm

Answer (2)

Sol. E_g = 1.42 eV

$$\Rightarrow \lambda = \frac{12400}{1.42} \text{ Å}$$

= 875 nm

45. The energy equivalent of 1 g of substance is

(1) 11.2×10^{24} MeV (2) 5.6 eV

(3) 5.6×10^{26} MeV (4) 5.6×10^{12} MeV

Answer (3)

Sol. $E = mc^2$

$$= \frac{1\times10^{-3}\times9\times10^{16}}{1.6\times10^{-19}} \text{ eV}$$

$= 5.6 \times 10^{26}$ MeV

46. A sphere of relative density σ and diameter D has concentric cavity of diameter d. The ratio of $\frac{D}{d}$ if it just floats on water in a tank is

(1) $\left(\frac{\sigma-2}{\sigma+2}\right)^{1/3}$ (2) $\left(\frac{\sigma}{\sigma-1}\right)^{1/3}$

(3) $\left(\frac{\sigma+1}{\sigma-1}\right)^{1/3}$ (4) $\left(\frac{\sigma-1}{\sigma}\right)^{1/3}$

Answer (2)

Sol. $mg = F_B$

$$\Rightarrow \frac{4}{3}\pi\frac{(D^3-d^3)}{8}\sigma = \frac{4}{3}\pi\frac{D^3}{8}$$

$\Rightarrow \sigma D^3 - \sigma d^3 = D^3$

$\Rightarrow (\sigma - 1)\, D^3 = \sigma d^3$

$$\Rightarrow \frac{D}{d} = \left(\frac{\sigma}{\sigma-1}\right)^{1/3}$$

47. A galvanometer has a coil of resistance 200 Ω with a full scale deflection at 20 μA. The value of resistance to be added to use it as an ammeter of range (0-20) mA is

(1) 0.20 Ω (2) 0.10 Ω

(3) 0.40 Ω (4) 0.50 Ω

Answer (1)

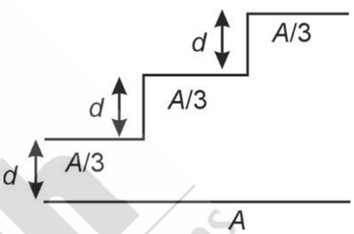
Sol. $20\mu A = \left(\frac{R_S}{R_G+R_S}\right)20 \text{ mA}$

$\Rightarrow 1000\, R_S = R_G + R_S$

$$\Rightarrow R_S = \frac{R_G}{999}$$

$$= \frac{200}{999} \approx 0.2\ \Omega$$

48. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure. If the area of each stair is arrangement is $\frac{A}{3}$ and the height is d, the capacitance of the arrangement is



(1) $\frac{13 \epsilon_0 A}{17d}$

(2) $\frac{18 \epsilon_0 A}{11d}$

(3) $\frac{11 \epsilon_0 A}{20d}$

(4) $\frac{11 \epsilon_0 A}{18d}$

Answer (4)

Sol. $C = C_1 + C_2 + C_3$

$$= \frac{A\epsilon_0}{3d} + \frac{A\epsilon_0}{6\,d} + \frac{A\epsilon_0}{gD}$$

$$= \frac{A\epsilon_0}{d}\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{9}\right)$$

$$= \frac{11}{18}\frac{A\epsilon_0}{d}$$

49. The dimensional formula of latent heat is

(1) $[M^0L^2T^{-2}]$ (2) $[ML^2T^{-2}]$

(3) $[MLT^{-2}]$ (4) $[M^0LT^{-2}]$

Answer (1)

Sol. $\because$ $Q = mL$

$$\Rightarrow L = \frac{Q}{m}$$

$\therefore$ Dimensions = $[L^2T^{-2}]$

50. A particle moving in a straight line covers half the distance with speed 6 m/s. The other half is covered in two equal time intervals with speeds 9 m/s and 15 m/s respectively. The average speed of the particle during the motion is

(1) 8.8 m/s (2) 9.2 m/s

(3) 10 m/s (4) 8 m/s

Answer (4)

Sol. $$V_{average} = \frac{\text{Distance}}{\text{Time interval}}$$

$$= \frac{d}{\frac{d}{2\times 6}+\frac{d}{(9+15)}}$$

$$= \frac{12\times 24}{36} = 8 \text{ m/s}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

51. A string is wrapped around the rim of a wheel of moment of inertia 0.40 kgm^2 and radius 10 cm. The wheel is free to rotate about its axis. Initially the wheel is rest. The string is now pulled by a force of 40 N. The angular velocity of the wheel after 10 s is x rad/s, where x is _______.

Answer (100)

Sol. $\tau = 40R$

$$\Rightarrow \alpha = \frac{\tau}{I}$$

$$\Rightarrow \alpha = \frac{40R}{I}$$

$$\therefore \omega = \alpha t = \frac{400R}{I}$$

$$= \frac{400\times 0.1}{0.4}$$

= 100

52. In Young's double slit experiment, the intensity at a point is $\left(\frac{1}{4}\right)^{th}$ of the maximum intensity, the minimum distance of the point from the central maximum is _______ μm.

(Given : λ = 600 nm, d = 1.0 mm, D = 1.0 m)

Answer (200)

Sol. $$\frac{I_0}{4} = I_0\cos^2\frac{d}{2}$$

and $$\phi = \frac{2\pi}{3}$$

$$\Rightarrow \Delta x = \frac{\lambda}{2\pi}\times\frac{2\pi}{3} = \frac{\lambda}{3}$$

$$\therefore \frac{\lambda}{3} = \frac{dy}{D}$$

$$\Rightarrow y = \frac{D\lambda}{3d} = \frac{1\times 600\times 10^{-9}}{3\times 1\times 10^{-3}}$$

$= 200 \times 10^{-6}$

53. At the centre of a half ring of radius R = 10 cm and linear charge density 4n C m^{-1}, the potential is $x\,\pi$ V. The value of x is _______.

Answer (36)

Sol. $V = \frac{KQ}{R}$

$= \frac{9 \times 10^9 \times 4 \times 10^{-9} \pi R}{R}$

$= 36\pi$

54. If $\vec{a}$ and $\vec{b}$ makes an angle $\cos^{-1}\left(\frac{5}{9}\right)$ with each other, then $|\vec{a}+\vec{b}| = \sqrt{2}\,|\vec{a}-\vec{b}|$ for $|\vec{a}| = n|\vec{b}|$. The integer value of n is ________.

Answer (3)

Sol. $|\vec{a}+\vec{b}| = \sqrt{2}\,|\vec{a}-\vec{b}|$

$\Rightarrow a^2 + b^2 + 2ab\cos\theta = 2(a^2 + b^2 - 2ab\cos\theta)$

$\Rightarrow a^2 + b^2 = 6ab\cos\theta$

$\Rightarrow a^2 + b^2 = \frac{10}{3}ab$

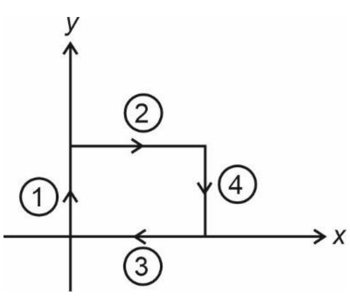
$\Rightarrow (n^2+1)b^2 = \frac{10}{3}nb^2$

$\Rightarrow n = 3$

55. A square loop of edge length 2 m carrying current of 2 A is placed with its edges parallel to the $x-y$ axis. A magnetic field is passing through the $x-y$ plane and expressed as $\vec{B} = B_0(1+4x)\hat{k}$, where B_0 = 5 T. The net magnetic force experienced by the loop is ________ N.

Answer (160)

Sol. Due to constant component of magnetic field $F = 0$



Due to variable component

$F_1 = 0$

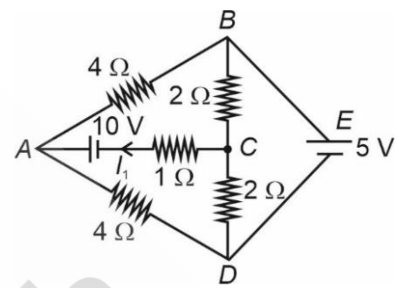
and, $F_2 + F_3 = 0$

and, $F_4 = (B_0 4_x) i L$

$= 5 \times 4 \times 2 \times 2 \times 2$

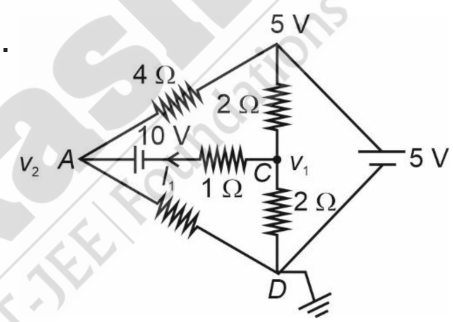
$= 160$ N

56. The current flowing through the 1 Ω resistor is $\frac{n}{10}$ A. The value of n is ________.



Answer (25)

Sol.



At C

$\frac{v_1}{2} + \frac{v_1 - 5}{2} + \frac{v_1 + 10 - v_2}{1} = 0$

$v_2 = 2v_1 + \frac{15}{2}$ ...(i)

At A

$\frac{v_2 - 5}{4} + \frac{v_2}{4} + \frac{v_2 - 10 - v_1}{1} = 0$

$6v_2 = 4v_1 + 45$ ...(ii)

$\Rightarrow\ v_1 = 0$

and $v_2 = \frac{15}{2}$

$\therefore\ i = \frac{5}{2}$ A

$\Rightarrow\ n = 25$

57. Two persons pull a wire towards themselves. Each person exerts a force of 200 N on the wire. Young's modulus of the material of wire is 1×10^{11} N m^{-2}. Original length of the wire is 2 m and the area of cross section is 2 cm^2. The wire will extend in length by ___________ μm.

Answer (20)

Sol. $\Delta L = \frac{FL}{AY}$

$= \frac{200 \times 2}{2 \times 10^{-4} \times 1 \times 10^{11}}$

$= 2 \times 10^{-5}$ m

$= 20\ \mu$m

58. A star has 100% helium composition. It starts to convert three ^{4}He into one ^{12}C via triple alpha process as $^4He + {}^4He + {}^4He \rightarrow {}^{12}C + Q$. The mass of the star is 2.0×10^{32} kg and it generates energy at the rate of 5.808×10^{30} W. The rate of converting these ^{4}He to ^{12}C is $n \times 10^{42}$ s^{-1}, where n is ________.

[Take, mass of ^{4}He = 4.0026 u, mass of ^{12}C = 12 u]

Answer (5)

Sol. $E = 5.808 \times 10^{30}$ W

$\Delta m = (3 \times 4.0026 - 12) \times 931.5$ MeV

$= 7.2657$ MeV

$= 7.2657 \times 1.6 \times 10^{-19} \times 10^6$ J

$= 1.1625 \times 10^{-12}$ J

Number of Reactions per second $= \frac{5.808 \times 10^{30}}{1.1625 \times 10^{-12}}$

$= 4.996 \times 10^{42}$

$\simeq 5 \times 10^{42}\ \text{s}^{-1}$

59. When a coil is connected across a 20 V dc supply, it draws a current of 5 A. When it is connected across 20 V, 50 Hz ac supply, it draws a current of 4 A. The self inductance of the coil is_______ mH.

(Take $\pi = 3$)

Answer (10)

Sol. $R = \frac{v}{i} = \frac{20}{5} = 4\ \Omega$

$\Rightarrow\ Z = \frac{v}{i} = \frac{20}{4} = 5\ \Omega$

$\therefore\ x_L = 3$

$\Rightarrow\ 3 = \omega L$

$L = \frac{3}{2\pi f} = \frac{1}{100}$

= 10 mH

60. The position, velocity and acceleration of a particle executing simple harmonic motion are found to have magnitudes of 4 m, 2 ms^{-1} and 16 ms^{-2} at a certain instant. The amplitude of the motion is $\sqrt{x}$, m where x is __________.

Answer (17)

Sol. $x = 4$

and, $a = \omega^2 x$

$\Rightarrow\ 16 = \omega^2 \times 4$

$\Rightarrow\ \omega = 2$

$\therefore\ v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow\ 2 = 2\sqrt{A^2 - 16}$

$\Rightarrow\ A^2 = 17$

$\Rightarrow\ A = \sqrt{17}$

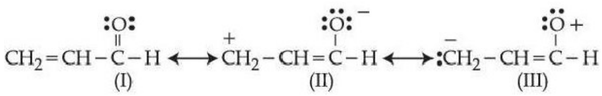
CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Relative stability of the contributing structures is



(1) (I) > (III) > (II)

(2) (II) > (I) > (III)

(3) (III) > (II) > (I)

(4) (I) > (II) > (III)

Answer (4)

Sol. Firstly higher the number of bonds or say complete octet higher will be stability, then the negative charge situated over more electronegative element more will be stability. Hence, overall order of stability is I > II > III.

62. The electronic configuration of Cu(II) is $3d^9$ whereas that of Cu(I) is $3d^{10}$. Which of the following is correct?

(1) Cu(II) is more stable

(2) Cu(II) is less stable

(3) Cu(I) and Cu(II) are equally stable

(4) Stability of Cu(I) and Cu(II) depends on nature of copper salts

Answer (1)

Sol. Cu^{2+} is more stable.

$Cu^{+} \longrightarrow Cu^{2+} + Cu$ disproportionation reaction

63. Given below are two statements :

Statement I : The oxidation state of an element in a particular compound is the charge acquired by its atom on the basis of electron gain enthalpy consideration from other atoms in the molecule.

Statement II : $p\pi - p\pi$ bond formation is more prevalent in second period elements over other periods.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) **Statement I** is correct but **Statement II** is incorrect

(2) Both **Statement I** and **Statement II** are correct

(3) Both **Statement I** and **Statement II** are incorrect

(4) **Statement I** is incorrect but **Statement II** is correct

Answer (4)

Sol. The electron gain enthalpy is defined for isolated atom while the oxidation number of an atom in a compound is the net real or hypothetical number of charge and it may be negative, positive or zero. Hence, Statement-I is incorrect and Statement-II is correct because of suitable size of atom the $p\pi - p\pi$ bond formation is more prevalent in second period elements.

64. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : S_N2 reaction of $C_6H_5CH_2Br$ occurs more readily than the S_N2 reaction of CH_3CH_2Br.

Reason (R) : The partially bonded unhybridized p-orbital that develops in the trigonal bipyramidal transition state is stabilized by conjugation with the phenyl ring.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) **(A)** is correct but **(R)** is not correct

(2) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(3) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**

(4) **(A)** is not correct but **(R)** is correct

Answer (2)

Sol. The transition state of S_N2 reaction of $C_6H_5CH_2Br$ is more stable as compared to the transition state of S_N2 reaction of CH_3CH_2Br. The extra stability attained by transition state of S_N2 of $C_6H_5CH_2Br$ is due to conjugation of electron deficient carbon with phenyl ring.

65. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Both rhombic and monoclinic sulphur exist as S_8 while oxygen exists as O_2.

Reason (R) : Oxygen forms $p\pi - p\pi$ multiple bonds with itself and other elements having small size and high electronegativity like C, N, which is not possible for sulphur.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(2) **(A)** is correct but **(R)** is **not** correct

(3) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

(4) **(A)** is not correct but **(R)** is correct

Answer (1)

Sol. Oxygen exists as O_2

Sulphur exists as S_8

Because of small size of oxygen, oxygen can form $2p_\pi - 2p_\pi$ bond.

$\Rightarrow$ Assertion (A) is correct.

$\Rightarrow$ Reason (R) is correct.

Reason (R) is correct explanation of Assertion (A).

66. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : The total number of geometrical isomers shown by $[Co(en)_2Cl_2]^+$ complex ion is three.

Reason (R) : $[Co(en)_2Cl_2]^+$ complex ion has an octahedral geometry.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) **(A)** is not correct but **(R)** is correct

(2) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**

(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(4) **(A)** is correct but **(R)** is not correct

Answer (1)

Sol. $[Co(en)_2Cl_2]^+$ has two geometrical isomers like other octahedral $[M(aa)_2b_2]$ coordination sphere $[Co(en)_2Cl_2]^+$ has d^2sp^3 hybridisation which is octahedral geometry.

67. Identify the **incorrect** statements regarding primary standard of titrimetric analysis.

(A) It should be purely available in dry form.

(B) It should not undergo chemical change in air.

(C) It should be hygroscopic and should react with another chemical instantaneously and stoichiometrically.

(D) It should be readily soluble in water.

(E) $KMnO_4$ & NaOH can be used as primary standard.

Choose the **correct** answer from the options given below.

(1) (C) and (E) only (2) (A) and (B) only

(3) (B) and (E) only (4) (C) and (D) only

Answer (1)

Sol. The substance to be used as a primary standard should also satisfy the following requirements.

(A) It must be easily available in pure and dry form.

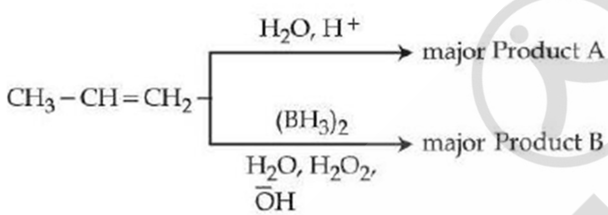
(B) It should not undergo change in air.

(C) It should not be hygroscopic.

(D) The substance should be readily soluble in water.

(E) It should not be affected by gases such as CO_2 present in the atmosphere it is why NaOH is not primary standard.

68. Identify the product A and product B in the following set of reactions.



(1) A - $CH_3CH_2CH_2 – OH$

B - $CH_3CH_2CH_2 – OH$

(2) A - $CH_3CH_2CH_3$

B - $CH_3CH_2CH_3$

(3) A - $CH_3CH_2CH_2 – OH$

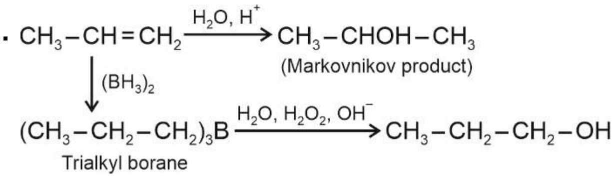
B - $CH_3CH(OH)–CH_3$

(4) A - $CH_3–CH(OH)–CH_3$

B - $CH_3CH_2CH_2 – OH$

Answer (4)

Sol.



69. 0.05M $CuSO_4$ when treated with 0.01M $K_2Cr_2O_7$ gives green colour solution of $Cu_2Cr_2O_7$. The two solutions are separated as shown below :

[SPM : Semi Permeable Membrane]

$K_2Cr_2O_7$	$CuSO_4$

Side X SPM Side Y

Due to osmosis :

(1) Green colour formation observed on side X.

(2) Green colour formation observed on side Y.

(3) Molarity of $CuSO_4$ solution is lowered.

(4) Molarity of $K_2Cr_2O_7$ solution is lowered.

Answer (3)

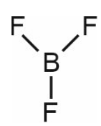
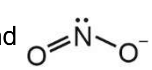
Sol. Osmosis leads to the net movement of water molecule from low osmotic pressure to high osmotic pressure. Since the iM value of 0.05 M $CuSO_4$ solution is higher hence it has higher osmotic pressure so the water moves towards $CuSO_4$ solution to drop of its molarity. The solute molecules do not cross S.P.M. via osmosis.

70. In which one of the following pairs the central atoms exhibit sp^2 hybridization?

(1) BF_3 and NO_2^- (2) NH_2^- and BF_3

(3) NH_2^- and H_2O (4) H_2O and NO_2

Answer (1)

Sol.  and  have central atom with sp^2 hybridisation NH_2^- and H_2O have sp^3 hybridised central atom.

71. On reaction of Lead Sulphide with dilute nitric acid which of the following is not formed?

(1) Sulphur (2) Lead nitrate

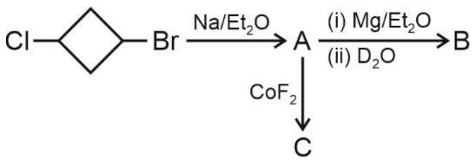
(3) Nitrous oxide (4) Nitric oxide

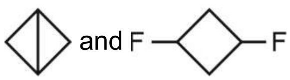
Answer (3)

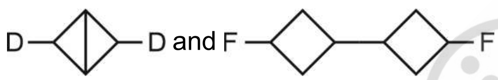
Sol. $3PbS\downarrow + 8HNO_3 \rightarrow 3Pb^{2+} + 6NO_3^- + 3S\downarrow + 2NO + 4H_2O$

Hence N_2O is not formed.

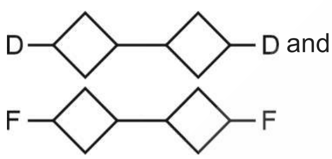
72. In the following sequence of reaction, the major products B and C Respectively are :



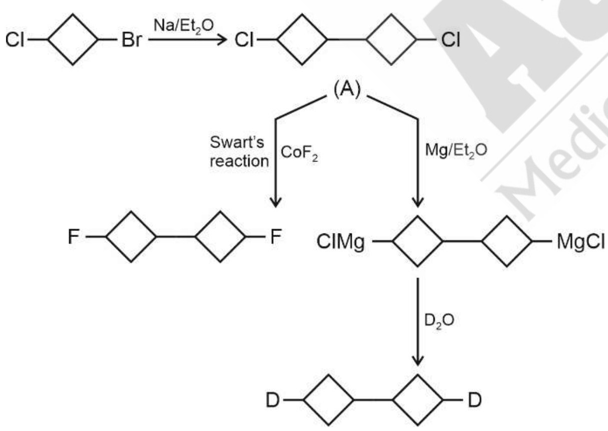
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. 

73. Methods used for purification of organic compounds are based on :

(1) nature of compound only.

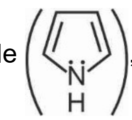
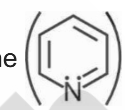
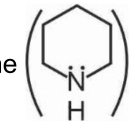
(2) neither on nature of compound nor on the impurity present.

(3) nature of compound and presence of impurity.

(4) presence of impurity only.

Answer (3)

Sol. Method used for purification of organic compound is based on the nature of various chemical species present in mixture but not only the nature of that organic compound.

74. Correct order of basic strength of Pyrrole (), Pyridine () and Piperidine () is :

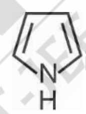
(1) Pyrrole > Pyridine > Piperidine

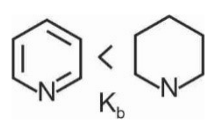
(2) Piperidine > Pyridine > Pyrrole

(3) Pyrrole > Piperidine > Pyridine

(4) Pyridine > Piperidine > Pyrrole

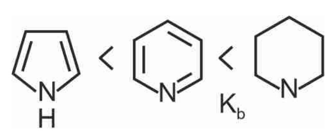
Answer (2)

Sol.  is weakest among the given nitrogenous bases because in this the lone pair is completely delocalised via cyclic resonance to show aromaticity.



: Due to strength of I effect of substituents

Hence the correct order of basic strength



75. Compare the energies of following sets of quantum numbers for multielectron system.

(A) n = 4, l = 1

(B) n = 4, l = 2

(C) n = 3, l = 1

(D) n = 3, l = 2

(E) n = 4, l = 0

Choose the **correct** answer from the options given below :

(1) (C) < (E) < (D) < (A) < (B)

(2) (E) > (C) > (A) > (D) > (B)

(3) (B) > (A) > (C) > (E) > (D)

(4) (E) < (C) < (D) < (A) < (B)

Answer (1)

Sol. Energy of subshell ∝ $(n + l)$

If $(n + l)$ comes equal then n dominates hence order of energy will be

(C) < (E) < (D) < (A) < (B)

76. The F^- ions make the enamel on teeth much harder by converting hydroxyapatite (the enamel on the surface of teeth) into much harder fluoroapatite having the formula.

(1) $[3(Ca_3(PO_4)_3)\cdot CaF_2]$

(2) $[3(Ca_3(PO_4)_2)\cdot CaF_2]$

(3) $[3(Ca_2(PO_4)_2)\cdot Ca(OH)_2]$

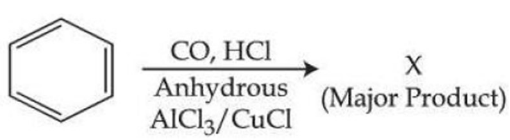
(4) $[3(Ca_3(PO_4)_2)\cdot Ca(OH)_2]$

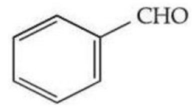
Answer (2)

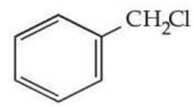
Sol. Hydroxyapatite : $[3(Ca_3(PO_4)_2)\cdot Ca(OH)_2]$

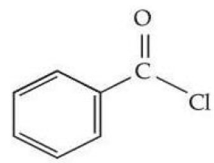
Fluoroapatite : $[3(Ca_3(PO_4)_2)\cdot CaF_2]$

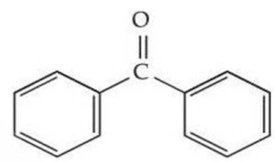
77. Identify major product "X" formed in the following reaction :



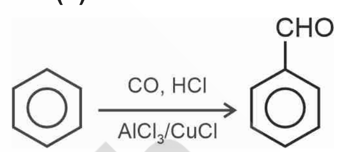
(1) 

(2) 

(3) 

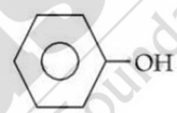
(4) 

Answer (1)

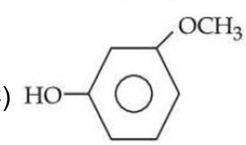
Sol. 

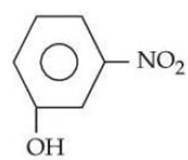
is known as Gattermann-Koch reaction.

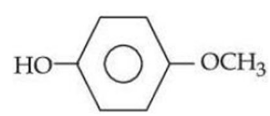
78. For the given compounds, the correct order of increasing pK_a value :

(A) 

(B) 

(C) 

(D) 

(E) 

Choose the **correct** answer from the options given below.

(1) (E) < (D) < (B) < (A) < (C)

(2) (D) < (E) < (C) < (B) < (A)

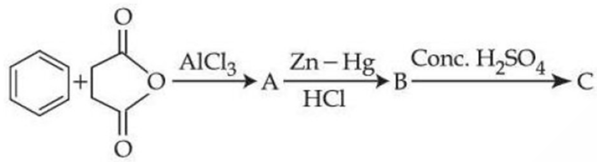
(3) (B) < (D) < (A) < (C) < (E)

(4) (E) < (D) < (C) < (B) < (A)

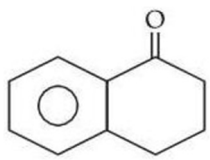
Answer (3)

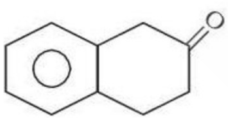
Sol. Higher the electron withdrawing strength of substituents higher will be acidic strength of phenols and lesser will be pK_a

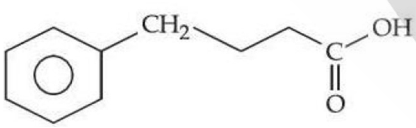
Acidic strength :

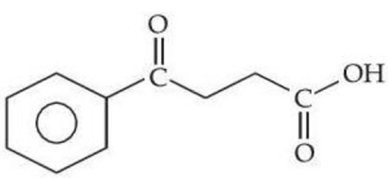
79. 

What is the structure of C?

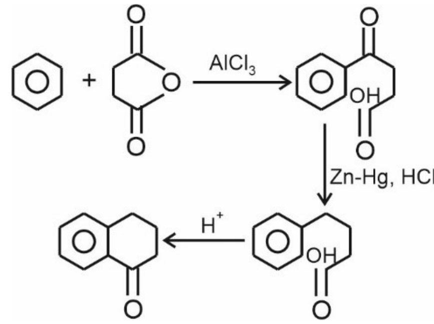
(1) 

(2) 

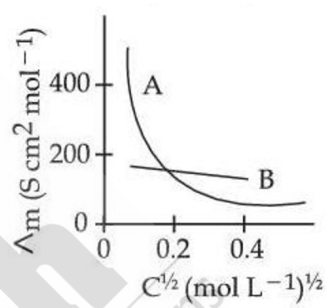
(3) 

(4) 

Answer (1)

Sol. 

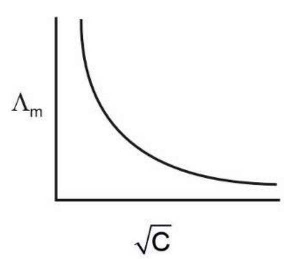
80. The molar conductivity for electrolytes A and B are plotted against $C^{1/2}$ as shown below. Electrolytes A and B respectively are



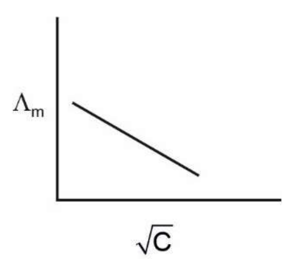
	A	**B**
(1)	Weak electrolyte	Strong electrolyte
(2)	Strong electrolyte	Strong electrolyte
(3)	Strong electrolyte	Weak electrolyte
(4)	Weak electrolyte	Weak electrolyte

Answer (1)

Sol.



Weak electrolyte



Strong electrolyte

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

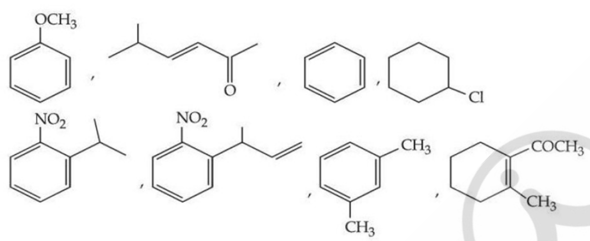
81. Number of ambidentate ligands among the following is ________.

$NO_2^-, SCN^-, C_2O_4^{2-}, NH_3, CN^-, SO_4^{2-}, H_2O.$

Answer (3)

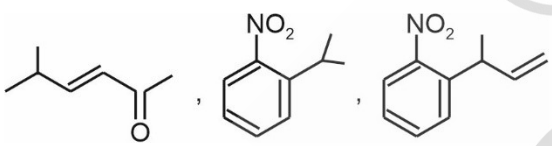
Sol. NO_2^-, SCN^- and CN^- are ambidentate ligands

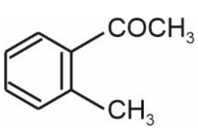
82. How many compounds among the following compounds show inductive, mesomeric as well as hyperconjugation effects?



Answer (4)

Sol.





are the compounds show inductive, mesomeric as well as hyperconjugation.

83. When equal volume of 1 M HCl and 1 M H_2SO_4 are separately neutralised by excess volume of 1 M NaOH solution. x and y kJ of heat is liberated respectively. The value of y/x is ________.

Answer (2)

Sol. $\frac{y}{x} = \frac{\text{Equivalents of } H_2SO_4}{\text{Equivalents of HCl}} = \frac{2}{1}$

84. The total number of species from the following in which one unpaired electron is present, is ______.

$N_2, O_2, C_2^-, O_2^-, O_2^{2-}, H_2^+, CN^-, He_2^+$

Answer (4)

Sol.

Species	Unpaired e
N_2	0
O_2	2
C_2^-	1
O_2^-	1
O_2^{2-}	0
H_2^+	1
CN^-	0
He_2^+	1

85. Number of colourless lanthanoid ions among the following is_______ Eu^{3+}, Lu^{3+}, Nd^{3+}, La^{3+}, Sm^{3+}

Answer (2)

Sol. Colour of these ions may be attributed to the presence of electrons. La^{3+} and Lu^{3+} are colourless.

86. The standard reduction potential at 298 K for the following half cells are given below:

$Cr_2O_7^{2-} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O, E° = 1.33$ V

$Fe^{3+}(aq) + 3e^- \rightarrow Fe$ E° = –0.04 V

$Ni^{2+}(aq) + 2e^- \rightarrow Ni$ E° = –0.25 V

$Ag^+(aq) + e^- \rightarrow Ag$ E° = 0.80 V

$Au^{3+}(aq) + 3e^- \rightarrow Au$ E° = 1.40 V

Consider the given electrochemical reactions,

The number of metal(s) which will be oxidized be $Cr_2O_7^{2-}$, in aqueous solution is _______.

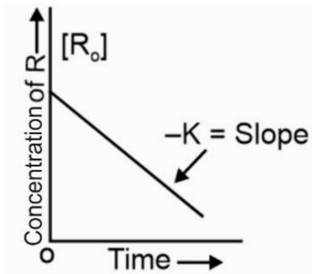
Answer (3)

Sol. The sum of standard reduction potential $Cr_2O_7^{2-}$ and standard oxidation potential of metal should be more than zero. Hence Fe, Ni and Ag can be oxidized.

87. Given below are two statements :

Statement-I : The rate law for the reaction A + B → C is rate (r) = $k[A]^2[B]$. When the concentration of both A and B is doubled, the reaction rate is increased "x" times.

Statement-II : The figure is showing "the variation in concentration against time plot" for a "y" order reaction.



The value of x + y is ________.

Answer (8)

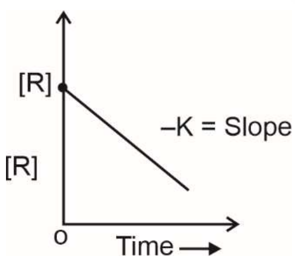
Sol. x = 8, y = 0

$$\frac{r_2}{r_1} = \left(\frac{C_{A_2}}{C_{A_1}}\right)^2 \left(\frac{C_{B_2}}{C_{B_1}}\right)$$

$$\frac{r_2}{r_1} = \left(\frac{2C_{A_1}}{C_{A_1}}\right)^2 \left(\frac{2C_{B_1}}{C_{B_1}}\right)$$

$r_2 = 8r_1$

x = 8



⇒ Zero order

y = 0

88. Total number of essential amino acid among the given list of amino acids is _____.

Arginine, Phenylalanine, Aspartic acid, Cysteine, Histidine, Valine, Proline

Answer (4)

Sol. Arginine, Phenylalanine, Histidine, Valine are essential amino acids.

89. The heat of solutions of anhydrous $CuSO_4$ and $CuSO_4{\cdot}5H_2O$ are –70 kJ mol^{-1} and +12 kJ mol^{-1} respectively.

The heat of hydration of $CuSO_4$ to $CuSO_4{\cdot}5H_2O$ is –x kJ. The value of x is ________. (Nearest integer)

Answer (82)

Sol. (I) $CuSO_4 + H_2O \longrightarrow CuSO_4$

Solution ΔH = –70 kJ

(II) $CuSO_4 \cdot 5H_2O + H_2O \longrightarrow CuSO_4$

ΔH = 12 kJ

(I) – (II)

$CuSO_4 + H_2O \longrightarrow CuSO_4 \cdot 5H_2O$

ΔH = –70 – 12 = –82

90. Molarity (M) of an aqueous solution containing x g of anhyd. $CuSO_4$ in 500 mL solution at 32°C is 2×10^{-1} M. Its molality will be _________ $\times 10^{-3}$ m. (nearest integer).

[Given density of the solution = 1.25 g/mL]

Answer (164)

Sol. $$m = \frac{1000 \times (0.2)}{1000(1.25) - (0.2)(159.5)}$$

$$m = \frac{200}{1250 - 31.9}$$

$$= \frac{200}{1218.1}$$

= 0.16419

= 164.19×10^{-3}

Nearest integer = 164

❑ ❑ ❑