05/04/2024

Evening

Aakash

Medical | IIT-JEE | Foundations

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2024 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is **Mathematics**, **Part-B** is **Physics** and **Part-C** is. **Chemistry** Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt **any 05 questions** out of 10 Questions.

(6) **Section – A : (01-20) / (31-50) / (61-80)** contains 20 multiple choice questions **(MCQs)** which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section – B: (21-30) / (51-60) / (81-90)** contains 10 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

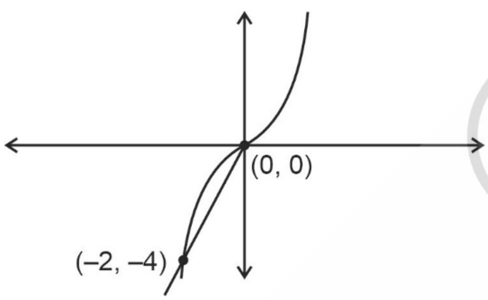
1. The area enclosed between the curves $y = x|x|$ and $y = x - |x|$ is:

(1) $\frac{2}{3}$ (2) $\frac{8}{3}$

(3) 1 (4) $\frac{4}{3}$

Answer (4)

Sol. $y = x|x|$ & $y = x - |x|$



$$y = x|x| = \begin{cases} x^2 & , x > 0 \\ -x^2 & , x < 0 \end{cases}$$

$$y = x - |x| = \begin{cases} 0 & , x > 0 \\ 2x & , x < 0 \end{cases}$$

Point of intersections are (0, 0) & (–2, 4).

$$\therefore \quad \text{Area} = \int_{-2}^{0} \left(-x^2 - 2x\right) dx$$

$$= \left[\frac{-x^3}{3} - \frac{2x^2}{2}\right]_{-2}^{0}$$

$$= -\left[\frac{8}{3} - 4\right] = \frac{4}{3} \text{ sq. unit}$$

2. The differential equation of the family of circles passing through the origin and having centre at the line $y = x$ is:

(1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$

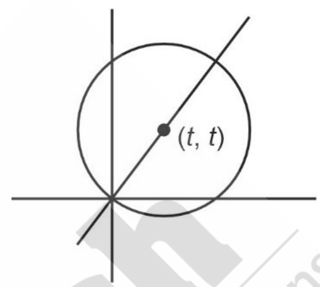
(2) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$

(3) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

(4) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$

Answer (1)

Sol. Equation of circle passing through origin & having centre at the line $y = x$ is



$(x - t)^2 + (y - t)^2 = 2t^2$

$x^2 + y^2 + t^2 + t^2 - 2tx - 2ty = 2t^2$

$x^2 + y^2 = 2t(x + y)$

Now differentiate

$2x + 2yy' = 2t(1 + y')$

$$t = \frac{x + yy'}{1 + y'}$$

$$\text{Now, } x^2 + y^2 = 2\left(\frac{x + yy'}{1 + y'}\right)(x + y)$$

$$x^2 + y^2 + x^2\frac{dy}{dx} + y^2\frac{dy}{dx}$$

$$= 2\left(x^2 + xy + xy\frac{dy}{dx} + y^2\frac{dy}{dx}\right)$$

$$dx\left(x^2 + y^2 - 2x^2 - 2xy\right) = dy(2xy + 2y^2 - x^2 - y^2)$$

$$dx(x^2 - y^2 + 2xy) = dy(x^2 - y^2 - 2xy)$$

3. If $y(\theta)=\dfrac{2\cos\theta+\cos 2\theta}{\cos 3\theta+4\cos 2\theta+5\cos\theta+2}$, then at $\theta=\dfrac{\pi}{2}$, $y''+y'+y$ is equal to:

(1) $\dfrac{3}{2}$ (2) $\dfrac{1}{2}$

(3) 1 (4) 2

Answer (4)

Sol. $y(\theta)=\dfrac{2\cos\theta+\cos 2\theta}{\cos 3\theta+4\cos 2\theta+5\cos\theta+2}$

$$=\frac{2\cos^2\theta+2\cos\theta-1}{4\cos^3\theta+8\cos^2\theta+2\cos\theta-2}$$

$$=\frac{2\cos^2\theta+2\cos\theta-1}{(2\cos^2\theta+2\cos\theta-1)(2\cos\theta+2)}$$

$$=\frac{1}{2(1+\cos\theta)}=\frac{1}{4\cos^2\theta/2}=\frac{\sec^2\theta/2}{4}$$

$$y'(\theta)=\frac{1}{4}\left(2\sec\frac{\theta}{2}\cdot\sec\frac{\theta}{2}\cdot\tan\frac{\theta}{2}\cdot\frac{1}{2}\right)$$

$$=\frac{1}{4}\sec^2\frac{\theta}{2}\cdot\tan\frac{\theta}{2}$$

$$y''(\theta)=\frac{1}{4}\left(\tan\frac{\theta}{2}\right)\left(\sec^2\frac{\theta}{2}\cdot\tan\frac{\theta}{2}\right)$$

$$+\frac{1}{4}\sec^2\frac{\theta}{2}\cdot\sec^2\frac{\theta}{2}\cdot\frac{1}{2}$$

at $\theta=\dfrac{\pi}{2}$, $y(\theta)=\dfrac{1}{2}$, $y'(\theta)=\dfrac{1}{2}$, $y''(\theta)=1$

$\therefore\ y+y'+y''=2$

4. The values of m, n for which the system of equations

$x+y+z=4$

$2x+5y+5z=17$

$x+2y+mz=n$

has infinitely many solutions, satisfy the equation:

(1) $m^2+n^2-mn=39$

(2) $m^2+n^2+mn=68$

(3) $m^2+n^2-m-n=46$

(4) $m^2+n^2+m+n=64$

Answer (1)

Sol. The given system of linear equations can be represented as,

$$\left(\begin{array}{ccc|c}1&1&1&4\\2&5&5&17\\1&2&m&n\end{array}\right)$$

$$\sim\left(\begin{array}{ccc|c}1&1&1&4\\0&3&3&9\\0&1&m-1&n-4\end{array}\right)$$

$$\sim\left(\begin{array}{ccc|c}1&1&1&4\\0&1&1&3\\0&0&m-2&n-7\end{array}\right)$$

$\because$ System of equations has infinitely many solutions

$\therefore$ $m=2$ & $n=7$

Which satisfy equation given in option (1).

(*i.e.*, $2^2+7^2-14=39$)

5. Let $\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, $m, n>0$. If $\int_0^1\left(1-x^{10}\right)^{20}dx=a\times\beta(b,c)$, then 100 $(a+b+c)$ equals ____________.

(1) 2012 (2) 1021

(3) 2120 (4) 1120

Answer (3)

Sol. Given integral, $I=\int_0^1(1-x^{10})^{20}dx$

take $x^{10}=t$

$\Rightarrow\ 10x^9dx=dt$

$\therefore\ I=\dfrac{1}{10}\int_0^1(1-t)^{20}t^{-9/10}dt$

$$= \frac{1}{10}\int_0^1 t^{\frac{1}{10}-1}(1-t)^{21-1}dt$$

$$= \frac{1}{10}\beta\left(\frac{1}{10}, 21\right)$$

$$\therefore \quad a = \frac{1}{10},\ b = \frac{1}{10}\ \&\ c = 21$$

$\therefore$ 100(*a* + *b* + *c*)

$$= 100\left(\frac{1}{10} + \frac{1}{10} + 21\right) = 2120$$

6. Let the set $S = \{2, 4, 8, 16, \ldots, 512\}$ be partitioned into 3 sets *A*, *B*, *C* with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of *S* is equal to:

(1) 1520 (2) 1680

(3) 1640 (4) 1710

Answer (2)

Sol. Given set $S = \{2^1, 2^2, \ldots 2^9\}$ which consist of 9 elements.

Maximum number of possible partitions (in set *A*, *B* and *C*)

$$= {}^9C_3 \cdot {}^6C_3 \cdot {}^3C_3 = 1680$$

7. Let the circle C_1: $x^2 + y^2 - 2(x + y) + 1 = 0$ and C_2 be a circle having centre at (–1, 0) and radius 2. If the line of the common chord of C_1 and C_2 intersects the *y*-axis at the point *P*, then the square of the distance of *P* from the centre of C_1 is:

(1) 6 (2) 1

(3) 2 (4) 4

Answer (3)

Sol. $C_1 : x^2 + y^2 - 2(x+y) + 1 = 0$

$C_2 : (x+1)^2 + y^2 = (2)^2$

$x^2 + y^2 + 2x - 3 = 0$

Common chord is

$C_1 - C_2 = 0$

$\Rightarrow 2x + y - 2 = 0$

also, this line intersects the *y*-axis at the point *P*(*y*, 0).

$\Rightarrow \boxed{y = 2}$

P(2, 0)

Distance of point *P* from (1, 1) is

$$d = \sqrt{(2-1)^2 + (0-1)^2}$$

$$= \sqrt{1^2 + 1^2}$$

$$d = \sqrt{2}$$

$$\Rightarrow \boxed{d^2 = 2}$$

8. Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of *A*, then det(*AB*) is equal to:

(1) 343 (2) 125

(3) 216 (4) 64

Answer (3)

Sol. $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$, $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$

Cofactor of *A*-matrix is

$$= \begin{bmatrix} 2\alpha^2 - \alpha\beta & -(2\alpha^2 + \beta^2) & \alpha^2 + \alpha\beta \\ -(2\alpha^2 - 3\alpha) & (2\alpha\beta + 3\beta) & -(2\alpha\beta) \\ \alpha\beta - 3\alpha & -(\beta^2 - 3\alpha) & \beta\alpha - \alpha^2 \end{bmatrix}$$

which is equal to matrix *B*

So, by comparing elements of two matrix

$\Rightarrow \alpha\beta - 3\alpha = -2\alpha$

$\Rightarrow \alpha\beta - \alpha = 0$

$\Rightarrow \alpha(\beta - 1) = 0$

$\Rightarrow \alpha = 0$ or $\beta = 1$ [$\because \alpha$ cannot be 0]

$\Rightarrow \beta = 1$

and $-\beta^2 + 3\alpha = 5$

$\Rightarrow 3\alpha = 6$

$\Rightarrow \alpha = 2$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{bmatrix}$$

$\text{Det}(AB) = |A||B| = |A||(\text{adj}A)^T|$

$= |A| \cdot |A|^2$

$= |A|^3$

$= (6 - 18 + 18)^3$

$= 6^3$

$= 216$

9. The coefficients a, b, c in the quadratic equations $ax^2 + bx + c = 0$ are from the set {1, 2, 3, 4, 5, 6}. If the probability of this equation having one real root bigger than the other p, then 216p equals

(1) 57 (2) 38

(3) 19 (4) 76

Answer (2)

Sol. Equation is $ax^2 + bx + c = 0$

D > 0 [for roots to be real & distinct]

$\Rightarrow b^2 - 4ac > 0$

For $b < 2$ no value of a & c are possible

For $b = 3 \Rightarrow ac < \frac{9}{4}$

$(a, c) \in \{(1, 1), (1, 2), (2, 1)\} \Rightarrow 3$ cases

For $b = 4 \Rightarrow ac < 4$

$(a, c) \in \{(1, 1), (1, 2), (2, 1), (3, 1), (1, 3)\} \Rightarrow 5$ cases

For $b = 5 \Rightarrow ac < \frac{25}{4}$

$(a, c) \in$ {(1, 1), (1, 2), (2, 1), (3, 1), (1, 3), (2, 2), (4, 1), (1, 4), (3, 2), (2, 3), (5, 1), (1, 5), (1, 6), (6, 1)} = 14 cases

For $b = 6 \Rightarrow ac < 9$

$(a, c) \in$ {(1, 1), (1, 2), (2, 1), (3, 1), (1, 3), (2, 2), (4, 1), (1, 4), (3, 2), (2, 3), (5, 1), (1, 5), (1, 6), (6, 1), (2, 4), (4, 2)} = 16 cases

Total cases = 3 + 5 + 14 + 16 = 38 cases

$\Rightarrow$ Probability, $p = \frac{38}{216}$

$\Rightarrow 216p = 38$

10. Let $S_1 = \{z \in C : |z| \le 5\}$,

$S_2 = \left\{ z \in C : \text{Im}\left(\frac{z + 1 - \sqrt{3}i}{1 - \sqrt{3}i} \right) \ge 0 \right\}$ and

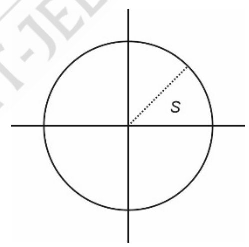
$S_3 = \{z \in C : \text{Re}(z) \ge 0\}$. Then the area of the region $S_1 \cap S_2 \cap S_3$ is :

(1) $\frac{125\pi}{6}$ (2) $\frac{125\pi}{4}$

(3) $\frac{125\pi}{24}$ (4) $\frac{125\pi}{12}$

Answer (4)

Sol. $S_1 = \{z \in C : |z| \le 5\}$



$S_2 = \text{Im}\left(\frac{z + 1 - \sqrt{3}i}{1 - \sqrt{3}i} \right) \ge 0$

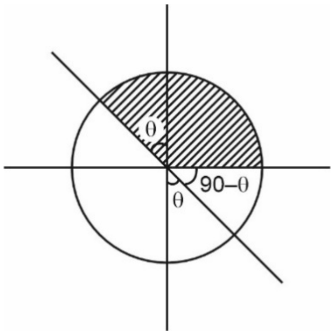
Take $z = x + iy$

$= \frac{x + iy + 1 - \sqrt{3}i}{1 - \sqrt{3}i} \times \frac{1 + \sqrt{3}i}{1 + \sqrt{3}i}$

$$= \frac{x + iy + 1 - \sqrt{3}i + \sqrt{3}ix - \sqrt{3}y + \sqrt{3}i + 3}{1+3}$$

$$= y + \sqrt{3}x \geq 0$$

$S_3 = x \geq 0$



$y + \sqrt{3}x = 0$

$y = -\sqrt{3}x$

Slope = $-\sqrt{3}$

90 + θ = 120°

θ = 30°

90 – θ = 60°

In first quadrant we have angle $\frac{\pi}{2}$

so, total angle 90° + 60° = 150°

So, area $\frac{\pi r^2}{2\pi} \times \frac{5\pi}{6} = \frac{125\pi}{12}$

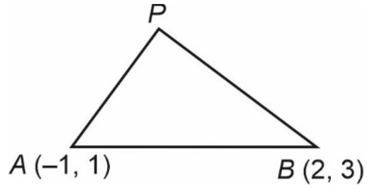
11. Let $A(-1, 1)$ and $B(2, 3)$ be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is $ax + by = 15$, then $5a + 2b$ is :

(1) $-\frac{12}{5}$ (2) $-\frac{6}{5}$

(3) 4 (4) 6

Answer (3)

Sol.



Locus of P,

$ax + by = 15$

Take point P as (h, k)

Area of ΔABP

$$= \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) | = 10$$

$$= \frac{1}{2} | -1(3 - k) + 2(k - 1) + h(1 - 3) | = 10$$

$$= | -3 + k + 2k - 2 + h - 3h | = 20$$

$$= | 3k - 2h - 5 | = 20$$

$$= -(3k - 2h - 5) = 20 \Rightarrow 2h - 3k = 15$$

$a = 2$, $b = -3$

$\therefore 5a + 2b = 4$

12. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c} - \vec{a}|^2$ is equal to :

(1) 110 (2) 121

(3) 105 (4) 124

Answer (4)

Sol. $\vec{a} = \vec{b} \times \vec{c}$

$|\vec{a}| = 2, |\vec{b}| = 3$

$\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \cdot \vec{c} = 0$

$|\vec{c} - \vec{a}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a}$

$= 4 + |\vec{c}|^2$

$|\vec{a}| = |\vec{b}\times\vec{c}| = |\vec{b}|\sin\alpha|\vec{c}|$

$\Rightarrow \sin\alpha|\vec{c}| = \dfrac{2}{3}$

$\Rightarrow \sin^2\alpha = \dfrac{4}{9|\vec{c}|^2}$

$\Rightarrow |\vec{c}|^2 = \dfrac{4}{9\sin^2\alpha}$

$\Rightarrow |\vec{c}-\vec{a}|^2 = 4 + \dfrac{4}{9\sin^2\alpha}$

For $|\vec{c}-\vec{a}|^2$ to be minimum for $\alpha \in \left[0, \dfrac{\pi}{3}\right]$

$\sin\alpha = \dfrac{\sqrt{3}}{2}$

$27|\vec{c}-\vec{a}|^2 = 27\left[4 + \dfrac{4.4}{9.3}\right]$

$= 27\left[\dfrac{124}{27}\right] = 124$

13. For $x \geq 0$, the least value of K for which $4^{1+x} + 4^{1-x}$, $\dfrac{K}{2}$, $16^x + 16^{-x}$ are three consecutive terms of an A.P., is equal to :

(1) 16 (2) 8

(3) 10 (4) 4

Answer (3)

Sol. $4^{1+x} + 4^{1-x}, \dfrac{K}{2}, 16^x + 16^{-x} \to$ A.P.

$\Rightarrow \quad K = 4^{1+x} + 4^{1-x} + 4^{2x} + 4^{-2x}$

Let $4^x = t$

$\Rightarrow \quad K = 4t + \dfrac{4}{t} + t^2 + \dfrac{1}{t^2}$

$= t^2 + \dfrac{1}{t^2} + 4\left(t + \dfrac{1}{t}\right)$

$= \left(t + \dfrac{1}{t}\right)^2 + 4\left(t + \dfrac{1}{t}\right) - 2$

Let $t + \dfrac{1}{t} = y \qquad \Rightarrow \quad y \in [2, \infty)$

$K = y^2 + 4y - 2$

$\dfrac{-b}{2a} = -2$

K is minimum at $y = 2$

$K_{min} = (2)^2 + 4 \times 2 - 2$

$= 10$

14. Let $f, g : \mathbf{R} \to \mathbf{R}$ be defined as:

$f(x) = |x - 1|$ and $g(x) = \begin{cases} e^x & x \geq 0 \\ x+1, & x \leq 0 \end{cases}$

Then the function $f(g(x))$ is

(1) Both one -one and onto

(2) Onto but not one-one

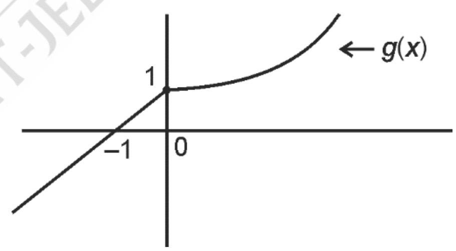
(3) One-one but not onto

(4) Neither one-one nor onto

Answer (4)

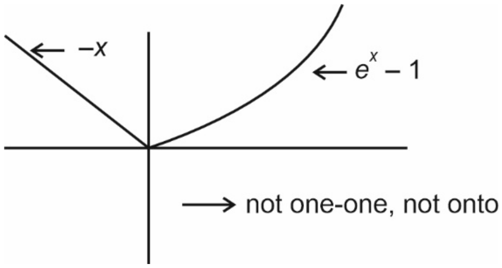
Sol. $f(x) = \begin{cases} x-1, & x \geq 1 \\ 1-x & x < 0 \end{cases}$

$g(x) = \begin{cases} e^x & ; \quad x \geq 0 \\ x+1 & ; \quad x \leq 0 \end{cases}$



$f(g(x)) = \begin{cases} g(x)-1, & g(x) \geq 1 \\ 1-g(x) & g(x) < 1 \end{cases}$

$= \begin{cases} e^x - 1 & x \geq 0 \\ -x & x < 0 \end{cases}$



15. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50th word is:

(1) HBBJO (2) JBBOH

(3) OBBHJ (4) OBBJH

Answer (4)

Sol. BHBJO

B, H, J, O

B _ _ _ _ $\to$ 4! = 24

H _ _ _ _ $\to \dfrac{4!}{2!} = 12$

J _ _ _ _ $\to \dfrac{4!}{2!} = \dfrac{12}{48}$

O B B H J $\to$ 49

O B B J H $\to$ 50

16. Let f: [–1, 2] $\to$ **R** be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where $[t]$ denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is:

(1) 4 (2) 3

(3) 6 (4) 5

Answer (1)

Sol. $f(x) = 2x^2 + x + [x^2] - [x] = 2x^2 + [x^2] + \{x\}$

$f(-1) = 2 + 1 + 0 = 3$

$f(-1^+) = 2 + 0 + 0 = 2$

$f(0^-) = 0 + 1 = 1$

$f(0^+) = 0 + 0 + 0 = 0$

$f(1^+) = 2 + 1 + 0 = 3$

$f(1^-) = 2 + 0 + 1 = 3$

$f(2^-) = 8 + 3 + 1 = 12$

$f(2^+) = 8 + 4 + 0 = 12$

$\therefore$ discontinuous at $x = 0, \sqrt{2}, \sqrt{3}, -1$

17. Let $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{c}$ be three vectors such that $(\vec{c} + \hat{i}) \times (\vec{a} + \vec{b} + \hat{i}) = \vec{a} \times (\vec{c} + \hat{i})$. If $\vec{a} \cdot \vec{c} = -29$, then $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k})$ is equal to:

(1) 12 (2) 5

(3) 15 (4) 10

Answer (2)

Sol. $(\vec{c} + \hat{i}) \times (\vec{a} + \vec{b} + \hat{i} + \vec{a}) = 0$

$\Rightarrow \vec{c} + \hat{i} = \lambda(\vec{a} + \vec{b} + \hat{i} + \vec{a})$

$= \lambda(2\vec{a} + \vec{b} + \hat{i})$

$= \lambda(7\hat{i} + 8\hat{j})$

$\Rightarrow \vec{c} = (7\lambda - 1)\hat{i} + 8\lambda\hat{j}$

$\vec{c} \cdot \vec{a} = -29$

$\Rightarrow 14\lambda - 2 + 40\lambda = -29$

$\Rightarrow 54\lambda = -27$

$\Rightarrow \lambda = -\dfrac{1}{2}$

$\therefore \vec{c} = \left(\dfrac{-7}{2} - 1\right)\hat{i} - 4\hat{j} = \dfrac{-9}{2}\hat{i} - 4\hat{j}$

$\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k}) = 9 - 4 = 5$

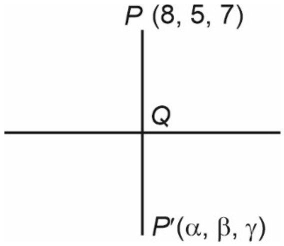
18. Let (α, β, γ) be the image of the point (8, 5, 7) in the line $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-2}{5}$. Then $\alpha + \beta + \gamma$ is equal to:

(1) 20 (2) 14

(3) 18 (4) 16

Answer (2)

Sol. $(x, y, z) \equiv (2\lambda + 1, 3\lambda + 1, 5\lambda + 2)$



DR of PQ:

$(2\lambda - 7, 3\lambda - 6, 5\lambda - 5)$

DR of line : (2, 3, 5)

$\therefore\ 2(2\lambda - 7) + 3(3\lambda - 6) + 5(5\lambda - 5) = 0$

$\Rightarrow\ \lambda = \frac{3}{2}$

$\therefore\ Q\left(4, \frac{7}{2}, \frac{19}{2}\right)$

$\therefore\ (\alpha, \beta, y) \equiv (0, 2, 12)$ (Q is mid point of P & P')

$\alpha + \beta + y \equiv 14$

19. Let $ABCD$ and $AEFG$ be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies:

(1) $r^2 - 8r + 8 = 0$

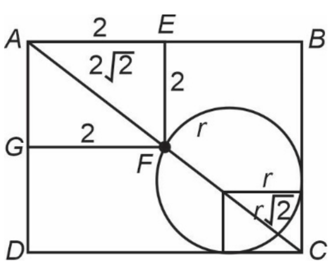
(2) $r = 1$

(3) $2r^2 - 4r + 1 = 0$

(4) $2r^2 - 8r + 7 = 0$

Answer (1)

Sol.



$CF = 4\sqrt{2} - 2\sqrt{2} = 2\sqrt{2} = r + r\sqrt{2}$

$\Rightarrow\ (2 - r)\sqrt{2} = r$

$\Rightarrow\ \sqrt{2} = \left(\frac{r}{2 - r}\right) \Rightarrow 2 = \frac{r^2}{(2 - r)^2}$

$\Rightarrow\ 2(r^2 - 4r + 4) = r^2$

$\Rightarrow\ r^2 - 8r + 8 = 0$

20. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x} + \frac{2x}{\sqrt[3]{5}}\right)^{12}$, $x \neq 0$, is $\alpha \times 2^8 \times \sqrt[5]{3}$, then 25α is equal to :

(1) 724

(2) 693

(3) 742

(4) 639

Answer (2)

Sol. $\left(\frac{\sqrt[5]{3}}{x} + \frac{2x}{\sqrt[5]{3}}\right)^{12}$

$T_{r+1} = {}^{12}Cr\left(\frac{\sqrt[5]{3}}{x}\right)^{12-r}\left(\frac{2x}{\sqrt[3]{5}}\right)^{r}$

For constant term – 12 + r + r = 0

$\Rightarrow\ r = 6$

$\therefore$ Constant term $= {}^{12}C_6 \frac{(3)^{\frac{6}{5}}}{(5)^{\frac{6}{3}}}(2)^6$

$= {}^{12}C_6 \times \frac{2^6}{25} \times 3.3^{\frac{1}{5}}$

$= \frac{231}{25} \times 2^8 \cdot 3^{\frac{1}{5}} \cdot 3$

$= \frac{693}{25} \cdot 2^8 \sqrt[5]{3}$

$\therefore\ \alpha = \frac{693}{25}$

$25\alpha = 693$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $a > 0$ be a root of the equation $2x^2 + x - 2 = 0$. If $\lim_{x\to\frac{1}{a}} \frac{16(1-\cos(2+x-2x^2))}{(1-ax)^2} = \alpha + \beta\sqrt{17}$, where α, $\beta \in Z$, then $\alpha + \beta$ is equal to_____.

Answer (170)

Sol. $\because\ 2x^2 + x - 2 = 0$ has two roots where

$a = \frac{\sqrt{17}-1}{4}$ and another root is $\frac{-\sqrt{17}-1}{4}$

And $2 + x - 2x^2 = -2\left(x - \frac{1}{a}\right)\left(x + \frac{4}{\sqrt{17}+1}\right)$

Now $\lim_{x\to\frac{1}{a}} \frac{16(1-\cos(2+x-2x^2))}{(1-ax)^2}$

$$= \lim_{x\to\frac{1}{a}} \frac{32\sin^2\left(\frac{2+x-2x^2}{2}\right)}{a^2\left(\frac{1}{a}-x\right)^2}$$

$$= \lim_{x\to\frac{1}{a}} \frac{\left(x+\frac{4}{\sqrt{17}+1}\right)^2 32\cdot\left(\sin\left(\frac{1}{2}\cdot(-2)\right)\left(x-\frac{1}{a}\right)\left(x+\frac{4}{\sqrt{17}+1}\right)\right)^2}{a^2\left(\left(x-\frac{1}{a}\right)\left(x+\frac{4}{\sqrt{17}+1}\right)\right)^2}$$

$$= 2\cdot\left(\frac{1}{a}+\frac{4}{\sqrt{17}+1}\right)^2\cdot\left(\frac{4}{\sqrt{17}-1}\right)^2$$

$$= 2\left(\frac{4}{\sqrt{17}-1}+\frac{4}{\sqrt{17}+1}\right)^2\cdot\left(\frac{4}{\sqrt{17}-1}\right)^2$$

$$= \frac{17\times 4}{18-2\sqrt{17}} = \frac{68}{9-\sqrt{17}}$$

$$= 17\left(9+\sqrt{17}\right)$$

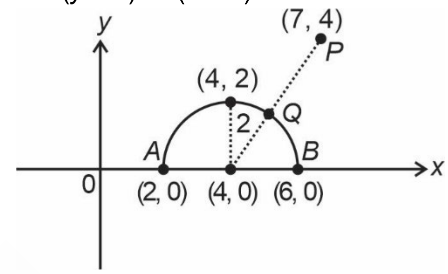
$\alpha + \beta = 170$

22. Let the maximum and minimum values of $\left(\sqrt{8x-x^2-12}-4\right)^2 + (x-7)^2$, $x \in R$ be M and m, respectively. Then $M^2 - m^2$ is equal to ______.

Answer (1600)

Sol. Let $y = \sqrt{8x - x^2 - 12} \Rightarrow (x-4)^2 + y^2 = 2^2$

$\Rightarrow\ d = (y-4)^2 + (x-7)^2$



$\Rightarrow\ M = PA^2 = 16 + 25 = 41$

$m = PQ^2 = \left(\sqrt{16+9}-2\right)^2 = 9$

$\Rightarrow\ M^2 - m^2 = 1681 - 81 = 1600$

23. Let the mean and the standard deviation of the probability distribution

X	α	1	0	–3
$P(X)$	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to ________.

Answer (5*)

Sol. Mean $(\mu) = \Sigma x_i P(x_i)$

Standard deviation $(\sigma) = \sqrt{\left(\Sigma x_i^2 P(x_i)\right) - \mu^2}$

$\Rightarrow\ \mu = \frac{1}{3}\alpha + K - \frac{3}{4}$

$$\sigma = \sqrt{\left(\frac{1}{3}\alpha^2 + K + 0 + \frac{9}{4}\right) - \left(\frac{1}{3}\alpha + K - \frac{3}{4}\right)^2}$$

$\because\ \Sigma P_i = 1 \Rightarrow \frac{1}{3} + K + \frac{1}{6} + \frac{1}{4} = 1$

$\Rightarrow\ K = \frac{1}{4} \Rightarrow \mu = \frac{1}{3}\alpha - \frac{1}{2}$

$\because\ \sigma - \mu = 2$

$\sigma^2 = (\mu + 2)^2$

$$\frac{1}{3}\alpha^2+\frac{5}{2}-\mu^2=(\mu+2)^2$$

$$\frac{1}{3}\alpha^2+\frac{5}{2}=\left(\frac{1}{3}\alpha-\frac{1}{2}\right)^2+\left(\frac{1}{3}\alpha+\frac{3}{2}\right)^2$$

$\Rightarrow\ \alpha = 0, 6$

If $\alpha = 0,\ K=\frac{1}{4}$	If $\alpha = 6,\ K=\frac{1}{4}$
$\mu=-\frac{1}{2},\ \sigma=\frac{3}{2}$	$\mu=\frac{3}{2},\ \sigma=\frac{7}{2}$
$\sigma+\mu=1$	$\sigma+\mu=5$

Both (1) and (5) are correct but according to NTA (5) is correct

24. If $f(t)=\int_0^{\pi}\frac{2xdx}{1-\cos^2 t\sin^2 x}$, $0 < t < \pi$, then the value of $\int_0^{\pi}\frac{\pi^2 dt}{f(t)}$ equals ________.

Answer (1)

Sol. $f(t)=\int_0^{\pi}\frac{2xdx}{1-\cos^2 t\sin^2 x}$

$x\to\pi-x$

$$f(t)=\int_0^{\pi}\frac{2(\pi-x)dx}{1-\cos^2 t\sin^2 x}=2\pi\int_0^{\pi}\frac{dx}{1-\cos^2 t\sin^2 x}-f(t)$$

$$\Rightarrow\ f(t)=\pi\int_0^{\pi}\frac{dx}{1-\cos^2 t\sin^2 x}$$

$$=2\pi\int_0^{\frac{\pi}{2}}\frac{dx}{1-\cos^2 t\sin^2 x}$$

$$f(t)=2\pi\int_0^{\frac{\pi}{2}}\frac{\sec^2 xdx}{\sec^2 x-\cos^2 t\tan^2 x}$$

$$I_1=\int\frac{\sec^2 xdx}{\sec^2 x-\cos^2 t\tan^2 x}$$

Put $\cos t\tan x=\lambda\Rightarrow\cos t\sec^2 xdx=d\lambda$

$$I_1=\int\frac{d\lambda}{\cos t\cdot(1+\lambda^2\sec^2 t-\lambda^2)}=\int\frac{d\lambda}{\cos t(1+\lambda^2\tan^2 t)}$$

$$=\frac{1}{\cos t\cdot\tan^2 t}\cdot\int\frac{d\lambda}{\lambda^2+\cos^2 t}=\frac{1}{\cos t\tan^2 t}\times\frac{1}{\cos t}\tan^{-1}(\lambda\tan t)$$

$$=\frac{1}{\sin t}\tan^{-1}(\sin t\tan x)$$

$$\Rightarrow\ f(t)=\frac{2\pi}{\sin t}\tan^{-1}(\sin t\tan x)\Big]_0^{\frac{\pi}{2}}=\frac{\pi^2}{\sin t}$$

$$\Rightarrow\ \int_0^{\frac{\pi}{2}}\frac{\pi^2 dt}{f(t)}=\int_0^{\frac{\pi}{2}}\sin tdt=1$$

25. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx}+\frac{2x}{(1+x^2)^2}y=xe^{\frac{1}{(1+x^2)}}$; $y(0) = 0$. Then the area enclosed by the curve $f(x)=y(x)e^{-\frac{1}{(1+x^2)}}$ and the line $y - x = 4$ is ________.

Answer (18)

Sol. $\frac{dy}{dx}+\frac{2x}{(1+x^2)^2}y=xe^{\frac{1}{1+x^2}}$; $y(0) = 0$

I.F. of linear differential equation,

$$\text{I.F.}=e^{\int\frac{2x}{(1+x^2)^2}dx}=e^{\left(\frac{-1}{1+x^2}\right)}$$

$$\Rightarrow y\left(e^{\left(\frac{-1}{1+x^2}\right)}\right)=\int x\cdot e^{\frac{1}{1+x^2}}\cdot e^{\left(\frac{-1}{1+x^2}\right)}dx$$

$$=\frac{x^2}{2}+c$$

$$\Rightarrow\ y(0)=0\ \Rightarrow 0(e^{-1})=c\Rightarrow c=0$$

$$\Rightarrow\ y=\frac{e^{\frac{1}{1+x^2}}\cdot x^2}{2}$$

Area between curve $ye^{\left(\frac{-1}{1+x^2}\right)} = \frac{x^2}{2}$ and $y - x = 4$

$\Rightarrow\ 2(x+4) = x^2 \Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow\ (x-4)(x+2) = 0$

$$\int_{-2}^{4}\left[(x+4) - \frac{x^2}{2}\right]dx = \left.\frac{x^2}{2} + 4x - \frac{x^3}{6}\right|_{-2}^{4}$$

$$= \left(8 + 16 - \frac{64}{6}\right) - \left(2 - 8 + \frac{8}{6}\right)$$

$= 30 - 12 = 18$

26. Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x - 9)$ at the point P. The distance of the point P from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is __________.

Answer (10)

Sol. Line perpendicular to $2x - y = 10$ have slope $= \frac{-1}{2}$

$\Rightarrow$ Line tangent to parabola $y^2 = 4(x - 9)$ with slope m is

$$y = m(x-9) + \frac{1}{m},\quad m = \frac{-1}{2}$$

$$\Rightarrow\ y = \frac{-(x-9)}{2} - 2 \ \Rightarrow\ 2y = -x + 9 - 4$$

$\Rightarrow 2y + x = 5$

Solving the tangent and parabola we get point P

$$\left(\frac{5-x}{2}\right)^2 = 4(x-9) \Rightarrow x^2 - 10x + 25 = 16x - 144$$

$\Rightarrow x^2 - 26x + 169 = 0 \Rightarrow (x - 13)^2 = 0$

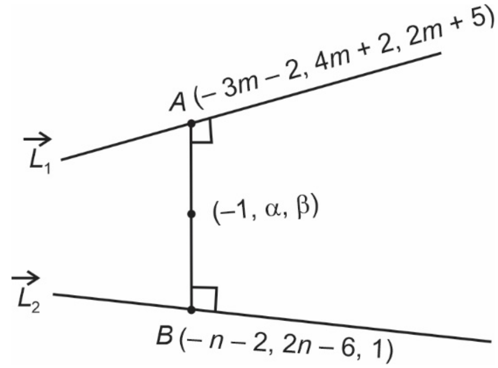
$\Rightarrow P \equiv (13, -4)$

Distance of P from the centre of circle $(7, 4)$ is $\sqrt{(13-7)^2 + (-4-4)^2} = \sqrt{36+64} = 10$ units.

27. Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$. Then $(\alpha - \beta)^2$ is equal to ____.

Answer (25)

Sol.



$\overrightarrow{AB} \perp \vec{L}_1$ and $\overrightarrow{AB} \perp \vec{L}_2$

$\overrightarrow{AB} = (-3m - 2 + n + 2, 4m + 2 - 2n + 6, 2m + 5 - 1)$

$= (-3m + n, 4m - 2n + 8, 2m + 4)$

$\overrightarrow{AB} \perp \vec{L}_1$

$\Rightarrow -3(-3m + n) + 4(4m - 2n + 8) + 2(2m + 4) = 0$

$(9m + 16m + 4m) + (-3n - 8n) + 32 + 8 = 0$

$\Rightarrow\ 29m - 11n + 40 = 0 \qquad \ldots(1)$

$\overrightarrow{AB} \perp \vec{L}_2$

$\Rightarrow -1(-3m + n) + 2(4m - 2n + 8) + 0(2m + 4) = 0$

$\Rightarrow\ 3m - n + 8m - 4n + 16 = 0$

$\Rightarrow\ 11m - 5n + 16 = 0 \ \Rightarrow m = -1, n = 1$

$\Rightarrow\ A \equiv (1, -2, 3),\quad B \equiv (-3, -4, 1)$

AB line $\Rightarrow \frac{x-1}{2} = \frac{y+2}{1} = \frac{z-3}{1}$

$\Rightarrow\ \alpha = -2,\quad \beta = 3$

$\Rightarrow\ (\alpha - \beta)^2 = 25$

28. If $1 + \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}} + \frac{5-2\sqrt{6}}{18} + \frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}} + \frac{49-20\sqrt{6}}{180} + \ldots\ldots$ upto $\infty = 2 + \left(\sqrt{\frac{b}{a}} + 1\right)\log_e\left(\frac{a}{b}\right)$, where a and b are integers with $gcd(a, b) = 1$ then $11a + 18b$ is equal to ______.

Answer (76)

Sol. $S = 1 + \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}} + \frac{5-2\sqrt{6}}{18} + \frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}} + \ldots\infty$

$$= 1 + \frac{\left(1-\sqrt{2}/\sqrt{3}\right)}{2} + \frac{\left(1-\sqrt{2}/\sqrt{3}\right)^2}{6} + \frac{\left(1-\sqrt{2}/\sqrt{3}\right)^3}{12} + \ldots\infty$$

let $1-\frac{\sqrt{2}}{\sqrt{3}}=a$

$$S=1+\frac{a}{2}+\frac{a^2}{6}+\frac{a^3}{12}+...$$

$$=1+\left(1-\frac{1}{2}\right)a+\left(\frac{1}{2}-\frac{1}{3}\right)a^2+\left(\frac{1}{3}-\frac{1}{4}\right)a^3+...$$

$$=1+\left(a+\frac{a^2}{2}+\frac{a^3}{3}...\infty\right)+\frac{1}{a}\left(\frac{-a^2}{2}-\frac{a^3}{3}-\frac{a^4}{4}...\infty\right)$$

$$=-\ln(1-a)+\frac{1}{a}\left(-a-\frac{a^2}{2}-\frac{a^3}{3}...\infty\right)+2$$

$$=-\ln(1-a)+\frac{1}{a}\ln(1-a)+2$$

$$=2+\left(\frac{1}{a}-1\right)\ln(1-a)$$

$$=2+\left(\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}-1\right)\ln\left(1-1+\sqrt{\frac{2}{3}}\right)$$

$$=2+\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}\ln\sqrt{\frac{2}{3}}$$

$$=2+\left(\frac{\sqrt{6}+2}{1}\cdot\frac{1}{2}\ln\frac{2}{3}\right)$$

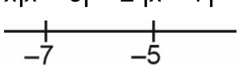
$$=2+\left(\sqrt{\frac{3}{2}}+1\right)\ln\frac{2}{3}$$

$\therefore\quad 11a+18b=76$

29. The number of real solutions of the equation $x|x+5|+2|x+7|-2=0$ is ___________.

Answer (03.00)

Sol. $x|x+5|+2|x+7|-2=0$



(i) $d\ x\geq -5 \Rightarrow x(x+5)+2(x+7)-2=0$

$x^2+7x+12=0 \Rightarrow x=-3,-4$

(ii) $x\in(-7,-5)$

$x(-x-5)+2(x+7)-2=0$

$-x^2-3x+12=0$

$\Rightarrow x^2+3x-12=0$

$\Rightarrow x=\frac{-3-\sqrt{57}}{2}$ satisfy

(iii) $x\leq -7$

$\Rightarrow x(-x-5)+2(-x-7)-2=0$

$-x^2-7x-16=0 \Rightarrow x^2+7x+16=0$

No solution

30. The number of solutions of $\sin^2 x+(2+2x-x^2)\sin x-3(x-1)^2=0$, where $-\pi\leq x\leq\pi$, is ________.

Answer (2)

Sol. $\sin^2 x+(3-(x-1)^2)\sin x-3(x-1)^2=0$

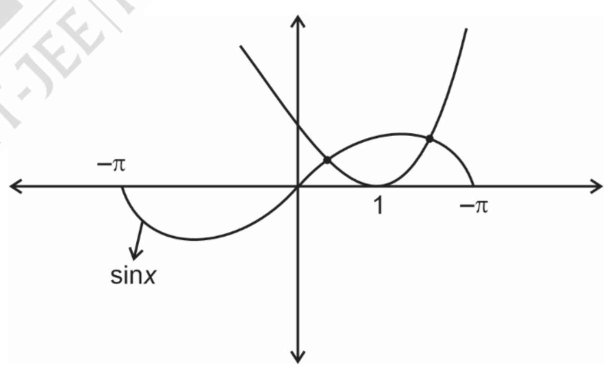
$\sin^2 x+3\sin x-(x-1)^2\sin x-3(x-1)^2=0$

$\sin x(\sin x+3)-(x-1)^2)[\sin x+3]=0$

$\underbrace{(\sin x+3)}_{\sin x\neq -3}\ \underbrace{(\sin x-(x-1)^2)}_{\sin x=(x-1)^2}=0$

↓

Not Possible



There are two intersections between this graph.

So, Number of solution will be 2.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. A galvanometer of resistance 100 Ω when connected in series with 400 Ω measures a voltage of upto 10 V. The value of resistance required to convert the galvanometer into ammeter to read upto 10 A is $x \times 10^{-2}$ Ω. The value of x is

(1) 800 (2) 2

(3) 20 (4) 200

Answer (3)

Sol. $I_g(100 + 400) = 10$

$\Rightarrow \quad I_g = 0.02$ A

Now,

$$\text{Shunt } (S) = \frac{100 I_g}{(10 - 0.02)}$$

≈ 0.2 A

$\therefore \quad x = 20$

32. If n is the number density and d is the diameter of the molecule, then the average distance covered by a molecule between two successive collisions (i.e. mean free path) is represented by

(1) $\frac{1}{\sqrt{2n\pi d^2}}$ (2) $\frac{1}{\sqrt{2}n\pi d^2}$

(3) $\sqrt{2}n\pi d^2$ (4) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$

Answer (2)

Sol. $\lambda_{\text{mean}} = \frac{1}{\sqrt{2}n\pi d^2}$

33. A man carrying a monkey on his shoulder does cycling smoothly on a circular track of radius 9 m and completes 120 resolutions in 3 minutes. The magnitude of centripetal acceleration of monkey is (in m/s²)

(1) $16\pi^2$ ms^{-2} (2) $4\pi^2$ ms^{-2}

(3) Zero (4) $57600\pi^2$ ms^{-2}

Answer (1)

Sol. $a_c = \omega^2 r$

$$= \left(\frac{120}{3 \times 60} \times 2\pi\right)^2 \times 9$$

$= 16\pi^2$ ms^{-2}

34. The vehicles carrying inflammable fluids usually have metallic chains touching the ground

(1) To alert other vehicles

(2) To conduct excess charge due to air friction to ground and prevent sparking

(3) To protect tyres from catching dirt from ground

(4) It is a custom

Answer (2)

Sol. Usually metallic chains are hung to allow the charge generated on the vehicles pass to the ground.

35. Match **List-I** with **List-II** :

List-I		List-II	
EM-Wave		**Wavelength Range**	
(A)	Infra-red	(I)	<10^{-3} nm
(B)	Ultraviolet	(II)	400 nm to 1 nm
(C)	X-rays	(III)	1 mm to 700 nm
(D)	Gamma rays	(IV)	1 nm to 10^{-3} nm

Choose the **correct** answer from the options given below

(1) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(3) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(4) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Answer (1)

Sol. Theoretical

Infra-red $\rightarrow$ 1 mm to 700 nm

Ultraviolet $\rightarrow$ 400 nm to 1 nm

X-rays $\rightarrow$ 1 nm to 10^{-3} nm

γ-rays $\rightarrow$ Less than 10^{-3} nm

36. What is the dimensional formula of ab^{-1} in the equation $\left(P+\frac{a}{V^2}\right)(V-b)=RT$, where letters have their usual meaning.

(1) $[M^6L^7T^4]$ (2) $[ML^2T^{-2}]$

(3) $[M^0L^3T^{-2}]$ (4) $[M^{-1}L^5T^3]$

Answer (2)

Sol. $[a] = [PV^2]$

$[b] = [V]$

$\therefore \left[\frac{a}{b}\right] = [PV]$

$= [\text{Work}]$

37. Which of the following statement is **not** true about stopping potential (V_0)?

(1) If depends on the nature of emitter material

(2) It depends upon frequency of the incident light

(3) It increases with increase in intensity of the incident light

(4) It is 1/e times the maximum kinetic energy of electrons emitted

Answer (3)

Sol. Stopping potential is independent of intensity of light. It depends on frequency of light.

38. During an adiabatic process, if the pressure of a gas is found to be proportional to the cube of its absolute temperature, then the ratio of $\frac{C_P}{C_V}$ for the gas is

(1) $\frac{5}{3}$ (2) $\frac{3}{2}$

(3) $\frac{7}{5}$ (4) $\frac{9}{7}$

Answer (2)

Sol. $P \propto T^3$

$\Rightarrow PT^{-3} = \text{const.}$

and, $PT^{\frac{\gamma}{1-\gamma}} = \text{const.}$

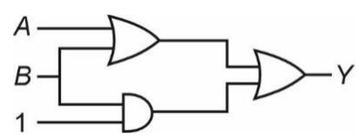
$\frac{\gamma}{1-\gamma} = -3$

$\Rightarrow \gamma = -3 + 3\gamma$

$\Rightarrow 3 = 2\gamma$

$\Rightarrow \boxed{\gamma = \frac{3}{2}}$

39. The output (Y) of logic circuit given below is 0 only when



(1) $A = 0, B = 0$

(2) $A = 0, B = 1$

(3) $A = 1, B = 0$

(4) $A = 1, B = 1$

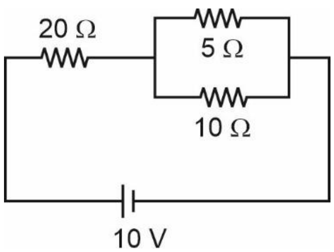
Answer (1)

Sol. $Y = (A + B) + (B \cdot 1)$

$= A + B + B$

$= A + B$

40. The ratio of heat dissipated per second through the resistance 5 Ω and 10 Ω in the circuit given below is



(1) 4 : 1 (2) 1 : 2

(3) 2 : 1 (4) 1 : 1

Answer (3)

Sol. In parallel

$$P \propto \frac{1}{R}$$

$$\therefore \quad \frac{P_{5\Omega}}{P_{10\Omega}} = \frac{10}{5} = 2$$

41. A heavy box of mass 50 kg is moving on a horizontal surface. If co-efficient of kinetic friction between the box and horizontal surface is 0.3 then force of kinetic friction is

(1) 1.47 N

(2) 14.7 N

(3) 147 N

(4) 1470 N

Answer (3)

Sol. $f_k = \mu_k mg$

$= 0.3 \times 50 \times 9.8$

$= 147$ N

42. A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to

(1) $t^{2/3}$ (2) $t^{3/2}$

(3) t^2 (4) t

Answer (2)

Sol. P = constant

$$\Rightarrow \frac{Fds}{dt} = C$$

$$m\frac{vdv}{ds} \cdot \frac{ds}{dt} = C$$

$$\Rightarrow v \propto \sqrt{t}$$

$$\Rightarrow \frac{ds}{dt} \propto \sqrt{t}$$

$$\Rightarrow s \propto t^{3/2}$$

43. A satellite revolving around a planet in stationary orbit has time period 6 hours. The mass of planet is one-fourth the mass of earth. The radius orbit of planet is

(Given = Radius of geo-stationary orbit for earth is 4.2×10^4 km)

(1) 1.05×10^4 km

(2) 1.4×10^4 km

(3) 1.68×10^5 km

(4) 8.4×10^4 km

Answer (1)

Sol. $$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} \sqrt{\frac{m_2}{m_1}}$$

$$\Rightarrow \frac{24}{6} = \left(\frac{4.2 \times 10^4}{r_2}\right)^{3/2} \sqrt{\frac{m/4}{m}}$$

$$\Rightarrow r_2 = 1.05 \times 10^4 \text{ km}$$

44. The electrostatic force $(\vec{F}_1)$ and magnetic force $(\vec{F}_2)$ acting on a charge q moving with velocity v can be written

(1) $\vec{F}_1 = q\vec{E},\ \vec{F}_2 = q(\vec{V} \times \vec{B})$

(2) $\vec{F}_1 = q\vec{B},\ \vec{F}_2 = q(\vec{B} \times \vec{V})$

(3) $\vec{F}_1 = q\vec{V} \cdot \vec{E},\ \vec{F}_2 = q(\vec{B} \cdot \vec{V})$

(4) $\vec{F}_1 = q\vec{E},\ \vec{F}_2 = q(\vec{B} \times \vec{V})$

Answer (1)

Sol. $\vec{F}_1 = \vec{F}_E = q\vec{E}$

$\vec{F}_2 = \vec{F}_B = q(\vec{v} \times \vec{B})$

45. A particle moves in x-y plane under the influence of a force $\vec{F}$ such that its linear momentum is $\vec{p}(t) = \hat{i}\cos(kt) - \hat{j}\sin(kt)$. If k is constant, then angle between $\vec{F}$ and $\vec{p}$ will be

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{4}$

(3) $\frac{\pi}{6}$

(4) $\frac{\pi}{2}$

Answer (4)

Sol. $\vec{p} = \cos(kt)\hat{i} - \sin(kt)\hat{j}$

$\vec{F} = \frac{d\vec{p}}{dt} = -k\sin(kt)\hat{i} - k\cos(kt)\hat{j}$

$\vec{F} \cdot \vec{p} = -k\sin(kt)\cos(kt) + k\sin(kt)\cos(kt)$

$= 0$

$\therefore\ \theta = 90° = \frac{\pi}{2}$ rad.

46. A vernier callipers has 20 divisions on the vernier scale, which coincides with 19th division on the main scale. The least count of the instrument is 0.1 mm. One main scale division is equal to _______ mm.

(1) 2

(2) 1

(3) 5

(4) 0.5

Answer (1)

Sol. 20 VSD = 19 MSD

1 VSD = $\frac{19}{20}$ MSD

$0.1\text{ mm} = \left(1 - \frac{19}{20}\right)$ MSD

$\therefore\ \text{MSD} = \frac{0.1 \times 20}{1}$

= 2 mm

47. A series LCR circuit is subjected to an ac signal of 200 V, 50 Hz. If the voltage across the inductor (L = 10 mH) is 31.4 V, then the current in this circuit is ________.

(1) 63 A

(2) 10 A

(3) 10 mA

(4) 68 A

Answer (2)

Sol. $V_L = I(\omega L) = 31.4$

$\Rightarrow\ I = \frac{31.4}{2 \times 3.14 \times 50 \times 10 \times 10^{-3}}$

$= 10A$

48. Match **List-I** with **List-II** :

	List-I		**List-II**
(A)	A force that restores an elastic body of unit area of its original state	(I)	Bulk modulus
(B)	Two equal and opposite forces parallel to opposite faces	(II)	Young's modulus
(C)	Forces perpendicular everywhere to the surface per unit area same everywhere	(III)	Stress
(D)	Two equal and opposite forces perpendicular to opposite faces	(IV)	Shear modulus

Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(2) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Answer (1)

Sol. Stress = $\dfrac{\text{restoring force}}{\text{area}}$

$\Rightarrow$ Two parallel equal and opposite forces cause shear.

49. Given below are two statements :

Statement I : When the white light passed through a prism, the red light bends lesser than yellow and violet.

Statement II : The refractive indices are different for different wavelengths in dispersive medium.

In the light of the above statements, chose the **correct** answer from the options given below :

(1) **Statement I** is false but **Statement II** is true

(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is true but **Statement II** is false

Answer (2)

Sol. $\lambda_{Red} > \lambda_{Blue}$

Also μ is different for different λ.

$\therefore$ Both statements are true.

50. The angular momentum of an electron in a hydrogen atom is proportional to :

(where r is the radius of orbit of electron)

(1) $\sqrt{r}$

(2) r

(3) $\dfrac{1}{\sqrt{r}}$

(4) $\dfrac{1}{r}$

Answer (1)

Sol. $L = mvr \propto n$

$\therefore \quad mvr \propto \sqrt{r}$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

51. A hollow sphere is rolling on a plane surface about its axis of symmetry. The ratio of rotational kinetic energy to its total kinetic energy is $\dfrac{x}{5}$. The value of x is ________.

Answer (2)

Sol. $K.E_R = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega^2 \quad \because V = R\omega$

$= \frac{1}{3}MV^2$

$K.E_{T_r} = \frac{1}{2}MV^2$

$\therefore \quad \frac{K.E_R}{K.E_R + K.E_{Tr}} = \frac{\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}} = \frac{2}{5}$

$\Rightarrow \quad x = 2$

52. The shortest wavelength of the spectral lines in the Lyman series of hydrogen spectrum is 915 Å. The longest wavelength of spectral lines in the Balmer series will be ________ Å.

Answer (6588)

Sol. $\frac{1}{915} = R_H\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$ (For Lyman)

$\Rightarrow \quad \frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$ (For Balmer)

$\Rightarrow \quad \lambda = 6588$ Å

53. The maximum height reached by a projectile is 64 m. If the initial velocity is halved, the new maximum height of the projectile is ________ m.

Answer (16)

Sol. $\frac{u^2 \sin^2\theta}{2g} = 64$

Now $u' = \frac{u}{2}$

$H' = \frac{1}{4} \times \frac{u^2 \sin\theta}{2g} = \frac{64}{4} = 16$ m

54. A sonometer wire of resonating length 90 cm has a fundamental frequency of 400 Hz when kept under some tension. The resonating length of the wire with fundamental frequency of 600 Hz under same tension __________ cm.

Answer (60)

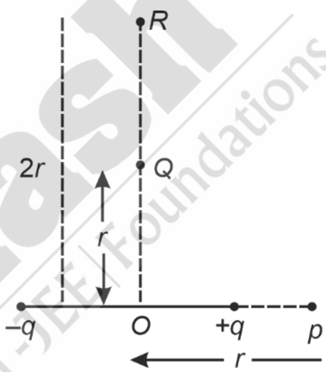
Sol. $f \propto \frac{1}{l}$

$\therefore \quad \frac{F_1}{F_2} = \frac{l_2}{l_1}$

$\Rightarrow \quad \frac{400}{600} = \frac{l_2}{90}$

$\Rightarrow \quad l_2 = 60$ cm

55. The electric field at point p due to an electric dipole is E. The electric field at point R on equatorial line will be $\frac{E}{x}$. The value of x :



Answer (16)

Sol. $E = \frac{2kp}{r^3}$

$E_R = \frac{kp}{(2r)^3} = \frac{1}{8}\left(\frac{E}{2}\right)$

$= \frac{E}{16}$

$\therefore \quad x = 16$

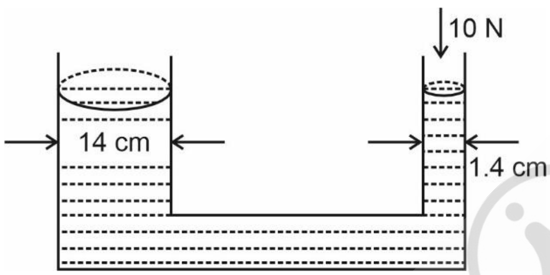
56. In a single slit experiment, a parallel beam of green light of wavelength 550 nm passes through a slit of width 0.20 mm. The transmitted light is collected on a screen 100 cm away. The distance of first order minima from the central maximum will be $x \times 10^{-5}$ m. The value of x is

Answer (275)

Sol. $y = \frac{n\lambda D}{a}$

$= \frac{1 \times (550 \times 10^{-9})(1)}{(0.2 \times 10^{-3})}$

$= 275 \times 10^{-5}$ m

57. 

A hydraulic press containing water has two arms with diameters as mentioned in the figure. A force of 10 N is applied on the surface of water in the thinner arm. The force required to be applied on the surface of water in the thicker arm to maintain equilibrium of water is _______ N.

Answer (1000)

Sol. $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ (By Pascal's law)

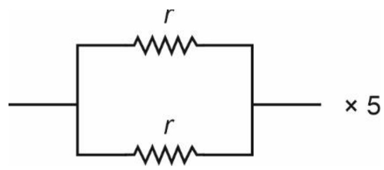
$\Rightarrow F_1 = 10\left(\frac{14^2}{1.4^2}\right)$

$= 10 \times 100$

$= 1000$ N

58. A wire of resistance 20 Ω is divided into 10 equal parts, resulting pairs. A combination of two parts are connected in parallel and so on. Now resulting pairs of parallel combination are connected in series. The equivalent resistance of final combination is _______ Ω.

Answer (5)

Sol. 

$\times 5 \qquad r = \frac{20}{10} = 2\Omega$

$\therefore \; R_{eq} = \left(\frac{r}{2}\right) \times 5 = 5\ \Omega$

59. The current in an inductor is given by $I = (3t + 8)$ where t is in second. The magnitude of induced emf produced in the inductor is 12 mV. The self-inductance of the inductor _______ mH.

Answer (4)

Sol. $\varepsilon = \left|L\frac{di}{dt}\right|, \frac{di}{dt} = 3$

$\Rightarrow 12 \times 10^{-3} = L\,(3)$

$\Rightarrow L = 4$ mH

60. A solenoid of length 0.5 m has a radius of 1 cm and is made up of 'm' number of turns. It carries a current of 5 A. If the magnitude of the magnetic field inside the solenoid is 6.28×10^{-3} T then the value of m is _______.

Answer (500)

Sol. $B = \mu_0 n i$

$6.28 \times 10^{-3} = (4\pi \times 10^{-7})\left(\frac{m}{0.5}\right)5$

$m = 500$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. The number of complexes from the following with no electrons in the t_2 orbital is _____.

$TiCl_4$, $[MnO_4]^-$, $[FeO_4]^{2-}$, $[FeCl_4]^-$, $[CoCl_4]^{2-}$

(1) 4 (2) 2

(3) 3 (4) 1

Answer (3)

Sol. $TiCl_4 \rightarrow Ti^{+4} \rightarrow [Ar]4s^0 3d^0 \rightarrow e^0 t_2^0$

$[MnO_4]^{\ominus} \rightarrow Mn^{+7} \rightarrow [Ar]\ 4s^0 3d^0 \rightarrow e^0 t_2^0$

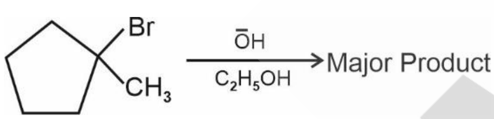
$[FeO_4]^{2-} \rightarrow Fe^{+6} \rightarrow [Ar]\ 4s^0 3d^0 \rightarrow e^0 t_2^0$

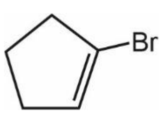
$[FeCl_4]^{\ominus} \rightarrow Fe^{3+} \rightarrow [Ar]\ 4s^0 3d^5 \rightarrow e^2 t_2^3$

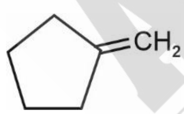
$[CoCl_4]^{2-} \rightarrow Co^{+2} \rightarrow [Ar]\ 4s^0 3d^7 \rightarrow e^4 t_2^3$

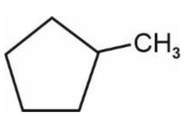
Three complexes have no electrons in t_2 orbital

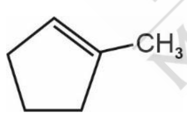
62. Identify the major product in the following reaction.



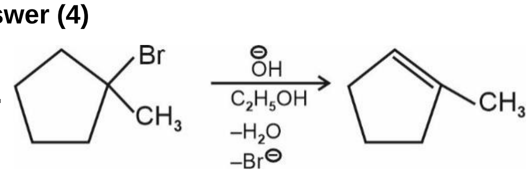
(1) 

(2) 

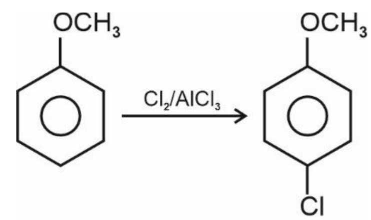
(3) 

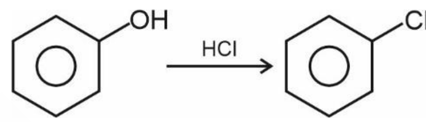
(4) 

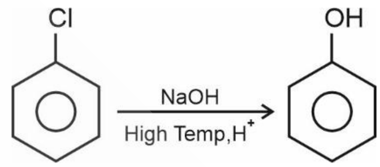
Answer (4)

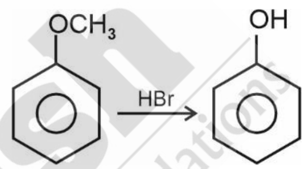
Sol. 

63. Which one of the following reactions is NOT possible?

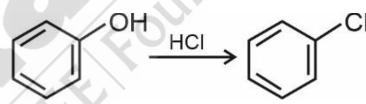
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

The above reaction is not possible as due to resonance C–O bond has partial double bond character. So, bond strength will increase and substitution will be less likely.

64. Coagulation of egg, on heating is because of :

(1) Biological property of protein remains unchanged

(2) The secondary structure of protein remains unchanged

(3) Breaking of the peptide linkage in the primary structure of protein occurs

(4) Denaturation of protein occurs

Answer (4)

Sol. On heating, denaturation of protein occurs in which only primary structure remains intact. This results in coagulation of egg.

65. The **correct** statements from the following are :

(A) The decreasing order of atomic radii of group 13 elements is Tl > In > Ga > Al > B.

(B) Down the group 13 electronegativity decreases from top to bottom.

(C) Al dissolves in dil. HCl and liberates H_2 but conc. HNO_3 renders Al passive by forming a protective oxide layer on the surface.

(D) All elements of group 13 exhibits highly stable +1 oxidation state.

(E) Hybridisation of Al in $[Al(H_2O)_6]^{3+}$ ion is sp^3d^2.

(1) (C) and (E) only

(2) (A) and (C) only

(3) (A), (C) and (E) only

(4) (A), (B), (C) and (E) only

Answer (1)

Sol. (A) Group-13 size : B < Ga < Al < In ≃ Tl

(B) In Group-13, Al has exceptional lower EN

EN : B > Tl > In > Ga > Al

(C) $2Al + 6HCl \rightarrow 2AlCl_3 + 3H_2$

$Al + HNO_3 \rightarrow$ No reaction

(Due to protective oxide layer)

(D) +1 oxidation state is observed mainly for heavier Group-13 element like Tl.

B and Al do not show +1 oxidation state

(E) $[Al(H_2O)_6]^{3+} \rightarrow$ Hybridisation : sp^3d^2

66. While preparing crystals of Mohr's salt, dil H_2SO_4 is added to a mixture of ferrous sulphate and ammonium sulphate, before dissolving the mixture in water, dil H_2SO_4 is added here to :

(1) Prevent the hydrolysis of ferrous sulphate

(2) Make the medium strongly acidic

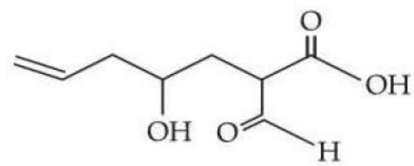
(3) Prevent the hydrolysis of ammonium sulphate

(4) Increase the rate of formation of crystals

Answer (1)

Sol. While preparing crystal of Mohr's salt, dil. H_2SO_4 is added to prevent hydrolysis of ferrous sulphate.

67. The correct nomenclature for the following compound is



(1) 2-carboxy-4-hydroxyhept-7-enal

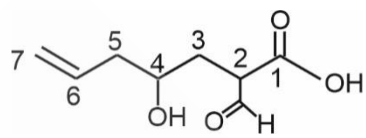
(2) 2-carboxy-4-hydroxyhept-6-enal

(3) 2-formyl-4-hydroxyhept-7-enoic acid

(4) 2-formyl-4-hydroxyhept-6-enoic acid

Answer (4)

Sol.



2-formyl-4-hydroxyhept-6-enoic acid

68. The metal atom present in the complex MABXL (where A, B, X and L are unidentate ligands and M is metal) involves sp^3 hybridization. The number of geometrical isomers exhibited by the complex is

(1) 2

(2) 4

(3) 0

(4) 3

Answer (3)

Sol. $[MABXL] \rightarrow sp^3 \rightarrow$ Tetrahedral

Tetrahedral complexes do not show G. I. as they have same relative position.

69. Match List-I with List-II.

	List-I		List-II
(A)	ICl	(I)	T-shape
(B)	ICl_3	(II)	Square pyramidal
(C)	ClF_5	(III)	Pentagonal bipyramidal
(D)	IF_7	(IV)	Linear

Choose the **correct** answer from the options given below:

(1) (A)-(IV), (B)-(III), (C)-(II), D-(I)

(2) (A)-(IV), (B)-(I), (C)-(II), D-(III)

(3) (A)-(I), (B)-(III), (C)-(II), D-(IV)

(4) (A)-(I), (B)-(IV), (C)-(III), D-(II)

Answer (2)

Sol. ICl $\rightarrow sp^3 \rightarrow$ 1 bp + 3 lp $\rightarrow$ Linear

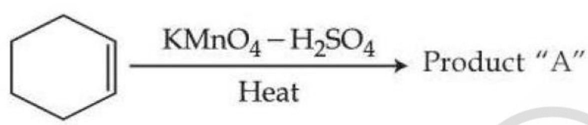
$ICl_3 \rightarrow sp^3d \rightarrow$ 3 bp + 2 lp $\rightarrow$ T-shape

$ClF_5 \rightarrow sp^3d^2 \rightarrow$ 5 bp + 1 lp $\rightarrow$ Square Pyramidal

$IF_7 \rightarrow sp^3d^3 \rightarrow$ 7 bp only $\rightarrow$ Pentagonal bipyramidal

(A)-(IV), (B)-(I), (C)-(II), D-(III)

70. Consider the given chemical reaction:



Product "A" is

(1) Oxalic acid

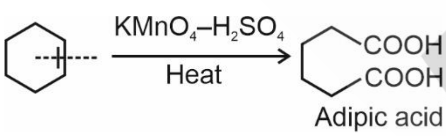
(2) Picric acid

(3) Adipic acid

(4) Acetic acid

Answer (3)

Sol.



71. The quantity of silver deposited when one coulomb charge is passed through $AgNO_3$ solution:

(1) 1 chemical equivalent of silver

(2) 1 g of silver

(3) 1 electrochemical equivalent of silver

(4) 0.1 g atom of silver

Answer (3)

Sol. $Ag^{+}(aq) + e^{-} \rightarrow Ag(s)$

w = Zq

q = 1C

w = Z = 1 electrochemical equivalent

72. $CH_3CH_2-OH \xrightarrow[\text{(ii) } KMnO_4 \text{ (iii) NaOH, CaO, } \Delta]{\text{(i) Jone's Reagent}} P$

Consider the above reaction sequence and identify the major product P.

(1) Methanal

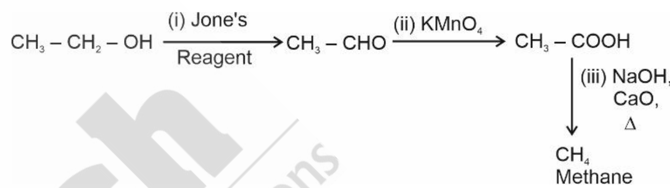
(2) Methoxymethane

(3) Methanoic acid

(4) Methane

Answer (4)

Sol.



73. Given below are two statements:

Statement I: On passing $HCl_{(g)}$ through a saturated solution of $BaCl_2$, at room temperature white turbidity appears.

Statement II: When HCl gas is passed through a saturated solution of NaCl, sodium chloride is precipitated due to common ion effect.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **statement I** and **Statement II** are incorrect

(2) **Statement I** is incorrect but **Statement II** is correct

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are correct

Answer (4)

Sol. $Cl^{\ominus}$ + saturated solution of $BaCl_2 \rightarrow \underset{(\text{white})}{BaCl_2}$ will precipitate.

$Cl^{\ominus}$ + saturated solution of NaCl $\rightarrow$ NaCl will precipitate.

74. The number of moles of methane required to produce 11 g $CO_2(g)$ after complete combustion is

(Given molar mass of methane in g mol^{-1} : 16)

(1) 0.5 (2) 0.25

(3) 0.75 (4) 0.35

Answer (2)

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

mol of $CO_2 = \frac{11}{44} = \frac{1}{4}$ mol

mol of CH_4 required = $\frac{1}{4} = 0.25$ mol

75. For the electro chemical cell

$M|M^{2+}||X|X^{2-}$

If $E^{\circ}_{(M^{2+}/M)} = 0.46$ V and $E^{\circ}_{(X/X^{2-})} = 0.34$ V

Which of the following is **correct**?

(1) $E_{cell} = -0.80$ V

(2) $M + X \rightarrow M^{2+} + X^{2-}$ is a spontaneous reaction

(3) $E_{cell} = 0.80$ V

(4) $M^{2+} + X^{2-} \rightarrow M + X$ is a spontaneous reaction

Answer (4)

Sol. $E^{\circ}_{cell} = E^{\circ}_{X|X^{2-}} - E^{\circ}_{M^{2+}|M}$

= 0.34 – 0.46

= –0.12 V (Non-spontaneous)

So, reverse reaction will be spontaneous.

$M^{2+} + X^{2-} \rightarrow M + X$

76. Match **List-I** with **List-II**

List-I (Pair of Compounds)		List-I (Isomerism)	
(A)	n-propanol and Isopropanol	(I)	Metamerism
(B)	Methoxypropane and ethoxyethane	(II)	Chain Isomerism
(C)	Propanone and propanal	(III)	Position Isomerism
(D)	Neopentane and Isopentane	(IV)	Functional Isomerism

(1) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

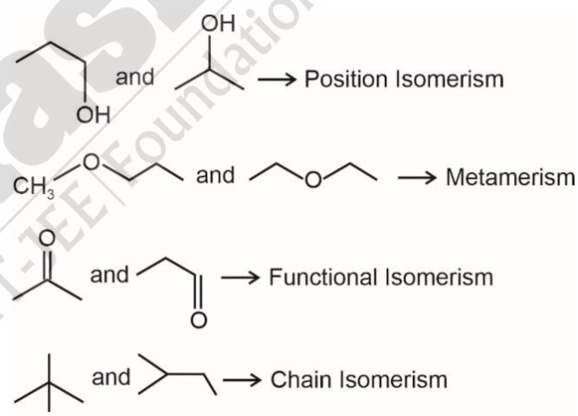
(2) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

(3) (A) – (I), (B) – (III), (C) – (IV), (D) – (II)

(4) (A) – (III), (B) – (I), (C) – (II), (D) – (IV)

Answer (2)

Sol.



77. The number of ions from the following that have the ability of liberate hydrogen from a dilute acid is______.

Ti^{2+}, Cr^{2+} and V^{2+}

(1) 3 (2) 0

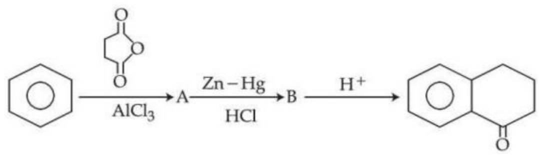
(3) 1 (4) 2

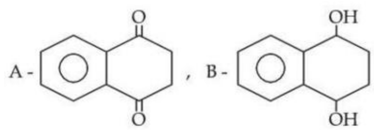
Answer (1)

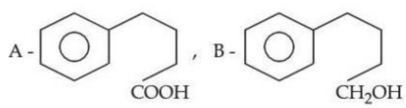
Sol. Ions with negative SRP can liberate H^2 from a dilute acid.

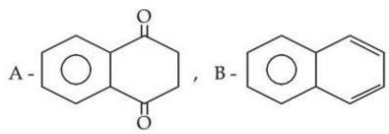
Ti^{2+}, Cr^{2+}, V^{2+} all can liberate H_2 gas from a dilute acid.

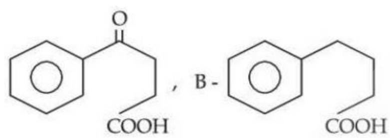
78. Identify A and B in the given chemical reaction sequence:



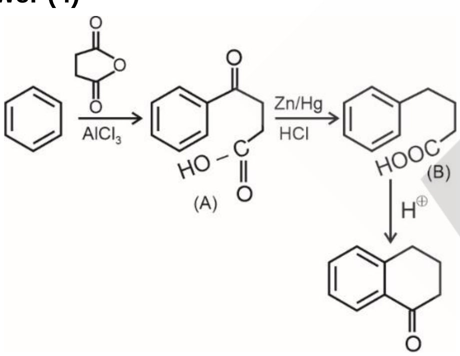
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. 

79. Given below are two statements :

Statement I : The metallic radius of Na is 1.86 Å and the ionic radius of Na^+ is lesser than 1.86 Å.

Statement II : Ions are always smaller in size than the corresponding elements.

In the light of the above statement, choose the **correct** answer from the options given below:

(1) **Statement I** is correct but **Statement II** is false

(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is incorrect but **Statement II** is true

Answer (1)

Sol. Cation are smaller in size than their parent atom due to more Z_{eff} while anion are bigger in size than their parent atom due to more inter electronic repulsion. So, Statement–I is correct & Statement–II is false

80. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : NH_3 and NF_3 molecule have pyramidal shape with a lone pair of electrons on nitrogen atom. The resultant dipole moment of NH_3 is greater than that of NF_3.

Reason (R) : In NH_3 the orbital dipole due to lone pair is in the same direction as the resultant dipole moment of the N – H bonds. F is the most electronegative element.

In the light of the above statement, choose the **correct** answer from the option given below:

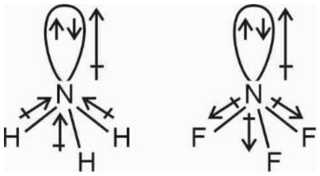
(1) **(A)** is false but **(R)** is true

(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true but **(R)** is NOT the correct explanation of **(A)**

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Answer (4)

Sol. 

Dipole moment of NH_3 is higher than NF_3 as μ of lone-pair is in same direction as the resultant dipole moment of N–H bonds.

So, both (A) & (R) are true , & (R) is the correct explanation of (A)

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

81. Considering acetic acid dissociates in water, its dissociation constant is 6.25×10^{-5}. If 5 mL of acetic acid is dissolved in 1 litre water, the solution will freeze at $-x \times 10^{-2}$ °C, provided pure water freezes at 0 °C.

 x = ________. (Nearest integer)

 Given : $(K_f)_{water} = 1.86$ K kg mol^{-1}.

 density of acetic acid is 1.2 g mol^{-1}.

 molar mass of water = 18 g mol^{-1}.

 molar mass of acetic acid = 60 g mol^{-1}.

 density of water = 1 g cm^{-3}

 Acetic acid dissociates as

$$CH_3COOH \rightleftharpoons CH_3COO^{\ominus} + H^{\oplus}$$

Answer (19)

Sol. $CH_3COOH(aq) \rightleftharpoons CH_3COO^-(aq) + H^+(aq)$

$$\alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{6.25\times10^{-5}}{\frac{1.2\times5}{60}}}$$

$$= \sqrt{625\times10^{-6}} = 25\times10^{-3}$$

$i = 1 + (2 - 1)(0.025)$

$= 1.025$

$\Delta T_f = 1.025 \times 1.86 \times 0.1$

$= 0.19065$

$T_f = -19.065 \times 10^{-2}$

$x \Rightarrow 19$

82. X g of ethanamine was subjected to reaction with $NaNO_2$/HCl followed by hydrolysis to liberate N_2 and HCl. The HCl generated was completely neutralised by 0.2 moles of NaOH. X is _____ g,

Answer (9)

Sol. $HCl + NaOH \rightarrow NaCl + H_2O$

 mol of HCl = 0.2 mol

$$CH_3CH_2NH_2 \xrightarrow[+HCl]{NaNO_2} CH_3CH_2N_2^+Cl^- \xrightarrow{H_2O} CH_3CH_2OH + N_2 + HCl$$

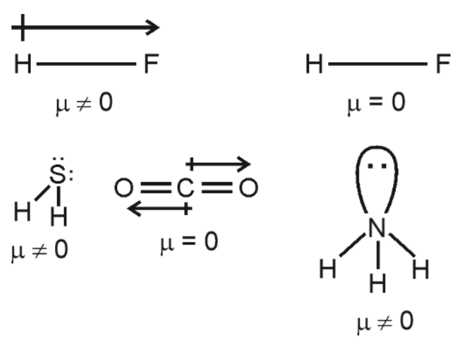
 0.2 mol HCl would be generated by 0.2 mol $C_2H_5NH_2$

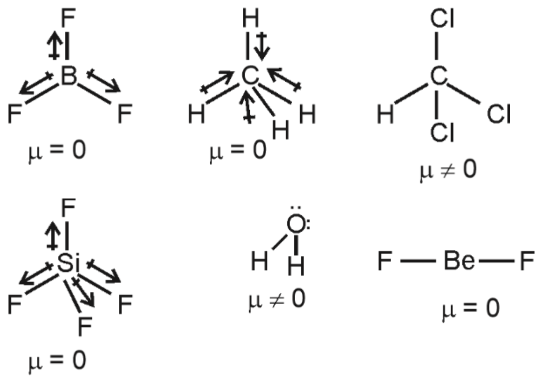
 $x = 0.2 \times 45 = 9$ g

83. Number of compounds from the following with zero dipole moment is ________.

 HF, H_2, H_2S, CO_2, NH_3, BF_3, CH_4, $CHCl_3$, SiF_4, H_2O, BeF_2

Answer (6)

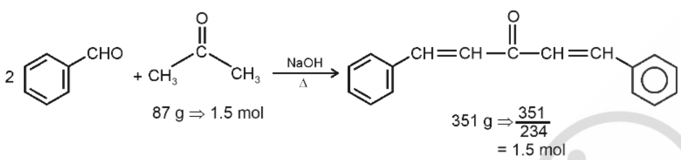
Sol. 



84. In the Claisen-Schmidt reaction to prepare 351 g of dibenzalacetone using 87 g of acetone, the amount of benzaldehyde required is _____ g. (Nearest integer)

Answer (318)

Sol.

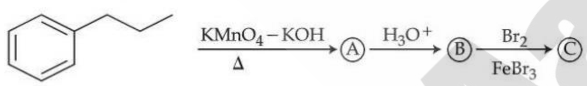


mol of benzaldehyde required = 1.5 × 2

= 3 mol

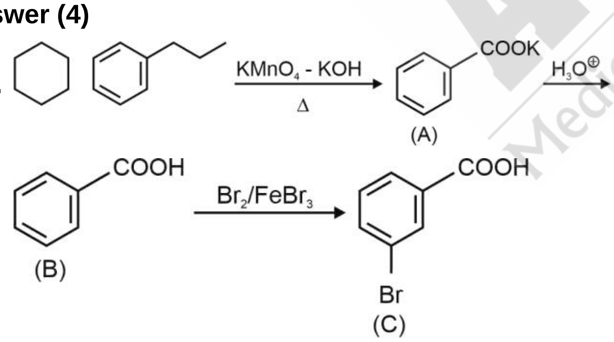
mass = 318 g

85. The product (C) in the following sequence of reactions has ______ π bonds.



Answer (4)

Sol.



Number of π bonds in (C) = 4

86. Using the given figure, the ratio of R_f values of sample A and sample C is $x \times 10^{-2}$. Value of x is ______

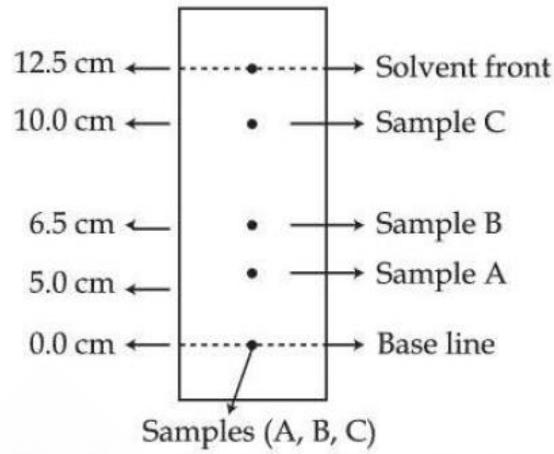


Fig : Paper chromatography of Samples

Answer (50)

Sol. $R_f = \frac{\text{Distance travelled by the substance from base line (x)}}{\text{Distance travelled by the solvent from base line (y)}}$

$$\frac{(R_f)_A}{(R_f)_C} = \frac{\frac{5}{12.5}}{\frac{10}{12.5}} = 0.5$$

$= 50 \times 10^{-2}$

x = 50

87. Consider the following single step reaction in gas phase at constant temperature.

$2A_{(g)} + B_{(g)} \rightarrow C_{(g)}$

The initial rate of the reaction is recorded as r_1 when the reaction starts with 1.5 atm pressure of A and 0.7 atm pressure of B. After some time the rate r_2 is recorded when the pressure of C becomes 0.5 atm. The ratio $r_1 : r_2$ is _______ $\times 10^{-1}$. (Nearest integer)

Answer (315)

Sol. $2A(g) + B(g) \rightarrow C(g)$

As this is single step reaction,

$r = k_f [A]^2 [B]^2$

	2A(g)	+	B(g)	→	C (g)
t = 0	1.5		0.7		0
t = 't'	1.5 – 2x		0.7 – x		x

$\Rightarrow x = 0.5$ atm

$r_1 = k_f(1.5)^2(0.7)$

$r_2 = k_f(0.5)^2(0.2)$

$\frac{r_1}{r_2} = \frac{9 \times 7}{2} = 31.5$

$= 315 \times 10^{-1}$

88. The fusion of chromite ore with sodium carbonate in the presence of air leads to the formation of products A and B along with the evolution of CO_2. The sum of spin-only magnetic moment values of A and B is _______ B.M. (Nearest integer)

[Given atomic number: C : 6, Na : 11, O : 8, Fe : 26, Cr : 24]

Answer (6)

Sol. $4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow 8Na_2CrO_4(A) + 2Fe_2O_3 + 8CO_2$
(B)

A : $Na_2\overset{+6}{Cr}O_4 \rightarrow Cr^{+6} \rightarrow Cr^{+6} \rightarrow [Ar]4s^03d^0 \Rightarrow \mu = 0$

B : $\overset{+3}{Fe_2}O_3 \rightarrow Fe^{+3} \rightarrow [Ar]4s^03d^5 \Rightarrow \mu = 5.9$ BM

Sum of magnetic moments of A & B $\simeq 6$

89. In an atom, total number of electrons having quantum numbers n = 4, $|m_l| = 1$ and $m_s = -\frac{1}{2}$ is_____.

Answer (6)

Sol. n = 4; $|m_l| = 1$ and $m_s = -\frac{1}{2}$

4s	4p	4d	4f
0	–1 0 +1	–2 –1 0 +1 +2	–3 –2 –1 0 +1 +2 +3
□	□□□	□□□□□	□□□□□□□

Total orbitals with $m_l = +1$ or -1

= 6

Total e^- with n = 4; $|m_l| = 1$; $m_s = -\frac{1}{2} = 6$

90. Combustion of 1 mole of benzene is expressed at

$$C_6H_6(l) + \frac{15}{2}O_2(g) \rightarrow 6CO_2(g) + 3H_2O(l).$$

The standard enthalpy of combustion of 2 mol of benzene is – 'x' kJ. x = _______.

Given :

1. Standard Enthalpy of formation of 1 mol of C_6H_6 (l), for the reaction 6 C (graphite) + $3H_2(g) \rightarrow C_6H_6(l)$ is 48.5 kJ mol^{-1}.
2. Standard Enthalpy of formation of 1 mol of $CO_2(g)$, for the reaction C(graphite) + $O_2(g) \rightarrow CO_2(g)$ is –393.5 kJ mol^{-1}.
3. Standard and Enthalpy of formation of 1 mol of $H_2O(l)$, for the reaction

   $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$ is –286 kJ mol^{-1}.

Answer (6535)

Sol. $C_6H_6(l) + \frac{15}{2}O_2(g) \rightarrow 6CO_2(g) + 3H_2O(l)$

$$\Delta_C H = 6\Delta_f H(CO_2) + 3\Delta_f H(H_2O) - \Delta_f H(C_6H_6)$$

= 6 × (–393.5) + 3(–286) – 48.5

= –3267.5 kJ/mol

For 2 mol of benzene = –6535 kJ/mol

x = 6535

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